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ÜNİVERSİTESİ

PROJECT 3 REPORT

Computation and Analysis (COMA) Capsule

PROJECT NAME: Angular Oscillations

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PROJECT TEAM

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1. INTRODUCTION

OBJECTIVE

The project's objective is to investigate calculations of the moment of inertia of an object. The main purpose is to measure torsional (angular) oscillations under the influence of the moment of elastic forces that arise in the wire when it is twisted.

BACKGROUND

The main structure of the project was discussed, and tasks were shared by the team members. Then, the necessary equipment (figure 1) is decided and collected. After that, the corresponding information collected by team members was decided and combined. Then, to collect data, experiments on a pendulum were done and recorded by a camera. After collecting data in the experiment, information was analyzed, and finally, analyzed data used in equations to compare results. This report analyzes the difference between real-life and theoretical data.

ANALYZE

1)Description of Experiment:

Figure 1

Equipment:

- A radially symmetric utensil
- String
- Camera
- Meter
- Adhesive tape

The utensil and string were combined into a kind of pendulum. A point was determined when the utensil was motionless. The utensil rotated at different angles. Then the utensil was released to rotate. During the oscillation, time for one period was measured. The experiment of oscillation was recorded by the camera and the experiment was repeated a few times to obtain accurate data. After that, the videos were analyzed.

2. CALCULUS I

2.1. Computation of the moment of inertia

While computing the moment of inertia, some steps were followed:

Firstly, by using the photograph of the handless utensil, a profile was created to derive functions that have the same shape as the object (Figure 2).

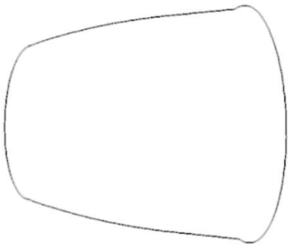

Figure 2

Secondly, the website "Desmos: Graphic Calculator" was used to derive a profile for the shape of the object. The obtained function and its graph from Desmos were used in calculations for the volume and the moment of inertia (Figure 3).

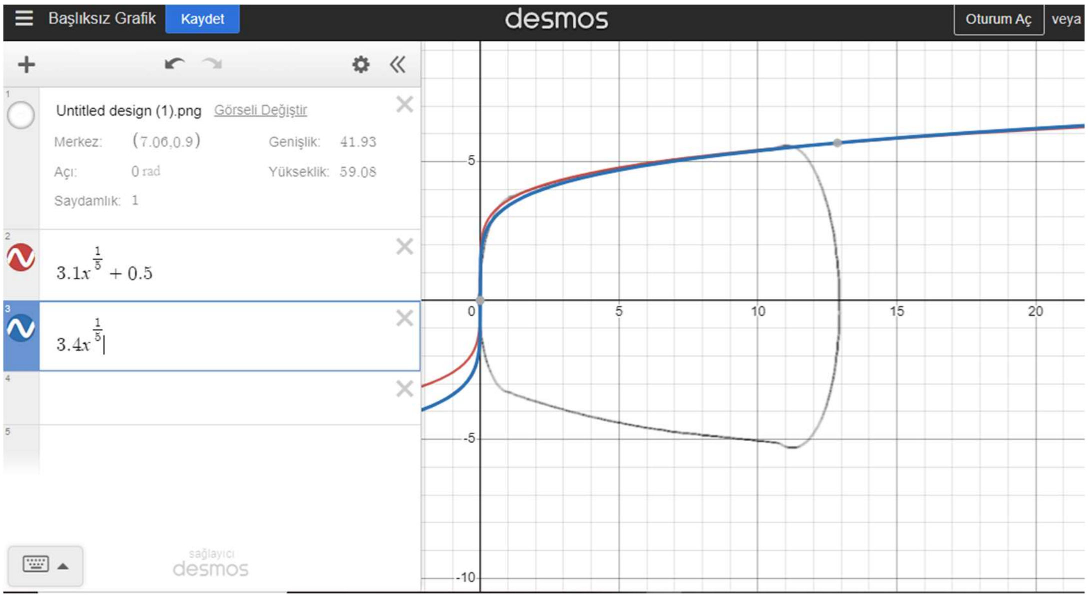

Figure 3

To calculate the volume of the object with theoretical data, derived functions f(x) and g(x) were used. The volume of the object was found by some integration techniques and the interval of the definite integral is 0 and 12.86.

$$\boldsymbol{F(x) = 3.1x^{\frac{1}{5}} + 0.5} \qquad\qquad \mathbf{G}\boldsymbol{(x) = 3.4x^{\frac{1}{5}}}$$

The intersection point of the two functions is 12.86 which is equal to the approximate height of the handless utensil.

