EX # 1: Power system stability

1.- The SG in Fig.1 delivers 0.8 p.u real power at 1.05 p.u terminal voltage. All system's reactance's are on a common system base. The system is illustrated at Fig. 1. Determine:

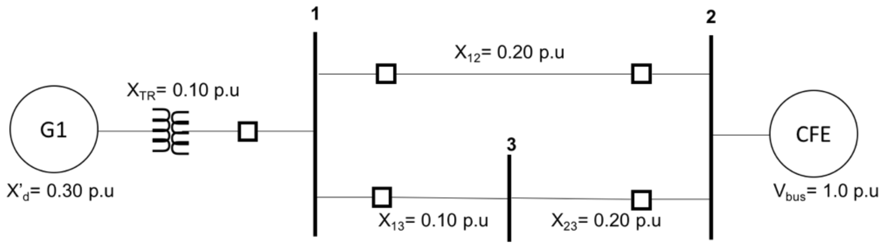


Fig. 1: Single-line diagram of a three-phase, 60 Hz (SG)

a. Reactive power output of the generator.
b. The generator internal voltage
c. An equation for electrical power delivered by the generator versus power angle δ.

2.- The SG in Fig. 1 is initially operating in the steady-state condition given previously at example 1. When a three-phase to ground bolted short circuit occurs at node #3. Determine:

a. The Thevenin equivalent of the faulted network of Fig. 1.
b. The equation for the electrical power delivered by the generator versus power angle δ during the fault.

3.- The inertia constant of SG is $H = 3.0$ per unit-seconds, $\omega_{p.u} = 1.0$, $p_m = 1.0\ p.u$ and the classic model is considered in the swing equation. Three-phase to ground bolted short circuits occur on line 1-2 at 50% of the transmission line, shown at arrow in Fig. 3. Four cycles later, the fault extinguishes by itself opening the line 1-2 indicated (circuit breakers in red) at Fig. 3. Determine:

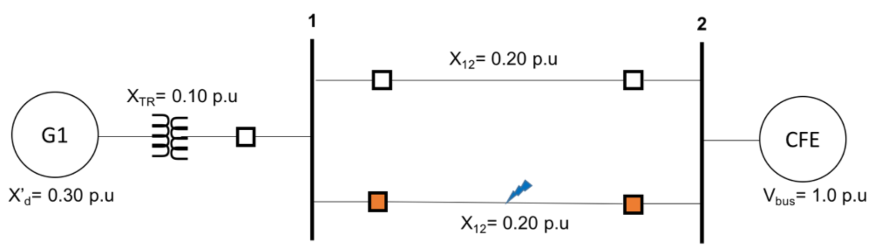


Fig. 3: Single-line diagram of a three-phase, 60 Hz (SG), 3-fault at transmission line

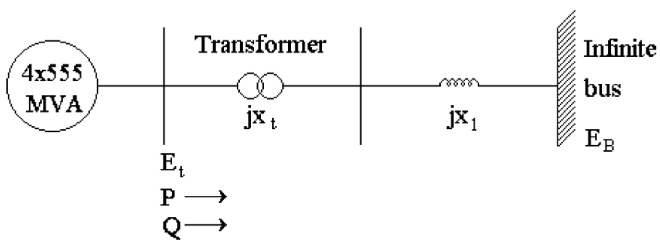
a. The Thevenin equivalent of the faulted network of Fig. 3.
b. Use the Equal-are criterion to determine whether stability is or not maintained.
c. Calculate the critical clearing angle.
d. Considering the case study previously described at the top paragraph. Assemble the power system display at Fig. 3 into the PowerWorld Simulator and compute the power flow solution and transient response. (plot speed, angle, power of the generator).

e. Analysis of the results

4.- A three-phase, 50 Hz, synchronous generator is connected to an infinite bus. The maximum real power that can be transferred to the infinite bus is 1 per unit. The mechanical input to the generator is 0.8 per unit. The inertia constant of the generator is 5 seconds.

a) Find the natural frequency of oscillations of the system and comment on stability.
b) If the damping coefficient is taken as 0.1 then find the natural frequency, damped frequency and damping ratio of the system oscillations.
c) For a small disturbance of $\Delta\delta = 10^0$, find the change in internal angle and speed of the generator with respect to time. Find the solution for t=0.0 and 1.0 using the analytic solution.

