Small signal stability For a SMIB

February 12^{th} 2025,

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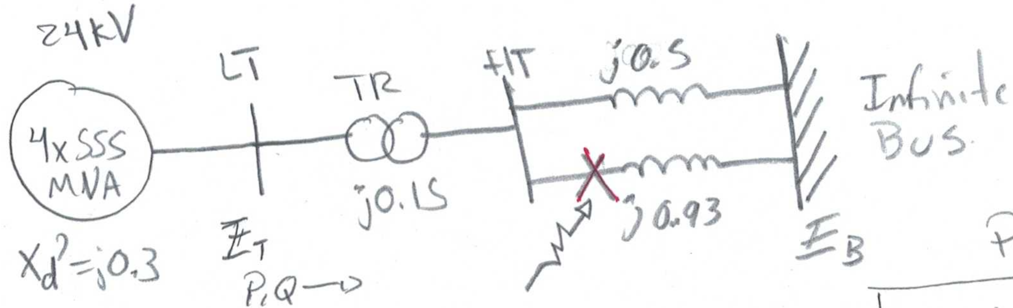
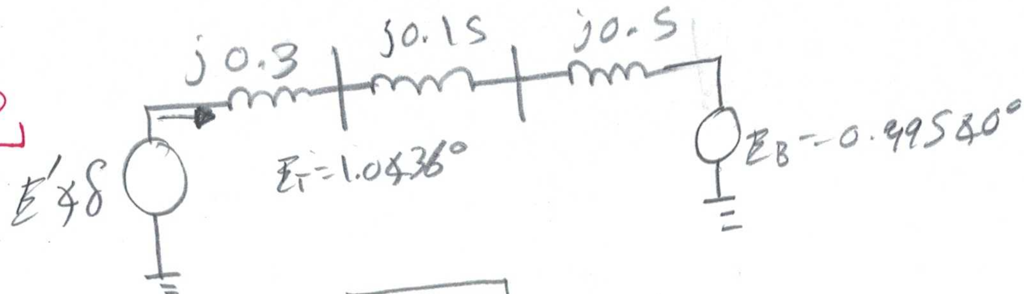


Post fault

$E_T = 1.0\angle 36°$, $E_B = 0.995\angle 0°$

$H = 3.5 MW\cdot s/MVA$, $P = 0.9$, $Q = 0.3$,

$\underline{K_d = 10}$



$X_T = 0.3 + 0.15 + 0.5 = \boxed{0.95j}$

Using $\underbrace{E_T \text{ as reference}}_{1\angle 0°}$ to get E':

$E_T = E' - jX_d'I \rightarrow E' = E_T + jX_d'I$; $I = \frac{S^*}{V} = \frac{(P+jQ)^*}{V}$

$I = \frac{0.9 - 0.3j}{1\angle 0°} = \boxed{0.9 - 0.3j}$; $E' = (1\angle 0°) + (0.3j)(0.9 - 0.3j)$

$\underbrace{E' = 1.123\angle 13.91°}$ Referenced to E_T

Now $\underline{E' \text{ referenced to } E_B}$:

$E' = 1.123\angle(13.91° + 36°) = \underline{1.123\angle 49.91°}$

* $\Delta\dot{\vec{x}} = A\cdot\vec{x} + \cancel{Bu}$, state variables. δ and ω $P_e = \frac{|V_1||V_2|}{X_T}\sin(\delta)$

*① $\frac{d\delta}{dt} = \omega_0 \cdot \omega_r$, *② $\frac{d\omega_r}{dt} = \frac{1}{2H}(P_m - P_e - K_d\omega_r)$

rated angular vel   rotor angular vel.   Damping factor

To linearize the differential equations (1) and (2) in page 1 we must apply Taylor series in order to construct the Jacobian

* Taylor series and the Jacobian:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$ * Jacobian

$$P_e = \frac{|V_1||V_2|}{x_T} \sin\delta$$

$$\omega_r = x_1, \; f_1(x_1) = \frac{1}{2H}(P_m - P_e - K_d\omega_r)$$

$$\delta = x_2, \; f_2(x_2) = \omega_0 \omega_r.$$

$$* \frac{\partial f_1}{\partial \omega_r} = \frac{1}{2H}(-K_D) \; ; \; * \frac{\partial f_1}{\partial \delta} = \frac{-1}{2H}\underbrace{\left(\frac{|V_1||V_2|}{x_T}\cos\delta\right)}_{K_s}$$

K_s ← synchronizing coefficient.

$$* \frac{\partial f_2}{\partial \omega_r} = \omega_0 \; ; \; * \frac{\partial f_2}{\partial \delta} = 0.$$

Δ linearized

$$\Delta\dot{x} = \underline{A\Delta x} + \cancel{B\Delta U}$$

$$A = \begin{bmatrix} -\frac{1}{2H}K_D & -\frac{1}{2H}K_S \\ \omega_0 & 0 \end{bmatrix}$$

$K_D = 10$

$\omega_0 = 2\pi(60) = 377 \frac{rad}{s}$. Radians.

$K_s = \frac{|E'||E_B|}{x_T} \cos(\delta) = \Big($ 0.8954 pu-torque / grad. Grades NOT Radians.

