ABDULLAH GÜL
ÜNİVERSİTESİ

ESD CAPSULE PROJECT 1 REPORT

PROJECT NAME: CAPACITOR

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PROJECT TEAM

Sıla DOĞAN
Hatice Kübra DURU
Ayşe Ece AKKURT

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INTRODUCTION

OBJECTIVE

The project's objective is to understand how a capacitor works and to investigate calculations of the two types of capacitors. The main purpose is to measure the capacitance with different laboratory equipment such as an oscilloscope and a function generator.

BACKGROUND

The main structure of the project was discussed, and tasks were shared by the team members. Then, the necessary equipment was decided and collected. After that, the corresponding information was collected by team members and combined. Then, to collect data, experiments on the two types of capacitors as parallel and cylindrical were done and data were collected. After collecting data in the experiment, information was analyzed, and finally, analyzed data used in equations to compare results. This report analyzes the difference between real-life and theoretical data.

ANALYZE

Equipment:

For parallel plate capacitor:

- A cardboard
- Sponge
- Aluminum foil

For parallel plate capacitor:

- A jar
- Aluminum foil
- A screw

* Resistor, crocodiles, jumpers, etc. used in both capacitors

1)Description of Experiment:

For the parallel capacitor:

Two plates were created by using cardboard. One side of each cardboard is covered by aluminum foil and combined with a sponge with the size of 10x10 cm. The jumpers were connected to cardboard plates. The parallel capacitor that was created is connected in series with a 10kΩ resistor to the oscilloscope and the function generator. To change the capacitance, the pressure was applied to the plates.

For the cylindrical capacitor:

A jar was filled with water and the jar lid was combined with a screw. Different width strips of 1, 2, 3, and 4 cm were created and pasted onto the surface of the jar respectively to change the capacitance of the capacitor. The crocodile cables were connected to the capacitor in series with a 10kΩ resistor the oscilloscope and the function generator.

1. **Designing Two Types of Capacitors**

Various equipment was used to design different types of capacitors as parallel (Image 1) and cylindrical (Image 2), such as a jar and a sponge. After designing the capacitors, a multimeter was used to test the variability of the capacitance. To calculate the capacitance an oscilloscope and a function generator were used. After collecting data from the equipment, some physical formulas such as the discharging equation were used to calculate the capacitance of the capacitor.

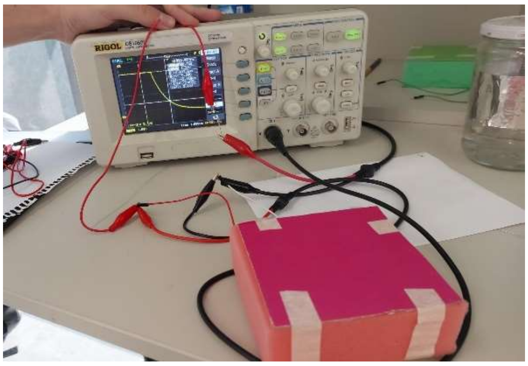


Image 1

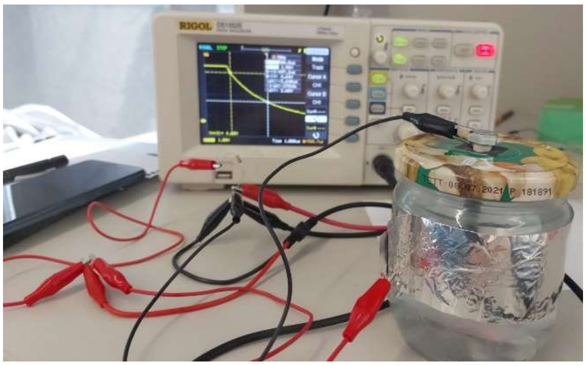


Image 2

2. **Calculating the Capacitance**

By using the discharging equation or discharging curve (eq. 1), the capacitance of the capacitor was calculated.

$$\mathrm{Vc(t)} = \mathrm{Vo}\left(\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{RC}}}\right) \text{ (eq. 1)}$$

In this equation, if the time was investigated when it equals RC, the equation becomes:

$$\mathrm{Vc(t)} = \mathrm{Vo} * \mathrm{e}^{-1}$$

$$\mathrm{Vc(t)} = \frac{\mathrm{Vo}}{e}$$

By using the data from the oscilloscope (Image 3), Vo and Vc(t) can be found. Since time was expressed as:

$$t = RC \text{ (eq. 2)}$$

The capacitance of the capacitor can be expressed as:

$$C = \frac{t}{R} \text{ (eq. 3)}$$

Data from the oscilloscope screen (Image 4), the corresponding data can be obtained.

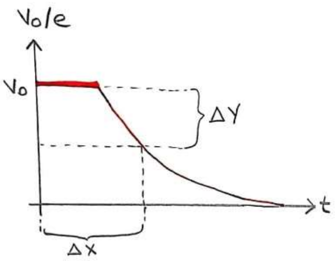


Image 3

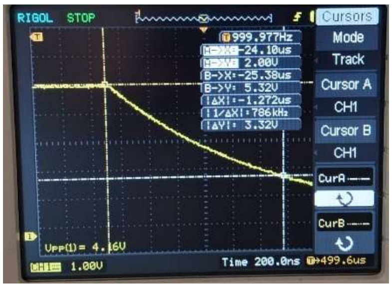


Image 4

2.1. Calculations for parallel plate capacitor

To calculate the capacitance for the parallel plate capacitor experimentally, the discharging equation (eq. 1) was used to obtain the correct calculations. The experiment was done four times with a different property which is the distance between cardboard plates. The corresponding data was collected from the oscilloscope for each of the four different distances. While calculating the capacitance, the capacitance of the oscilloscope and cables was subtracted from the total capacitance that was calculated.

$$C = C_{total} - C_{oscilloscope}$$

$$Vc(t) = Vo\left(e^{-\frac{t}{RC}}\right) \text{ (eq. 1)}$$

$$C = \frac{t}{R} \text{ (eq. 3)}$$

$$C = \frac{Ttotal - Toscilloscope}{R}$$

Time (t) equals 1.272 μs.

10kΩ resistor was used in the circuit.

For d = 4 cm: (d represents the distance between the plates)

$$C = \frac{1.352 - 1.272}{10^4} = 8\,pF$$

For d = 3 cm:

$$C = \frac{1.4 - 1.272}{10^4} = 12.8\,pF$$

For d = 2 cm:

$$C = \frac{1.472 - 1.272}{10^4} = 20\,pF$$

For d = 1 cm:

$$C = \frac{1.488 - 1.272}{10^4} = 21.6\,pF$$

To calculate the capacitance for the parallel plate capacitor theoretically, the equation which is related to the dielectric constant of the material and the size of the capacitor.

$$C = \varepsilon_0 * \varepsilon_r * \frac{A}{d} \text{ (eq. 4)}$$

C: the capacitance of the capacitor

ε_0: an electric field capability to permeate a vacuum (≈$8.854*10^{-12}$)

ε_r: the dielectric constant of the material between the plates

A: the surface area of the plates

d: the distance between the plates

The surface area of the plates is 0.01 m^2 (the size of the cardboard is 10x10 cm)

To find the dielectric constant ε_r, a multimeter was used to measure the capacitance when the distance between the plates equals 4 cm, and the result was found as 0.01 nF.