5.- From the power system described in the next figure:

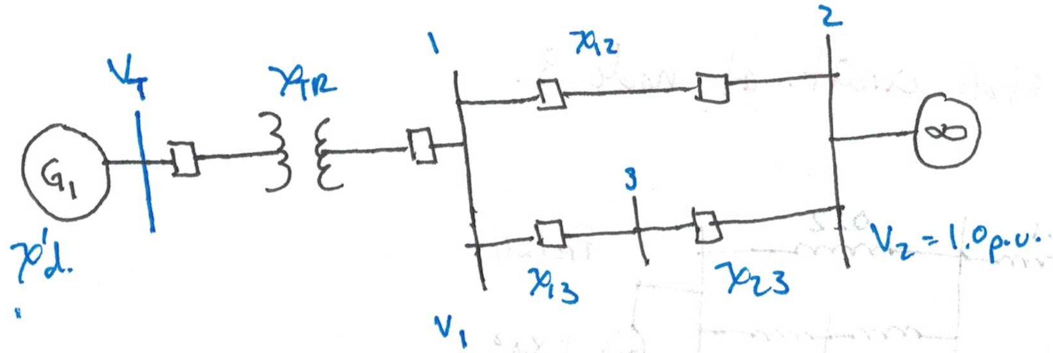


Considere el modelo clásico:

$$\frac{\partial\delta}{\partial t} = \omega_o - \omega$$

$$\frac{\partial\omega}{\partial t} = \frac{1}{2H}\left(P_m - D\omega - P_{max}sin(\delta)\right)$$

a) Symbolically obtain the plant matrix A.
b) Symbolically obtain the Hessian matrix H2.
c) Symbolically obtain the matrix H3.
d) Obtain the third-order linearized model obtained from the Taylor series expansion.



P = 0.8 p.u at terminals.

a) Reactive power

$$P = \frac{|V_1||V_2|}{x_{eq}} \sin(\theta_1 - \theta_2) ; \quad \theta_1 = \sin^{-1}\left(\frac{P \cdot x_{eq}}{|V_1||V_2|}\right), \quad x_{eq} = 0.22 \text{ p.u.}$$

$\theta_2 = 0$

$x_{eq} = x_{TR} + x_{12} \| (x_{13} + x_{23})$

$$\theta_1 = \sin^{-1}\left(\frac{0.8(0.22)}{(1.05)(1.0)}\right) = 9.65° \qquad V_T = 1.05 \angle 9.65°$$

$$S_T = V_T I_T^* ; \quad I_T = \frac{V}{Z} = \frac{V_T - V_2}{j x_{eq}} = \frac{(1.05 \angle 9.65°) - 1\angle 0°}{0.22 \angle 90°}$$

$$I_T = 0.8158 \angle -11.29°.$$

$$S_T = (1.05 \angle 9.65)(0.8158 \angle 11.29°) = \underbrace{0.8}_{P} + \underbrace{j0.3061}_{Q}$$

$$\boxed{Q_T = 0.3061 \text{ p.u.}}$$

b) Internal Voltage

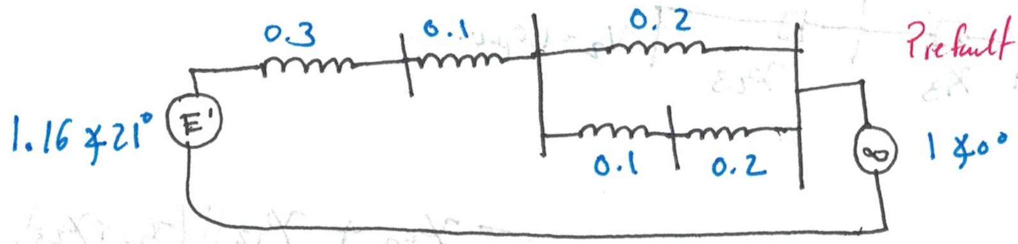
$$E' = V_T + x'_d I_T = (1.05 \angle 9.65) + (0.3j)(0.8158 \angle -11.29°)$$

