DC Power Flow and Sensitivity Indices for N-1 Contingency Analysis in Power Systems

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***Abstract*—Ensuring the reliability and security of modern power systems requires efficient N-1 contingency analysis to assess the impact of single-element outages on grid stability. This paper presents a sensitivity-based assessment framework that leverages DC power flow analysis and sensitivity indices, including Power Transfer Distribution Factors (PTDFs) and Line Outage Distribution Factors (LODFs), to rapidly evaluate post-contingency power redistributions and identify critical network vulnerabilities. By integrating PowerWorld and Python-based simulations, the proposed methodology describes contingency ranking, accelerates overload detection, and improves decision-making for both real-time operations and long-term planning. Theoretical foundations of sensitivity indices are outlined.**

I. Introduction

The security and reliability of modern power systems are fundamental concerns for system operators and planners. With increasing system complexity, the ability to assess and mitigate contingencies effectively has become crucial in ensuring stable grid operation. Among these contingencies, the N-1 security criterion is important, it evaluates whether a power system can withstand the loss of a single element such as a transmission line or a generator without violating operational limits.

Traditionally, AC power flow simulations have been used for contingency analysis, but their computational intensity makes them less suitable for large-scale, time-sensitive applications. The DC power flow provides a computationally efficient alternative by making several key assumptions: voltage magnitudes are fixed at 1.0 per unit, reactive power flows and voltage variations are neglected, line resistances are ignored, and phase angle differences are assumed to be small, allowing for a linearized power flow model. While these simplifications reduce accuracy, particularly in low-voltage or heavily loaded systems, they enable fast and scalable contingency screening, making DC power flow highly applicable for real-time operational assessments and preliminary contingency ranking.

To further enhance contingency analysis, sensitivity indices such as Power Transfer Distribution Factors (PTDFs) and Line Outage Distribution Factors (LODFs) provide a systematic approach to quantifying the impact of contingencies on system flows. These indices allow operators to estimate power redistributions following line outages without the need for re calculate a new power flow case. By leveraging these indices, contingency screening can be accelerated, ranked, and prioritized, facilitating decision making in system operations.

II. Case study system

The case study system is an interconnected power network consisting of 6 buses, 11 transmission lines, 3 generators, and 3 loads. The generator located at Bus 1 is designated as the slack bus, which balances the system by absorbing any real power mismatches. Figure 1 shows the one-line diagram of the network.

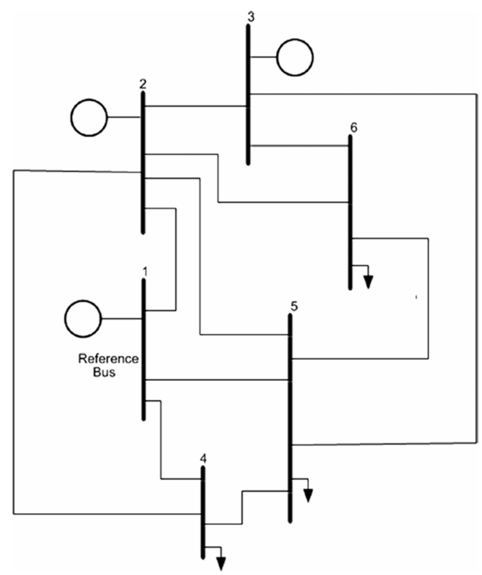

Fig. 1. One-Line Diagram of the Case Study System

The detailed system parameters are presented in Tables I, II, and III.

A. System Overview

- **Buses:** 6
- **Transmission Lines:** 11
- **Generators:** 3
- **Loads:** 3
- **Slack Bus:** Generator at Bus 1

B. Generator Data

Table I presents the generator parameters. The table includes the minimum and maximum active power limits, reactive power limits, and the voltage setpoints for each generator.

TABLE I
GENERATOR PARAMETERS

Bus	Pmin (MW)	Pmax (MW)	Qmax (MVAR)	Qmin (MVAR)	Vg (p.u.)	Pset (MW)
1	50	200	150	-100	1.07	—
2	37.5	150	150	-100	1.05	80
3	45	180	120	-100	1.05	60

C. Transmission Line Data

Table II shows the parameters of the transmission lines. For each line, the table lists the "From Bus" and "To Bus", the resistance (R), reactance (X), line charging susceptance (B_{cap}), and the flow limit.

