ABDULLAH GÜL
ÜNİVERSİTESİ

PROJECT REPORT

Computation and Analysis (COMA) Capsule

PROJECT NAME: PENDULUM

Submitted at 24.12.2023

PROJECT TEAM

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1. INTRODUCTION

OBJECTIVE

The project's objective is to investigate a pendulum depending on different properties. The main purpose is to measure the period of oscillation of the pendulum.

BACKGROUND

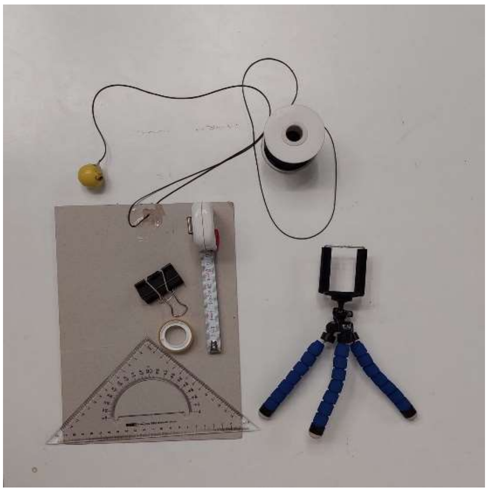
The main structure of the project was discussed, and tasks were shared by the team members. Then, the necessary equipment is decided and collected. After that, the corresponding information collected by team members was decided and combined. Then, to collect data, experiments on a pendulum were done and recorded by a camera. After collecting data, information was analyzed by using the “Tracker Simulation” program, and finally, analyzed data used in equations to compare results. This report analyzes the difference between real-life and theoretical data.

ANALYZE

1)Description of Experiment:

Equipment:

- A ball
- String
- Camera
- Meter
- Tripod
- Adhesive tape
- Protractor



The ball and string were combined into a pendulum. The upper and lower points of the pendulum between the ground and the angle between these two points were measured. The ball of the pendulum is left to oscillate at a height of 1 meter which is the highest point. During the oscillation, time for one period was measured. The experiment of oscillation with different properties was recorded by the camera and uploaded to the Tracker program. After that, the videos were analyzed by sensitive measurements.

2. CALCULUS I

2.1. Check the function to solve for equation 2.

Equation 2 given as: $\frac{d^2\theta\,(t)}{dt^2} + \omega^2\theta(t) = 0$

Function (equation 3) given as: $\theta(t) = B\cos(\omega t)$

The first derivative of equation 3: $-\omega B\sin(\omega t)$

The second derivative of equation 3: $-\omega^2 B\cos(\omega t)$

The second derivative of the function for θ(t) equals to $-w^2Bcos(wt)$. If the second derivative is written in equation 2, the result of equation 2 equals 0 as given above.

$$\theta''(t) + \omega^2\theta(t) = 0$$

$$-\omega^2 Bcos(\omega t) + \omega^2 Bcos(\omega t) = 0$$

As a result of the calculations, equation 3 proves equation 2.

2.2. Function of the path S(t) and the area function A(t)

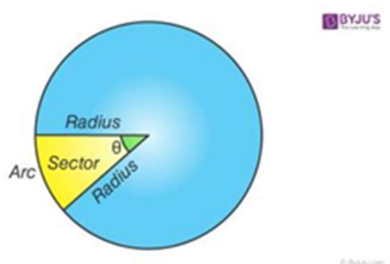


Figure 2. Geometrical definition of an arc and a sector.

The length of an arc found by the formula of $S(t) = \theta(t) \cdot L$

The formula of function θ(t) given as $\theta(t) = B\cos(\omega t)$

The constant B is found with calculations as θ_{max}.

Therefore, the function of path S(t) equal to:

$$S(t) = B\cos(\omega t) \cdot L$$

$$S(t) = \theta\max\cos(\omega t) \cdot L$$

The area of the circle is found by the formula of $A_{circle} = \pi r^2$. To find the area of a sector created by the path of the pendulum, the unit of angle in the circle is converted into radians.

$$A_{sector} = \left(\frac{\theta}{2\pi}\right).A_{circle}$$

$$A_{sector} = \left(\frac{\theta}{2\pi}\right).\ \pi r^2 \text{ (π are eliminated)}$$

$$A(t) = \frac{1}{2}\theta(t)L^2 \qquad A(t) = \frac{1}{2}B\cos(\omega t)L^2 \qquad A(t) = \frac{1}{2}\theta_{max} \cdot \cos(\omega t)L^2$$

3. PHYSICS I

3.1 Theoretical Tasks

3.1.1. Deriving the equation for mechanical energy

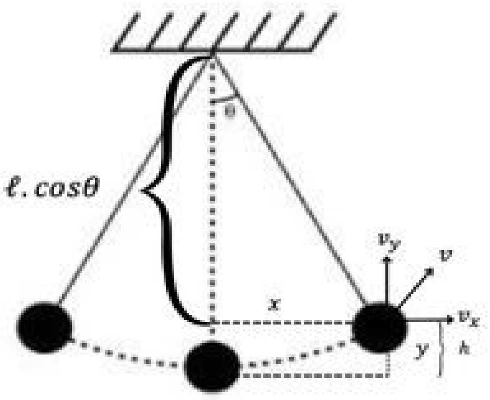
Mechanical energy (E) is the sum of kinetic (K) and potential (U) energy. The conservation of mechanical energy applies the cases such as frictionless motions. While deriving the equation for mechanical energy E, friction was neglected.