0.01 nF= 10^{-11} F

$$10^{-11} = 8.854*10^{-12} * \varepsilon_r * \frac{0.01}{0.04}$$

ε_r= 5.517

For d= 4 cm:

C= $8.854*10^{-12}*5.517*\frac{0.01}{0.04}$

C≈ 9.99 pF

For d= 3 cm:

C= $8.854*10^{-12}*5.517*\frac{0.01}{0.03}$

C≈ 13.197 pF

For d= 2 cm:

C= $8.854*10^{-12}*5.517*\frac{0.01}{0.02}$

C≈ 19.996 pF

For d= 1 cm:

C= $8.854*10^{-12}*5.517*\frac{0.01}{0.01}$

C≈ 39.993 pF

*The results are approximate values and may change due to ε_r of the sponge is changing when the pressure applied to the capacitor.

2.2. Calculations for cylindrical capacitor

The discharge equation (equation 1) was used to experimentally calculate the capacitance of the cylinder capacitor. The experiment was performed four times by varying the length of the aluminum foil forming the outer cylinder of the cylinder capacitor. Relevant data were collected from the oscilloscope for each of the four different distances. When calculating the capacitance, the capacitance of the oscilloscope and the cables were subtracted from the total capacitance calculated.

$$C = \frac{Ttotal - Toscilloscope}{R}$$

$Toscilloscope$ = 1.272 µs.

10kΩ resistor was used in the circuit.

For L = 5 cm: (L represents the length of the aluminum foil)

$$C = \frac{5.600 - 1.272}{10^4} = 432.8\, pF$$

For L = 4 cm:

$$C = \frac{4.120 - 1.272}{10^4} = 284.8\, pF$$

For L = 3 cm:

$$C = \frac{3.640 - 1.272}{10^4} = 236.8\, pF$$

For L= 2 cm:

$$C = \frac{3.160 - 1.272}{10^4} = 188.8\, pF$$

The following equation is used to theoretically calculate the capacitance of the cylinder capacitor.

$$C = \varepsilon_0 * \varepsilon_r * \frac{2\pi . L}{ln(\frac{r_o}{r_i})}$$

ε_r calculated in this way:

Capacitance was measured as 445.6 pF with a multimeter.

$$\mathbf{445.6 * 10^{-12}} = \frac{\mathbf{6.28} * \varepsilon_r \; * 8.85 * 10^{-12} * \; 0.05}{\mathbf{2.650}}$$

$$\boldsymbol{\varepsilon_r} \approx 425$$

C: the capacitance of the capacitor

ε_0: an electric field capability to permeate a vacuum ($\approx 8.854*10^{-12}$)

ε_r: The permittivity of the dielectric material

L: The length of the aluminum foil

r_o: The radius of the outer cylinder ($\approx$ 0.0425 m)

r_i: The radius of the inner cylinder ($\approx$0.003 m)

π: The pi number ($\approx$3.14)

$$ln(\frac{r_o}{r_i}) \approx 2.650$$

For L= 5 cm:

$$C = 8.854*10^{-12}*425*\frac{6.28*0.05}{2.650} = 445.8\, pF$$

For L= 4 cm:

$$C = 8.854*10^{-12}*425*\frac{6.28*0.04}{2.650} = 356.6\, pF$$

For L= 3 cm:

$$C = 8.854*10^{-12}*425*\frac{6.28*0.03}{2.650} = 267.5\, pF$$

For L= 2 cm:

$$C = 8.854*10^{-12}*425*\frac{6.28*0.02}{2.650} = 178.3\, pF$$

* The results are approximate and may vary due to the uncertainty of ε_r values.

2.3. LTspice Simulation

To control the discharge time data of the two types of capacitors from the oscilloscope, the LTspice simulation program (Images 5 and 6) was used. The data from the simulation and the oscilloscope were compared for both capacitors.

For parallel plate capacitor (Table 1):

Table 1: data from simulation

D (the distance between plates) (cm)	1	2	3	4
Time data from the simulation (s)	$0.25*10^{-6}$	$0.23*10^{-6}$	$0.15*10^{-6}$	$0.09*10^{-6}$

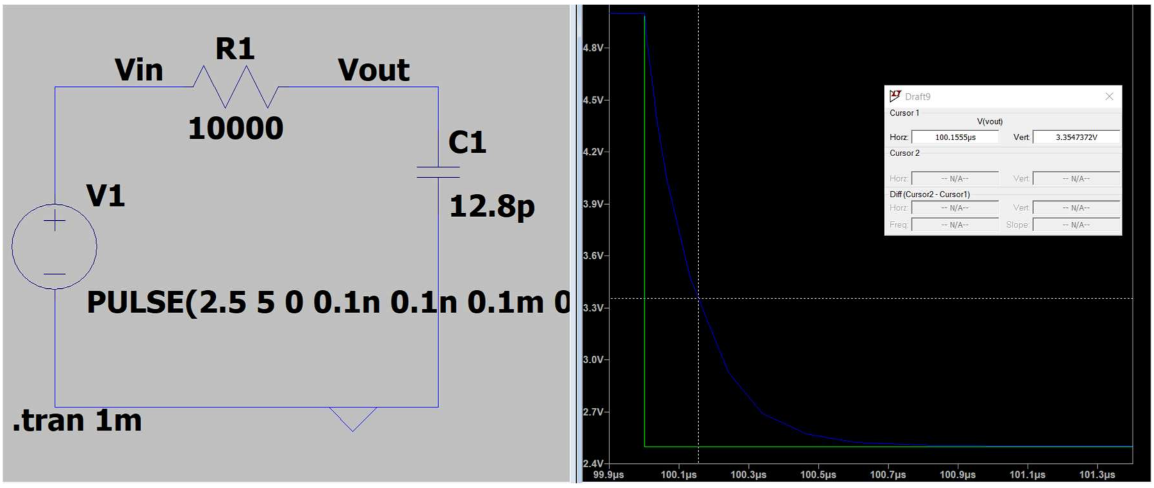


Image 5

For cylindrical capacitor (Table 2):

Table 2: data from simulation

L (the length of the aluminum foil) (cm)	2	3	4	5
Time data from the simulation (s)	$2.11*10^{-6}$	$2.36*10^{-6}$	$2.84*10^{-6}$	$4.32*10^{-6}$

