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Forced Oscillations in Power Systems: Mechanisms, Linear and Nonlinear Analysis.

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Summary: This research addresses the challenge of forced oscillations in electric power systems, which are sustained oscillations driven by external sources like equipment malfunctions. The primary risk occurs when the forcing frequency resonates with a system's natural mode, leading to dangerously amplified oscillations that can threaten grid stability and cause blackouts. While detection methods exist, the underlying nonlinear dynamics of these phenomena are not fully understood, as most current analytical tools rely on linear models. To bridge this gap, this work proposes a nonlinear analytical approach using the multiple scale method to obtain closed-form solutions.

Keywords: **Forced Oscillations, Power Systems, Stability, Resonance, Dynamical Systems, Multiple Scales, Linear Analysis, Closed Solutions, Analytical Approach.**

Motivation

In modern electric power systems (EPS), forced oscillations (FO) often originate from the misoperation or malfunction of control systems and mechanical components. These oscillations can become critically amplified when the frequency of the external perturbation is close or equal to the system's natural modes, leading to resonance conditions.

Forced oscillations can limit power transfer, cause protective devices to trip elements out of the system, damage equipment, and in severe cases, lead to massive blackouts.

Background

The operation of electric power systems is becoming more complex due to the interconnection of large scale networks, the growing demand for energy, and the massive installation of converter-based equipment. This has led to an increase in oscillation events. Oscillatory phenomena in power systems can be classified into two main types: natural oscillations (NO) and forced oscillations (FO). While the natural response reflects the characteristics of the system, the forced response reflects the characteristics of the external disturbance [1].

Unstable or poorly damped systems can naturally exhibit sustained oscillations. However, when the natural oscillation modes of the system are stable, the natural oscillation is damped, and the system returns to a new operating point. In the presence of an external input, the system may exhibit a sustained oscillation even if it is stable [2]. Figure 1 shows this fact.

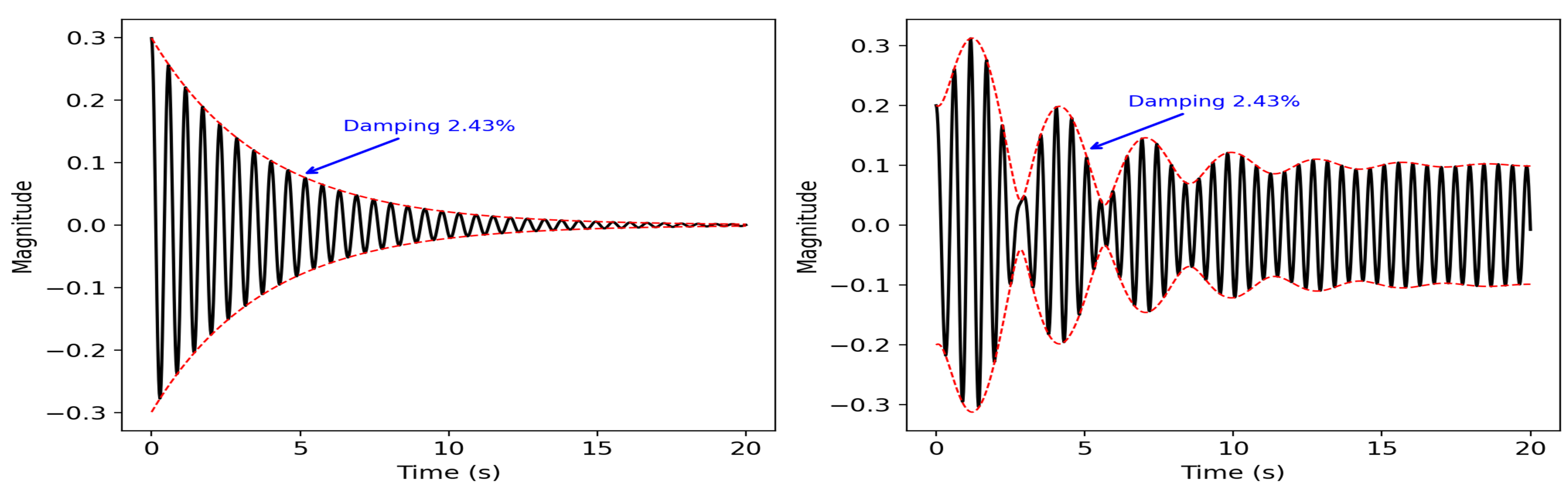

Figure 1: Comparison between the natural response without forcing and the system response under a periodic input at 1.5 times the natural frequency.

