ABDULLAH GÜL
ÜNİVERSİTESİ

FINAL REPORT

ELECTRIC CIRCUIT DESIGN CAPSULE

PROJECT NAME:

DESIGN AND PRODUCE INDUCTOR

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INTRODUCTION

In this project, the design and analysis of Helmholtz coils were carried out to generate a uniform magnetic field. This study, conducted by Burak Bekir, Ahmet Burak, and Ayşe Ece, aimed to achieve a nearly uniform magnetic field within a certain volume using Helmholtz coils.

First, the magnetic field induction on the z-axis of a single circular coil was computed. Next, the ideal spacing between the coils was determined, and a general formula for the magnetic field of two identical coils was obtained. In addition, the partial derivatives of the magnetic field, the vector function of the field shifted from the axis in cylindrical coordinates, and the magnetic field of multi-turn coils were investigated. Additionally, a calculation of the coils' theoretical inductance was made, and suggested measuring devices were presented. As part of the project, Helmholtz coil analysis within a circuit, LT-SPICE simulations, and frequency response analysis were also carried out.

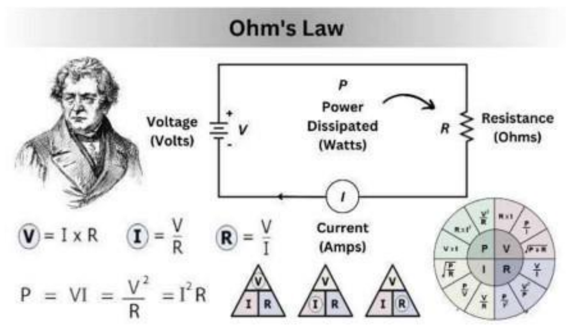
Team members gained useful knowledge in magnetic field theory, electromagnetic design, circuit simulations, and analytical calculations from this project. Researching science and engineering was made easier with the reinforcement of both theoretical and practical knowledge. Furthermore, a small mishap throughout the project served as a reminder to us of the value of exercising greater caution when working in laboratories.

BACKGROUND

Before starting the project, we need to know the physics rules that we will need to use in the project.

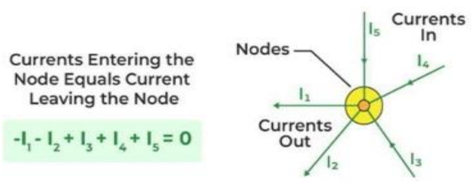
Ohm's Law: Ohm's Law is a fundamental principle in electronics that describes the relationship between voltage (V), current (I), and resistance (R) in an electrical circuit.

This law states that in a circuit with a given resistance, if you increase the voltage, the current will increase proportionally and vice versa. Similarly, if the resistance increases while the voltage remains constant, the current will decrease.



Ohm's Law is expressed by the equation:
$V=I\times R$
Where:
V (Voltage) is the potential difference or electromotive force measured across a component in volts (V).
I (Current) is the flow of electric charge measured in amperes (A).
R (Resistance) is the opposition to the flow of electric current measured in ohms (Ω).

Kirchoff Current Law: Kirchhoff's Current Law states that *the total current or charge entering a junction or node is precisely equal to the total current or charge exiting the node, as no charge is lost at the node.*



Kirchhoff's First law is like the ***Law of Conservation of charge.*** *As a result, a Nord or junction is a point in a circuit that does not serve as a charge source or sink. Therefore,*

$$^{n}\sum_{k=1} I_K = 0$$

Where n denotes the total number of branches at the node with currents flowing toward or away from it.

$$I_{(exiting)}+I_{(entering)} = 0$$

Calculating Theory Henry Formula: In physics, inductance, especially electromagnetic inductance, is measured in Henry (H) units. In a circuit, inductance causes changes in electric current to produce a magnetic field, and this magnetic field, in turn, induces a back electromotive force (EMF) that opposes the change in current. Inductance is denoted by the symbol LL.
The formula for inductance is as follows:

$$V(t) = L.\frac{dI(t)}{dt}$$

Here:

$V(t)$: Time-varying voltage (V)
L: Inductance (in Henrys, H)
$dI(t)$: The time derivative of the current (A/s)

This formula indicates that the voltage across an inductor is equal to the inductance times the rate of change of current through the inductor. In other words, the voltage of an inductor depends on the rate of change of the current passing through it.

Inductance of a Solenoid (Straight Wire Coil):

$$L = \mu_0\mu_{r^1}\, {N^2A}/_{l}$$

Here:

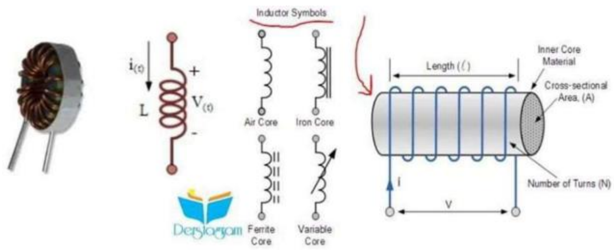
$\mu0$: Permeability of free space ($4\pi\times10^{-7}$ H/m$4\pi\times10^{-7}$H/m)
μr: Relative permeability of the material
N: Number of turns
A: Cross-sectional area of the coil (m²)
l: Length of the coil (m)

Inductance of a Toroid (Ring-Shaped Solenoid)

$$L = \mu_0\mu_{r^1}\, {N^2A}/_{2\pi r}$$

r: Radius of the toroid (m)

These formulas are used to calculate the inductance values of different types of inductors and are applied in electrical circuits, electromagnetic devices, and many other applications.



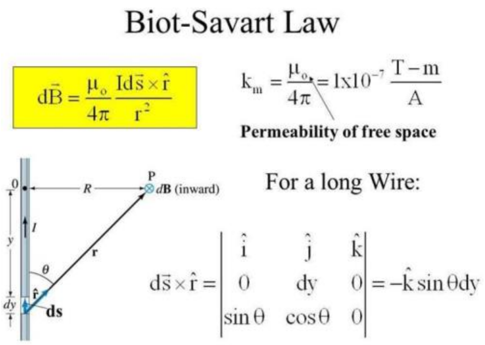
Biot Savart Law: The Biot-Savart law is a fundamental principle used in calculating magnetic fields. This law is used to determine the magnetic field generated around an electric current. The law describes the magnetic field produced by a current element (a small segment of a current-carrying wire). Mathematically, it is expressed as follows:

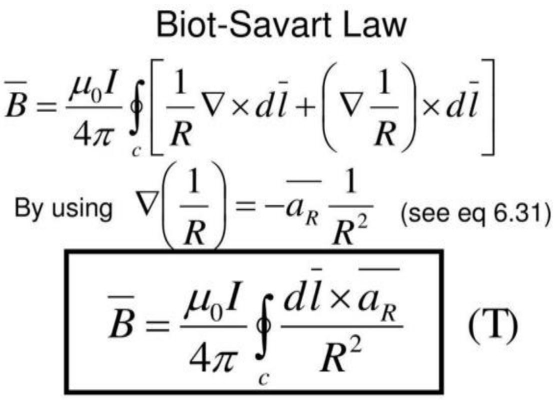
$$dB = {}^{\mu}/_{4\pi}\ {}^{(dlxr).I}/_{r^3}$$

Here:

*d***B**: The infinitesimal magnetic field produced by the current element.
$\mu 0$: The permeability of free space ($4\pi \times 10^{-7}$ T·m/A).
I: The current.
*d***l**: The length and direction of the current element.
r: The vector from the current element to the point where the magnetic field is being calculated.
r: The distance from the current element to the point where the magnetic field is being calculated.

This law is used to find the magnetic field of each small segment of a current-carrying wire, and the total magnetic field is the vector sum of these small magnetic fields.





Ampere's Law: Ampère's law is a fundamental principle used to relate magnetic fields and electric currents. Ampère's law states that the line integral of the magnetic field around a closed loop is proportional to the total electric current passing through the loop. Mathematically, it is expressed as:

$$\oint \mathbf{Bdl} = \mu_0 \mathbf{I}_{enc}$$

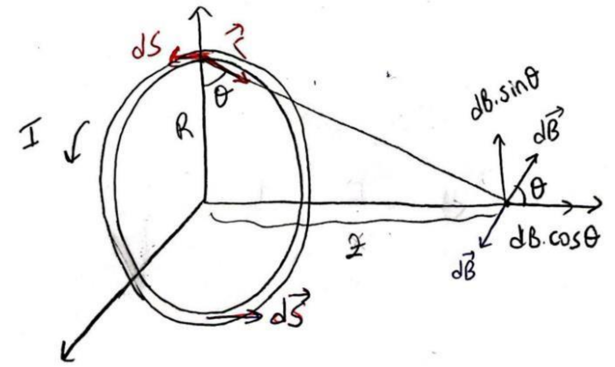
Here:

- $\oint$B·*d*l: The line integral of the magnetic field *B*B around a closed path (contour integral).
- μ_0: The permeability of free space ($4\pi \times 10^{-7}$ T·m/A).
- I_{enc}: The total electric current enclosed by the path.

