Steady State Representation of the Synchronous Generator for Multimachine Stability Studies

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***Abstract*—This document presents the steady state operating point and initializes multimachine stability simulations for a 4-machine, 12 bus benchmark. A power flow solution provides bus voltages/angles and branch flows, the full Y_{bus} is assembled and a Kron-reduced network is derived to retain couplings. Generator terminal currents are calculated and contrasted for GENCLS and GENROU. Power flow results are used as initial conditions (δ, ω), internal EMFs, and for GENROU, the flux states. A 10 seconds transient stability study validates that the initialized states reproduce the steady solution and compares stable responses of both models. Findings highlight model fidelity tradeoffs: GENROU better captures electromagnetic dynamics, while GENCLS yields similar rotor/angle trends with lower complexity.**

I. Introduction

The Kundur two area, 4 machine system is a standard power system model for analyzing inter area electromechanical oscillations and evaluating model fidelity in multimachine studies. The transient stability study starts from a power flow solution that gives the steady state operating points for each generator. These quantities are useful to define the algebraic constraints of the dynamic model and provide all the parameters needed to compute the differential algebraic equations of the system.

The general formulation of power flow equations is derived from *Kirchhoff's Current Law (KCL)* and expressed in terms of real and reactive power at each bus:

$$P_{Gi} - P_{Li} - P_i = 0 \tag{1}$$

$$Q_{Gi} - Q_{Li} - Q_i = 0 \tag{2}$$

$$P_i = \sum_{j \in N_i} |V_i|\,|V_j| \Big(G_{ij}\,\cos(\theta_{ij}) \;+\; B_{ij}\,\sin(\theta_{ij}) \Big) \tag{3}$$

$$Q_i = \sum_{j \in N_i} |V_i|\,|V_j| \Big(G_{ij}\,\sin(\theta_{ij}) \;-\; B_{ij}\,\cos(\theta_{ij}) \Big) \tag{4}$$

where:

- P_i and Q_i are the real and reactive power injections at bus i.
- V_i and V_j are the voltage magnitudes at buses i and j.
- θ_{ij} is the phase angle between buses.

The system is an interconnected power network consisting of 11 buses, 8 transmission lines, 4 generators, and 2 loads. Figure 1 shows the one-line diagram of the network.

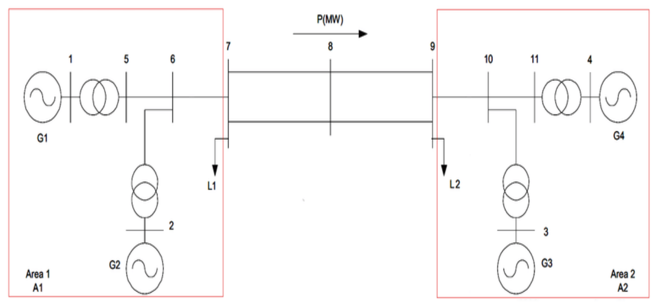

Fig. 1. One-line Diagram of the Case Study System

The network is represented by an admittance matrix named $Y_{bus} = G + jB$. Since we're interested in the terminal voltages of each generator, to improve the computation time a matrix reduction technique can be used. In electrical engineering the Kron reduction (Schur complement) is widely used to reduce the network while preserving the coupling between external and internal nodes, Kron reduction is shown in equation (5).

$$Y_{kron} = Y_{gg} - Y_{gl} Y_{ll}^{-1} Y_{lg} \tag{5}$$

The generator models used here are taken directly from PowerWorld and represents two levels of fidelity. The classical model (GENCLS) has a constant internal voltage behind a transient reactance X'_d. On the other hand, a round rotor detailed model (GENROU) is used, it includes the transient and subtransient electromagnetic dynamics, damper and field winding effects.

This document is organized as follows: i) Power flow solution is solved using PowerWorld to obtain initial conditions. ii) The admittance matrix is assembled and its Kron reduction is calculated. iii) Classical model is presented, then differential equations are coded in python, then a brief comparison between both simulations is conducted. iii) GENROU model is introduced, the methodology is same as the previous case, the model is assembled in PowerWorld and coded. iv) Conclusions are reported.

II. Power Flow Solution

The power flow solution used throughout this document is shown in table I. The currents were computed as $I = (P - jQ)/(S_{\text{base}} V^*)$ with $S_{\text{base}} = 100$ MVA and $V = |V|e^{j\theta}$ in per unit. Positive MW/MVAR denote net injection (generators) and negative values denote net consumption (loads), yielding current angles near π.

