Acronyms

AVR	Automatic Voltage Regulator
BPA	Bonneville Power Administration
BPE	Branch Potential Energy
CAISO	California Independent System Operator
CNN	Convolutional Neural Networks
ERCOT	Electric Reliability Council of Texas
FDNF	Forced Decoupled Normal Form
FO	Forced Oscillation
HVDC	High Voltage Direct Current
ISO-NE	Independent System Operator of New England
LSTM	Long Short-Term Memory
NO	Natural Oscillation
NYISO	New York Independent System Operator
OG&E	Oklahoma Gas & Electric
PMU	Phasor Measurement Unit
RMS	Root Mean Square
RNN	Recurrent Neural Networks
RPCA	Robust Principal Component Analysis
SALO	System-Agnostic Localization of Oscillations
SINDy	Sparse Identification of Nonlinear Dynamics
SMIB	Single-Machine Infinite-Bus
TEF	Transient Energy Function
WAMS	Wide-Area Measurement Systems
WECC	Western Electricity Coordinating Council

Table of Contents

Chapter 1
Introduction

1.1. Background

Forced oscillations (FO) have become a significant and recurring phenomenon in power systems, characterized by sustained periodic responses driven by persistent disturbances rather than by intrinsic modal dynamics [1]. Unlike natural oscillations, which decay due to system damping, forced oscillations persist while the external disturbance remains active and can reach amplitudes substantially larger than the forcing when its frequency aligns with the system's natural frequency (resonance), posing a significant threat to power system stability by causing cascading failures due to resonance [2].

A catastrophic event in which forced oscillations played a significant role is the Sayano-Shushenskaya incident in Russia. In this event, nine out of ten generating units, with a combined capacity of approximately 6,400 MW, were severely damaged or destroyed, resulting in the loss of 75 lives [3]. More recently, on January 11, 2019, a forced oscillation event occurred in the Eastern Interconnection of the United States. A 0.25 Hz oscillation, caused by a faulty control input of a steam turbine, propagated through the system. The resulting active power oscillations had amplitudes of 200 MW at the unit terminals and persisted for nearly 18 minutes before disconnection [4].

Distinguishing between natural and forced oscillations is crucial, as they exhibit fundamentally different characteristics, consequences and mitigation strategies [5]. Forced oscillations may occur at unpredictable frequencies over a wide spectral range, can experience frequency drift and may appear or disappear spontaneously depending on the forcing source nature. They usually contain harmonics and can have different waveforms like sinusoids, limit cycles, pulse trains, lasting from minutes to hours [6].

The analysis, detection and localization of forced oscillations are critical tasks for reliable operation of power systems. Taking mitigation actions not only enhance system stability and increase power transfer, but also support asset management and reduce operational costs for utilities [7]. Disconnection or repair of the forcing source is generally the preferred mitigation action. However, increasing system damping can also help reduce the impact of forced oscillations.

1.1.1. Sources of forced oscillations

A broad range of mechanisms can generate forced oscillations, including cyclic loads, turbine–governor interactions, malfunctioning of control loops/mechanical devices and converter-based generation [8] [9] [10]. In general, sources can be classified in 4 groups, where each group has a frequency range associated with the nature of the source as presented in Fig. 1.1.

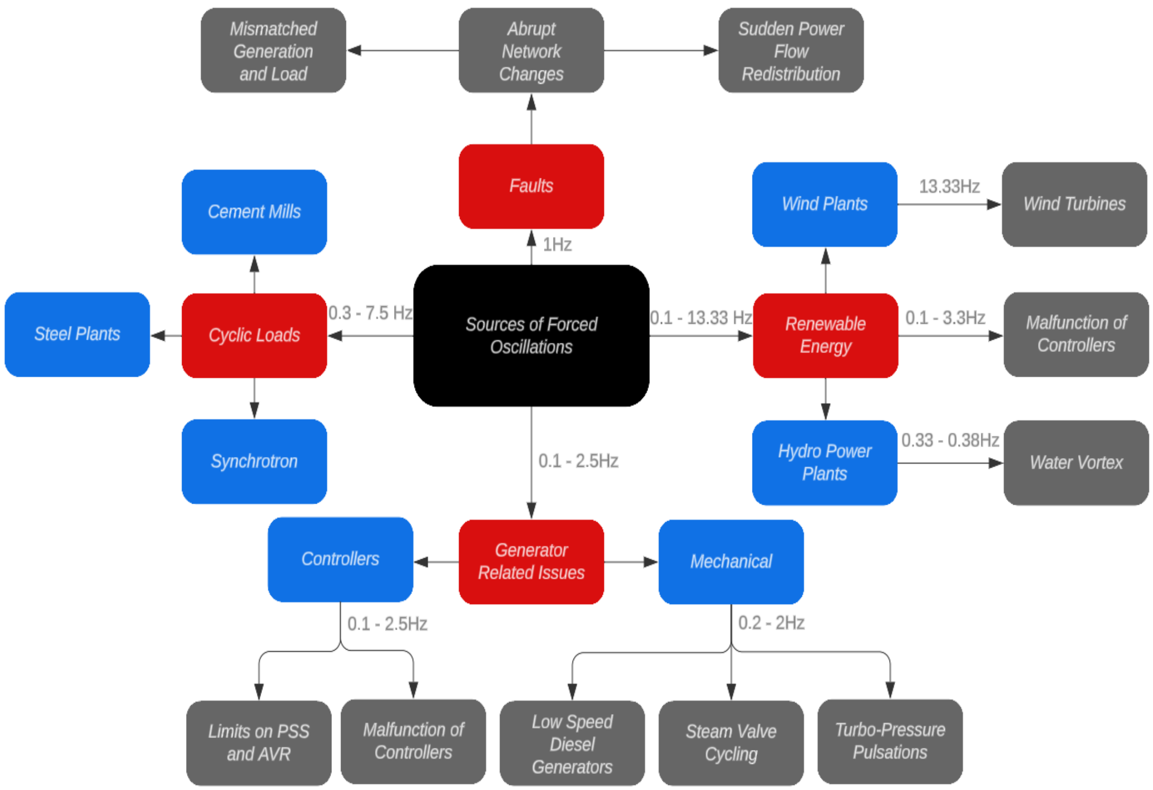

Fig. 1.1: Sources of forced oscillations and their documented frequency range

1.1.1.1. Cyclic loads

In 1966, J. Ness investigated the behavior of an electric power network in the presence of cyclic loads [11]. Several industrial processes require large amounts of energy in a cyclic manner, such as those used in cement plants, steel mills and particle accelerators [12] [13]. These loads can introduce periodic fluctuations with frequency ranges typically between 0.3 Hz and 7.5 Hz [8].

Most of the existing information on the impact of cyclic loads on electric power systems is derived from simulation studies rather than from documented field events. For instance, [14] analyzes the voltage and frequency behavior of generators supplying power to a nuclear accelerator connected at 345 kV for planning purposes.

1.1.1.2. Conventional generation units

Generation units, such as thermal units, hydroelectric plants, or diesel generator sets, may experience faults or malfunctions in various mechanical/electrical components. For instance, when excitation control systems (AVR and PSS) or frequency control systems (governors) fail or are improperly tuned, they can act as sources of forced oscillations.

In [15], the influence of excitation system and governor parameters on sustained forced oscillations is evaluated using the Kundur two-area system. Imposing limits on the field voltage in a single-machine infinite-bus (SMIB) system may induce limit cycles, which can lead to forced oscillations through Hopf bifurcations [16].

Mechanical devices associated with turbines can also introduce oscillations, these include fuel control valves in gas turbines and pressure-release pulsations in turbogenerators [17]. In hydroelectric power plants, vortices formed at the water intakes can put air into the turbine, reducing its efficiency and causing cavitation, which may lead to oscillations in the turbine. In [2], a case study reported in the western North American power system describes a forced oscillation at 0.38 Hz attributed to a vortex phenomenon in a hydroelectric power plant. Typical frequency ranges of forced oscillations associated with conventional generation units lie between 0.1 and 2.5 Hz [18] [19].

1.1.1.3. Renewable energy sources

In general, renewable energy plants exhibit oscillatory behavior associated with generation intermittency. In addition, power converters can introduce various types of oscillations into the electric power system. Forced oscillations arise from interactions between the fast control loops of converters and the grid, as well as from couplings with nearby power electronic devices. These oscillations may manifest at low frequencies, interacting with electromechanical modes, or at higher frequencies, where resonance phenomena involving control devices may occur [20].

