HW2 Economic Dispatch of Thermal Units

Master of Science in Electrical Engineering

Control and Operation of Power Systems

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General Landscape.

An engineer is always concerned with the cost of products and services. For a power system to return a profit on the capital invested, proper operation is very important. We shall determine how the output of each generation unit is scheduled to achieve minimum cost of power delivered to the load.

Distribution of the load among units.

To determine the economic distribution of load among different units with different technologies the variable operating costs of each unit must be expressed in terms of the power output. Fuel cost is the principal factor in fossil-fuel plant and since most of our electric energy will continue to come from fossil fuel our discussion will be based on this assumption, expressing all the involved costs as fuel and maintenence, etc as a function of power output.

The criteria for distribution of the load between any two units is based upon wheter increasing the load on one unit reduces the same amount on other unit.

We are concerned about the incremental cost, which is determined by the slopes of the input-output curves of the units.

F_n = Input of unit n \$/hr

P_n = output of unit n MW.

The incremental cost of the unit in dollars per megawatthour is dF_n / dP_n.

Case 1 → Three generators have the following cost f and power limits.

$F_1(P_1) = 0.00165P_1^2 + 7.7P_1 + 550$, $P_{min_1} = 150MW$, $P_1^{MAX} = 600MW$

$F_2(P_2) = 0.002P_2^2 + 7.88P_2 + 300$, $P_2^{min} = 125MW$, $P_2^{MAX} = 500MW$

$F_3(P_3) = 0.005P_3^2 + 7.99P_3 + 80$, $P_3^{min} = 75MW$, $P_3^{MAX} = 600MW$.

Total load = 600MW. All units are commited. Determine O.P.

* Solving Using Lambda Iterative Method

For this method we must assign an starting point for λ.

A "λ" of 8 \$/MWh is chosen.

The incremental cost functions are:

$\lambda_1 = 0.0033P_1 + 7.7$

$\lambda_2 = 0.004P_2 + 7.88$

$\lambda_3 = 0.01P_3 + 7.99$

Obtaining P_1, P_2, P_3 with $\lambda^{(1)}$ equal to 8 \$/MWh

$$P_1 = \frac{8-7.7}{0.0033} = 90.909 \text{ MW}$$

$$P_2 = \frac{8-7.88}{0.004} = 30 \text{ MW}$$

$$P_3 = \frac{8-7.99}{0.01} = 10 \text{ MW.}$$

$$e^{(1)} = P_L^{TOTAL} - \sum_{i=1}^{ng} P_i = 600\text{MW} - 90.909 - 10 - 30 = 469.091\text{MW.}$$

Now due to the big error obtained we raise the value of λ 10% to achieve higher Power values.

$\lambda^{(2)} = 8.8$

$P_1 = 333.34$ MW $P_2 = 230$ MW, $P_3 = 81$ MW.

$e^{(2)} = -44.34$ MW.

Using the secant method to approximate $\lambda^{(3)}$ which makes the error ≈ 0.

A slope between $(e^{(1)}, \lambda^{(1)})$ and $(e^{(2)}, \lambda^{(2)})$ must be obtained.

$$\cancel{e^{(3)}} - e^{(2)} = \frac{\lambda^{(2)} - \lambda^{(1)}}{e^{(2)} - e^{(1)}} (\lambda^{(3)} - \lambda^{(2)})$$

So $e^{(3)}$ must be driven to 0. and solve for $\lambda^{(3)}$

Solving for $\lambda^{(3)}$ we set:

$$\lambda^{(3)} = \lambda^{(2)} - \frac{\lambda^{(2)} - \lambda^{(1)}}{e^{(2)} - e^{(1)}} e^{(2)}$$

$$\lambda^{(3)} = 8.8 - \frac{8.8 - 8}{-44.34 - 469.091}(-44.34) = \boxed{8.73}$$

Using $\lambda^{(3)} = 8.73$

$P_1 = 312.12\,MW$, $P_2 = 212.5\,MW$, $P_3 = 74\,MW$.

$e^{(3)} = 1.38\,MW$

P_3 is under its minimum limit we shall fix the minimum supply at $75\,MW$ and only look for G_1 and G_2 balance, Thus the $P_L^{TOTAL} - P_3 = P_1 + P_2$.

$P_1 + P_2$ must meet $600 - 75\,MW = 525\,MW$ demand.

with this assumption the error is $e^{(3)} = 0.38\,MW$

$$\lambda^{(4)} = 8.8 - \frac{8.8 - 8.73}{-44.34 - 0.38}(-44.34) = 8.730595.$$

$P_1 = 312.30\,MW$, $P_2 = 212.64875\,MW$, $P_3 = 75\,MW$.

$e^{(4)} = 525\,MW - 312.30 - 212.64875 = \boxed{0.05125\,MW.}$

In four (4) iterations a 0.009% error was achieved. Generator 1 is at it's half capacity, G_2 is working at less than its half capacity and G_3 at it's minimum.

The incremental cost per each MWh injected to the network is:

$$\lambda_F = 8.730595 \frac{\$}{MWh}$$

The total cost is the sum of each generator cost function:

$$C_{TOT} = [0.00165(312.30)^2 + 7.7(312.30) + 550] + [0.002(212.649)^2 + 7.88(212.649) + 300] + [0.005(75)^2 + 7.99(75) + 80]$$

$$C_{TOT} = 5839.1249 \$$$

CASE 1

Results obtained from the developed python script:

Preview of generator data:

Unit	x^2 term	x term	constant	Minimum MW	Maximum MW
1	0.00165	7.70	550	150	600
2	0.00200	7.88	300	125	500
3	0.00500	7.99	80	75	600

Iteration 1: λ = 8.0000, Total Gen = 350.0000 MW, Error = 250.0000 MW

Iteration 2: λ = 8.8000, Total Gen = 644.3333 MW, Error = -44.3333 MW

Iteration 3: λ = 7.9200, Total Gen = 350.0000 MW, Error = 250.0000 MW

Iteration 4: λ = 8.6675, Total Gen = 565.0302 MW, Error = 34.9698 MW

Iteration 5: λ = 8.7890, Total Gen = 637.1552 MW, Error = -37.1552 MW

Iteration 6: λ = 8.7264, Total Gen = 597.6238 MW, Error = 2.3762 MW

Iteration 7: λ = 8.7302, Total Gen = 599.7054 MW, Error = 0.2946 MW

Iteration 8: λ = 8.7307, Total Gen = 600.0000 MW, Error = 0.0000 MW

Convergence achieved.

Final dispatch schedule:

Unit 1: P = 312.3288 MW

Unit 2: P = 212.6712 MW

Unit 3: P = 75.0000 MW

Final λ = 8.7307 $/MWh, Total Generation = 600.0000 MW, Load = 600.0000 MW, Error = 0.0000 MW

Total Cost: $5889.57

Case 2 Unit Commitment and Dynamic Programming.

- Unit Commitment.

Consist of deciding which generators turn on or turn off over multiple time periods in order to meet a given load while minimizing costs. Costs not only include fuel or operating costs but also start-up and shutdown costs incurred.

- Dynamic Programming.

This method breaks down multi-period decisions into sequential stages, it breaks down a problem into subproblems. This optimization technique starts from the key idea that complex problems have an inherent structure called "optimal structure", this means the overall optimal solution can be optained from subproblems optimal solutions. Dynamic Programing solves each subproblem and stores its solution to avoid redundant computations.

The economic dispatch problem is addressed by DP as an allocation problem, this computes a table of optimal generator outputs for a range of load values. It's applied to problems where generators have cost functions that are non-convex or have discontinuities. Traditional methods based on equal incremental cost may not work because single incremental cost value can correspond to multiple power outputs.

In this formulation, we assume there are no transmission losses. Thus, the sum of the power outputs from all committed generators equals to the demanded load P_L.

* Economic Dispatch per Period.

For each generator i that is on in period t, the optimal power output $P_{i,t}$ is given by the dispatch formula:

$$P_{i,t} = \max\left\{ P_i^{min}, \min\left\{ \frac{\lambda_t - b_i}{2a_i}, P_i^{MAX} \right\} \right\}$$

a_i and b_i are cost coefficients from the cost function:

$$C_i(P_{i,t}) = a_i P_{i,t}^2 + b_i P_{i,t} + c.$$

P_i^{min} and P_i^{MAX} are the minimum and maximum power outputs of each generator. λ_t is the system incremental cost at time t.