TABLE I
POWER FLOW SOLUTION WITH INJECTED CURRENTS (PER UNIT)

Bus	**V (p.u.)**	**V (kV)**	**Angle (deg)**	**MW**	**Mvar**	I **(p.u.)**
1	1.0300	20.60	0.00	420.68	104.47	$4.2083\angle -13.95°$
2	1.0100	20.20	-6.33	356.62	124.12	$3.7386\angle -25.52°$
3	1.0300	20.60	-8.11	596.30	326.34	$6.5996\angle -36.80°$
4	1.0100	20.20	0.60	596.30	105.39	$5.9955\angle -9.42°$
5	1.0154	233.53	-3.85	0.00	0.00	$0.0\angle 0.0°$
6	0.9912	227.98	-9.74	0.00	0.00	$0.0\angle 0.0°$
7	0.9727	223.72	-14.26	-667	-100	$6.9340\angle 157.21°$
8	0.9727	223.72	-17.60	0.00	0.00	$0.0\angle 0.0°$
9	0.9554	219.73	-20.88	-1267	-100	$13.3034\angle 154.61°$
10	0.9819	225.83	-13.76	0.00	0.00	$0.0\angle 0.0°$
11	0.9975	229.42	-5.08	0.00	0.00	$0.0\angle 0.0°$

III. Admittance Matrix and Kron Reduction

Table II summarizes the parameters of all network branches, including both transformers and transmission lines. Transformer branches and loads are also presented explicitly, as they will be incorporated into the admittance matrix construction.

TABLE II
NETWORK BRANCH PARAMETERS (TRANSFORMERS AND TRANSMISSION LINES)

Branch	**Device**	**From**	**To**	R **(p.u.)**	X **(p.u.)**	B **(p.u.)**
1	Transformer	1	5	0.00000	0.01670	0.00000
2	Transformer	2	6	0.00000	0.01670	0.00000
3	Transformer	3	10	0.00000	0.01670	0.00000
4	Transformer	11	4	0.00000	0.01670	0.00000
5	Line	5	6	0.00250	0.02500	0.04370
6	Line	7	6	0.00100	0.01000	0.01750
7	Line	7	8	0.01100	0.11000	0.19250
8	Line	7	8	0.01100	0.11000	0.19250
9	Line	9	8	0.01100	0.11000	0.19250
10	Line	8	9	0.01100	0.11000	0.19250
11	Line	9	10	0.00100	0.01000	0.01750
12	Line	10	11	0.00250	0.02500	0.04370

The complete Y_{bus} is a 11x11 matrix since 11 is the number of buses and it is obtained by summing the corresponding admittances of each branch (table III).

The constant impedance loads are in function of the voltage at terminals, the impedance for each load is:

TABLE IV
BUS LOAD DATA

Bus	**Model**	V (p.u)	P$_L(p.u.)$	Q$_L(p.u.)$	Y$_L(p.u.)$
7	Constant Impedance	0.9727	7.05	1.06	$7.05 - j1.057$
9	Constant Impedance	0.9554	13.88	1.10	$13.879 - j1.095$

The matrix is then ready for Kron reduction, which eliminates non generator buses to produce a reduced order admittance matrix suitable for solving large grids and conducting oscillation and resonance analysis. Reorder buses as $G = \{1, 2, 3, 4\}$ (generators) and $L = \{5, \ldots, 11\}$ (network and loads).

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix}, \quad \begin{matrix} Y_{GG} \in \mathbb{C}^{4\times4},\ Y_{GL} \in \mathbb{C}^{4\times7}, \\ Y_{LG} \in \mathbb{C}^{7\times4},\ Y_{LL} \in \mathbb{C}^{7\times7} \end{matrix} \tag{6}$$

For the given $Y_{\text{bus}} \in \mathbb{C}^{11\times11}$ with $G = \{1, 2, 3, 4\}$, the numerical 4×4 Kron-reduced matrix using equation (5) is:

$$Y_{\text{kron}} = \begin{bmatrix} 1.25 - j17.67 & 0.47 + j15.54 & 0.32 + j1.18 & 0.10 + j0.48 \\ 0.47 + j15.54 & 3.51 - j21.15 & 0.97 + j2.89 & 0.32 + j1.18 \\ 0.32 + j1.18 & 0.97 + j2.89 & 5.58 - j22.10 & 1.32 + j15.22 \\ 0.10 + j0.48 & 0.32 + j1.18 & 1.32 + j15.22 & 1.60 - j17.78 \end{bmatrix} \tag{7}$$

The diagonal of the Y_{kron} embeds the entire network, lines and loads connected to that generator. While the off diagonal entries represent the mutual admittances between generator buses.

