HW1 Modeling Power System Behavior - Power Flow Study

Master of Science in Electrical Engineering

Control and Operation of Power Systems

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* Power Flow Study.

A power flow study is a steady-state analysis that determines the operating conditions of an electrical power system. In such study, a set of non linear algebraic equations are solved, these equations provide the voltage magnitude and phase angle at every bus ande active power (P) and reactive power (Q) flowing in each branch.
This study is useful for planning and operational studies, also for contingency analysis and stability assessments.

* Assumptions.

In order to perform this study general assumptions must be made:

* Steady State Operations. (Balanced, Sinusoidal Conditions).
* Per Unit System.
* Network modeled using an equivalent circuit [Y_{BUS}] which encapsulates all the information of the network.

* Bus Injection Equations.

At each bus i, the net injected complex Power S_i is given by:

$$S_i = P_i + jQ_i = V_i\left(\sum_{j=1}^{n} V_j Y_{ij}\right)^* \qquad 1)$$

Using the phasor definition $V_i = |V_i| e^{j\theta_i}$ and the elements of the admitance $Y_{ij} = G_{ij} + jB_{ij}$ we get:

$$S_i = |V_i| e^{j\theta_i} \left[\sum_{j=1}^{n} |V_j| e^{j\theta_j} (G_{ij} + jB_{ij}) \right]^* \quad 2)$$

Hence, separating real and complex component (P_i, Q), using $e^{j\theta} = \cos(\theta) + j\sin(\theta)$:

$$P_i = \sum_{j=1}^{n} |V_i||V_j| \left[G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j) \right] \quad 3)$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j| \left[G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j) \right] \quad 4)$$

* Polar Form
* Note these equations are in function of the voltage magnitude and angle to describe power injections.
* They are nonlinear due to the trigonometric functions involved.

* Bus Types.

Each bus in the network must satisfy a power balance equation

$$f(x_1, x_2) = K_1$$

$$f(x_1, x_2) - K_1 = 0 \quad 5)$$

Where x_1 and x_2 are the state variables and K_1 is a constant that could be P or Q specified at each bus. So we must drive the function to be 0, or in other words to find the roots of the functions. We consider 3 types of buses:

* Slack (Reference) * Generation (PV) * Load (PQ)

* Known and Unknown variables for each bus:

Bus Type	Known V.	Unknown V.	Number of Equations per bus
Slack	V, θ	None	0
Generation PV	V, P	θ, Q	1
Load PQ	P, Q	θ, V	2

The total number of equations must be the same as the total number of unknown state variables. It is:

$$\#\text{equations} = \#\text{state variables}$$

$$\#\text{equations} = 2n_{PQ} + n_{PV}. \qquad 6)$$

The total number of buses is: $n_T = n_{PQ} + n_{PV} + \underbrace{1}_{\text{SLACK}} \quad 7)$

Note that P and Q for the slack bus are not fixed. Hence, this node "absorbs" the net mismatch by providing extra real and reactive power needed to make the total power balance zero. It acts as a balancing node, thus it must be the largest generator in the system to provide complex power without changing its angle nor magnitude.

* Formulation of Power Flow Equations.

As seen in 5) we must drive a function to be zero, in this case we have two functions 3) and 4), which both are function of V and θ. Hence:

$$* \; P_i(\vec{x}) - P_i^{spec} = 0 \qquad 8)$$

$$* \; Q_i(\vec{x}) - Q_i^{spec} = 0 \qquad 9)$$

Since the equations are non linear they are solved by using iterative methods like Newton-Raphson.
First we need to linearize the functions by using Taylor Series.

$$F(\vec{x}^{(k)}) + J(\vec{x}^{(k)})\,\Delta\vec{x} = 0 \qquad 10)$$

$$\Delta\vec{x} = \vec{x}^{(k+1)} - x^{(k)}$$

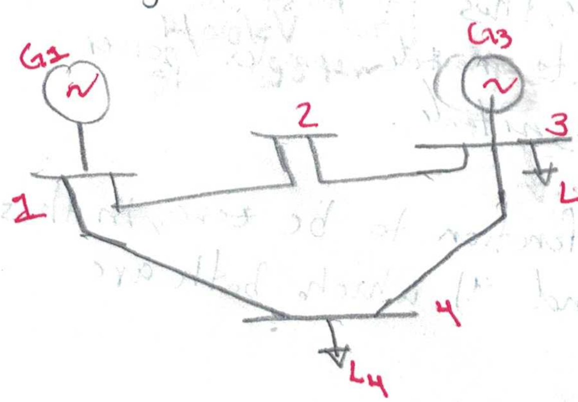
$$\vec{x}^{(k+1)} = \Delta\vec{x} + x^{(k)} \qquad 11)$$

In 10), J is the Jacobian which is a matrix that collects all the partial derivatives of the nonlinear functions with respect to the state variables. It's purpose is to linearize the equations at a given operating point to determine how small changes in the state variables affect the functions.

$$J(x) = \frac{\partial F(\vec{x})}{\partial \vec{x}} \qquad 12)$$

* Test Systems.

