Power Flow-Based Prefault Condition Estimation for Digital Protection: IEEE 9-Bus Case Study

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***Abstract*—Power flow analysis is a fundamental tool for determining steady-state operating conditions in a power system. In digital protection, pre-fault conditions such as bus voltages, power injections, and line flows are a reference for relay settings and coordination. This study presents the implementation of a MATLAB-based Newton-Raphson power flow solver and compares its results with those obtained from PowerWorld for the IEEE 9-bus system. The objective is to validate the accuracy of the self-developed script by benchmarking it against established software. The comparison focuses on key power system variables, evaluating discrepancies and computational efficiency. The results confirm the MATLAB script's reliability in computing prefault conditions, supporting its applicability in protection studies.**

I. INTRODUCTION

In digital protection schemes, accurate prefault conditions are necessary for setting protective relays, ensuring proper coordination, and enhancing fault detection reliability. Steady-state power flow solutions serve as the baseline conditions from which protective relays detect deviations and disturbances in the system. Therefore, obtaining reliable prefault voltage and current levels is crucial for effective protection strategies.

The case study focuses on the IEEE 9-bus system, a well-established benchmark for power system analysis. This system consists of three generators, three loads, nine buses, three transformers, and six transmission lines. It represents a simplified but realistic test system suitable for validating power flow methods. The Newton-Raphson algorithm is selected due to its efficiency and robustness in handling nonlinear power flow equations. Fig. 1 provides a one-line diagram of the IEEE 9-bus system, showing the interconnections among elements.

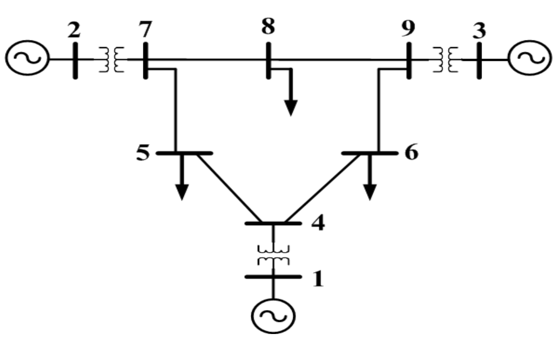

Fig. 1. 9 Bus IEEE Test System.

Several software tools exist for performing power flow analysis, including PowerWorld, PSSE, and ETAP, which are widely used in industry and academia. However, the ability to develop a customized power flow solver provides deeper insight into the numerical methods behind these calculations and enables adaptability for specific applications. This work presents the implementation of a MATLAB-based Newton-Raphson power flow solver and compares its results with those obtained using PowerWorld. Fig. 2 shows a conceptual flowchart that will be used for solving power flow equations by using Newton-Raphson algorithm.

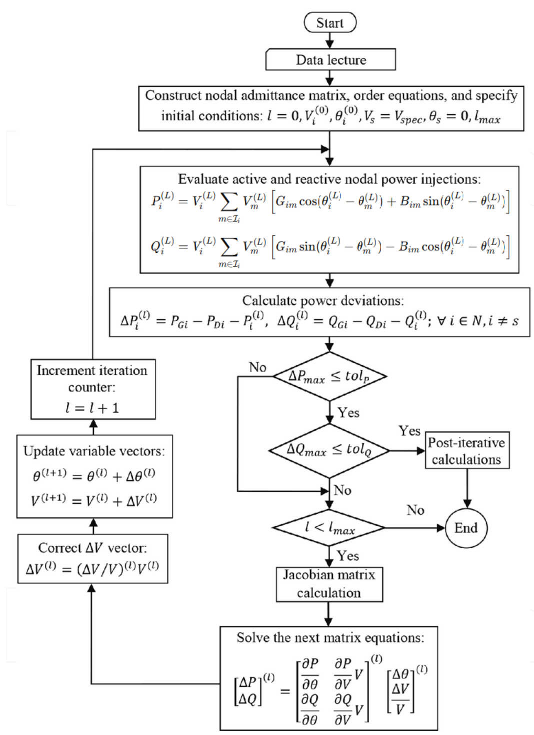

Fig. 2. Flow Chart to Compute Power Flow Using N-R Method.

II. POWER FLOW METHODS

The primary objective is to solve for the unknown voltage magnitudes and angles at system buses, ensuring proper operation, planning, and control of the power grid. Power flow analysis typically relies on iterative numerical techniques due to the nonlinear nature of power system equations.