IV. Classical model case study (GENCLS)

The current injection of each generator is calculated using the Kron-reduced matrix obtained in (7).

$$I_G = Y_{\text{kron}} V_G = Y_{\text{kron}} \begin{bmatrix} 1.0300 + j0.0000 \\ 1.0038 - j0.1114 \\ 1.0197 - j0.1453 \\ 1.0099 + j0.0106 \end{bmatrix} \tag{8}$$

$$I_G = \begin{bmatrix} 4.085 - j1.015 \\ 3.372 - j1.610 \\ 5.282 - j3.953 \\ 5.915 - j0.981 \end{bmatrix} = \begin{bmatrix} 4.209\angle -13.95° \\ 3.737\angle -25.52° \\ 6.598\angle -36.81° \\ 5.996\angle -9.41° \end{bmatrix} p.u \tag{9}$$

TABLE III
SYSTEM'S Y_{bus} USING TABLES II AND IV

$$Y_{\text{bus}} = \begin{bmatrix} -j59.88 & 0 & 0 & 0 & j59.88 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j59.88 & 0 & 0 & 0 & j59.88 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j59.88 & 0 & 0 & 0 & 0 & 0 & 0 & j59.88 & 0 \\ 0 & 0 & 0 & -j59.88 & 0 & 0 & 0 & 0 & 0 & 0 & j59.88 \\ j59.88 & 0 & 0 & 0 & 3.96 - j99.46 & -3.96 + j39.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & j59.88 & 0 & 0 & -3.96 + j39.6 & 13.86 - j198.46 & -9.9 + j99 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -9.9 + j99 & 20.15 - j117.61 & -1.8 + j18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.8 + j18 & 3.6 - j35.62 & -1.8 + j18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.8 + j18 & 28.18 - j117.61 & -9.9 + j99 & 0 \\ 0 & 0 & j59.88 & 0 & 0 & 0 & 0 & 0 & -9.9 + j99 & 13.86 - j198.46 & -3.96 + j39.6 \\ 0 & 0 & 0 & j59.88 & 0 & 0 & 0 & 0 & 0 & -3.96 + j39.6 & 3.96 - j99.46 \end{bmatrix}$$

Equation (9) shows how current injections are in agreement with those calculated directly from the power flow solution (table I). It confirms the correctness and consistency of the network reduction. Overall, these results validate that the Kron reduction preserves the electrical behavior of the original system at the generator buses.

To have a fair comparison between initializing the model manually and using PowerWorld, the exact model used in PowerWorld is displayed in equation (10).

$$\dot{\delta} = \omega\ \omega_0, \quad \dot{\omega} = \frac{1}{2H}\left(\frac{P_{mech} - D\omega}{1+\omega} - P_{elec}\right) \tag{10}$$

$$P_{elec} = \frac{E' V_{bus}}{X'_d} sin(\delta - \theta_{bus}) \tag{11}$$

Where $\omega = (\omega_m - \omega_0)/(\omega_0)$ is the per unit speed deviation, so $\omega = 0$ means the machine is at synchronous speed and $\omega = 1$ would mean it is spinning at double the synchronous speed. For this case G1 and G2 have a value of $H = 6.5s$ and $D = 3$, G3 and G4 a value of $H = 3$ and $D = 1$.

Before solving the differential equations (10) we must initialize it, to do so, internal voltages and angles are required. Obtaining E' and δ is straightforward using equation (12) and (9).

$$E'_i e^{j\delta_i} = V_i + (R_a + jX'_d)\, I_i \tag{12}$$

The internal voltages and angles are displayed in table V. In steady state the machine is rotating at synchronous velocity, thus, $\dot{\delta}(0) = 0$, $\dot{\omega}(0) = 0$ and $P_{mech} = P_G$. The direct axis transient reactance is $X'_d = 0.2$ p.u and resistance $R_a = 0$. These values become the initial conditions for integrating the differential algebraic equations in (10) and (13).

$$\left(Y_{kron} - \frac{1}{jX'_d}\right) V_G = -\frac{1}{jX'_d} E'(\delta) \tag{13}$$

Where $E'(\delta)$ is a vector with the internal complex voltage of each generator. Equation (13) is the algebraic current balancing condition, left hand side is the current entering the network and right hand side the current injected by the generator.