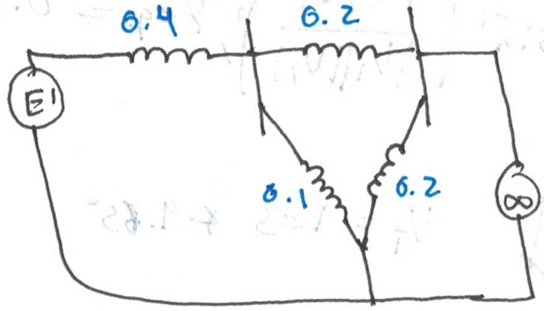
$$\boxed{E' = 1.1602 \angle \underbrace{21.0124°}_{\delta}}$$

c) Electric Power function with δ as parameter.

$$P_e = \frac{|E||V_2|}{x_T} \sin(\delta - \underset{0}{\theta_2}) = \frac{|1.16||1|}{0.52} \sin(\delta) = \boxed{2.2308 \sin(\delta)}$$

② 3 phase bolted short circuit at node 3.

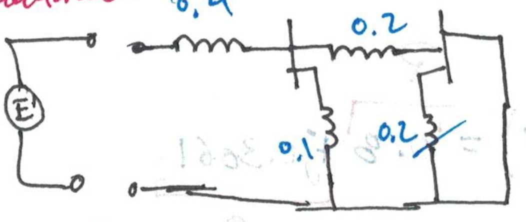


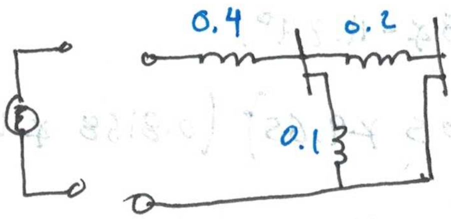


$\Delta \to Y \quad , \quad Y \to \Delta$

Since Thevenin is required:

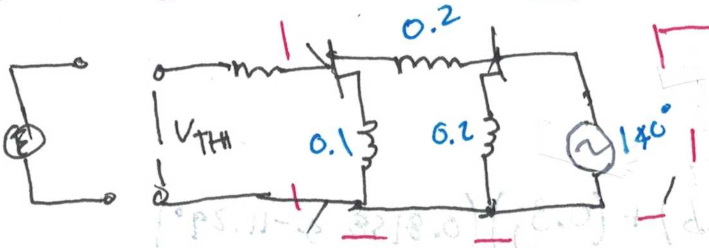
Thevenins reactance.





$X_{TH} = 0.4667$ p.u.

Thevenins voltage

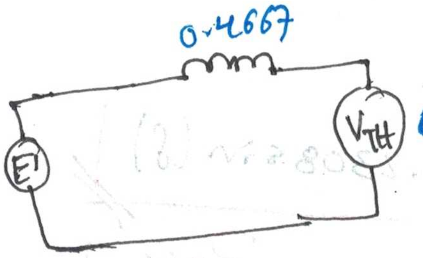


$I_{12} = I_{13}$

$$I_{13} = \frac{V_2}{j(X_{12}+X_{13})} = \frac{1}{j\,0.3}$$

$I_{13} = 3.33 \angle -90°$

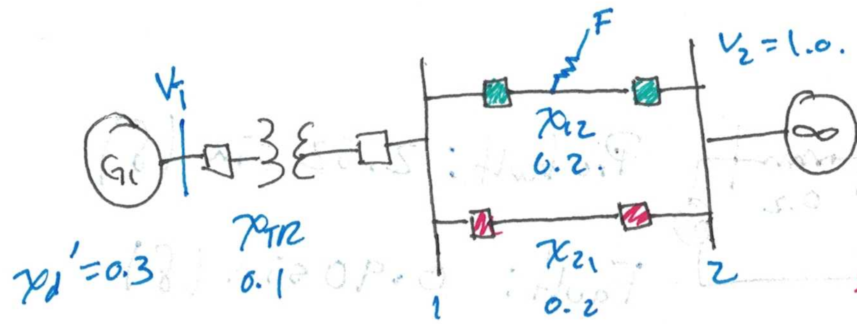
$V_{13} = I_{13} \cdot X_{13} = 3.33 \angle -90° \cdot j0.1 = 0.333 \angle 0° = V_{TH}$