A forced oscillation by itself does not represent instability. The problem arises when the FO interacts with the system's modes. This interaction can lead to resonance conditions, significantly increasing the oscillation amplitude and leading to a potential risk of system collapse.

The **mitigation** of forced oscillations consists of repairing or disconnecting the source that causes the oscillation, although increasing damping can help reduce its impact on the system. Figure 2 shows the sources of forced oscillations reported in scientific literature with their characteristic frequency range.

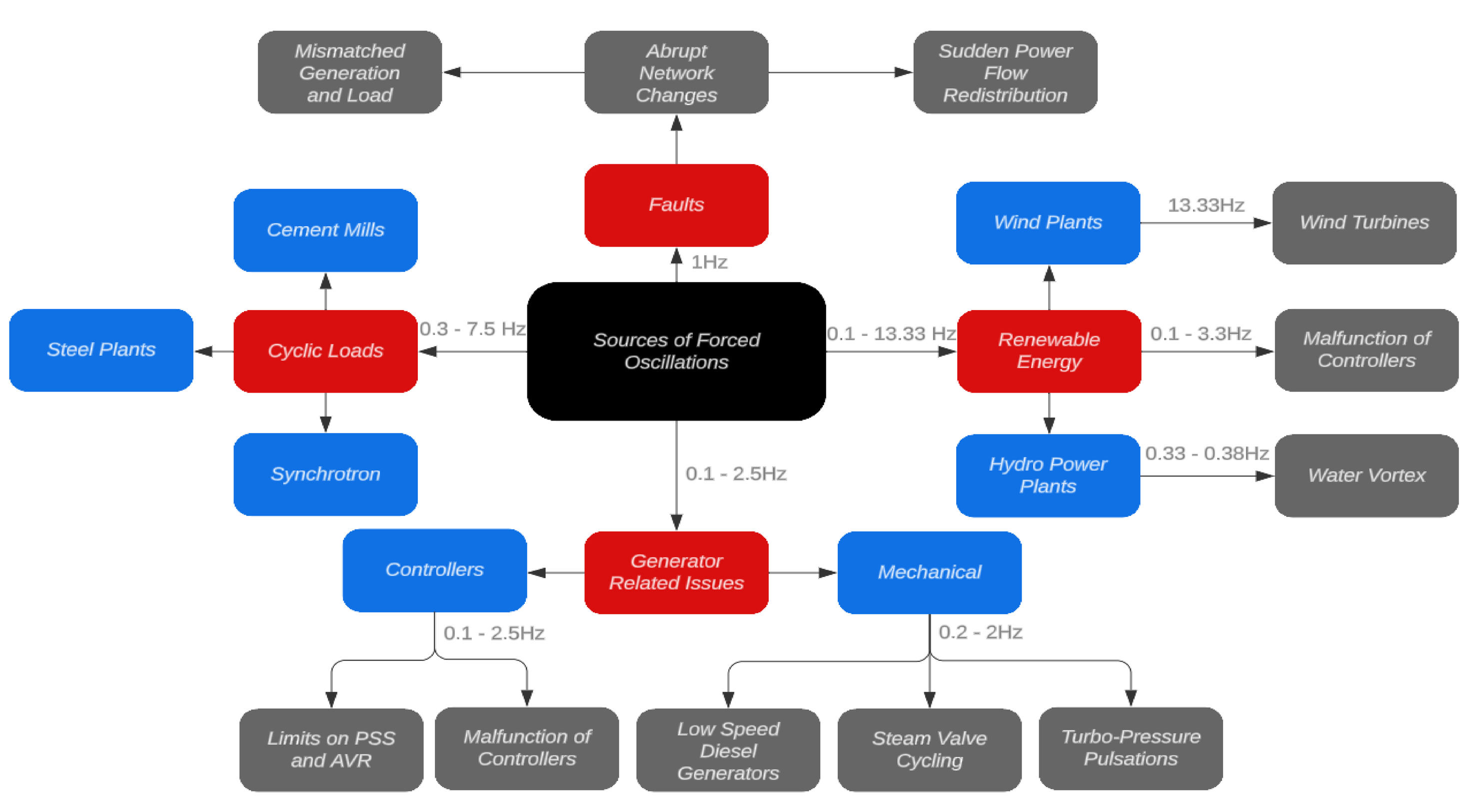

Figure 2: Sources of Forced Oscillations and Their Characteristic Frequencies

Figure 2 indicates that forced oscillations in electric power systems may arise from various components within the network, including both the generation and load sides. These sources cover a wide range of frequencies, from fractions of a hertz up to more than 13 Hz, and can interact with the system's natural modes and generate resonance phenomena. Small forced resonant oscillations can excite large amplitude responses, an example of this is the case reported in Kundur's two-area system, where a forced oscillation of 10MW produced a response of 477MW [3].

On one hand, table 1 presents some of the events documented in the literature that have been associated with forced oscillations. On the other hand, table 2 shows a brief comparison of methods to analyze, detect and locate forced oscillations.

Table 1: Selected Documented Forced Oscillation Events

No.	Date	Location	Source	Magnitude	Duration	Freq (Hz)
1	June 1992	Rush Island (East)	Failure of an insulator, line switching, system re-configuration	280 MW	37 min	1.0
2	Oct. 2009	South Texas, ERCOT	Sub-synchronous control interaction between DFIG wind turbines and series compensated line	Voltages, currents 300%	400 ms	20
3	June 17, 2016	GGNS, Mississippi	Control valve malfunction	200 MW	45 min	0.27

Table 2: Comparison of Forced Oscillation Analysis, Detection, and Localization Methods

Method	Analysis	Detection	Localization	Method	Analysis	Detection	Localization
Small Signal Linearization	✓	X	X	Metrics Based	X	✓	✓
Transfer Function (Linear)	✓	X	X	Data-Driven Algorithms	X	✓	✓