Since no losses are taken into account:

$$\sum_{i: U_{i,t}=1} P_{i,t} = P_L .$$

$U_{i,t}$ is binary vector with k elements where k is the number of generators, its the commitment vector for that stage.

* Transition Costs

When transitioning from stage $t-1$ to t, additional costs are incurred if a generator is started or shut down.

$$C^{trans}_{i,t} = S_i \max\{0, u_{i,t} - u_{i,t-1}\} + D_i \max\{0, u_{i,t-1} - u_{i,t}\}$$

wher S_i is the start-up cost and D_i the shut-down cost for each unit.

* Stage cost.

The cost incurred in each stage t is the sum of:

- Operational Cost

$$\sum_{i: u_{i,t}=1} C_i(P_{i,t})$$

- Transition Cost

$$\sum_{i=1}^{K} C^{trans}_{i,t}.$$

The total cost per stage t can be written as:

$$C_T(S_{t-1}, S_t) = \sum_{i; u_{i,t}=1} \left[a_i P^2_{i,t} + b_i P_{i,t} + C\right] + \sum_{i=1}^{K} C^{trans}_{i,t}.$$

where S_T is the state vector representing the on/off status of each generator at stage t. $S_t = \{u_{1,t}, u_{2,t} \dots u_{K,t}\}$.

* Dynamic Programming Recursion.

A cost-to-go function $J_t(S_t)$ represents the minimum total cost incurred from time period t until the final period T, given that the system is in stage S_t at time t.

The recursion is defined as follows:

$$J_t(s_t) = \min_{s_{t+1} \in S} \left\{ \underbrace{C_{t+1}(s_t, s_{t+1})}_{\text{Immediate cost at } t+1\text{: Operational and transition costs.}} + \overbrace{J_{t+1}(s_{t+1})}^{\text{Minimum future cost from time } t+1 \text{ onward to } T.} \right\}$$

* Boundary Condition.

At the final stage T, there is no future cost. Therefore the cost-to-go function is:

$$J_T(s_T) = C_T(s_{T-1}, s_T)$$

* Backward Recursion and Optimal Path Recovery.

- Feasible States.

For each stage t, a list with all possible on/off combinations must be created. For k generators there are 2^k combinations (potential states). Feasibility is restricted by constraints as minimum and maximum up/down times (unit commitment).

- Compute Stage Costs.

For every transition from s_t to s_{t+1}:

* Determine Dispatch $\left\{ \sum_{i: u_{i,t+1}=1} P_{i,t+1} = P_L \right\}$
* Calculate Costs $\left\{ C_{t+1}(s_t, s_{t+1}) \right\}$

- Fill the Dynamic Programing Table Backwards.

Starting at $t=T$:

* Initialization.

Set $J_T(s_T) = C_T(s_{T-1}, s_T)$ for every feasible final state s_T

* Backward Recursion.

For $t = T-1, T-2, \dots 1$ compute:

$$J_t(s_t) = \min_{s_{t+1} \in S} \left\{ C_{t+1}(s_t, s_{t+1}) + J_{t+1}(s_{t+1}) \right\}$$

From the above equation the values obtained for each state s_t are stored. It minimize the s_{t+1} stage.

* Optimal Path Recovery.

After completing the backward recursion:
Begin at $t=1$ from the given chosen state, here the unit 1 and unit 2 are a must run unit, so $u_{1,1} = 1$ and $u_{2,1} = 1$.
The binary vector $u_{i,t}$ has the form: $[1, 1, 0, 0]$ for stage 1.

For each period t, select the state s_{t+1} that has the minimal value obtained in the DP table. This segence has the optimal segrence of states. which minimizes the total cost over 24 hours (24 stages)

$$(s_1^*, s_2^*, s_3^* \dots s_T^*)$$

DP systematically accounts for both immediate costs and transition cost through a recursive cost-to-go function. The method optimizes the overall operating cost while meeting load demands and respecting generator constraints.

CASE 2

Results obtained from the developed python script:

Optimal Stage-by-Stage Results:

Stage 1:

Commitment: 1100

Load: 1000.0 MW

ED λ: 11.9365 $/MWh

Dispatch:

Unit 1: 423.2804 MW

Unit 2: 576.7196 MW

Unit 3: 0.0000 MW

Unit 4: 0.0000 MW

Transition Cost: $6000.00

ED Cost: $10406.88

Stage Cost: $16406.88

Stage 2:

Commitment: 1110

Load: 1400.0 MW

ED λ: 11.9248 $/MWh

Dispatch:

Unit 1: 422.0210 MW

Unit 2: 575.4995 MW

Unit 3: 402.4795 MW

Unit 4: 0.0000 MW

Transition Cost: $3000.00

ED Cost: $14966.83

Stage Cost: $17966.83

Stage 3:

Commitment: 1110

Load: 1800.0 MW

ED λ: 13.6137 $/MWh

Dispatch:

Unit 1: 603.6269 MW

Unit 2: 625.0000 MW

Unit 3: 571.3731 MW

Unit 4: 0.0000 MW

Transition Cost: $0.00

ED Cost: $20044.50

Stage Cost: $20044.50

Stage 4:

Commitment: 1110

Load: 800.0 MW

ED λ: 9.9998 $/MWh

Dispatch:

Unit 1: 215.0352 MW

Unit 2: 374.9820 MW

Unit 3: 209.9828 MW

Unit 4: 0.0000 MW

Transition Cost: $0.00

ED Cost: $8389.45

Stage Cost: $8389.45

Full Total Cost over 4 stages: $62807.66

Case 3 This case is similar to case 1 but for this case a gradient method will be used instead of using the secant method to approximate λ.

In the dispatch problem when neglecting losses the power demanded by the load can be expressed as:

$$P_D = \sum_{i=1}^{ng} \frac{\lambda - b_i}{2a_i} = \sum_{i=1}^{ng} P_i$$

$$P_D = f(\lambda) = \sum_{i=1}^{ng} P_i$$

Using Taylor's series expansion about an operating point $\lambda^{(k)}$ and neglecting high order terms we get:

$$f(\lambda)^{(k)} + \left(\frac{d\,f(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D$$

Solving for $\Delta\lambda^{(k)}$:

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \frac{1}{2a_i}} = \frac{P_D - f(\lambda)^k}{\left(\frac{d\,(f\lambda)}{d\lambda}\right)^{(k)}}$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$$

$$\Delta P^{(k)} = P_D - \sum_{i=1}^{ng} P_i^{(k)}$$

i) $P_D = 320\,MW$, $P_i^{min} = 39\,MW$, $P_i^{max} = 150\,MW$.

To find the economic schedule:

$$\frac{\partial F_1(P_1)}{\partial P_1} = 0.25P_1 + 215 \quad ; \quad \frac{\partial F_2(P_2)}{\partial P_2} = 2P_2 + 270$$

$$\frac{\partial F_3(P_3)}{\partial P_3} = 0.49P_3 + 160$$

*LAGRANGE MULTIPLIERS

Remember, the objective function is the cost function in this case $F_i(P_i)$ and the restriction is that the load and generation are equal.

$$\mathcal{L} = \sum_{i=1}^{ng} F_i(P_i) + \lambda\left(P_D - \sum_{i=1}^{ng} P_i\right)$$

To minimize the $\mathcal{L}$ function one must take the derivative and it needs to be zero, it yields:

$$\frac{\partial \mathcal{L}(P_i,\lambda)}{\partial P_i} = \sum_{i=1}^{ng} \frac{dF_i(P_i)}{dP_i} - \lambda = 0$$

$$\sum_{i=1}^{ng} \frac{dF_i(P_i)}{dP_i} = \lambda.$$

Lagrange multipliers are mainly an analytical tool to handle constraints and the gradient methods are numerical approaches. Both are tools for optimization problems

Using the gradient method:

For this method a starting λ of 8 is proposed $\lambda = 8$. Using the cost functions to identify a and b coefficients and using the following equation.

$$\sum_{i=1}^{ng} \frac{\lambda - b_i}{2a_i} = \sum_{i=1}^{ng} P_i = P_D$$

$$\left(\frac{8-215}{2(0.3)}\right) + \left(\frac{8-270}{2(1)}\right) + \left(\frac{8-160}{2(0.7)}\right) = -446.57 \text{ MW}$$

$$\Delta P^{(1)} = 320 - (-446.57) = 766.57 \text{ MW}$$

$$\Delta \lambda^{(1)} = \frac{766.57}{\frac{1}{2(0.5)} + \frac{1}{2(1)} + \frac{1}{2(0.7)}} = 346.192$$

$$\lambda^{(2)} = 8 + 346.192 = 354.192 \text{ \$/MWh}$$

$$\lambda^{(2)} = 354.192$$

$$\left(\frac{354.192-215}{2(0.5}\right) + \left(\frac{354.192-270}{2}\right) + \left(\frac{354.192-160}{2(0.7)}\right) = 319.996$$

$$\Delta P^{(2)} = 320 - 319.996 = 0.004$$

$P_1 = 139.192$ MW

$P_2 = 42.096$ MW $\qquad \lambda = 354.192$ \$/mwh.