$$P_{eF} = \frac{|E'||V_{TH}|}{0.4667}\sin(\delta - \theta_{TH})$$

$$P_{eF} = \frac{(1.16)0.333}{0.4667}\sin(\delta) = \boxed{0.8277 \sin(\delta)}$$

(3)



P.u.s $H = 3.0$, $\omega = 1.0$ p.u.

$p_m = 1.0$

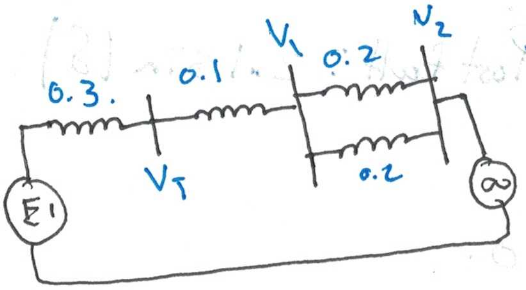
50% line, 3ϕ fault.

4 cycles cleared

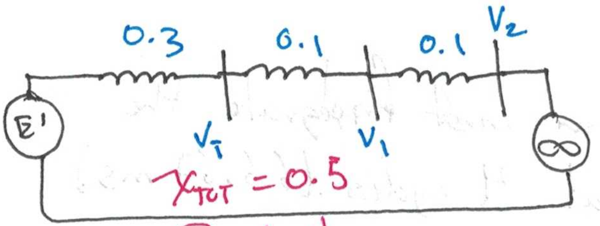
Circuit Breakers do open.

OPEN

CLOSED



a)



Pre fault.

steady state

$$E' = V_2 + jX_{TOT} I_2 \; ; \; P_e = P_m = 1.0 \text{ p.u.}$$

$$P_e = \frac{|E'|\,|V_2|}{X_T} \sin(\delta_0) \; ; \; S = VI^*$$

Since no power factor is given, a value of 0.95 lagging is proposed. $|P_e| = |S|\cos(\theta) \Rightarrow 1 = S(0.95)$, $S = 1.0526$.

correct but wrong way, prone to errors.

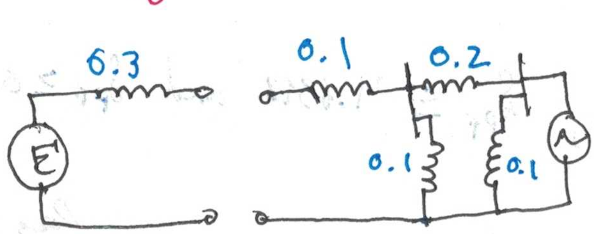
$$I_2 = \left(\frac{S}{V_2}\right)^* = \left(\frac{1.0526}{1\angle 0^\circ}\right)^* = 1.0526 \angle \underbrace{-18.19^\circ}_{0.95 \text{ pf Lagging}}$$

$$E' = 1\angle 0^\circ + j0.5(1.05 \angle -18.19^\circ) = 1.2663 \angle 23.19^\circ$$

correct way: $\theta = \cos^{-1}(0.95) = 18.19^\circ$, $Q = P\tan(\theta) = 1\tan(18.19) = 0.3287$ p.u.