Two generators



Test system ①
400kV, 200-500km T.L. □

* Assumption: The real part of the □ impedance is negligible.

Bus #	Bus Type	Description
1	Slack	Arbitrary Chosen. (Doesn't have load).
2	PQ	Modeled as load but P=0, Q=0.
3	PV	Generation Unit
4	PQ	Has a load.

It's a four node system, hence, 2(2)+1=5 equations are needed. and there are also 5 state variables.

$$\vec{X} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ V_2 \\ V_4 \end{bmatrix} \qquad \Delta P_i = P_G^{esp} - P_L^{esp} - V_i \sum_{j=1}^{n} V_j \left(G_{ij}\cos(\theta_{ij}) + B_{ij}\sin(\theta_{ij}) \right) = 0$$

$$\Delta Q_i = Q_G^{esp} - Q_L^{esp} - V_i \sum_{j=1}^{n} V_j \left(G_{ij}\sin(\theta_{ij}) - B_{ij}\cos(\theta_{ij}) \right) = 0$$

(1) * Slack $V_1 = 1.0pu \measuredangle 0°$ (3) * Generation $V_3 = 1.0pu \measuredangle \theta_3$

ΔP

* $\Delta P_2 = \cancelto{0}{P_G^{esp}} - \cancelto{0}{P_2^{esp}} - V_2 \Big[(1) B_{21}\sin(\theta_2 - 0) + \cancelto{0}{V_2 B_{22}\sin(0)} + \dots$

$\cancelto{1}{V_3} B_{23}\sin(\theta_2 - \theta_3) + V_4 B_{24}\sin(\theta_2 - \theta_4) \Big]$

* $\Delta P_3 = P_{3G}^{esp} - P_{L3}^{esp} - \cancelto{1}{V_3} \Big[\cancelto{1}{V_1} B_{31}\sin(\theta_3 - 0) + V_2 B_{32}\sin(\theta_3 - \theta_2) + \dots$

$\cancelto{0}{V_3 B_{33}\sin(0)} + V_4 B_{34}\sin(\theta_3 - \theta_4) \Big]$

* $\Delta P_4 = \cancelto{0}{P_{4G}^{esp}} - P_{L4}^{esp} - V_4 \Big[\cancelto{1}{V_1} B_{41}\sin(\theta_4 - 0) + V_2 B_{42}\sin(\theta_4 - \theta_2) + \dots$

$\cancelto{1}{V_3} B_{43}\sin(\theta_4 - \theta_3) + \cancelto{0}{V_4 B_{44}\sin(0)} \Big]$

ΔQ

* $\Delta Q_2 = \cancelto{0}{Q_{2G}^{esp}} - \cancelto{0}{Q_{2L}^{esp}} + V_2 \Big[\cancelto{1}{V_1} B_{21}\cos(\theta_2 - 0) + V_2 B_{22}(1) + \dots$

$\cancelto{1}{V_3} B_{23}\cos(\theta_2 - \theta_3) + V_4 \cdot B_{24}\cos(\theta_2 - \theta_4) \Big]$

$$\Delta Q_4 = \cancelto{0}{Q_{4G}^{esp}} - Q_{L4}^{esp} + V_4 \Big[\cancelto{1}{V_1} B_{41} \cos(\theta_4 - 0) + V_2 B_{42} \cos(\theta_4 - \theta_2)$$

$$+ \cancelto{1}{V_3} B_{43} \cos(\theta_4 - \theta_3) + V_4 B_{44}(1) \Big]$$

The susceptances B_{ij} are constant values which are extracted from the Y_{BUS}. The Y_{BUS} is a square matrix that has all the information of the connectivity of the system. It can be obtained using nodal analysis as follows:

$$I_1 = y_{12}(V_1 - V_2) + y_{14}(V_1 - V_4)$$

$$0 = y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$$

$$I_3 = y_{32}(V_3 - V_2) + y_{34}(V_3 - V_4) + y_L(V_3)$$

$$0 = y_{43}(V_4 - V_3) + y_{41}(V_4 - V_1)$$

$$V_1 \underbrace{(y_{12} + y_{14})}_{Y_{11}} + V_2 \underbrace{(-y_{12})}_{Y_{12}} + V_3 \underbrace{(0)}_{Y_{13}} + V_4 \underbrace{(-y_{14})}_{Y_{14}} = I_1$$

$$V_1 \underbrace{(-y_{21})}_{Y_{21}} + V_2 \underbrace{(y_{21} + y_{23})}_{Y_{22}} + V_3 \underbrace{(-y_{23})}_{Y_{23}} + V_4 \underbrace{(0)}_{Y_{24}} = 0$$

$$V_1 \underbrace{(0)}_{Y_{31}} + V_2 \underbrace{(-y_{32})}_{Y_{32}} + V_3 \underbrace{(y_{32} + y_{34} + y_L)}_{Y_{33}} + V_4 \underbrace{(-y_{34})}_{Y_{34}} = I_3$$

$$V_1 \underbrace{(-y_{41})}_{Y_{41}} + V_2 \underbrace{(0)}_{Y_{42}} + V_3 \underbrace{(-y_{43})}_{Y_{43}} + V_4 \underbrace{(y_{43} + y_{41})}_{Y_{44}} = 0$$

