Examen II: TSP course

Preguntas demostrativas: 50 pts

1.- Es correcto asumir que para cada modo λ_j, hay un generador directamente asociado. Demuestre matemáticamente si la afirmación es correcta o incorrecta.
2.- Demuestre que la solución analítica obtenida del análisis de pequeñas señales está compuesta por valores reales. Sugerencia, recuerde la importancia de las propiedades de ortogonalidad de los eigenvectores derechos e izquierdos obtenidos en la descomposición modal.
3.- Que errores surgen al realizar la integración numérica de una ecuación diferencial de primer orden, empleando algún método de integración numérica explicita. Proporcione un ejemplo ilustrativo.
4.- Demuestre, si hay o no acoplamiento, entre las variables de estado x_j y los modos obtenidos durante la descomposición modal en el espacio z_j.
5.- Explique que es la controlabilidad y observabilidad de un modo. Emplee ecuaciones para responder la pregunta.

Preguntas conceptuales: 20 pts

1.- Enuncie las ventajas y desventajas del análisis de pequeñas señales con respecto a estabilidad transitoria. Sugerencia utilice una tabla comparativa.
2.- Explique las diferencias que resultan durante un proceso oscilatorio electromecánico causado por modos modos inter-area, locales y globales.
3.- Que diferencia existe entre los sistemas de monitoreo WAMS, AMI y SCADA. Utiliza una tabla para enunciar sus características y describa sus diferencias.
4.- Qué diferencia hay entre una respuesta natural y una respuesta forzada en el sistema eléctrico de potencia. Ilustre con una grafico de una variable contra tiempo y utilice una tabla para describir sus diferencias.

Ejercicio basado en el modelo matemático: 30 pts

Plantee y considere que el Sistema Eléctrico de Potencia (SEP) está formado por “M” nodos; de los cuales “N” son nodos de generación, y el resto están relaciones a nodos de carga y transición en la red eléctrica. Asuma que el modelo del SEP esta descrito por: “N” Generadores con modelo clásico, “L” Cargas con modelo constante y el sistema tiene una conectividad con líneas de transmisión largas que conectan sistemas de consumo-generación.

a).- Presente el modelo **Ax+Bu** asociado al modelo de generación multimaquina

b) Explique matemáticamente el mecanismo oscilatorio electromecánico, a partir de la respuesta analítica de estabilidad de pequeña señal. Sugerencia, grafique el modo-shape para los modos locales e inter-area y analice los gráficos propuestos desde el marco de oscilaciones electromecánicas.

c) Explique matemáticamente que sucede en la respuesta analítica obtenida del análisis de pequeña señal, cuando $\lambda j \approx \lambda k$. Sugerencia, grafique el modo-shape cuando $\lambda j \approx \lambda k$ y describa las implicaciones en el contexto de oscilaciones electromecánicas.

Demastrative Questions.

1. The linearized multimachine system $\dot{x} = Ax$. where $x \in \mathbb{R}^n$ contains all the system states.

Let's suppose we have m generators and the generator K has n_k states.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} ; \; x_k \in \mathbb{R}^{n_k} \; . \; \underbrace{\sum_{k=1}^{m} n_k = n}$$

each generator have the same number of states n

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}$$

$\underline{A_{kk}}$ describes the dyanmics of each generator and its contols, with no interaction to other parts of the system.

A_{kj}, where $k \neq j$ are the couplings due to the network and contals, it is the way how generator j affects to k.

We want to show wheter or not is correct to assume that mode λ_j only affects one generator.

$$v_j = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ v_{j,k} \\ \vdots \\ 0 \\ 0 \end{bmatrix} , \; v_{jk} \neq 0$$

The eigenvector v_j only affects or is related to the states of generator k.

$$Av_j = \lambda_j v_j, \quad A_{r1}v_{j1} + A_{r2}v_{j2} + + A_{rm}v_m = \lambda v_r.$$

r just denotes the number of generators, same as k but for rows.