OBJECTIVE

The goal of this project is to create, analyze, and simulate a Helmholtz coil system and examine its behavior in a circuit. The Helmholtz coil system consists of two similar coils of radius R on the same z-axis that are used to generate a tiny region of space with a nearly uniform magnetic field. The project comprises a preliminary work to calculate the size of the magnetic field induction B on the axis of a single circular coil of radius R with current I at a distance z orthogonal to its plane. The fundamental Helmholtz coil model is then developed by considering two identical circular coils of radius R, each with current I flowing in a single direction. The coils are organized with parallel planes and centers on the same z-axis at a distance of d from each other. The magnetic field generated by the Helmholtz coil is then examined and optimized to provide the most uniform magnetic field in the system's center. The Helmholtz coil is linked in series with a 10Ω resistor. A 0-5V symmetric square wave is delivered to explore voltage responses between the coil and resistor while adjusting the frequency of the square wave. The Helmholtz coil is also modelled in LTspice and simulated to validate the theoretical conclusions. The inductance of one of the coils is also predicted theoretically, and a measurement device consisting of an oscilloscope, a signal generator, and basic circuit elements is presented to measure it.

1.The General Equation for the Magnetic Field B(z)



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} \longrightarrow \text{Biot-Savart Law}$$

$$B(z) = \frac{\mu_0 I}{4\pi} \int \frac{dS}{(R^2+z^2)} \cdot \overbrace{\frac{R}{(R^2+z^2)^{1/2}}}^{\cos\theta}$$

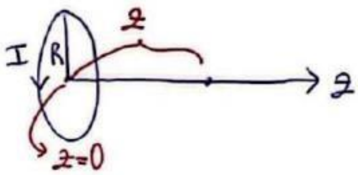
$$B(z) = \frac{\mu_0 I . R}{4\pi} \int \frac{dS}{(R^2+z^2)^{3/2}}$$

$$B(z) = \frac{\mu_0 I R}{4\pi (R^2+z^2)^{3/2}} \int dS$$

$$B(z) = \frac{\mu_0 I R}{4\pi (R^2+z^2)^{3/2}} \cdot 2\pi R$$

$$\boxed{B(z) = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}} \cdot \hat{z}} \longrightarrow \text{Our general equation}$$

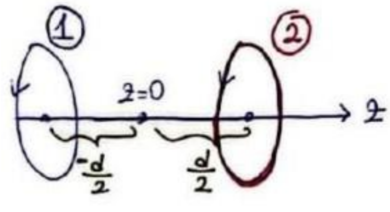
$$B(z) = \frac{\mu_0}{2} \frac{I R^2}{(R^2+z^2)^{3/2}}$$



$$B(z) = B_1(z) + B_2(z)$$

$$B_1(z) = \frac{\mu_0}{2} \frac{I R^2}{\left[R^2+\left(z+\frac{d}{2}\right)^2\right]^{3/2}}$$

$$B_2(z) = \frac{\mu_0}{2} \frac{I R^2}{\left[R^2+\left(z-\frac{d}{2}\right)^2\right]^{3/2}}$$



$$B_{Total}(z) = \frac{\mu_0}{2} \frac{I R^2}{\left[R^2+\left(z+\frac{d}{2}\right)^2\right]^{3/2}} + \frac{\mu_0}{2} \frac{I R^2}{\left[R^2+\left(z-\frac{d}{2}\right)^2\right]^{3/2}}$$

$$= \frac{\mu_0}{2} I R^2 \left(\frac{1}{\left[R^2+\left(z+\frac{d}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2+\left(z-\frac{d}{2}\right)^2\right]^{3/2}} \right)$$

2.Determine in Which d/R Ratio the Magnetic Field is as Uniform as Possible

for two identical coils, each radius R and separated by a distance d, the magnetic field along the axis z is:

$$B(z) = \frac{\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}} + \frac{1}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}}\right]$$

<u>first Derivative</u>

$$B(z) = B_1(z) + B_2(z)$$

where

$$B_1(z) = \frac{\mu_0 I R^2}{2\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}}$$

$$B_2(z) = \frac{\mu_0 I R^2}{2\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}}$$

$$\frac{dB(z)}{dz} = \frac{dB_1(z)}{dz} + \frac{dB_2(z)}{dz}$$

using the chain rule:

$$\frac{dB_1(z)}{dz} = \frac{\mu_0 I R^2}{2}\cdot\frac{-3\left(z-\frac{d}{2}\right)}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{5/2}}$$

$$\frac{dB_2(z)}{dz} = \frac{\mu_0 I R^2}{2}\cdot\frac{-3\left(z+\frac{d}{2}\right)}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{5/2}}$$

$$\frac{dB(z)}{dz} = -\frac{3\mu_0 I R^2}{2}\left[\frac{\left(z-\frac{d}{2}\right)}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{5/2}} + \frac{\left(z+\frac{d}{2}\right)}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{5/2}}\right]$$

At $z=0$:

$$\left.\frac{dB(z)}{dz}\right|_{z=0} = -\frac{3\mu_0 I R^2}{2}\left[\frac{-d/2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} + \frac{d/2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}}\right] = 0$$

Second Derivative

$$\frac{d^2B(z)}{dz^2} = \frac{d^2B_1(z)}{dz^2} + \frac{d^2B_2(z)}{dz^2}$$

$$\frac{d^2B_1(z)}{dz^2} = \frac{d}{dz}\left[-\frac{3\mu_0 I R^2\left(z-\frac{d}{2}\right)}{2\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{5/2}}\right]$$

$$= -\frac{3\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{5/2}} - \frac{5\left(z-\frac{d}{2}\right)^2}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{7/2}}\right]$$

$$\frac{d^2B_2(z)}{dz^2} = \frac{d}{dz}\left[-\frac{3\mu_0 I R^2\left(z+\frac{d}{2}\right)}{2\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{5/2}}\right]$$

$$= -\frac{3\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{5/2}} + \frac{5\left(z+\frac{d}{2}\right)^2}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{7/2}}\right]$$

$$\left.\frac{d^2B(z)}{dz^2}\right|_{z=0} = -\frac{3\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} - \frac{5\left(\frac{d}{2}\right)^2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{7/2}} + \frac{1}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} - \frac{5\left(\frac{d}{2}\right)^2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{7/2}}\right]$$

$$\left.\frac{d^2B(z)}{dz^2}\right|_{z=0} = -\frac{3\mu_0 I R^2}{2}\left[\frac{2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} - \frac{10\left(\frac{d}{2}\right)^2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{7/2}}\right]$$

for the second derivative to be zero, the following must:

$$\frac{2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{5/2}} = \frac{10\left(\frac{d}{2}\right)^2}{\left(R^2+\left(\frac{d}{2}\right)^2\right)^{7/2}}$$

$$2\left(R^2+\left(\frac{d}{2}\right)^2\right) = 10\left(\frac{d}{2}\right)^2$$

$$2R^2 + 2\left(\frac{d}{2}\right)^2 = 10\left(\frac{d}{2}\right)^2$$

$$2R^2 + \frac{d^2}{2} = \frac{5d^2}{2}$$

$$2R^2 = \frac{5d^2}{2} - \frac{d^2}{2}$$

$$2R^2 = \frac{4d^2}{2}$$

$$2R^2 = 2d^2$$

$$R^2 = d^2 \quad \boxed{R = d} \quad \boxed{\frac{d}{R} = 1}$$

The size of a Helmholtz coil's uniform zone can be approximated by the superposition of three pairs of uniaxial coils in three axes. As an example, this section will look at the size of the uniform zone of the coils in each z-axis Helmholtz coil. The Taylor series approximation approach is added to the solution procedure to make it easier to calculate the coil's uniform size.

3.Guess About the Homogeneity of the Magnetic Field in the Direction Perpendicular to the z-axis

In the Helmholtz coil arrangement, a very homogeneous magnetic field is obtained at the centre along the z axis. This homogeneity is largely maintained in the centre region also in directions perpendicular to the z-axis (x- or y-direction). Due to the symmetrical and optimised arrangement of the coils, the homogeneity of the magnetic field in the central region is quite high. However, this homogeneity starts to decrease as you move towards the edges. Due to edge effects, the homogeneity of the field deteriorates as you move away from the centre. Nevertheless, the main purpose of Helmholtz coils is to provide the most homogeneous magnetic field possible over a large area at the centre.

4.Magnetic Field as a Function of μ0, I and R for the Determined d/R

The magnetic field at the center of the Helmholtz coils, which is at z=0, is obtained by summing the contributions from both coils.