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

We have the functions and the Y_{BUS}, now is neccesary to fill the Jacobian matrix.

$$
\begin{array}{c}
\begin{array}{cc} \qquad\qquad H \qquad\qquad & \qquad N \qquad \end{array} \\
\begin{bmatrix}
\frac{\partial \Delta P_2}{\partial \theta_2} & \frac{\partial \Delta P_2}{\partial \theta_3} & \frac{\partial \Delta P_2}{\partial \theta_4} & \frac{\partial \Delta P_2}{\partial V_2} & \frac{\partial \Delta P_2}{\partial V_4} \\
\frac{\partial \Delta P_3}{\partial \theta_2} & \frac{\partial \Delta P_3}{\partial \theta_3} & \frac{\partial \Delta P_3}{\partial \Delta \theta_4} & \frac{\partial \Delta P_3}{\partial V_2} & \frac{\partial \Delta P_3}{\partial V_4} \\
\frac{\partial \Delta P_4}{\partial \theta_2} & \frac{\partial \Delta P_4}{\partial \theta_3} & \frac{\partial \Delta P_4}{\partial \theta_4} & \frac{\partial \Delta P_4}{\partial V_2} & \frac{\partial \Delta P_4}{\partial V_4} \\
\frac{\partial \Delta Q_2}{\partial \theta_2} & \frac{\partial \Delta Q_2}{\partial \theta_3} & \frac{\partial \Delta Q_2}{\partial \theta_4} & \frac{\partial \Delta Q_2}{\partial V_2} & \frac{\partial \Delta Q_2}{\partial V_4} \\
\frac{\partial \Delta Q_4}{\partial \theta_2} & \frac{\partial \Delta Q_4}{\partial \theta_3} & \frac{\partial \Delta Q_4}{\partial \theta_4} & \frac{\partial \Delta Q_4}{\partial V_2} & \frac{\partial \Delta Q_2}{\partial V_4}
\end{bmatrix} \\
\begin{array}{cc} \qquad\qquad J \qquad\qquad & \qquad L \qquad \end{array}
\end{array}
\begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta\theta_4 \\ \Delta V_2 \\ \Delta V_4 \end{bmatrix}
$$

$H = \frac{\partial P}{\partial \theta}$; $N = \frac{\partial P}{\partial V}$; $J = \frac{\partial Q}{\partial \theta}$; $L = \frac{\partial Q}{\partial V}$.

* $\frac{\partial \Delta P_2}{\partial \theta_2} = -V_2 B_{21} \cos(\theta_2) - V_2 B_{23} \cos(\theta_2 - \theta_3) - V_2 V_4 B_{24} \cos(\theta_2 - \theta_4)$

$\frac{\partial \Delta P_2}{\partial \theta_3} = V_2 B_{23} \cos(\theta_2 - \theta_3)$

$\frac{\partial \Delta P_2}{\partial \theta_4} = V_2 V_4 \cos(\theta_2 - \theta_4)$

$$\frac{\partial \Delta P_2}{\partial V_2} = -B_{21}\sin(\theta_2) - B_{23}\sin(\theta_2-\theta_3) - V_4 B_{24}\sin(\theta_2-\theta_4)$$

$$\frac{\partial \Delta P_2}{\partial V_4} = -V_2 B_{24}\sin(\theta_2-\theta_4)$$

$$\frac{\partial \Delta P_3}{\partial \theta_2} = V_2 B_{32}\cos(\theta_3-\theta_2)$$

$$* \frac{\partial \Delta P_3}{\partial \theta_3} = -B_{31}\cos(\theta_3) - V_2 B_{32}\cos(\theta_3-\theta_2) - V_4 B_{34}\cos(\theta_3-\theta_4)$$

$$\frac{\partial \Delta P_3}{\partial \theta_4} = V_4 B_{34}\cos(\theta_3-\theta_4)$$

$$\frac{\partial \Delta P_3}{\partial V_2} = -B_{32}\sin(\theta_3-\theta_2)$$

$$\frac{\partial \Delta P_3}{\partial V_4} = -B_{34}\sin(\theta_3-\theta_4).$$

$$\frac{\partial \Delta P_4}{\partial \theta_2} = V_4 V_2 B_{42}\cos(\theta_4-\theta_2)$$