$$E(t) = K(t) + U(t)$$

Potential energy $U(t) = mgh$

Kinetic energy $K(t) = \frac{1}{2}mv^2$

$$E(t) = \frac{1}{2}mv^2 + mgh$$



$$x = \ell.sin\theta$$
$$y = \ell - \ell.cos\theta \ (h)$$
$$v^2 = {v_x}^2 + {v_y}^2$$
$$KE = \frac{1}{2}.m.v^2 = \frac{1}{2}.m\left({v_x}^2 + {v_y}^2\right)$$
$$v_x = \frac{d_x}{d_t} = x' = \ell.cos\theta.\theta'$$
$$v_y = \frac{d_y}{d_t} = y' = \ell.sin\theta.\theta'$$

$$KE = \frac{1}{2}.m[\ell^2.\theta'^2.cos^2\theta + \ell^2.\theta'^2.sin^2\theta]$$

$$KE = \frac{1}{2}.m[\ell^2.\theta'^2(cos^2\theta + sin^2\theta)]$$

$$KE = \frac{1}{2}.m.\ell^2.(\theta'(t))^2$$

$h = \ell - \ell.cos\theta$ (h is the vertical height between the upper and lower points)

$$PE = m.g.h = m.h.\ell(1 - \ cos\theta(t))$$

$$E(t) = mg[\ell(1 - \ cos\theta(t)] + \frac{1}{2}m[\theta'(t)\ell]^2$$

θ is given as $\theta = (\omega t)$

3.1.2. Mechanical energy E conservation for the model (2)

Model 2 given as: $\frac{d^2\theta\ (t)}{dt^2} + \omega^2\theta(t) = 0$

The equation (model 2) means the formula for oscillator energy formula.

Given that the equation describes simple harmonic motion, the total mechanical energy (E) of the pendulum remains constant throughout its motion. This is because, in simple harmonic motion, the kinetic and potential energies interchange cyclically, but the total energy remains constant as long as no external forces are acting on the system. Therefore, the conservation of mechanical energy holds for the model described by equation (2) representing the motion of a simple pendulum.

3.1.3. Investigation of frequency (ω) depends on the parameters: m, L, θmax, and g (the gravity acceleration)

The formula of the frequency is given as:

$\omega = 1/T$

The period T equals to $T= 2\pi\sqrt{\frac{l}{g}}$ (the analyze of units to find T will be explained in 3.1.5)

Therefore, the period T does not depend on mass and θmax.

3.1.4. Physical meaning of B

The equation is given as: $\theta(t) = B\cos(\omega t)$

At the upper point where the motion started, time (t) equals 0, and angle θ is maximum. Time (t) is written as 0 in the formula to find B. The result of this calculation gives:

$$\mathbf{B\cos 0 = \theta_{max}} \qquad \cos 0 = 1 \Rightarrow B = \boldsymbol{\theta_{max}}$$

3.1.5. The relationship between different parameters

To investigate the relationship between different parameters in a pendulum such as mass, length, and gravity acceleration:

$$T = k.m^x.l^y.g^z$$

k represents a coefficient,

m represents the mass of the bob (kg),

l is the length of the rope of the pendulum (m),

g is the gravity acceleration (m/s^2).

By doing dimensional analysis, units are investigated. If units are written in the equation above:

$$s = (kg)^x (m)^y (m.s^{-2})^z$$

$x=0 \qquad y=0 \qquad -2z=1 \Rightarrow z=-1/2$

Therefore, $T \sim \sqrt{\frac{l}{g}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$

3.2. Experimental Tasks

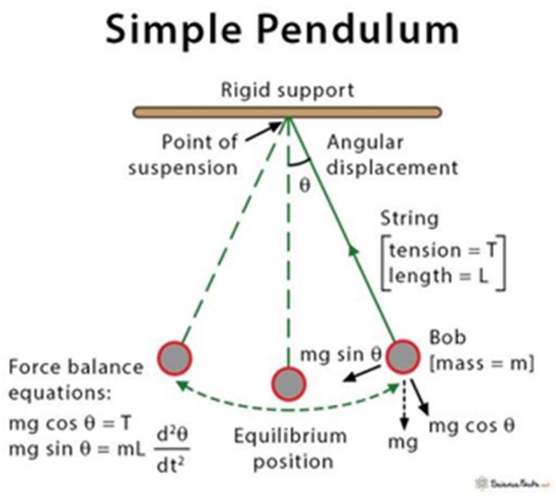


Figure 1. *Principle scheme of the experimental setup.*

Hooke's law, which states that as long as the angle of displacement is small, the restoring force of the displaced mass follows a simple harmonic motion, given by the formula: **Freturn = -kθ**

3.2.1. Real θ(t) follows Equation (3)

By experimenting with a pendulum, different data were collected. Collected data were used to achieve a conclusion to compare whether real θ(t) follows equation (3) or not.

Equation (3): θ(t) = **B cos(ωt)**

	0.4m rope	0.55m rope	0.4m rope	0.55m rope
Experimental Results:	17.5	20.2	14	26.5
Theoretical Results:	23.5363	22.1	17.7926	32.0633

*This experiment was done with different properties of the pendulum such as the length of the rope and angle.

As a result of the calculation by using equation 3, the data collected in the experiment and the values that are found with calculations are very close to each other. That means real θ follows equation 3 with small margins of error.

3.2.2. Average % of mechanical energy loss per a single period.

There are differences in experimental and theoretical data due to the conditions in real-life experiments. Theoretical calculations assume every force exerted on the pendulum and conditions are stable. However, the results of the calculations according to the data from experiments are different from theoretical results. Mechanical energy is one of the results that is different from the theoretical results. In theoretical calculations, mechanical energy is conserved but, in the experiment, there is a loss of mechanical energy.

The experiment was done to observe the behavior of a pendulum in real life. By using data from the experiment mechanical energy was calculated per a single period. While calculations were done, the upper point of the pendulum where the bob starts the motion was used to calculate potential energy.

Mechanical energy (ME)=Potential energy(U) + Kinetic energy(K)

U(t) = $\boldsymbol{mgh}$ K(t) $= \frac{1}{2}\boldsymbol{mv^2}$

At the upper point, kinetic energy equals 0 due to the velocity being 0, so mechanical energy equals potential energy at the upper point of the motion of the pendulum.