Nonlinear Perturbation Methods	✓	X	X	Transform-Based Algorithms	X	✓	✓
Koopman Operator	✓	X	X	Machine Learning Algorithms	X	✓	✓

Problem Statement

Numerous incidents of forced oscillations have motivated a wide range of measurement-based detection and localization algorithms. However, the phenomenon itself remains insufficiently understood, particularly the variables that initiate, sustain, or amplify these oscillations. Moreover, most analytical studies and practical tools rely on linear models. Hence, a systematic investigation of the nonlinear characteristics of forced oscillations in power systems is still required.

Justification

Forced oscillations in power systems, typically ranging from 0.1 and 2.5 Hz, pose a latent risk when their frequency coincides with poorly damped modes. This coincidence can produce large sustained oscillations that may evolve toward instability leading to thousands of families affected and economic losses. In México, a 45 minutes, 200MW forced oscillation can be translated into approximately 200,000 families affected and 7,140,900 MXN in economic losses [4].

This research proposes an analytical approach based on linear analysis and the nonlinear multiple-scale method, to provide insights of the phenomena and develop indices that quantify interactions among variables and modes.

Objectives

General objective: Conduct a nonlinear analysis to a power system to obtain closed-form solutions and find insights about its forced response.

Specific objectives: i) Analyze the dynamics of linear systems, including the single machine infinite bus (SMIB) system, under periodic excitation. ii) Conduct an analysis to a nonlinear forced power system using the method of multiple scales, derive closed-solutions and propose impact quantification indices. iii) Validate the multiple-scales equations and indices through software simulations. iv) Propose a mitigation action to address forced oscillations in power systems based on the analytical results.

Hypothesis

Through perturbation analysis of nonlinear power system models, closed-form expressions can be derived that offer a predictive and causal understanding of forced resonance phenomena and their dependence on system parameters.