TABLE II
TRANSMISSION LINE PARAMETERS

From	To	R (p.u.)	X (p.u.)	B_{cap} (p.u.)	Limit (MVA)
1	2	0.1	0.2	0.04	100
1	4	0.05	0.2	0.04	100
1	5	0.08	0.3	0.06	100
2	3	0.05	0.25	0.06	60
2	4	0.05	0.1	0.02	60
2	5	0.1	0.3	0.04	60
2	6	0.07	0.2	0.05	60
3	5	0.12	0.26	0.05	60
3	6	0.02	0.1	0.02	60
4	5	0.2	0.4	0.08	60
5	6	0.1	0.3	0.06	60

D. Load Data

Table III provides the load data. The table lists the bus numbers where loads are connected and shows the active and reactive power demands.

TABLE III
LOAD PARAMETERS

Load Bus	P_{load} (MW)	Q_{load} (MVAR)
4	100	15
5	100	15
6	100	15

III. STATIC SECURITY ANALYSIS AND THE DC POWER FLOW METHOD

Static security analysis aims to determine whether the power system can remain in a safe operating state following specific contingencies or equipment failures. For example, one might ask: "What happens if a 500 kV transmission line is disconnected?" By examining the steady-state response of the system to such hypothetical events, operators can verify whether load flows remain within safe limits. In practice, static security analysis allows us to anticipate possible overloads or voltage violations, enabling proactive measures before real emergencies occur.

A key tool in this process is the DC power flow method, which provides a simplified, linearized representation of the grid for quick screening of numerous potential outages. While it does not capture all AC effects, its computational efficiency makes it well suited for contingency enumeration and preliminary security checks.

A. The DC Power Flow Method

The DC power flow method is a widely used approximation of the full AC power flow model. By linearizing the nonlinear equations under a set of simplifying assumptions, it reduces the problem to a system of linear equations that can be solved rapidly. This method is particularly effective in high-voltage transmission networks and is commonly applied in contingency analysis and real-time security assessments. The assumptions taken into account are:

1) **Flat Voltage Magnitudes:** All bus voltage magnitudes are assumed to be fixed at 1.0 per unit.
2) **Small Angle Differences:** The differences in voltage angles between connected buses are assumed to be small. Consequently,

$$\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j \quad \text{and} \quad \cos(\theta_i - \theta_j) \approx 1. \tag{1}$$

3) **Negligible Line Resistance:** Transmission lines are modeled as purely reactive elements, so that the impedance of line ij is

$$Z_{ij} = jX_{ij}. \tag{2}$$

4) **Lossless Network:** With resistance neglected, the network is assumed to be lossless.

Under these assumptions, the real power flow on a transmission line connecting buses i and j is given by

$$P_{ij} \approx \frac{\theta_i - \theta_j}{X_{ij}}, \tag{3}$$

where X_{ij} is the reactance of the line.

The nodal power balance at each bus i (excluding the slack bus) can be expressed as

$$P_i = \sum_{j \in \mathcal{N}_i} \frac{\theta_i - \theta_j}{X_{ij}}, \tag{4}$$

where P_i is the net real power injection at bus i and $\mathcal{N}_i$ denotes the set of buses connected to bus i.

These equations can be expressed in matrix form. Let $\boldsymbol{\theta}$ be the vector of bus voltage angles (with the slack bus angle fixed, typically at 0) and $\mathbf{P}$ be the vector of net power injections. Then the DC power flow equations become:

$$\mathbf{P} = \mathbf{B}\,\boldsymbol{\theta}, \tag{5}$$

where $\mathbf{B}$ is the reduced susceptance matrix. Its elements are defined as follows:

$$B_{ij} = -\frac{1}{X_{ij}} \quad \text{for } i \neq j, \tag{6}$$

$$B_{ii} = \sum_{j \in \mathcal{N}_i} \frac{1}{X_{ij}}. \tag{7}$$

Once the matrix equation (5) is established, standard linear algebra techniques (such as Gaussian elimination or LU factorization) can be used to solve for the voltage angles $\boldsymbol{\theta}$. With these angles known, the power flow on any line is computed using (3).

