100%

Homework IB

Control Modern Theory.

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40% Problem 1) Which of the following sets of vectors are linearly independent?

a) $\begin{bmatrix} 4 \\ -9 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 13 \\ 10 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ in $(\mathbb{R}^3, \mathbb{R})$

If the determinant of a matrix is zero, the columns of the matrix are linearly dependent. Calculating the determinant by cofactors:

$$A = \begin{bmatrix} 4 & 2 & 2 \\ -9 & 13 & -4 \\ 1 & 10 & 1 \end{bmatrix}$$

✓

$$\text{Det}(A) = (-1)^{1+1} 4\left(13\cdot 1 - 10(-4)\right) + (-1)^{1+2} 2\left(-9\cdot 1 - 1(-4)\right)$$

$$= (-1)^{1+3} 2\left(-9(10) - (1)(13)\right) = 212 + 10 - 206 = \boxed{16}$$

Due to Det(A) = 16, the vectors that forms A are linearly independent

✓

b) $\begin{bmatrix} 1+i \\ 2+3i \end{bmatrix}, \begin{bmatrix} 10+2i \\ 4-i \end{bmatrix}, \begin{bmatrix} -i \\ 3 \end{bmatrix}$ in $(\mathbb{C}^2, \mathbb{R})$.

linear independency:

A set of vectors $v_1, v_2 - v_n \in V$ are linearly independent if the unique solution to $\alpha_1 v_1 + \alpha_2 v_2 \dots \alpha_n v_n$ is the trivial one, where $\alpha_1, \alpha_2, \dots \alpha_n$ are 0 these elements are scalars from the $\mathbb{R}$ field.

$$\alpha_1 \begin{bmatrix} 1+i \\ 2+3i \end{bmatrix} + \alpha_2 \begin{bmatrix} 10+2i \\ 4-i \end{bmatrix} + \alpha_3 \begin{bmatrix} -i \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

① $\alpha_1(1+i) + \alpha_2(10+2i) + \alpha_3(-i) = 0$

② $\alpha_1(2+3i) + \alpha_2(4-i) + \alpha_3(3) = 0$

For reals in ① and ②

③ $\alpha_1 + 10\alpha_2 + 0\alpha_3 = 0$

④ $2\alpha_1 + 4\alpha_2 + 3\alpha_3 = 0$

For imaginary part in ① and ②

⑤ $(\alpha_1 + 2\alpha_2 - \alpha_3)i = 0$

⑥ $(3\alpha_1 - \alpha_2 + 0\alpha_3)i = 0$

From ③

$\alpha_1 = -10\alpha_2$

From ⑥

$\alpha_2 = 3\alpha_1$

Then

$\alpha_1 = -10(3\alpha_1) \rightarrow \alpha_1 + 30\alpha_1 = 0 \rightarrow 31\alpha_1 = 0 \rightarrow \boxed{\alpha_1 = 0}$

④

$0 + 10\alpha_2 + 0\alpha_3 = 0 \rightarrow \boxed{\alpha_2 = 0}$ Hence $\boxed{\alpha_3 = 0}$

The set is linearly independent $(\mathbb{C}^2, \mathbb{R})$

c) e^{-t}, te^{-t}, e^{-2t} in $(\mathcal{U}, \mathbb{R})$

set of all piecewise continuous functions defined on $[0, \infty)$

① $\alpha_1 e^{-t} + \alpha_2 t e^{-t} + \alpha_3 e^{-2t} = 0 \quad \forall \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}.$

② $e^{-t}(\alpha_1 + t\alpha_2 + \alpha_3 e^{-t}) = 0$

③ $\alpha_1 + \alpha_2 t + \alpha_3 e^{-t} = 0$ Setting $t = 0$.