Rate of change in ME (%)	MEinitial Joule(J)	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
0.4m rope θmax=43.9	0.01221	7.2%	1.42%	1.7%	3.18%	6.9%	4.3%	3%	5.58%	4.16%	5.26%
0.55m rope θmax=42.1	0.01551	8.96%	7.93%	8.3%	5.78%	6.7%	7.14%	5.86%	5.9%	6.22%	6.38%

*All data are approximate values

	Average % of mechanical energy loss per a single period
0.4m rope θmax=43.9	4.27%
0.55m rope θmax=42.1	6.937%

3.2.3. The difference between the experimental and the theoretical solution

According to the results of the experiment and calculations by using equation 3, there are some differences between the behavior of the pendulum and theoretical results. The theoretical solution assumes the pendulum has a massless rope. However, in real life, the rope has a mass even if it is as small as that can be neglected. Due to some conditions in real life, getting different results with minor deviations is possible.

Reasons that may cause differences in data:

1- Imperfect Construction:

The conditions of the experiment and the quality of the equipment may affect the results of the accuracy of the experiment.

2- External Forces and environmental factors:

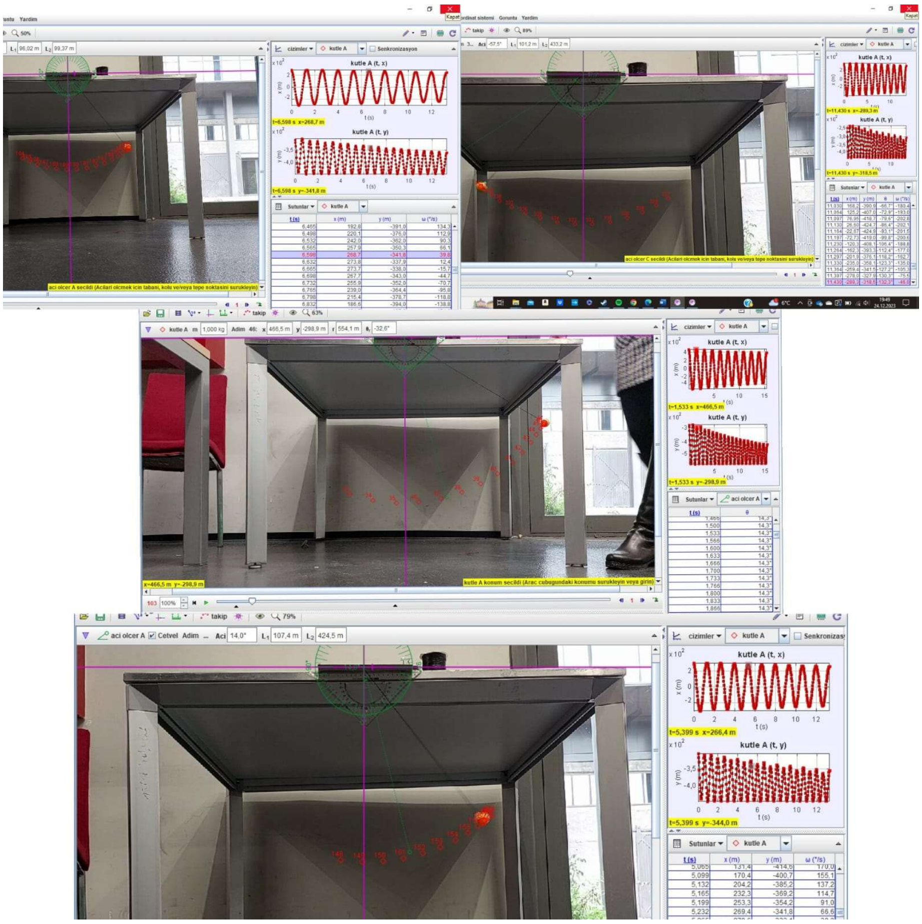
External forces and environmental factors such as air resistance, temperature, and wind affect the results of the experiment. The theoretical solution assumes conditions as no friction and no air resistance. However, in real life, these forces cannot be calculated exactly without laboratory conditions.

3- Initial conditions:

In the experiment, the initial velocity of the bob must be equal to zero (0) to observe the mechanical energy conservation. Because potential energy is converted into kinetic energy and mechanical energy is conserved.

4- Approximations:

While calculations were done, a calculator was used. Calculators do not give the result exactly due to making approximations in numbers.



4. LINEAR ALGEBRA

4.1. Construct a matrix.

4.1.1. The inverse of the matrix A

$t_1 = 1 \qquad t_2 = 10 \qquad t_3 = 20$

$$A = \begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} \end{bmatrix}$$

$$\left(\begin{array}{ccc|ccc} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} & 1 & 0 & 0 \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} & 0 & 1 & 0 \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} & 0 & 0 & 1 \end{array}\right)$$

$$R_1 \,/ \left(\frac{258083}{25000}\right) \rightarrow R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} & 0 & 1 & 0 \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} & 0 & 0 & 1 \end{array}\right)$$

$$R_2 - \left(\frac{3158319}{100000}\right).R_1 \rightarrow R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & \frac{-64992317937}{25808300000} & \frac{31916891}{7373800000} & \frac{-3158319}{1032332} & 1 & 0 \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} & 0 & 0 & 1 \end{array}\right)$$

$$R_3 - \left(\frac{15441}{10000}\right).R_1 \rightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & \frac{-64992317937}{25808300000} & \frac{31916891}{7373800000} & \frac{-3158319}{1032332} & 1 & 0 \\ 0 & \frac{5786188151}{6452075000} & \frac{-87289}{921725000} & \frac{-77205}{516166} & 0 & 1 \end{array}\right)$$