The DC power flow method is especially valuable for:

- **Contingency Analysis:** Its linearity enables rapid evaluation of many N-1 contingency scenarios.
- **Security Assessments:** It offers a fast screening tool to identify potential overloads and voltage issues.
- **Operational Studies:** Its computational efficiency makes it ideal for real-time system monitoring.

Although the DC model neglects reactive power and voltage magnitude variations, its accuracy is sufficient for many high-voltage transmission applications where the assumptions hold reasonably well.

B. Outage Analysis by DC Load Flow Method

One advantage of the DC load flow model is that it allows quick solution updates when a transmission line is added or removed. In particular, the network's nodal impedance matrix can be modified via a single outer-product operation between two vectors. In matrix algebra, an update of the form

$$\mathbf{A}' \;=\; \mathbf{A} \;+\; \mathbf{u}\,\mathbf{v}^T \tag{8}$$

(where $\mathbf{u}$ and $\mathbf{v}$ are vectors) is called a *rank-1 update* because it changes the dimension (rank) of $\mathbf{A}$ by at most 1.

Suppose the original network's nodal impedance matrix is $\mathbf{X} \in \mathbb{R}^{n\times n}$. We introduce a new branch k with reactance x_k between buses i and j and define $\mathbf{e}_k$:

$$\mathbf{e}_k \;=\; \begin{bmatrix} 0 \\ \vdots \\ 1\ (\text{row } i) \\ \vdots \\ -1\ (\text{row } j) \\ \vdots \\ 0 \end{bmatrix}, \tag{9}$$

which has all zeros except $+1$ at entry i and -1 at entry j. Then set

$$\mathbf{X}_L \;=\; \mathbf{X}\,\mathbf{e}_k, \quad X_{LL} \;=\; \mathbf{e}_k^T\,\mathbf{X}_L \;=\; \mathbf{e}_k^T\,\mathbf{X}\,\mathbf{e}_k. \tag{10}$$

Under the DC approximation, adding the new line k modifies the matrix $\mathbf{X}$ to a new nodal impedance matrix $\mathbf{X}'$ via

$$\mathbf{X}' \;=\; \mathbf{X} \;-\; \frac{\mathbf{X}_L\,\mathbf{X}_L^T}{X_{LL} + x_k}, \tag{11}$$

where x_k is the (purely reactive) impedance of the new branch.

$$\chi_k \;=\; \mathbf{e}_k^T\,\mathbf{X}\,\mathbf{e}_k, \quad \beta_k \;=\; -\big(x_k + \chi_k\big)^{-1}, \tag{12}$$

so that

$$\mathbf{X}' \;=\; \mathbf{X} \;+\; \beta_k\,\mathbf{X}\,\mathbf{e}_k\,\mathbf{e}_k^T\,\mathbf{X}. \tag{13}$$

This last form highlights the outer-product ($\mathbf{X}\,\mathbf{e}_k\,\mathbf{e}_k^T\,\mathbf{X}$), confirming it is a rank-1 update.

Because nodal injections are assumed constant in DC load flow, the change from $\mathbf{X}$ to $\mathbf{X}'$ directly provides a fast update of bus angles. The DC solution $\boldsymbol{\theta}$ satisfies $\boldsymbol{\theta} = \mathbf{X}\,\mathbf{P}$ (or equivalently $\mathbf{P} = \mathbf{B}\,\boldsymbol{\theta}$). When a line is either added or removed, the new angle vector $\boldsymbol{\theta}'$ differs by

$$\Delta\boldsymbol{\theta} = \boldsymbol{\theta}' - \boldsymbol{\theta} \tag{14}$$

From the updated formula, we obtain

$$\Delta\boldsymbol{\theta} \;=\; \beta_k\,\mathbf{X}\,\mathbf{e}_k\,\big(\mathbf{e}_k^T\,\boldsymbol{\theta}\big), \tag{15}$$

so that

$$\boldsymbol{\theta}' \;=\; \boldsymbol{\theta} \;+\; \Delta\boldsymbol{\theta} \tag{16}$$

In summary, whether a line is being added or removed, the update requires only the pre-change angle difference $\mathbf{e}_k^T\,\boldsymbol{\theta}$ across the buses connected by the affected line, which is then scaled by the product $\mathbf{X}\,\mathbf{e}_k$ and β_k to yield the change in bus angles.