The functions f(x) and g(x) that intersect at point 12.86, are rotated around the x-axis. The inner radius of the object becomes g(x), and the outer radius becomes f(x). If the new shape created by rotation around the X-axis was thought of as like thin slices, one of the slices' volumes was calculated and all the volume of the object was found.

The volume for one, thin slice:

$\boldsymbol{\pi(r_{outer}^2 - r_{inner}^2)\, dx}$ (dx is thickness)

Integration to calculate all volume of the object:

$$\int_0^{12.86} \boldsymbol{\pi(\mathbf{router}^2 - \mathbf{rinner}^2)}\,dx$$

$$\int_0^{12.86} \boldsymbol{\pi}\left((\mathbf{3.1x^{\frac{1}{5}} + 0.5}) - (\mathbf{3.4x^{\frac{1}{5}}})^2\right) dx$$

$$\boldsymbol{\pi} \int_0^{12.86} \left(-\frac{\mathbf{195}}{\mathbf{100}} \boldsymbol{x^{\frac{2}{5}}} - \frac{\mathbf{31}}{\mathbf{10}} \boldsymbol{x^{\frac{1}{5}}} + \mathbf{0.25}\right) dx$$

$$= \boldsymbol{\pi\left(-\frac{195}{100}.\frac{5}{7}x^{\frac{7}{5}} - \frac{31}{10}.\frac{5}{6}x^{\frac{6}{5}} + 0.25x\right) \Big|_0^{12.86}}$$

$\approx \mathbf{27.73\ cm^3 \approx 0.0002773\ m^3}$

The area of the basement of the object should be add. To find the basement area:

$\boldsymbol{\pi r^2 h = \pi(0.046)^2(0.0005)}$

$\approx \mathbf{0.0000033238\ m^3}$

The sum of the areas: $\mathbf{0.0002773 + 0.0000033238}$ $\approx \mathbf{0.00003103\ m^3}$

Calculations to calculate the moment of inertia:

A similar method for the calculation of the volume was used while the moment of inertia was calculated. The moment of inertia was found for one slice of the object.

$$\frac{dM}{M} = \frac{dV}{V} \qquad dV = 2\pi y\, dx \qquad dM = \frac{M}{V} dV$$

The radius of the slice equals y. To find y, the outer and inner radii are subtracted from each other. The outer radius is f(x) and the inner radius is g(x).

$$f(x) - g(x) = -0.3x^{\frac{1}{5}} + 0.5$$

$$dV = 2\pi\left(-0.3x^{\frac{1}{5}} + 0.5\right) dx \qquad dM = 1.78 * 2\pi\left(-0.3x^{\frac{1}{5}} + 0.5\right) dx$$

$$dI = \frac{1}{2} dmy^2$$

$$\boldsymbol{dI = 11.18 \int_0^{12.86} \frac{1}{2}\left(-\frac{27}{1000}x^{\frac{3}{5}} + \frac{27}{200}x^{\frac{2}{5}} - \frac{9}{40}x^{\frac{1}{5}} + 0.125\right) dx}$$

$$\boldsymbol{\approx 11.18 * \frac{1}{2} * 9.01}$$

$$\boldsymbol{\approx 50.36}$$

2.2. Method of integration

While the volume of the object (handless utensil) was calculated, a kind of integration technique was used named as “Washer method”.

The washer method is a technique used to find the volume of a solid of revolution obtained by rotating a region between two curves around an axis, generally the x-axis or y-axis. This method involves subtracting the volume of a smaller region from the volume of a larger region to find the volume of the entire solid. According to the definition of the method, the most proper method among other methods (such as disk, shell, cross-sections…) for the object was the washer method.