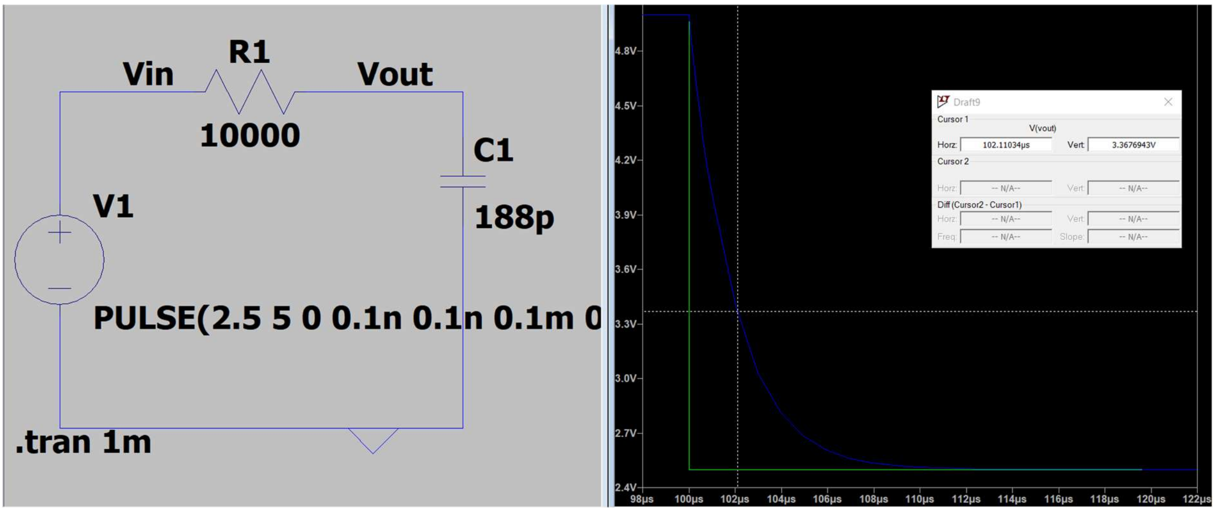


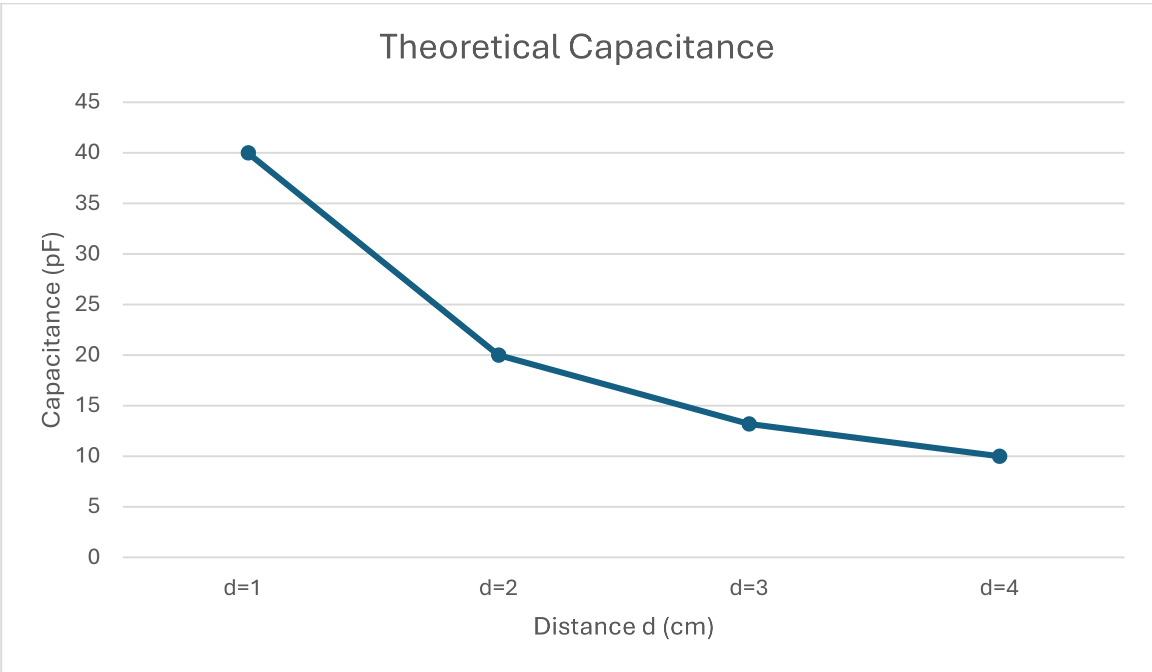
Image 6

As a result of the simulation, the experimental time data and the simulation data are very close to each other with small differences. The reason behind the difference is the conditions of the experiment. Due to experimenting under non-optimum conditions, deflections were observed.

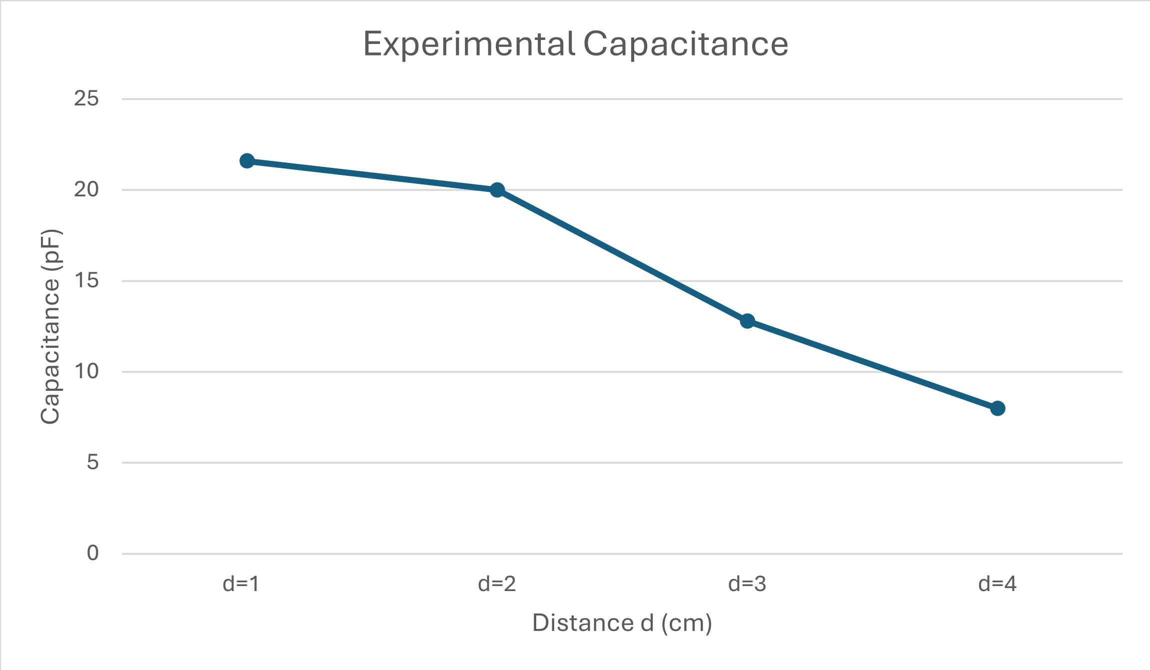
2.4. Graphs for two types of capacitors

- For parallel plate capacitor,

Graph 1: theoretical capacitance vs. the distance between plates



Graph 2: measured capacitance vs. the distance between the plates



To calculate the charge, the basic capacitor charge formula (eq. 5) was used.