The ratio d/R was found to be 1. R was written instead of d in the magnetic field formula.

$$\frac{d}{R} = 1$$

$$d = R$$

$$B(z) = \frac{\mu_0}{2} I R^2 \left(\frac{1}{\left[R^2 + \left(z + \frac{d}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(z - \frac{d}{2}\right)^2\right]^{3/2}} \right)$$

$$B(z) = \frac{\mu_0}{2} I R^2 \left(\frac{1}{\left[R^2 + \left(z + \frac{R}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(z - \frac{R}{2}\right)^2\right]^{3/2}} \right)$$

For z=0:

$$B(0) = \frac{\mu_0}{2} I R^2 \left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(-\frac{R}{2}\right)^2\right]^{3/2}} \right)$$

$$B(0) = \frac{\mu_0}{2} I R^2 2 \left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}} \right)$$

$$B(0) = \mu_0 IR^2\left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}}\right)$$

$$B(0) = \mu_0 IR^2\left(\frac{1}{\left[\frac{5R^2}{4}\right]^{3/2}}\right)$$

$$B(0) = \mu_0 IR^2 \frac{1}{\frac{5\sqrt{5}}{8}R^3}$$

$$B(0) = \mu_0 I \frac{8}{5\sqrt{5}R}$$

$$B(0) \sim 0.7155 \frac{\mu_0 I}{R}$$

This result shows the magnetic field at the center of the system in terms of the permeability of free space (μ0), the current (I) and the radius of the coils (R).

5.The Magnetic Field of Two Identical Thin Coils of N Turns Each for a Given d/R Ratio

The ratio d/R was found to be 1. R was written instead of d in the magnetic field formula.

$$\frac{d}{R} = 1$$

$$d = R$$

$$B(z) = \frac{\mu_0}{2} INR^2\left(\frac{1}{\left[R^2 + \left(z + \frac{d}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(z - \frac{d}{2}\right)^2\right]^{3/2}}\right)$$

$$B(z) = \frac{\mu_0}{2} INR^2\left(\frac{1}{\left[R^2 + \left(z + \frac{R}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(z - \frac{R}{2}\right)^2\right]^{3/2}}\right)$$

For z=0:

$$B(0) = \frac{\mu_0}{2} INR^2\left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}} + \frac{1}{\left[R^2 + \left(-\frac{R}{2}\right)^2\right]^{3/2}}\right)$$

$$B(0) = \frac{\mu_0}{2} INR^2 2\left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}}\right)$$

$$B(0) = \mu_0 INR^2\left(\frac{1}{\left[R^2 + \left(\frac{R}{2}\right)^2\right]^{3/2}}\right)$$

$$B(0) = \mu_0\, INR^2 \left(\frac{1}{[\frac{5R^2}{4}]^{3/2}} \right)$$

$$B(0) = \mu_0 \mathrm{INR}^2 \frac{1}{\frac{5\sqrt{5}}{8}\mathrm{R}^3}$$

$$B(0) = \mu_0 IN \frac{8}{5\sqrt{5}R}$$

$$B(0) \sim 0.7155 \frac{\mu_0 IN}{R}$$

6.Cylindrical Coordinate System

General $B(z)$ formula

$$B(z) = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(R^2+(z-\frac{d}{2})^2)^{3/2}} + \frac{1}{(R^2+(z+\frac{d}{2})^2)^{3/2}}\right]$$

$$B_{total}(\Delta, z) = B(0,z).\left[\frac{\partial B}{\partial r}\bigg|_{r=0}\Delta + \frac{\partial^2 B}{\partial^2 r}\bigg|_{r=0}\frac{\Delta^2}{2}\right]$$

$$B_{total} = B(0).\left[\begin{array}{l}\text{We take Taylor up to the second}\\ \text{derivative of } B(z)\end{array}\right]$$

$$B(0) = \frac{\mu_0 I R^2}{2}\left[\frac{1}{R^2+(-\frac{d}{2})^{\frac{3}{2}}} + \frac{1}{(R^2+(+\frac{d}{2})^{\frac{3}{2}}}\right]$$

First Derivative:

$$\frac{dB(z)}{dz} = -\frac{3\mu_0 I R^2}{2}\left[\frac{(z-\frac{d}{2})}{((R^2+(z-\frac{d}{2})^2)^{\frac{5}{2}}} + \frac{(z+\frac{d}{2})}{(R^2+(z+\frac{d}{2})^2)^{5/2}}\right]$$

At $r=0$

Second Derivative:

$$\frac{dB^2(z)}{d(z^2)} = -\frac{3\mu_0 I R^2}{2}\left[\frac{1}{(R^2+(z+\frac{d}{2})^2)^{\frac{5}{2}}} - \frac{5(\frac{d}{2})^2}{(R^2+(\frac{d}{2})^2)^{7/2}}\right.$$

$$\left. + \frac{1}{(R^2+(\frac{d}{2})^2)^{\frac{5}{2}}} - \frac{5(d/2)^2}{(R^2+(d/2)^2)^{7/2}}\right]$$

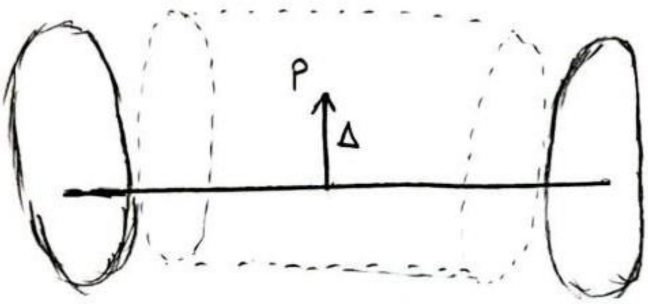
Taylor Formula:

$$\left[\frac{\partial B}{\partial r}\bigg|_{r=0}\Delta + \frac{\partial^2 B}{\partial^2 r^2}\frac{\Delta^2}{2}\bigg|_{r=0}\right]$$

$\Delta = r$

$z = z$ $\quad B_{total} = B(0)$. [we take Taylor up to the second derivative of $B(z)$]

$\varphi = 0$



* Here, we think of a cyclinder in a horizontal shape.

P point

Magnetic field acting at point P.

$$B_{total}(\Delta, z) = B(0) + \frac{\partial B}{\partial r}\Big|_{r=0} + \frac{\partial^2 B}{\partial r^2}$$

Taylor Formula (2)

$$B(z) = \frac{B'(z) \cdot (z - z_0)^1}{1!} + \frac{B''(z)\,(z - z_0)^2}{2!}$$

$$B_{total}(\Delta, z) = (r, \theta, z)$$

$\Delta = r$, $\theta = 0$, $z = z$

As a result, our answer is ;

$$= \frac{8\mu_0 N I}{5\sqrt{5}} - \frac{3\mu_0 N I R^2 \Delta^2}{4 \cdot (R^2 + z^2)^{5/2}}$$

7.Partial Derivative

$$f(r,z) = \left(\frac{8 \mu_0 N I}{5\sqrt{5}} \right) - \left(\frac{3 \mu_0 N I r^2 R^2}{4 (R^2+z^2)^{5/2}} \right)$$

$$f(r,z) = A - B \cdot \frac{r^2}{(R^2+z^2)^{5/2}}$$

$$A = \left(\frac{8 \mu_0 N I}{5\sqrt{5}} \right) \qquad B = \left(\frac{3 \mu_0 N I R^2}{4} \right)$$

$$\frac{\partial f}{\partial r} = -B \left(\frac{2r (R^2+z^2)^{5/2} - r^2 \, 5/2 \cdot 2z \, (R^2+z^2)^{3/2}}{(R^2+z^2)^5} \right)$$

$$\frac{\partial f}{\partial r} = -B \left(\frac{2r (R^2+z^2) - 5r^2 z}{(R^2+z^2)^{7/2}} \right)$$

$$\frac{\partial f}{\partial r} = -\left(\frac{3 \mu_0 N I R^2}{4} \right) \left(\frac{2r (R^2+z^2) - 5r^2 z}{(R^2+z^2)^{7/2}} \right)$$

$$\frac{\partial f}{\partial z} = -B \cdot \left(\frac{0 (R^2+z^2)^{5/2} - r^2 \cdot 5/2 \; 2 \; z \; (R^2+z^2)^{3/2}}{(R^2+z^2)^5} \right)$$

$$\frac{\partial f}{\partial z} = \left(\frac{3 \mu_0 N I R^2}{4} \right) \left(\frac{5 r^2 z}{(R^2+z^2)^{7/2}} \right)$$

As a result, our answer are;

$$\frac{\partial f}{\partial r} = -\left(\frac{3 \mu_0 N I R^2}{4} \right) \left(\frac{2r (R^2+z^2) - 5r^2 z}{(R^2+z^2)^{7/2}} \right)$$

$$\frac{\partial f}{\partial z} = \left(\frac{3 \mu_0 N I R^2}{4} \right) \left(\frac{5 r^2 z}{(R^2+z^2)^{7/2}} \right)$$