If line k is removed, one replaces x_k with $-x_k$ to achieve a similar update. However, if removing line k disconnects the network, $\mathbf{X}'$ becomes non-invertible. In practical terms, this corresponds to

$$x_k + \mathbf{e}_k^T\,\mathbf{X}\,\mathbf{e}_k = 0 \tag{17}$$

making β_k blow up. Hence, one must check connectivity ($\mathbf{X}'$ remains nonsingular) before applying this approach.

C. N-1 Checking and Contingency Ranking Method

This procedure ensures the system meets the requirement that no single outage (N-1) will drive the grid into an insecure state. The approach involves evaluating how each possible line outage affects overall power flows and ranking these outages by severity.

Step 1: Define a Performance Index (PI). One way to quantify system loading is to sum the normalized power flows over all lines. For L total lines, define

$$\mathrm{PI} \;=\; \sum_{l=1}^{L} \alpha_l\,w_l\left(\frac{P_l}{\overline{P}_l}\right)^2, \tag{18}$$

where:

- P_l is the active power flow on line l,
- $\overline{P}_l$ is the thermal limit (rating) of line l,
- α_l is the number of parallel circuits associated with line l,
- w_l is a weighting factor indicating the importance of line l.

When all lines operate below their ratings, each term $P_l/\overline{P}_l$ is less than 1, keeping PI small.

Step 2: Perform a Base Case DC Load Flow. Solve the DC load flow to find the vector of bus voltage angles $\boldsymbol{\theta}$ and compute each line flow P_l. From these flows, calculate the base performance index PI using the expression above.

Step 3: Simulate Each N-1 Outage. For each line k in the system:

- Remove line k from the network model.
- Solve the DC load flow again to obtain the new bus angles $\boldsymbol{\theta}'$ and the new line flows P_l'.

Step 4: Compute the New Performance Index. With the updated flows P_l', define

$$\text{PI}' \; = \; \sum_{l=1}^{L} \alpha_l \, w_l \left(\frac{P_l'}{\overline{P}_l} \right)^2 . \tag{19}$$

This PI' captures the system's loading after the outage of line k.

Step 5: Calculate the Severity of Each Outage. The change in performance index is

$$\Delta \text{PI}_k \; = \; \text{PI}' \; - \; \text{PI}. \tag{20}$$

A larger ΔPI_k indicates a more severe impact on system loading.

Step 6: Rank All Contingencies. Repeat Steps 3–5 for every candidate line k, then sort the resulting ΔPI_k values in descending order. The top entries in this list correspond to the most critical single-line outages. These should be investigated more carefully, as they are most likely to cause line overloads or other security violations.

D. PTDF by DC Power Flow Method

The Power Transfer Distribution Factor (PTDF) represents the sensitivity of the active power flow on a given transmission line to a change in power injection between two buses. Under the DC load flow approximation, the flow on a transmission line l connecting buses i and j is given by

$$P_l = \frac{1}{x_l} \, (\theta_i - \theta_j), \tag{21}$$

where x_l is the reactance of line l and θ_i and θ_j are the voltage angles at buses i and j, respectively.

Consider a transaction in which a power ΔP is injected at bus m and withdrawn at bus n. The corresponding injection vector is defined as

$$\Delta \mathbf{P} = \Delta P \, (\mathbf{e}_m - \mathbf{e}_n), \tag{22}$$

where $\mathbf{e}_m$ and $\mathbf{e}_n$ are the standard basis vectors corresponding to buses m and n. They are unit vectors, $\mathbf{e}_m$ is the vector that has a 1 in the position corresponding to bus m (where the extra injection is applied) and zeros elsewhere, and $\mathbf{e}_n$ is the similar unit vector for bus n (where the same amount of power is withdrawn). The difference $\mathbf{e}_m - \mathbf{e}_n$ therefore represents a net injection of power at bus m and an equal withdrawal at bus n, which is used to model the transaction in the DC load flow analysis.