$P_3 = 138.708$ MW

The three generators are within the limits.

ii) $P_D = 200$ MW, $P^{min} = 39$ MW, $P^{MAX} = 150$ MW.

$\lambda = 300$

$$\left(\frac{300-215}{2(0.5)}\right)+\left(\frac{300-270}{2(1)}\right)+\left(\frac{300-160}{2(0.7}\right)=200$$

$\Delta P = 200 - 200 = 0$

$P_1 = 85$ MW, $P_2 = 15$ MW, $P_3 = 100$ MW,

Note that P_2 is below it's minimum, hence we fix it to its minimum $P_2 = 39$ MW, now $P_D = 161$ MW.

$\Delta P^{(1)} = 161 - 85 - 100 = -24$ MW.

$$\Delta\lambda^{(1)} = \frac{\cancel{-2}+24}{\frac{1}{1}+\frac{1}{2}+\frac{1}{1.4}} = -10.83$$

$\lambda^{(2)} = 300 - 10.83 = 289.17$ \$/MWH

$$\left(\frac{289.17-215}{1}\right)+\left(\frac{289.17-160}{1.4}\right)=166.43\,MW$$

$$\Delta P^{(2)} = 161-166.43 = -5.43$$

$$\Delta\lambda^{(2)} = \frac{-5.43}{1+\frac{1}{1.4}} = -3.167$$

$$\lambda^{(3)} = 289.17-3.167 = 286 \ \$/MWh$$

$$\left(\frac{286-215}{1}\right)+\left(\frac{286-160}{1.4}\right)=161\,MW$$

$$\Delta P^{(3)} = 161-161 = 0.$$

$P_1 = 71\,MW$ $\qquad \lambda = 286 \ \$/MWh.$

$P_2 = 39\,MW$

$P_3 = 90\,MW$

The incremental cost of ~~i~~ ii) is lower than i) for obvious reasons, the units are running in a lower stress state, there is less demand in energy. Also Unit 2 is fixed and does not contribute to the incremental cost.

CASE 3

Results obtained from the developed python script:

For Total Load = 320 MW:

- $P1$ = 139.19 MW
- $P2$ = 42.10 MW
- $P3$ = 138.71 MW
- Incremental Cost (λ) = 354.19 $/MWh
- Converged in 9 iterations

For Total Load = 200 MW:

- $P1$ = 71.00 MW
- $P2$ = 39.00 MW
- $P3$ = 90.00 MW
- Incremental Cost (λ) = 286.00 $/MWh
- Converged in 12 iterations

Case 4

In an unconstrained economic dispatch the goal is to equalize the incremental costs of the two units while satisfying the total load.

$$0.8P_1+160 = 0.9P_2+120$$
$$P_1+P_2=L$$

The unconstrained solution yields:

$$P_1=L-P_2\ ;\ P_2=\frac{L+50}{2.15}$$

$$0.8(L-P_2)+160=0.9(P_2)+120$$

$$0.8L-0.8P_2+40=0.9P_2$$

$$P_2=\frac{0.8L+40}{1.7}$$

$$P_2=\frac{0.8L+0.8(50)}{1.7}\left(\frac{\frac{1}{0.8}}{\frac{1}{0.8}}\right)$$

$$\boxed{P_2=\frac{L+50}{2.125}}$$

However because of the limits we shall see the power output when the load is at it's minimum and its maxim to be sure that the units are within their limits.

When the load is 50MW we have:

$$P_2 = \frac{50+50}{2.125} = 47.06\,MW$$

$$P_1 = 50-47.06 = 2.94$$

Clearly, P_1 cant be less then 20MW, Set $P_1 = 20MW$.

$$P_2 = L-20$$
$$P_1 = 20MW$$

The incremental cost is:

$$\lambda = 0.9(L-20)+120 \ \$/MWh.$$

Note that Unit 1 has a fixed power output thus, it has no incremental cost. Unit 2 sets the marginal price.

For a 250 MW load:

$$P_2 = \frac{250+50}{2.125} = 141.18\,MW$$

Unit 2 is above its capacity so P_2 must be fixed to 125MW.

$$P_2 = 125MW$$
$$P_1 = L-125$$

The incremental cost is:

$$\lambda = 0.8(L-125)+160 \ \$/MWh.$$

In this case the unit 2 has fixed output power, so unit 1 determines the marginal cost.

From the lower and upper bounds, we find the feasible interval for loads where the unconstrained solution applies:

$$P_1 = L - \frac{L+50}{2.125} \; ; \; P_1 \geq 20$$

$$(2.125)20 \leq 2.125L - (L+50)$$

$$42.5 \leq 2.125L - L - 50$$

$$92.5 \leq 1.125L$$

$$82.22 \leq L$$

$$L \geq 82.22 \quad \text{Lower bound}$$

$$\frac{L+50}{2.125} \leq 125$$

$$L + 50 \leq 265.625$$

$$L \leq 215.625 \text{ MW} \quad \text{Upper bound}$$

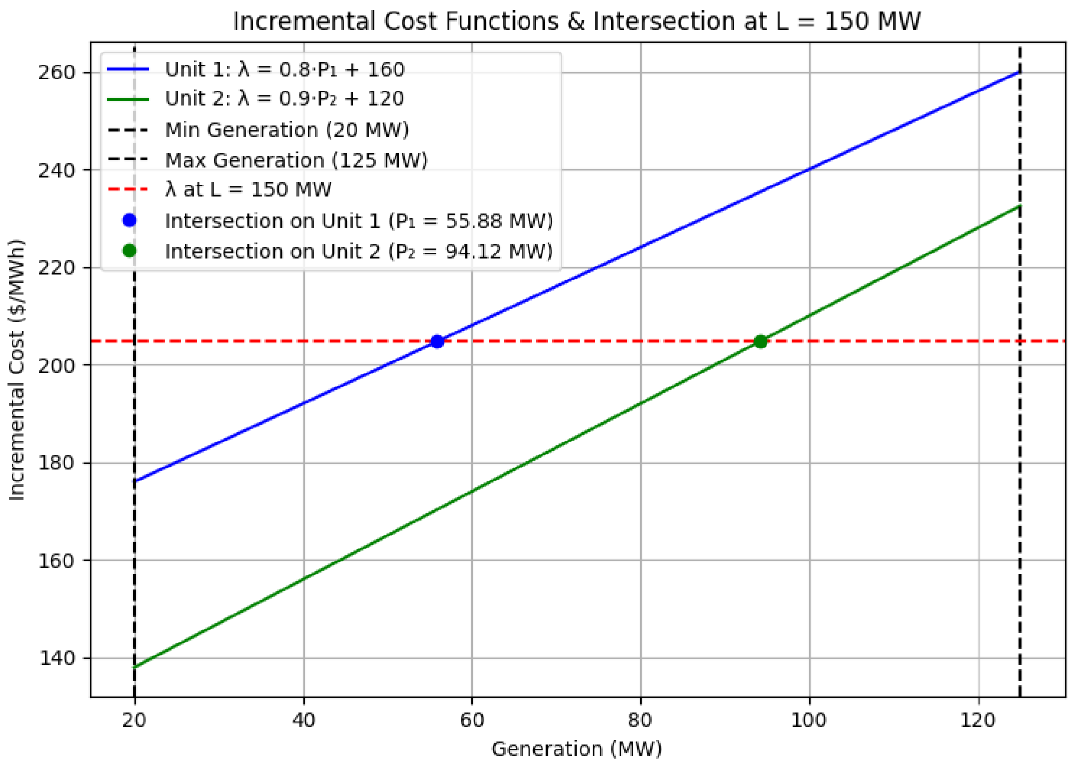
When the load is between 82.22 and 215.63 MW we can use the unconstrained solution. Both units share the load