$\alpha_1 + \alpha_2(0) + \alpha_3(1) = 0 \longrightarrow \alpha_1 = -\alpha_3$ ④

$t = 1$

$\alpha_1 + \alpha_2 + \alpha_3 e^{-1} = 0$ ⑤

$t = 2$

$\alpha_1 + 2\alpha_2 + \alpha_3 e^{-2} = 0$ ⑥

with ④ and ⑤

$-\alpha_3 + \alpha_2 + \alpha_3 e^{-1} = 0 \longrightarrow \alpha_3(e^{-1} - 1) + \alpha_2 = 0$

$\alpha_2 = -\alpha_3(e^{-1} - 1)$ ⑦

with ⑦ and ⑥ and ④

$-\alpha_3 + 2(-\alpha_3(e^{-1} - 1)) + \alpha_3 e^{-2} = 0$

$-\alpha_3 - 2\alpha_3(e^{-1} - 1) + \alpha_3 e^{-2} = 0$

$\alpha_3(e^{-2} - 2(e^{-1} - 1) - 1) = 0$ The only way to make this equation 0 is that $\alpha_3 = 0$. Then $\alpha_1 = -\alpha_3$, $\alpha_1 = 0$ so $\alpha_2 = 0$ so the set is linearly independent.

d) $3s^2+s-10$, $-2s+3$, $s-5$ in $(\mathbb{R}_3[s]; \mathbb{R})$

$$\alpha_1(3s^2+s-10)+\alpha_2(-2s+3)+\alpha_3(s-5)=0$$

$$s^2(3\alpha_1)+s^1(\alpha_1-2\alpha_2+\alpha_3)+s^0(-10\alpha_1+3\alpha_2-5\alpha_3)=0$$

① $3\alpha_1=0 \longrightarrow \boxed{\alpha_1=0}$

② $\alpha_1-2\alpha_2+\alpha_3=0 \longrightarrow$ ④ $\alpha_3=2\alpha_2$

③ $-10\alpha_1+3\alpha_2-5\alpha_3=0$

Using ④ and ③

$$\cancel{-10\alpha_1}^{0}+3\alpha_2-5(2\alpha_2)=0$$

$$3\alpha_2-10\alpha_2=0$$

$$-7\alpha_2=0 \longrightarrow \boxed{\alpha_2=0}$$

Using ②

$$\cancel{\alpha_1}^{0}-\cancel{2\alpha_2}^{0}+\alpha_3=0 \longrightarrow \boxed{\alpha_3=0}$$

$\alpha_1, \alpha_2, \alpha_3=0$ Thus the set is linearly independent.

e) $\dfrac{3s^2-12}{2s^3+4s-1}$, $\dfrac{4s^5+s^3-2s-1}{1}$, $\dfrac{1}{s^2+s+1}$ in $(\mathbb{R}(s), \mathbb{R})$

Linear Combination:

$\forall\, \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R} \quad \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 = 0$

$$\alpha_1\left(\frac{3s^2-12}{2s^3+4s-1}\right) + \alpha_2\left(\frac{4s^5+s^3-2s-1}{1}\right) + \alpha_3\left(\frac{1}{s^2+s+1}\right) = 0$$

Common denominator $(2s^3+4s-1), (s^2+s+1)$

$$\alpha_1\left(\frac{(3s^2-12)(s^2+s+1)}{(2s^3+4s-1)(s^2+s+1)}\right) + \alpha_2\left(\frac{(4s^5+s^3-2s-1)(s^2+s+1)(2s^3+4s-1)}{(s^2+s+1)(2s^3+4s-1)}\right)$$

$$+\ \alpha_3\left(\frac{(1)(2s^3+4s-1)}{(s^2+s+1)(2s^3+4s-1)}\right) = 0$$

Since the denominator is non zero the numerator must be zero.

$$\alpha_1(3s^2-12)(s^2+s+1) + \alpha_2(4s^5+s^3-2s-1)(s^2+s+1)(2s^3+4s-1)$$

$$+\ \alpha_3(2s^3+4s-1) = 0$$

$$s^{10}(8\alpha_2) + s^9(8\alpha_2) + s^8(26\alpha_2) + s^7(14\alpha_2) + s^6(14\alpha_2) - s^5(7\alpha_2)$$

$$+ s^4(3\alpha_1 - 11\alpha_2) + s^3(3\alpha_1 - 13\alpha_2 + 2\alpha_3) - 9s^2(\alpha_1 + \alpha_2)$$

$$+ s(-12\alpha_1 - \alpha_2 + 4\alpha_3) + (-12\alpha_1 + \alpha_2 - \alpha_3) = 0 \quad (1)$$

From equation ① we have: $s^{10}(8\alpha_2)$ so the only way to make this term 0 is $8\alpha_2 = 0 \rightarrow \boxed{\alpha_2 = 0}$ ✓