0.757 " /rads.

$$\begin{bmatrix} \Delta\dot{\omega}_r \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2H}K_D & -\frac{1}{2H}K_S \\ \omega_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega_r \\ \Delta\delta \end{bmatrix} \; ; \; A = \begin{bmatrix} -1.43 & -0.108 \\ 377 & 0 \end{bmatrix}$$

* Eigen values.

$$\det(A - \lambda I) = 0$$

$$\boxed{\zeta = \frac{1.43}{(2)6.38} = 0.112}$$

$$\omega_n = \sqrt{40.716}$$

$$\boxed{\omega_n = 6.387 \frac{rad}{s}}$$

$$\boxed{\omega_n = 1.0165 \text{ Hz}}$$

$$\det\begin{bmatrix} -1.43-\lambda & -0.108 \\ 377 & -\lambda \end{bmatrix} = \lambda^2 + \underbrace{1.43\lambda}_{2\zeta\omega_n} + \underbrace{40.716}_{\omega_n^2}.$$

$$\boxed{\lambda_{1,2} = -0.715 \pm 6.34j}$$

* Damped frequency.

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 1.0165\left(\sqrt{1-0.112^2}\right) = \boxed{1.0101\,Hz}$$

* Eigenvectors.

* Right eigenvectors

$\lambda = -0.715 + 6.34j$

$$(A - \lambda I)\phi = 0 \qquad \begin{bmatrix} -1.43-\lambda & -0.108 \\ 377 & -\lambda \end{bmatrix} \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} (-0.715-6.34j)\phi_{11} - 0.108\,\phi_{21} &= 0 \\ 377\,\phi_{11} + (0.715-6.34j)\phi_{22} &= 0 \end{aligned}\right\}$$ n-1 linearly independent must chose 1.

$\phi_{21} = 1$, $\phi_{11} = -0.0019 + 0.0168j$

$\phi_{22} = 1$ $\phi_{12} = -0.0019 - 0.0168j$

$$\phi = \begin{bmatrix} -0.0019+0.0168j & -0.0019-0.0168j \\ 1 & 1 \end{bmatrix}$$

* Left eigenvecters

$$\psi = \phi^{-1} = \begin{bmatrix} -29.7619j & 0.5-0.0565j \\ 29.7619j & 0.5+0.0565j \end{bmatrix}$$

* Participation factor.

eigenvalues

$$P_i^{\circ} = \begin{bmatrix} \phi_{11}\psi_{11} & \phi_{12}\psi_{21} \\ \phi_{21}\psi_{12} & \phi_{22}\psi_{22} \end{bmatrix}$$

$$P = \begin{bmatrix} 0.5+0.0565j & 0.5-0.0565j \\ 0.5-0.0565j & 0.5+0.0565j \end{bmatrix}$$

* Time response.

$$\Delta x(t) = \sum_{i=1}^{n} \phi_i c_i e^{\lambda_i t} \longrightarrow \Delta x(t) = \sum_{i=1}^{2} \phi_i c_i e^{\lambda_i t}$$

$$\begin{bmatrix} \Delta \dot{\omega}_r \\ \Delta \dot{\delta} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ c_2 e^{\lambda_2 t} \end{bmatrix}$$

* Applying initial conditions to get the constants.

$$\begin{bmatrix} \Delta \dot{\omega}_r(0) \\ \Delta \dot{\delta}(0) \end{bmatrix} \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$\Delta \dot{\omega}_r(0) = 0$

$\Delta \dot{\delta}(0) = 0.0873 \text{ rad}.$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.0436 - j0.0049 \\ 0.0436 + j0.0049 \end{bmatrix}$$

$$\Delta \omega_r(t) = c_1 \phi_{11} e^{\lambda_1 t} + c_2 \phi_{12} e^{\lambda_2 t}$$

$$\Delta \delta(t) = c_1 \phi_{21} e^{\lambda_1 t} + c_2 \phi_{22} e^{\lambda_2 t}$$

$$* \Delta \omega_r(t) = (0.0436 - 0.0049j)(-0.0019 + 0.0168j) e^{(-0.715+6.34j)t}$$
$$+ (0.0436 + 0.0049j)(-0.0019 - 0.0168j) e^{(0.715-6.34j)t}$$

$$* \Delta \omega_r(t) = (-0.0015) e^{-0.714t} e^{j6.35t} + (-0.0015) e^{-0.714t} e^{-j6.35t}$$

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$$

$$\Delta\omega_r(t) = -0.0015\, e^{-0.714t}\left(2j\sin(6.35t)\right)$$

$$\Delta\omega_r(t) = \mathrm{Re}\left\{-0.0015\, e^{-0.714t}\left(2j\sin(6.35t)\right)\right\}$$