TABLE V
INTERNAL VOLTAGES AND ANGLES ($X'_d = 0.2$, $R_a = 0$)

Bus	$\lvert E' \rvert$ (p.u.)	δ (deg)
1	1.4789	33.52°
2	1.4407	23.02°
3	2.0269	26.72°
4	1.6969	44.40°

Figures 2 and 3 presents the internal voltages and angles obtained directly from a transient stability simulation on PowerWorld. This confirms the approach of obtaining initial conditions by using the Kron reduction.

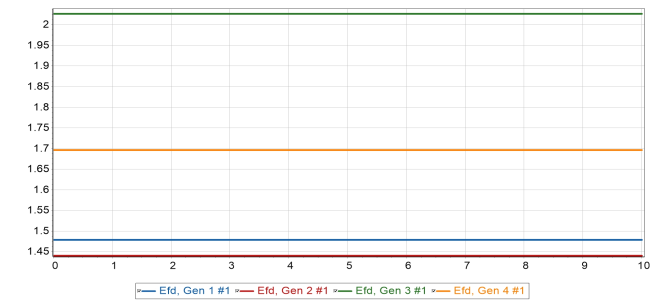

Fig. 2. PowerWorld transient stability internal voltages E' (GENCLS)

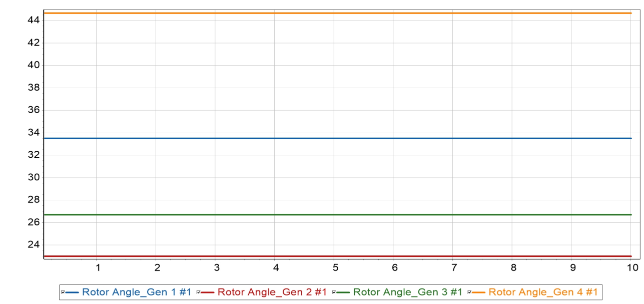

Fig. 3. PowerWorld transient stability internal angles δ (GENCLS)

V. ROUND ROTOR DETAILED MODEL CASE (GENROU)

The GENROU model augments the classical representation by adding transient and subtransient electromagnetic dynamics on both d and q axes (field and damper windings). The state is

$$x = \left[\ \delta,\ \omega,\ \psi''_d,\ \psi''_q,\ E'_q,\ E'_d\ \right]^T \tag{14}$$

with speed deviation ω as in (10). Parameters are $\{R_a, X_\ell, X_d, X_q, X'_d, X'_q, X''_d, T'_{d0}, T'_{q0}, T''_{d0}, T''_{q0}, H, D\}$, and q leads d axis by $+\pi/2$. It is considered that $\psi''_d = E''_d$, $\psi''_q = E''_q$.

A. Electromagnetic dynamics

$$\dot{\psi}''_q = \frac{-\psi''_q + (X'_q - X_\ell)I_q + E'_d}{T''_{q0}}, \tag{15}$$

$$\dot{\psi}''_d = \frac{-\psi''_d - (X'_d - X_\ell)I_d + E'_q}{T''_{d0}}. \tag{16}$$

$$\dot{E}'_q = \frac{1}{T'_{d0}}\Bigg(E_{fd} - \Big[E'_q + (X_d - X'_d)\big(I_d + \frac{X'_d - X''_d}{(X'_d - X_\ell)^2}\left(-\psi''_d - (X'_d - X_\ell)I_d + E'_q\right)\big)\Big]\Bigg) \tag{17}$$

$$\dot{E}'_d = \frac{1}{T'_{q0}}\Bigg(-\Big[E'_d - (X_q - X'_q)\big(I_q - \frac{X'_q - X''_q}{(X'_q - X_\ell)}\left(-\psi''_q + (X'_q - X_\ell)I_q + E'_d\right)\big)\Big]\Bigg) \tag{18}$$

