CIMPA Research School:
Algebraic, Enumerative and Geometric Combinatorics - ECCO 2016

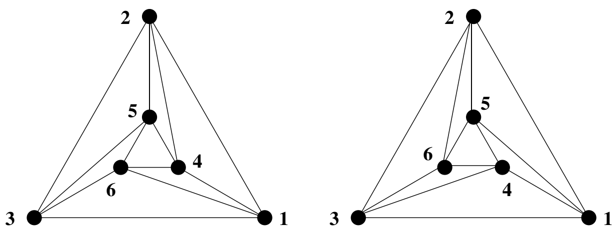
Triangulations of polytopes. Problem sheet.

4 Regular triangulations and subdivisions

1. *Let $\alpha, \beta \in \mathbb{R}^n$ be two lifting vectors such that $\alpha - \beta$ is an affine evaluation on the set V. Show that α and β produce the same regular triangulation of V.

2. *Deduce from the previous problem that to construct all the regular triangulations of a set V, or to check whether a particular triangulation is regular or not, there is no loss of generality in choosing a priori a $d+1$ affinely independent points of V (a d-simplex) and prescribing those $d+1$ coordinates in the lifting vector to be zero.

3. *Show that the following two triangulations of m.o.a.e. are not regular. Clue: by the previous problem, you can assume height zero to the three interior points.



4. *Show that, except for the two non-regular triangulations in the previous exercise, all other triangulations of the sets in Problem 1.1 are lexicographic (hence regular).

5. Construct a regular but not lexicographic triangulation.

6. *Let B be a subset of V. Show that there is a subdivision of V (in fact, a regular one) having B as a face.

7. Let $B1$ and B_2 be two subsets of A with $\text{conv}(B_1) \cap \text{conv}(B_2) = \emptyset$. Show that there is a subsdivision of A (in fact, a regular one) in which both B_1 and B_2 are faces.

8. Show that the same is not necessarily true for *three* subsets: taking B the vertex set of a triangular prism, find three subsets B_1, B_2, B_3 of it with $\text{conv}(B_i) \cap \text{conv}(B_j) = \emptyset$ for all $i, j \in \{1, 2, 3\}$, but such that no subdivision of V has B_1, B_2 and B_3 as cells. Remark: the m.o.a.e. is an example where no *regular* subdivision exists using B_1, B_2 and B_3.