Reported frequency ranges go from 1.5 Hz up to 14 Hz [8]. For example, [21] analyzes a case in which, under high wind conditions, a wind power plant generated a forced oscillation at 13.33 Hz. A more recent case occurred on April 28, 2025, in Spain, where a forced oscillation at 0.6 Hz was detected as part of a series of events that eventually led to a blackout, with a possible origin attributed to a photovoltaic power plant in the province of Badajoz, Spain [22].

1.1.1.4. Changes in system topology

The disconnection of system elements and other network alterations can fragment the power system into subsystems that attempt to reach their own equilibrium points. This separation may trigger oscillatory responses, creating or exacerbating forced oscillations. In [1], it is reported that incomplete islanding can lead to very severe forced oscillations. This occurs when a portion of the network becomes partially isolated while maintaining connections with low short-circuit strength. Systems based on power electronics may exhibit low short-circuit strength, which can cause the fast control loops of inverter-based resources (IBRs) to become unstable [23].

1.1.2. Forced resonance

Forced resonance occurs when the disturbance frequency is close or equal to the system natural frequency, this allows maximum energy exchange from the source to the system [24]. If the system has weak damping even a low-magnitude input can lead to a very large amplitude power oscillations due to the lack of energy dissipation provided by system damping [25]. The research in [2] documents a case where a 10 MW FO caused a resonant response of 477 MW in the two-areas Kundur power system.

Power systems have electromechanical modes associated with the inertial dynamics of the generation units, these are usually represented by complex conjugates with frequencies around 0.1 to 0.7 Hz for inter-area modes and 0.7 to 3 Hz for local modes [10]. Torsional and control modes are often located above 3 Hz, so forced resonance can appear in a wide frequency range, from low frequencies often linked with mechanical components to high frequencies related to electronic devices [26].

1.1.3. Documented forced oscillation events

Table 1.1 summarizes some of the real-world events documented in the literature that have been associated with forced oscillations. Where ▲ stands for cyclic loads, ★ corresponds to conventional generation, ■ is assigned to renewable energy sources and ♦ to changes in system topology.

Table 1.1: Documented forced oscillation events

No	Date	Location	Source	Magnitude	Time	Hz	Ref
1	June 1992	Rush Island ♦	Failure of an insulator & re-configuration	280 MW	37 min	1.0	[27]
2	Nov 2005	Alberta, BPA-CAISO ★	Steam extractor valve	20 MW (source), 200 MW (COI)	6 min	0.28	[28]

Continued on the next page

Table 1.1 (continuation)

No	Date	Location	Source	Magnitude	Time	Hz	Ref
3	Jan 2008	BPA-SCE-CAISO ■	Pacific DC intertie oscillations	150 MW, 200 MVA	55 min	4.0	[29]
4	October 2009	South Texas, ERCOT ■	Sub-synchronous control interaction between DFIG wind turbines and series compensated line	voltages and currents > 300 % normal operation	400 ms	20	[30]
5	May 2010	No info ■	Voltage control problem in PMSG	80 MW	120 min	5	[26]
6	Dec 2010	Oregon ■	Active control malfunction of converter (WPP)	15 MW	560 min	0.04	[26]
7	Dec 2010	Woodward Oklahoma ■	Interaction between two wind plants	No info	32 min	13.5	[26]
8	Apr 2011	OG&E, Oklahoma ■	Two types of wind turbines during high-wind output	5.1 kV	25 min	13.33	[21]
9	2011	Virginia ★	Voltage setting in a nuclear generator	250 MW	12 min	No info	[29]
10	July 2011	Central Washington ■	Rotor-resistance control (WPP)	15 MW	8.3 mins	0.01	[26]
11	Before 2012	MISO (Eastern) ★	Governor controller causing valve malfunction at a large coal-fired plant	40 MW	2.5 min	0.285	[29]
12	2012	Western US ★	Combined cycles in gas and steam turbines	100 MW	20 min	1	[26]
13	Dec 2012	OG&E Wind farm ■	69kV line outage instigated FO	Voltage variations up to 18%	30 min	3	[7]

Continued on the next page

Table 1.1 (continuation)

No	Date	Location	Source	Magnitude	Time	Hz	Ref
14	Summer 2013	WECC ★	Hydro generator	10 MVA	several hours	0.38	[2]
15	Apr 2013	ISO-NE ★	A generator	15.5 MW	45 min	0.22-0.28	[7]
16	May 2013	NYISO (Eastern) ★	AVR controller malfunction of a generator	No info	2 min	1.25	[8]
17	Apr 2014	BPA Northeast Oregon ■	Voltage controller of a wind plant	70 MVAR	2 hours	14	[7]
18	Feb 2014	ERCOT ★	Faulty control card for a hydro power plant	No info	3 mins	1.80	[29]
19	2014	ERCOT ■	High power output and controller setting of a wind plant	1% voltage	17 hours	3.3	[31]
20	Oct 2014	No info ★	Whirling and spiral vortex in HPP	30 MW	60 min	0.38	[26]
21	Oct 2014	BPA (Western) ★	Surging water vortex on turbine at a hydro generator	No info	1 min	0.33	[8]
22	Before 2015	Oklahoma OG&E ▲	Oscillation induced by a refinery	2MVA, 1% voltage	25 min	4.62–5.0	[29]
23	Sep 2015	Nevada ★	Damping controller issues	40 MW	30 min	0.398	[26]
24	Oct 2015	Dalles, BPA (West) ★	Interaction between an under-excited limiter and a PSS of a generator	No info	60 min	1.00	[8]
25	Apr 2016	Oregon Pacific DC Intertie ■	Equipment failure at Sylmar converter station	250 MVAR, 50MW	5.5 mins	No info	[7]

Continued on the next page

Table 1.1 (continuation)

No	Date	Location	Source	Magnitude	Time	Hz	Ref
26	May 2016	Southern Wisconsin	Unknown, oscillation appears and dissapears	<1% voltage	Long term	0.6-0.75	[7]
27	June 2016	Grand Gulf Nuclear Station ★	Control valve malfunction	200 MW FO at GGNS	45 min	0.27	[32]
28	Jan 2019	Florida ★	Faulty input to a steam turbine's control system	200 MW and 4–8 kV at unit bus	18 mins	0.25	[4]
29	Apr 2025	Badajoz, Spain ■	Solar power plant, under investigation	0.5-1% voltage	2 hours	0.6	[22]
30	No info	ISO-NE ★	Single coal-fired generator, unknown but common on this plant	100 MW (1st) 70 MW (2nd)	8 min (1st) 44 min (2nd)	0.03 (1st) 0.05 (2nd)	[7]

Overall, the events summarized in Table 1.1 indicate that forced oscillations have been reported under a wide range of geographical areas and source types. Conventional generation represents a large fraction of the documented cases (50%), with many events traced back not only to control systems, but also to mechanical devices such as steam valves, turbine components and water columns. These mechanisms typically give rise to low-frequency oscillations and are often characterized by relatively large power or voltage variations. In parallel, a substantial number of events are associated with renewable energy sources (40%), where converter controls and electrical interactions can excite oscillations over a broader frequency range.

From a temporal and spectral standpoint, the table shows that forced oscillations are not limited to short duration disturbances. Several events persist for seconds, minutes or even hours, particularly at very low frequencies, while higher-frequency oscillations tend to be shorter but potentially severe. Many reported frequencies lie close to known local or inter-area electromechanical modes, increasing the probability of resonance. The wide diversity of sources, timescales and physical mechanisms involved, highlights the need for analytical models that explicitly represent external periodic excitations. This diversity also underscores the importance of developing identification and localization methods capable of distinguishing different types of driven forced oscillations.

1.1.4. Forced oscillation detection methods

A critical challenge in forced oscillation mitigation is distinguishing between natural and forced oscillations, since poorly damped natural modes may exhibit sustained oscillatory behavior that closely resembles that of forced oscillations. Hence, a dedicated distinction stage is necessary to correctly classify the oscillation type prior to any control or mitigation action [33].

Methods for detecting forced oscillations can be broadly classified into three categories: 1) based on increases in the energy of the measured signal, 2) based on increased coherence and 3) based on signal characteristics.