$v_j = 0$, except v_k : $A_{rk}v_{jk} = \lambda_j v_{jr}$

$r = k$: $A_{kk}v_{jk} = \lambda_j v_{jk}$, $r \neq k$: $A_{rk}v_{jk} = \lambda_j \cdot 0 = 0$

Strictly. $A_{rk}v_{jk} \doteq 0 \quad \forall r \neq k$

if $v_{jk} \neq 0$: $A_{rk} = 0 \quad \forall r \neq k$.

This means that if we want 1:1 association of a mode j with a generator k the matrix A, must be diagonal. So zero coupling exist and each generator only affects itself.

In a real power system it is not true. A is not diagonal due to couplings through transmission lines and controls couplet to the network.

For a single machine the statement is TRUE. BUT, for a multimachine system it is FALSE. To be true the system must be decoupled.

v_j is the so called right eigenvector or ϕ_j.

② ϕ_j is the right eigenvector and ψ_j the left eigenvector.

$A \in \mathbb{R}^{m \times m}$ — $A\phi_j = \lambda_j \phi_j$, $\psi_j^T A = \lambda_j \psi_j^T$

Φ and Ψ^T are vectors with $\mathbb{R}^n$ dimensions. They have the property :

$\Psi^T \Phi = I$, ψ and ϕ are orthogonal.

The state in time can be obtained as.

$$x(t) = \sum_{j=1}^{m} [\psi_j^T x(0)] \phi_j e^{\lambda_j t}$$

└ Real initial condition.

Because the matrix A is real, the eigenvalues comes in conjugate pairs. So $x(t)$ becomes.

$$x(t) = \sum_{j=1}^{m} [\psi_j^T x(0)] \phi_j e^{\lambda_j t} + \underbrace{\sum_{j=1}^{m} [\psi_j^T x(0)]^* \phi_j^* e^{\lambda_j^* t}}_{\text{conjugates}}$$

$$x(t) = 2\mathbb{R}\left\{\sum_{j=1}^{m} [\psi_j^T x(0)] \phi_j e^{\lambda_j t}\right\} \in \mathbb{R}^n, \forall t$$

3 Consider a first order ordinary ODE

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0$$

An explicit numerical method such as Euler, Heuns, Runge Kutta advances the solution as.

$$y_{n+1} = y_n + h\, g(t_n, y_n, h)$$

time step. explicit function built from evaluations of f.

There are several sources of errors such as truncation, Round-off error and stability method errors.
Truncation refers to the discretization of the function, replacing the derivative by a finite difference approximation.
The round off means that machines have finite precision, talking about floating-point calculations. Stability error, each method has its limitations, so some methods are prone to instability under certain conditions, mainly due to time step h.

Example $\frac{dy}{dt} = -2y \quad y(0) = 1 \longrightarrow \underline{y(t) = e^{-2t}}$

step index

Explicit Euler: $y_{n+1} = y_n + h(-2y_n) = y_n(1-2h)$. time step size

$y_n = (1-2h)^n$ so, $|1-2h| < 1$, $0 < h < 1$ #Stable.

Stability error can be mitigated using the correct method and time step.

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$\dot{x} = Ax$, $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$

$A\phi_j = \lambda_j \phi_j$, $\psi_j^T A = \lambda_j \psi_j^T$.

$\underbrace{\Psi^T \Phi}_{\text{vectors.}} = \underbrace{I}_{\text{identity matrix}}$, $A = \Phi \Lambda \Phi^{-1}$ ↳ Diagonizable.

Define model coordinates $z = \Psi^T x \longrightarrow \underline{x = \Phi z}$.

Coupling in modal space z:

$$\dot{z} = \Psi^T \dot{x} = \Psi^T A x = \Psi^T A \Phi z = (\Psi^T \Phi) \Lambda z = \Lambda z$$

where $\Lambda = diag(\lambda_1, \lambda_2, \dots \lambda_m)$

$$\dot{z}_j = \underline{\lambda_j z_j} \longrightarrow z_j(t) = z_j(0)\, e^{\lambda_j t}.$$

Each mode evolves independently of the other, z_j mode evolves in λ_j direction. In modal space there is no dynamic coupling among meds for digonizable A matrix.