$$R_2 / \left(\frac{-64992317937}{25808300000}\right) \rightarrow R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & 1 & \frac{-223418237}{129984635874} & \frac{26319325000}{21664105979} & \frac{-25808300000}{64992317937} & 0 \\ 0 & \frac{5786188151}{6452075000} & \frac{-87289}{921725000} & \frac{-77205}{516166} & 0 & 1 \end{array}\right)$$

$$R_3 - \left(\frac{5786188151}{6452075000}\right).R_2 \rightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & 1 & \frac{-223418237}{129984635874} & \frac{26319325000}{21664105979} & \frac{-25808300000}{64992317937} & 0 \\ 0 & 0 & \frac{940252828339}{649923179370000} & \frac{-53686847327}{43328211958} & \frac{23144752604}{649923179371} & 1 \end{array}\right)$$

$$R_3 / \left(\frac{940252828339}{649923179370000}\right) \rightarrow R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & 1 & \frac{-223418237}{129984635874} & \frac{26319325000}{21664105979} & \frac{-25808300000}{64992317937} & 0 \\ 0 & 0 & 1 & \frac{-805302709905000}{940252828339} & \frac{231447526040000}{940252828339} & \frac{649923179370000}{940252828339} \end{array}\right)$$

$$R_2 - \left(\frac{-223418237}{129984635874}\right).R_3 \to R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & \frac{5}{73738} & \frac{25000}{258083} & 0 & 0 \\ 0 & 1 & 0 & \frac{-241862377500}{940252828339} & \frac{24440920000}{940252828339} & \frac{1117091185000}{940252828339} \\ 0 & 0 & 1 & \frac{-805302709905000}{940252828339} & \frac{231447526040000}{940252828339} & \frac{649923179370000}{940252828339} \end{array}\right)$$

$$R_1 - \left(\frac{5}{73738}\right).R_3 \to R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{24701}{258083} & 0 & \frac{37599118131812500}{242663270696214137} & \frac{-578618815100000}{34666181528030591} & \frac{-1624807948425000}{34666181528030591} \\ 0 & 1 & 0 & \frac{-241862377500}{940252828339} & \frac{24440920000}{940252828339} & \frac{1117091185000}{940252828339} \\ 0 & 0 & 1 & \frac{-805302709905000}{940252828339} & \frac{231447526040000}{940252828339} & \frac{649923179370000}{940252828339} \end{array}\right)$$

$$R_1 - \left(\frac{24701}{258083}\right).R_2 \to R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{168834680000}{940252828339} & \frac{-18033140000}{940252828339} & \frac{-150986020000}{940252828339} \\ 0 & 1 & 0 & \frac{-241862377500}{940252828339} & \frac{24440920000}{940252828339} & \frac{1117091185000}{940252828339} \\ 0 & 0 & 1 & \frac{-805302709905000}{940252828339} & \frac{231447526040000}{940252828339} & \frac{649923179370000}{940252828339} \end{array}\right)$$

$$A^{-1} = \begin{bmatrix} \frac{168834680000}{940252828339} & \frac{-18033140000}{940252828339} & \frac{-150986020000}{940252828339} \\ \frac{-241862377500}{940252828339} & \frac{24440920000}{940252828339} & \frac{1117091185000}{940252828339} \\ \frac{-805302709905000}{940252828339} & \frac{231447526040000}{940252828339} & \frac{649923179370000}{940252828339} \end{bmatrix}$$

4.1.2 LDU form

$$A = \begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} \end{bmatrix}$$

$$\begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} \end{bmatrix}$$

$$R_2 - \left(\frac{3158319}{1032332}\right).R_1 \rightarrow R_2$$

$$\begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ 0 & \frac{-64992317937}{25808300000} & \frac{31916891}{7373800000} \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} \end{bmatrix}$$

$$R_3 - \left(\frac{772055}{516166}\right).R_1 \rightarrow R_3$$

$$\begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ 0 & \frac{-64992317937}{25808300000} & \frac{31916891}{7373800000} \\ 0 & \frac{5786188151}{6452075000} & \frac{-87289}{921725000} \end{bmatrix}$$

$$R_3 - \left(\frac{-23144752604}{64992317937}\right).R_2 \rightarrow R_3$$

$$\begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ 0 & \frac{-64992317937}{25808300000} & \frac{31916891}{7373800000} \\ 0 & 0 & \frac{940252828339}{649923179370000} \end{bmatrix}$$

$$A = L \cdot D \cdot U$$

$$A = \begin{bmatrix} \frac{258083}{25000} & \frac{24701}{25000} & \frac{7}{10000} \\ \frac{3158319}{100000} & \frac{25227}{50000} & \frac{647}{100000} \\ \frac{15441}{10000} & \frac{52229}{50000} & \frac{1}{100000} \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3158319}{1032332} & 1 & 0 \\ \frac{77205}{516166} & \frac{-23144752604}{64992317937} & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{258083}{25000} & 0 & 0 \\ 0 & \frac{-64992317937}{25808300000} & 0 \\ 0 & 0 & \frac{940252828339}{649923179370000} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & \frac{24701}{258083} & \frac{5}{73738} \\ 0 & 1 & \frac{-1718804961}{1000000000000} \\ 0 & 0 & 1 \end{bmatrix}$$