1.1.4.1. Energy-based detection

Under normal conditions, the energy of measured signals remains relatively constant across consecutive data windows. When a forced oscillation occurs, additional energy is injected into the data window in which the oscillation happened. Therefore, by comparing the energy of adjacent windows, the presence of a disturbance can be inferred.

One of the simplest approaches to analyze signal energy in the frequency domain is to estimate the power spectral density (PSD) using a periodogram. The PSD quantifies how the signal power is distributed across different frequency bands.

In [34], the root-mean-square (RMS) energy of the measured signal is estimated and then filtering is applied to analyze the energy within specific frequency bands. The practical deployment of this type of detector is described in [35]. Follum and Pierre [36] propose a statistical method in which general expressions for the periodogram distributions are derived under the hypotheses of the presence and absence of a forced oscillation. This analysis yields a detection threshold that explicitly relates the probabilities of false alarm and detection. Establishing the detection threshold required one hour of ambient measurements.

1.1.4.2. Coherency-based detection

Engineers have observed that when a power system is subjected to a disturbance, the machines in the system tend to separate into groups, with each group oscillating against the others. This behavior is known as coherency and can be exploited to detect forced oscillations. In signal processing, coherence is a statistical measure used to examine the relationship between two signals and is commonly applied to estimate power transfer between the input and output of linear systems. The coherence between two signals is a real-valued function calculated from their power spectral densities (PSDs) and their cross-spectral density.

In [37], Zhou applied a coherence-based technique to PMU measurements from different areas of the western United States power system, demonstrating superior accuracy compared to the simple periodogram method. Subsequently, in [38], this approach was extended to a single data channel through the development of self-coherency, which analyzes the correlation between the original measurement and its time-delayed versions.

1.1.4.3. Characteristic-based detection

Characteristic-based methods can be regarded as hybrid approaches, as they rely on PMU measurements while leveraging linearized system models to extract physically interpretable indices. These indices are compared against predefined thresholds to distinguish between natural and forced oscillations.

The parameters required to construct these indices are usually: modal frequency, damping ratio, phase or spectral content and are typically obtained using well-established signal processing and system identification techniques such as Prony, Stochastic Subspace Identification (SSI), Welch's method and the Hilbert–Huang Transform (HHT). A detailed comparison of these signal processing techniques is provided in [39]. For example, in [40] and [41], modal parameters estimated using Prony and the Eigensystem Realization Algorithm (ERA) are used to construct decision metrics that exploit expected differences in damping or frequency content under forced excitation.

Several representative characteristic-based methods, together with their principles and their associated decision criteria are summarized in Table 1.2 based on the IEEE technical report in [42]. These methods illustrate how some features of linear models are mapped to simple thresholds for practical forced oscillation detection.

Table 1.2: Characteristic-based methods presented in [42]

Method	**Principle**	**Data Type**	**Criterion**	**Limitations**
Initial-phase envelope	Analysis of the slope σ of the exponential envelope $Ae^{\sigma t}$	Initial peak-to-peak values, ring-down data	$S = \sigma T$: $S < 0 \rightarrow$ forced, $S > 0 \rightarrow$ natural	Requires capturing the onset of the event, sensitive to noise and window selection
Angle between power and frequency phasors	Relationship between damping and the angle ϕ between the P and f phasors	Active power and frequency phasors, ring-down data	$\phi < (\phi_0 \approx 0.25\pi) \rightarrow$ forced $\phi > \phi_0 \rightarrow$ natural	Reliable only when the oscillation is clearly observable, sensitive to noise and low modal participation
Harmonic content index	Ratio of harmonic components: $h = (m_2 + m_3)/m_1$	Harmonic content of ambient data	$h > 0.11 \rightarrow$ forced, $h \leq 0.11 \rightarrow$ natural	Cannot detect purely sinusoidal oscillations, nonlinearities may generate harmonics

1.1.4.4. Relevant industrial methods

Industrial forced oscillation monitoring systems commonly combine low complexity screening tools with more advanced identification algorithms. One widely deployed example is the power dynamic extraction (PDX) method developed by General Electric, which performs autoregressive identification of oscillatory components using a sliding time window. PDX tracks the evolution of frequency, damping and amplitude within predefined modal bands and explicitly identifies harmonic content, a feature that is particularly effective for distinguishing poorly damped natural oscillations from FO [43].

As a first line of defense, system operators use computationally efficient detectors based on signal energy or spectral magnitude obtained from the FFT. These detectors have been implemented by OG&E and Peak Reliability, enabling the real-time monitoring of a large number of signals. While their low computational burden is advantageous, these methods are prone to higher false alarm rates. Consequently, industry practice recommends complementing these tools with more robust techniques, such as spectral coherence, characteristic-based or PDX methods, to improve reliability [43].

1.1.5. Forced oscillation source identification methods

To effectively address the mitigation of forced oscillations, the source must be located, disconnected or repaired. With the widespread deployment of phasor measurement units (PMUs) and the wide-area measurement systems (WAMS), a broad range of measurement-driven approaches has been proposed for the localization of forced oscillations in power systems. A key advantage of these approaches is that they do not rely on detailed system models, which are often unavailable or inaccurate in large interconnected grids. Challenging problems in this area are related with uncertainties in the models, noise and the diversity of mechanisms that can generate forced oscillations [44].

To try addressing these challenges, a wide variety of source identification techniques have been developed over the past decade by exploiting different physical and statistical properties of power systems. Broadly, existing methods can be classified into five main categories: 1) mode shape and damping torque methods, 2) energy flow methods, 3) hybrid approaches, 4) data-driven techniques and 5) artificial intelligence methods. Each category offers distinct advantages and limitations in terms of modeling requirements, robustness to noise and applicability across different system conditions.

1.1.5.1. Mode shape and damping torque methods

The mode shape is defined by the right eigenvectors of the state matrix of a linearized system model and represents the relative magnitudes and phases of oscillations across the system. However, obtaining reliable system models is often difficult. For this reason, mode shapes are commonly estimated using measurement data collected from multiple locations in the power system.

For example, in [45], it was observed that the leading generator in mode shape phase contributed with less damping. Hence, it was concluded that this leading generator was the oscillation source. An overview on mode shape techniques using either ring-down signals or ambient signals can be found in [46]. Although mode shape methods are able to identify the source these may fail, especially under resonance cases [47].

On the other hand, damping torque methods focus on generators that have an estimated negative damping torque coefficient. The damping torque concept was first introduced in [48] to offer a physical insight to system stability problem on a single-machine infinite-bus system (SMIB). To estimate the damping torque coefficient of a SMIB using the system response the least square method [49] [50] or the Kalman filter [51] can be used. Damping torque methods often require the electromagnetic torque, rotor angle and speed of the generator, which are not always directly measured by PMUs. These methods may fail to identify the source under certain conditions [47].

1.1.5.2. Energy flow methods

Energy flow methods have been widely explored for oscillation source localization [52] [53] [54] [55]. The concept was initially introduced for synchronous generators, where the source is identified by tracking the sign of the energy produced by the machine. The fundamental assumption relies on treating the forced oscillation as a wave that propagates through the power system. Then, by calculating the dissipated or injected energy at various points, the physical location of the disturbance can be traced.

In [56], an universal transient energy function (TEF), which is a particular application of the Lyapunov function [57] was formulated to quantify the exchange and dissipation of energy in power systems. The TEF approach was extended in [58] using Port-Controlled Hamiltonian theory to consider excitation and governor models.

When applying the energy-based methods to monitor the energy flow on branches, the directions of the energy flow may be significantly be affected by nearby loads [44]. Hence, energy-based localization methods are highly affected by the model of the load and the frequency of the perturbation [59].

1.1.5.3. Traveling wave methods

Traveling wave is a concept based in the principle of electromechanical wave propagation [60], this theory can be used to locate the oscillation source. In general, it assumes that the oscillation travels through the electric network as a continuum. Thus, by measuring location and the time of arrival, the source can be determined. Only the first period of the wave are required to locate the source [61]. Therefore, traveling wave methods have shorter processing time than other methods. However, an accurate arrival time is required from the measuring device, this may be challenging due to the impact of the non-constant wave speed, also different network topologies can influence in the performance of these methods [62].