If A has repeated eigenvalues and no linearly independ eigenvectors are present coupling can exist within a Jorden block as polynomial factors. $t^k e^{\lambda t}$ but different eigenvalues are decoupled.

Coupling between states x_i and modes z_j:

$$x(t) = \Phi z(t) = \sum_{j=1}^{m} z_j(t)\phi_j \; , \quad \underline{x_i(t) = \sum_{j=1}^{m} \phi_{ij}\, z_j(t).}$$

state x_i is influenced by all modes j with $\phi_{ij} \neq 0$

The modal initial amplitudes:

$$z_j(0) = \psi_j^T x(0) = \sum_{i=1}^{m} \psi_{ij} x_i(0).$$

Mode j is initially excited by those states with $\psi_{ij} \neq 0$

A standard index is the participation factor:

$$\underbrace{p_{ij} = \phi_{ij} \psi_{ij}}$$

It's a relative coupling index, it says how strongly mode j shapes the state i and how effective state i excites mode j. States and modes are coupled through Φ and Ψ, even though modal dynamics are decoupled

ϕ governs response shape (output composition) and ψ governs excitation (input decomposition).

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For this case lets expand the linear system.

$$\dot{x} = Ax + Bu, \quad y = Cx + Du.$$

Same procedure as in previous case.

$$\dot{z} = \Psi^T \dot{x} = \Lambda z + \Psi^T B u, \quad y = C \Phi z + Du$$

$$\dot{z}_j = \lambda_j z_j + \underbrace{\psi_j^T B}_{\text{controllability}} u, \quad y = \sum_{j=1}^{m} \overbrace{(C \phi_j)}^{\text{observability}} z_j + Du.$$

$\psi_j^T B$ tells us that the mode j is controllable if $\psi_j^T B \neq 0$. The input affects the dynamics (modal) $\dot{z}_j$.

Likewise, $C\phi_j$ tells us that the mode z_j appears in the measured output y.
So mode j is obserable if $C\phi_j \neq 0$.

From a transfer function point of view:

$$A = \Phi \Lambda \Phi^{-1}, \quad \Psi^T \Phi = I$$

Diagonalizable.

$$G(s) = C[sI - A]^{-1} B + D.$$

$$(sI - A)^{-1} = \Phi [sI - \Lambda]^{-1} \Psi^T = \sum_{j=1}^{m} \frac{\phi_j \psi_j^T}{s - \lambda_j}$$

$$G(s) = \sum_{j=1}^{m} \frac{C \phi_j \psi_j^T B}{s - \lambda_j} + D.$$

If $\psi_j^T B = 0$ the input can not excite the eigenvector j, so no energy reaches the mode j

If $C\phi_j = 0$, the output is blind to the eigenvector, even if the mode is excited, it does not appear at the output. Basically:

$$\operatorname*{Res}_{s = \lambda_j} G(s) = \lim_{s \to \lambda_j} (s - \lambda_j)\, G(s) = C\phi_j \psi_j^T B.$$

If the mode is uncontrollabe or unobservable it does not appear in the transfer function. In a minimal realization, these hidden modes are removed.

Concepts.

1.

Aspect	Small-Signal Analysis	Transient Stability.
Principle.	Small perturbations, LTI approximation.	Large disturbances, full nonlinear models, protections, etc.
Insights.	Modal content, sensitivity analysis, participation factor controllability/observabillity. [Freguency analysis]	Is the system stable after a disturbance? Clearing times and angles, trajectories (post fault). [Time analysis.]
Model structure.	Linearized DAEs → ODE state space. Neglects hard nonlinearities.	Detailed nonlinear DAE's. One can add discrete logic to simulations.
Computational cost.	Low to moderate can be improved by matrix factorizations	Moderate to high depending on models.
Oriented to	Residue placement, mode controllability. and observability tuning controlls	Operational margins and remedial actions. Study out of step risk and loss of synchronism. Contingencies.
Use when:	Fast diagnosis of oscillations, modal damping and resonance risks.	Evaluate faults, protections and nonlinear interactions.