3. PHYSICS I

3.1. The torsion modulus k

The torsional modulus k (Table 1), also known as torsional stiffness, is a measure of the resistance of a beam or shaft to torsion or torsional deformation.

$$T = 2\pi\sqrt{\frac{I}{k}} \qquad \omega_0 = \sqrt{\frac{k}{I}}$$

Table 1: Torsion modulus k

Angle (rad)	2π	2.25π	2.5π
Period (T)	8	9	10
Torsion modulus k	0.03216838761	0.04070691025	0.05023828435

3.2. Measuring ω, φmax, and conservation of the energy

The experiment was done (figure 4) by measuring some conditions such as time and angle. φmax was measured by using some equations. To measure ω, which is the angular velocity of the pendulum, time is measured. By using obtained time data, the angular velocity (ω) of the pendulum was calculated (Table 2).

Figure 4

Table 2: Angular velocity

Angle (rad)	2 π	2.25 π	2.5 π
ω (rad/s)	0.7853	0.6981	0.6283

To find φmax, some equations were used. φmax will be used while checking the energy conservation (Table 3).

$$\varphi=\varphi\max*\sin(\omega t)$$

Table 3: Value of φmax

Angle (rad)	2 π	2.25 π	2.5 π
φmax(rad)	29.74	25.69	23.59

According to conservation law, the kinetic energy of the rotational motion of the pendulum is converted into potential energy (Table 4).

$$\frac{I\omega^2}{2}=\frac{k\varphi^2\max}{2}$$

Table 4: Conservation of energy

	2π	2.25π	2.5π
Kinetic Energy	30.63	23.97	19.35
Potential Energy	28.32	21.13	17.82

4. LINEAR ALGEBRA

4.1 How is linear algebra applied in real-world problems, and what are the key areas where its applications have proven to be essential in the context of Electrical & Electronics Engineering?

For many scientists, engineers, and mathematicians, linear algebra is an essential instrument. Modeling physical phenomena, solving systems of linear equations, and representing and analyzing linear relationships between variables are all done with it. From computer graphics to financial analysis, from robotics to data science, its applications are wide and diverse. Examples include cryptography and quantum mechanics. For electrical electronics engineers, linear algebra is a fundamental tool that lays the groundwork for comprehending and forecasting the behavior of complex systems. In the study of electrical circuits, linear algebra is employed to resolve sets of linear equations. This aids engineers in grasping the behavior of various circuit components under diverse conditions and their interplay with one another. For example, when analyzing an AC circuit, engineers can use linear algebra to find the current and voltage in each component. Feedback control systems are also designed and optimized using linear algebra. Differential equations are used to model the behavior of the system and these equations are transformed into matrices that can then be studied using linear algebra. For example, engineers can use linear algebra to design the feedback loop in a robotic arm control system that regulates the movements of the arm. For the purpose of applying Fourier transforms and filtering signals to remove noise in audio and video processing, linear algebraic concepts such as linear functions, vectors, and matrices are crucial. By representing signals as vectors and filters as matrices or linear functions, engineers perform various filtering operations using linear algebra operations such as scalar multiplication, vector addition, and matrix multiplication. Digital signal processing algorithms in communication systems are designed and optimized using linear algebra. These include data encoding and decoding techniques like digital signal processing filters and error-correcting codes. For instance, coding schemes for wireless communication systems that can consistently send data over noisy channels can be designed by engineers using linear algebra. Linear algebra is an essential tool for machine learning techniques such as principal component analysis and linear regression. These methods manipulate data matrices to derive insights and predict outcomes. For example, engineers can use linear algebra to identify important facial features for facial recognition systems and create classification functions that use those features to classify images. Linear algebra in optimization is used to solve both linear and non-linear optimization problems. Optimization variables, constraints, and objective functions are represented and manipulated using matrices and linear transformations. Image and video processing uses linear algebra for analysis and operations such as compression, segmentation, and filtering. Pixel values, feature vectors, and transformation coefficients are examples of image and video data that can be represented and modified using matrices and linear transformations. Linear algebra is used in robotics to analyze and design robotic systems, including kinematics, dynamics, and control. Engineers apply linear algebra in this field using matrices and linear transformations.

[1] This question is based on this source.