1.1.5.4. Hybrid methods

Hybrid methods formulate the problem of source localization as a model-fitting problem, in which the measurement is compared with the dynamic power system model to asses if there is a source of forced oscillations by minimizing or maximizing the discrepancy between the measurement and the dynamic model. Because these methods rely on detailed representations of generators, controls and the network, their performance depends strongly on model accuracy.

A class of hybrid methods employs state observers, such as unknown input observers [63] or decoupling observers [64]. When a persistent periodic disturbance is reconstructed at a particular generator or control device, that component is identified as the source. Other approaches rely on effective generator impedance [65] or equivalent circuit models [44], in which a source generator is characterized by the presence of additional effective current injection beyond its admittance. While these methods might provide potentially device-level localization, they are sensitive to modeling errors.

1.1.5.5. Data-driven spatiotemporal methods

Data-driven spatiotemporal methods localize forced oscillation sources by exploiting statistical structure and data relations in multichannel time-synchronized PMU measurements collected across the power system. These approaches operate directly on measured data and do not require explicit physical models or detailed system topology. Instead, oscillation sources are inferred from dominant spatiotemporal patterns and low-dimensional representations that emerge from the measurements.

The primary advantage of these methods is their independence from detailed system models, which are often unavailable, inaccurate or difficult to maintain in large interconnected grids. However, this advantage comes at the cost of increased sensitivity to data quality, observability and noise. In many cases, reliable performance requires extensive PMU coverage and long data records. Moreover, since localization is based on statistical dominance rather than explicit physical mechanisms, these methods may offer limited physical interpretation.

In [66], PMU measurements are assembled into a data matrix and decomposed using robust principal component analysis (RPCA) into a low-rank component representing coherent system behavior and a sparse component associated with localized disturbances. The source is identified from the dominant entries of the sparse component. The method was validated on the WECC 179-bus system and demonstrated robustness in resonant scenarios where mode-shape and energy-based methods often fail.

Building on this framework, [67] proposed an online approach based on sparse identification of nonlinear dynamics (SINDy). Using measured rotor angles and rotor speeds, an effective sparse dynamical representation is inferred directly from data, where generators exhibiting persistent external forcing are identified as oscillation sources. Although the method reconstructs an approximate dynamical model, it does not require prior knowledge of system topology or parameters. The method was evaluated on the WECC 240-bus system for forced oscillations with frequencies below 1 Hz.

Recently, in [68], a two-tier dynamic mode decomposition (DMD) method is proposed for oscillation source localization in power systems exhibiting sustained low-frequency oscillations. By reformulating DMD to incorporate the initial measurement state, in a way analogous to DMD with controls. The method combines full-time and early peak-time analyses of modal energy distribution to identify the bus that consistently spreads out energy during the initial stage of the oscillation.

Other methods include the system agnostic localization of oscillations (SALO) algorithm proposed in [69], which formulates source localization as a maximum likelihood problem, as well as causality approaches using Granger theory [70] and graph theory localization methods [71]. These techniques infer source location based on statistical influence, likelihood scores or transfer function residue patterns estimated from data.

Overall, data-driven spatiotemporal methods are motivated by the difficulty of obtaining reliable system models in large power systems. Nevertheless, their reliance on signal processing and statistical properties of the data can lead to ambiguity in complex scenarios involving resonance, multiple interacting sources or limited PMU coverage.

1.1.5.6. Artificial intelligence methods

Artificial intelligence (AI) has developed rapidly in recent years, leading to increased interest in AI methods for FO source localization in power systems. Most AI approaches formulate the localization task as a supervised classification or regression problem, in which multichannel, time-synchronized PMU measurements or features extracted from these signals are used to infer the location of the oscillation source, typically represented as a bus, generator or system area. This formulation relies on learning relationships between measurement patterns and known source locations rather than on explicit physical modeling. Early studies mainly employed conventional machine learning techniques, such as decision trees, support vector machines (SVMs) and random forests, trained offline using labeled data collected under diverse operating conditions [61] [72] [73].

More recently, deep learning methods have gained attention due to their strong capability to capture nonlinear dynamics and complex temporal and spatial correlations in power system measurements [74]. Approaches based on convolutional neural networks (CNN) [75] and recurrent neural networks (RNN), including long short-term memory (LSTM) architectures [76], have been proposed to learn spatiotemporal signatures of forced oscillations from time-series data, frequency-domain features or image-like representations derived from PMU measurements [77]. To mitigate the lack of labeled data in real power systems, transfer learning is commonly adopted, where models are pretrained on simulated systems and subsequently fine-tuned using real-world data [78] [79].

Despite their high reported accuracy in complex scenarios, artificial intelligence methods generally require large and representative training datasets and extensive offline training. Their performance may degrade under unseen operating conditions and the learned mappings often lack clear physical interpretation compared to hybrid or energy-based approaches where the localization metric is often linked with physical variables, this can limit their applicability in practical power system operation [80].

1.1.6. Closed-form analytical methods for forced oscillations

Most analytical studies of FO adopt small-signal models in which external disturbances are modeled as single sinusoidal inputs [81]. This assumption enables closed-form solutions based on transfer functions or eigenvalue analysis and provides a direct relation between the forcing frequency and resonance with lightly damped system modes. Such models are particularly effective for showing amplification mechanisms and for developing detection principles under narrowband excitation. However, their validity is limited when disturbances are large or broadband, as is often observed in real power systems.

Field observations indicate that many FO sources, generate nonsinusoidal but periodic waveforms, which contain multiple harmonic components [33]. Analytical studies that incorporate harmonic forcing show that, under linear assumptions, the system response can be expressed as a weighted superposition of harmonic components shaped by the system dynamics [82]. In practice, however, power systems exhibit inherent nonlinearities that introduce amplitude-dependent behavior. As a result, forced oscillations cannot be fully characterized using linear models alone. In particular, linear models fail to capture interactions between external forcing and additional harmonic content generated by nonlinear system dynamics.

1.1.6.1. Small-signal analysis

Within the small-signal framework, a key limitation of classical eigenvalue analysis is that it characterizes only the intrinsic modal properties of the system and does not directly quantify the response to external forcing. Frequency-domain formulations based on transfer functions explicitly address this limitation by coupling external inputs to system outputs. From this perspective, it has been shown that significant amplification of forced oscillations can occur even when all system modes are well damped, explaining why certain events are not revealed by conventional modal stability analysis [18].

In [41], an explicit single-tone multi-machine formulation is developed and detection criteria based on modal components and response envelope characteristics are proposed. Validation using the IEEE 39-bus New England system and a real forced oscillation event demonstrates that oscillation severity increases when the forcing frequency is close to lightly damped modes, confirming the combined influence of modal damping and frequency proximity on the forced response. Although these approaches provide valuable analytical insight and quantitative tools for characterizing forced oscillation amplification in linearized systems, they inherently neglect amplitude-dependent effects and nonlinear interactions. This motivates the use of nonlinear analytical methods.

1.1.6.2. Nonlinear methods

To address the limitations of linear analysis, nonlinear analytical methods have been applied to the study of forced oscillations. Perturbation techniques approximate the response of nonlinear systems by introducing corrections through a small parameter, enabling analytical insight beyond the linear regime. Common approaches include harmonic balance, which computes steady-state periodic solutions in the frequency domain and the method of multiple scales, which separates time scales to eliminate secular terms and obtain valid approximations near resonance.

A nonlinear modal decomposition technique based on normal form theory is presented in [83], where a third-order forced decoupled normal form formulation is introduced to analyze FOs in nonlinear multi-converter systems under external inputs. This approach captures higher-order modal interactions, nonlinear resonance and amplitude and frequency shifts that are not represented by linear models or classical normal form. Alternatively, in [81], the method of multiple scales is applied to a single-machine infinite-bus system with quadratic nonlinearity under sinusoidal forcing. The resulting analytical expressions captured nonlinear resonance phenomena such as amplitude jumps and shifts in the resonance frequency that are absent in linear theory. Related studies show that even weak nonlinearities can substantially modify forced response characteristics near resonance, altering both amplitude and frequency content [84] [85].