4.2. Complete solution for the system

$A = x.b$

$Ab \to Rd \qquad RREF$

$$t_1 = 5 \qquad t_2 = 15 \qquad t_3 = 25$$

$$A = \begin{bmatrix} 43.798 & 0.048 & 0.012 & 17.519 & 3.503 \\ 42.987 & 0.430 & 0.011 & 17.195 & 3.439 \\ 41.381 & 1.165 & 0.010 & 16.552 & 3.310 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

$$\left(\begin{array}{ccccc|c} 43.798 & 0.048 & 0.012 & 17.519 & 3.503 & 1 \\ 42.987 & 0.430 & 0.011 & 17.195 & 3.439 & 3 \\ 41.381 & 1.165 & 0.010 & 16.552 & 3.310 & 0 \end{array}\right)$$

$$\frac{500}{21899} . R_1 \to R_1$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ \frac{42987}{1000} & \frac{43}{100} & \frac{11}{1000} & \frac{3439}{200} & \frac{3439}{1000} & 3 \\ \frac{41381}{1000} & \frac{233}{200} & \frac{1}{100} & \frac{2069}{125} & \frac{331}{100} & 0 \end{array}\right)$$

$$\frac{1000}{42987}.R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 1 & \frac{430}{42987} & \frac{11}{42987} & \frac{17195}{42987} & \frac{3439}{42987} & \frac{1000}{14329} \\ \frac{41381}{1000} & \frac{233}{200} & \frac{1}{100} & \frac{2069}{125} & \frac{331}{100} & 0 \end{array}\right)$$

$$\frac{1000}{41381}.R_3 \rightarrow R_3$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 1 & \frac{430}{42987} & \frac{11}{42987} & \frac{17195}{42987} & \frac{3439}{42987} & \frac{1000}{14329} \\ 1 & \frac{1165}{41381} & \frac{10}{41381} & \frac{16552}{41381} & \frac{3310}{41381} & 0 \end{array}\right)$$

$$R_2 - 1.R_1 \rightarrow R_2$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & \frac{8384882}{941372313} & \frac{-17033}{941372313} & \frac{17357}{1882744626} & \frac{37861}{1882744626} & \frac{14734500}{313790771} \\ 1 & \frac{1165}{41381} & \frac{10}{41381} & \frac{16552}{41381} & \frac{3310}{41381} & 0 \end{array}\right)$$

$$R_3 - 1.R_1 \rightarrow R_3$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & \frac{8384882}{941372313} & \frac{-17033}{941372313} & \frac{17357}{1882744626} & \frac{37861}{1882744626} & \frac{14734500}{313790771} \\ 0 & \frac{24519191}{906202519} & \frac{-29296}{906202519} & \frac{-9243}{1812405038} & \frac{13737}{1812405038} & \frac{-500}{21899} \end{array}\right)$$

$$\frac{941372313}{8384882}.R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & 1 & \frac{-17033}{8384882} & \frac{17357}{16769764} & \frac{37861}{16769764} & \frac{2009250}{381131} \\ 0 & \frac{24519191}{906202519} & \frac{-29296}{906202519} & \frac{-9243}{1812405038} & \frac{13737}{1812405038} & \frac{-500}{21899} \end{array}\right)$$

$$\frac{906202519}{24519191}.R_3 \rightarrow R_3$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & 1 & \frac{-17033}{8384882} & \frac{17357}{16769764} & \frac{37861}{16769764} & \frac{2009250}{381131} \\ 0 & 1 & \frac{-29296}{24519191} & \frac{-9243}{49038382} & \frac{13737}{49038382} & \frac{-20690500}{24519191} \end{array}\right)$$

$$R_3 - 1.R_2 \rightarrow R_3$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & 1 & \frac{-17033}{8384882} & \frac{17357}{16769764} & \frac{37861}{16769764} & \frac{2009250}{381131} \\ 0 & 0 & \frac{171991877231}{205590523270462} & \frac{-503081062513}{411181046540924} & \frac{-813137966417}{411181046540924} & \frac{-57150975472250}{9345023785021} \end{array}\right)$$

$$\frac{205590523270462}{171991877231}.R_3 \rightarrow R_3$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & 1 & \frac{-17033}{8384882} & \frac{17357}{16769764} & \frac{37861}{16769764} & \frac{2009250}{381131} \\ 0 & 0 & 1 & \frac{-22972787}{15707738} & \frac{-37131283}{15707738} & \frac{-57414560500}{7853869} \end{array}\right)$$

$$R_2 + \frac{17033}{8384882}.R_3 \rightarrow R_2$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & \frac{6}{21899} & \frac{17519}{43798} & \frac{3503}{43798} & \frac{500}{21899} \\ 0 & 1 & 0 & \frac{-30409}{15707738} & \frac{-39965}{15707738} & \frac{-75227500}{7853869} \\ 0 & 0 & 1 & \frac{-22972787}{15707738} & \frac{-37131283}{15707738} & \frac{-57414560500}{7853869} \end{array}\right)$$

$$R_1 - \frac{6}{21899}.R_3 \rightarrow R_1$$

$$\left(\begin{array}{ccccc|c} 1 & \frac{24}{21899} & 0 & \frac{137729767733}{343983754462} & \frac{27734890805}{343983754462} & \frac{348414297500}{171991877231} \\ 0 & 1 & 0 & \frac{-30409}{15707738} & \frac{-39965}{15707738} & \frac{-75227500}{7853869} \\ 0 & 0 & 1 & \frac{-22972787}{15707738} & \frac{-37131283}{15707738} & \frac{-57414560500}{7853869} \end{array}\right)$$

$$R_1 - \frac{24}{21899}.R_2 \rightarrow R_1$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{6289351}{15707738} & \frac{1266535}{15707738} & \frac{15992500}{7853869} \\ 0 & 1 & 0 & \frac{-30409}{15707738} & \frac{-39965}{15707738} & \frac{-75227500}{7853869} \\ 0 & 0 & 1 & \frac{-22972787}{15707738} & \frac{-37131283}{15707738} & \frac{-57414560500}{7853869} \end{array}\right)$$

First 3 columns are pivot columns and last 2 columns are free columns.

$$X_c = X_n + X_p$$

X_p

$x_4 = 0$ $\quad \frac{6289351}{15707738} \cong 0.40 \quad \frac{1266535}{15707738} \cong 0.08 \quad \frac{15992500}{7853869} \cong 2.03$

