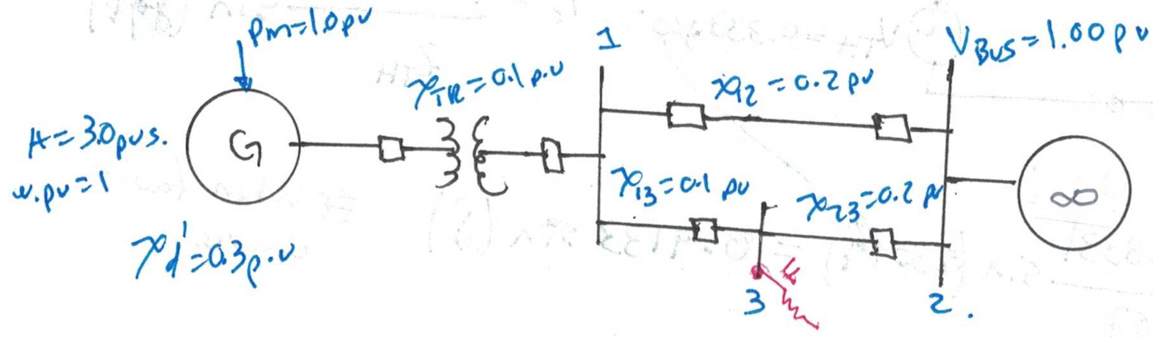
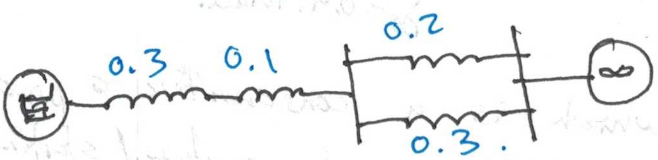
Equal Area Criterion and Transient Stability.

Davalos Gonzalez Erick Christopher



a) Steady state equivalent. $\boxed{x_{eq}=j0.52 \text{ p.u.}}$

Internal voltage E' seen by system.



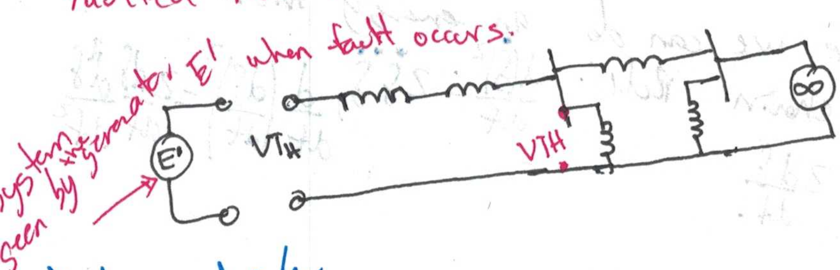
$$E' = V_{Bus} + j\,x_{eq} I,\quad E' = 1\angle 0 + j(0.52) I,\quad I = \left(\frac{S}{V_{Bus}}\right)^{*} = \frac{\frac{P}{\cos\theta}}{V} = \frac{P}{\cos\theta\, V_{Bus}}$$

$$E' = 1\angle 0^\circ + j(0.52)(1.0526\angle -18.19^\circ) \qquad I = \frac{1}{(0.95)(1\angle 0^\circ)} = 1.0526\angle -18.19^\circ$$

$\boxed{E' = 1.28\angle 23.94^\circ}$ Internal voltage seen from the infinite bus.

Faulted network. Thevenin Voltage.

Thevenin equivalent seen by generator E' when fault occurs.



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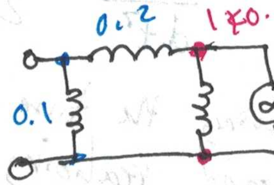
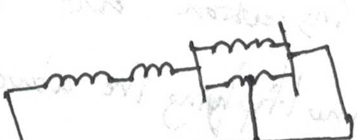
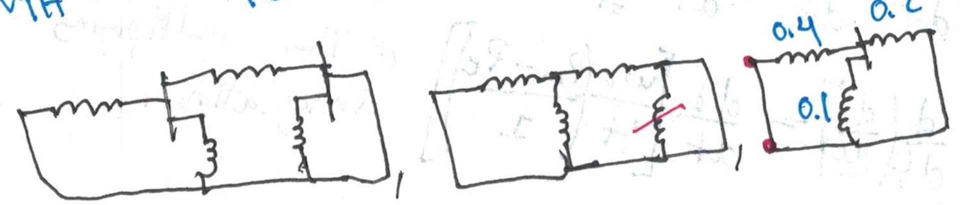


Voltage divider

$$V_{TH} = 1\angle 0^\circ\left(\frac{0.1}{0.3}\right) = \boxed{0.333\angle 0^\circ \text{ p.u.}},\quad Z_{TH} =$$