1.1.7. Discussion on the reviewed methods

The previous sections reviewed a wide range of methods for forced oscillation detection, source localization and analytical characterization. Given the diversity of approaches and underlying assumptions, it is useful to synthesize their main strengths and limitations to obtain a broader perspective on the forced oscillation problem in power systems and on the solution strategies reported in the scientific literature. To this end, the following tables summarize the advantages and disadvantages of the reviewed methods at a categorical level. This high-level comparison highlights common trends and trade-offs among detection, localization and analytical approaches.

1.1.7.1. Summary on forced oscillation detection methods

Table 1.3 summarizes the main advantages and limitations of FO detection methods. These approaches primarily aim at distinguishing FO from natural oscillations (NO) using measured data, with an emphasis on timely and reliable identification.

Table 1.3: Advantages and disadvantages of FO detection methods

Method	**Advantages**	**Disadvantages**
Energy-based [34–36]	Simple, fast, low computational burden, proven deployment	Noise sensitive, threshold dependent, higher false alarm rate
Coherency-based [37, 38]	Improved FO–NO discrimination by exploiting correlation structure	Requires synchronized data, sensitive to data quality and windowing
Characteristic-based [39–42]	Physically interpretable indices, model-consistent decision metrics	Sensitive to noise and parameter estimation, threshold selection is case-dependent
Industrial screening [43]	Scalable, real-time, proven deployment in operations	Limited accuracy alone, typically needs complementary methods
	Common: *early identification, PMU-based, computationally efficient screening*	***Common:*** *PMU coverage, threshold tuning, noise and data-quality sensitivity*

1.1.7.2. Summary on source localization methods

Table 1.4 compares forced oscillation source localization methods, which aim to identify the physical origin of the disturbance to support mitigation actions. These methods differ in terms of modeling requirements, data dependency and interpretability.

1.1.7.3. Summary on closed-form analytical methods

Table 1.5 summarizes closed-form analytical approaches, which provide theoretical insight into forced oscillation mechanisms, resonance and system response. These methods play a key role in the understanding of the phenomenon and development of detection, localization and mitigation techniques.

Table 1.4: Advantages and disadvantages of FO source localization methods

Method	Advantages	Disadvantages
Mode shape / damping torque [45–47]	Clear physical interpretation, leverages oscillatory patterns across the system	Model/estimation dependent, unreliable under resonance
Energy flow [44, 56, 58, 59]	Physically representative decision metrics	Sensitive to load modeling, forcing frequency and operating conditions
Traveling wave [60–62]	Fast localization using short time windows (early wavefronts)	Precise timing, grid topology and wave-speed variability
Hybrid [63–65]	Potential device-level localization via model-fitting or observers	Model dependent, sensitive to parameter and modeling errors
Data-driven spatiotemporal [66–69]	Model-agnostic inference from dominant spatiotemporal patterns, robust in resonant cases	High data and coverage requirements, limited physical interpretation
Artificial intelligence [61, 72, 74]	Can learn complex spatiotemporal signatures in challenging scenarios	Requires large training data, generalization issues, limited physical interpretation
	Common: *enables targeted mitigation, measurement-driven, scalable to large grids*	***Common:*** *PMU coverage and noise sensitivity, model uncertainty, interpretation limits*

Table 1.5: Advantages and disadvantages of closed-form analytical methods for FOs

Method	Advantages	Disadvantages
Small-signal analysis [18, 41]	Rigorous, closed-form insight, clarifies resonance and amplification mechanisms	Relies on linear assumptions, limited for amplitude-dependent phenomena
Frequency analysis [33, 82]	Accounts for nonsinusoidal periodic forcing via superposition (linear)	Neglects nonlinear interactions that can generate or shape harmonics
Nonlinear methods [81, 83–85]	Captures nonlinear features and higher-order interactions	Higher mathematical complexity, harder to scale to large models
	Common: *physics-based, links forcing-mode proximity, supports interpretation and design*	***Common:*** *simplifying assumptions, nonlinear methods add complexity*

1.2. Research problem statement

Low-frequency forced oscillations are normally induced by mechanical components or associated controls and can threaten system stability due to the their likely interaction with the natural modes of the system. To mitigate their impact, measurement-based detection and localization algorithms have been widely developed. Although effective for real-time operation, these methods provide limited insight into the underlying interactions among system variables that drive the onset and temporal evolution of forced oscillations. Furthermore, most analytical studies rely on linear models and single-frequency inputs, limiting their ability to capture nonlinear dynamics under multi-frequency forcing. Therefore, a systematic investigation of the nonlinear characteristics of forced oscillations under multi-frequency mechanical excitation is required.

1.3. Contribution to the national strategic program of Mexico

Forced oscillations in the 0.1–2.5 Hz range pose a significant risk when their frequency coincides with lightly damped modes, leading to sustained oscillations that can threaten system stability causing social and economic impacts. In Mexico, a 200 MW FO such as the one reported as event 27 in Table 1.1 can affect approximately 200,000 families and result in economic losses of 7,140,900 MXN [86]. This phenomenon is aligned with the priorities of PRONACE on energy transition, which aims to strengthen the reliability of the Mexican electric power system. The present research proposes an analytical approach based on the nonlinear analysis of a power system subjected to periodic mechanical multi-frequency disturbances to characterize its response and quantify interactions among system variables to support current measurement-based methods.

1.4. Objectives

To model and analyze a nonlinear electric power system under multi-frequency mechanical inputs using a perturbation method, with the aim of deriving closed-form expressions that characterize forced oscillations, resonance phenomena and their dependence on system parameters.

1.4.1. Specific Objectives

- To assess the limitations of single-frequency linear models with traditional analytical methods commonly used to study electric power systems.
- To use the limitations of single-frequency models to propose a multi-frequency model for analyzing forced oscillations.
- To model and analyze a multi-frequency forced electric power system using the proposed strategy and derive closed-form expressions employing both linear and nonlinear frameworks.
- To validate and compare the analytical results using numerical integration and software simulations.

1.5. Hypothesis

In a nonlinear power system under periodic multi-frequency mechanical forcing, the system dynamics and resonance behavior can be analytically characterized through nonlinear analysis, providing insight into the time evolution of the system by deriving closed-form expressions that depend on key system parameters such as damping, nonlinearity strength and forcing characteristics.

1.6. Methodology

The general methodology to carry out this research is explicitly presented in Fig. 1.2.