8.The Inductance of a Coil

$$L = \frac{\mu_0 \mu_r N^2 A}{\ell} \qquad \pi = 3.1416 \quad N = 90 \quad \ell = 1.2 cm \qquad \mu_0 = 4\pi \cdot 10^{-7} \quad \mu_r = 0.38 \quad r = 2.75 cm$$

$$= \frac{4\pi \cdot 10^{-7} \cdot (0,38) \cdot 90^2 \cdot \pi (2.75)^2 \cdot 10^{-4}}{(1.2) \cdot 10^{-2}}$$

$$= 7.657949425 \times 10^{-4} H$$

25 kHz

$V_{in} = 4.96$ ↗ all circuit (Vpp)

$V_{out} = 0.424$

↳ resistor (Vpp)

$$L = \sqrt{\left(\frac{V_{in}^2 \cdot R^2}{V_{out}^2} - R^2\right)\left(\frac{1}{(2\pi f)^2}\right)}$$

$$L = \sqrt{\frac{(4.96)^2 \cdot 10^2}{(0.424)^2} - 10^2 \cdot \frac{1}{(2 \cdot (3.1416) \cdot (25000))^2}}$$

$$= 7.419989914 \times 10^{-4} H$$

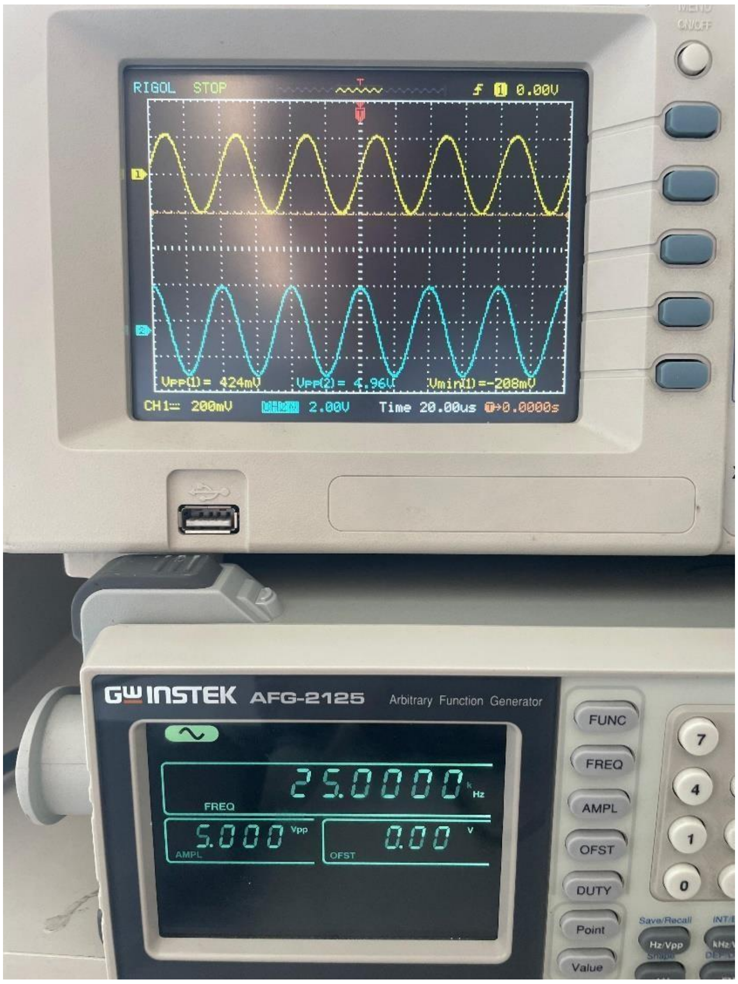


Image 1

As a result of the formula used in the theoretical part, the inductance value was found as 7.657949425×10^-4 H. For the experimental part, phasor domain was used and the inductance value was calculated as 7.419989914×10^-4 H. The reason for the difference in this value is the experimental conditions and the sensitivity of the oscilloscope.

For Phasor Domain:

Series R-L Circuit:

Consider an AC circuit with a series connected resistor (R) and inductor (L). In this circuit:

Voltage across the resistor : V_R

Voltage across the inductor: V_L

Total input voltage : V_{in}

frequency angular velocity : W

Impedances can be defined as:

Impedance of a resistor : $Z_R = R$

Impedance of the inductor : $Z_L = jWL$

Total impedance:

$$Z_{Total} = Z_R + Z_L = R + jWL$$

Magnitude of Total Impedance can be calculated using Pythagoras' theorem

$$|Z_{Total}| = \sqrt{R^2 + (WL)^2}$$

Ohm's Law and Voltage Division Rule:

According to Ohm's Law, the current passing through the circuit (I):

$$I = \frac{V_{in}}{Z_{Total}}$$

According to the voltage division rule, the voltage across the resistor (V_R) can be calculated as follows:

$$V_R = I \cdot R = \left(\frac{V_{in}}{Z_{Total}}\right) \cdot R$$

$$\frac{V_{in}}{V_R} = \frac{V_{in}}{\left(\frac{V_{in}}{Z_{Total}}\right) \cdot R} = \frac{Z_{Total}}{R} = \frac{\sqrt{R^2 + (WL)^2}}{R} = \sqrt{\frac{R^2 + (WL)^2}{R^2}}$$

$$= \sqrt{1 + \left(\frac{WL}{R}\right)^2}$$

$$\frac{V_R}{V_{in}} = \frac{R}{\sqrt{R^2+(WL)^2}}$$

$$\frac{V_R}{V_{in}} = \frac{WL}{WL} \cdot \frac{R}{\sqrt{R^2+(WL)^2}} = \frac{WL}{\sqrt{R^2+(WL)^2}}$$

$$\frac{V_{in}}{V_R} = \frac{\sqrt{R^2+(WL)^2}}{WL}$$

$$\frac{V_{in}}{V_R} = \frac{WL}{\sqrt{R^2+(WL)^2}} \longrightarrow L = \sqrt{\frac{V_{in}^2 \cdot R^2}{V_{out}^2} \cdot R^2 \cdot \left(\frac{1}{(2\pi f)^2}\right)}$$

Double Coil

12kHz

$V_{in} = 4.96$

$V_{out} = 400$

$$L = \sqrt{\left(\frac{V_{in}^2 \cdot R^2}{V_{out}^2} \cdot R^2\right)\left(\frac{1}{(2\pi f)^2}\right)}$$

$$L = \sqrt{\left(\frac{(4.96)^2 \cdot 10^2}{400^2} \cdot 10^2 \cdot \left(\frac{1}{(2 \cdot (3{,}1416) \cdot 12000)^2}\right)\right)}$$

$$= 1.63924407 \times 10^{-3}\ H$$
$$= 16.3924407 \times 10^{-4}\ H$$

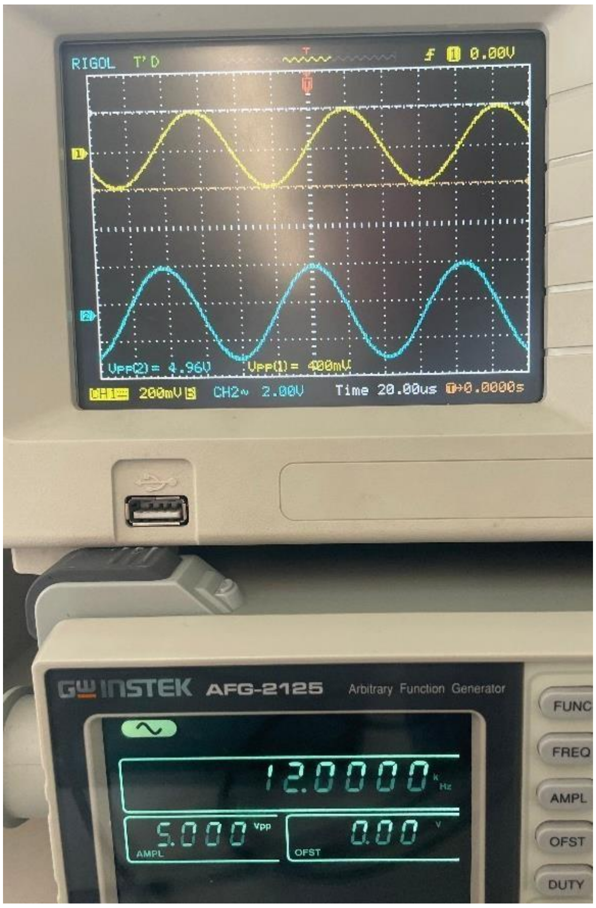
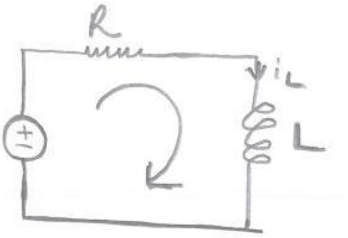