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Feature.	Interarea	Local Mode	Global
Frequency.	$\approx 0.1-0.8Hz$	$0.8-3Hz$	very low $\approx < 0.1Hz$,
Mode Shape	Groups of gen swinging against other groups.	One gen or plant vs its local infinite bus.	Bulk kinetic energy vs load or system.
Excitation	Large transfers, weak network control mistuning.	Control mistuning, aggressive AVR, torsional effects, poor damping.	Large power imbalance in big zones.
Mitigation	Power system stabilizer (PSS), adjust energy transfer.	Retune controllers and damping devices.	Primary and secondary control and load shedding.

(3)

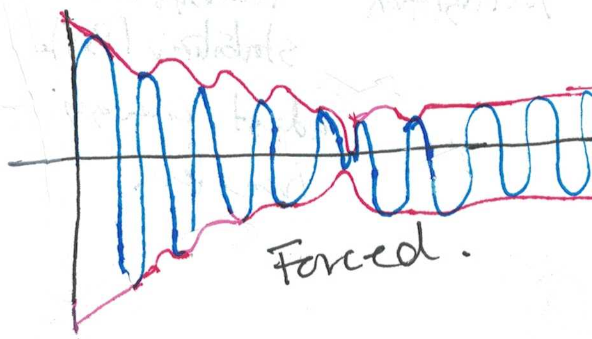
Feature.	Wide Area Monitoring Sys. (WAMS)	Advance Metering Infr. (AMI).	Supervisory Control and data acq. (SCADA).
Purpose	Real time monitoring. of oscillations and dynamics.	Data exchange between consumers and utilities.	Real time monitoring and control of substations and equipment.
Data.	PMU, GPS sync.	Smart meters. No GPS sync.	RTU and PLCs Local time stamps.
Resolution	High (20-100 samples/second)	Low, minutes to hours.	Moderate (seconds)
Coverage.	Wide area, transmission level.	Distribution and end-user level.	Local/substation.

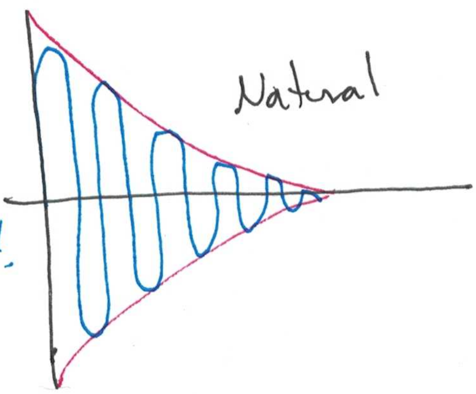
(4) Natural Response

- Intrinsically associated to the system
- Depends on initial conditions (energy)
- Decays over time if stable, governs transient response (state).

Forced Response.

- Caused by an external input.
- It governs the steady state.
- Persist as long the input is present.
- Intrinsically zero damping.





* Model Based

(a) Multimachine model $Ax+Bu$.

Take N classical generators. For generator i:

$$\dot{\delta}_i = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\Delta\dot{\omega}_i = \underbrace{\Delta P_{m,i}}_{\text{Mech disturbance}} - \overbrace{\Delta P_{e,i}}^{\text{elect. Disturbance}} - D_i \underbrace{\Delta\omega_i}_{\text{speed deviation.}}$$

With constant power loads, the electrical power of machine i is:

$$P_{e,i} = \sum_{j=1}^{N} V_i V_j |Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$$

When linearized around (δ_0, ω_s):

$$\Delta P_e = K_s \underbrace{\Delta\delta}_{\Delta\delta_i = \delta_i - \delta_{i(0)}}, \quad (K_s)_{ij} = \left.\frac{\partial P_{e,i}}{\partial \delta_j}\right|_0$$

$$\Delta\dot{\delta} = \Delta\omega, \quad M\Delta\dot{\omega} = -K_s\Delta\delta - D\Delta\omega + \Delta P_m$$

where M and D: $M = \operatorname{diag}\left(\frac{2H_i}{\omega_s}\right)$ and $D = \operatorname{diag}(D_i)$.