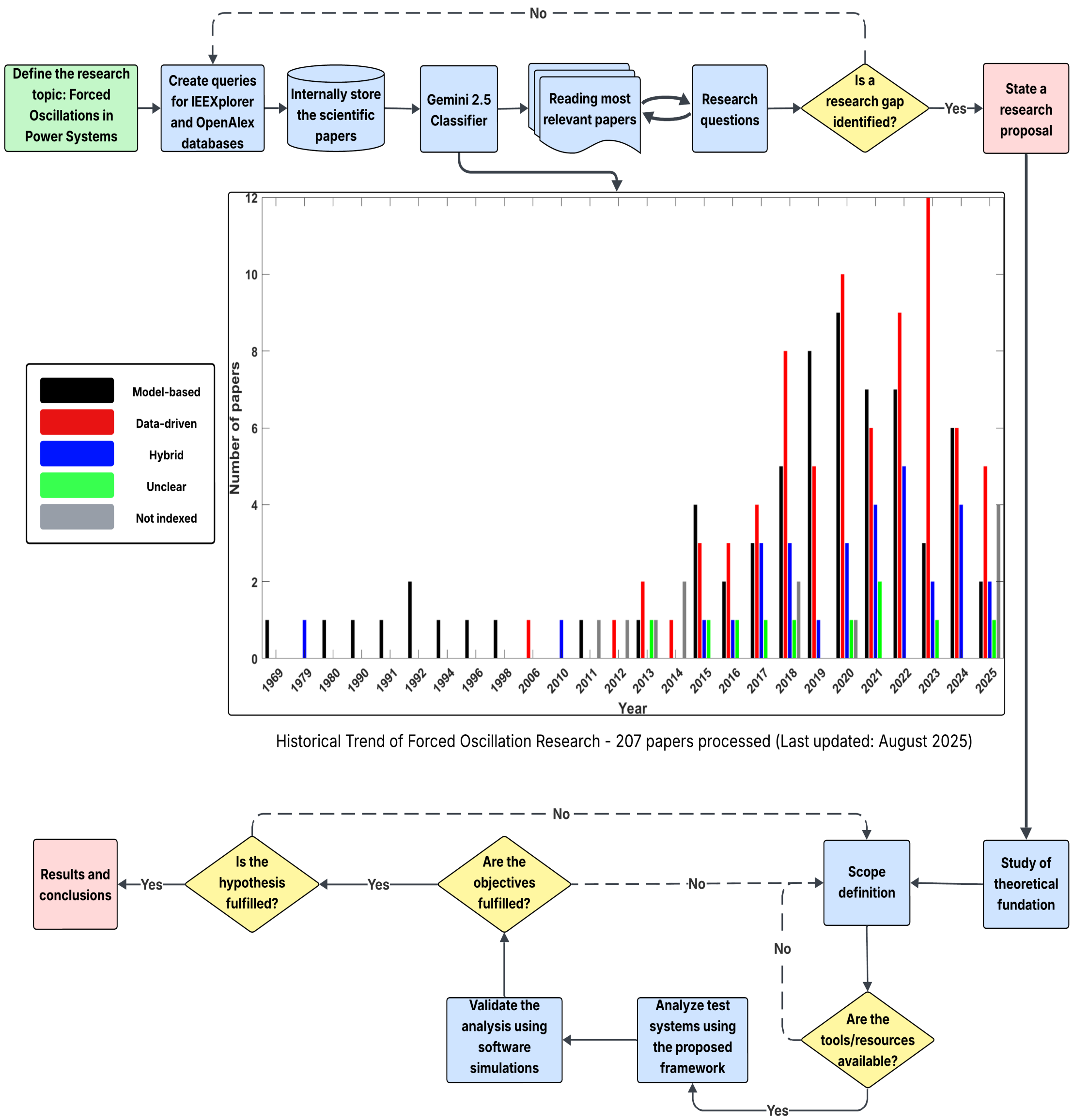

Fig. 1.2: Proposed general methodology for the development of this research

The methodology begins by defining the research topic, which in this case is: Forced Oscillations in Power Systems. Afterwards a systematic search of the scientific literature using IEEXplorer and OpenAlex databases through queries is performed. The retrieved publications are downloaded and stored in a local machine. Then, Python and the Gemini 2.5 API are used to build a document classifier based on the type of content. This turns to be helpful to realize the amount of existing information and to get a simple approximation of the historical path followed by researchers. Notice how it is also handy to possibly recognize research gaps.

After having a broader perspective about the available information, the task is to read the most relevant papers based on cites and type of paper. This allow us to iteratively refine research questions until a research gap is identified.

Before defining the scope of this thesis, the theoretical foundation to tackle the type of problems involved in the identified gap must be satisfied. The next process is straightforward, define the scope of the research with the tools and resources available, conduct the research and verify the objectives and hypothesis.

1.7. Thesis Organization

This thesis is organized as follows: Chapter 1 presents the background and motivation of this research, it provides the direction of this thesis and states the objectives and hypothesis that guide the investigation.

Chapter 2 introduces the fundamental concepts of linear dynamical systems under external forcing, with the aim of establishing the scope and limitations of linear analysis.

Chapter 3 presents a single-machine infinite-bus (SMIB) system subjected to different multi-frequency waveforms to analyze harmonic-induced resonance. Closed-form solutions describing the system dynamics are derived and the concept of envelope is introduced to identify possible patterns.

Chapter 4 extends the analysis of Chapter 3 using nonlinear theory for dynamical systems to analyze the nonlinear system and derive closed-form expressions that relate system parameters to its dynamics.

Chapter 5 documents and discuss the results obtained in Chapters 3 and 4. Finally, Chapter 6 summarizes the main conclusions of this thesis and outlines possible directions for future work.

References

[1] Ruichao Xie and D. Trudnowski, "Distinguishing features of natural and forced oscillations," in *2015 IEEE Power & Energy Society General Meeting*. Denver, CO, USA: IEEE, Jul. 2015, pp. 1–5.

[2] S. A. N. Sarmadi and V. Venkatasubramanian, "Inter-Area Resonance in Power Systems From Forced Oscillations," *IEEE Trans. Power Syst.*, vol. 31, no. 1, pp. 378–386, Jan. 2016.

[3] L. Zhu, W. Yu, Z. Jiang, C. Zhang, Y. Zhao, J. Dong, W. Wang, Y. Liu, E. Farantatos, D. Ramasubramanian, A. Arana, and R. Quint, “A Comprehensive Method to Mitigate Forced Oscillations in Large Interconnected Power Grids,” *IEEE Access*, vol. 9, pp. 22 503–22 515, 2021.

[4] “Eastern Interconnection Oscillation Disturbance January 11, 2019 Forced Oscillation Event,” North American Electric Reliability Coordination (NERC), 2019.

[5] J. Follum and J. Pierre, “Time-localization of forced oscillations in power systems,” in *2015 IEEE Power & Energy Society General Meeting*. Denver, CO, USA: IEEE, Jul. 2015, pp. 1–5.

[6] U. Agrawal and J. W. Pierre, “Detection of Periodic Forced Oscillations in Power Systems Incorporating Harmonic Information,” *IEEE Trans. Power Syst.*, vol. 34, no. 1, pp. 782–790, Jan. 2019.

[7] “Reliability Guideline Forced Oscillation Monitoring & Mitigation,” North American Electric Reliability Coordination (NERC), 2017.

[8] M. Ghorbaniparvar, “Survey on forced oscillations in power system,” *J. Mod. Power Syst. Clean Energy*, vol. 5, no. 5, pp. 671–682, Sep. 2017.

[9] Y. Xu, Z. Gu, K. Sun, and X. Xu, “Understanding a Type of Forced Oscillation Caused by Steam-Turbine Governors,” *IEEE Trans. Energy Convers.*, vol. 35, no. 3, pp. 1719–1722, Sep. 2020.

[10] L. G. Meegahapola, S. Bu, D. P. Wadduwage, C. Y. Chung, and X. Yu, “Review on Oscillatory Stability in Power Grids With Renewable Energy Sources: Monitoring, Analysis, and Control Using Synchrophasor Technology,” *IEEE Trans. Ind. Electron.*, vol. 68, no. 1, pp. 519–531, Jan. 2021.

[11] J. Van Ness, “Response of Large Power Systems to Cyclic Load Variations,” *IEEE Trans. on Power Apparatus and Syst.*, vol. PAS-85, no. 7, pp. 723–727, Jul. 1966.

[12] K. Rao and L. Jenkins, “Studies on power systems that are subjected to cyclic loads,” *IEEE Trans. Power Syst.*, vol. 3, no. 1, pp. 31–37, Feb. 1988.

[13] N. Rostamkolai, R. Piwko, and A. Matusik, “Evaluation of the impact of a large cyclic load on the LILCO power system using time simulation and frequency domain techniques,” *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1411–1416, Aug. 1994.

[14] J. Pinneilo and J. Van Ness, “Dynamic Response of a Large Power System to a Cyclic Load Produced by a Nuclear Accelerator,” *IEEE Trans. on Power Apparatus and Syst.*, vol. PAS-90, no. 4, pp. 1856–1862, Jul. 1971.

[15] N. L. Thotakura, C. R. Burge, and Y. Liu, “Impact of the Exciter and Governor Parameters on Forced Oscillations,” *Electronics*, vol. 13, no. 16, p. 3177, Aug. 2024.

[16] P. B. Reddy and I. A. Hiskens, “Limit-induced stable limit cycles in power systems,” in *2005 IEEE Russia Power Tech.* St. Petersburg, Russia: IEEE, Jun. 2005, pp. 1–5.

[17] Z.-x. Han, Z.-l. Zhu, X.-s. Tian, and F. Li, “Analysis and Simulation Research on Power System Low Frequency Oscillation,” in *2010 Second International Conference on Computer Modeling and Simulation.* Sanya, China: IEEE, Jan. 2010, pp. 223–228.

[18] M. Magdy and F. Coowar, “Frequency domain analysis of power system forced oscillations,” *IEE Proc. C Gener. Transm. Distrib. UK*, vol. 137, no. 4, p. 261, 1990.

[19] W. Xuanyin, L. Xiaoxiao, and L. Fushang, “Analysis on oscillation in electro-hydraulic regulating system of steam turbine and fault diagnosis based on PSOBP,” *Expert Systems with Applications*, vol. 37, no. 5, pp. 3887–3892, May 2010.