Image 2

9.Measurement System Employing Oscilloscope, Signal Generator, and Basic Circuit Elements to Measure the Inductance

$$V_L = L\frac{di_L}{dt}$$



for $t<0$: $i_L(t)=0$, $V_L(t)=0$

for $t>0$: at $t=0$ $\underbrace{i_L(t)=0,\ V_L(t)=0}_{\text{initial conditions}}$

$$V_0 = i_L(t)R + L\frac{di_L}{dt}$$

$V_{in}(t) = V_R(t) + V_L(t)$

$V_{in}(t) = i_L(t)R + L\frac{di_L}{dt}$

$V_{in}(t) = \begin{cases} 0, & t<0 \\ V_0, & t>0 \end{cases}$

$V_L(t)\ \forall t$

$\Rightarrow$ homogeneous sol'n $i_{LH}(t)$

$\Rightarrow 0 = i_{Lh}R + L\frac{di_{Lh}}{dt} \Rightarrow$ Let's try $i_{Lh} = I_0 e^{-t/\tau}$

$\frac{di_{Lh}}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$

$$0 = RI_0 e^{-t/\tau} - \frac{LI_0}{\tau}e^{-t/\tau}$$

$$0 = I_0 e^{-t/\tau}\left(R - \frac{L}{\tau}\right) \Rightarrow \tau = \frac{L}{R}$$

$$\Rightarrow i_{LH} = I_0 e^{-t/L/R}$$

general solution $=$ $\underbrace{\text{homogen solution}}$ $+$ $\underbrace{\text{particular solution}}$

$i_{Lh} = I_0 e^{-t/L/R}$

$i_L(t)R = V_0$; a = constant

$\Rightarrow i_{Lp} = \frac{V_0}{R}$

$$i_L(t) = I_0 e^{-t/L/R} + \frac{V_0}{R}$$

Use $i_L(t=0) = 0$ to find I_0 :

$i_L(t) = 0 = I_0 + \frac{V_0}{R} \Rightarrow I_0 = -\frac{V_0}{R}$

$$\boxed{\Rightarrow i_L(t) = -\frac{V_0}{R}e^{-t/L/R} + \frac{V_0}{R}}$$

$\Rightarrow$ We get Δx from ociloscope (Δx) which is time constant

$\Delta x = t$

$t = \tau$

$\tau = \frac{L}{R}$ $R = 10\,\Omega$

τ = values were taken from ociloscope

10.OSCILLOSCOPE

Double Coil (for Circuit)