$$P_2 = \frac{L+50}{2.125} \;,\quad P_1 = L - P_2$$

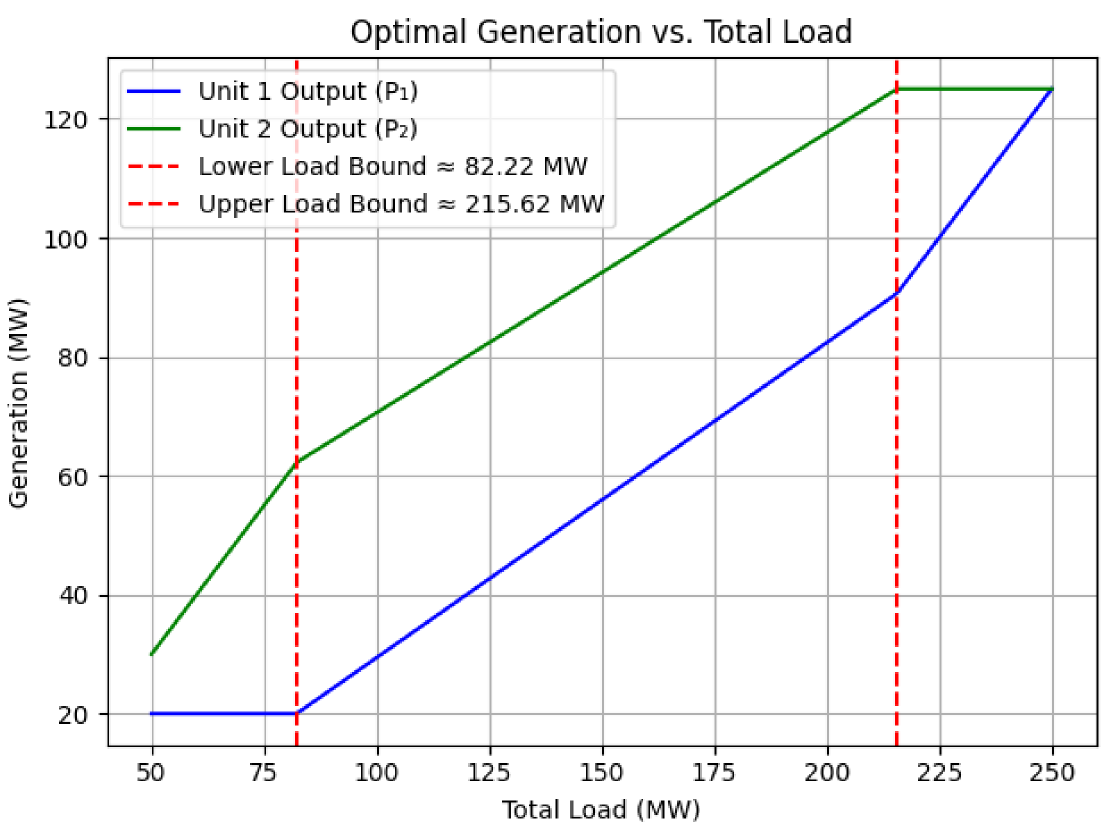
$$\lambda = 0.9P_2 + 120 = 0.8P_1 + 160.$$

Python was used to plot the functions, the results are in the following pages.