$$Q=CV \text{ (eq. 5)}$$

Q represents charge,

C is the capacitance,

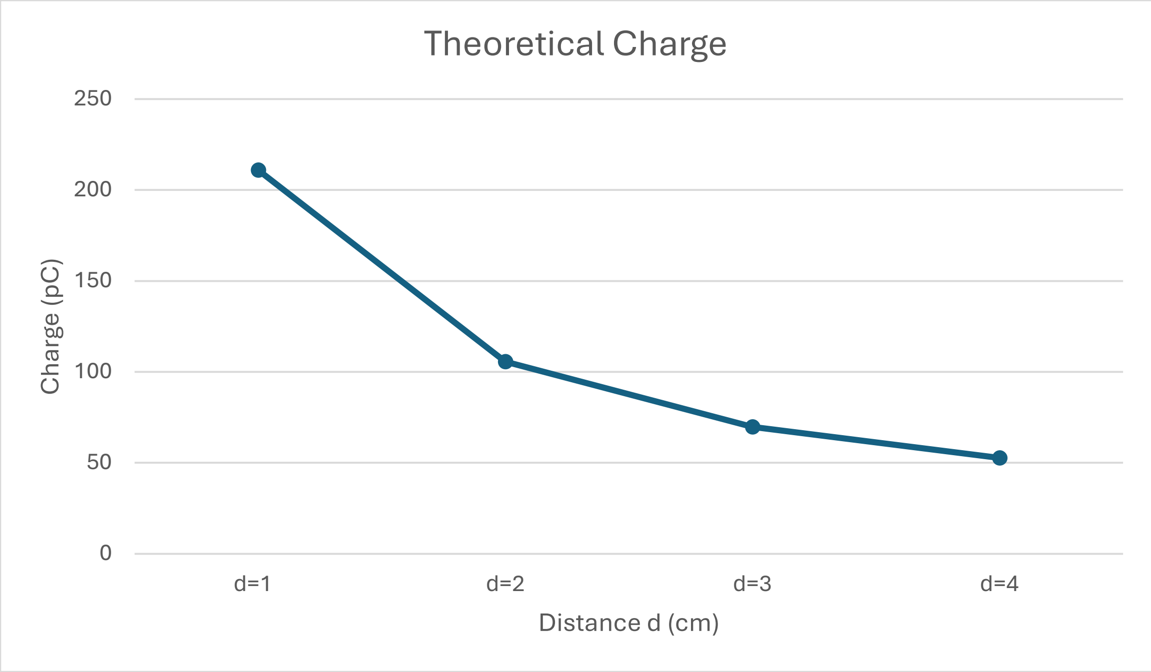
V is the voltage.

The voltage is fixed at 5.28V.

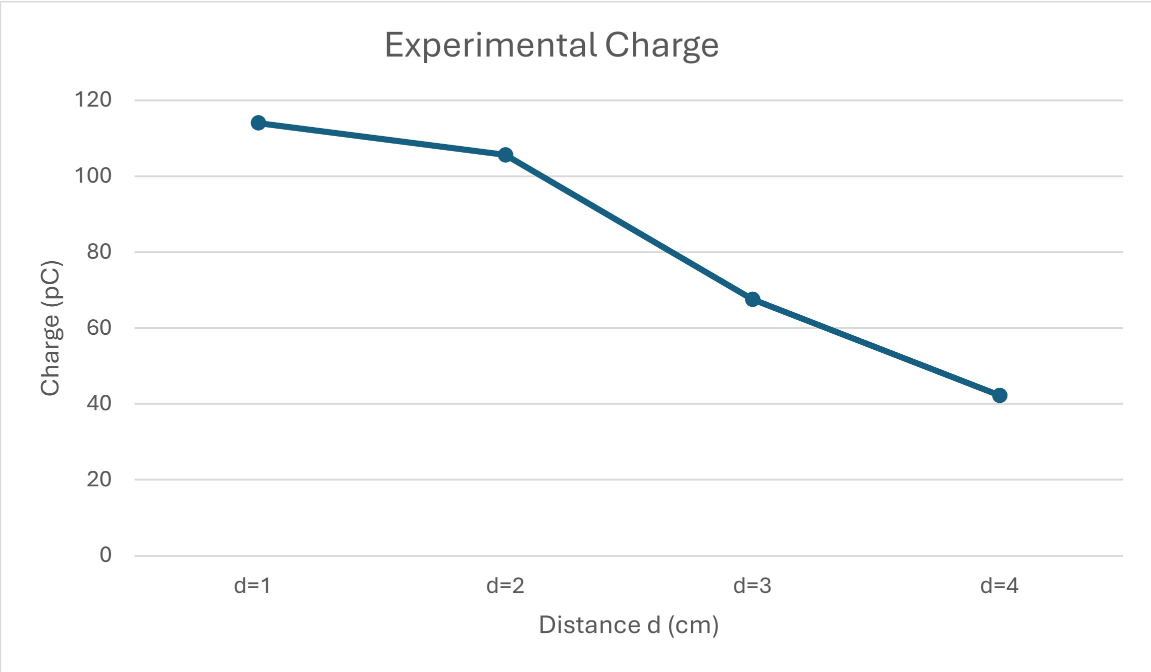
Table 3: charge values in different d (the distance between plates) values

	d=1	d=2	d=3	d=4
Theoretical Charge Values (pC)	211	105.57	69.68	52.7472
Experimental Charge Values (pC)	114.048	105.6	67.58	42.24