Methodology

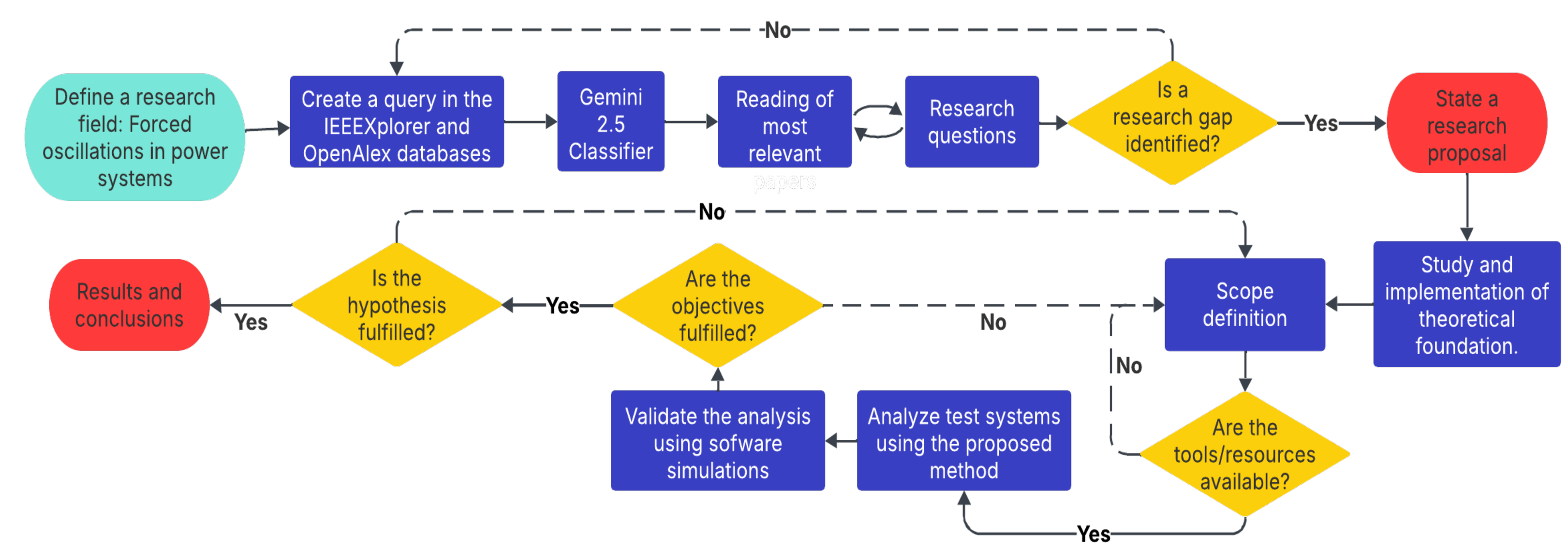

Figure 3: Methodology and general workflow

Preliminary Results

There are several physical systems that can be modeled with this mathematical model, such as a mass, spring and damper, the small angle simple pendulum, a torsional oscillator and also the linearized SMIB. The canonical second order system under resonance is presented in equation (1).

$$\left(\frac{d^2\theta}{dt^2}\right) + 2\zeta\omega_0\left(\frac{d\theta}{dt}\right) + \omega_0^2\theta = \gamma\cos(\omega_0 t) \tag{1}$$

The closed solutions for position and velocity are displayed in equations 2 and 4. To obtain the envelopes, the analytic signals are required, these are obtained by performing the Hilbert transform of each term. Then we add them up and calculate the magnitude of the phasor sum.