$x_5 = 0$

$1.x_1 + 0.x_2 + 0.x_3 + (0.40).x_4 + (0.08).x_5 = 2.03$

$x_1 = 2.03$ $\quad \frac{-30409}{15707738} \cong -0.001 \quad \frac{-39965}{15707738} \cong -0.002 \quad \frac{-75227500}{7853869} \cong -9.57$

$0.x_1 + 1.x_2 + 0.x_3 + (-0.001).x_4 + (-0.002).x_5 = -9.57$

$x_2 = -9.57$ $\quad \frac{-22972787}{15707738} \cong -1.46 \quad \frac{-37131283}{15707738} \cong -2.36 \quad \frac{-57414560500}{7853869} \cong -7310$

$0.x_1 + 0.x_2 + 1.x_3 + (-1.46).x_4 + (-2.36).x_5 = -7310$

$x_3 = -7310$

$$X_p = \begin{bmatrix} 2.03 \\ -9.57 \\ -7310 \\ 0 \\ 0 \end{bmatrix}$$

X_n

$x_4 = 0$
$x_5 = 1$

$1. x_1 + 0. x_2 + 0. x_3 + (0.40). x_4 + (0.08). x_5 = 0$

$x_1 = -0.08$

$0. x_1 + 1. x_2 + 0. x_3 + (-0.001). x_4 + (-0.002). x_5 = 0$

$x_2 = 0.002$

$0. x_1 + 0. x_2 + 1. x_3 + (-1.46). x_4 + (-2.36). x_5 = 0$

$x_3 = 2.36$

$$x_5 \begin{bmatrix} -0.08 \\ 0.002 \\ 2.36 \\ 0 \\ 1 \end{bmatrix}$$

$x_4 = 1$

$x_5 = 0$

$1. x_1 + 0. x_2 + 0. x_3 + (0.40). x_4 + (0.08). x_5 = 0$

$x_1 = 0.40$

$0. x_1 + 1. x_2 + 0. x_3 + (-0.001). x_4 + (-0.002). x_5 = 0$

$x_2 = 0.001$

$$0.x_1 + 0.x_2 + 1.x_3 + (-1.46).x_4 + (-2.36).x_5 = 0$$

$$x_3 = 1.46$$

$$x_4 \begin{bmatrix} 0.40 \\ 0.001 \\ 1.46 \\ 1 \\ 0 \end{bmatrix}$$

$$X_n = x_4 \begin{bmatrix} 0.40 \\ 0.001 \\ 1.46 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -0.08 \\ 0.002 \\ 2.36 \\ 0 \\ 1 \end{bmatrix}$$

$$X_c = X_n + X_p$$

$$X_c = x_4 \begin{bmatrix} 0.40 \\ 0.001 \\ 1.46 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -0.08 \\ 0.002 \\ 2.36 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2.03 \\ -9.57 \\ -7310 \\ 0 \\ 0 \end{bmatrix}$$

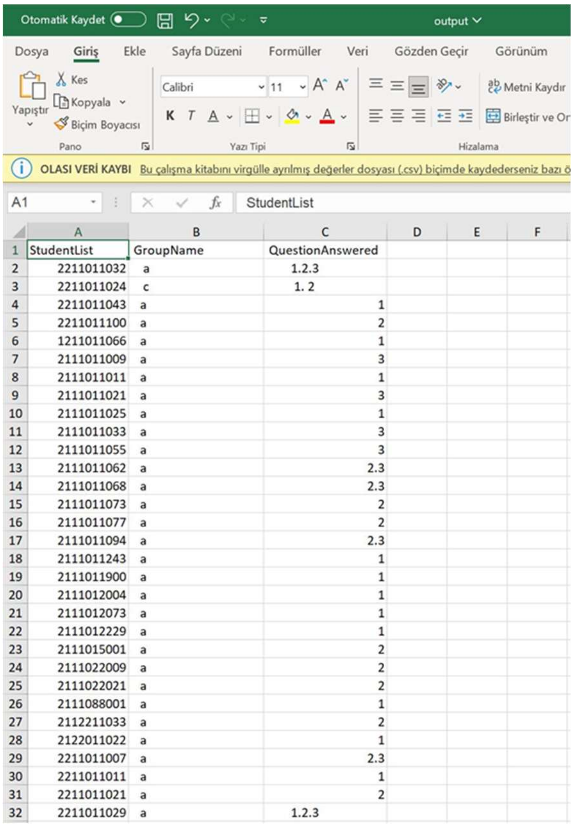
5. INTRODUCTION TO PROGRAMMING

```
#include <iostream>

#include <vector>

#include <sstream>

#include <fstream>

#include <string>

#include <regex>

#include <map>

//These are the standard C++ header files that are included for vectors, maps, regular expressions, input/output, and string manipulation.

using namespace std;

std::vector<std::string> split(std::string s, std::string delimiter) {

   size_t pos_start = 0, pos_end, delim_len = delimiter.length();

   std::string token;

   std::vector<std::string> res;
```