Graph 3: charge (theoretically calculated) vs. the distance between plates

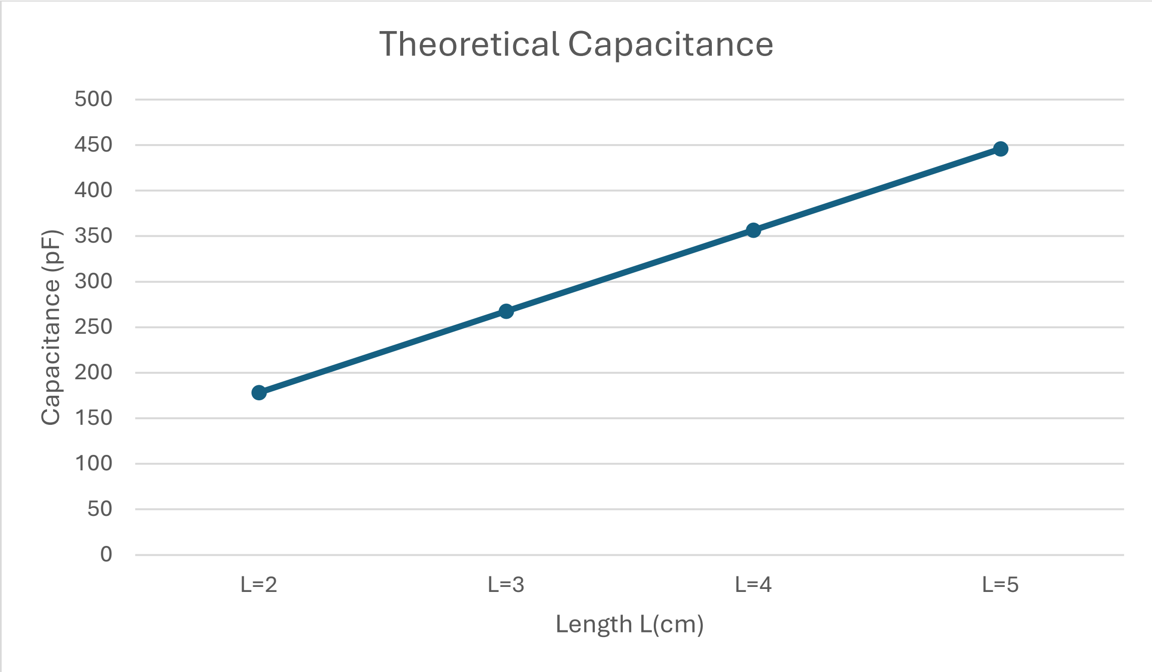


Graph 4: charge (calculated based on measured capacitance) vs. the distance between plates

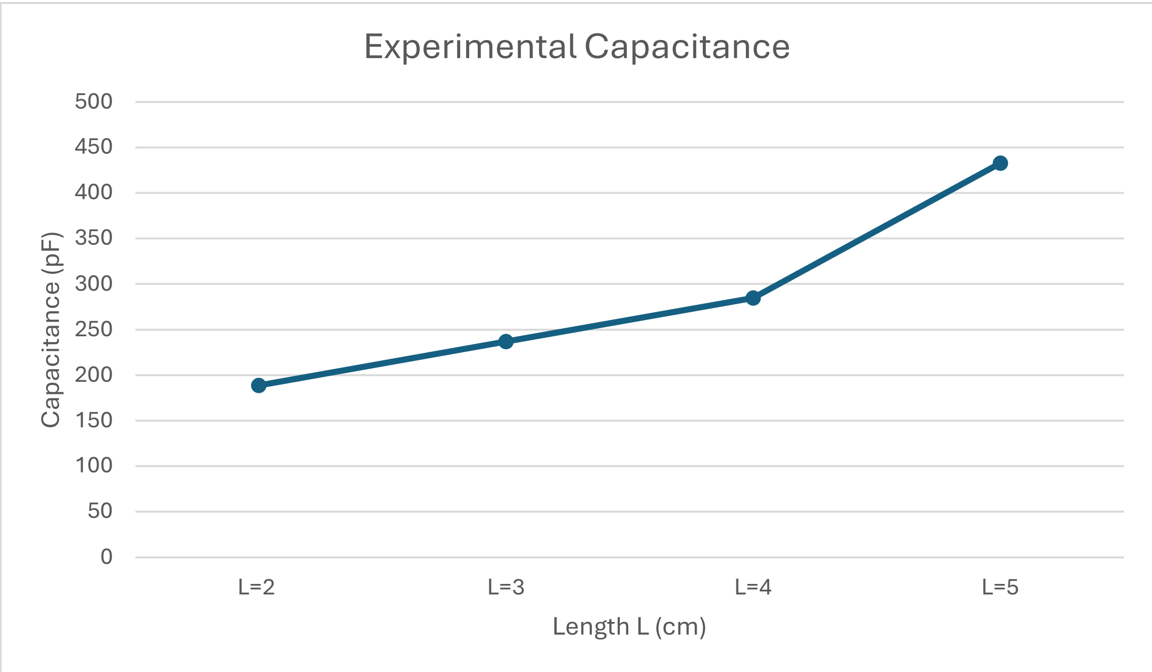


- For cylindrical capacitor,

Graph 5: theoretical capacitance vs. the length of the aluminum foil



Graph 6: measured capacitance vs. the length of the aluminum foil



To calculate the charge, the basic capacitor charge formula (eq. 5) was used.

$$Q=CV \text{ (eq. 5)}$$

Q represents charge,

C is the capacitance,

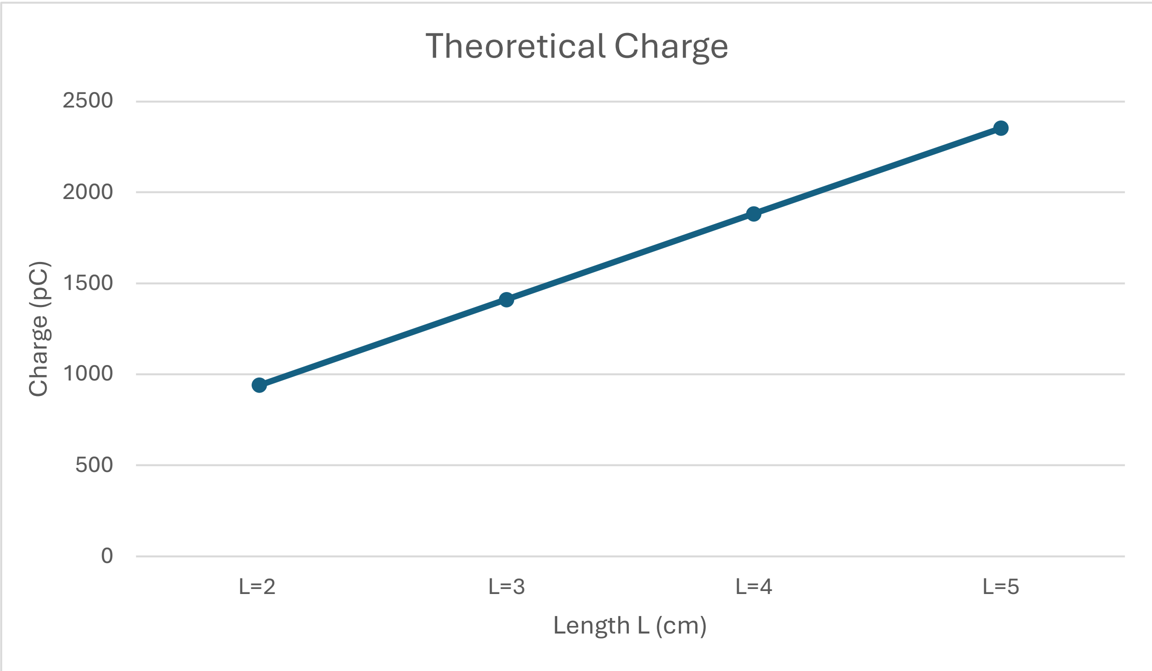
V is the voltage.

The voltage is fixed at 5.28V.