$Z_{TH} =$

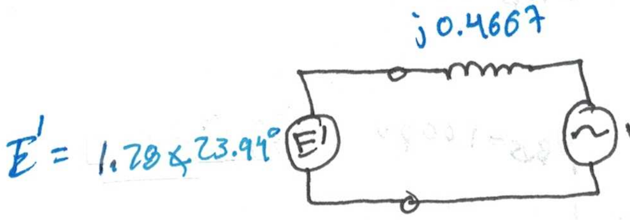


seen by the generator.

$$Z_{TH} = \frac{0.2(0.1)}{0.3} + 0.4$$

$$\boxed{Z_{TH} = 0.4667}$$

The thevenin equivalent is



$$P_e = \frac{|E'||V_{TH}|}{Z_{TH}} \sin(\delta - \phi)$$

$$P_{e_F} = \frac{|1.28||0.333|}{0.4667} \sin(\delta - \phi) = 0.9133 \sin(\delta)$$ # when fault occurs.

$$P_e = \frac{|1.28||1|}{0.52} \sin(\delta_0) = 2.4615 \sin(\delta_0) \; ; \quad \delta_0 = 23.94°$$ # Before and after the fault.

$\delta_0 = 0.4178$ rad.

Using the equal area criterion which is a conservative approach where no damping is considered. The stability can be analyzed starting from the swing equation:

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \bar{P}_M - \bar{P}_e$$ per unit. # $\left(\frac{T_m - T_e}{VA_{BASE}}\right)\omega_{om} = \bar{P}_m - \bar{P}_e$

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}\left(\bar{P}_m - \bar{P}_e\right)$$ Non linear term.

Instead of solving the time response we can do an energy trick. By inspection and realizing the chain Rule: $\frac{d(\dot{\delta})^2}{dt} = 2\dot{\delta}\ddot{\delta}$, $\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2}$

By multiplying the above expression by $2\frac{d\delta}{dt}$.

$$\frac{2d^2\delta}{dt^2}\frac{d\delta}{dt} = \frac{d\delta}{dt}\frac{\omega_0}{H}\left(\bar{P}_m - \bar{P}_e\right)$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{d\delta}{dt}\left[\frac{\omega_0}{H}\left(\frac{P_m - P_e}{1}\right)\right]$$ # Then multiplying by dt and integrating..

$$\int_{\delta_0}^{\delta_F} d\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta_F} \frac{w_0}{H}\left(\bar{P}_m - \bar{P}_e\right) d\delta$$

$$\underbrace{\left(\frac{d\delta}{dt}\right)^2}_{\text{velocity}}\Bigg|_{\delta_0}^{\delta_F} = \underbrace{\int_{\delta_0}^{\delta_F} \frac{w_0}{H}\left(\bar{P}_m - \bar{P}_e\right) d\delta}_{\text{work done}}$$

Because we want to reach the angular equilibrium $\frac{d\delta}{dt} = 0$. It'll seems obvious that the initial point δ_0 and the maximum point δ_F it's derivative is zero $\frac{d\delta(\delta_0)}{dt} = 0$ and $\frac{d\delta(\delta_F)}{dt} = 0$.

$$\int_{\delta_0}^{\delta_F} \frac{w_0}{H}\left(\bar{P}_m - \bar{P}_e\right) d\delta = 0.$$

The above integral can be separated into two integrals one corresponding for the acceleration and the deacceleration when $P_m > P_e$ and $P_e > P_m$ respectively.

$$\int_{\underset{\text{initial steady state}}{\delta_0}}^{\overset{\text{Fault cleared}}{\delta_1}} \left(\bar{P}_m - \bar{P}_e\right) d\delta + \int_{\delta_1}^{\overset{\text{final angle}}{\delta_2}} \left(\bar{P}_m - \bar{P}_e\right) d\delta = 0$$

b) $\delta_0 = 0.4178\,rad.$, To get δ_1 we must know how the angle behaves during the fault as a function of time. The swing equations needs to be solved numerically for a time $0 < t < 0.05\,seg.$ (3 cycles). After finding δ_1, δ_2 can be obtained from the equal area criterion to find the maximum reaching angle for that disturbance. Note that the swing equation during the fault is still nonlinear since there is still electrical power because the fault isnt at generator terminals. Thus, numerical approximation is useful.