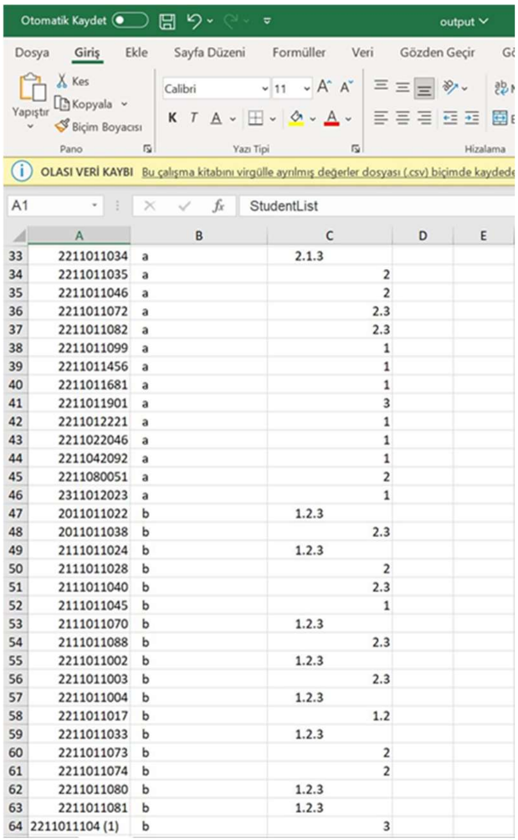
```
    while ((pos_end = s.find(delimiter, pos_start)) != std::string::npos) {
        token = s.substr (pos_start, pos_end - pos_start);
        pos_start = pos_end + delim_len;
        res.push_back (token);
    }

    res.push_back (s.substr (pos_start));
    return res;
}
int main() {

    try {
         ifstream file("file_list.csv");
        ofstream secondFile("output.csv");
        map<string,map<string,std::vector<string>>> student_list;
        /*
         * Map data structure works as key-value.
         * The key of this map is student id and the value is defined as vector.
         * This vector represents an ordered sequence, holding the student's answers.
         */

        string line = "";
        std::getline(file, line,'\n');
        while (std::getline(file, line, '\n')) {
            line = regex_replace(line, regex("\n"),
                                 "");
            if (!line.empty()) {

                std::vector<std::string> group_student_answer = split(line,
                                                                      "_"); //'' was split with this character.
```

```
            if (group_student_answer.size() == 3) {
                string group = group_student_answer[0];//group name
                group=regex_replace(group, regex("group|Group|grup|Grup|qroup"),
                                    "");

                transform(group.begin(), group.end(), group.begin(), ::tolower);
                string answer = group_student_answer[1];//answer
                answer=regex_replace(answer,
regex("QUESTION|q|quesiton|question|Question|Q|Quesiton|uestion|uesiton"),
                                    "");
                string studentID = group_student_answer[2];//studentID
                studentID = regex_replace(studentID, regex(".cpp|.txt"),
                                         "");//The .cpp.txt at the end of studentID has been cleared.

                if (student_list.find(group) == student_list.end()) {
                    student_list[group] = map<string, vector<string>>();

                }
                student_list[group][studentID].push_back(
                        answer);//From the incoming group information to the student, the answers from the
student were accessed and the new answer was added to the existing answer list.
            }
        }
    }
    secondFile<<"StudentList , "<<"GroupName , "<<"QuestionAnswered"<<endl;
    for (const auto &group: student_list) {//All groups are drawn from map, group.first represents
the group name, group.second is the map inside.
        for (const auto &student: group.second) {
            string answers="";
            int index=0;
            for (string answer: student.second) {
                if(student.second.size()-1!=index){
```

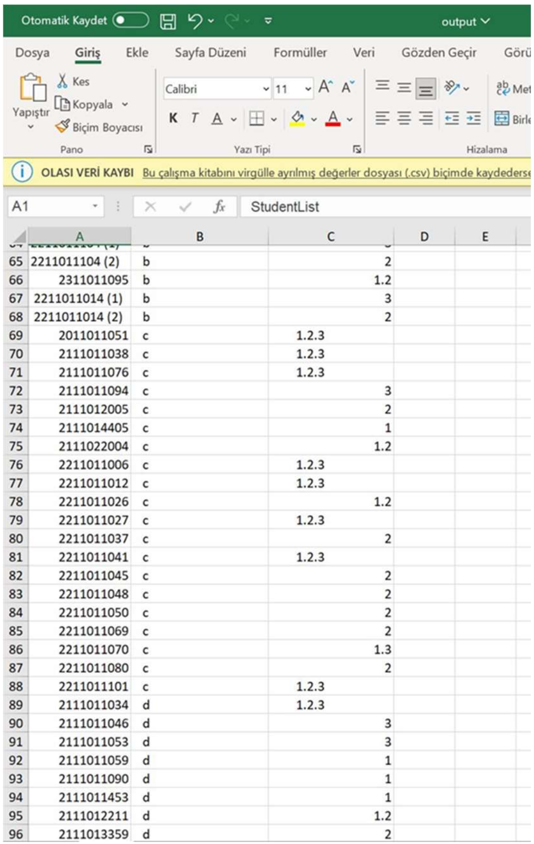
```
                answers+=answer+'.';
            }
            else{
                answers+=answer;
            }
            index++;
        }
        secondFile<<student.first+"   ,   "<<group.first+"       ,       "<<answers<<endl;// recorded as studentID-group name-answer.
      }
    }
    file.close();//the reading file is closed.
    secondFile.close();//the writing file is closed.
  }
  catch (exception ex){
    cout<<"Exception: "<< ex.what()<<std::endl;
  }
  return 0;
}
//This code was run with MinGW on visual studio. It may give errors in other compilers.
```