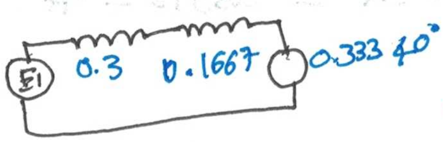
$$S = 1 + j0.3287 \; ; \; I_2 = \left(\frac{1 + 0.3287j}{1\angle 0^\circ}\right)^* = 1.0526\angle -18.19^\circ.$$

$$\delta_0 = 23.19^\circ \quad |E'| = 1.2663.$$

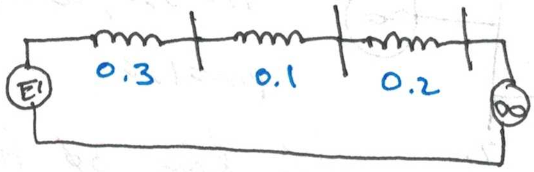


$Z_{TH} = 0.1667$; $I = \frac{1\angle 0^\circ}{0.2 + 0.1} = 3.333$

$V_{TH} = I(0.1) = 0.333\angle 0^\circ$



a)



Pre fault: $2.52 \sin(\delta)$

Fault: $0.90 \sin(\delta)$

Post fault: $2.1 \sin(\delta)$

b) $\int_{\delta_0}^{\delta_f} (P_m - P_e)\, d\delta + \int_{\delta_f}^{\delta_{pf}} (P_m - P_e)\, d\delta = 0.$

$w_r(0) = 1$ p.u. or $377 \frac{rad}{s}$.

$\delta_0 = 0.4047$ rad. To obtain δ_f we must integrate the swing equation numerically to know δ_f at 4 cycles (66.67 ms)

Using modified Euler (Heuns), python script:

After 4 cycles. $\boxed{\delta_F = 0.4931 \text{ rad}}$ $\overline{}$ 28.25°, $w_r = 379.69 \frac{rad}{s}$.

$$\int_{0.4047}^{0.4931} (1 - 0.9\sin(\delta))\, d\delta + \int_{0.4931}^{\delta_{PF}} (1 - 2.1\sin(\delta))\, d\delta = 0$$

$$0.05388. + (\delta_{pf} + 2.1\cos(\delta_{pf})) - (2.3429) = 0$$

$$\delta_{pf} + 2.1\cos(\delta_{pf}) - 2.2890 = 0$$

Using Newton Raphson to solve the above equation that satisfies equal area criterion.: Our inital guess $\delta_{pf_{old}} = 0.$ rad.

There is an interesting fact: note that the clearing angle at 4 cycles is 0.4931 rad, and the steady state for the post fault case is

$$\delta_S = \sin^{-1}\left(\frac{1}{2.1}\right) = 0.4963 \text{ rad} \quad 28.43^\circ$$

$$\delta_{S} - \delta_F = 0.0032 \text{ rad.}$$

The machine is still accelerating because $\delta_F < \delta_S$, so we must account for that part in the equal area criterion.

$$\underbrace{\int_{\delta_0}^{\delta_F} (1-0.95\sin(\delta))\,d\delta + \int_{\delta_F}^{\delta_S}(1-2.1\sin(\delta))\,d\delta}_{\text{acceleration}} + \underbrace{\int_{\delta_S}^{\delta_{PF}} (1-2.1\sin(\delta))\,d\delta}_{\text{deceleration}} = 0$$

$$0.05388 + 9.56\times10^{-6} + \left(\delta_{PF} + 2.1\cos(\delta_{PF})\right) - (2.3429) = 0.$$

$$\delta_{PF} + 2.1\cos(\delta_{PF}) - 2.2891 = 0 = \Delta$$

The small accelerating portion is no very important in this case.

To chose the starting point for Newton Raphson we can check the plot of the function to see it's roots or directly calculate

$$\delta_{n+1} = \delta_n - \frac{f(\delta_n)}{f'(\delta_n)}$$

42.62°

$$\boxed{\delta_{PF} \approx 0.744 \text{ rad}}$$

Maximum reaching angle that satisfies equal area criterion.

c) $\delta_{MAX} = \pi - \delta_s = 2.6453$ rad., and $\delta_s = 0.4963$ rad.