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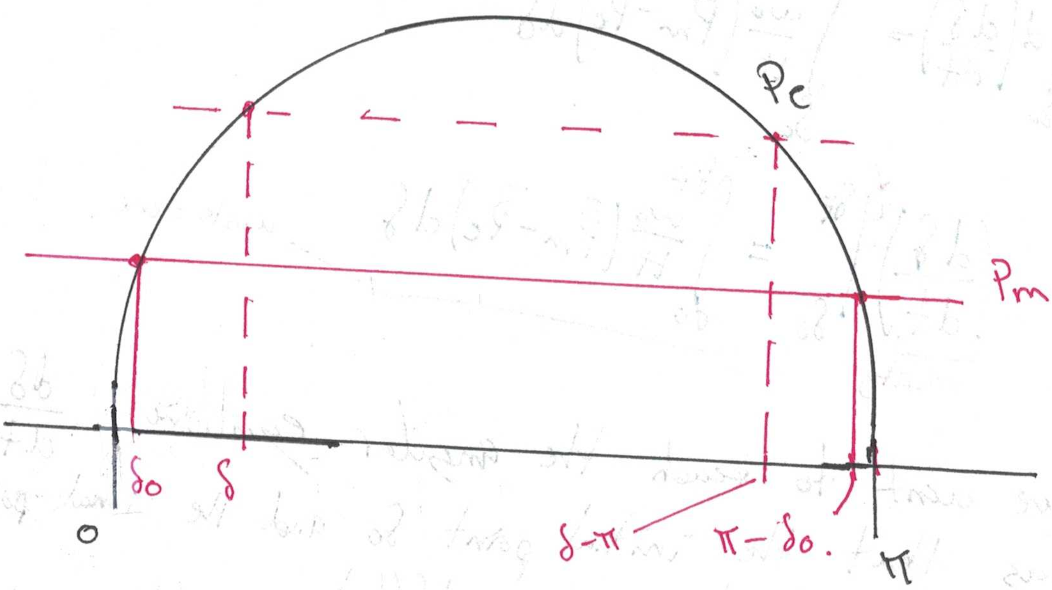


For circular motion and a constant value as P_m there are 2 intersection points, each intersection point represents a stable steady state operation point where $P_m - P_e = 0$. Thus we can define the maximum final angle as $\delta_F = \pi - \delta$. Now the task is to find the critical angle, its the one that satisfies the equal area criterion which is the area between P_m and the faulty P_e curve must be the same as the area between the past fault and P_m.

It's $P_m > P_e$, accelerates. $P_e > P_m$ deaccelerats until a stable operating point.

$\pi - \delta_0 = 2.7238$

prefault and post fault are equal (P_e)

$$\int_{0.4178}^{\delta_{CR}} (P_m - 0.9133\sin(\delta))\,d\delta + \int_{\delta_{CR}}^{2.7238} (P_m - 2.4615\sin(\delta))\,d\delta = 0.$$

$$\left(P_m \cdot \delta + 0.9133\cos(\delta)\right)\Big|_{0.4178}^{\delta_{CR}} + \left(P_m\delta + 2.4615\cos(\delta)\right)\Big|_{\delta_{CR}}^{2.7238} = 0.$$

$$\left[\delta_{CR} + 0.9133\cos(\delta_{CR}) - \underbrace{(0.4178 + 0.8347)}_{1.2525}\right] + \left[\underbrace{(2.7238 - 2.2498)}_{0.4740} - (\delta_{CR} + 2.461\cos(\delta_{CR}))\right] = 0.$$

$$\underbrace{\delta_{CR}} + 0.9133\cos(\delta_{CR}) - 1.2525 + 0.4740 - \underbrace{\delta_{CR}} - 2.461\cos(\delta_{CR}) = 0$$

$$0.9133\cos(\delta_{CR}) - 0.7785 - 2.461\cos(\delta_{CR}) = 0.$$

$$\cos(\delta_{CR})\left[\underbrace{(0.9133 - 2.461)}_{-1.5477}\right] = 0.7785$$

c) $$\cos(\delta_{CR}) = \frac{0.7785}{-1.5477}.$$

$$\delta_{CR} = \cos^{-1}\left(\frac{0.7785}{-1.5477}\right) = \underline{2.0979 \text{ rad} = 120.19°}$$

Since the power transfered is different from 0, a direct approach to calculate the clearing time isnt possible. So, the fastest the better.

b) For 3 cycles (0.05 s), Is neccesary to find δ_1 first which is the angle reached by the machine when the fault is self cleared. Afterwards the equal area criterion can be used to determine stability.

Solving numerically for $t = 0.05$ s. Using modified Euler.

$$\delta = x_1 \qquad \frac{d^2\delta}{dt} = \frac{w_0}{2H}\left(P_m - P_{MAX}\sin(\delta)\right)$$ # Can be seperated in two equations.

$$\dot{\delta} = x_2$$

$$x_2 = \dot{x}_1 = \frac{d\delta}{dt} = w_r - w_o \quad , \quad \frac{dw_r}{dt} = \frac{w_0}{2H}\left(P_m - P_{MAX}\sin(\delta)\right) = \dot{x}_2$$

Defining the initial conditions. * $\delta(0) = \delta_0 = 0.4178$ rad. * $\dot{\delta}(0) = 0$, since $w_r = w_s$.