[20] N. Hatziargyriou, J. Milanovic, C. Rahmann, V. Ajjarapu, C. Canizares, I. Erlich, D. Hill, I. Hiskens, I. Kamwa, B. Pal, P. Pourbeik, J. Sanchez-Gasca, A. Stankovic, T. Van Cutsem, V. Vittal, and C. Vournas, “Definition and Classification of Power System Stability – Revisited & Extended,” *IEEE Trans. Power Syst.*, vol. 36, no. 4, pp. 3271–3281, Jul. 2021.

[21] Y.-H. Wan, “Synchronized Phasor Data for Analyzing Wind Power Plant Dynamic Behavior and Model Validation,” Tech. Rep. NREL/TP–5500-57342, 1067916, Jan. 2013.

[22] “Incidente en el Sistema Eléctrico Peninsular Español el 28 de abril de 2025,” Red Eléctrica, Spain, Tech. Rep., Jun. 2025.

[23] D. Bowman, D. Ramasubramanian, R. McCann, E. Farantatos, A. Gaikwad, and J. Caspary, “SPP Grid Strength Study with High Inverter-Based Resource Penetration,” in *2019 North American Power Symposium (NAPS).* Wichita, KS, USA: IEEE, Oct. 2019, pp. 1–6.

[24] B. Shao, Q. Xiao, L. Wang, P. Han, Z. Bin, C. Wang, F. Blaabjerg, and Z. Chen, "Review on power system generalized modal resonance analysis," *International Journal of Electrical Power & Energy Systems*, vol. 154, p. 109417, Dec. 2023.

[25] T. Surinkaew, K. Emami, R. Shah, N. Mithulananthan, S. M. Muyeen, and T. Fernando, "A unified damping controller for non-stationary forced oscillation," *International Journal of Electrical Power & Energy Systems*, vol. 143, p. 108410, 2022.

[26] T. Surinkaew, K. Emami, R. Shah, S. Islam, and N. Mithulananthan, "Forced Oscillation in Power Systems With Converter Controlled-Based Resources–A Survey With Case Studies," *IEEE Access*, vol. 9, pp. 150 911–150 924, 2021.

[27] K. Kim, H. Schattler, V. Venkatasubramanian, J. Zaborszky, and P. Hirsch, "Methods for calculating oscillations in large power systems," *IEEE Trans. Power Syst.*, vol. 12, no. 4, pp. 1639–1648, Nov. 1997.

[28] S. A. Nezam Sarmadi, V. Venkatasubramanian, and A. Salazar, "Analysis of November 29, 2005 Western American Oscillation Event," *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 5210–5211, Nov. 2016.

[29] "Diagnosing Equipment Health and Mis-operations with PMU Data," North American SynchroPhasor Initiative (NASPI), Tech. Rep., May 2015.

[30] J. Adams, C. Carter, and S.-H. Huang, "ERCOT experience with Sub-synchronous Control Interaction and proposed remediation," in *PES T&D 2012*. Orlando, FL, USA: IEEE, May 2012, pp. 1–5.

[31] K. Chen, "Wind Farm Oscillation Detection and Mitigation White Paper," Salt Lake City, UT, USA, Apr. 2014.

[32] "Interconnection Oscillation Analysis Reliability Assessment," Electric Reliability Council of Texas, 2019.

[33] J. D. Follum, F. K. Tuffner, L. A. Dosiek, and J. W. Pierre, "Power System Oscillatory Behaviors: Sources, Characteristics, & Analyses," Tech. Rep. PNNL–26375, 1411936, May 2017.

[34] M. Donnelly, D. Trudnowski, J. Colwell, J. Pierre, and L. Dosiek, "RMS-energy filter design for real-time oscillation detection," in *2015 IEEE Power & Energy Society General Meeting*. Denver, CO, USA: IEEE, Jul. 2015, pp. 1–5.

[35] D. Kosterev, A. Donahoo, N. Leitschuh, J. Dagle, F. Tuffner, and D. Trudnowski, “Deployment and Initial Experience with Oscillation Detection Application at Bonneville Power Administration,” 2016.

[36] J. Follum and J. W. Pierre, “Detection of Periodic Forced Oscillations in Power Systems,” *IEEE Trans. Power Syst.*, vol. 31, no. 3, pp. 2423–2433, May 2016.

[37] Ning Zhou, “A coherence method for detecting and analyzing oscillations,” in *2013 IEEE Power & Energy Society General Meeting*. Vancouver, BC: IEEE, 2013, pp. 1–5.

[38] N. Zhou and J. Dagle, “Initial Results in Using a Self-Coherence Method for Detecting Sustained Oscillations,” *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 522–530, Jan. 2015.

[39] J. Thambirajah, E. Barocio, and N. Thornhill, “Comparative review of methods for stability monitoring in electrical power systems and vibrating structures,” *IET Gener. Transm. Distrib.*, vol. 4, no. 10, pp. 1086–1103, Oct. 2010.

[40] K. Abhinav, P. Rai, A. Prakash, and S. K. Parida, “Comparative Assessment of Prony Analysis and Eigensystem Realization Algorithm for Forced Oscillation Detection and Mode Estimation Considering PMU Noise,” in *2023 IEEE 3rd International Conference on Smart Technologies for Power, Energy and Control (STPEC)*. Bhubaneswar, India: IEEE, Dec. 2023, pp. 1–6.

[41] H. Ye, Y. Liu, P. Zhang, and Z. Du, “Analysis and Detection of Forced Oscillation in Power System,” *IEEE Trans. Power Syst.*, pp. 1–1, 2016.

[42] “TR-110 Forced Oscillations in Power Systems,” IEEE Power & Energy Society, Technical Report, 2023.

[43] “Recommended Oscillation Analysis for Monitoring and Mitigation Reference Document,” North American Electric Reliability Coordination (NERC), Tech. Rep., Nov. 2021.

[44] B. Wang and K. Sun, “Location methods of oscillation sources in power systems: a survey,” *J. Mod. Power Syst. Clean Energy*, no. 2, pp. 151–159, Mar. 2017.

[45] N. Ashwal, D. Wilson, and M. Parashar, “Identifying sources of oscillations using wide area measurements,” in *Grid of the Future Syumposium*, 2014.

[46] L. Dosiek, N. Zhou, J. W. Pierre, Z. Huang, and D. J. Trudnowski, “Mode shape estimation algorithms under ambient conditions: A comparative review,” *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 779–787, May 2013.

[47] S. Maslennikov, B. Wang, Q. Zhang, a. Ma, a. Luo, a. Sun, and E. Litvinov, "A test cases library for methods locating the sources of sustained oscillations," in *2016 IEEE Power and Energy Society General Meeting (PESGM)*. Boston, MA, USA: IEEE, Jul. 2016, pp. 1–5.

[48] F. Demello and C. Concordia, "Concepts of Synchronous Machine Stability as Affected by Excitation Control," *IEEE Trans. on Power Apparatus and Syst.*, vol. PAS-88, no. 4, pp. 316–329, Apr. 1969.

[49] R. Alden and A. Shaltout, "Analysis of Damping and Synchronizing Torques Part I-A General Calculation Method," *IEEE Trans. on Power Apparatus and Syst.*, vol. PAS-98, no. 5, pp. 1696–1700, Sep. 1979.

[50] A. Shaltout and R. Alden, "Analysis of Damping and Synchronizing Torques Part II-Effect of Operating Conditions and Machine Parameters," *IEEE Trans. on Power Apparatus and Syst.*, vol. PAS-98, no. 5, pp. 1701–1708, Sep. 1979.

[51] E. A. Abu-Al-Feilat, N. Younan, and S. Grzybowski, "Estimating the synchronizing and damping torque coefficients using Kalman filtering," *Electric Power Systems Research*, vol. 52, no. 2, pp. 145–149, 1999.

[52] S. Feng, B. Zheng, P. Jiang, and J. Lei, "A Two-Level Forced Oscillations Source Location Method Based on Phasor and Energy Analysis," *IEEE Access*, vol. 6, pp. 44 318–44 327, 2018.

[53] S. Maslennikov and E. Litvinov, "ISO New England Experience in Locating the Source of Oscillations Online," *IEEE Trans. Power Syst.*, vol. 36, no. 1, pp. 495–503, Jan. 2021.