Now with $s^4(3\alpha_1 - 11\alpha_2) \rightarrow 3\alpha_1 - 11\alpha_2 = 0$

$3\alpha_1 - 11(0) = 0 \rightarrow 3\alpha_1 = 0 \rightarrow \boxed{\alpha_1 = 0}$. ✓

The term $s(-12\alpha_1 - \alpha_2 + 4\alpha_3)$ must be zero

$-12(0) - (0) + 4\alpha_3 = 0 \rightarrow 4\alpha_3 = 0 \rightarrow \boxed{\alpha_3 = 0}$

Hence the set e) is linearly independent ✓

Problem 1b in $(\mathbb{C}^2, \mathbb{C})$ 5%

From page 2 eq ① $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{C}$

$$(x_1+y_1 i)(1+i) + (x_2+y_2 i)(10+2i) + (x_3+y_3 i)(-i) = 0$$

$$(\underline{x_1} + \underline{x_1 i} + \underline{y_1 i} - \underline{y_1}) + (\underline{10x_2} + \underline{2x_2 i} + \underline{10y_2 i} - \underline{2y_2}) +$$

$$(\underline{-x_3 i} + \underline{y_3}) = 0$$ Grouping Re and Im

① $(x_1 - y_1 + 10x_2 - 2y_2 + y_3) + i$ ② $(x_1 + y_1 + 2x_2 + 10y_2 - x_3) = 0$

eq ② page 2.

$$(x_1+y_1 i)(2+3i) + (x_2+y_2 i)(4-i) + x_3+y_3 i\,(3) = 0$$

$$(2x_1 + 3x_1 i + 2y_1 i - 3y_1) + (4x_2 - x_2 i + 4y_2 i + y_2) +$$

$$(3x_3 + 3y_3 i) = 0$$ Grouping

③ $(2x_1 - 3y_1 + 4x_2 + y_2 + 3x_3) + i$ ④ $(3x_1 + 2y_1 - x_2 + 4y_2 + 3y_3) = 0$

We can keep operating until we find if the system of equations formed by ①, ②, ③, ④ has other solutions.

But Lets consider the following, eq ① page 2.

⋮

Next Page

Eq ① page 2.

$\alpha_1(1+i) + \alpha_2(10+2i) + \alpha_3(-i) = 0$

Let $\alpha_1 = (0+i)$, $\alpha_2 = 0$ and $\alpha_3 = (1+i)$

$(0+i)(1+i) + 0 + (1+i)(-i) = 0$

$\left[(0)(1) + (0)(i) + i(1) + (i)(i)\right] + 0 + \left[(-i)(1) + (i)(-i)\right] = 0$

$(0 + 0 + i - 1) + 0 + (-i - (i)^2) = 0$

$(i-1) - (i + (-1)) = 0$

$(i-1) + (1-i) = 0 \longrightarrow \cancel{i} - \cancel{1} + \cancel{1} - \cancel{i} = 0$

$\boxed{0=0}$ When $\alpha_1 = (0+i)$, $\alpha_2 = 0$ and $\alpha_3 = (1+i)$ the equation gives 0 so there is other solution then the trivial one. The set l_b is linearly dependent in $(\mathbb{C}^2, \mathbb{C})$!

Pas 8

Problem 1d in $(\mathbb{R}(s), \mathbb{R}(s))$ $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}(s) \mid \frac{P(s)}{Q(s)}$

From Page 4 problem 1d but now with $\mathbb{R}(s)$.

$$\frac{s^2(3\alpha_1)}{1} + \frac{s^1(\alpha_1 - 2\alpha_2 + \alpha_3)}{1} + \frac{s^0(-10\alpha_1 + 3\alpha_2 - 5\alpha_3)}{1} = 0$$

$$3s^2\left(\frac{P_1(s)}{Q_1(s)}\right) = 0 \quad \text{for all } s \neq 0 \Rightarrow \frac{P_1(s)}{Q_1(s)} = 0 \rightarrow \boxed{\alpha_1 = 0}$$

$$s^1\left(0 - 2\left(\frac{P_2(s)}{Q_2(s)}\right) + \frac{P_3(s)}{Q_3(s)}\right) = 0 \quad \text{Suppose } s = 1$$

$$\frac{+2P_2(s)}{Q_2(s)} = \frac{+P_3(s)}{Q_3(s)} = \alpha_3 = 2\alpha_2$$

Using s^0 term

$$-10(0) + 3\alpha_2 - 5(2\alpha_2) = 0 \rightarrow -7\alpha_2 = 0$$

$$\frac{-7P_2(s)}{Q_2(s)} = 0, \ \alpha_2 \text{ must be } 0 \ \boxed{\alpha_2 = 0}$$

$$\alpha_3 = 2(0) \longrightarrow \boxed{\alpha_3 = 0}$$ The set is still linearly independent in $(\mathbb{R}(s), \mathbb{R}(s))$

$$\alpha_1(3s^2 + s - 10) + \alpha_2(-2s + 3) + \alpha_3(s - 5) = 0$$

$\alpha_i \in \mathbb{R}[s]$ Función racional.