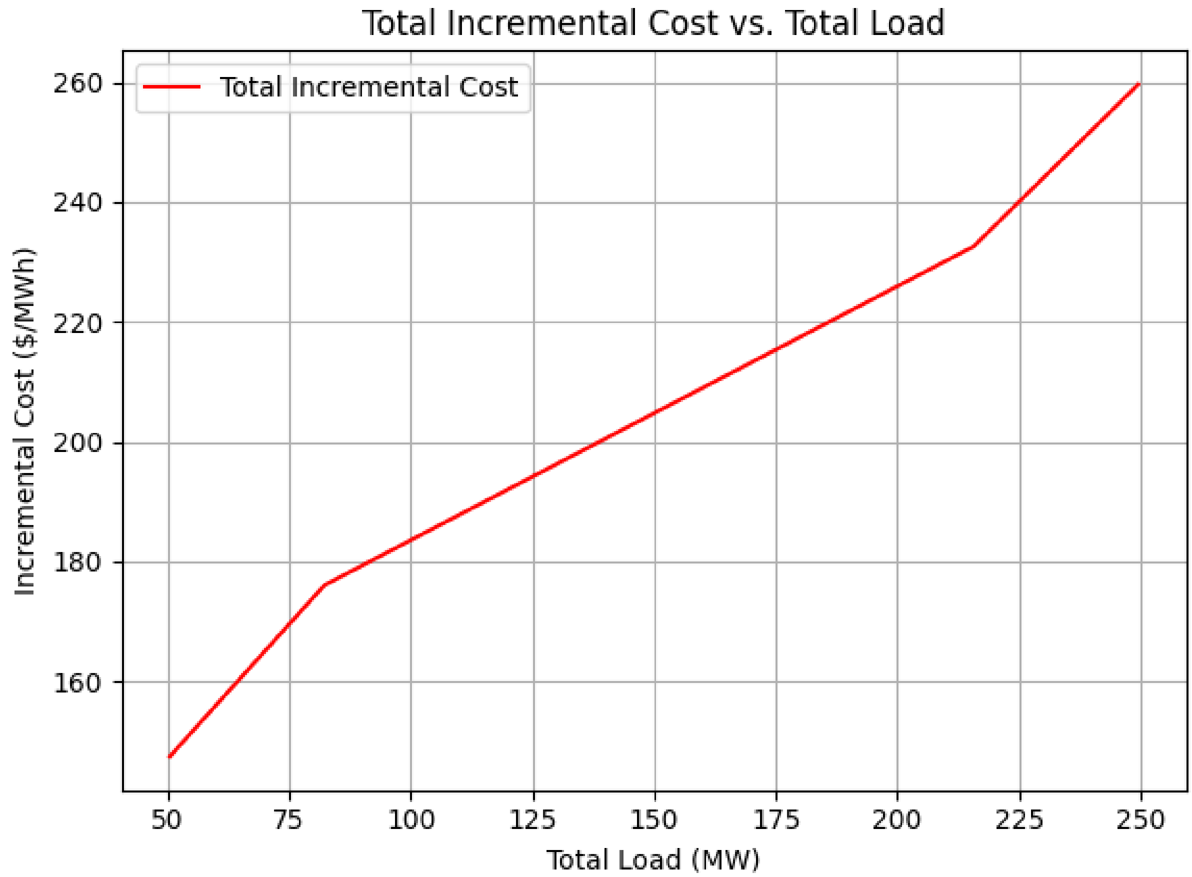
CASE 4

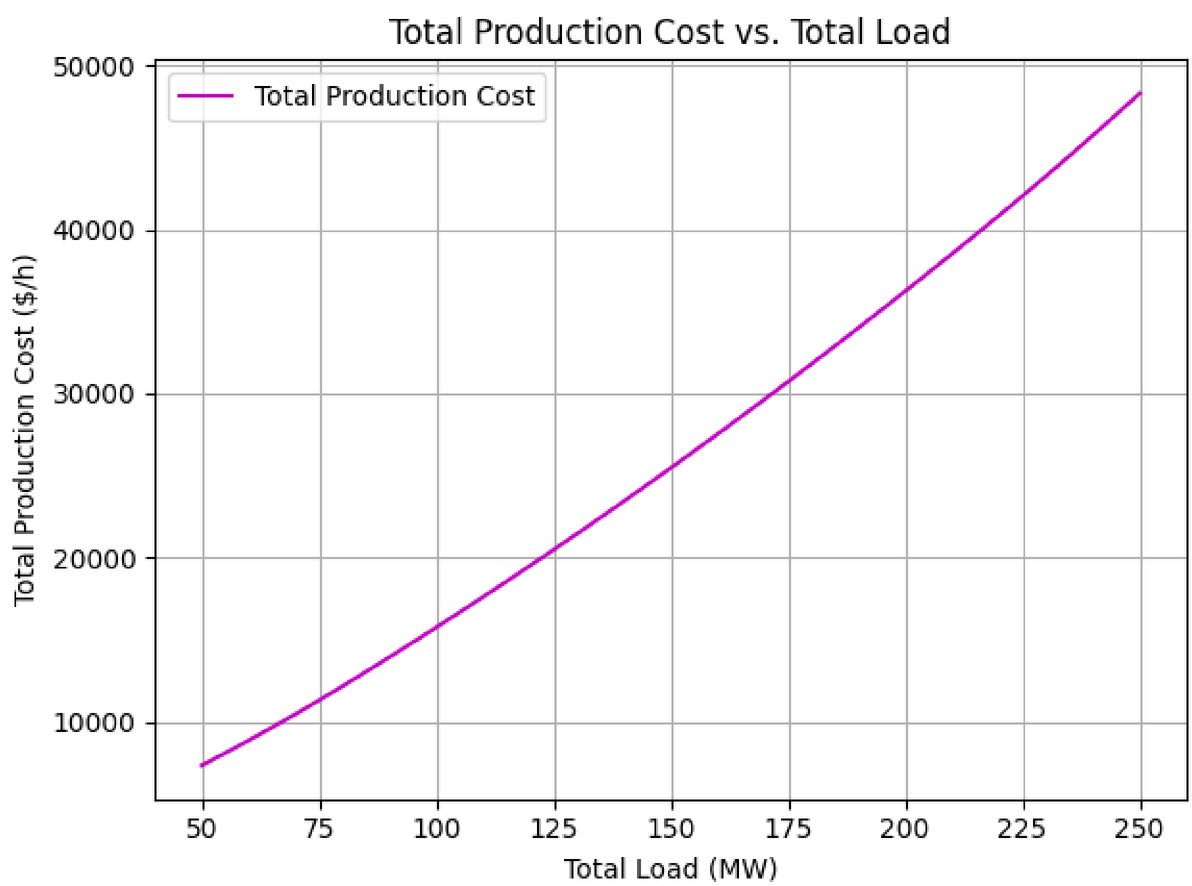


Incremental Cost Functions & Intersection at L = 150 MW
Unit 1: λ = 0.8·P₁ + 160
Unit 2: λ = 0.9·P₂ + 120
Min Generation (20 MW)
Max Generation (125 MW)
λ at L = 150 MW
Intersection on Unit 1 (P₁ = 55.88 MW)
Intersection on Unit 2 (P₂ = 94.12 MW)
Incremental Cost ($/MWh)
Generation (MW)



Optimal Generation vs. Total Load
Unit 1 Output (P₁)
Unit 2 Output (P₂)
Lower Load Bound ≈ 82.22 MW
Upper Load Bound ≈ 215.62 MW
Generation (MW)
Total Load (MW)


Total Incremental Cost vs. Total Load
Total Incremental Cost
Incremental Cost ($/MWh)
160
180
200
220
240
260
50
75
100
125
150
175
200
225
250
Total Load (MW)


Total Production Cost vs. Total Load
Total Production Cost
Total Production Cost ($/h)
10000
20000
30000
40000
50000
50
75
100
125
150
175
200
225
250
Total Load (MW)

Case 5 In this case the fuel cost is not assumed as 1 \$/MBtu for all units.

The fuel cost for unit 1 is 0.8 \$/MBtu, 1.02 for unit 2 and 0.9 for unit 3. The actual cost function $F_i(P_i)$ for each unit i is:

$$F_i(P_i) = \text{Fuel Cost} \times \underbrace{H_i(P_i)}_{\text{Heat input as a function of } P}$$

The incremental cost for each unit is:

$$IC_i(P_i) = \text{Fuel Cost}_i \times \frac{d}{dP_i}\left(H_i(P_i)\right)$$

- Unit 1

$$IC_1(P_1) = 0.8\,(8.4 + 0.005P_1) = 6.72 + 0.004P_1$$

- Unit 2

$$IC_2(P_2) = 1.02\,(6.3 + 0.0162P_2) = 6.426 + 0.016524P_2$$

- Unit 3

$$IC_3(P_3) = 0.9\,(7.5 + 0.005P_3) = 6.75 + 0.0045P_3$$

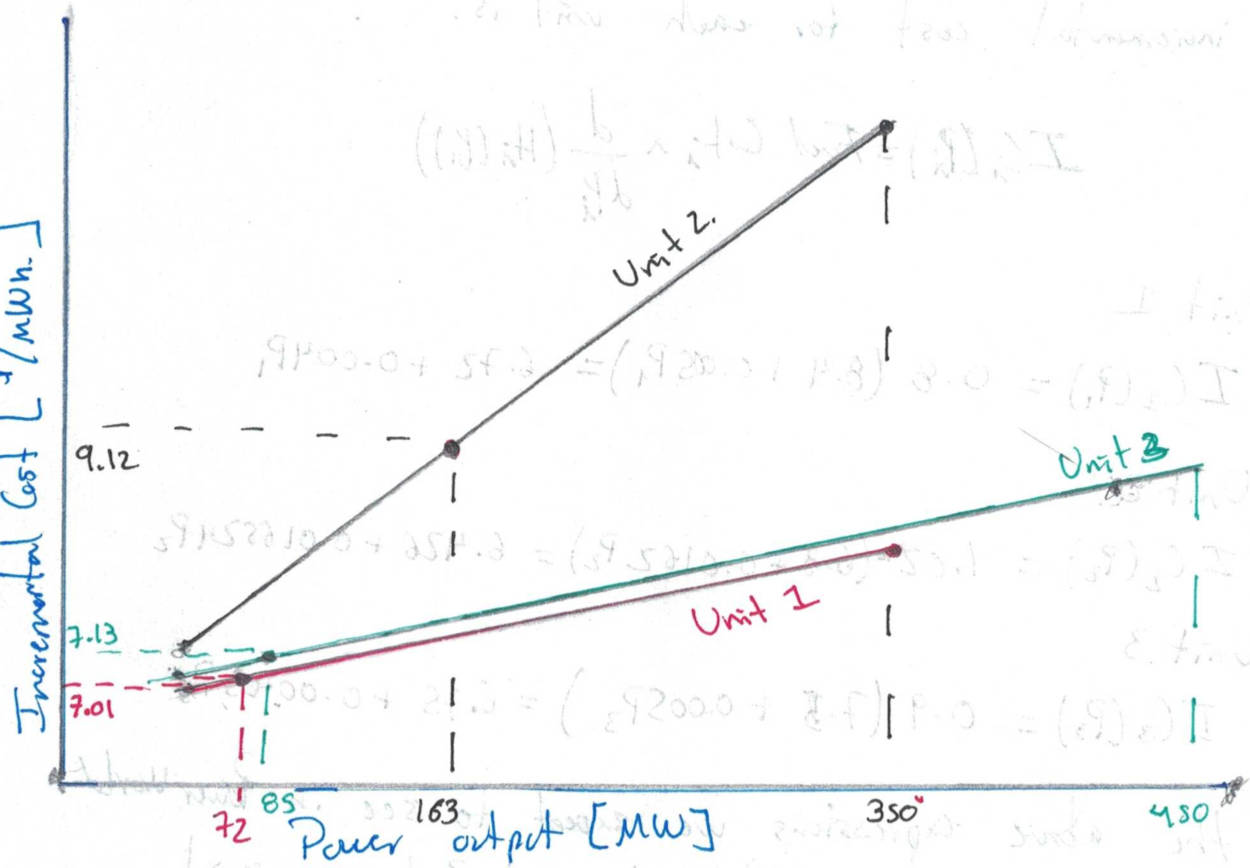
From the above expressions we expect to see in PowerWorld three different slopes where slope for unit 2 has a rapid increment of its incremental cost. Unit 1 and Unit 3 are almost equal in growth but Unit 3 has a slightly more increment than unit 1.