Evaluating at $t = 0$., First Run.

$$\frac{d\delta_0}{dt} = 0 \text{ and}$$

$$\frac{d\,dw_0}{dt} = \frac{377}{2(3)}\left(1 - 0.9133\sin(0.4178)\right) = 39.54\ \frac{\text{rad}}{s^2}$$

Modified Euler's predictor: $\Delta t = 0.025$

For $k=0$.

$$\delta_{k+1}^{(P)} = \delta^{k} + \frac{d\delta^{k}}{dt}\Delta t \Rightarrow \delta_1^{(P)} = 0.4178 + 0(0.025) = 0.4178 \text{ rad.}$$

$$\Delta\omega r_{k+1}^{(P)} = \Delta\omega r^{k} + \frac{d\Delta\omega r^{k}}{dt}\Delta t \Rightarrow \Delta\omega r_1^{(P)} = 0 + 39.54(0.025) = 0.9885 \frac{\text{rad}}{s}$$

Obtaining the slope of the predicted point.

$$\frac{d\delta_1^{(P)}}{dt} = \Delta\omega r_1^{(P)} = 0.9885 \frac{\text{rad}}{s}.$$

$$\frac{d\Delta\omega r_1^{(P)}}{dt} = \frac{377}{2(3)}\left(1 - 0.9133\sin(0.4178)\right) = 39.54 \frac{\text{rad}}{s^2}$$

Corrector, slope average.

$$\delta_{k+1} = \delta_0 + \frac{\Delta t}{2}\left[\frac{d\delta_0}{d(t_0)} + \frac{d\delta_1^{(P)}}{dt|_{t_0+\Delta t}}\right]$$

$$\delta_1 = 0.4178 \text{ rad} + \frac{(0.025)}{2}\left[0 + 0.9885 \frac{\text{rad}}{s}\right]$$

$$\delta_1 = 0.4302 \text{ rad.}$$

$$\Delta\omega r_1 = 0 + \frac{(0.025)}{2}\left[39.54 \frac{\text{rad}}{s} + 39.54 \frac{\text{rad}}{s}\right] = 0.9885 \frac{\text{rad}}{s}$$

We will achieve $t=0.05s$ in two iterations due to the time step.

* Second iteration:
  * Predictor.

$$\delta_2^{(P)} = 0.4302 + 0.9885(0.025) = 0.4549 \text{ rad.}$$

$$\Delta\omega r_2^{(P)} = 0.9885 + 38.9(0.025) = 1.9610 \frac{\text{rad}}{s}$$

* Slope of prediction.

⋮

Slope of prediction:

$$\frac{d\delta_2^{(P)}}{dt} = 1.9610 \frac{rad}{s}$$

$$\frac{d\Delta w r_2^{(P)}}{dt} = \frac{377}{2(3)}\left[1 - 0.9133\sin(0.4549)\right] = 37.6196 \frac{rad}{s^2}.$$

Corrector, slope average of the second iteration

$$\delta_2 = 0.4302\,rad + \frac{0.025}{2}\left[0.9885\frac{rad}{s} + 1.9610\frac{rad}{s}\right] = 0.4671\,rad$$

$$\Delta w_{r_2} = 0.9885\frac{rad}{s} + \frac{0.025}{2}\left[38.9\frac{rad}{s^2} + 37.6196\frac{rad}{s^2}\right] = 1.945\frac{rad}{s}.$$

To know if the system is stable after a 3 cycles fault. (0.05s)
The equal area criterion can be used.

$\delta_0 = 0.4178$ rad., $\delta_{FC} = 0.4671$ rad, $\delta_R = ?$

initial angle. steady state.

Fault clearing angle $t = 0.05s$.

Final angle reached by the machine.

$$\int_{0.4178}^{0.4671}(P_m - 0.9133\sin(\delta))d\delta + \int_{0.4671}^{\delta_R}(P_m - 2.4615\sin(\delta))d\delta = 0.$$

Fault

Post fault.

$$\left[(0.4671 + 0.8155) - (0.4178 + 0.8347)\right] + \left[(\delta_R + 2.4615\cos(\delta_R)) - (0.4671 + 2.1978)\right] = 0.$$

$$0.0301 + (\delta_R + 2.4615\cos(\delta_R)) - 2.6649 = 0$$

$$\delta_R + 2.4615\cos(\delta_R) = 2.6348.$$ # Non linear equation.

To find δ_R Newton Raphson can be used to solve the nonlinear equation.

$$\delta_r + 2.4615\cos(\delta_R) = 2.6348.$$

To narrow down the interval of real solution before trying to solve it using Numerical methods:

$$\cos(\delta_R) \in [-1, 1]$$

$$-2.4615 \leq 2.4615\cos(\delta_R) \leq 2.4615$$

$$\delta_R - 2.4615 \leq \delta_R + 2.4615\cos(\delta_R) \leq \delta_R + 2.4615$$

since $\delta_R + 2.4615\cos(\delta_R)$ demands to be 2.6348

$$\delta_R - 2.4615 \leq 2.6348 \leq \delta_R + 2.4615.$$

$\delta_R \leq 5.0963$ and $\delta_R \geq 0.1733$. For our case $\delta_{R(0)} = 0.2$

Newton Raphson
$$\delta_{K+1} = \delta_K - \frac{f(\delta_K)}{f'(\delta_K)}$$ # Until Converge.