4.2. What role does linear algebra play in data science and machine learning applications, and how does it contribute to advancements in these fields?

Linear algebra is a field of mathematics that works with matrices and vectors and is used as a fundamental mathematical tool in data science and machine learning. It provides the mathematical basis for many methods, procedures, techniques, and algorithms used in these fields. It has a fundamental place in almost all areas of machine learning, such as image recognition, natural language processing, and artificial neural networks. In data science, linear algebra enables the manipulation of data points and data sets. With linear algebra, you can operate on entire data sets in a single operation, making it significantly more efficient than performing these operations on individual data points. Vectors and matrices are used to represent multidimensional data, so linear algebra helps with data representation. For example, an image on a computer is basically a matrix of pixel values. Example mentioned in an article: Image X consists of small squares, each with four pixels. Each of these squares has an edge in a particular direction. To record this edge direction, we use a 2D array. A 2D array is a type of array that is often referred to as a matrix. However, arrays are more general than matrices and allow more types of operations and manipulations [2]. Machine learning algorithms, which generally involve transforming inputs into a more useful representation, perform these transformations using linear algebra. On the other hand, linear algebra is used to compute machine learning algorithms. This is especially true for unsupervised learning techniques such as Principal Component Analysis (PCA) and supervised learning algorithms such as Support Vector Machines and Linear Regression. These algorithms utilize linear algebra to compute distances, directions, and relationships in high-dimensional spaces, and linear algebra is also used in optimization techniques used in machine learning. One source states that "When solving an optimization problem, linear algebra requires the use of linear algebra as a tool because linear algebra involves an understanding of vector and matrix operations and allows for effective and efficient calculations on large data sets. Therefore, linear algebra must be understood and applied to optimize the performance of the machine learning model and improve the overall accuracy of the model."[3]. A popular optimization method, gradient descent, involves computing the derivative of a function, which requires knowledge of linear algebra. In addition, linear algebra has an important place in the training process of neural networks, which are the cornerstone of deep learning. Vectors or matrices are used to represent the inputs, outputs, weights, and bases of the network. Operations such as addition, multiplication, and transposition performed during forward and backward propagation steps are all matrix operations. Consequently, linear algebra is essential to the advancement of data science and machine learning. It provides effective representation, optimization, model evaluation, transformation, and dimension reduction of data. Without linear algebra, large datasets could not be efficiently processed. Therefore, a good understanding of linear algebra is essential for the machine learning practitioner or data scientist. Advances in these fields are in many ways due to the power and efficiency of linear algebra.

4.3. In what ways is linear algebra used in computer graphics and image processing, and how does it enhance the quality of visual representations in various applications?

"Digital image processing" (DIP) refers to techniques that use computers and algorithms to manipulate digital images. Digital images need to be enhanced, interpreted, and analyzed by applying multiple techniques. DIP is connected with linear algebra. C.M.R. Caridade claims that the DIP aids in the development of students' visual and intuitive comprehension of the concepts, which are typically abstract and completely new to them. Using Digital Image Processing (DIP), LA-related concepts such as inverse transformations, matrix operations, linear equations, algebraic matrices, and matrix transformations can be studied. In this way, applications of these mathematical ideas in image segmentation, compression, enhancement, restoration, and representation can be investigated. By using an interdisciplinary approach, the knowledge gap between DIP and linear algebra can be bridged and a deeper understanding of the mathematical concepts and their real-world applications in image processing can be gained. C.M.R. Caridade said that enhancing instructional strategies and enhancing student learning can be achieved by applying matrix operations to remove the image background or isolate specific areas of the image. C.M.R. Caridade adds that this simultaneous teaching approach enables the integration of both Linear Algebra (LA) and Digital Image Processing (DIP), increasing students' motivation and skills in these areas. In the article written by C.M.R. Caridade, DIP was used to teach LA to engineering students, and their interest in students' achievement was evaluated. A digital image is an image consisting of picture elements, also known as pixels, which is an output from two-dimensional functions that can be represented as a matrix where each coordinate (x,y) represents a pixel. According to this article, Grayscale images use integer values ranging from 0 (black) to 255 (white) to represent pixel intensity, binary images use pixel values of 0 (black) or 1 (white) and color images consist of three matrices representing the red, green and blue color spaces and the values of each pixel are vectors of three integers ranging from 0 to 255. C.M.R. Caridade indicates that facilitating matrix operations with digital images allows students to perform basic operations of their choosing, requiring them to verify constraints such as careful choice of matrix dimensions in operations such as matrix multiplication. Linear algebra plays an important role in improving visual representations. Linear algebra operations such as convolution are often used in image processing. This operation allows for features such as blurring and sharpening, thus improving the quality of visual representations. Operations such as translation, rotation, and scaling create realistic 3D visualizations and animations. In addition, linear algebra is used to represent colors in different color spaces, helping to improve the color quality of the image. techniques such as singular value decomposition reduce file sizes without sacrificing too much visual quality. Matrix operations group pixels according to similar characteristics, thus improving segmentation quality. In conclusion, linear algebra plays a major role in the advancement of visual representations in a variety of fields, including computer graphics, machine learning, and image processing.