B. Algebraic stator equations

The total stator subtransient fluxes in the rotor frame (near synchronous speed) are:

$$V_d = -R_a I_d + \psi_q'' + X_\ell I_d, \tag{19}$$

$$V_q = -R_a I_q - \psi_d'' - X_\ell I_q. \tag{20}$$

The rotation to the network reference is

$$V = (V_d + jV_q)\, e^{j(\delta - \pi/2)}, \quad I_t = (I_d + jI_q)\, e^{j(\delta - \pi/2)} \tag{21}$$

Define the subtransient internal EMF components

$$E_q'' = E_q' - (X_d' - X_d'')I_d, \quad E_d'' = E_d' + (X_q' - X_q'')I_q \tag{22}$$

and the complex internal voltage

$$E'' = (E_d'' + jE_q'')\, e^{j(\delta - \pi/2)}. \tag{23}$$

Where the current current injected by each generator I_t can be written as:

$$I_t = Y_s\,(E'' - V), \tag{24}$$

If $X_d'' = X_q''$ due to the round rotor configuration Y_s yields:

$$Y_s = \frac{1}{R_a + jX_d''} = G + jB \tag{25}$$

With Y_{kron} from (7)

$$\big[Y_{\text{kron}} + \text{diag}(Y_s)\big]\, V = \text{diag}(Y_s)\, E''. \tag{26}$$

C. Initialization of the GENROU model

Given the power flow solution $V = |V|e^{j\theta}$ and $S = P + jQ$ and using the model relations already stated in the text (15)–(18) and the algebraic/stator and rotation equations, the initialization proceeds as follows.

1) **Terminal current (from power flow).**

$$I_t = \frac{S^*}{V^*}. \tag{27}$$

2) **Rotor angle via the transient X_d'.**

$$\delta = \arg\big(V + (R_a + jX_d')\, I_t\big) \tag{28}$$

3) **Rotate to the rotor dq frame.**

$$V_d + jV_q = V\, e^{-j(\delta - \pi/2)} \tag{29}$$

$$I_d + jI_q = I_t\, e^{-j(\delta - \pi/2)} \tag{30}$$

4) **Subtransient air-gap flux linkages from V, I.**

$$\psi_q'' = V_d + R_a I_d - X_\ell I_d \tag{31}$$

$$\psi_d'' = -\,V_q - R_a I_q - X_\ell I_q \tag{32}$$

5) **Form the subtransient internal emf and its dq components.**

$$E'' = V + (R_a + jX_d'')\, I_t \tag{33}$$

$$E_d'' + jE_q'' = E''\, e^{-j(\delta - \pi/2)} \tag{34}$$

6) **Recover the transient emfs from the subtransient split.**

$$E_q' = E_q'' + (X_d' - X_d'')\, I_d \tag{35}$$

$$E_d' = E_d'' - (X_q' - X_q'')\, I_q \tag{36}$$

7) **Mechanical speed at equilibrium.**

$$\omega(0) = 0. \tag{37}$$

When initializing multiple machines, use the subtransient injection relation and the Kron reduced network.

$$I_t = Y_s\,(E'' - V) \tag{38}$$

$$\big[Y_{\text{kron}} + \text{diag}(Y_s)\big]\, V = \text{diag}(Y_s)\, E'' \tag{39}$$

D. Results of the initialization (GENROU)

The parameters of the model are chosen as $H = 6.5, 3$ and $D = 3, 1$ for zone 1 (G1,G2) and zone 2 (G3,G4). The other parameters are set using PowerWorld's default settings.

TABLE VI
GENROU MACHINE PARAMETERS (PER UNIT)

Bus	H	D	X_d	X_q	X_d'	X_q'	X_d''	X_l	T_{do}'	T_{qo}'	T_{do}''	T_{qo}''
1	6.5	3	2.1	0.5	0.2	0.5	0.18	0.15	7	0.75	0.04	0.05
2	6.5	3	2.1	0.5	0.2	0.5	0.18	0.15	7	0.75	0.04	0.05
3	3	1	2.1	0.5	0.2	0.5	0.18	0.15	7	0.75	0.04	0.05
4	3	1	2.1	0.5	0.2	0.5	0.18	0.15	7	0.75	0.04	0.05

Using all the equations presented through this section (15)-(39) and the table VI, the initial conditions for the GENROU model can be computed and are shown in table VII.