PowerWorld was used to perform the following Analysis.

In the study case system Unit 2 has a fixed power of 163 MW, Unit 3 has 85 MW. Hence, the slack is varying to compensate the total load. The slack Unit (1) is supplying 72 MW. Because unit 2 and unit 3 are fixed the incremental cost only depends of Unit 1.

$$IC_1(P_1) = 6.72 + 0.004(72) = \underline{7.008 \text{ \$/mwh}}$$

The incremental cost obtained from PowerWorld is $\underline{7.01 \text{ \$/mwh}}$.



The input-output curves are shown in the next pages.
The above incremental cost curves are for fixed units, only the slack is varying but if we set all three generators to balance themselves to achieve a minimum cost we will have different incremental costs and power outputs.

In this case study the total load in the system is 315MW. Due to this the generation constraints are always meet. Even a single generator can supply the whole load if we dont consider losses in transmission lines.

* Impact of one contingency.

Suppose the generator 2 is out due to a fault, the Unit 1 must compensate the 163MW lost. The output power of Unit 1 is 235MW and Unit 3 remains fixed at 85MW. In this case Unit 1 is still within the limits.

Now, the incremental cost raised to 7.66 $/MWh. This means, an additional MWh delivered by Unit 1 will cost 7.66$ instead of 7.01 as in the previous case.

An important thing to notice is if neither generator has fixed power output the system will balance the power of each generator to accomplish an equal incremental cost for each Unit, for example in our case study system if we allow to each generator to move freely to adjust its power output the total incremental cost would be: 7.32 $/MWh and the power output of each unit is: 152MW - Unit 1, 111MW Unit 3 and 55MW Unit 2. this makes sense since Unit 1 has the lower costs and Unit 2 the highest incremental cost curve. The generators are still within the limits.

If Unit 2 is out and Unit 1 and 3 moves freely, the power outputs and incremental costs are as follows:

$P_1 = 178MW$, $P_2 = 141MW$, $\lambda = 7.43$ $/MWh.

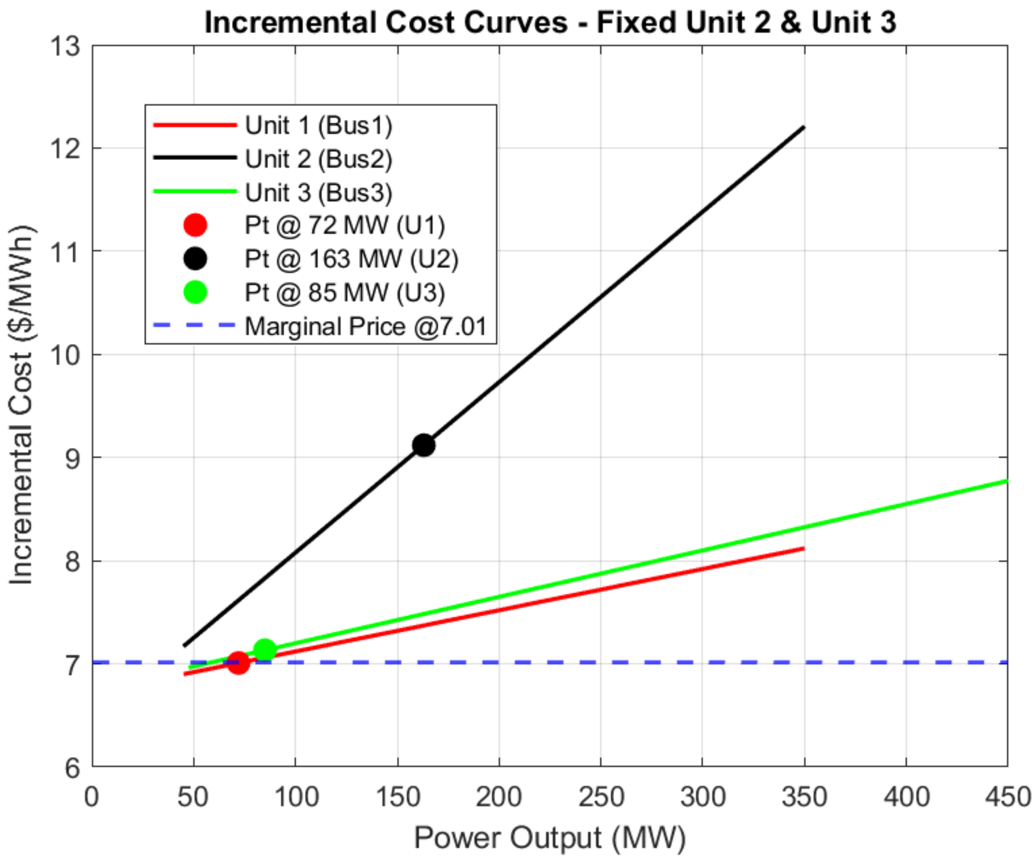
From the previous discussion is important to note that with the units fixed the incremental cost decreases, in this case the reason is because the unit 1 has the total control of the marginal price and its the chepeast. But, it does not mean that the total production cost is the minimum, instead it will be higher, since Unit 2 and Unit 3 has higher cost functions but in this study case they remain fixed.

Just remember if the incremental cost is lower in certain states with generator outputs fixed doesnt mean that state is the chepeastes overall.

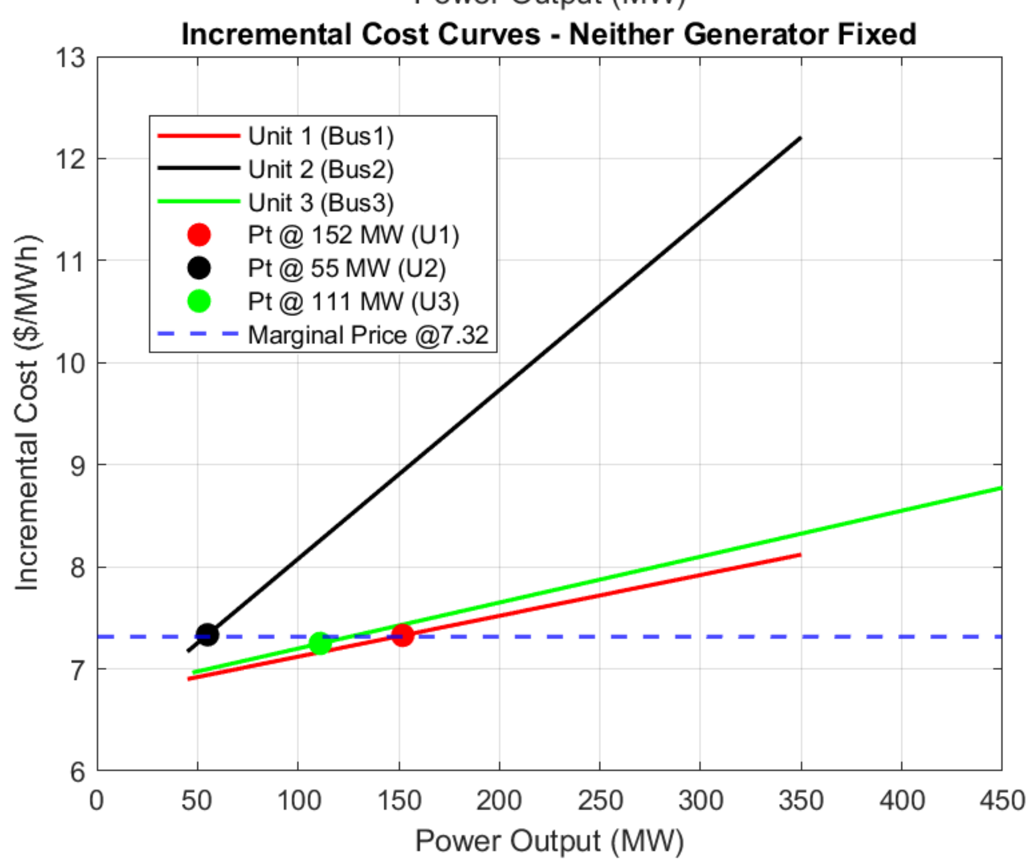
Curves presented:

* Incremental Cost Curves with U_2 and U_3 fixed.
* Incremental Cost Curves with Neither generator fixed.
* Generator Input-Output Curves.
* Incremental Cost Curves for the Contingency (Unit 2 out). U_3 fixed and not fixed.