[4] This question is based on this source.

4.4. In what ways is linear algebra utilized in engineering disciplines, and what specific applications does it have in Electrical & Electronics Engineering?

Linear algebra is widely used in electrical electronics and various engineering disciplines. Linear algebra is used in a variety of applications in general engineering disciplines: in civil engineering for the analysis of structures, in mechanical engineering for fluid dynamics, and aerospace engineering for control systems. The areas where linear algebra is used in electrical and electronic engineering are as follows: Circuit Analysis, Control Systems, Signal Processing, Communication Systems, Electromagnetics, Image and Video Processing. Linear algebra in circuit analysis is used in the calculation of linear equation systems consisting of equations of complex electrical circuits. In one article, systems of linear equations consisting of the equations of electrical circuits are considered, and instant solutions are obtained, computed, and tested using MATLAB. MATLAB stands for Matrix Laboratory, it is a programming language that includes a library of mathematical functions that allow you to perform linear algebra and matrix calculations [5]. Linear algebra is used in the mathematical modeling and analysis of the dynamic behavior of control systems. The state space representation is used in linear transformations and subspace methods. In a related study, "There is no need for explicit model parametrization, which is a highly complex issue for multi-output linear systems. A second numerical advantage is the elegance and computational efficiency of subspace algorithms. The size and numerical representation of the previously mentioned subspaces are computed using QR and singular value decomposition. "[6]. Linear algebra is used to represent signals and signal processing, which means matrices and vectors are used. A research paper on the relationship between signal processing and linear algebra mentions Töplitz and Henkel matrices. The paper states that "In radar or sonar (or more generally in antenna processing), Töplitz matrices arise when far-field sources propagate in a homogeneous medium and then hit a series of regularly spaced sensors."[7] It is also converted into a mathematical formula for analyzing signals and systems using the Fourier transform. The Fourier transform is a mathematical formula that converts a signal sampled in time or space into the same signal sampled at a temporal or spatial frequency. Matrices are also used to analyze signals and systems. In communication systems, linear algebra is used in the analysis and design of systems using matrices and linear transformations and is also used in our understanding of the transmission and reception of signals and the basis for coding schemes [8]. The use of linear algebra in the field of electromagnetism is used in the calculation of algebraic equations arising from the decomposition of Maxwell's equations and their calculation and is used to solve systems of electromagnetic field equations. In one paper the method of moments, in which the functional equations of field theory are reduced to matrix equations, is used to solve field problems [9]. The use of linear algebra in image and video processing is as follows. It is used in techniques such as image transformation and filtering, which involve operations on matrices and vectors. In a related study," Students build new images (matrices) by using elementary operations between matrices. Test their properties and constraints. Thus, it is possible with the images, for example, to verify the associative property between 3 matrices and the fact that the sum is only possible between matrices of the same dimensions."[10]. Matrix operations are used to remove image background or identify parts of an image, and image processing techniques also include linear algebra. Consequently, linear algebra is used for computation in various fields of engineering, especially in electrical and electronics engineering, and is an important mathematical tool for engineering.