Let $\mathbf{a}_l$ be the incidence vector for line l, which has a $+1$ at the sending bus, a -1 at the receiving bus, and zeros elsewhere. The change in the flow on line l is then

$$\Delta P_l = \frac{1}{x_l} \, \mathbf{a}_l^T \, \Delta \boldsymbol{\theta} = \frac{\Delta P}{x_l} \, \mathbf{a}_l^T \, \mathbf{B}^{-1} (\mathbf{e}_m - \mathbf{e}_n). \tag{23}$$

Thus, the PTDF for line l with respect to a transaction from bus m to bus n is defined as

$$\text{PTDF}_l^{(m,n)} = \frac{\Delta P_l}{\Delta P} = \frac{1}{x_l} \, \mathbf{a}_l^T \, \mathbf{B}^{-1} (\mathbf{e}_m - \mathbf{e}_n). \tag{24}$$

Equation (24) shows that the PTDF depends only on the network topology and the line reactances. Its linearity make it especially useful for congestion analysis and market simulations.

E. Analysis of LODF by DC Power Flow Method

The Line Outage Distribution Factor (LODF) quantifies the sensitivity of the flow on a remaining line l to the outage of another line k. Let P_k denote the pre-outage flow on line k. The LODF is defined as the ratio of the change in flow on line l to the pre-outage flow on line k:

$$\text{LODF}_{l,k} = \frac{\Delta P_l}{P_k}. \tag{25}$$

When line k is outaged, the network's susceptance matrix is perturbed. Employing a similar approach to the PTDF derivation, if the incidence vector of line k is $\mathbf{a}_k$, then the change in flow on line l due to the outage of line k can be expressed as

$$\Delta P_l = \left(\frac{\frac{1}{x_l} \mathbf{a}_l^T \mathbf{B}^{-1} \mathbf{a}_k}{1 - \frac{\mathbf{a}_k^T \mathbf{B}^{-1} \mathbf{a}_k}{x_k}} \right) P_k. \tag{26}$$

Thus, the LODF becomes

$$\text{LODF}_{l,k} = \frac{\frac{1}{x_l} \mathbf{a}_l^T \mathbf{B}^{-1} \mathbf{a}_k}{1 - \frac{\mathbf{a}_k^T \mathbf{B}^{-1} \mathbf{a}_k}{x_k}}. \tag{27}$$

Alternatively, if the PTDFs are known, the LODF can be related to them by

$$\text{LODF}_{l,k} = \frac{\text{PTDF}_l^{(m,n)}}{1 - \text{PTDF}_k^{(m,n)}}, \tag{28}$$

where the transaction (m,n) is chosen such that P_k is represented in the corresponding PTDF. Equations (27) and (28) demonstrate that the LODF, as the PTDF, is only a function of the network topology and line parameters. This factor is essential in contingency analysis to quickly estimate how flows are redistributed due to an outage.

IV. Case Study: Simulation and Results

This section presents the simulation results of the contingency analysis conducted through three distinct studies.

Study A – Contingency Analysis via Direct Line Disconnection: This study involves simulating N-1 contingencies by disconnecting individual transmission lines and monitoring for any violations in transmission line loadings. The analysis includes a comparison between the results generated by PowerWorld and those produced by the Python script, facilitating the identification of the worst case contingency scenario.

Study B – Sensitivity Indices from PowerWorld: This study focuses on the sensitivity indices provided by PowerWorld, specifically FTPD and LODF. The obtained values are interpreted to assess the system's response to line outages, providing insight into critical lines and the redistribution of power flows under contingency conditions.

Study C – Python Script and PowerWorld Comparison: This study compares the calculations of the Python script, based on the theoretical equations (24) for PTDFs and (27) for LODFs, with the corresponding values from PowerWorld. This validates the Python implementation and evaluates if there are discrepancies between the analytical approach and the simulation results obtained from the industry standard software.