$f'(\delta_R) = 1 - 2.4615\sin(\delta_R)$ and $K=0$.

$$\delta_{1R} = 0.2 - \left(\frac{0.2 + 2.4615\cos(0.2) - 2.6348}{1 - 2.4615\sin(0.2)}\right) = 0.2438.$$

$\delta_{R1} = 0.2438$ isnt a real possible angle in this sense becase to accomplish the egual area criterion, the machine needs to deacelerate in the final angle and $\delta_R < \delta_0$ which says the machine tends to accelerate, $\delta_0 \rightarrow \delta_{FC}$ corresponds to acceleration becase $P_m > P_e$ while the fault exist. Then δ_R needs to be langer than δ_{FC} so the machine deacelerates becase $P_e > P_m$ when the fault is cleared. A new initial point: $\delta_{R(0)} = 0.5$

If one plots the non linear equation one can see 3 roots. (9)
at 0.249, 0.5915 and 4.08. The 0.5915 root seems to be our solution since 0.249 is not realistic neither 4.08.
This is an important reminder of how important the starting point in numerical methods is. Just reinforcing the previously said.

$$\delta_{R(1)} = 0.5 - \left(\frac{0.5 + 2.4615\cos(0.5) - 2.6348}{1 - 2.4615\sin(0.5)} \right) = 0.6410.$$

$$\delta_{R(2)} = 0.6410 - \left(\frac{0.6410 + 2.4615\cos(0.6410) - 2.6348}{1 - 2.4615\sin(0.6410)} \right) = 0.5967.$$

Error $\simeq$ 0.0443 or 7.42% relative error

$$\delta_{R(3)} = 0.5967 - \left(\frac{0.5967 + 2.4615\cos(0.5967) - 2.6348}{1 - 2.4615\sin(0.5967)} \right) = 0.5916.$$

Error $\approx$ 0.0051 or 0,86%

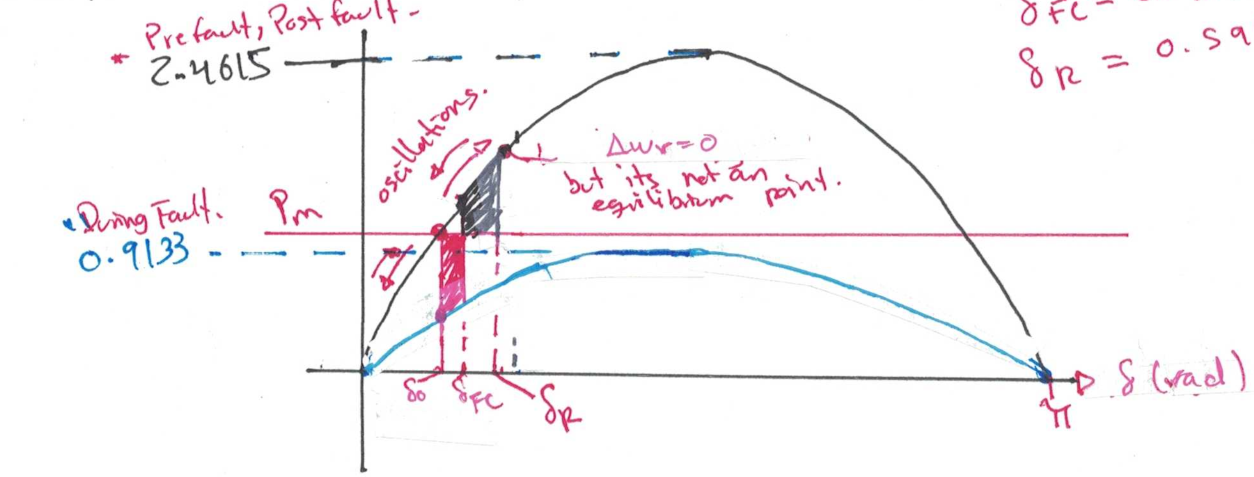
$$\delta_{R(4)} = 0.5916 - \left(\frac{0.5916 + 2.4615\cos(0.5916) - 2.6348}{1 - 2.4615\sin(0.5916)} \right) = \underline{0.5915}$$

$$\delta_R = 0{,}5915$$

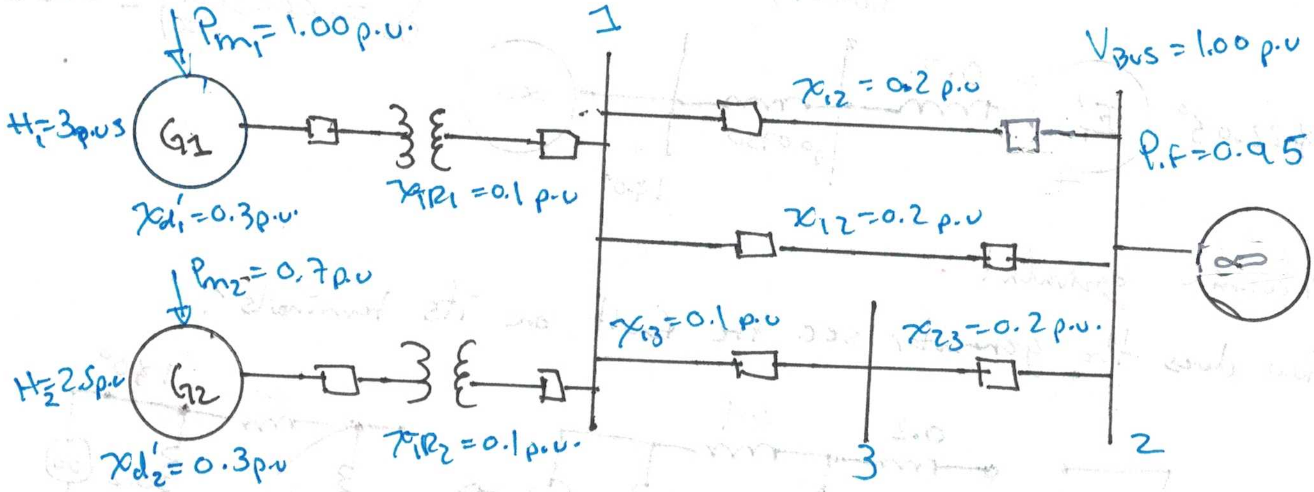
Error $\approx$ 0.0001 or 0.017% which is sufficient.