The factor of 2 dissapears because the real physical responce comes from the sum of two complex exponentials so we only take the real part. $(e^{j\omega t}, e^{-j\omega t})$

$$\boxed{\Delta\omega_r(t) = -0.0015\, e^{-0.714t}\left(\sin(6.35t)\right)}$$

$$\Delta\delta(t) = (0.0436 - j0.0049)(1)\, e^{(-0.715+6.34j)t} + (0.0436 + 0.0049)(1)\, e^{(-0.715-6.34j)t}$$

$$\underline{\Delta\delta(t) = 0.0872\, e^{-0.714t}\cos(6.35t)} \qquad e^{j\theta} + e^{-j\theta} = 2\cos\theta.$$

$\boxed{K_D = 0}$

$$A = \begin{bmatrix} 0 & -\frac{1}{2H} K_S \\ \omega_0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -0.108 \\ 377 & 0 \end{bmatrix}$$

Calculating eigenvalues $\det(A - \lambda I) = 0$ to form th char polynomial

$$\boxed{\lambda_{1,2} = 0 \pm 6.38j} \quad \boxed{\omega_n = 1.0165\ Hz} \quad \zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 1.0165\sqrt{1-0} \boxed{= 1.0165 Hz} \quad \boxed{\zeta = 0}$$

* Obtaing the eigenvectors. $(A - \lambda I)\Phi = 0$

$$\Phi = \begin{bmatrix} 0 + 0.0169j & 0 - 0.0169j \\ 1 & 1 \end{bmatrix} ; \quad \Psi = \begin{bmatrix} -29.545j & 0.5 + 0j \\ 29.545j & 0.5 + 0j \end{bmatrix}$$

* Participation factor.

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

* Time response

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -29.545j & 0.5 \\ 29.545j & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0.0873 \end{bmatrix} = \begin{bmatrix} 0.0437 \\ 0.0437 \end{bmatrix}$$

$$\Delta\omega_r(t) = (0.0437)(0.0169j)e^{6.38jt} - 0.0437(0.0169j)e^{-6.38jt}$$

$$\Delta\omega_r(t) = -0.00147 \sin(6.38t).$$

$$\Delta\delta(t) = (0.0437)(1)e^{6.38jt} + (0.0437)(1)e^{-6.38jt}$$

$$\Delta\delta(t) = 0.0874 \cos(6.38t)$$

$\boxed{K_D = -10}$

$$A = \begin{bmatrix} \frac{5}{H} & -\frac{1}{2H} K_S \\ \omega_0 & 0 \end{bmatrix} = \begin{bmatrix} 1.428 & -0.108 \\ 377 & 0 \end{bmatrix}$$

* Eigen values $\det(A - \lambda I) = 0$

$\boxed{\lambda_{1,2} = 0.7140 \pm 6.3408j}$ $\boxed{\omega_n = 1.0092 Hz}$ $\boxed{\zeta = -0.112}$

$$\omega_d = 1.0092\sqrt{1-(-0.112)^2} = \boxed{1.0029 Hz}$$

* Eigenvectors

$$\Phi = \begin{bmatrix} 1 & 1 \\ 0.0019 + 0.0168j & 0.0019 - 0.0168j \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.5 + 0.0563j & -29.73j \\ 0.5 - 0.0563j & 29.73j \end{bmatrix}$$

* Participation factor

$$P = \begin{bmatrix} 0.5 + 0.0563j & 0.5 - 0.0563j \\ 0.5 - 0.0563j & 0.5 + 0.0563j \end{bmatrix}$$

* Time response

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0.5 + 0.0563j & -29.73j \\ 0.5 - 0.0563j & 29.73j \end{bmatrix} \begin{bmatrix} 0 \\ 0.0873 \end{bmatrix} = \begin{bmatrix} -2.5956j \\ 2.5956j \end{bmatrix}$$