$$\frac{\partial \Delta P_4}{\partial \theta_3} = V_4 B_{43}\cos(\theta_4-\theta_3)$$

$$* \frac{\partial \Delta P_4}{\partial \theta_4} = -V_4 B_{41}\cos(\theta_4) - V_4 V_2 B_{42}\cos(\theta_4-\theta_2) - V_4 B_{43}\cos(\theta_4-\theta_3)$$

$$\frac{\partial \Delta P_4}{\partial V_2} = -V_4 B_{42} \sin(\theta_4 - \theta_2)$$

$$\frac{\partial \Delta P_4}{\partial V_4} = -B_{41} \sin(\theta_4) - V_2 B_{42} \sin(\theta_4 - \theta_2) - B_{43} \sin(\theta_4 - \theta_3)$$

$$\frac{\partial \Delta Q_2}{\partial \theta_2} = -V_2 B_{21} \sin(\theta_2) - V_2 B_{23} \sin(\theta_2 - \theta_3)$$
$$-V_2 V_4 B_{24} \sin(\theta_2 - \theta_4)$$

$$\frac{\partial \Delta Q_2}{\partial \theta_3} = V_2 B_{23} \sin(\theta_2 - \theta_3)$$

$$\frac{\partial \Delta Q_2}{\partial \theta_4} = V_2 V_4 B_{24} \sin(\theta_2 - \theta_4)$$

$$\frac{\partial \Delta Q_2}{\partial V_2} = B_{21} \cos(\theta_2) + B_{22} + B_{23} \cos(\theta_2 - \theta_3) + 2V_2 B_{22}$$
$$+ B_{23} \cos(\theta_2 - \theta_3) + V_4 B_{24} \cos(\theta_2 - \theta_4)$$

$$\frac{\partial \Delta Q_2}{\partial V_4} = V_2 B_{24} \cos(\theta_2 - \theta_4)$$

$$\frac{\partial \Delta Q_4}{\partial \theta_2} = V_4 V_2 B_{41} \sin(\theta_4 - \theta_2)$$

$$\frac{\partial \Delta Q_4}{\partial \theta_3} = V_4 B_{43} \sin(\theta_4 - \theta_3)$$

$$\frac{\partial \Delta Q_4}{\partial \theta_4} = -V_4 B_{41} \sin(\theta_4) - V_4 V_2 B_{41} \sin(\theta_4 - \theta_2) - \dots$$
$$V_4 B_{43} \sin(\theta_4 - \theta_3)$$

$$\frac{\partial \Delta Q_4}{\partial V_4} = B_{41} \cos(\theta_4) + V_2 B_{42} \cos(\theta_4 - \theta_2) - \dots$$
$$B_{43} \cos(\theta_4 - \theta_3) + 2 V_4 B_{44}.$$

$$\frac{\partial \Delta Q_4}{\partial V_2} = V_4 B_{42} \cos(\theta_4 - \theta_2)$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} + \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = - \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

$$\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = - \begin{bmatrix} H & N \\ J & L \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

13)

The negative signs in $\frac{\partial P_i}{\partial \theta_i}$ indicates that if the angle increases the net active power will decrease. The positive signs in $\frac{\partial Q_i}{\partial V_i}$ says increase in voltage – increase Q.
In reality the angles between buses arent too large because of this fact the sine terms are smaller than the cosine ones. Thus the most relevant submatrices of the Jacobian are H and L. which link active and reactive power respectively.

* Decoupled Power Flow - Two generator Test System.

In high voltage above 161KV and transmission systems that connects far away substations (>200km), the system can be considered purely reactive.

$$G_{ij} << B_{ij}$$

Also other assumption can be made, the angular difference between the nodes is very small. (The sine terms are almost negligible).

$$\sin(\theta_i - \theta_j) << \cos(\theta_i - \theta_j)$$

Through the equations from page 7 to 10, all the equations with sine terms dissapear. Hence the Jacobian for the decoupled approach is:

$$J = \begin{vmatrix} H & 0 \\ 0 & L \end{vmatrix}$$

* These simplifications reduce the computational load in power flow.

The decoupled power flow is:

* H and L are still updated every iteration
* Faster than full NR and has good accuracy.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \frac{\Delta V}{V} \end{bmatrix}$$

$$\Delta\theta = H^{-1}\Delta P$$

$$\frac{\Delta V}{V} = L^{-1}\Delta Q$$

Normalization factor.

$$\theta^{k+1} = \theta^k + \Delta\theta^k$$

$$V^{k+1} = V^k + \left(\frac{\Delta V}{V}\right)^k$$

Key insights for the decoupled approach.

* Active power (P) is much more sensitive to voltage angle differences than to voltage magnitude changes.
* Reactive power (Q) is primarily sensitive to voltage magnitudes rather than angles.