Since 0.5915 rad is less than $\pi - \delta_0$, $\pi - 0.4178 = 2.7238$ the system mantains stability.

$\delta_0 = 0.4178$ rad
$\delta_{Fc} = 0.4671$ rad
$\delta_R = 0.5915$ rad.



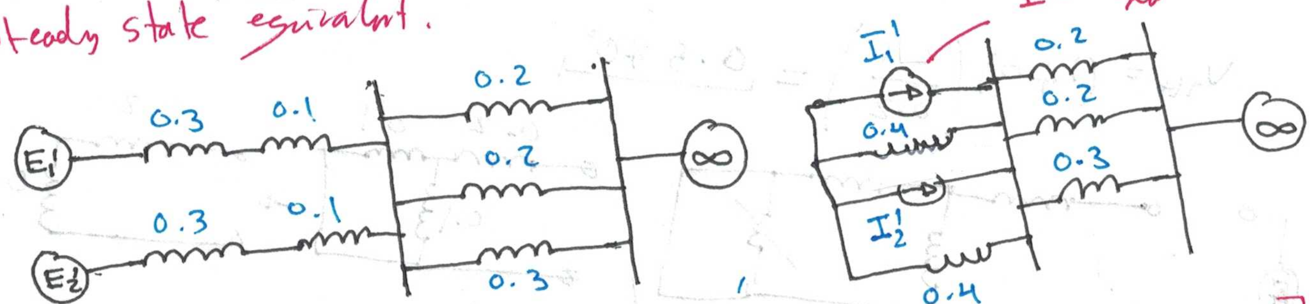
Test case pro:



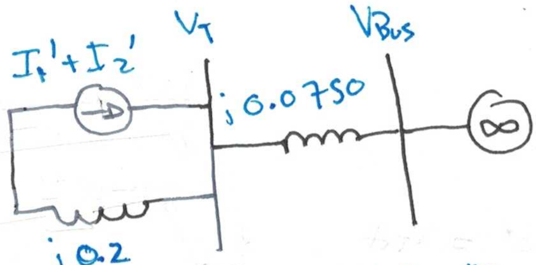
$$H_{eq} = \frac{H_1 + H_2}{2} = \frac{3 + 2.5}{2} = \underline{2.75 \text{ p.u}} \; ; \; P_{mq} = P_{m_1} + P_{m_2} = \underline{1.7 \text{ p.u.}}$$

$$I = \frac{E'}{x_d'}$$

* Steady state equivalent.



$I_T = I_1' + I_2'$



$$I_T' = I_1' + I_2' = \frac{V_T - V_{Bus}}{j0.0750} = \frac{V_T - 1\angle 0^\circ}{j0.0750}$$

$$S_T = V_{Bus} \cdot I_T^* \; ; \; P = S_T \cos\theta = S_T (p.f).$$

$$S_T = \frac{P}{p.f} \; ; \; I_T' = \left(\frac{S_T}{V_{Bus}}\right)^* = \frac{1.7}{(0.95)(1\angle 0^\circ)} \underset{p.f = 0.95}{\underline{\angle -18.19}} = 1.7895 \angle -18.19$$

$$V_T = I_T'(j0.0750) + V_{Bus} = (1.7895 \angle -18.19)(j0.0750) + (1\angle 0^\circ) = \underline{1.0497 \angle 6.97^\circ}$$