[54] P. G. Estevez, P. Marchi, C. Galarza, and M. Elizondo, "Complex Dissipating Energy Flow Method for Forced Oscillation Source Location," *IEEE Trans. Power Syst.*, vol. 37, no. 5, pp. 4141–4144, Sep. 2022.

[55] W. Yu, Y. Liu, L. Zhu, and E. Farantatos, "Dissipating Potential-Based Method for Forced Oscillation Location Estimation," in *2024 International Conference on Smart Grid Synchronized Measurements and Analytics (SGSMA)*. Washington, DC, USA: IEEE, May 2024, pp. 1–5.

[56] L. Chen, Y. Min, and W. Hu, "An energy-based method for location of power system oscillation source," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 828–836, May 2013.

[57] Y. Min and L. Chen, "A transient energy function for power systems including the induction motor model," *SCI CHINA SER E*, vol. 50, no. 5, pp. 575–584, Oct. 2007.

[58] Li Ying, Shen Chen, and Liu Feng, "An energy-based methodology for locating the source of forced oscillations in power systems," in *2012 IEEE International Conference on Power System Technology (POWERCON)*. Auckland: IEEE, Oct. 2012, pp. 1–6.

[59] Y. Zhi and V. Venkatasubramanian, "Analysis of Energy Flow Method for Oscillation Source Location," *IEEE Trans. Power Syst.*, vol. 36, no. 2, pp. 1338–1349, Mar. 2021.

[60] J. Thorp, C. Seyler, and A. Phadke, "Electromechanical wave propagation in large electric power systems," *IEEE Trans. Circuits Syst. I*, vol. 45, no. 6, pp. 614–622, Jun. 1998.

[61] Y. Chen, Z. Fan, D. Gregory, X. Zhou, and R. Rabbani, "A Survey of Oscillation Localization Techniques in Power Systems," *IEEE Access*, vol. 13, pp. 28 836–28 860, 2025.

[62] M. A. Aftab, S. M. S. Hussain, I. Ali, and T. S. Ustun, "Dynamic protection of power systems with high penetration of renewables: A review of the traveling wave based fault location techniques," *International Journal of Electrical Power & Energy Systems*, vol. 114, p. 105410, 2020.

[63] M. Luan, D. Gan, Z. Wang, and H. Xin, "Application of unknown input observers to locate forced oscillation source," *Int Trans Electr Energ Syst*, vol. 29, no. 9, Sep. 2019.

[64] S. Li, M. Luan, D. Gan, and D. Wu, "A model-based decoupling observer to locate forced oscillation sources in mechanical power," *International Journal of Electrical Power & Energy Systems*, vol. 103, pp. 127–135, Dec. 2018.

[65] S. C. Chevalier, P. Vorobev, and K. Turitsyn, "Using Effective Generator Impedance for Forced Oscillation Source Location," *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 6264–6277, Nov. 2018.

[66] T. Huang, N. M. Freris, P. R. Kumar, and L. Xie, "A Synchrophasor Data-Driven Method for Forced Oscillation Localization Under Resonance Conditions," *IEEE Trans. Power Syst.*, vol. 35, no. 5, pp. 3927–3939, Sep. 2020.

[67] Y. Cai, X. Wang, G. Joós, and I. Kamwa, "An Online Data-Driven Method to Locate Forced Oscillation Sources From Power Plants Based on Sparse Identification of Nonlinear Dynamics (SINDy)," *IEEE Trans. Power Syst.*, vol. 38, no. 3, pp. 2085–2099, May 2023.

[68] M.-S. Ko, W. Shin, K. Sun, and K. Hur, "Locating the Source of Oscillation With Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy," *IEEE Trans. Power Syst.*, vol. 39, no. 4, pp. 5535–5547, Jul. 2024.

[69] R. Delabays, A. Y. Lokhov, M. Tyloo, and M. Vuffray, "Locating the Source of Forced Oscillations in Transmission Power Grids," *PRX Energy*, vol. 2, no. 2, p. 023009, Jun. 2023.

[70] M. Luan, S. Li, and D. Gan, "Locating Forced Oscillation Source Using Granger Causality Analysis and Delay Estimation," in *2019 4th International Conference on Intelligent Green Building and Smart Grid (IGBSG)*. Hubei, Yi-chang, China: IEEE, Sep. 2019, pp. 502–507.

[71] T. R. Nudell and A. Chakrabortty, "Graph-Theoretic Methods for Measurement-Based Input Localization in Large Networked Dynamic Systems," *IEEE Trans. Automat. Contr.*, vol. 60, no. 8, pp. 2114–2128, Aug. 2015.

[72] T. Zhou, J. Sun, H. Quan, D. Zou, and Q. Peng, "Oscillation Source Location of Ultra-low Frequency Oscillations Based on Random Forest Algorithm," in *2022 International Conference on Cyber-Physical Social Intelligence (ICCSI)*. Nanjing, China: IEEE, Nov. 2022, pp. 460–465.

[73] Kejun Mei, S. Rovnyak, and Chee-Mun Ong, "Clustering-Based Dynamic Event Location Using Wide-Area Phasor Measurements," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 673–679, May 2008.

[74] M. Matar, P. G. Estevez, P. Marchi, F. Messina, R. Elmoudi, and S. Wshah, "Transformer-based deep learning model for forced oscillation localization," *International Journal of Electrical Power & Energy Systems*, vol. 146, p. 108805, Mar. 2023.

[75] L. Li, B. Wang, S. Liu, J. Li, S. Teng, D. Liu, X. Peng, and S. Feng, "Forced oscillation location based on temporal graph convolutional network," *Energy Reports*, vol. 9, pp. 646–654, 2023.

[76] X. Yang, J. Wang, X. Huang, Y. Wang, and X. Xiao, "Forced Oscillation Detection via a Hybrid Network of a Spiking Recurrent Neural Network and LSTM," *Sensors*, vol. 25, no. 8, p. 2607, Apr. 2025.

[77] L. Huo and X. Chen, "Higher-order motif-based time series classification for forced oscillation source location in power grids," *Nonlinear Dyn*, vol. 111, no. 21, pp. 20 127–20 138, Nov. 2023.

[78] S. Feng, J. Chen, Y. Ye, X. Wu, H. Cui, Y. Tang, and J. Lei, "A two-stage deep transfer learning for localisation of forced oscillations disturbance source," *International Journal of Electrical Power & Energy Systems*, vol. 135, p. 107577, 2022.

[79] Z. Cui, W. Hu, G. Zhang, Q. Huang, Z. Chen, and F. Blaabjerg, "Knowledge-Informed Deep Learning Method for Multiple Oscillation Sources Localization," *IEEE Trans. Power Syst.*, vol. 40, no. 3, pp. 2811–2814, May 2025.

[80] Z. Ping, X. Li, W. He, T. Yang, and Y. Yuan, "Sparse learning of network-reduced models for locating low frequency oscillations in power systems," *Applied Energy*, vol. 262, p. 114541, 2020.

[81] Y. Zhou, J. Wu, H. Li, and Y. Li, "Analysis of Nonlinear Characteristics for Forced Oscillation Affected by Quadratic Nonlinearity," *IEEE Trans. Power Syst.*, vol. 37, no. 1, pp. 804–807, Jan. 2022.

[82] Lei Chen, Dan Trudnowski, Luke Dosiek, Sukumar Kamalasadan, Yanhui Xu, and Xiaozhe Wang, "Forced Oscillations in Power Systems (TR 110)," publisher: IEEE.

[83] T. Tian, X. Kestelyn, and O. Thomas, "Normal form based analytical investigation of nonlinear power system dynamics under excitation," in *2017 IEEE Power & Energy Society General Meeting*. Chicago, IL: IEEE, Jul. 2017, pp. 1–5.

[84] J. Xiong, H. Ye, W. Pei, L. Kong, Y. Li, and K. Strunz, "Characteristics Analysis of Current-Controlled VSCs for Periodic Forced Oscillations Excited by Grid Frequency Disturbance," *IEEE Trans. Smart Grid*, vol. 15, no. 1, pp. 545–556, Jan. 2024.

[85] S. Starrett and A. Fouad, "Nonlinear measures of mode-machine participation [transmission system stability]," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 389–394, May 1998.

[86] (2025, Jan.) Precio tope de los precios marginales locales (PML) del mercado de energía de corto plazo. CENACE.