Defining the state and input.

$$\underbrace{x}_{} = \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} \in \mathbb{R}^{2N}, \quad \underbrace{u}_{} = \Delta P_m \in \mathbb{R}^N \longrightarrow \boxed{\dot{x} = Ax + Bu}$$

Identity matrix.

Pase 12.

$$A = \begin{bmatrix} 0_{N\times N} & I_N \\ -M^{-1}K_s & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{N\times N} \\ M^{-1} \end{bmatrix} \longrightarrow \underline{\dot{x} = Ax + Bu}$$

This is the multimachine small signal model, using the classical model. Network variables dissapear after linearization.

b) Electromechanical Oscillations and Mode Shape

The linear solution can be written in modal form. If A is diagonalizable with right eigenvectors ϕ_j and left eigenvectors ψ_j, eigenvalues λ_j.

identity - $j \times i$

$$A\phi_j = \lambda_j \phi_j, \quad \psi_j^T A = \lambda_j \psi_j^T, \quad \psi_j^T \phi_i = I$$

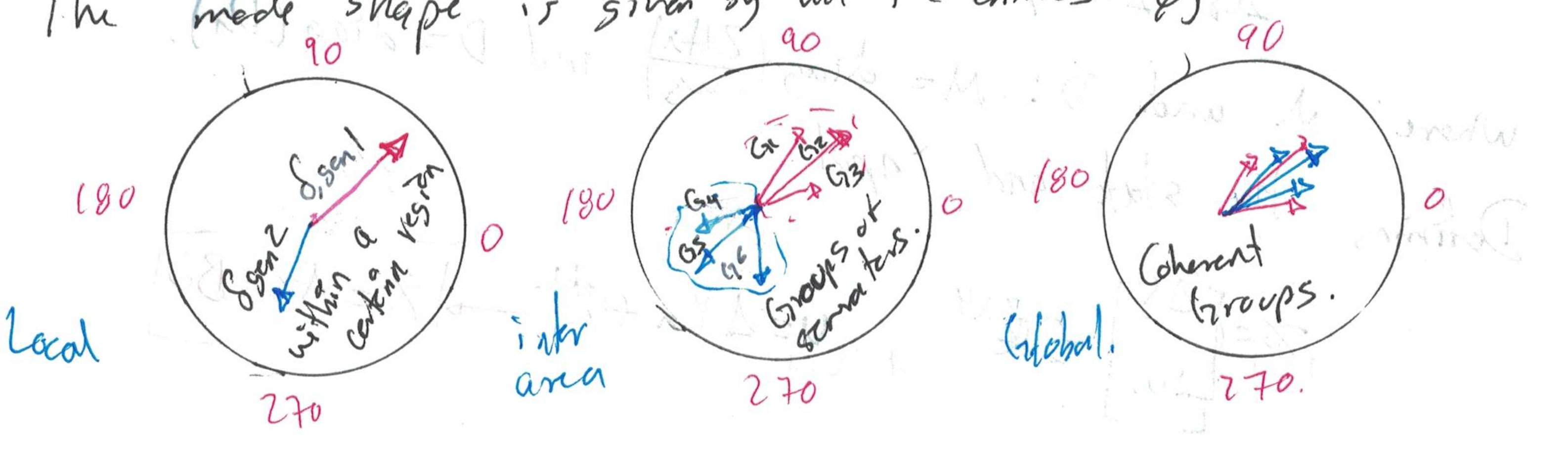
For zero input. $x(t) = \sum_{j=1}^{2N} e^{\lambda_j t} \phi_j (\psi_j^T x(0))$

The mode is: $\lambda_j = -\sigma_j \pm j\omega_j \quad \sigma_j > 0$.