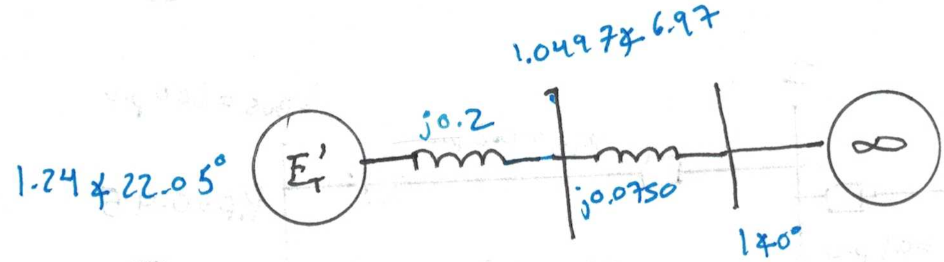
$$E_1' = (1.0497 \angle 6.97^\circ) + j(0.4)\left(\frac{1}{0.95(1\angle 0^\circ)} \angle -18.19\right) = 1.2865 \angle 24.20^\circ$$

$$E_2' = (1.0497 \angle 6.97) + j(0.4)\left(\frac{0.7}{0.9(1)} \angle -18.19\right) = 1.2049 \angle 19.76^\circ$$

$$E_T' = (j0.2)(I_T') = (j0.2)\left[\left(\frac{E_1'}{x_{d_1}' + x_{TR_1}} + \frac{E_2'}{x_{d_2}' + x_{TR_2}}\right)\right] = (j0.2)\left(\frac{E_1' + E_2'}{j0.4}\right)$$

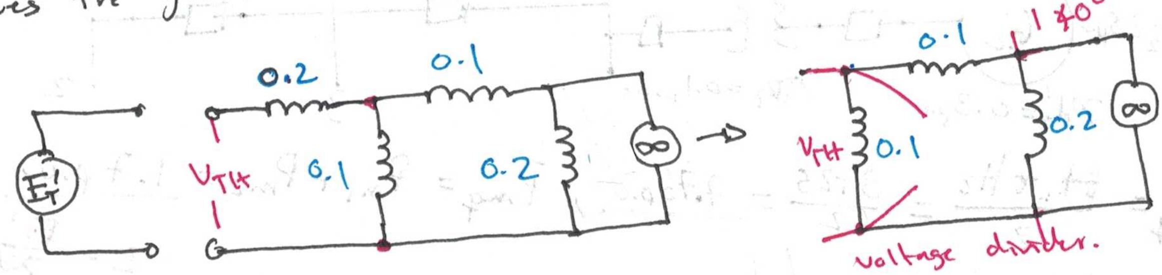
$$E_T' = \frac{E_1' + E_2'}{2} = \frac{(1.2865 \angle 24.20) + (1.2049 \angle 19.76)}{2} = \underline{1.24 \angle 22.05^\circ}$$

$$\delta_0 = 22.05\left(\frac{\pi}{180}\right) = 0.3848 \text{ rad}$$

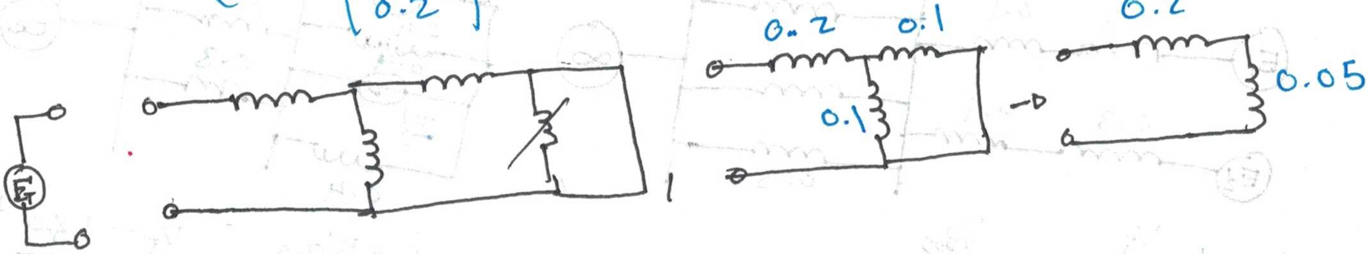


* Thevenin equivalent.

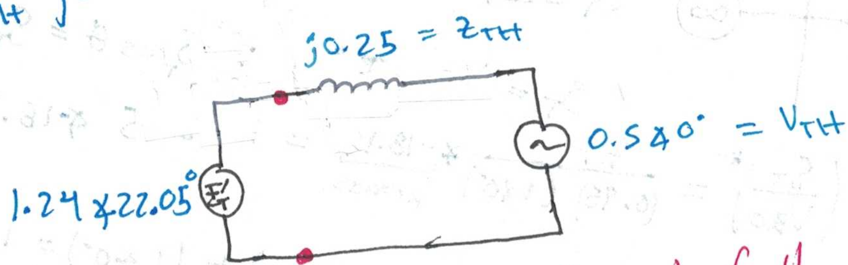
How does the generator see the fault at its terminals?



$$V_{TH} = (1\angle 0)\left(\frac{0.1}{0.2}\right) = \underline{0.5\angle 0^\circ}$$



- $Z_{TH} = j0.25$



* Power transfered during pre and post fault.

$$P_e = \frac{|1.24||1|}{0.2750}\sin(\delta) = 4.5091\sin(\delta)$$

Power transfered during fault.

$$P_{eF} = \frac{|1.24||0.5|}{0.25}\sin(\delta_F) = 2.48\sin(\delta_F).$$

Solving numerically to find δ_F at instant $t = 0.05\,s$ (3 cycles). Using modified euler; $\delta_0 = 0.3848\,rad$, $\dot{\delta}(0) = 0$ and $\Delta t = 0.025\,s$ (2 iterations).