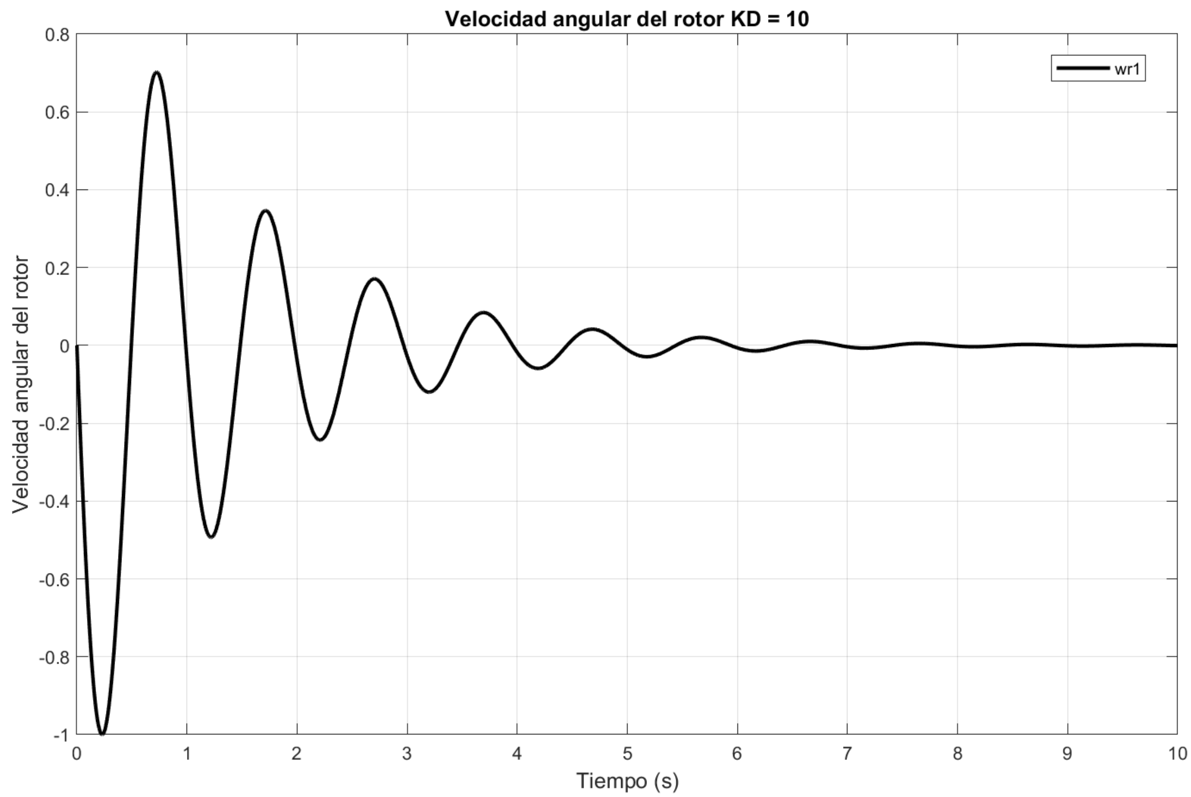
*Time Response

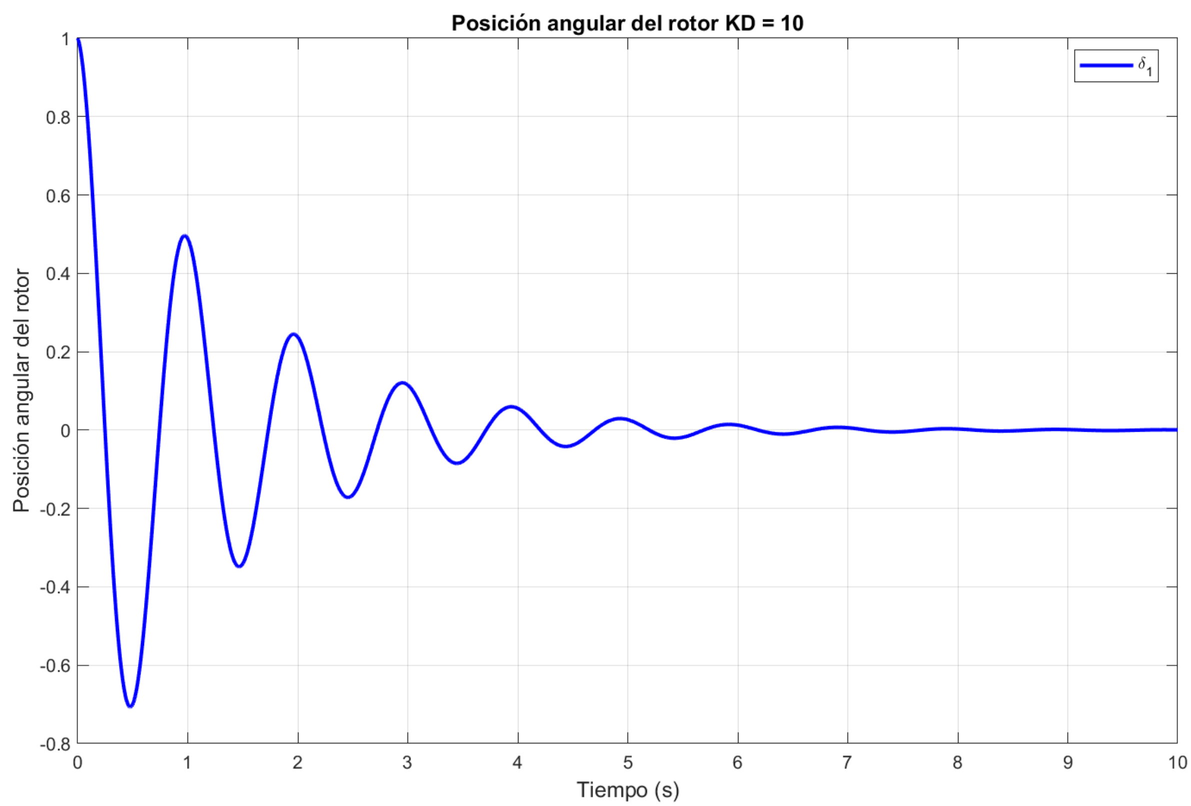
$$\Delta wr(t) = (-2.5956j)(1)e^{(0.7140+6.34j)t} + (2.5956j)(1)e^{(0.7140-6.34)t}$$

