LC Filter Design for Harmonic Mitigation in a Switched Circuit: Analysis and Simulation

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***Abstract*—This document studies a switched circuit, focusing on harmonic characterization and the design of an LC filter to achieve a maximum Total Harmonic Distortion (THD) of 5%. A Fourier-based approach is used to identify how the switching frequency and duty cycle influence both the average voltage and harmonic distribution. The system's transfer function is derived from the state-space mode. PSIM and MATLAB simulations validate the model.**

I. Introduction

Filters play a critical role in power converters by mitigating unwanted harmonic content, reducing electromagnetic interference, and ensuring power quality and efficiency. In converter applications, LC filters are widely used to smooth the output waveform. The case study circuit is shown in (Fig. 1) The load specifications, set at 1 kW power and 24 V.

In this study, a sweep of duty cycles and switching frequencies is performed and the Fourier series method is applied to decompose and analyze the signal harmonic components. For further analysis, a switching frequency of 100 kHz and a duty cycle of 0.5 are chosen.

Time-domain waveforms from PSIM and MATLAB are compared to validate the models and confirm that the designed filter effectively attenuates unwanted harmonics. The following sections detail the methodology, analysis, simulation results, and conclusions.

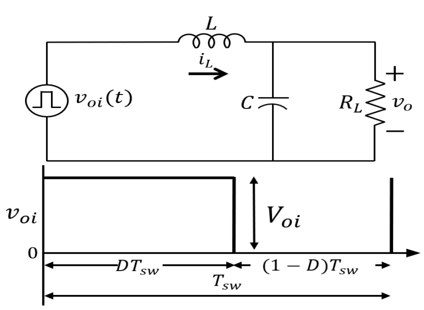

Fig. 1. Case Study Circuit.

II. Methodology and Analytical Approach

A. Fourier Series Analysis of a Periodic Switching Signal

The switching source can be described as a pulse that starts at zero with a duty cycle D. The source has a switching frequency f_{sw} (hence a switching period $T_{sw} = 1/f_{sw}$) and an amplitude V_{oi} (48V). The waveform is high (with amplitude V_{oi}) during a time interval $D\,T_{sw}$ and low (zero) for the remaining part of the period. The Fourier coefficients (harmonic spectrum) are defined in general as follows:

The DC (average) component:

$$a_0 = \frac{1}{T_{sw}} \int_0^{t_{on}} f(t)\,dt = V_{oi}\,D \tag{1}$$

The cosine (even) coefficients:

$$a_n = \frac{2}{T_{sw}} \int_0^{t_{on}} V_{oi} \cos\bigl(n\omega_o t\bigr)\,dt = \frac{V_{oi}}{\pi\,n} \sin\bigl(2\pi\,n\,D\bigr) \tag{2}$$

The sine (odd) coefficients:

$$b_n = \frac{2}{T_{sw}} \int_0^{t_{on}} V_{oi} \sin\bigl(n\omega_o t\bigr)\,dt = \frac{V_{oi}}{\pi\,n}\Bigl[1-\cos\bigl(2\pi\,n\,D\bigr)\Bigr] \tag{3}$$

B. Duty Cycle and Switching Frequency Sweep

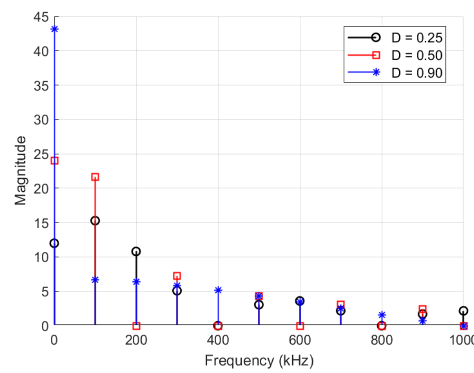

Fig. 2. Source harmonic content with 100kH switching frequency.

Parameter	Observed Effects
Duty Cycle (D)	**0.25:** Lower average value, moderate harmonics, spectrum asymmetry. 87% THD. **0.5:** Cancellation of even harmonics, balanced spectrum, 43% THD. **0.9:** Higher average value, increased higher-order harmonics, spectrum, asymmetry 177% THD.
Switching Frequency (f_{sw})	**25 kHz:** Harmonics concentrated in lower frequencies 43% THD for all frequencies. **50 kHz:** Intermediate distribution. **100 kHz:** Harmonics shifted to higher frequencies

TABLE I
Summary of the Duty cycle and switching frequency effects.

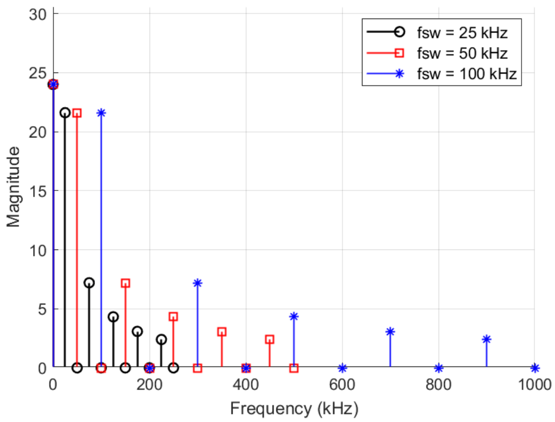

Fig. 3. Source harmonic content with 0.5 duty cycle .