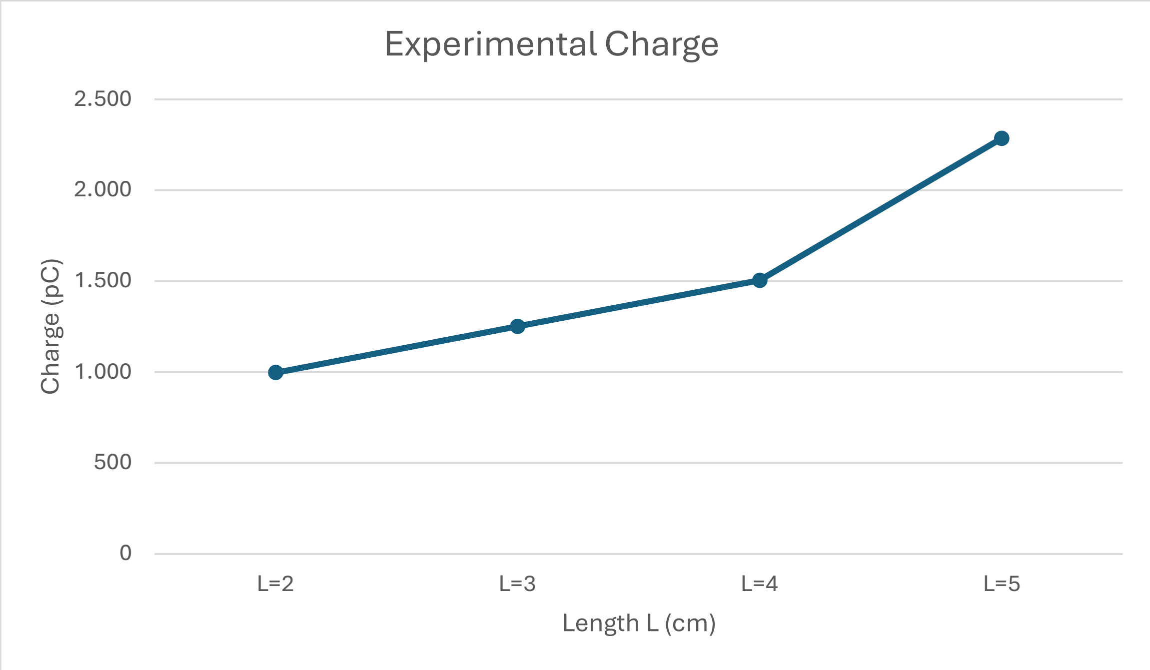
Table 4: charge values in different L (the length of the aluminum foil) values

	L=2	L=3	L=4	L=5
Theoretical Charge Values (pC)	941.424	1412,4	1882,848	2353,824
Experimental Charge Values (pC)	996,864	1250,3	1503,744	2285,184

Graph 7: charge (theoretically calculated) vs. the length of the aluminum foil



Graph 8: charge (calculated based on measured capacitance) vs. the length of the aluminum foil



As a result of the experiment, it was observed that the theoretical data and experimental data gave very similar results. There were small differences between the theoretical data and experimental data due to some reasons such as ambient conditions, not knowing the exact Er, resistance of the cables, tolerance of the resistance.

Table 5: theoretical and experimental comparison

	d=1	d=2	d=3	d=4
Theoretical result	≈ 39.993 pF	≈ 19.996 pF	≈ 13.197 pF	≈ 9.99 pF
Experimental result	≈ 21.6 pF	≈ 20pF	≈ 12.8 pF	≈ 8 pF

Table 6: theoretical and experimental comparison

	L=2	L=3	L=4	L=5
Theoretical result	≈ 188.8 pF	≈ 236.8 pF	≈ 284.8 pF	≈ 432.8 pF
Experimental result	≈ 178.3 pF	≈ 267.5 pF	≈ 356.6 pF	≈ 445.8 pF

3. Triangular Waves and FFT Analysis

A circuit was constructed as a parallel plate capacitor and a resistor connected in series to the circuit. Then the circuit was fed with a voltage in the form of a periodic triangular wave of a chosen frequency and amplitude (Image 8).

3.1. The triangular wave as sums of sinusoidal (Fourier Series)

A triangular wave can be expressed as a sum of sinusoidal waves (Image 7) by using Fourier series. The formula for a triangular wave can be written as (eq. 6):

$$V(t) = \frac{8A}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\sin(2\pi nft)\ (eq. 6)$$

V(t) is the voltage at time t,

A is the amplitude,

f is the frequency,

n is the harmonic number.

The amplitude was chosen as 5 Vpp, and the frequency was chosen as 5 kHz.

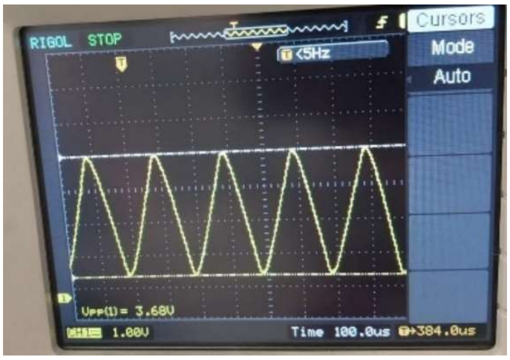


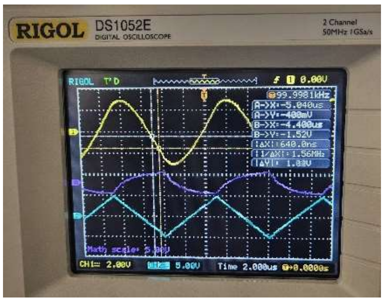
Image 7

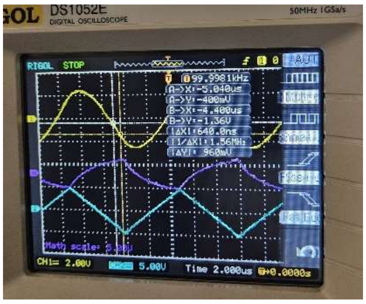
Image 8

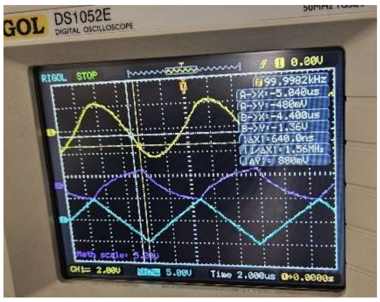
3.2. Waveform experiment

The voltage (Vc) across the capacitor decreases over time due to its charging and discharging cycle, resulting in a breakdown of DC voltage. A higher capacitance (C) increases the circuit's time constant (τ = RC), causing Vc to rise and fall more slowly during these phases.

The voltage across the resistor (Vr) mirrors the input triangular wave (Vin) (Images 9,10,11), though with a slightly lower amplitude because of voltage division across the resistor and capacitor. This amplitude reduction is less pronounced at higher frequencies due to the capacitor's lower reactance.







Images 9,10,11

3.3. Fast Fourier transform (FFT Analysis)

The Fast Fourier Transform (FFT) is a mathematical algorithm used to analyze the frequency content of a signal, such as the voltage across a capacitor in a circuit. When the voltage is applied across a capacitor, the FFT can help break down the signal into its sinusoidal components at different frequencies.