TABLE VII
GENROU INITIAL CONDITIONS

Bus	δ (deg)	E_q' (p.u.)	E_d' (p.u.)	ψ_d'' (p.u.)	ψ_q'' (p.u.)
1	53.03°	1.394265	0.000000	1.200558	0.576020
2	41.04°	1.369644	0.000000	1.198226	0.520181
3	39.79°	1.974108	0.000000	1.653217	0.535094
4	63.18°	1.609382	0.000000	1.323315	0.627659

Figures 4 through 7 presents the states of each generator.

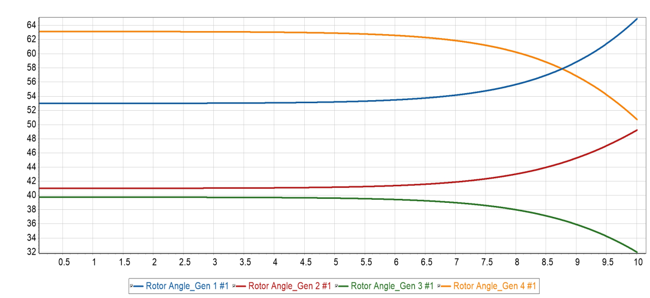

Fig. 4. PowerWorld transient stability angles δ (GENROU)

Observe that the system goes unstable after approximately 6 seconds. The GENROU model adds damper and field dynamics, so if the field setpoint is not initialized, the excitation can drift on the T_{d0}' timescale and slowly push to instability. Additionally, the parameters look aggressive: large saliency ($X_d \gg X_q$) weakens synchronizing torque at the relatively high initial angles, and very small X_d'' close to X_ℓ makes the electrical torque stiff and numerically sensitive.

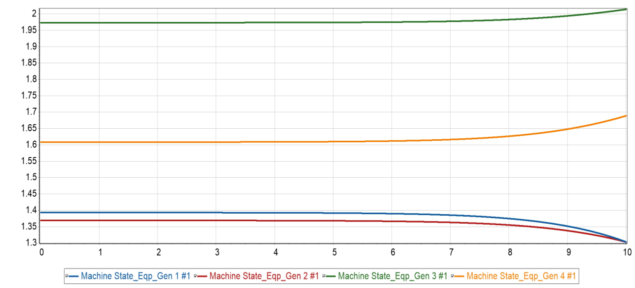

Fig. 5. PowerWorld transient stability transient quadrature voltage (GENROU)

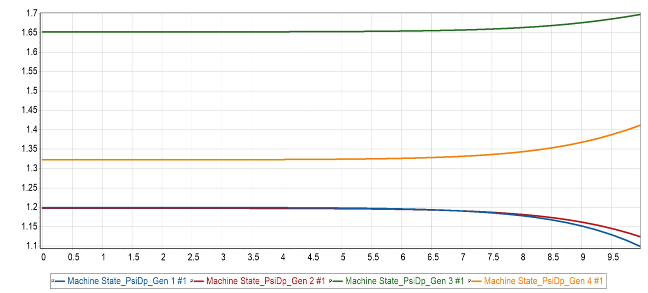

Fig. 6. PowerWorld transient stability subtransient flux linkage direct axis (GENROU)

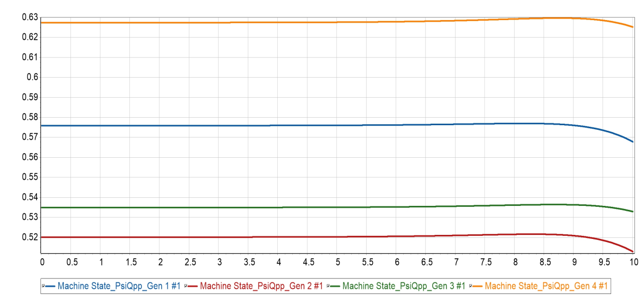

Fig. 7. PowerWorld transient stability subtransient flux linkage quadrature axis (GENROU)

VI. Conclusion

The workflow: power flow $\rightarrow$ Y_{bus} assembly $\rightarrow$ Kron reduction $\rightarrow$ initialization was consistent: currents from $I_G = Y_{\text{kron}} V_G$ matched the power flow injections, confirming that the reduction preserved generator bus couplings. These quantities set (δ, ω), internal EMFs, and (for GENROU) flux states that reproduce the steady operating point at t=0.

The fidelity complexity tradeoff is clear. GENCLS (constant E' behind X'_d) captured rotor angle/speed trends with strong numerical robustness, while GENROU solved transient/subtransient electromagnetic effects but became sensitive to excitation and saliency which led to instability.

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