A. Study A

This section compares the base case results and contingency violations obtained from PowerWorld with those from the Python DC power flow script. Tables IV and V summarize the base-case (no outage) line flows and bus voltage angles. Tables VI and VII then present the contingency analysis results for each line outage. Although there are slight numerical differences between the two methods, the lines identified as violated in the Python script also appear in the PowerWorld results.

PowerWorld can incorporate generator re-dispatch or more detailed features, which adjust generator outputs to maintain system balance across multiple machines. However, the same critical lines tend to show violations in both approaches, confirming the validity of the Python based contingency analysis.

TABLE IV
BASE-CASE LINE FLOWS (MW): POWERWORLD VS. PYTHON

From Bus	To Bus	PowerWorld	Python
1	2	42.5	42.50
1	4	64.3	64.27
1	5	53.2	53.23
2	3	11.9	11.90
2	4	43.5	43.53
2	5	24.9	24.90
2	6	42.2	42.18
3	5	17.3	17.29
3	6	54.6	54.61
4	5	7.8	7.79
5	6	3.2	3.22

TABLE V
BASE-CASE BUS VOLTAGE ANGLES (DEGREES): POWERWORLD VS. PYTHON

Bus	PowerWorld	Python
1	0	0.00
2	-4.87	-4.87
3	-6.57	-6.57
4	-7.36	-7.36
5	-9.15	-9.15
6	-9.7	-9.70

Tables IV and V show a close match in both line flows and angles for most lines and buses.

Table VI illustrates the lines experiencing flow violations (in MW) when certain lines or the generator (G1) is out of service, according to PowerWorld. For instance, an outage on line L12 (Bus 1–2) shows a significant overload on line L14.

As shown in Table VII, the Python-based DC power flow also identifies significant overloads on lines such as 2–4, 2–6, and 3–6 under specific outages.

TABLE VI
POWERWORLD CONTINGENCY ANALYSIS: VIOLATIONS IDENTIFIED

Contingency	L12	L14	L24	L26	L35	L36
L12	–	108.51	–	–	–	–
L14	103.97	–	92.94	–	–	–
L15	–	103.18	–	–	–	–
L26	–	–	–	–	–	78.94
L36	–	–	–	–	74.46	–
G1	–	76.97	–	–	67.02	120.09

TABLE VII
PYTHON DC CONTINGENCY ANALYSIS: VIOLATIONS IDENTIFIED

Outage	Violated Lines (Flow, Limit, Ratio)
1–2	No violations
1–4	2–4 (92.67 MW / 60.00 MW, 1.54)
1–5	No violations
2–3	No violations
2–4	No violations
2–5	No violations
2–6	3–6 (81.52 MW / 60.00 MW, 1.36)
3–5	3–6 (64.99 MW / 60.00 MW, 1.08)
3–6	2–6 (74.08 MW / 60.00 MW, 1.23)
4–5	No violations
5–6	No violations

Critical contingencies are consistently identified in both PowerWorld and the Python script. These discrepancies can primarily be attributed to the differing handling of power imbalances.

B. Study B

Tables VIII and IX present the PTDF and LODF values, respectively, obtained from PowerWorld.

The PTDF values (Table VIII) reveal the sensitivity of line flows to power transactions between specific buses. Higher absolute PTDF values indicate a greater share of the transaction being carried by a particular line, thereby pointing out its importance in the network.

The LODF values (Table IX) quantify the redistribution of flows when a given line is removed. A large positive LODF indicates that the outage of a line significantly increases the flow on another, potentially causing overloads.