$t=0$. First run $\frac{d\delta}{dt} = 0$ and $\frac{d\Delta w_{r0}}{dt} = \frac{377}{2(2.75)}\left(1.7 - 2.48\sin(0.3848)\right)$

$= 52.7165\ \frac{rad}{s^2}$.

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* Predictor

$\delta_1^{(P)} = 0.3848 + (0.025)(0) = 0.3848\ rad.$

$\Delta w_{r1}^{(P)} = 0 + (0.025)(52.7165) = 1.3179\ \frac{rad}{s}.$

* Slope of the predicted point.

$$\frac{d\Delta w_{r1}^{(P)}}{dt} = \frac{377}{2(2.75)}\left(1.7 - 2.48\sin(0.3848)\right) = 52.7165\ \frac{rad}{s^2}$$

$$\frac{d\delta_1^{(P)}}{dt} = 1.3179\ \frac{rad}{s}.$$

* Corrector, Slope average

$$\delta_1 = 0.3848 + \frac{0.025}{2}\left[0 + 1.3179\right] = 0.4013\ rad.$$

$$\Delta w_{r1} = 0 + \frac{0.025}{2}\left[2(52.7165)\right] = 1.3179\ \frac{rad}{s}$$

Second iteration

$$\delta_2^{(P)} = 0.4013 + 0.025(1.3179) = 0.4342\ rad$$

$$\Delta w_{r2}^{(P)} = 1.3179 + 0.025(50.1255) = 2.5710\ \frac{rad}{s}$$

Slope of the second prediction.

$$\frac{d\delta_2^{(P)}}{dt} = 2.5710\ \frac{rad}{s}$$

$$\frac{d\Delta w_{r2}^{(P)}}{dt} = 45.0139\ \frac{rad}{s^2}$$

- Corrector, slope average of the second iteration.

$$\delta_2 = 0.4013 + \frac{0.025}{2}\left(1.3179 + 2.5710\right) = 0.4499 \approx 0.45 \text{ rad}$$

$$\Delta\omega_{r_2} = 1.3179 + \frac{0.025}{2}\left(50.1255 + 45.0139\right) = 2.5071 \ \frac{\text{rad}}{s}$$

Equal area criterion.

$$\int_{0.3848}^{0.45} (1.7 - 2.48 \sin(\delta)) d\delta + \int_{0.45}^{\delta_R} (1.7 - 4.5091 \sin(\delta)) d\delta = 0$$

$$\left[\left(1.7(0.45) + 2.48 \cos(0.45)\right) - \left(1.7(0.3848) + 2.48 \cos(0.3848)\right)\right] + \ldots$$

$$\left[\left(1.7(\delta_R) + 4.5091 \cos(\delta_R)\right) - \left(1.7(0.45) + 4.5091 \cos(0.45)\right)\right] = 0.$$

$$(2.9981 - 2.9528) + \left[\left(1.7(\delta_R) + 4.5091 \cos(\delta_R)\right) - 4.8252\right] = 0$$

$$0.0453 + 1.7(\delta_R) + 4.5091 \cos(\delta_R) - 4.8252 = 0$$

$$1.7(\delta_R) + 4.5091 \cos(\delta_R) - 4.78 = 0.$$

- Newton Raphson to solve the above equation $\delta_{R(0)} = 0.5$ rad.

$$\delta_{R(1)} = 0.5 - \left(\frac{1.7(0.5) + 4.5091 \cos(0.5) - 4.78}{1.7 - 4.5091 \sin(0.5)}\right) = 0.5592 \text{ rad.}$$

$$\delta_{R(2)} = 0.5592 - \left(\frac{1.7(0.4626) + 4.5091 \cos(0.4626) - 4.78}{1.7 - 4.5091 \sin(0.4626)}\right) = 0.5489 \text{ d.}$$

$\delta_{R(3)} = 0.5487$. The error is 0.0002. $\delta_0 = 0.3848$, $\delta_{FC} = 0.45$, $\delta_R = 0.5487$.

initial

Fault cleared.

maximum angle reached.

Recalling from page 11 the power transfered during the fault is higher than the total mechanical input.

$$P_{mq} < P_{eF}$$

$$1.7 < 2.48$$

After some angle (0.75 rad) the power transfered and the mechanical input are the same so the system tends to stabilize at this point, the problem is, this point corresponds to the fault power transfer curve.

At this operation point with this topology, parameters and assumptions the conclusion is: the fastest, the better, for achieving stability. There might be a critical angle that stands before reaching 0.75 rad, for practical purposes, software simulations can be conducted. But the later the fault is cleared the severity of the first oscillation will increase.

The critical time can't be calculated since $P_e \neq 0$. The standard method of calculating δ_{cr} as supposing the next stable point as $\pi - \delta_0$ can't be done.

This can be "fixed" by removing one j0.2 p.u parallel line. So the power transfered during the fault goes below 1.7, which is the sum of the mechanical power.