$$\int_{\delta_s}^{\delta_{cR}} (P_m - P_{MAX}\sin(\delta))\,d\delta + \int_{\delta_{cR}}^{\delta_{MAX}} (P_m - P_{MAX}\sin(\delta))\,d\delta = 0.$$

$$\Big[\big(\delta_{cR} + 0.9\cos(\delta_{cR})\big) - \big(0.4963 + 0.9\cos(0.4963)\big)\Big] + \Big[\big(2.6453 + 2.1\cos(2.6453)\big)$$

$$- \big(\delta_{cR} + 2.1\cos(\delta_{cR})\big)\Big] = 0$$

$$(\cancel{\delta}_{cR} + 0.9\cos(\delta_{cR})) - 1.2859 + 0.7987 - (\cancel{\delta}_{cR} + 2.1\cos(\delta_{cR})) = 0$$

$$-1.2\cos(\delta_{cR}) - 0.4872 = 0$$

$$\delta_{cR} = \cos^{-1}\left(\frac{0.4872}{-1.2}\right) = 1.9889 \text{ rad.}$$

Since the equal area criterion is satisfied within the margins (conditions) imposed (δ_{MAX}), the system will stabilize at 2.6453 rad. if the fault is cleared at 1.9889 rad.

(4) SMIB system, $H = 5s$, $50\,Hz$, $P_m = 0.8$.

$x_1 = \delta$

$$\dot{x}_1 = \frac{d\delta}{dt} = \omega_r - \omega_s = \dot{\delta} = x_2$$

$$\dot{x}_2 = \frac{d\dot{\delta}}{dt} = \frac{\omega_s}{2H}\left(P_m - \underbrace{P_e}_{\frac{|E'||V|}{x}\sin(\delta)} - D \cdot x_2\right) = W$$

$\dot{x} = Ax + Bu$; $y = Cx + Du$. Since we're interested on the autonomous system, matrices B and D are not considered, neither matrix C. since we're not interested in using the output for any feedback loops.

Non linear model

$$\dot{x} = f(x, u)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{\omega_s}{2H}\left(P_m - \frac{|E'||V|}{x_{Line}}\sin(x_1) - Dx_2\right) \end{bmatrix}$$

For the linearized model; set $\delta = \delta_0 + \Delta\delta$, $\Delta\delta = \delta - \delta_0$ and also $\omega_r = \omega_s + \Delta\omega_r$, $\Delta\omega_r = \omega_r - \omega_s$. $\Delta P_m = P_m - P_{m0}$ and $\Delta E' = E' - E'_0$, $\Delta V = V - V_0$.

$$\dot{x} = f(x_0 + \Delta x, u_0 + \Delta u)$$

Taylor Series linear Terms.

$$f(x_0 + \Delta x, u_0 + \Delta u) = f(x_0, u_0) + \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \dots + \frac{\partial f}{\partial x_n}\Delta x_n + \frac{\partial f}{\partial u_1}\Delta u_1 + \dots + \frac{\partial f}{\partial u_n}\Delta$$

linearized. $\Delta\dot{x} = A\Delta x + B\Delta u$; $\Delta y = C\Delta x + D\Delta u$.

Matrices A, B, C, D are constructed by using the Jacobian.
For our case:

$$\Delta\dot{x} = A\,\Delta x.$$

$$A = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\omega_s}{2H}\underbrace{\left(\overbrace{\frac{|E''||V|}{X_{line}}}^{P_{MAX}}\cos(\delta_0)\right)}_{K_s} & -\frac{\omega_s}{2H}D \end{bmatrix} \quad a)$$

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\omega_s}{2H}K_s & -\frac{\omega_s}{2H}D \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$\omega_s = 2\pi f_s.$
$\omega_s = 2\pi(50\,Hz)$
$P_{MAX} = 1.0\ p.u.$
$\omega_s = 100\pi$
$H = 5$
$\frac{\omega_s}{2H} = \frac{100\pi}{2(5)} = 10\pi$

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10\pi\cos(\delta_0) & -10\pi\cdot D \end{bmatrix}\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max}}\right) =$
$\cos(\delta_0) = 0.6$

$$\det(A - I\lambda) = \det\begin{bmatrix} -\lambda & 1 \\ -10\pi\cos(\delta_0) & -\lambda - 10\pi\cdot D \end{bmatrix} = \lambda^2 + \underline{10\pi\cdot D}\,\lambda + 10\pi\cos(\delta_0) = 0$$