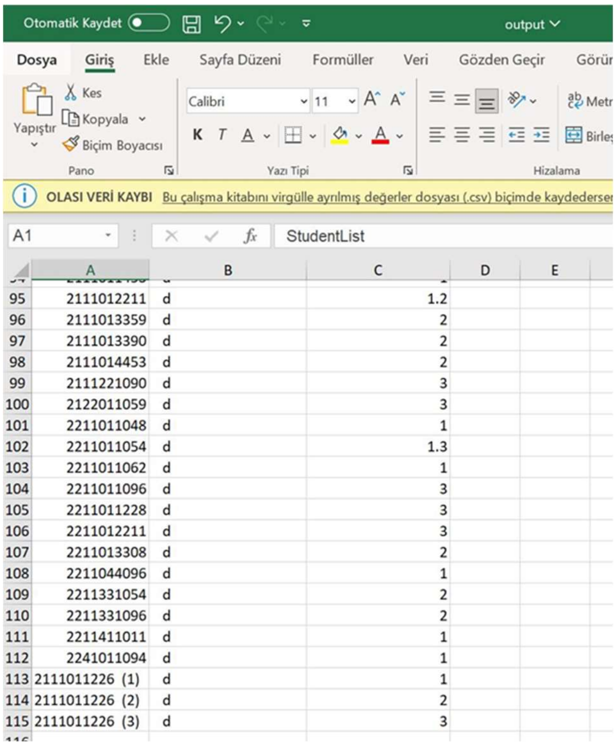
	A	B	C
1	StudentList	GroupName	QuestionAnswered
2	2211011032	a	1.2.3
3	2211011024	c	1. 2
4	2211011043	a	1
5	2211011100	a	2
6	1211011066	a	1
7	2111011009	a	3
8	2111011011	a	1
9	2111011021	a	3
10	2111011025	a	1
11	2111011033	a	3
12	2111011055	a	3
13	2111011062	a	2.3
14	2111011068	a	2.3
15	2111011073	a	2
16	2111011077	a	2
17	2111011094	a	2.3
18	2111011243	a	1
19	2111011900	a	1
20	2111012004	a	1
21	2111012073	a	1
22	2111012229	a	1
23	2111015001	a	2
24	2111022009	a	2
25	2111022021	a	2
26	2111088001	a	1
27	2112211033	a	2
28	2122011022	a	1
29	2211011007	a	2.3
30	2211011011	a	1
31	2211011021	a	2
32	2211011029	a	1.2.3



	A	B	C
33	2211011034	a	2.1.3
34	2211011035	a	2
35	2211011046	a	2
36	2211011072	a	2.3
37	2211011082	a	2.3
38	2211011099	a	1
39	2211011456	a	1
40	2211011681	a	1
41	2211011901	a	3
42	2211012221	a	1
43	2211022046	a	1
44	2211042092	a	1
45	2211080051	a	2
46	2311012023	a	1
47	2011011022	b	1.2.3
48	2011011038	b	2.3
49	2111011024	b	1.2.3
50	2111011028	b	2
51	2111011040	b	2.3
52	2111011045	b	1
53	2111011070	b	1.2.3
54	2111011088	b	2.3
55	2211011002	b	1.2.3
56	2211011003	b	2.3
57	2211011004	b	1.2.3
58	2211011017	b	1.2
59	2211011033	b	1.2.3
60	2211011073	b	2
61	2211011074	b	2
62	2211011080	b	1.2.3
63	2211011081	b	1.2.3
64	2211011104 (1)	b	3



	A	B	C
65	2211011104 (2)	b	2
66	2311011095	b	1.2
67	2211011014 (1)	b	3
68	2211011014 (2)	b	2
69	2011011051	c	1.2.3
70	2111011038	c	1.2.3
71	2111011076	c	1.2.3
72	2111011094	c	3
73	2111012005	c	2
74	2111014405	c	1
75	2111022004	c	1.2
76	2211011006	c	1.2.3
77	2211011012	c	1.2.3
78	2211011026	c	1.2
79	2211011027	c	1.2.3
80	2211011037	c	2
81	2211011041	c	1.2.3
82	2211011045	c	2
83	2211011048	c	2
84	2211011050	c	2
85	2211011069	c	2
86	2211011070	c	1.3
87	2211011080	c	2
88	2211011101	c	1.2.3
89	2111011034	d	1.2.3
90	2111011046	d	3
91	2111011053	d	3
92	2111011059	d	1
93	2111011090	d	1
94	2111011453	d	1
95	2111012211	d	1.2
96	2111013359	d	2



	A	B	C
95	2111012211	d	1.2
96	2111013359	d	2
97	2111013390	d	2
98	2111014453	d	2
99	2111221090	d	3
100	2122011059	d	3
101	2211011048	d	1
102	2211011054	d	1.3
103	2211011062	d	1
104	2211011096	d	3
105	2211011228	d	3
106	2211012211	d	3
107	2211013308	d	2
108	2211044096	d	1
109	2211331054	d	2
110	2211331096	d	2
111	2211411011	d	1
112	2241011094	d	1
113	2111011226 (1)	d	1
114	2111011226 (2)	d	2
115	2111011226 (3)	d	3

6. CONCLUSION

The main objective of the project is to analyze the difference between the real-life and theoretical motion of a pendulum by using experiments, measurements, and theoretical calculations. As a result of this analysis, the relation between variables such as period, the length of the rope, and θmax can be realized. The purpose of theoretical calculations is to create mathematical formulas and understand the conditions to apply them in real life. The purpose of experiments with the pendulum is to observe the behavior of the pendulum and to confirm the theoretical data. Experiments compare data under different conditions to real-life and theoretical predictions and confirm the accuracy of data. Creating mathematical equations for the motion of the pendulum requires several operations such as differentiation, algebra, and matrix inversion. These tasks allow students to improve their knowledge and skills, gain the ability to make mathematical operations and understand the practicability of the motion of the pendulum in real life.

6.1 References and Resources

YouTube link:

https://youtube.com/playlist?list=PLzN6IaX8b1uG9vR7xQ3V4qnziDGp8R9E2&si=y_4g2FQJfyaIXRsj

https://www.cuemath.com/geometry/arclength/#:~:text=The%20arc%20length%20of%20a%20circle%20can%20be%20calculated%20with,where%20%CE%B8%20is%20in%20degree.

http://labman.phys.utk.edu/phys135core/modules/m9/The%20pendulum.html#:~:text=%CF%892%20%3D%20k%2Fm%20%3D,m%20is%20F%3D%20%2Dmgsin%CE%B8

https://matrixcalc.org/

https://planetcalc.com/

https://www.sciencefacts.net/simple-pendulum.html

https://tuhsphysics.ttsd.k12.or.us/Research/IB22/Fig/index.htm

https://unacademy.com/content/nda/study-material/physics/pendulum/