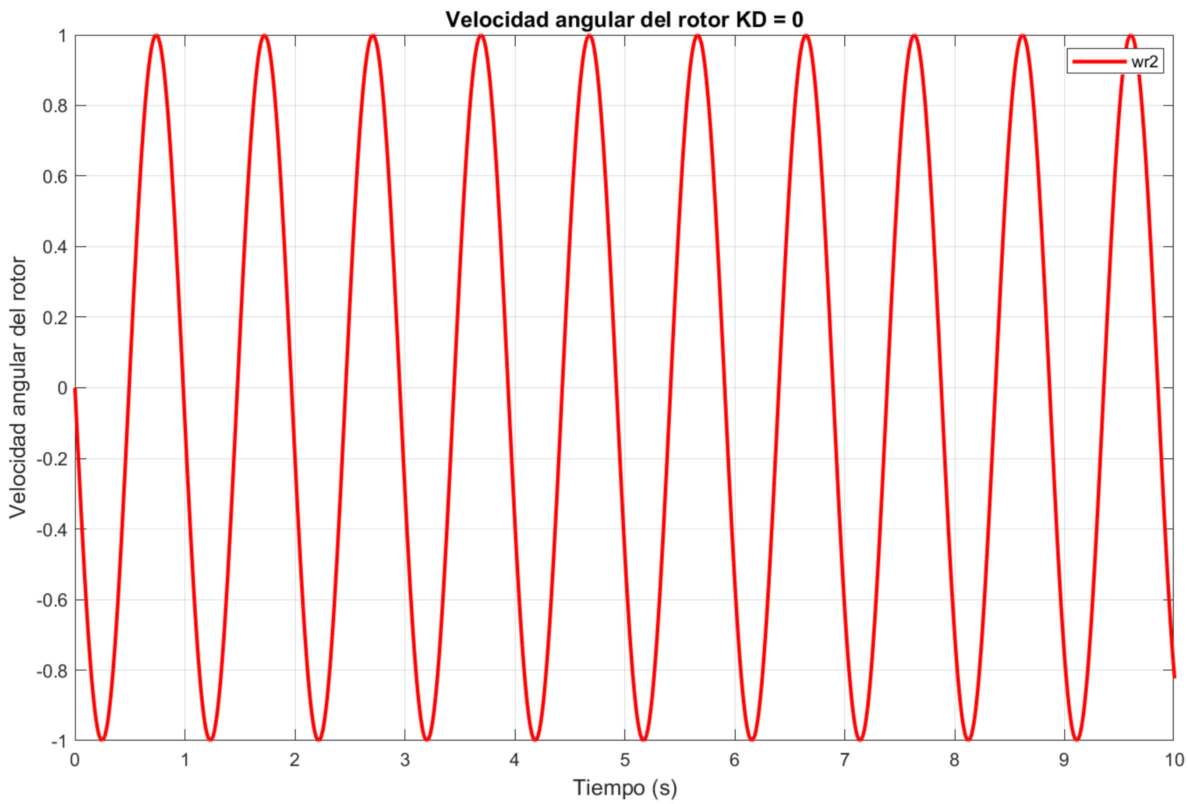
$$\underline{\Delta wr(t) = 5.1912e^{0.7140t}\sin(6.34t)}$$

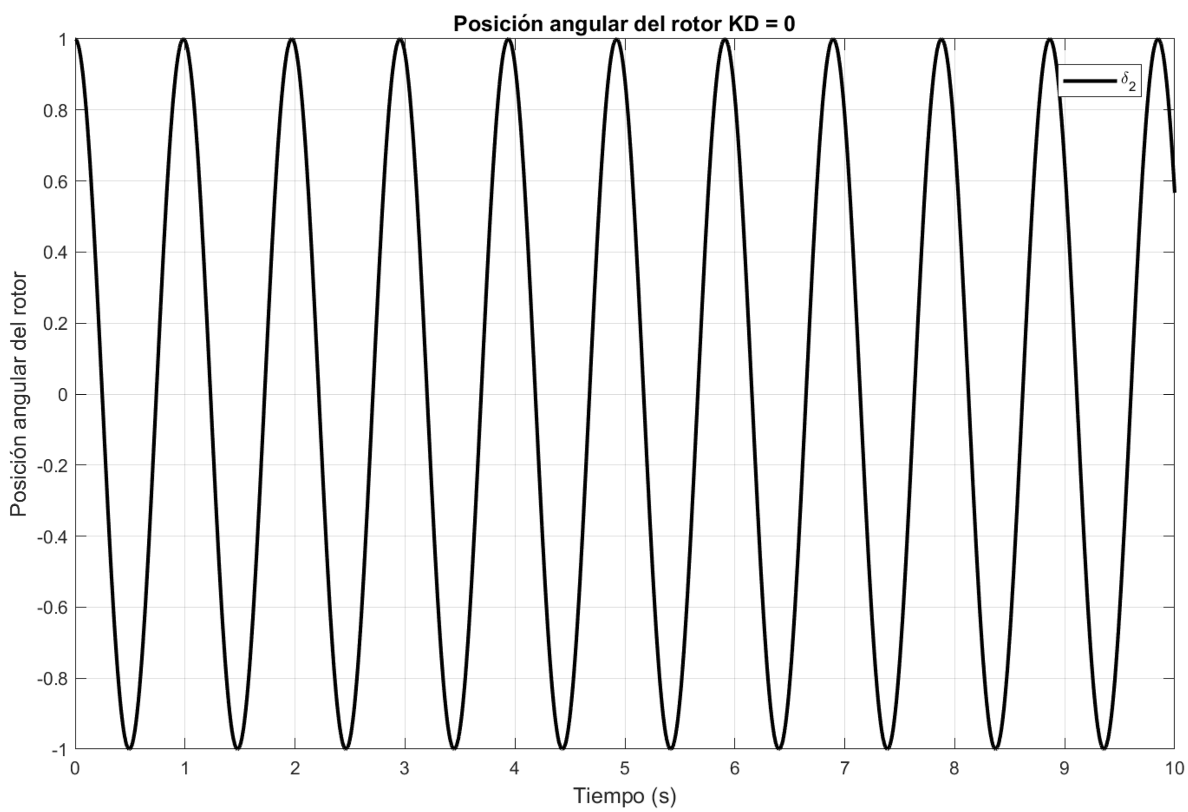
$$\Delta\delta(t) = (-2.5956j)(0.0019 + 0.0168j)e^{(0.7140+6.34j)t}$$
$$+ (2.5956j)(0.0019 - 0.0168j)e^{(0.7140+6.34j)t}$$