In conseccuence $N = \frac{\partial P}{\partial V}$ and $\frac{\partial Q}{\partial \theta}$ are very small.

* Fast Decoupled for Two Generators Test System.

Further physically-justificable simplifications may be made: These assumptions are almost always valid:

$$V \approx 1.0\,pu.$$

$$\cos(\theta_i - \theta_j) \approx 1$$

Long transmission lines Suceptance is higher then condutance.

$$\underbrace{G_{ij}}\,\underbrace{\sin(\theta_i - \theta_j)} << B_{ij}$$

Both G_{ij} and the sine term are almost negligible

$$\underbrace{Q_i} << \underbrace{B_{ii} V_i^2}$$

$Im(Y_{ii})$

$Q_i^{Gen} - Q_i^{LOAD}$

inherent reactive coupling

The net reactive power injected at a bus is much smaller than the inherent reactive capacity of the transmission system.

Applying these restrictions. we get:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} B' & 0 \\ 0 & B'' \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$

Note that B' and B'' are constant matrizes which means no need to recalculate each iteration is needed.

$$\Delta P = B' \Delta \theta \quad \text{and} \quad \Delta Q = B'' \Delta V$$

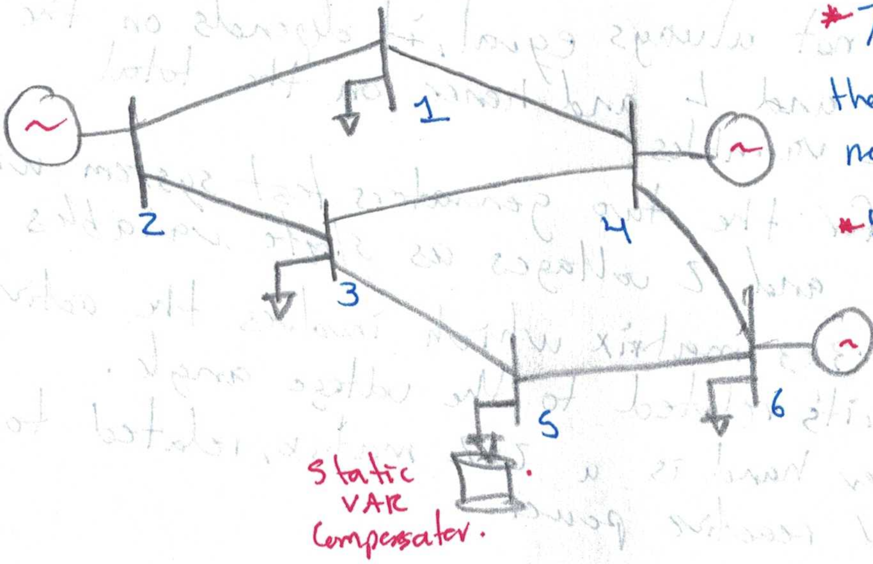
B' and B'' are not always equal, it depends on the dimension of H and L and hence on the total number of state variables.
In this case for the two generators test system we have 3 angles and 2 voltages as state variables so, B' is a 3x3 matrix which involves the active power flow (P), it's related to the voltage angle. B'' in the other hand is a 2x2 matrix, related to voltages and reactive power.

$$B' \in \mathbb{C}^{(n-1)(n-1)}$$
$$B'' \in \mathbb{C}^{(n-n_L-1)(n-n_L-1)}$$

* Summary.

* Decoupled method
  * Coupling submatrices N an J are neglected and H, L are updated each iteration
  * $\Delta P \approx H \Delta \theta$ and $\Delta Q \approx L \Delta V$.
  * High accuracy if coupling is moderate (not high).
* Fast decoupled method.
  * $V \approx 1$ pu, small changes and reactive net injection assumptions must be made to approximate H and L by constant matrices B' and B''
  * B' and B'' are computed once.
  * $\Delta P \approx B' \Delta \theta$ and $\Delta Q \approx B'' \Delta V$
  * High accuracy for transmission networks where approximations held.

* Three generators test system:



* The real part of the impedance is negligible.

* 500 kV, 200-500 km T.L

Bus #	Bus Type	Description	Known Variables	Unknown Variables.
1	PQ	Load.	P, Q	V, θ
2	Slack	Compensator.	V, θ	P, Q
3	PQ	Load.	P, Q	V, θ
4	PV	Generator	V, P	θ, Q
5	PV	SVC, P=0, V fixed	V, P	θ, Q
6	PV	Generator V fixed and load.	V, P	θ, Q

$$\vec{X} = \begin{bmatrix} \theta_1 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ V_1 \\ V_3 \end{bmatrix}$$

equations $= 2N_L + N_G = 2(2) + 3 = 7$.

There will be 5 ΔP and 2 ΔQ equations.