Una posible solución; $\alpha_1[s] = 0$, $\alpha_2 = \frac{-1}{-2s+3}$ y $\alpha_3 = \frac{1}{s-5}$

Problem 1e in $(\mathbb{R}(s), \mathbb{R}(s))$ $\left(\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}(s) \mid \frac{P(s)}{Q(s)}\right.$

$Q(s) \neq 0$

From page 5:

$$\frac{P_1(s)}{Q_1(s)}\left(\frac{3s^2-12}{2s^3+4s-1}\right)+\frac{P_2(s)}{Q_2(s)}\left(\frac{4s^5+s^3-2s-1}{1}\right)+\frac{P_3(s)}{Q_3(s)}\left(\frac{1}{s^2+s+1}\right)=0$$

There is not such a function of the form $\frac{P(s)}{Q(s)}$ that multiplied by other function $\frac{Z(s)}{R(s)}$ gives us 0 only $\frac{0(s)}{Q(s)}$.

Example.

$$\frac{P_1(s)(3s^2-12)}{Q_1(s)(2s^3+4s-1)}=0$$

$$P_1(s)(3s^2-12)=(0)\left[Q_1(s)\cdot(2s^3+4s-1)\right]$$

$$P_1(s)(3s^2-12)=0$$

$$P_1(s)=\frac{0}{3s^2-12}$$

$\boxed{P_1(s)=0}$ The same demonstration applies for the other terms.

It's still linearly independent in $(\mathbb{R}(s), \mathbb{R}(s))$

Revisar la solución anterior.

Problem 3

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Exercise 3.12 Is the vector $u = (3,-1,0,-1)$ in the subspace of $\mathbb{R}^4$ spanned by the vectors $(2,-1,3,2)$ $(-1,1,1,-3)$, $(1,1,9,-5)$?

We need to prove that a linear combination of the basis vectors gives us the vector u.

$$\alpha_1(2,-1,3,2) + \alpha_2(-1,1,1,-3) + \alpha_3(1,1,9,-5) = (3,-1,0,-1)$$

① $(2\alpha_1 - \alpha_2 + \alpha_3) = 3$ ③ $(3\alpha_1 + \alpha_2 + 9\alpha_3) = 0$

② $(-\alpha_1 + \alpha_2 + \alpha_3) = -1$ ④ $(2\alpha_1 - 3\alpha_2 - 5\alpha_3) = -1$

$$\underset{4\times3}{\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 1 \\ 3 & 1 & 9 \\ 2 & -3 & -5 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}} = \underset{4\times1}{\begin{bmatrix} 3 \\ -1 \\ 0 \\ -1 \end{bmatrix}}$$ Constructing Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ -1 & 1 & 1 & -1 \\ 3 & 1 & 9 & 0 \\ 2 & -3 & -5 & -1 \end{array}\right] \begin{array}{l} R_1(\tfrac{1}{2}) + R_2 \\ R_1(\tfrac{-3}{2}) + R_3 \\ R_1(-1) + R_4 \end{array} \left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 0 & \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2} \\ 0 & \tfrac{5}{2} & \tfrac{15}{2} & -\tfrac{9}{2} \\ 0 & -2 & -6 & -4 \end{array}\right] R_2(2)$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{15}{2} & -\frac{9}{2} \\ 0 & -2 & -6 & -4 \end{array}\right] \begin{array}{l} R_2 + R_1 \\ \\ R_2\left(-\frac{5}{2}\right) + R_3 \\ R_2(2) + R_4 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & -4 & -2 \end{array}\right] R_3\left(\frac{1}{5}\right)$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -\frac{7}{5} \\ 0 & 0 & -4 & -2 \end{array}\right] R_3(4) + R_4$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -\frac{7}{5} \\ 0 & 0 & 0 & -\frac{38}{5} \end{array}\right]$$ From Row 4: $0 = -\frac{38}{5}$

$2\alpha_1 + 2\alpha_3 = 4$

$\alpha_2 + \alpha_3 = 1$

$\alpha_3 = -\frac{7}{5}$

$\boxed{0 = -\frac{38}{5}}$ There is no solution ✓

The vector u isnt in the subspace of $\mathbb{R}^4$ spanned by the set of vectors.