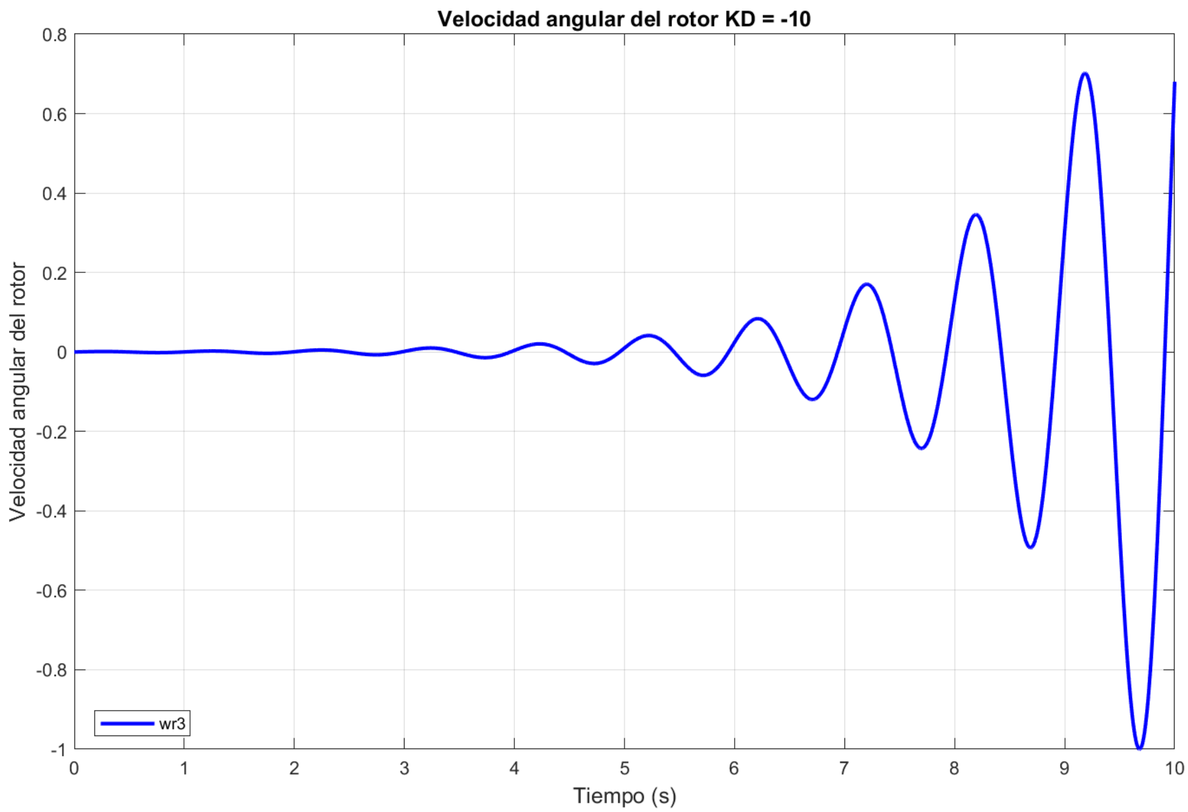
$$\underline{\Delta\delta t = 0.0872e^{0.7140t}\cos(6.34t) + 0.00986e^{0.7140t}\sin(6.34t)}$$