Node 5 is modeled as a PV node with P=0 injection due to the SVC which maintains the bus voltage constant by injecting certain amount of reactive power

Slack = $V_2 = 1.00\,pu \angle 0°$ • Generation Buses 4, 5, 6 ■ No conection

$V_{4,5,6} = 1.00\,pu \angle \theta$

B_{13} 0 No conection

ΔP Equations

$$\Delta P_1 = \cancelto{0}{P^{esp}_{1G}} - P^{esp}_{1L} - V_1\Big[V_1 B_{11}\cancelto{0}{\sin(0)} + \cancelto{1}{V_2} B_{12}\sin(\theta_1 - 0) + V_3 \cancelto{0}{B_{13}\sin}(\theta_1-\theta_3)$$

$$+ \cancelto{1}{V_4} B_{14}\sin(\theta_1-\theta_4) + \cancelto{1}{V_5} B_{15}\cancelto{0}{\sin(\theta_1-\theta_5)} + \cancelto{1}{V_6} B_{16}\cancelto{0}{\sin(\theta_1-\theta_6)}\Big]$$

$$* \Delta P_1 = -P^{esp}_{1L} - V_1\Big[B_{12}\sin(\theta_1) + B_{14}\sin(\theta_1-\theta_4)\Big]$$

$$\Delta P_3 = \cancelto{0}{P^{esp}_{3G}} - P^{esp}_{3L} - V_3\Big[V_1 \cancelto{0}{B_{31}\sin(\theta_3-\theta_1)} + \cancelto{1}{V_2} B_{32}\sin(\theta_3 - 0) + V_3 \cancelto{0}{B_{33}\sin(0)} + \ldots$$

$$\cancelto{1}{V_4} B_{34}\sin(\theta_3-\theta_4) + \cancelto{1}{V_5} B_{35}\sin(\theta_3-\theta_5) + V_6 B_{36}\cancelto{0}{\sin(\theta_3-\theta_6)}\Big]$$

$$* \Delta P_3 = -P^{esp}_{3L} - V_3\Big[B_{32}\sin(\theta_3) + B_{34}\sin(\theta_3-\theta_4 + B_{35}\sin(\theta_3-\theta_5)\Big]$$

$$\Delta P_4 = P^{esp}_{4G} - \cancelto{0}{P^{esp}_{4L}} - \cancelto{1}{V_4}\Big[V_1 B_{41}\sin(\theta_4-\theta_1) + V_2 \cancelto{0}{B_{42}\sin}(\theta_4 - 0) + \ldots$$

$$V_3 B_{43}\sin(\theta_4-\theta_3) + V_4 \cancelto{0}{B_{44}}\sin(0) + V_5 \cancelto{0}{B_{45}\sin}(\theta_3-\theta_5) + \cancelto{1}{V_6} B_{46}\sin(\theta_4-\theta_6)\Big]$$

$$* \Delta P_4 = P^{esp}_{4G} - \Big[V_1 B_{41}\sin(\theta_4-\theta_1) + V_3 B_{43}\sin(\theta_4-\theta_3) + B_{46}\sin(\theta_4-\theta_6)\Big]$$

$$\Delta P_5 = \cancelto{0}{P^{esp}_{5G}} - \cancelto{0}{P^{esp}_{5L}} - \cancelto{1}{V_5}\Big[V_1 \cancelto{0}{B_{51}\sin}(\theta_5-\theta_1) + V_2 \cancelto{0}{B_{52}}\sin(\theta_5 - 0) + V_3 B_{53}\sin(\theta_5-\theta_3) + \ldots$$

$$V_4 \cancelto{0}{B_{54}}\sin(\theta_5-\theta_4) + V_5 \cancelto{0}{B_{55}}\sin(0) + \cancelto{1}{V_6} B_{56}\sin(\theta_5-\theta_6)\Big]$$

$$* \Delta P_5 = -\Big[V_3 B_{53}\sin(\theta_5-\theta_3) + B_{56}\sin(\theta_6-\theta_6)\Big]$$

$$\Delta P_6 = P^{esp}_{6G} - P^{esp}_{6L} - \cancelto{1}{V_6}\Big[V_1 \cancelto{0}{B_{61}}\sin(\theta_6-\theta_1) + V_2 \cancelto{0}{B_{62}}\sin(\theta_6 - 0) + V_3 \cancelto{0}{B_{63}}\sin(\theta_6-\theta_3)$$

$$+ \cancelto{1}{V_4} B_{64}\sin(\theta_6-\theta_4) + \cancelto{1}{V_5} B_{65}\sin(\theta_6-\theta_5) + V_6 \cancelto{0}{B_{66}\sin(0)}\Big]$$

$$*\Delta P_6 = P_{6G}^{esp} - P_{6L}^{esp} - \left[B_{64}\sin(\theta_6-\theta_4) + B_{65}\sin(\theta_6-\theta_5) \right]$$