The Fast Fourier Transformation of Vc shows an evident peak at DC (0 Hz) that represents the average voltage across the capacitor. The smaller peaks in the FFT represent the harmonics of the input triangular wave's fundamental frequency (f), though their amplitudes are particularly lower due to the capacitor's filtering effect. Regarding the resistor voltage (Vr), its FFT will be similar to that of the input triangular wave but might show decreased amplitudes for higher harmonics due to RC filtering. Increasing capacitance (C) accentuates this reduction in higher harmonic amplitudes.

When the capacitance was increased, it indicated the increasing ability of the capacitor to block higher frequency components of the triangular wave, so as C increases, the blocking ability increases.

The FFT graphs for parallel plate capacitor in different distances between the plates.

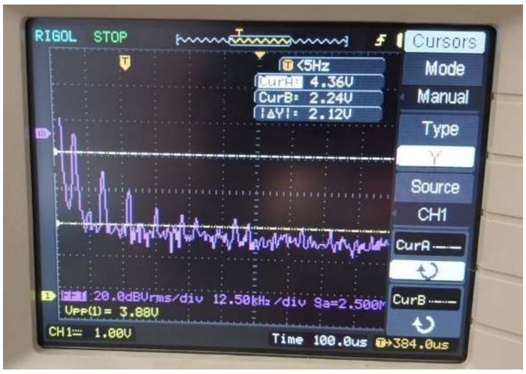


Image 12 (d=1)

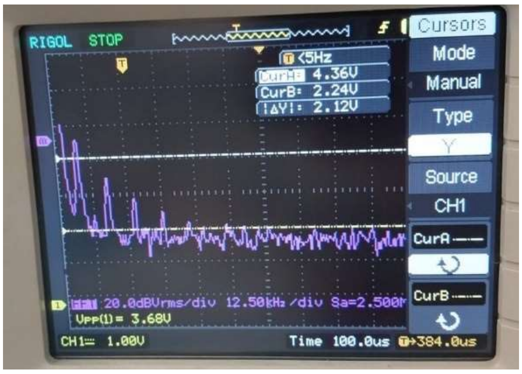


Image 13 (d=4)

3.4. Explanation by Using Circuit Theory Principles

In circuit theory, increasing capacitance in a series circuit causes a higher reactance ($Xc = 1 / (2\pi fC)$), where f is the frequency of the input waveform. Higher reactance means the capacitor more effectively blocks higher-frequency components, acting as a low-pass filter. As a result, the FFT analysis shows reduced amplitudes for higher harmonics because these frequencies are reduced by the capacitor. This behavior is arranged with the concept of RC filtering, where the resistor (R) and capacitor (C) combination influences the frequency response of the circuit, affecting the FFT spectrum of the output voltage.

4. The Kirchhoff Loop Rule

This question describes the analysis of a capacitor charging circuit and the related equations. The circuit consists of a capacitor, a resistor, and a constant voltage source, and the capacitor is not initially charged. By applying Kirchhoff's voltage law and the loop rule, the following equation is solved. The expression for capacitor charging as a function of time and circuit parameters is derived. The graph of q(t) concerning the variables is plotted. R-value changes were examined and interpreted by creating a graph.

The terms used in the solution of the equations you have seen below are explained:
$\boldsymbol{V}$ = The constant voltage applied to the circuit
$\boldsymbol{V_C}$ = The voltage across the capacitor
$\boldsymbol{V_R}$ = The voltage across the resistor
I = Total current value passing through the circuit

$\boldsymbol{q(t)}$ = The charge on the capacitor at time t
$\boldsymbol{t}$ = Time
$\boldsymbol{C}$ = Value of the capacitor
$\boldsymbol{R}$ = Value of resistance

$$V - V_R - V_C = 0$$

$$V = V_R(t) + V_C(t)$$

Ohm's Law was used for calculations. This equality is which represents the current through the resistor at time t.

$V_R(t) = I.R$ $\qquad$ $I(t) = \frac{dq(t)}{dt}$

$$V_C(t) = \frac{q(t)}{C}$$

$$V = I.R + \frac{q(t)}{C}$$

The equations are combined.

$$I = \frac{dq(t)}{dt} = \frac{V - \frac{q(t)}{C}}{R}$$

Differential equations are solved by integration.

$$\int \frac{dq(t)}{V - \frac{q(t)}{C}} = \int \frac{dt}{R}$$

$$\int \frac{dq(t)}{V - \frac{q(t)}{C}}$$

Integration with the substitute has been done.

$$u = V - \frac{q(t)}{C}$$

$$du = \frac{d}{dq}\left[V - \frac{q(t)}{C}\right]$$

$$du = \frac{d}{dq}[V] - \frac{1}{C}.\frac{d}{dq}[q(t)]$$

$$du = 0 - \frac{1}{C}$$

$$du = -\frac{1}{C}$$

$$du = -\frac{1}{C}dq$$

$$= -C \int \frac{1}{u} \, du$$

$$\int \frac{1}{u} \, du = ln \, (u)$$

$$= -C \, ln(u)$$

Undo substitution has been made.

$$u = V \; - \; \frac{q(t)}{C}$$

$$= -C \, ln\left(\; \left| V \; - \; \frac{q(t)}{C} \right| \; \right) + C_2$$

$$\int \frac{dt}{R} = \frac{t}{R} + \; C_2$$

$$-C \, ln\left(\; \left| V \; - \; \frac{q(t)}{C} \right| \; \right) + C_1 \; = \; \frac{t}{R} + C_2$$

C_1 and C_2 are integral constant and has been ignored in the calculations.

$$ln\left(\; \left| V \; - \; \frac{q(t)}{C} \right| \; \right) \; = \; -\frac{t}{R.C}$$

$$V \; - \; \frac{q(t)}{C} \; = \; e^{-\frac{t}{R.C}}$$

$$V \; - \; e^{-\frac{t}{R.C}} \; = \; \frac{q(t)}{C}$$

$$C.\left(V - e^{-\frac{t}{R.C}}\right) = q(t)$$

$$C.V = q(t) + C.e^{-\frac{t}{R.C}}$$

$$V \; - \frac{q(t)}{C} = C.e^{-\frac{t}{R.C}}$$

As a result of the operations performed, the equation q(t) is as follows.

$$q(t) = C.V \; (1 - e^{-\frac{t}{R.C}})$$

The constant value R, V and C values used are given below.

$$V = 5 \, V$$

$$R = 100 \; k \; \Omega$$

$$C = 20 \; x \; 10^{-12} \; F$$

$$t = 5 \; x \; 10^{-7} \; sec$$

Calculations were made by changing t values.