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Exercise 3.14. Show that: $(1-t)^3, (1-t)^2, (1-t), 1$

Spans the $\mathbb{R}^3[t]$ space

$\mathrm{Dim}\,(P_3(\mathbb{R}))=4 \quad ; P(t)=a_0+a_1t+a_2t^2+a_3t^3.$

$$a_0+a_1t+a_2t^2+a_3t^3=\alpha_1(1)+\alpha_2(1-t)+\alpha_3(1-t)^2+\alpha_4(1-t)^3$$

Expanding the polynomial.

$$\alpha_1+(\alpha_2-\alpha_2t)+(\alpha_3-2\alpha_3t+\alpha_3t^2)+(\alpha_4-3\alpha_4t+3\alpha_4t^2-\alpha_4t^3)$$

Grouping

$$(\alpha_1+\alpha_2+\alpha_3+\alpha_4)=a_0$$

$$t(-\alpha_2-2\alpha_3-3\alpha_4)\longrightarrow(-\alpha_2-2\alpha_3-3\alpha_4)=a_1$$

$$t^2(\alpha_3+3\alpha_4)\longrightarrow(\alpha_3+3\alpha_4)=a_2$$

$$t^3(-\alpha_4)\longrightarrow -\alpha_4=a_3$$

$$\underset{A}{\begin{bmatrix}1&1&1&1\\0&-1&-2&-3\\0&0&1&3\\0&0&0&-1\end{bmatrix}}\underset{\vec{\alpha}}{\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{bmatrix}}=\underset{\vec{a}}{\begin{bmatrix}a_0\\a_1\\a_2\\a_3\end{bmatrix}}$$

$\vec{\alpha}=A^{-1}\vec{a}$

Since its a squared matrix we can calculate its inverse to find $\alpha_1,\alpha_2,\alpha_3,\alpha_4$.

The inverse of A is the same matrix $A=A^{-1}$

$$\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{bmatrix}=\begin{bmatrix}1&1&1&1\\0&-1&-2&-3\\0&0&1&3\\0&0&0&-1\end{bmatrix}\begin{bmatrix}a_0\\a_1\\a_2\\a_3\end{bmatrix}\longrightarrow\begin{array}{l}\alpha_1=a_0+a_1+a_2+a_3\\\alpha_2=-a_1-2a_2-3a_3\\\alpha_3=a_2+3a_3\\\alpha_4=-a_3\end{array}$$

With the values of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$:

$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0$

$(a_0 + a_1 + a_2 + a_3) + (-a_1 - 2a_2 - 3a_3) + (a_2 + 3a_3) + (-a_3) = a_0$

$a_0 + a_1(\cancel{1-1}) + a_2(\cancel{1-2+1}) + a_3(\cancel{1-3+3-1}) = a_0$

$\boxed{1a_0 = a_0}$ The same applies for the other constants.

$\alpha_1(1) + \alpha_2(1-t) + \alpha_3(1-t)^2 + \cancel{\alpha_4(1-t)^3} = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

We conclude that the set spans the entire $\mathbb{R}^3[t]$ space because the linear combination of the vectors gives us an $\mathbb{R}^3[t]$ polynomial.

We can see that $(1-t)^3$ can be ignored since we want to span $\mathbb{R}^3[t]$ not $\mathbb{R}^4[t]$.