$$\theta(t) = \underbrace{e^{-\zeta\omega_0 t}\left[\theta_0\cos(\omega_d t) + \frac{1}{\omega_d}\left(v_0 + \zeta\omega_0(\theta_0) - \frac{\gamma}{2\zeta\omega_0}\right)\sin(\omega_d t)\right]}_{\text{Natural Response}} + \underbrace{B_p\sin(\omega t)}_{\text{Particular Response}} \tag{2}$$

$$a_{pn}(t) = \sqrt{|C_h|^2 e^{-2\zeta\omega_0 t} + B_p^2 + 2|C_h|B_p e^{-\zeta\omega_0 t}cos\Big(-\frac{\omega_0\zeta^2}{2}t + (\phi_h - \phi_p)\Big)} \tag{3}$$

$$\frac{d\theta}{dt} = \underbrace{e^{-\zeta\omega_0 t}\Big[\big(-\zeta\omega_0\theta_0 + \omega_d B_h\big)\cos(\omega_d t) - \big(\zeta\omega_0 B_h + \omega_d\theta_0\big)\sin(\omega_d t)\Big]}_{\text{Natural Response}} + \underbrace{B_p\omega\cos(\omega t)}_{\text{Particular Response}} \tag{4}$$

$$a_{Vpn}(t) = \omega_0\sqrt{|C_h|^2(1-\zeta^2)e^{-2\zeta\omega_0 t} + B_p^2 + 2|C_h|B_p e^{-\zeta\omega_0 t}cos\Big(-\frac{\omega_0\zeta^2}{2}t + (\phi_h - \phi_p)\Big)} \tag{5}$$

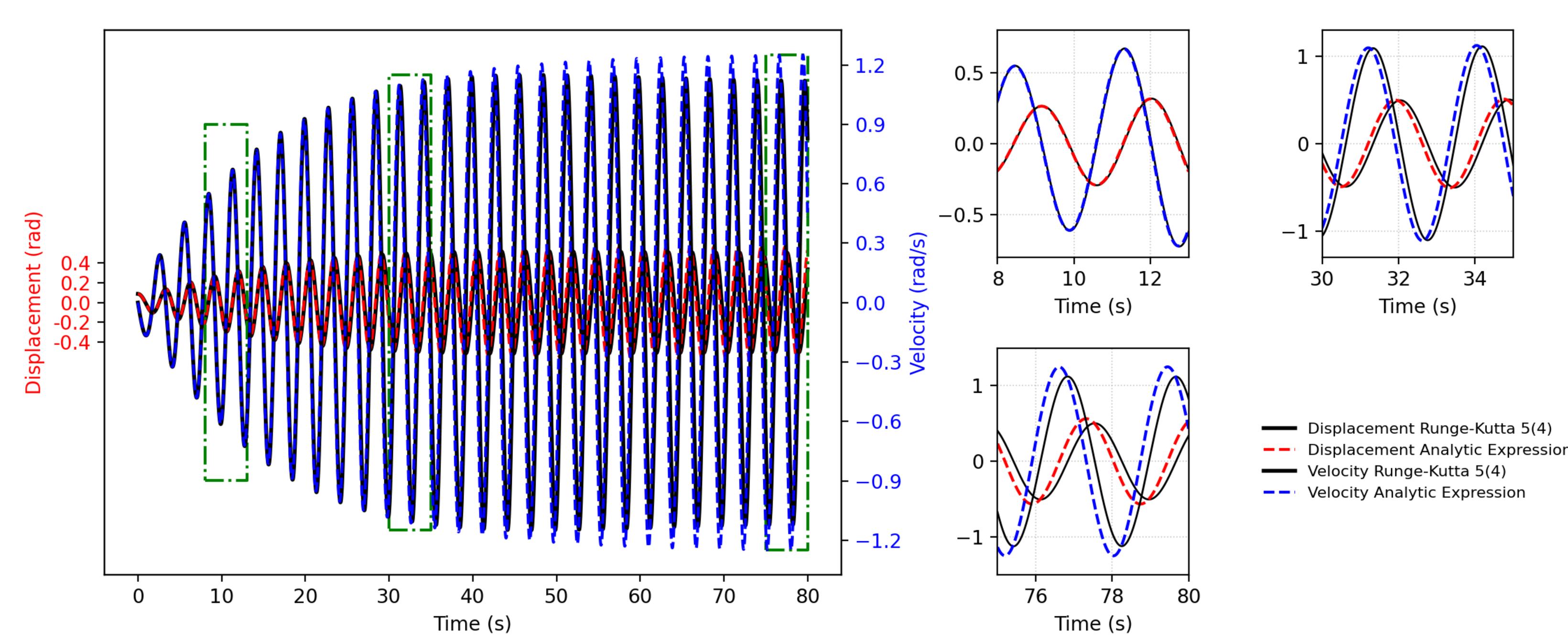

Figure 4: Comparison between the resonant linear and nonlinear system using RK45, $\gamma = 0.167$ and $(\theta_0, v_0) = (5°, 0)$.