$$q(5\ x\ 10^{-7}) = 1\ x\ 10^{-10}\ (1 - e^{-\frac{5\ x\ 10^{-7}}{2x10^{-6}}}) = 2.21199217 \times 10^{-11}$$

$$t = 1\ x\ 10^{-6}\ sec$$

$$q(1\ x\ 10^{-6}) = 1\ x\ 10^{-10}\ (1 - e^{-\frac{1\ x\ 10^{-6}}{2x10^{-6}}}) = 3.9346934 \times 10^{-11}$$

$$t = 1.5\ x\ 10^{-6}\ sec$$

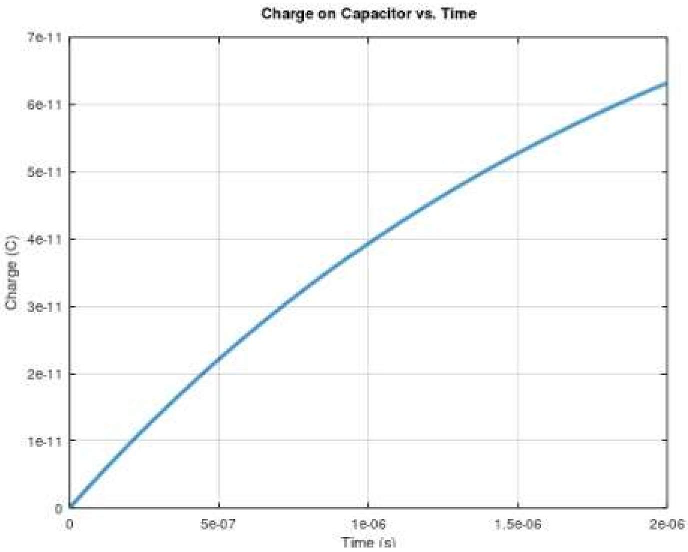
$$q(1.5\ x\ 10^{-6}) = 1\ x\ 10^{-10}\ (1 - e^{-\frac{1.5\ x\ 10^{-6}}{2x10^{-6}}}) = 5.27633447 \times 10^{-11}$$

$$t = 2\ x\ 10^{-6}\ sec$$

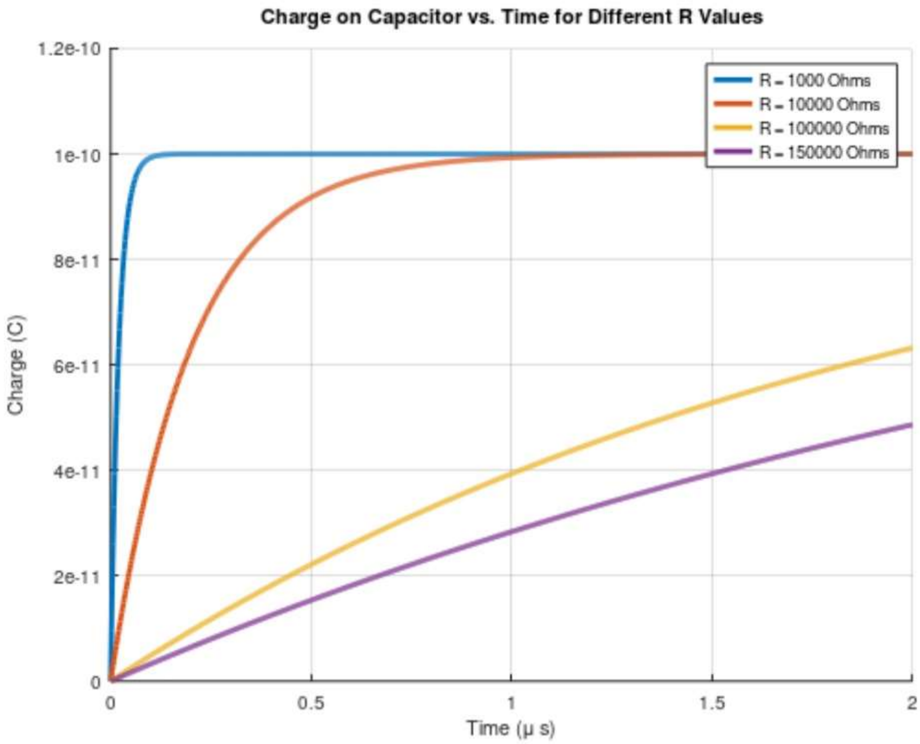
$$q(2\ x\ 10^{-6}) = 1\ x\ 10^{-10}\ (1 - e^{-\frac{2\ x\ 10^{-6}}{2x10^{-6}}}) = 6.32120559 \times 10^{-11}$$

According to the data obtained, the following graph was obtained.

q(t) - time graph



This graph provides information on how the rate of charge accumulation on a capacitor varies with different resistor values in an RC circuit.



As resistance increases, the curve shows a more gradual rise, reflecting a slower rate of charge accumulation. Higher resistance results in a larger time constant, causing the capacitor to charge more slowly over time. For lower resistance values, the curve exhibits a steeper initial rise, indicating a faster rate of charge accumulation. This is because lower resistance leads to a smaller time constant, causing the capacitor to charge faster. $\tau = RC$ is the charging time constant.

5. Infinite Sequence for Resistances

This question focuses on the sequence q_{max},n, which represents the maximum charge of a capacitor as a function of R.

Maximum Charge of Capacitor (q_{max})

A capacitor's capacitance (C) dictates its maximum charge capacity. This is represented by the formula:

$$q_{max} = C.V$$

Infinite Sequence of Resistances (R_n)

As a capacitor charges through a resistor, the voltage across it increases, following a rising curve. This can be represented by the resistance sequence:

R_1= R
R_2= 2R
Rn=n*R

In this sequence, the nth resistance can be expressed as Rn=n * R. Replacing Rn in the formula V=I*R gives V=I.n.R. With this, we can express qmax as

$$q_{max} = \sum_{n=1}^{\infty} C.V.n$$

The convergence of the infinite series for qmax,n needs to be investigated using the limit as n approaches infinity for C.V.n.

$$lim_{n\to\infty}^{C.V.n}$$

As n increases, the capacitance linearly increases, leading to a larger maximum charge. As n approaches infinity, the sequence qmax,n diverges, i.e., it increases without bound.

The infinite series for qmax,n is therefore expressed as the limit as n approaches infinity for C.V.n. This series also diverges, in line with the sequence.

The infinite series does not converge because its sequence also does not.

To summarize, as the capacitance increases according to the sequence Rn=n.R, the maximum charge of a capacitor rises indefinitely. This phenomenon is characterized by a diverging infinite series.

6. CONCLUSION

The main objective of the project is to understand how a capacitor works and analyze the difference between real-life and theoretical capacitors by using experiments, measurements, and theoretical calculations. As a result of this analysis, the relation between variables such as the distance between plates for the parallel plate capacitor, the material that was used between cylinders, and the surface area of the plates can be realized. The purpose of theoretical calculations is to verify mathematical formulas and understand the conditions to apply them in real life. The purpose of experiments with the two types of capacitors is to observe the behavior of a capacitor to compare data from the experiment and real life. Experiments compare data under different conditions to real-life and theoretical predictions and confirm the accuracy of data. While calculating the capacitance of the capacitor, mathematical equations were used. Creating two types of capacitors requires knowledge of physics, circuit analysis, etc. These tasks allow students to improve their knowledge and skills, gain the ability to make and produce mathematical operations, using theoretical knowledge in real life, and understand the practicability of the calculation of the capacitance of the capacitor.

7. REFERENCES

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