ΔQ equations

$$\Delta Q_1 = \cancelto{0}{Q_{1G}^{esp}} - Q_{1L}^{esp} + V_1 \left[V_1 B_{11}\cos\cancelto{1}{(0)} + \cancelto{1}{V_2} B_{12}\cos(\theta_1 - 0) + V_3 \cancelto{0}{B_{13}}\cos(\theta_1-\theta_3) \right.$$

$$\left. + \cancelto{1}{V_4} B_{14}\cos(\theta_1-\theta_4) + V_5 \cancelto{0}{B_{15}}\cos(\theta_1-\theta_5) + V_6 \cancelto{0}{B_{16}}\cos(\theta_1-\theta_6) \right]$$

$$*\Delta Q_1 = -Q_{1L}^{esp} + V_1^2 B_{11} + V_1 B_{12}\cos(\theta_1) + V_1 B_{14}\cos(\theta_1-\theta_4)$$

$$\Delta Q_3 = \cancelto{0}{Q_{3G}^{esp}} - Q_{3L}^{esp} + V_3 \left[V_1 \cancelto{0}{B_{31}}\cos(\theta_3-\theta_1) + \cancelto{1}{V_2} B_{32}\cos(\theta_3 - 0) + V_3 B_{33}\cos\cancelto{1}{(0)} \right.$$

$$\left. + \cancelto{1}{V_4} B_{34}\cos(\theta_3-\theta_4) + \cancelto{1}{V_5} B_{35}\cos(\theta_3-\theta_5) + V_6 \cancelto{0}{B_{36}}\cos(\theta_3-\theta_6) \right]$$

$$*\Delta Q_3 = -Q_{3L}^{esp} + V_3 B_{32}\cos(\theta_3) + V_3^2 B_{33} + V_3 B_{34}\cos(\theta_3-\theta_4)$$

$$+ V_3 B_{35}\cos(\theta_3-\theta_5)$$

$$Y_{BUS} = \begin{bmatrix} Y_{12}+Y_{14}+Y_{1L} & -Y_{12} & 0 & -Y_{14} & 0 & 0 \\ -Y_{21} & Y_{21}+Y_{23} & -Y_{23} & 0 & 0 & 0 \\ 0 & -Y_{32} & Y_{32}+Y_{34}+Y_{35} & -Y_{34} & -Y_{35} & 0 \\ -Y_{41} & 0 & -Y_{43} & Y_{46}+Y_{43}+Y_{41} & 0 & -Y_{46} \\ 0 & 0 & -Y_{53} & 0 & Y_{56}+Y_{53}+Y_{5svc} & -Y_{56} \\ 0 & 0 & 0 & -Y_{64} & -Y_{65} & Y_{64}+Y_{65} \end{bmatrix}$$

The Jacobian Matrix will be a 7x7 matrix. The calculation for each element is as follows:

- $\frac{\partial \Delta P_1}{\partial \theta_1} = -V_1 B_{12} \cos(\theta_1) - V_1 B_{14} \cos(\theta_1 - \theta_4)$.

- $\frac{\partial \Delta P_1}{\partial \theta_3} = 0$ / The change in angle θ_3 has no effect in the active power of bus 1.

- $\frac{\partial \Delta P_1}{\partial \theta_4} = V_1 B_{14} \cos(\theta_1 - \theta_4)$

- $\frac{\partial \Delta P_1}{\partial \theta_5} = 0$; - $\frac{\partial \Delta P_1}{\partial \theta_6} = 0$

- $\frac{\partial \Delta P_1}{\partial V_1} = -B_{12} \sin(\theta_1) - B_{14} \sin(\theta_1 - \theta_4)$

- $\frac{\partial \Delta P_1}{\partial V_2} = 0$ // The voltage V_2 is fixed then there is no active power difference due to change in V_2, because it doesnt change.

- $\frac{\partial \Delta P_3}{\partial \theta_1} = 0$; - $\frac{\partial \Delta P_3}{\partial \theta_2} = 0$.

- $\frac{\partial \Delta P_3}{\partial \theta_3} = -V_3 B_{32} \cos(\theta_3) - V_3 B_{34} \cos(\theta_3 - \theta_4) - V_3 B_{35} \cos(\theta_3 - \theta_5)$

- $\frac{\partial \Delta P_3}{\partial \theta_4} = V_3 B_{34} \cos(\theta_3 - \theta_4)$