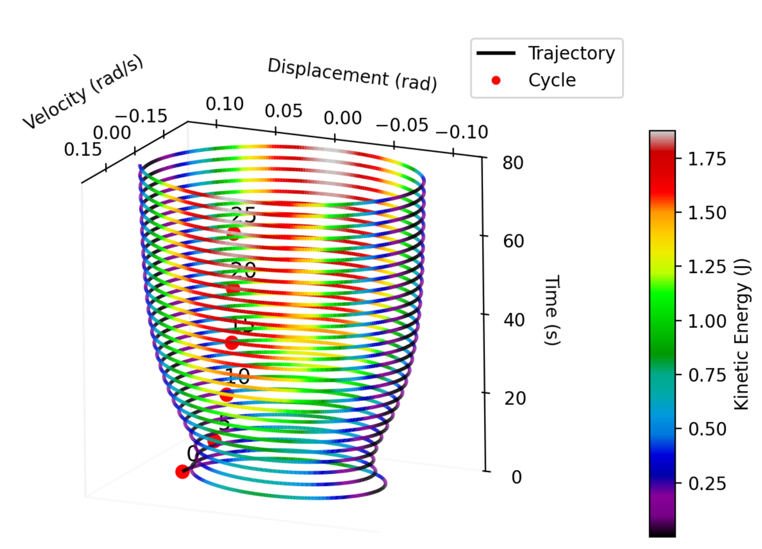

Figure 5: 3D Phase portrait and Poincaré map of the second order system total response with NZIC $(\theta_0, v_0) = (5°, 0)$ and $\gamma = 0.034$.

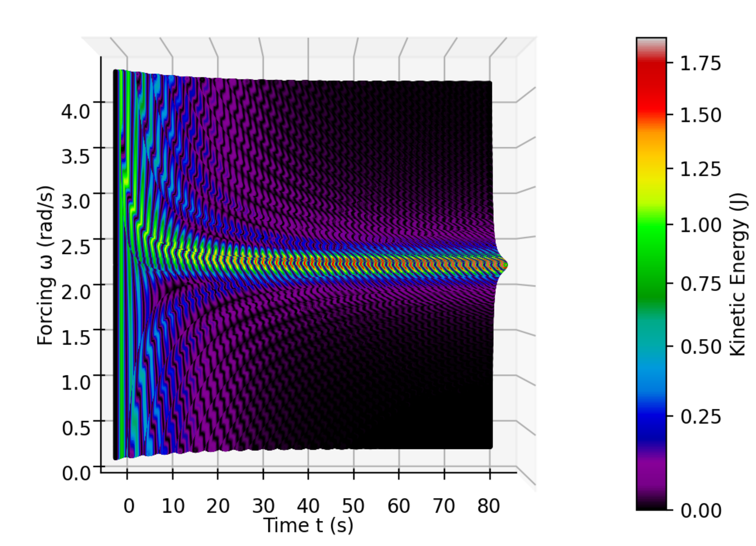

Figure 6: Second order system forcing frequency vs time with NZIC $(\theta_0, v_0) = (5°, 0)$, $\gamma = 0.034$ and its kinetic energy over time.

Preliminary Conclusions

The analytical framework for linear second order systems has been successfully established and validated. Closed-form solutions for the system's displacement, velocity, and their envelopes under resonant forcing have been derived and confirmed through numerical simulation. This provides a crucial benchmark and a verified mathematical foundation for the subsequent analysis of more complex system models.

Furthermore, the initial comparative analysis between linear and nonlinear system responses has clearly demonstrated a significant divergence in behavior for larger disturbances. This finding critically validates the core research problem, confirming that linear models are insufficient for capturing the full dynamics of forced oscillations. Thus, nonlinear analytic approaches such as the multiple-scale method are required.

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