C. LC Selection Criteria

The LC values were chosen to ensure a resonant (cutoff) frequency at approximately one-fifth of the switching frequency ($f_{sw}/5$) while keeping the impedance $Z_0 = \sqrt{L/C}$ comparable, but not excessively higher than the load resistance. A quality factor less than 1 was targeted to minimize transient oscillations. Specifically, an inductance of 65 μH was arbitrarily selected. Using the resonant frequency criterion

$$f_o = \frac{1}{2\pi\sqrt{LC}}, \tag{4}$$

The required capacitance was calculated as $C \approx 970\,\mu$F. The load has a resistance of $0.576\,\Omega$ and draws 41.67 A. These values ensure the LC filter can effectively attenuate harmonics above the switching frequency, maintaining an acceptable level of THD.

D. State-Space Model, Transfer Function, and Bode Plot

For the circuit with the LC filter connected to a load, the state-space representation is:

$$\begin{bmatrix} \dot{v}_L \\ \dot{i}_C \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} v_{oi}. \tag{5}$$

The output equation is:

$$v_R = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} i_L \\ v_C \end{bmatrix} \tag{6}$$

From this model, the transfer function between the input voltage and the output voltage across the resistor is:

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}. \tag{7}$$

A Bode plot (Fig. 4) of $H(s)$ shows that around 20 kHz, the magnitude reaches -3 dB and the phase approaches $-90°$. Beyond this frequency, the filter's attenuation increases significantly, confirming that high-frequency harmonics are effectively suppressed. This behavior aligns with design expectations, ensuring a reduced harmonic content.

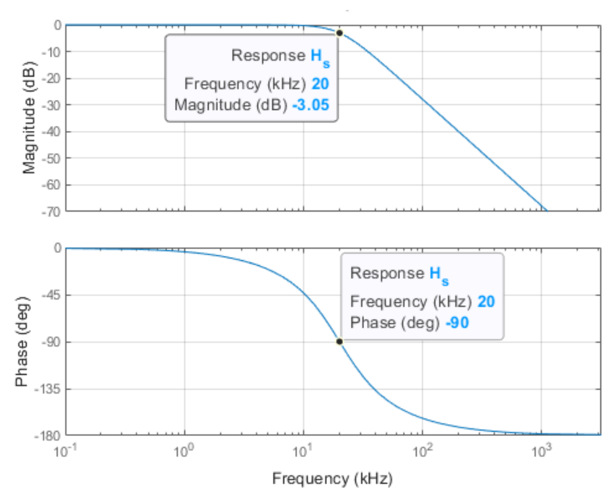

Fig. 4. Bode plot of the transfer function $H(s)$.

III. Simulation Results

The simulation was performed with a time step of 1×10^{-8} s over a period of 3 ms (300 switching cycles) in both PSIM and MATLAB.

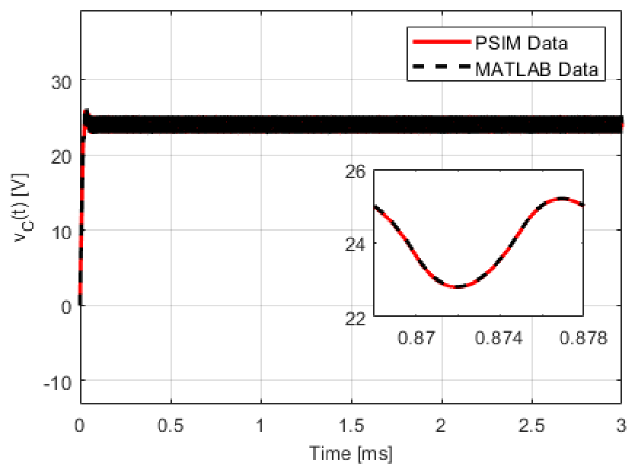

Fig. 5. Resistor voltage comparison using PSIM and MATLAB.

IV. Conclusions

- A 50% duty cycle cancels even harmonics and higher frequencies shift harmonic content upward.
- The LC values were selected to achieve a cutoff frequency of $f_{sw}/5$, ensuring effective attenuation of harmonics.
- Bode plot confirms the state-space model and circuit dynamics, PSIM confirms the accuracy of the analytical model.
- Resistor THD of 4.7% and Inductor THD of 11.7%.
- The designed LC filter successfully maintains an acceptable level of THD, thereby demonstrating the viability of the proposed approach in power electronics applications.

References

[1] R. W. Erickson y D. Maksimovic, Fundamentals of Power Electronics. Springer, 2020.