- $\frac{\partial \Delta P_3}{\partial \theta_5} = V_3 B_{35} \cos(\theta_3 - \theta_5)$; - $\frac{\partial \Delta P_3}{\partial \theta_6} = 0$

$$\frac{\partial \Delta P_3}{\partial V_1} = 0 \ ; \ \frac{\partial \Delta P_3}{\partial V_2} = 0$$ V_1 doesn't change. its fixed

$$\frac{\partial \Delta P_4}{\partial \theta_1} = V_1 B_{41} \cos(\theta_4 - \theta_1)$$

$$\frac{\partial \Delta P_4}{\partial \theta_3} = V_3 B_{43} \cos(\theta_4 - \theta_3)$$

$$\frac{\partial \Delta P_4}{\partial \theta_4} = -V_1 B_{41} \cos(\theta_4 - \theta_1) - V_3 B_{43} \sin(\theta_4 - \theta_3) - B_{46} \cos(\theta_4 - \theta_6)$$

$$\frac{\partial \Delta P_4}{\partial \theta_5} = 0 \ ; \ \frac{\partial \Delta P_4}{\partial \theta_6} = B_{46} \cos(\theta_4 - \theta_6)$$

$$\frac{\partial \Delta P_4}{\partial V_1} = -B_{41} \sin(\theta_4 - \theta_1) \ ; \ \frac{\partial \Delta P_4}{\partial V_2} = 0 .$$

$$\frac{\partial \Delta P_5}{\partial \theta_1} = 0 \ , \ \frac{\partial \Delta P_5}{\partial \theta_3} = V_3 B_{53} \cos(\theta_5 - \theta_3)$$

$$\frac{\partial \Delta P_5}{\partial \theta_4} = 0 \ ; \ \frac{\partial \Delta P_5}{\partial \theta_5} = -V_3 B_{53} \cos(\theta_5 - \theta_3) - B_{56} \cos(\theta_5 - \theta_6)$$

$$\frac{\partial \Delta P_5}{\partial \theta_6} = B_{56} \cos(\theta_5 - \theta_6) \ ; \ \frac{\partial \Delta P_5}{\partial V_1} = 0$$

$$\frac{\partial \Delta P_5}{\partial V_3} = -B_{53} \sin(\theta_5 - \theta_3) .$$

$$\frac{\partial \Delta P_6}{\partial \theta_1} = 0 \; ; \; \frac{\partial \Delta P_6}{\partial \theta_3} = 0 \; ; \; \frac{\partial \Delta P_6}{\partial \theta_4} = B_{64} \cos(\theta_6 - \theta_4)$$

$$\frac{\partial \Delta P_6}{\partial \theta_5} = B_{65} \cos(\theta_6 - \theta_5)$$

$$\frac{\partial \Delta P_6}{\partial \theta_6} = -B_{64} \cos(\theta_6 - \theta_4) - B_{65} \cos(\theta_6 - \theta_5)$$

$$\frac{\partial \Delta P_6}{\partial V_1} = 0 \; ; \; \frac{\partial \Delta P_6}{\partial V_3} = 0$$

$$\frac{\partial Q_1}{\partial \theta_1} = -V_1 B_{12} \sin(\theta_1) - V_1 B_{14} \sin(\theta_1 - \theta_4)$$

$$\frac{\partial Q_1}{\partial \theta_3} = 0 \; ; \; \frac{\partial Q_1}{\partial \theta_4} = V_1 B_{14} \sin(\theta_1 - \theta_4)$$

$$\frac{\partial Q_1}{\partial \theta_5} = 0 \; ; \; \frac{\partial Q_1}{\partial \theta_6} = 0$$

$$\frac{\partial Q_1}{\partial V_1} = 2 V_1 B_{11} + B_{12} \cos(\theta_1) + B_{14} \cos(\theta_1 - \theta_4)$$

$$\frac{\partial Q_1}{\partial V_2} = 0.$$

$$\frac{\partial Q_3}{\partial \theta_1} = 0 \; ; \; \frac{\partial Q_3}{\partial \theta_2} = 0$$

$$\frac{\partial Q_3}{\partial \theta_3} = -V_3 B_{32} \sin(\theta_3) - V_3 B_{34} \sin(\theta_3 - \theta_4)$$

$$\frac{\partial Q_3}{\partial \theta_4} = V_3 B_{34} \sin(\theta_3 - \theta_4)$$

$$\frac{\partial Q_3}{\partial \theta_5} = 0 \; ; \; \frac{\partial Q_3}{\partial \theta_6} = 0.$$

$$\frac{\partial Q_3}{\partial V_1} = 0;$$

$$\frac{\partial Q_3}{\partial V_3} = B_{32} \cos(\theta_3) + 2V_3 B_{33} + B_{34} \cos(\theta_3 - \theta_4).$$

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_3 \\ \Delta\theta_4 \\ \Delta\theta_5 \\ \Delta\theta_6 \\ \Delta V_1 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \cdots & \frac{\partial P_1}{\partial V_3} \\ \vdots & & \vdots \\ \frac{\partial Q_3}{\partial \theta_1} & \cdots & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_1 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta P_6 \\ \Delta Q_1 \\ \Delta Q_3 \end{bmatrix}$$