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

$$\omega_n = \sqrt{10\pi\cos(\delta_0)} = 5.60\sqrt{\underbrace{\cos(\delta_0)}_{0.6}}. = \sqrt{\frac{\omega_s}{2H}K_s} = 4.3377\ \frac{rad}{s}.$$

$$\boxed{\omega_n = 0.6966\ Hz} \qquad 10\pi\cdot D = 2\zeta\omega_n,\ (10\pi)(0.1) = 2\zeta(4.3377).$$

$$\zeta = \frac{\pi}{2(4.3377)} = 0.3621 \times 100\% = 36.21\% = \frac{\omega_s D}{4H\omega_n}$$

b) $\lambda^2 + 10\pi D\lambda + 10\pi \cos(\delta_0) = 0$

$\lambda^2 + \pi\lambda + 6\pi = 0.$

$\lambda_1 = -1.5708 + 4.047j$
$\lambda_2 = -1.5708 - \underbrace{4.047j}_{\omega_d}$ $\Big\}$ $-\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$,

$\boxed{\omega_d = 0.6441 \text{ Hz}}$

The damped frequency is little less then the natural frequency. The difference is 0.0525 Hz.

c) Right eigenvectors. $(A - \lambda I)\phi = 0$, for mode λ_1

$$\begin{bmatrix} -\lambda_1 & 1 \\ -6\pi & -\lambda_1 - \pi \end{bmatrix}\begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = 0$$

$-\lambda\phi_{11} + \phi_{21} = 0$,

$-6\pi\phi_{11} - (\lambda+\pi)\phi_{21} = 0$. # These are not linearly independent.

Let $\phi_{21} = 1.0$, then: $-\lambda\phi_{11} = -1$, $\phi_{11} = \frac{1}{\lambda} = -0.0834 - 0.2147j$

$\phi_{12} = -0.0834 + 0.2147j$

Time response: $\Delta x(t) = \sum_{i=1}^{n} \phi_i c_i e^{\lambda_i t} = \sum_{i=1}^{2} \phi_i c_i e^{\lambda_i t}$ $c_i = \Psi_i \Delta x(0)$ Left eigenvectors $\Psi = \phi^{-1}$

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$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ c_2 e^{\lambda_2 t} \end{bmatrix} = \underbrace{\phi_{11} c_1 e^{\lambda_1 t} + \phi_{12} c_2 e^{\lambda_2 t}}_{x_1} + \underbrace{\overset{(1)}{\phi_{21}} c_1 e^{\lambda_1 t} + \overset{(1)}{\phi_{22}} c_2 e^{\lambda_2 t}}_{x_2}$$

$$\Psi = \phi^{-1} = \begin{bmatrix} 2.3288j & 0.5+0.1942j \\ -2.3288j & 0.5-0.1942j \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{x}_1(0) \\ \Delta \dot{x}_2(0) \end{bmatrix} \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad \Delta \dot{x}_1(0) = 10° = 0.1745 \text{ rad}, \quad \Delta \dot{x}_2(0) = 0.$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} 0.1745 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \Psi_{11}(0.1745) + \Psi_{21}(0.1745)$$

$c_1 = 0.4064j$, $c_2 = -0.4064j$

$$\Delta\delta(t) = \Delta x_1(t) = \phi_{11}(0.4064j) e^{(-1.5708+4.04j)t} + \phi_{12}(-0.4064j) e^{(-1.5708-4.04j)t}$$

$$\Delta\delta(t) = \Delta x_1(t) = e^{-1.5708t}\left[0.1745 \cos(4.047t) + 0.0677 \sin(4.047t)\right]$$

$$\Delta w(t) = -0.8128\, e^{-1.5708t} \sin(4.07t)$$

$t=0$: $x_1(0) = 0.174508$ rad $\approx 10°$, $x_2(0) = 0$

$t = 1s$: $x_1(0) = -0.0334$ rad $\approx -1.918°$, $x_2(0) = 0.1329 \frac{\text{rad}}{s}$