To illustrate the impact of these factors, consider a scenario where a 50 MW transaction occurs from bus 1 to bus 2. The power flow on line 1–2 can be estimated using:

$$P_{\text{flow}} = \text{PTDF} \times P_{\text{transaction}} \tag{29}$$

Substituting the values:

$$P_{\text{flow, 1–2}} = 0.4706 \times 50 = 23.53 \text{ MW} \tag{30}$$

Similarly, if line 1–2 is disconnected, the power redistribution on line 1–4 can be estimated using:

$$\Delta P_{\text{affected line}} = \text{LODF} \times P_{\text{flow on disconnected line}} \tag{31}$$

Substituting the values:

$$\Delta P_{1\text{–}4} = 0.543 \times 23.53 = 12.78 \text{ MW} \tag{32}$$

TABLE VIII
POWERWORLD PTDFS

Branch		Modified Lines Due to B Injection - Withdrawn														
From	**To**	**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**2-3**	**2-4**	**2-5**	**2-6**	**3-4**	**3-5**	**3-6**	**4-5**	**4-6**	**5-6**
1	2	47.06	40.26	31.49	32.17	40.64	-6.81	-15.57	-14.89	-6.42	-8.77	-8.08	0.39	0.68	9.15	8.47
1	4	31.49	29.49	50.44	27.11	29.60	-2.00	18.95	-4.38	-1.89	20.95	-2.38	0.11	-23.33	-20.84	2.49
1	5	21.45	30.26	18.07	40.72	29.76	8.81	-3.38	19.27	8.31	-12.18	10.46	-0.50	22.64	11.68	-10.96
2	3	-5.44	34.16	-1.60	10.57	19.07	39.60	3.84	16.01	24.51	-35.76	-23.59	-15.09	12.17	20.67	8.50
2	4	-31.15	-21.54	37.90	-10.13	-22.08	9.61	69.04	21.02	9.06	59.44	11.41	-0.55	-48.02	-59.98	-11.96
2	5	-9.93	3.42	-2.92	19.27	2.66	13.35	7.01	29.19	12.59	-6.34	15.85	-0.76	22.19	5.58	-16.61
2	6	-6.42	24.22	-1.89	12.46	41.00	30.64	4.53	18.88	47.42	-26.11	-11.76	16.78	14.35	42.89	28.54
3	5	-6.22	-28.90	-1.83	12.07	-15.26	-22.68	4.39	18.29	-9.05	27.07	40.97	13.63	13.90	-13.43	-27.33
3	6	0.77	-36.95	0.23	-1.50	34.33	-37.72	-0.55	-2.27	33.56	37.18	35.45	71.28	-1.73	34.10	35.83
4	5	0.34	7.95	-11.66	16.98	7.52	7.61	-12.01	16.64	7.17	-19.61	9.03	-0.43	28.65	19.18	-9.47
6	5	-5.65	-12.73	-1.66	10.96	-24.67	-7.08	3.99	16.61	-19.02	11.07	23.69	-11.94	12.62	-23.01	-35.63

TABLE IX
POWERWORLD LODFS

Branch	Modified Lines Due to Line Outage										
Outage	**L12 (1–2)**	**L14 (1–4)**	**L15 (1–5)**	**L23 (2–3)**	**L24 (2–4)**	**L25 (2–5)**	**L26 (2–6)**	**L35 (3–5)**	**L36 (3–6)**	**L45 (4–5)**	**L56 (5–6)**
1–2	-100	63.5	54.3	-11.3	-50.3	-21	-12.2	-13.7	1.3	1	13.2
1–4	59.5	-100	45.7	-3.3	61.2	-6.2	-3.6	-4	0.4	-32.7	3.9
1–5	40.5	36.5	-100	14.6	-10.9	27.2	15.8	17.7	-1.7	31.7	-17
2–3	-10.3	-3.2	17.8	-100	12.4	22.6	46.6	-40	-52.5	17.1	13.2
2–4	-58.8	76.5	-17.1	15.9	-100	29.7	17.2	19.3	-1.9	-67.3	-18.6
2–5	-18.8	-5.9	32.5	22.1	22.6	-100	23.9	26.8	-2.6	31.1	-25.8
2–6	-12.1	-3.8	21	50.7	14.6	26.7	-100	-19.9	58.4	20.1	44.3
3–5	-11.7	-3.7	20.4	-37.5	14.2	25.8	-17.2	-100	47.5	19.5	-42.5
3–6	1.5	0.5	-2.5	-62.5	-1.8	-3.2	63.8	60	-100	-2.4	55.7
4–5	0.6	-23.5	28.6	12.6	-38.8	23.5	13.6	15.3	-1.5	-100	-14.7
5–6	-10.7	-3.4	18.5	-11.7	12.9	23.5	-36.2	40.1	-41.6	17.7	100