The general formulation of power flow equations is derived from *Kirchhoff's Current Law (KCL)* and expressed in terms of real and reactive power at each bus:

$$P_{Gi} - P_{Li} - P_i = 0 \tag{1}$$

$$Q_{Gi} - Q_{Li} - Q_i = 0 \tag{2}$$

$$P_i = \sum_{j \in N_i} |V_i|\, |V_j| \Big(G_{ij}\, \cos(\theta_{ij}) \;+\; B_{ij}\, \sin(\theta_{ij}) \Big) \tag{3}$$

$$Q_i = \sum_{j \in N_i} |V_i|\, |V_j| \Big(G_{ij}\, \sin(\theta_{ij}) \;-\; B_{ij}\, \cos(\theta_{ij}) \Big) \tag{4}$$

where:

- P_i and Q_i are the real and reactive power injections at bus i.
- V_i and V_j are the voltage magnitudes at buses i and j.
- θ_{ij} is the phase angle between buses.

A. Decoupled Power Flow

The *decoupled power flow* approach leverages the observation that real power flows (P) are predominantly influenced by voltage phase angles, and reactive power flows (Q) are primarily influenced by voltage magnitudes. Under typical operating conditions, certain assumptions simplify the Jacobian matrix into block-diagonal form:

$$\mathbf{J} = \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix},$$

where $\mathbf{H}$ relates changes in θ to changes in P and $\mathbf{L}$ relates changes in V to changes in Q. The decoupling is based on assumptions such as $G_{ij} \ll B_{ij}$ (conductance much smaller than susceptance) and $\sin(\theta_{ij}) \ll \cos(\theta_{ij})$, thereby simplifying cross-coupling terms between real and reactive power equations. This results in two smaller decoupled systems to be solved iteratively, enhancing computational efficiency.

B. Fast Decoupled Power Flow

The *fast decoupled power flow* method further refines the decoupled approach by making additional simplifying assumptions:

- Bus voltage magnitudes are close to 1.0 p.u.
- $\cos(\theta_{ij}) \approx 1$ and $\sin(\theta_{ij}) \approx \theta_{ij}$ for small angles.
- $G_{ij}\, \sin(\theta_{ij}) \ll B_{ij}$ for real power flows.
- $Q_i \ll B_{ii} V_i^2$ for reactive power flows.

These lead to constant $\mathbf{B}'$ and $\mathbf{B}''$ matrices formed primarily by network susceptances, decoupling the power flow equations as:

$$\Delta P \approx \mathbf{B}'\, \Delta\delta, \quad \Delta Q \approx \mathbf{B}''\, \Delta V, \tag{5}$$

where $\mathbf{B}'$ and $\mathbf{B}''$ are treated as constant, significantly speeding up each iteration. While the fast decoupled method can sometimes reduce accuracy compared to Newton-Raphson for heavily loaded or ill-conditioned systems, it remains popular due to its simplicity and high computational efficiency in large-scale networks.

III. Solving Methods for Power Flow

Different methods exist to solve the power flow equations, including:

- Gauss-Seidel Method: A simple iterative approach that updates bus voltages but has slow convergence for large systems.
- Newton-Raphson Method: A fast, robust iterative method that uses a Jacobian matrix to refine voltage estimates.

Between these two methods, the Newton-Raphson method is widely preferred due to its quadratic convergence and numerical stability. It linearizes the power equations using a Taylor series expansion and iteratively calculates voltage estimates.

The system of nonlinear power flow equations is expressed in matrix form as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}, \tag{6}$$

where:

- ΔP and ΔQ are the mismatches in active and reactive power.
- $J_{11}, J_{12}, J_{21}, J_{22}$ represent elements of the Jacobian matrix, which contains partial derivatives of power equations.
- $\Delta\delta$ and ΔV are the corrections applied to the voltage angles and magnitudes.

Each iteration updates the voltage estimates as:

$$\begin{bmatrix} \delta^{(k+1)} \\ V^{(k+1)} \end{bmatrix} = \begin{bmatrix} \delta^{(k)} \\ V^{(k)} \end{bmatrix} + J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}. \tag{7}$$

IV. Study Case: Simulation and Results

The accuracy of the implemented Newton-Raphson power flow solver was validated using the IEEE 9-bus system, depicted in Fig. 1. This system has three generators, three loads, nine buses, three transformers, and six transmission lines. To maintain consistency in system parameters, the bus admittance matrix ($\mathbf{Y}_{bus}$) was directly exported from PowerWorld and utilized in the MATLAB based solver.