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Exercise 3.15

Find a vector in $\mathbb{R}^3$ which span the intersection of U and W where U is the plane xy: $U = \{(a,b,0) \mid a,b \in \mathbb{R}\}$ and W is the space spanned by the vectors (0,2,3) and (1,-1,1)

To find a vector in W we do a linear combination

$$\alpha_1(1,2,3) + \alpha_2(1,-1,1)$$

$$(\alpha_1+\alpha_2,\ 2\alpha_1-\alpha_2,\ 3\alpha_1+1\alpha_2) =$$

The form of the elements in U is $(x,y,0)$ so the third element is always 0

$$(\alpha_1+\alpha_2,\ 2\alpha_1-\alpha_2,\ 3\alpha_1+\alpha_2) = (x,y,0)$$

$x = \alpha_1+\alpha_2$; $y = 2\alpha_1-\alpha_2$; $0 = 3\alpha_1+\alpha_2$

$x = \alpha_1 + (-3\alpha_1)$ $\quad$ $y = 2\alpha_1 - (-3\alpha_1) \leftarrow \alpha_2 = -3\alpha_1$

$x = -2\alpha_1$ $\quad$ $y = 5\alpha_1$ $\quad$ $\alpha_2 = -\frac{3y}{5}$

$\alpha_1 = \frac{-x}{2}$ $\quad$ $\alpha_1 = \frac{y}{5}$

$$\frac{-x}{2} = \frac{y}{5} \longrightarrow -x = \frac{2y}{5} \rightarrow x = \frac{-2y}{5}$$

The vector that intersects U and W is:

$\left(\frac{-2y}{5}, y, 0\right)$ $\quad$ $\left\{y\left(\frac{-2}{5},1,0\right) \mid y \in \mathbb{R}\right\}$ Revisar como definir los subespacios.

pdo Exercise 4.5 Let $V=\mathbb{R}^3[t]$ in the vector space of polynomial with grade ≤ 3 over $\mathbb{R}$. Determine whter $u,v,w \in V$ are linearly independent or dependent if:

$u=t^3-4t^2+2t+3$, $v=t^3+2t^2+4t-1$, $w=2t^3-t^2-3t+5$

$$\alpha_1(t^3-4t^2+2t+3)+\alpha_2(t^3+2t^2+4t-1)+\alpha_3(2t^3-t^2-3t+5)=0$$

$$t^3(\alpha_1+\alpha_2+2\alpha_3)=0$$
$$t^2(-4\alpha_1+2\alpha_2-\alpha_3)=0$$
$$t^1(2\alpha_1+4\alpha_2-3\alpha_3)=0$$
$$t^0(3\alpha_1-\alpha_2+5\alpha_3)=0$$

$$\underset{4\times3}{\begin{bmatrix}1&1&2\\-4&2&-1\\2&4&-3\\3&-1&5\end{bmatrix}}\underset{3\times1}{\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}}=\underset{4\times1}{\begin{bmatrix}0\\0\\0\\0\end{bmatrix}}$$

$$\left[\begin{array}{ccc|c}1&1&2&0\\-4&2&-1&0\\2&4&-3&0\\3&-1&5&0\end{array}\right]\begin{array}{l}\\R_1(4)+R_2\\R_1(-2)+R_3\\R_1(-3)+R_4\end{array}\left[\begin{array}{ccc|c}1&1&2&0\\0&6&7&0\\0&2&1&0\\0&-4&-1&0\end{array}\right]\begin{array}{l}R_2(-1)+R_1\\R_2(\frac{1}{6})\\R_2(-2)+R_3\\R_2(4)+R_4\end{array}$$

$$\left[\begin{array}{ccc|c}1&0&\frac{5}{6}&0\\0&1&\frac{7}{6}&0\\0&0&-\frac{4}{3}&0\\0&0&\frac{11}{3}&0\end{array}\right]\begin{array}{l}R_3(\frac{-5}{6})+R_1\\R_3(\frac{-7}{6})+R_2\\R_3(\frac{-3}{4})\\R_3(-\frac{11}{3})+R_4\end{array}\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{array}\right]\begin{array}{l}\to\alpha_1=0\\\to\alpha_2=0\\\to\alpha_3=0\\\to 0=0\end{array}$$

The vectors u, v and w are linearly independent ✓

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Exercise 5.21 Determine whether the set $S=\{1, 1-t, (1-t)^2, (1-t)^3\}$ is a basis for the vector space of polynomial of grade ≤ 3. $\mathbb{R}^4[t]$

In page 13. Exercise 3.14 we did show that the set vectors spans $\mathbb{R}^3[t]$, so now we need to prove linear independency because these vectors to be a basis needs to be linearly independent and also span the whole $\mathbb{R}^4[t]$.