1- For 610hz and 5v

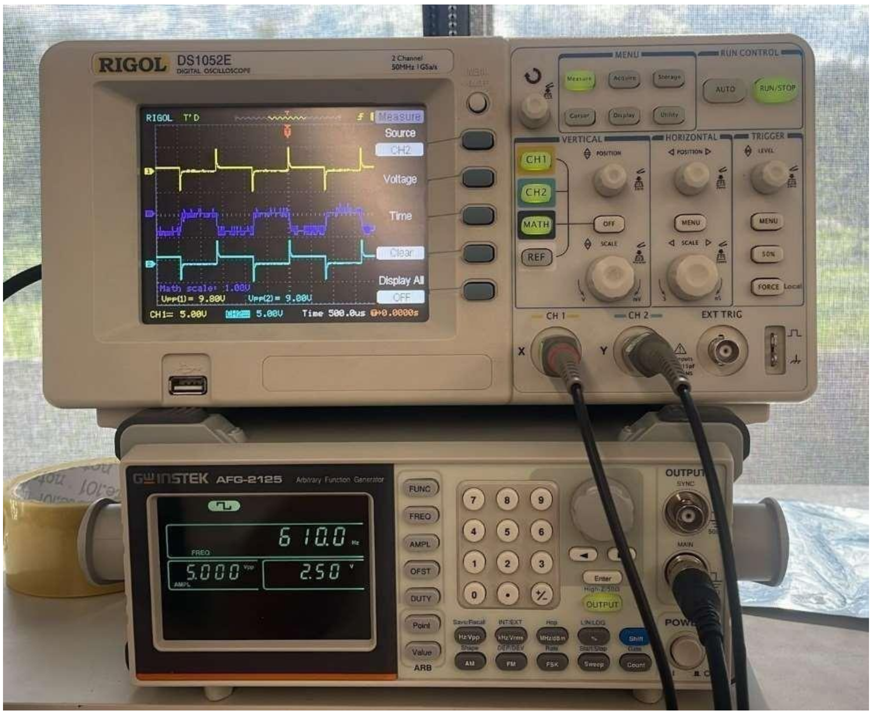


Image 3

2- For 610hz and 2.5v

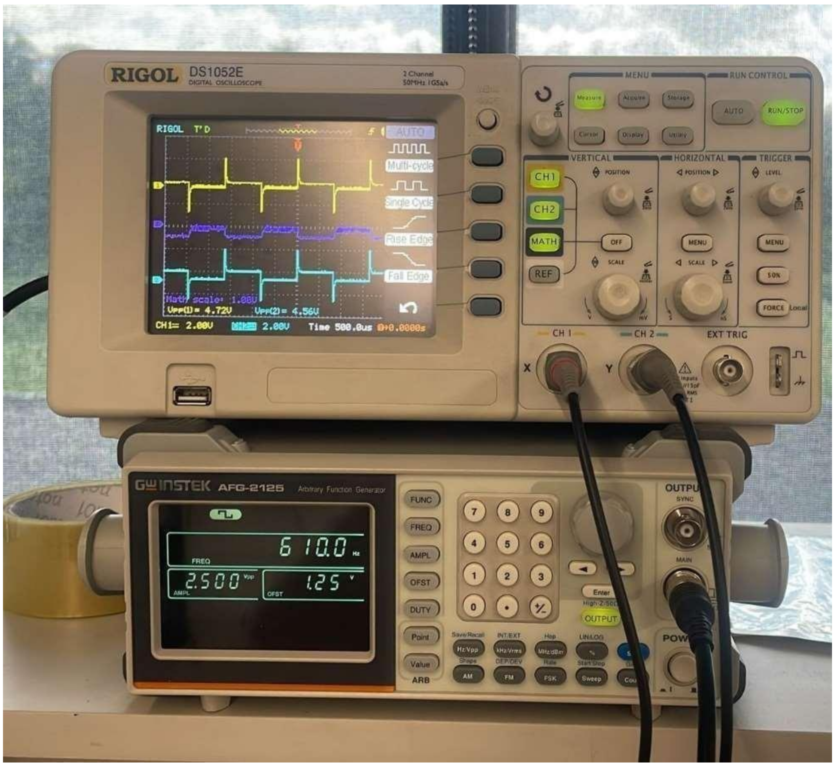


Image 4

3- For 6100.3hz and 5v

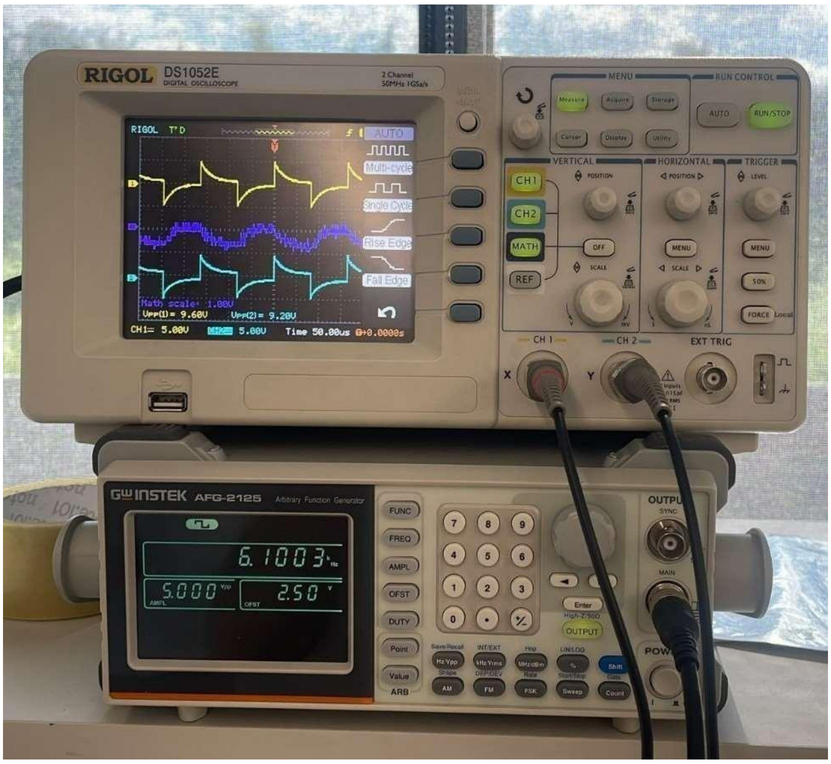


Image 5

4- For 6100.3hz and 2.5v

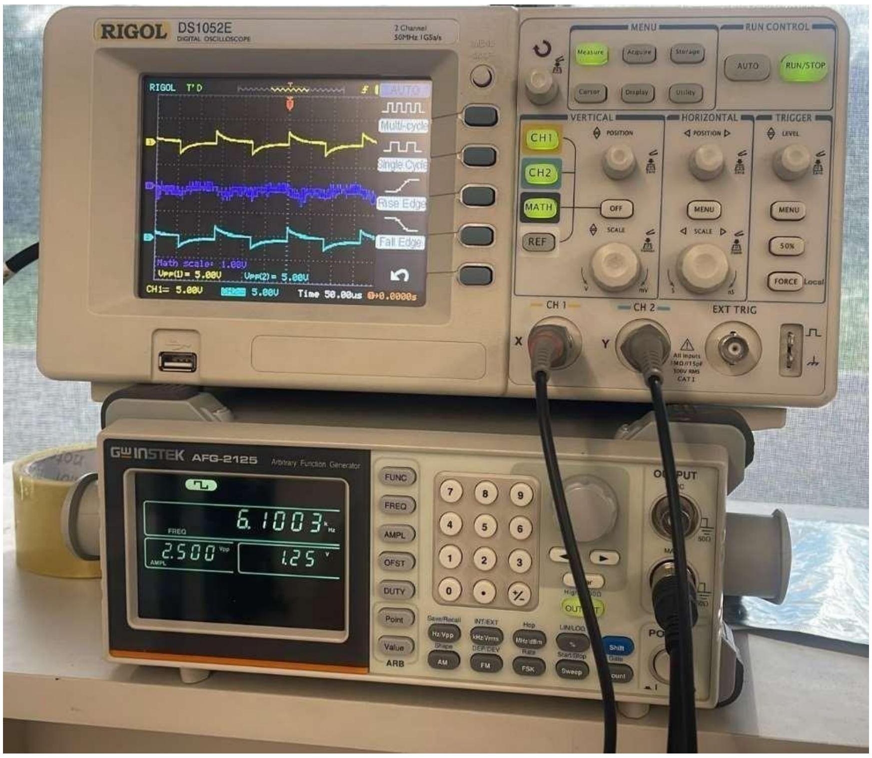


Image 6

5- For 61030.0hz and 5v

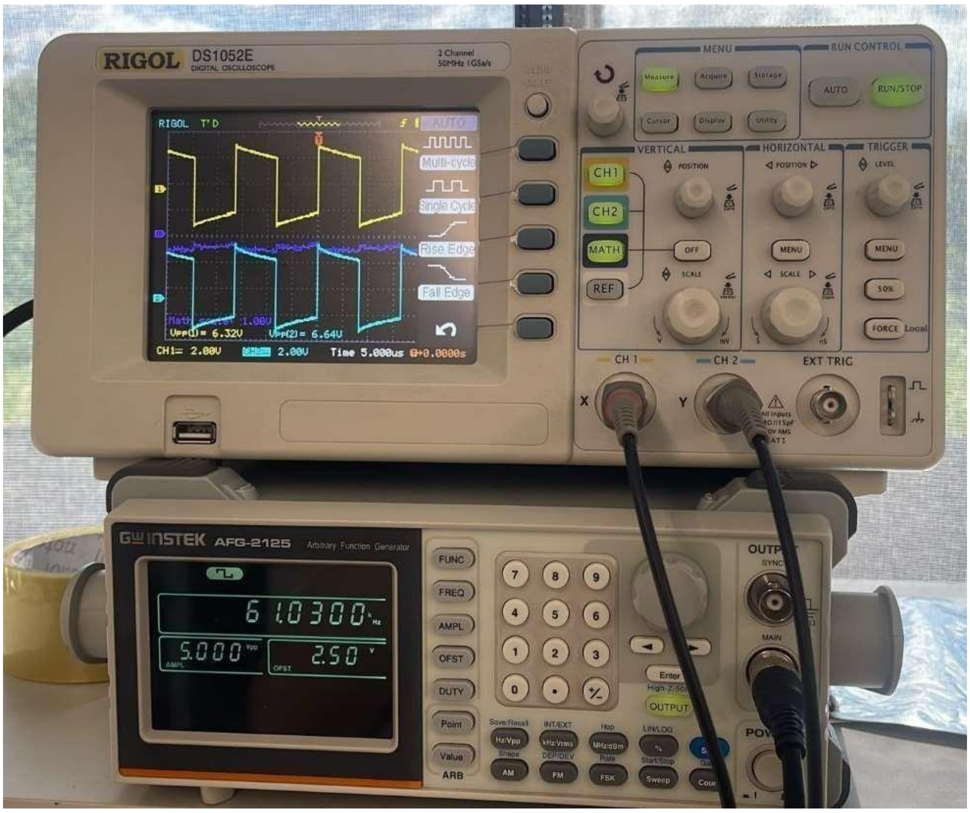


Image 7

6- For 61030.0hz and 2.5v

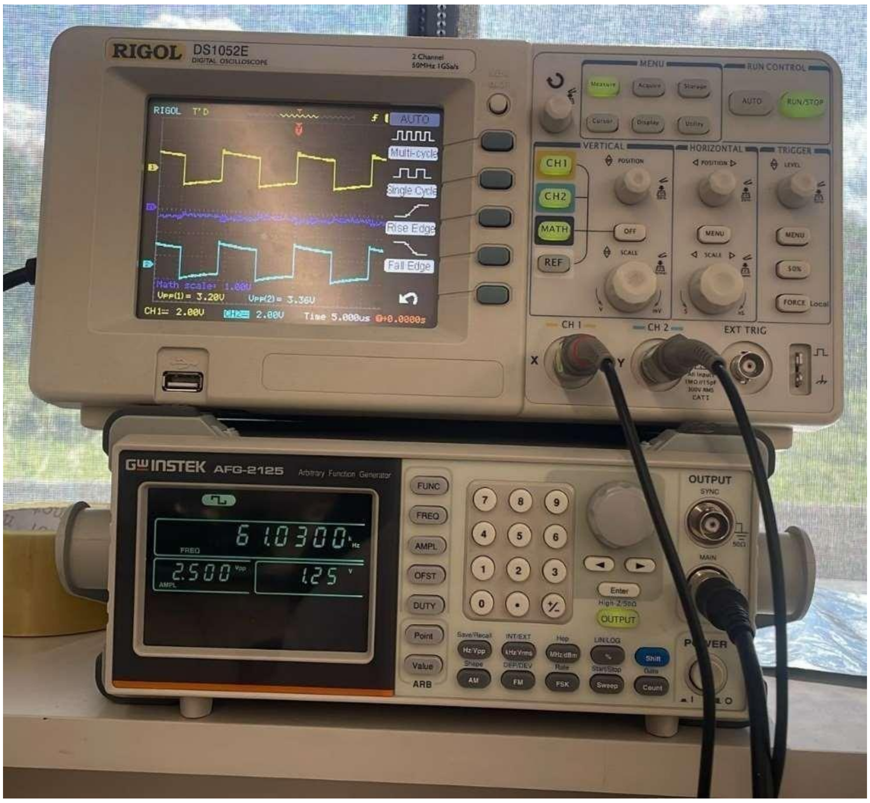


Image 8

Single Coil (for Physic)

1- Circuits

Image 9-10

1- Oscilloscope for Physic

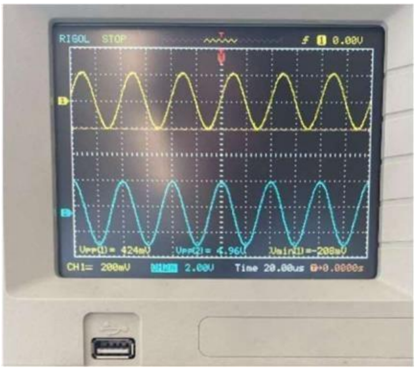


Image 11-12

Double Coil (for Physic)

1- Circuit

Image 13

2- Oscilloscope for Physic:

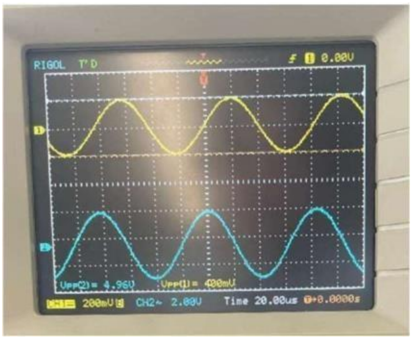


Image 14-15

The differences between LT-spice and oscilloscope at the timeof measurement are due to ambient conditions, internal resistances, etc.