A. Bus Data

The system comprises different types of buses:

- **Slack Bus**: A reference bus with a fixed voltage magnitude and angle.
- **PV Buses**: Generator buses with specified voltage magnitude and real power injection.
- **PQ Buses**: Load buses where active and reactive power are specified.

Table I presents the input bus data, including bus type, real and reactive power demands (P_d, Q_d), real and reactive power generation (P_g, Q_g), and voltage setpoints (V_{set}).

TABLE I
BUS DATA FOR IEEE 9-BUS SYSTEM

Bus	Type	P_d (MW)	Q_d (MVAR)	P_g (MW)	V_{set} (p.u.)
1	1 (Slack)	0	0	0	1.040
2	2 (PV)	0	0	163	1.025
3	2 (PV)	0	0	85	1.025
4	3 (PQ)	0	0	0	-
5	3 (PQ)	125	50	0	-
6	3 (PQ)	90	30	0	-
7	3 (PQ)	0	0	0	-
8	3 (PQ)	100	35	0	-
9	3 (PQ)	0	0	0	-

B. Simulation Process and Convergence Criteria

The power flow solution was obtained using the Newton-Raphson method, which iteratively calculates the voltage estimates until the power mismatch falls below a predefined tolerance. The simulation parameters were set as follows:

- Tolerance for convergence: 1×10^{-6}.
- Maximum iterations: 50.
- Convergence achieved in: 4 iterations.
- Final mismatch: 3.42×10^{-7}.

C. Comparison of Bus Voltage and Power Results

Table II compares the bus voltage magnitudes, angles, and power injections obtained from MATLAB and PowerWorld. The results indicate a high degree of accuracy between both methods.

TABLE II
COMPARISON OF BUS VOLTAGES AND POWER INJECTIONS

Bus	Voltage (p.u.)	Angle (°)	P (MW)	Q (MVAR)
PowerWorld Results				
1	1.0400	0.00	71.64	27.04
2	1.0250	9.28	163.00	6.65
3	1.0250	4.67	85.00	-10.86
4	1.0258	-2.22	0.00	0.00
5	0.9956	-3.99	-125.00	-50.00
6	1.0127	-3.69	-90.00	-30.00
7	1.0258	3.72	0.00	0.00
8	1.0159	0.73	-100.00	-35.00
9	1.0324	1.97	0.00	0.00
MATLAB Results				
1	1.0400	0.00	71.651	27.045
2	1.0250	9.28	163.00	6.65
3	1.0250	4.66	85.00	-10.857
4	1.0258	-2.22	0.00	0.00
5	0.9956	-3.99	-125.00	-50.00
6	1.0127	-3.69	-90.00	-30.00
7	1.0258	3.72	0.00	0.00
8	1.0159	0.73	-100.00	-35.00
9	1.0324	1.97	0.00	0.00

D. Comparison of Power Flow Results

The power flow results obtained from MATLAB were also compared with PowerWorld in terms of power transfer between transmission lines. Table III summarizes the real and reactive power flows between selected transmission lines.

TABLE III
COMPARISON OF LINE POWER FLOWS

From Bus	To Bus	P (MW)	Q (MVAR)
PowerWorld Results			
4	1	-71.6	-23.9
2	7	163.0	6.6
9	3	-85.0	15.0
5	4	-40.7	-38.7
6	4	-30.5	-16.5
7	5	86.6	-8.4
9	6	60.8	-18.1
7	8	76.4	-0.8
8	9	-24.1	-24.3
MATLAB Results			
4	1	-71.651	-23.921
2	7	163.000	6.650
9	3	-85.000	14.952
5	4	-40.686	-38.685
6	4	-30.541	-16.543
7	5	86.614	-8.380
9	6	60.813	-18.075
7	8	76.376	-0.801
8	9	-24.099	-24.289

V. CONCLUSION

The implementation of a Newton-Raphson power flow solver in MATLAB has demonstrated high accuracy and has been verified through comparison with PowerWorld simulations for the IEEE 9-bus system. The solver achieved a solution in four iterations, with a final mismatch well within the defined tolerance, highlighting its numerical stability and efficiency. The full power flow method used ensures precise computation of bus voltages, power injections, and line flows.

This project emphasizes the importance of understanding the power flow model when implementing numerical methods for analysis. By implementing the Newton-Raphson method from scratch, a deeper understanding of the solution techniques for power flow analysis was achieved. The results between MATLAB and PowerWorld validate the reliability of the developed solver.

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