From the system of equations in page 13.

$$(\alpha_1+\alpha_2+\alpha_3+\alpha_4)=0 \longrightarrow \alpha_1=0$$

$$(-\alpha_2-2\alpha_3-3\alpha_4)=0 \longrightarrow -\alpha_2=0 \longrightarrow \alpha_2=0$$

$$(\alpha_3+3\alpha_4)=0 \quad \alpha_3=0$$

$$(-\alpha_4)=0 \longrightarrow \alpha_4=0$$

The set of vectors are linearly independent.

Although we did show that $\mathbb{R}^3[t]$ is spanned by the set of vectors in Page 13 we did the problem considering $\mathbb{R}^4[t]$:

$$\mathbb{R}^3[t]=a_0t^0+a_1t^1+a_2t^2 \; ; \; \mathbb{R}^4[t]=a_0t^0+a_1t^1+a_2t^2+a_3t^3$$

In page 13 we got $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ values which generates a polynomial grade ≤ 3 with that set of vectors. Hence the set is a basis

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Problem 4 | Exercise 6.4

Find the solution of an homogeneous system generated by:

$$\{(1,-2,0,3,-1),(2,-3,2,5,-3),(1,-2,1,2,-2)\}$$

$Ax=b \rightarrow Ax=0$ Solution set to find

$$\underset{3\times5}{\begin{bmatrix}1 & -2 & 0 & 3 & -1\\ 2 & -3 & 2 & 5 & -3\\ 1 & -2 & 1 & 2 & -2\end{bmatrix}}\underset{5\times1}{\begin{bmatrix}a\\b\\c\\d\\e\end{bmatrix}}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Row one times -2 plus Row 2.
Row one times -1 plus Row 3.

$$\left[\begin{array}{ccccc|c}1 & -2 & 0 & 3 & -1 & 0\\ 0 & 1 & 2 & -1 & -1 & 0\\ 0 & 0 & 1 & -1 & -1 & 0\end{array}\right]$$

$a-2b+0c+3d-e=0$

$b+2c-d-e=0$

$c-d-e=0 \rightarrow \boxed{c=d+e}$

$b+2(d+e)-d-e=0 \rightarrow b=-2(d+e)+d+e$

$b=-2d-2e+d+e=\boxed{-d-e}$ $-5d-e$

$a-2(-d-e)+3d-e=0 \rightarrow a+2d+2e+3d-e=0$

$a+5d+e=0 \rightarrow \boxed{a=-5d-e}$ $\boxed{d=d}$ $\boxed{e=e}$

$$\left\{\begin{bmatrix}-5d-e\\-d-e\\d+e\\d\\e\end{bmatrix}\right\} \rightarrow \; = d\begin{bmatrix}-5\\-1\\1\\1\\0\end{bmatrix}+e\begin{bmatrix}-1\\-1\\1\\0\\1\end{bmatrix}.$$

Revisar la definición de los conjuntos

Base

10%

Problem 5 | Exercise 7.6: Let V and W be the following subspaces in $\mathbb{R}^4$

$$V = \{(a,b,c,d) \mid b-2c+d=0\},$$

$$W = \{(a,b,c,d) \mid a=d,\ b=2c\}$$

Intersection..

Find a basis and dimension of V, W and $V \cap W$

V) $b = 2c - d$

$(a, 2c-d, c, d)$

V basis $= \{(1,0,0,0), (0,2,1,0), (0,-1,0,1)\}$

$a(1,0,0,0) + c(0,2,1,0) + d(0,-1,0,1) = \underline{(a, 2c-d, c, d)}$

$\underline{\dim(V) = 3}$

W) $a = d \quad b = 2c - (d, 2c, c, d)$

W basis $= \{(1,0,0,1), (0,2,1,0)\}$

$d(1,0,0,1) + c(0,2,1,0) = \underline{(d, 2c, c, d)}$

$\underline{\dim(W) = 2}$

$V \cap W)$

(a,b,c,d)

$(a,2c-d,c,d) \in V$ and (a,b,c,d)

$(d,2c,c,d) \in W$

$(a,2c-d,c,d) = (d,2c,c,d)$

$a=d \Rightarrow a=0$

$2c-\cancel{d}^{\,0}=2c \rightarrow -d=0 \rightarrow \boxed{d=0}$ $\quad 2c=2c$

$c=c$

$d=d \Rightarrow 0=0$ $\quad (0,2c,c,0)$

$V \cap W = \{(0,2,1,0)\}$

$\dim(V \cap W)=1$