Consider the following SMIB model:

$$x = \begin{bmatrix} \delta \\ \omega \end{bmatrix} ; \dot{\delta} = \omega , \dot{\omega} = \frac{1}{2H}\left(P_m - D\omega - P_{MAX}\sin(\delta)\right) = f_2(\delta, \omega)$$

$f_1(\delta, \omega) = \omega$. Then $f(x) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$.

a)

An equilibrium point (δ_0, ω_0) ; $\omega_0 = 0$ and $\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{MAX}}\right)$

$\Delta\delta = \delta - \delta_0$, $\Delta\omega = \omega - \omega_0$, The goal is to expand $f(x)$ and write everything in deviations to get the dynamics.

From the previous problem it can be easily seen that the plant matrix A is:

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{P_{MAX}}{2H}\cos(\delta_0) & -\frac{D}{2H} \end{bmatrix} = J(\delta_0, \omega_0)$$

For a vector field $\mathbb{R}^n \to \mathbb{R}^m$, The Jacobian matrix of first partial derivatives is

$$J(x) = \left[\frac{\partial f_i}{\partial x_j}(x)\right]_{i,j}$$

$$\Delta\dot{x} \simeq A\,\Delta x .$$

b) Hessian (H_2)

The Hessian $\nabla^2 f$ is a $n \times n$ matrix of all second partial derivatives It encodes local curvature.

$$(H_2)_{i,jk} = \left.\frac{\partial^2 f_i}{\partial x_j \partial x_k}\right|_{(\delta_0, \omega_0)}$$

$f_1 = \omega$, It's linear, thus, the H_2 matrix is zero.

$$H_2^1 = \nabla^2 f_1(\delta_0, \omega_0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f_2 = \frac{1}{2H}(P_m - D\omega - P_{MAX}\sin(\delta))$$

$$\frac{\partial^2 f_2}{\partial \delta^2} = \frac{P_{MAX}}{2H}\sin(\delta), \quad \frac{\partial^2 f_2}{\partial \delta \partial \omega} = 0, \quad \frac{\partial^2 f_2}{\partial \omega^2} = 0.$$

$$H_2^2 = \nabla^2 f_2(\delta_0, \omega_0) = \begin{bmatrix} \frac{P_{MAX}}{2H}\sin(\delta_0) & 0 \\ 0 & 0 \end{bmatrix}$$

Third order derivative H_3.

$$(H_3)_{i,jkl} = \left. \frac{\partial^3 f_i}{\partial x_j \partial x_k \partial x_l} \right|_{(\delta_0, \omega_0)}$$

The above representation is in tensorial form but it can be separated H_3^i, similarly as in H_2.

$$\frac{\partial^3 f_2}{\partial \delta^3} = \frac{P_{MAX}}{2H}\cos\delta_0 = (H_3)_{2,111}$$

all other $(H_3)_{2,jkl} = 0$ and $(H_3)_{1,jkl} = 0$.

Building the third order linearized model:

$$\Delta\dot{x} = A\Delta x + \frac{1}{2} H_2(\Delta x_j, \Delta x_k) + \frac{1}{6} H_3(\Delta x_j, \Delta x_k, \Delta x_l) \; ; \; A = J_1(x_0)$$

$$H_2(\Delta x_j, \Delta x_k) = \left[0, \frac{P_{MAX}}{2H} \sin(\delta_0)(\Delta\delta)^2\right]^T$$

$$H_3(\Delta x_j, \Delta x_k, \Delta x_l) = \left[0, \frac{P_{MAX}}{2H} \cos(\delta_0)(\Delta\delta)^3\right]^T$$

$$\boxed{\begin{aligned} \Delta\dot{\delta} &= \Delta\omega. \\ \Delta\dot{\omega} &= -\frac{D}{2H}\Delta\omega - \frac{P_{MAX}}{2H}\cos(\delta_0)\Delta\delta + \frac{P_{MAX}}{4H}\sin(\delta_0)(\Delta\delta)^2 + \frac{P_{MAX}}{12H}\cos(\delta_0)(\Delta\delta)^3 \end{aligned}}$$