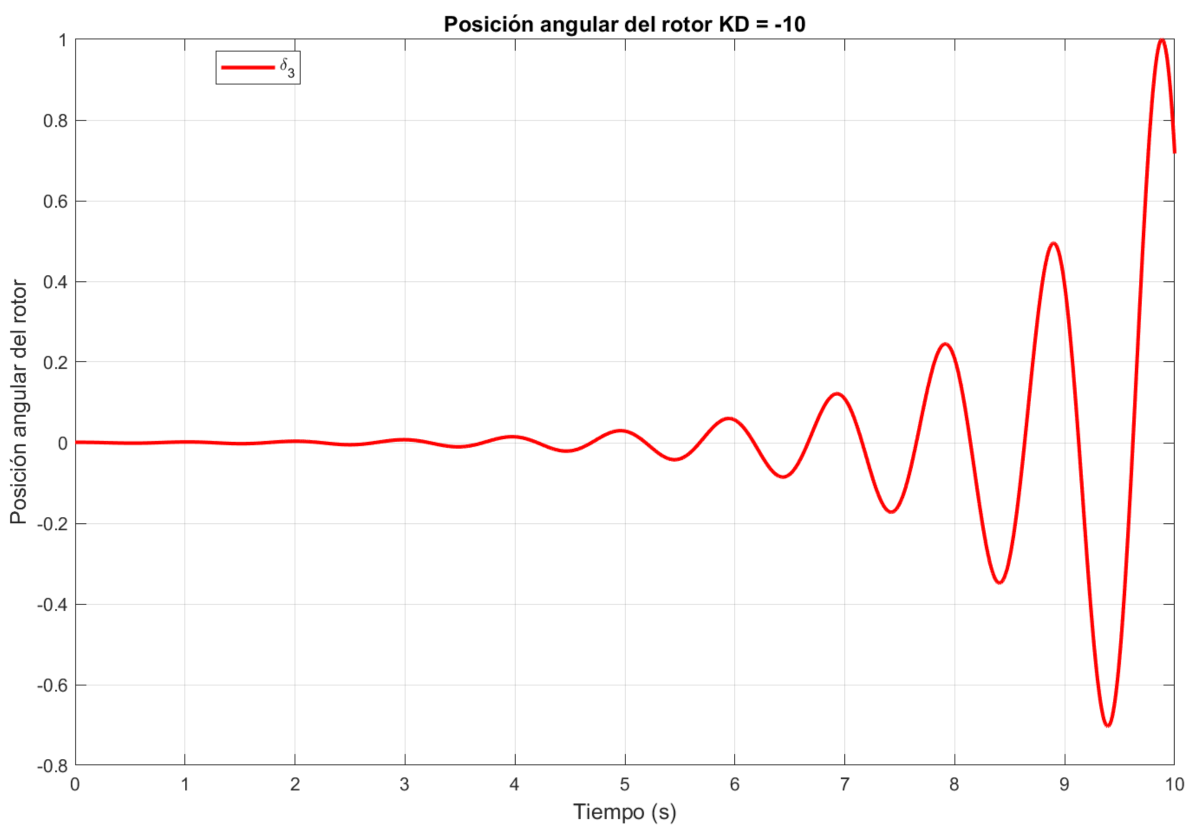

Velocidad angular del rotor KD = 10
wr1
Velocidad angular del rotor
Tiempo (s)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10