5. INTRODUCTION TO PROGRAMMING

```
#include<iostream>
#include<string>
#include<cmath>
using namespace std;
//Pendulum class definition
class pendulum {
private:
	double gravity, length, answer;
	const double pi = 3.14;
public:
	//Constructor to initialize gravity and length
	pendulum(double L, double G) {
		gravity = G;
		length = L;
	}
	//Function to calculate and output pendulum information
	void output() {
		answer = 2 * pi * sqrt(length / gravity);
		cout << "Pendulum information: " << endl;
		cout << "Length: " << length << endl;
		cout << "Gravity: " << gravity << endl;
		cout << "Display period: " << answer << endl;

	}
};
//Main function
int main() {
	//Creating an instance of the pendulum class
	pendulum P1(1.5, 9.8);
	//Calling the output function to display pendulum information
	P1.output();
	return 0;
}
```

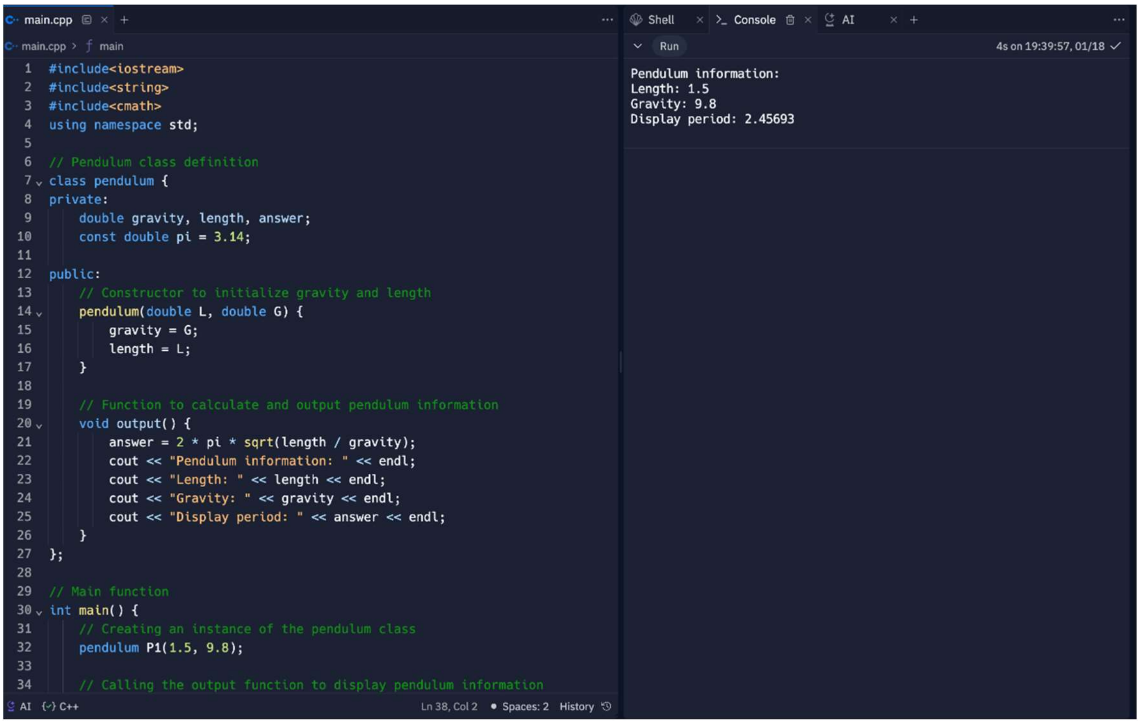

Figure 5.output

6. CONCLUSION

The main objective of the project is to analyze the difference between the real-life and theoretical motion of a pendulum by using experiments, measurements, and theoretical calculations. As a result of this analysis, the relation between variables such as volume, the moment of inertia, and φmax can be realized. The purpose of theoretical calculations is to create mathematical formulas and understand the conditions to apply them in real life. The purpose of experiments with the pendulum is to observe the behavior of the pendulum to compare data from experiment and real life. Experiments compare data under different conditions to real-life and theoretical predictions and confirm the accuracy of data. Creating mathematical equations for the motion of the pendulum requires several operations such as differentiation, algebra, and matrix inversion. These tasks allow students to improve their knowledge and skills, gain the ability to make mathematical operations and understand the practicability of the calculation of volume and the moment of inertia.

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