Since $f = \frac{1}{T}$:

$$\frac{R}{10L} = \frac{10}{10 \cdot (1.63924407 \times 10^{-3})} \simeq 610,03 \text{ hz}$$

$$\frac{R}{L} = \frac{10}{(1.63924407 \times 10^{-3})} \simeq 6100,3 \text{ hz}$$

$$\frac{10R}{L} = \frac{10 \cdot 10}{(1.63924407 \times 10^{-3})} \simeq 61003 \text{ hz}$$

11.LT-SPICE

For T=10L/R
(5V)

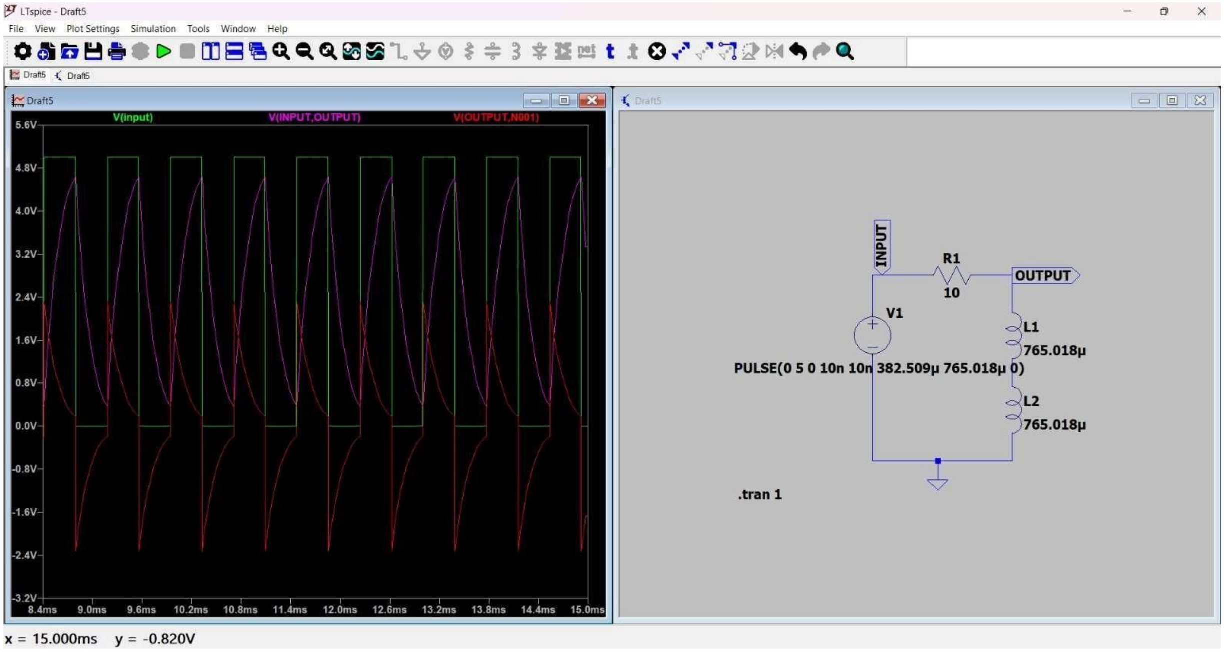


Image 16

For T= 10L/R (2.5V)

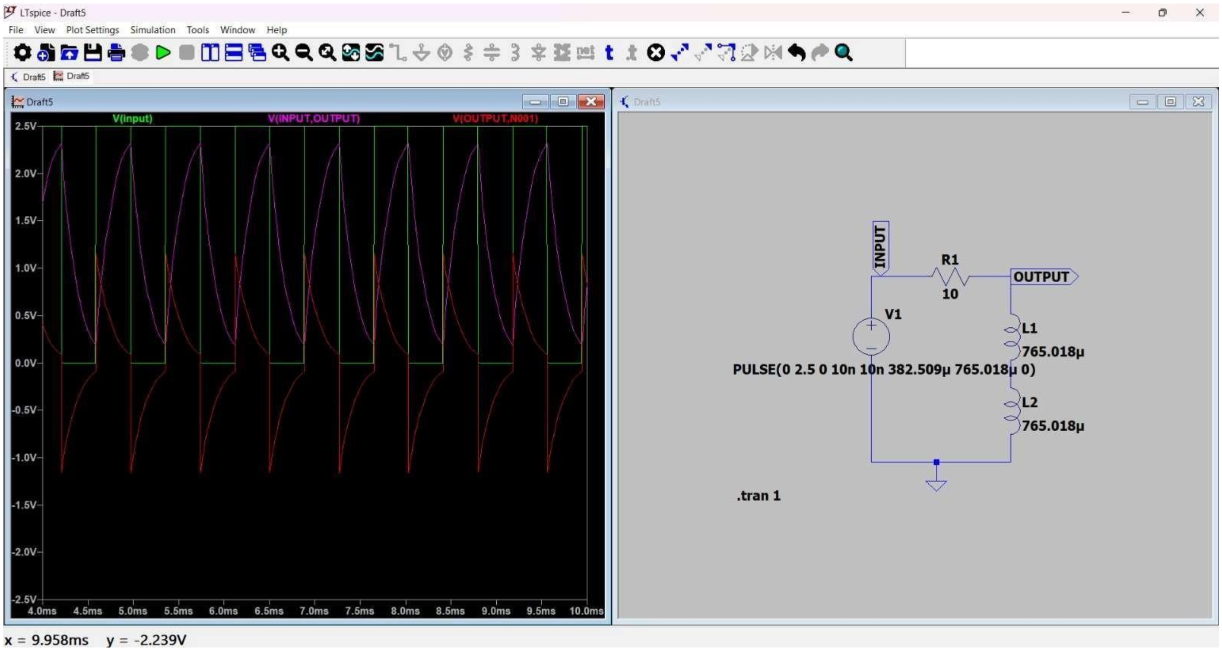


Image 17

For T= L/R (5V)

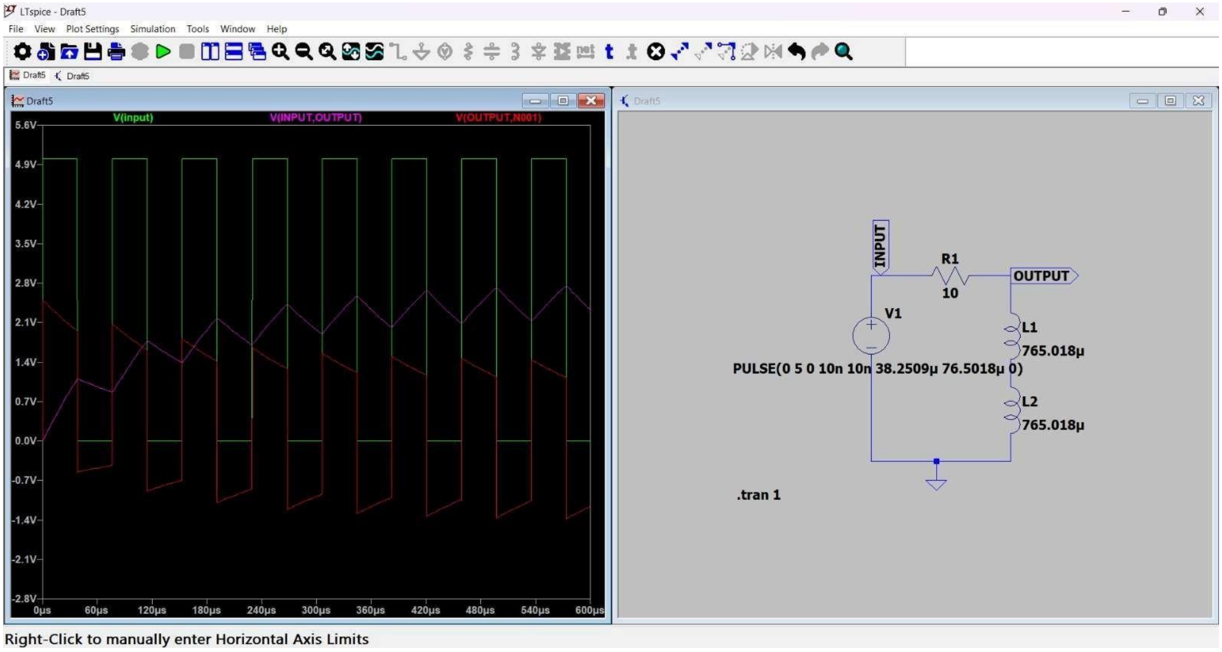


Image 18

For T= L/R (2.5V)