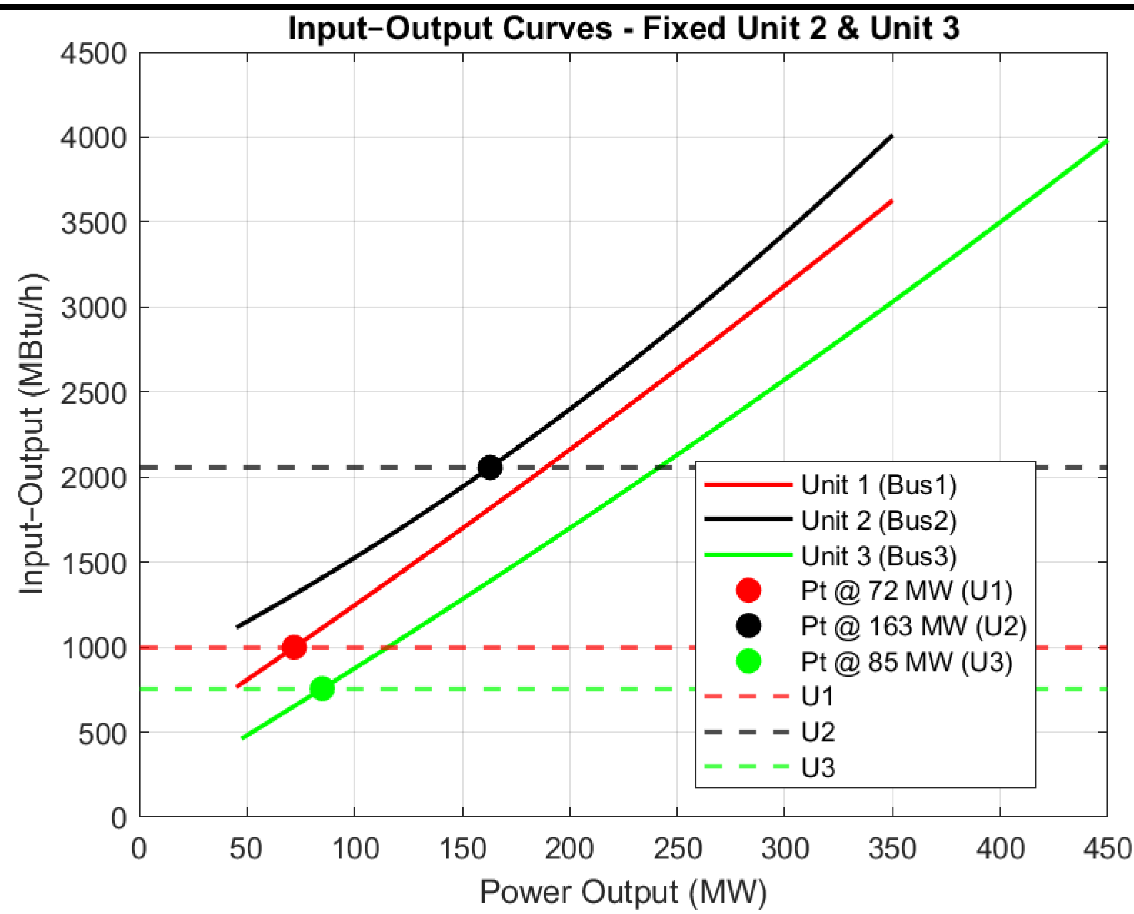
CASE 5

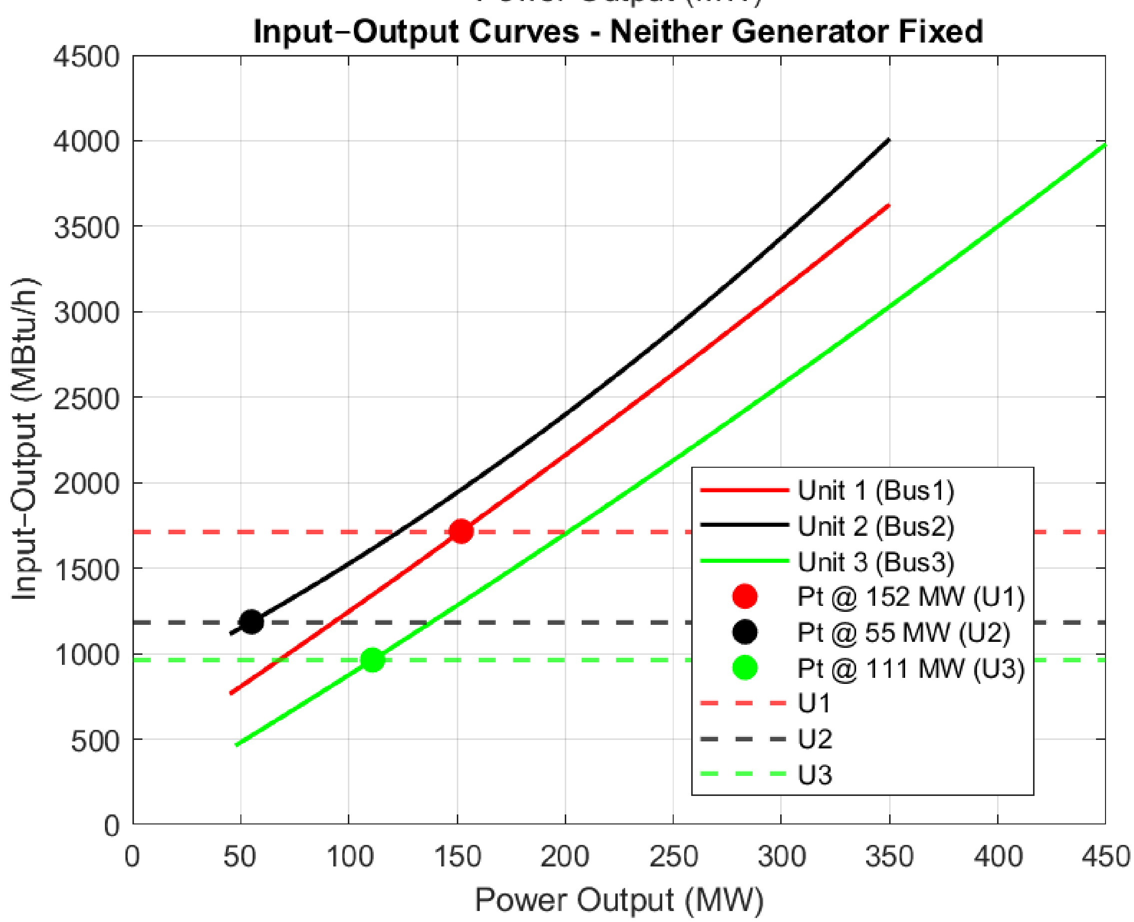


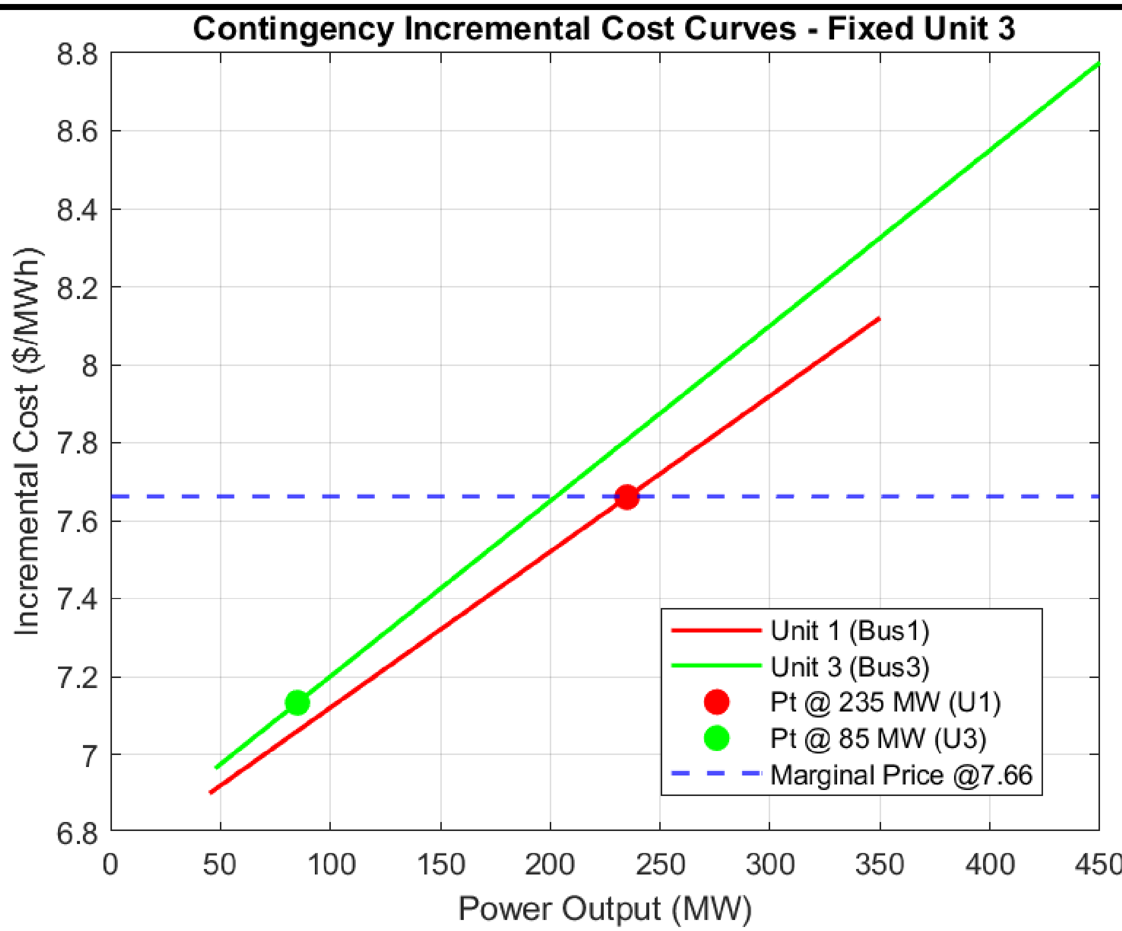
Incremental Cost Curves - Fixed Unit 2 & Unit 3
Incremental Cost ($/MWh)
Power Output (MW)
Unit 1 (Bus1)
Unit 2 (Bus2)
Unit 3 (Bus3)
Pt @ 72 MW (U1)
Pt @ 163 MW (U2)
Pt @ 85 MW (U3)
Marginal Price @7.01
6 7 8 9 10 11 12 13
0 50 100 150 200 250 300 350 400 450

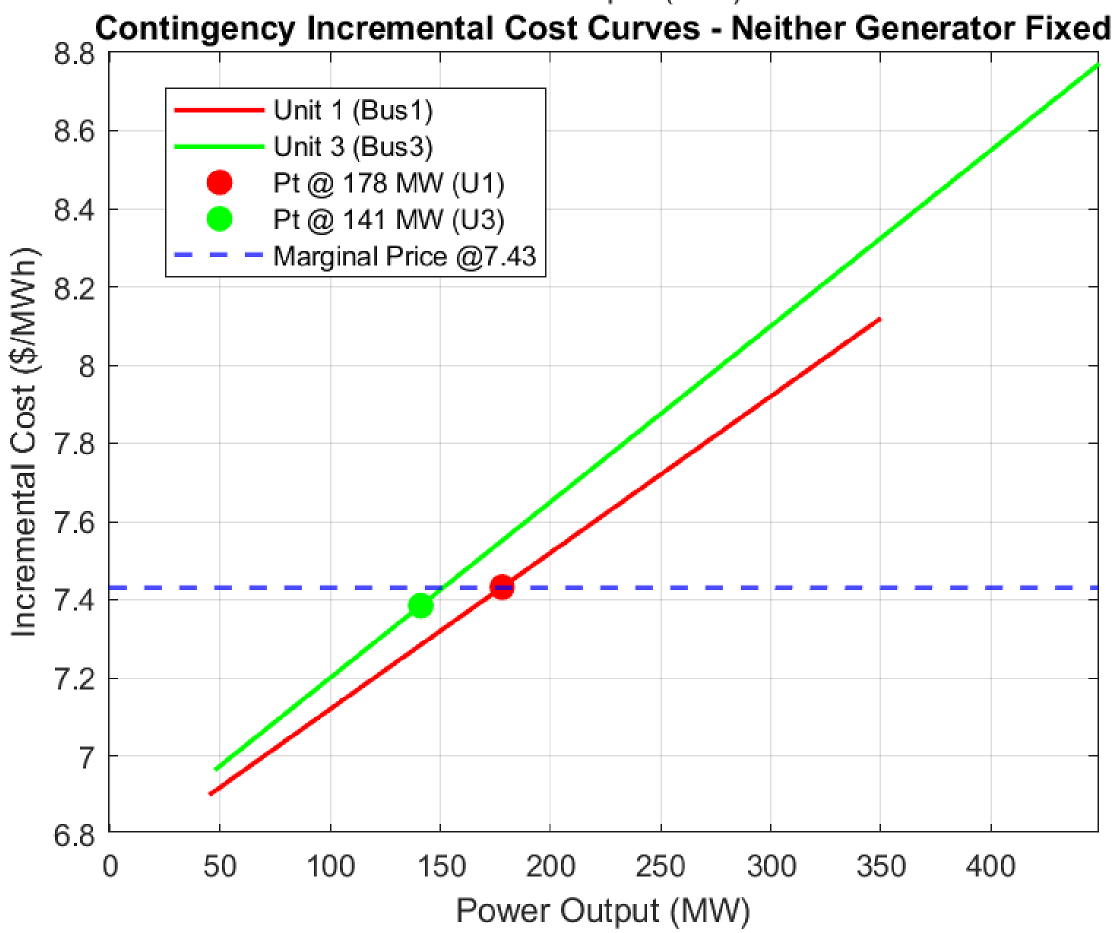


Incremental Cost Curves - Neither Generator Fixed
Incremental Cost ($/MWh)
Power Output (MW)
Unit 1 (Bus1)
Unit 2 (Bus2)
Unit 3 (Bus3)
Pt @ 152 MW (U1)
Pt @ 55 MW (U2)
Pt @ 111 MW (U3)
Marginal Price @7.32
6 7 8 9 10 11 12 13
0 50 100 150 200 250 300 350 400 450


Input−Output Curves - Fixed Unit 2 & Unit 3
Input-Output (MBtu/h)
Power Output (MW)
Unit 1 (Bus1)