TABLE X
POWER TRANSFER DISTRIBUTION FACTORS (PTDF) FOR 50 MW INJECTION FROM BUS 1

Transaction	L12	L13	L14	L15	L16
$1 \rightarrow 2$	23.5	18.5	17.2	14.3	19.1
$1 \rightarrow 3$	20.0	30.0	13.8	12.6	16.4
$1 \rightarrow 4$	15.5	14.7	25.2	9.7	13.7
$1 \rightarrow 5$	16.0	12.9	10.1	22.8	15.9
$1 \rightarrow 6$	20.0	19.0	14.5	17.6	27.3

TABLE XII
PYTHON SELECTED PTDFS FOR LINES AND TRANSACTIONS

Line	1–2	1–4	1–5	2–3	2–4
1–2	0.4706	0.3149	0.2145	-0.0544	-0.3115
1–4	0.3149	0.5044	0.2711	-0.0200	0.1895
1–5	0.2145	0.1807	0.4072	0.0881	-0.0338
2–3	-0.0544	-0.0160	0.1057	0.3960	0.0384
2–4	-0.3115	0.3790	-0.1013	0.0961	0.6904

TABLE XIII
PYTHON SELECTED LODFS (%) FOR LINES UNDER DIFFERENT OUTAGES

Affected Line	1–2	1–4	1–5	2–3	2–4
1–2	-100.00	63.53	54.27	-11.27	-50.31
1–4	59.48	-100.00	45.73	-3.31	61.21
1–5	40.52	36.47	-100.00	14.58	-10.90
2–3	-10.29	-3.23	17.83	-100.00	12.42
2–4	-58.84	76.47	-17.08	15.91	-100.00

C. Study C

This study compares the sensitivity indices computed using the Python script with those obtained from PowerWorld. A performance index (PI) is first established for the base case (PI = 3.0401), and ΔP_i is calculated for each line outage to rank the severity of contingencies. In addition, selected Power Transfer Distribution Factors (PTDFs) and Line Outage Distribution Factors (LODFs) are provided to illustrate the redistribution of power flows when a line is removed.

TABLE XI
CONTINGENCY RANKING BY ΔPI

Outage	ΔPI	Outage	ΔPI	Outage	ΔPI
(1–4)	2.1386	(2–6)	1.2055	(1–5)	1.1644
(3–6)	0.8117	(2–5)	0.3341	(3–5)	0.3257
(2–4)	0.1199	(5–6)	0.0484	(2–3)	-0.0375
(4–5)	-0.0512	(1–2)	-0.2464		

The performance index ranking clearly identifies the most critical contingencies, with the outage of line 1-4 yielding in a large increase in PI. The selected PTDFs and LODFs further illustrate how power flows are redistributed under each outage condition. These indices, computed from the Python script using theoretical formulations, are consistent with the results observed in PowerWorld.

V. CONCLUSION

This project effectively demonstrates the application of the DC power flow method and sensitivity indices, such as Power Transfer Distribution Factors (PTDF) and Line Outage Distribution Factors (LODF), for contingency analysis in power systems. Through a case study of a 6-bus network with 11 transmission lines, 3 generators, and 3 loads, the study evaluated the system's security under N-1 contingencies where the loss of a single component is assesed. By comparing results from PowerWorld with a custom Python script, the project confirmed the reliability of both approaches in identifying critical contingencies and ranking them using a performance index (PI).

The use of PTDF and LODF indices provides an efficient way to estimate power flow changes due to transactions or outages without requiring full simulations for each case. These sensitivity factors enabled fast identification of vulnerable lines, such as those experiencing overloads after outages like 1–4 or 2–6. While the DC power flow method's assumptions like fixed voltage magnitudes and negligible reactive power limit its accuracy in systems with significant voltage variations, its speed and scalability make it ideal for real-time assessments and initial contingency screening.

The comparison between the python script and PowerWorld validated the analytical approach taken in this document.

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