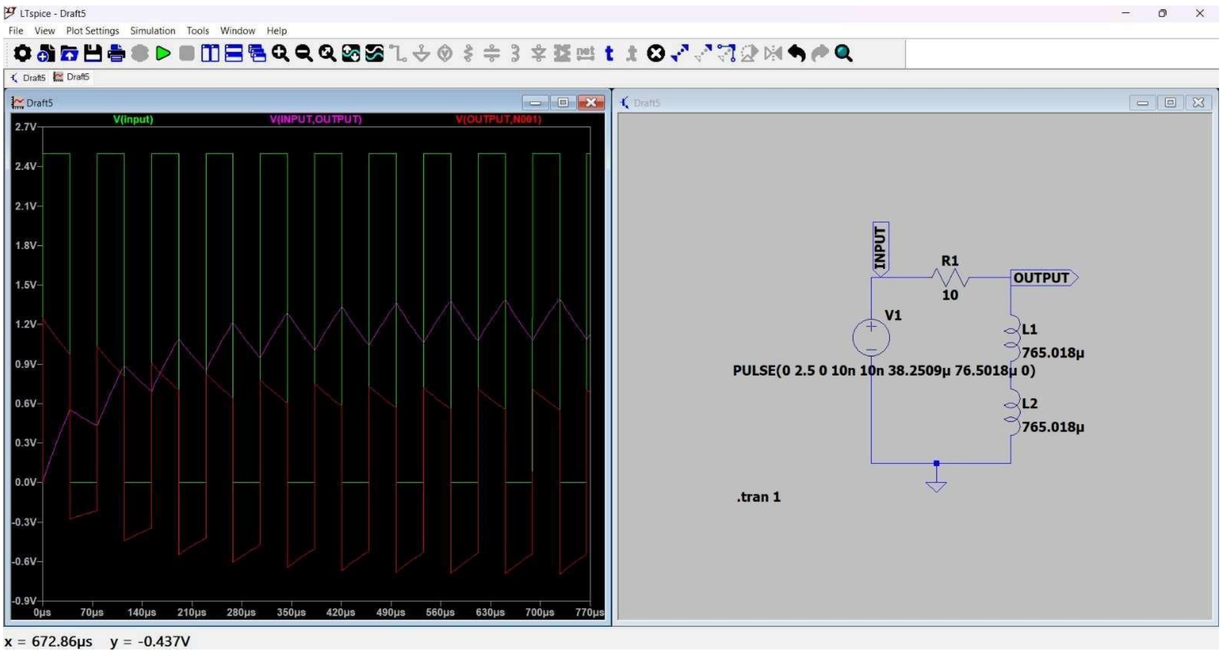


Image 19

For T= L/10R (5V)

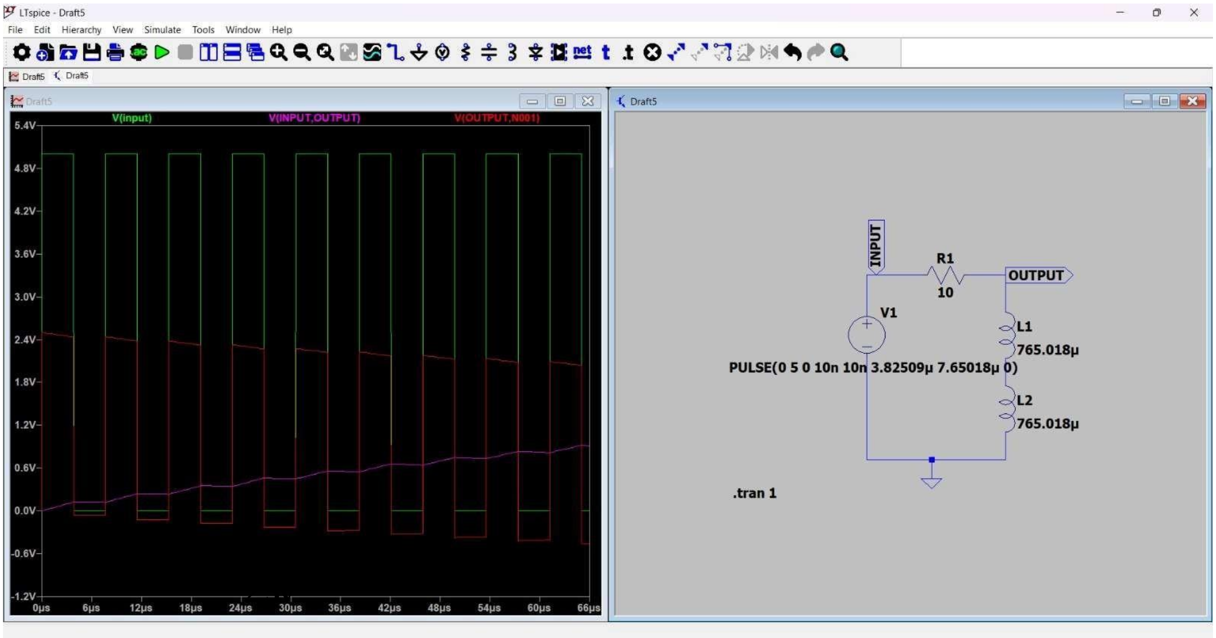


Image 20

For T=
L/10R (2.5V)

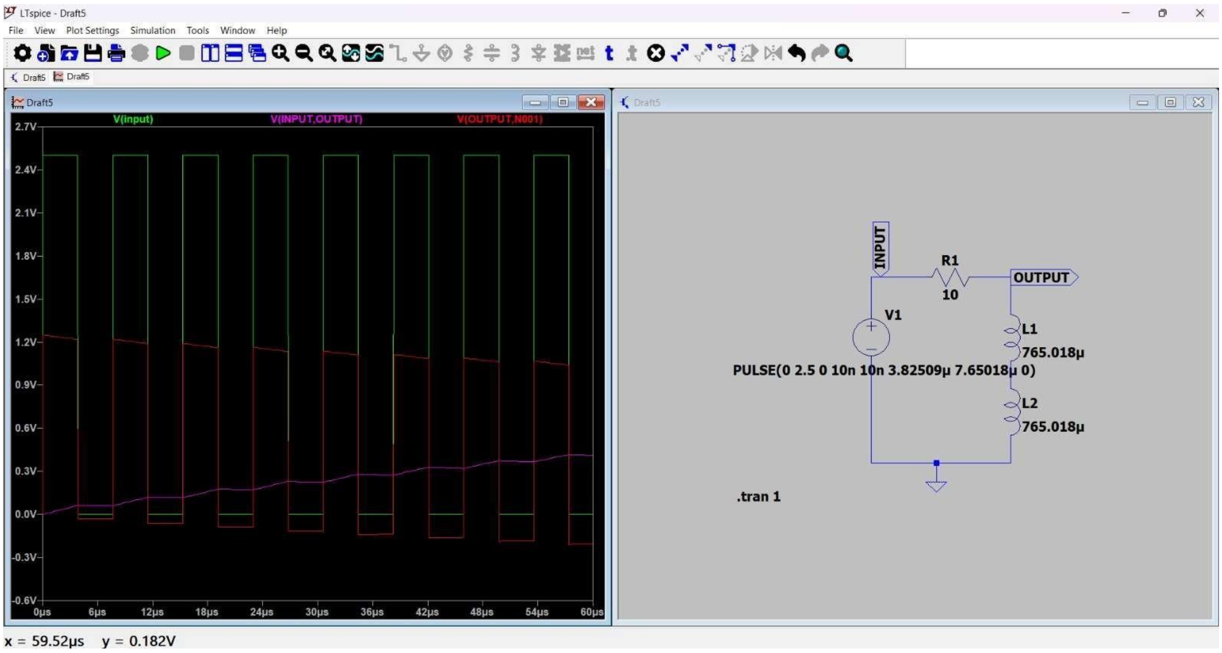


Image 21

$$\frac{10L}{R} = 765.0186885 \times 10^{-6} \simeq 765.018\ \mu s$$

$$\frac{L}{R} = 76.50186885 \times 10^{-6} \simeq 76.5018\ \mu s$$

$$\frac{L}{10R} = 7.650186885 \times 10^{-6} \simeq 7.65018\ \mu s$$

The data obtained as a result of the experiment and the values obtained in the circuit simulation made by means of the LTspice program are close to each other.
Due to the intrinsic resistance values and error margins of the devices used in the laboratory environment, the laboratory data obtained has a small margin of deviation compared to the simulation.

12.CONCLUSION

As a result of the experiments and calculations, the inductance values of Helmholtz coils, a type of coil, and the magnetic field generated by each coil were calculated. All the data obtained were tested both theoretically and experimentally, and a simulation of the circuit in the report and the circuit in the same tasks was made by the program called Ltspice. In order to calculate the magnetic field generated by the Helmholtz coils in a more stable way and to be suitable for linearity and homogeneity, the Taylor Series method was used and this calculation was also made according to the cylindrical coordinate system.

The data obtained as a result of the calculations have slight inequalities in terms of data with the circuit copy provided by the simulation due to a number of factors such as the oscilloscope used in the laboratory environment and the resistivity of the signal-centered function generator device.

13.REFERENCE LIST

1) https://www.researchgate.net/publication/369024492_Design_of_A_Three-axis_Helmholtz_Coil_for_Magnetic_Sensor_Calibration
2) https://www.youtube.com/watch?v=C-JRTQAZ8sY
3) https://en.wikipedia.org/wiki/Main_Page