Unit 2 (Bus2)
Unit 3 (Bus3)
Pt @ 72 MW (U1)
Pt @ 163 MW (U2)
Pt @ 85 MW (U3)
U1
U2
U3


Input−Output Curves - Neither Generator Fixed
Input-Output (MBtu/h)
Power Output (MW)
Unit 1 (Bus1)
Unit 2 (Bus2)
Unit 3 (Bus3)
Pt @ 152 MW (U1)
Pt @ 55 MW (U2)
Pt @ 111 MW (U3)
U1
U2
U3


Contingency Incremental Cost Curves - Fixed Unit 3
Incremental Cost ($/MWh)
8.8
8.6
8.4
8.2
8
7.8
7.6
7.4
7.2
7
6.8
0
50
100
150
200
250
300
350
400
450
Power Output (MW)
Unit 1 (Bus1)
Unit 3 (Bus3)
Pt @ 235 MW (U1)
Pt @ 85 MW (U3)
Marginal Price @7.66


Contingency Incremental Cost Curves - Neither Generator Fixed
Incremental Cost ($/MWh)
8.8
8.6
8.4
8.2
8
7.8
7.6
7.4
7.2
7
6.8
0
50
100
150
200
250
300
350
400
Power Output (MW)
Unit 1 (Bus1)
Unit 3 (Bus3)
Pt @ 178 MW (U1)
Pt @ 141 MW (U3)
Marginal Price @7.43