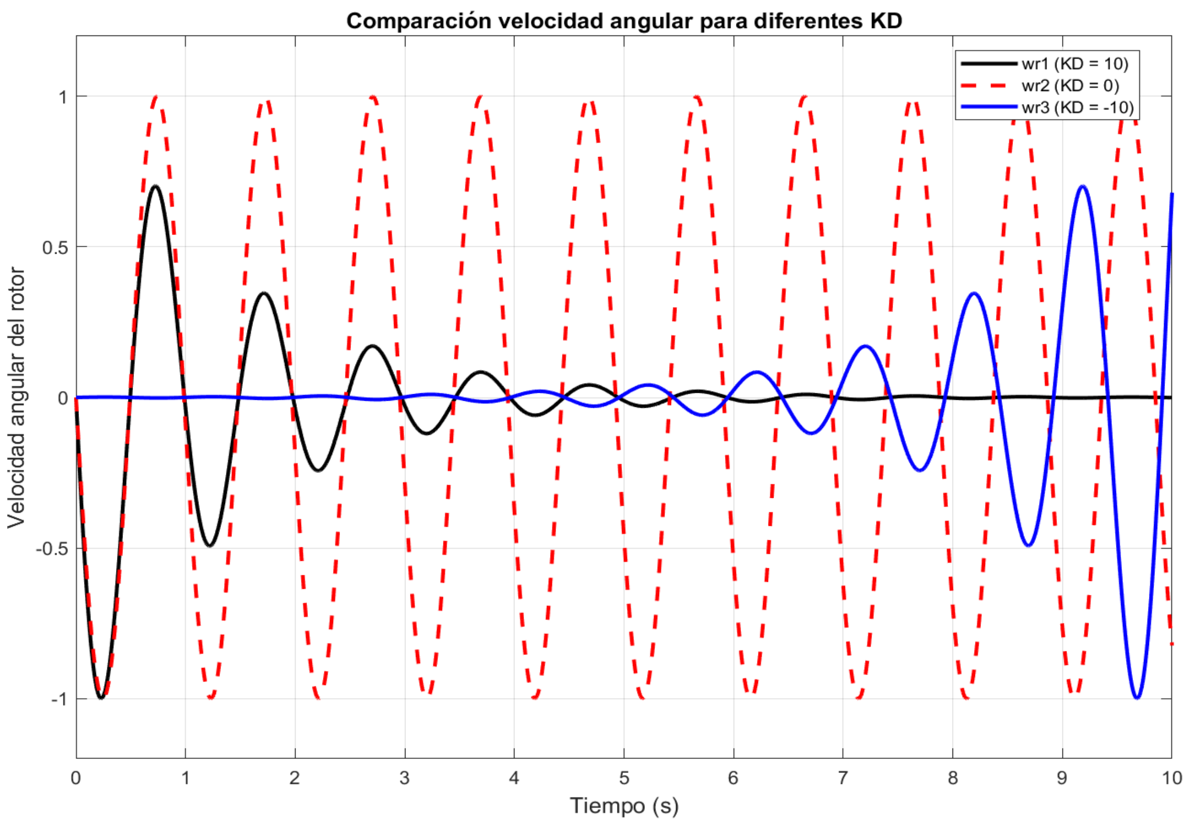

Posición angular del rotor KD = 10
δ_1
Posición angular del rotor
Tiempo (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1
2
3
4
5
6
7
8
9
10

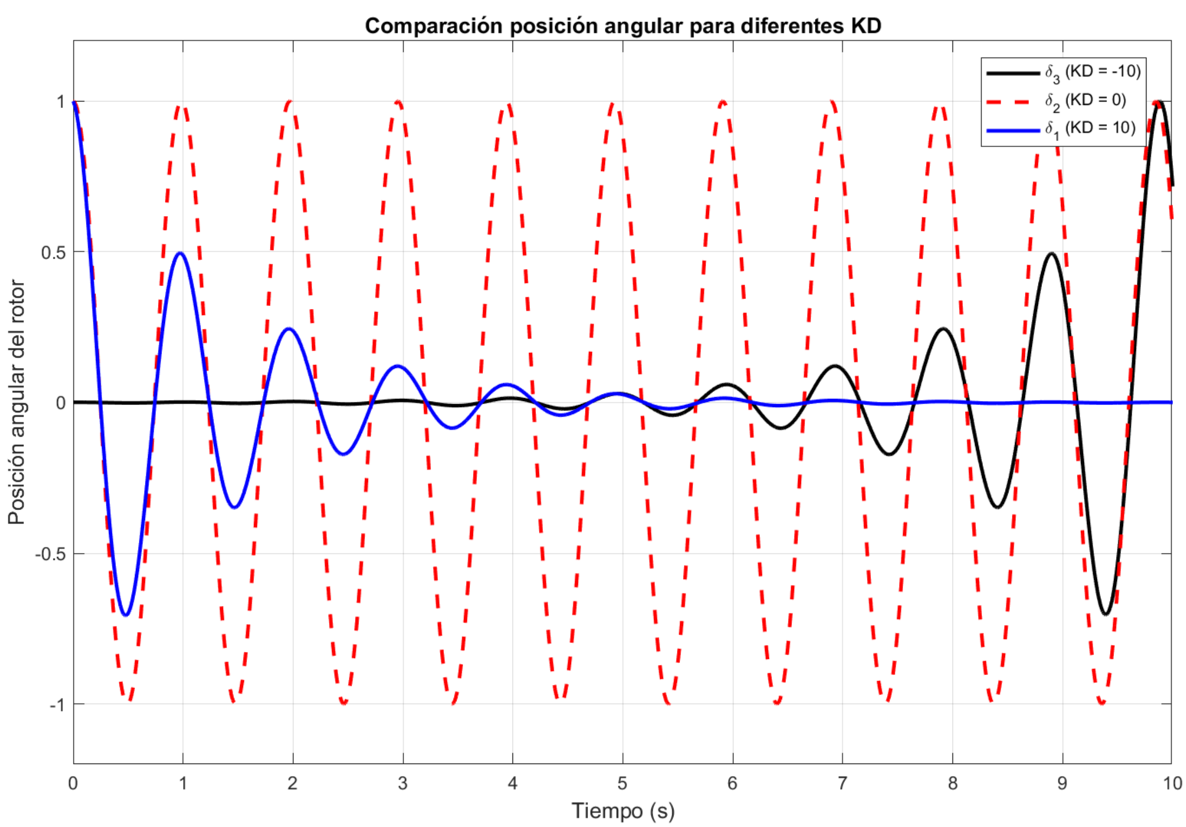

Velocidad angular del rotor KD = 0
wr2
Velocidad angular del rotor
Tiempo (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10


Posición angular del rotor KD = 0
δ_2
Posición angular del rotor
Tiempo (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10


Velocidad angular del rotor KD = -10
Velocidad angular del rotor
Tiempo (s)
wr3
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10


Posición angular del rotor KD = -10
Posición angular del rotor
Tiempo (s)
δ_3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1
2
3
4
5
6
7
8
9
10


Comparación velocidad angular para diferentes KD
wr1 (KD = 10)
wr2 (KD = 0)
wr3 (KD = -10)
Velocidad angular del rotor
Tiempo (s)
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9
10


Comparación posición angular para diferentes KD
δ3 (KD = -10)
δ2 (KD = 0)
δ1 (KD = 10)
Posición angular del rotor
Tiempo (s)
1
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9
10