So each state component behaves approximately as.

$$x_i^{(j)}(t) \approx |\phi_{ij}||\psi_j^T x(0)| e^{-\sigma_j t} \sin(\omega_j t + \theta_{ij})$$

The mode shape is given by all the entries ϕ_j



Ⓒ What happens when $\lambda_j \approx \lambda_k$, close eigenvalues. Page 13.

Consider two complex modes with similar eigenvalues.

$$\lambda_j \approx \lambda_k \approx -\sigma \pm j\omega$$

The contribution of these two to a state component is:

$$x_i^{(j+k)}(t) = e^{-\sigma t}\left[a_i \cos(\omega_j t + \alpha_i) + b_i \cos(\omega_k t + \beta_i)\right]$$

coefficients a_i, b_i, determined by $\phi_j, \phi_k, \psi_j^T x(0), \psi_k^T x(0)$.

If ω_j and ω_k are close, let $\omega_j = \omega + \Delta\omega/2$, $\omega_k = \omega - \Delta\omega/2$, substituting in $x_i^{(j+k)}(t)$. and simplifying.

$$x_i^{(j+k)}(t) = e^{-\sigma t}\,\mathbb{R}\{\underbrace{C_{i1}}_{a_i e^{j\alpha_i}} e^{j\omega_j t} + \underbrace{C_{2i}}_{b_i e^{j\beta_i}} e^{j\omega_k t}\}$$

$$x_i^{(j+k)}(t) = e^{-\sigma t}\,\mathbb{R}\left\{e^{j\omega t}\underbrace{\left[C_1 e^{j\frac{\Delta\omega t}{2}} + C_2 e^{-j\frac{\Delta\omega t}{2}}\right]}_{E_i(t)}\right\}$$

$$x_i^{(j+k)}(t) = e^{-\sigma t}\,\mathbb{R}\{E_i(t) e^{j\omega t}\}$$

$A_i(t) = |E_i(t)|$
$\theta_i(t) = \arg(E_i(t))$

$$x_i^{(j+k)}(t) = e^{-\sigma t}\underbrace{A_i(t)}_{\text{Amplitude modulation.}}\cos(\omega t + \underbrace{\theta_i(t)}_{\text{Frequency envelope}})$$ # Beating.

When two eigenvalues are close, there is a strong modal interaction the response is a mixture of both frequencies.

Eigenvectors ϕ_j and ϕ_k become nearly linearly dependent. Page 14.
Any linear combination of them is almost an eigenvector.

If both are equal matrix A is defective, its eigenvectors does not span all the eigenspace, its geometric multiplicity is less than its algebraical one. Creating Jordan blocks, introducing secular terms that makes the response blow up as time goes to infinity.

Mode shapes become ill conditioned, from the perturbed eigenvalue problem $A \rightarrow A + \Delta A$ we set $K(\lambda)$ which is the conditioning equation.

$$K(\lambda_j) = \frac{\|\psi_j\|\,\|\phi_j\|}{|\psi_j^T \phi_j|} \qquad \|\cdot\| \text{ — Norm.}$$

$$|\Delta\lambda_j| \leq K(\lambda_j)\,\|\Delta A\|$$

. To satisfy orthogonality between modes:

$\psi_j^T \phi_K = 0$. This condition says that left eigenvectors of distinct eigenvalues are in an orthogonal invariant subspace. If $\phi_K = \phi_j + \varepsilon v$, then $\psi_j^T(\phi_j + \varepsilon v) = \psi_j^T\phi_j + \varepsilon\psi_j^T v$ therefore $\psi_j^T\phi_j = -\varepsilon(\psi_j^T v) \rightarrow 0$ as $\varepsilon \rightarrow 0$.

So as ϕ_K approaches ϕ_j, $\psi_j^T\phi_j$ tends to 0, causing the denominator blow up. Larger $K(\lambda)$, small parameter perturbations ΔA can cause large changes in eigenvalues $\Delta\lambda$. When eigenvalues are clustered the mode shapes become ill conditioned.