

# Fuzzy logic

## Definitions

Each task contains sets of 1. *observations*, 2. *conclusions*, and 3. *rules*.

The sets of *observations* might not be defined for the same domain (though, you will notice that they often are), so they are marked with different variables:

$\mu_A(x) \leftarrow$  fuzzy set  $A$  containing values of  $x$  from its domain.

$\mu_B(y) \leftarrow$  fuzzy set  $B$  containing values of  $y$  from its domain.

A set of observations will look like this:

$$\mu_A(x) = \left\{ \frac{0.0}{1}, \frac{0.1}{2}, \frac{0.4}{3}, \frac{0.7}{4}, \frac{1.0}{5} \right\}$$

Please note that the set does not contain fractions. This is notation describing each element's degree of belonging. The objects of the set are read as follows:  $\frac{0.4}{3}$  - "element 3 belongs to set  $A$  to a degree of 0.4".

As opposed to set theory you are used to, elements can "partially" belong to a set. Elements with a degree of 0.0 fully do not belong to the set, while element of a degree of 1.0 fully belong to the set. The degree is always a number between 0 and 1, both inclusive.

Sets of conclusions are always defined on the same domain, as they describe the same variable.

The set of rules is defined in a similar fashion to classical logic:

1. if  $x$  is  $\neg A$  then  $z$  is  $\neg N$
2. if  $x$  is  $\neg A \vee y$  is  $B$  then  $z$  is  $M$
3. if  $x$  is  $A \wedge y$  is  $\neg B$  then  $z$  is  $K$

Where variables  $x$  and  $y$  are inputs found in the observation sets  $A$  and  $B$  respectively, and  $z$  is an output we are trying to determine, found in conclusion sets  $N$ ,  $M$ , and  $K$ .

The goal of each task is to calculate the value of  $z$  for some known values of  $x$  and  $y$ .

Remember that your answer on the value of  $z$  must be in the domain of  $z$ . If the value is fractional, round it to the nearest value in the domain (halfway point is rounded up).

## Practice tasks

Using the Zadeh s and t-norm and Mandami's implication, calculate the value of  $z$  for given values  $x$  and  $y$ .

1

$$\mu_A(x) = \left\{ \frac{0.0}{1}, \frac{0.1}{2}, \frac{0.4}{3}, \frac{0.7}{4}, \frac{1.0}{5} \right\}$$

$$\mu_B(y) = \left\{ \frac{0.1}{0}, \frac{0.5}{25}, \frac{0.8}{50}, \frac{0.7}{75}, \frac{0}{100} \right\}$$

$$\mu_N(z) = \left\{ \frac{0.5}{1}, \frac{0.7}{2}, \frac{1.0}{3}, \frac{0.0}{4}, \frac{0.5}{5} \right\}$$

$$\mu_M(z) = \left\{ \frac{0.2}{1}, \frac{0.3}{2}, \frac{0.9}{3}, \frac{0.7}{4}, \frac{0.0}{5} \right\}$$

$$\mu_K(z) = \left\{ \frac{1.0}{1}, \frac{0.3}{2}, \frac{0.1}{3}, \frac{0.2}{4}, \frac{1.0}{5} \right\}$$

1. if  $x$  is  $\neg A$  then  $z$  is  $\neg N$
2. if  $x$  is  $\neg A \vee y$  is  $B$  then  $z$  is  $M$
3. if  $x$  is  $A \wedge y$  is  $\neg B$  then  $z$  is  $K$

$x = 4; y = 0; z = ?$

## 2

$$\mu_A(x) = \left\{ \frac{0.1}{1}, \frac{0.5}{2}, \frac{0.0}{3}, \frac{0.3}{4}, \frac{0.7}{5} \right\}$$

$$\mu_B(y) = \left\{ \frac{1.0}{15}, \frac{0.0}{30}, \frac{0.2}{45}, \frac{0.3}{60} \right\}$$

$$\mu_N(z) = \left\{ \frac{0.2}{25}, \frac{0.3}{50}, \frac{1.0}{75} \right\}$$

$$\mu_M(z) = \left\{ \frac{0.0}{25}, \frac{0.5}{50}, \frac{0.7}{75} \right\}$$

$$\mu_K(z) = \left\{ \frac{0.2}{25}, \frac{0.6}{50}, \frac{0.9}{75} \right\}$$

1. if  $x$  is  $\neg A \vee y$  is  $B$  then  $z$  is  $N$
2. if  $x$  is  $\neg A$  then  $z$  is  $M$
3. if  $x$  is  $\neg A \wedge y$  is  $\neg B$  then  $z$  is  $\neg K$

$$x = 1; y = 45; z = ?$$

## 3

$$\mu_A(x) = \left\{ \frac{0.5}{1}, \frac{0.7}{2}, \frac{0.3}{3}, \frac{0.1}{4}, \frac{0.9}{5} \right\}$$

$$\mu_B(y) = \left\{ \frac{0.8}{15}, \frac{0.4}{30}, \frac{0.1}{45}, \frac{1.0}{60} \right\}$$

$$\mu_N(z) = \left\{ \frac{0.6}{25}, \frac{0.4}{50}, \frac{0.2}{75} \right\}$$

$$\mu_M(z) = \left\{ \frac{0.1}{25}, \frac{0.8}{50}, \frac{0.9}{75} \right\}$$

$$\mu_K(z) = \left\{ \frac{0.7}{25}, \frac{0.1}{50}, \frac{0.7}{75} \right\}$$

1. if  $x$  is  $A$  then  $z$  is  $N$
2. if  $x$  is  $\neg A \wedge y$  is  $\neg B$  then  $z$  is  $\neg M$
3. if  $x$  is  $\neg A \vee y$  is  $B$  then  $z$  is  $\neg K$

$$x = 3; y = 30; z = ?$$

## Example solution - task 3

If  $x$  is  $A$  then  $z$  is  $N$ .

$$x \text{ is } A = \mu_A(x) = \mu_A(3) = 0.3$$

$$z \text{ is } N = \left\{ \frac{0.6}{25}, \frac{0.4}{50}, \frac{0.2}{75} \right\} \quad (1)$$

$$\text{If } 0.3 \text{ then } z \text{ is } N = \hat{\mu}_N(z) = \{..|m(z) = \min(m(z), 0.3)\} = \left\{ \frac{0.3}{25}, \frac{0.3}{50}, \frac{0.2}{75} \right\}$$

If  $x$  is  $\neg A \wedge y$  is  $\neg B$  then  $z$  is  $\neg M$ .

$$x \text{ is } \neg A = \mu_{\neg A}(x) = \neg \mu_A(x) = \neg \mu_A(3) = \neg 0.3 = 1 - 0.3 = 0.7$$

$$y \text{ is } \neg B = \mu_{\neg B}(y) = \neg \mu_B(y) = \neg \mu_B(30) = \neg 0.4 = 1 - 0.4 = 0.6$$

$$x \text{ is } \neg A \wedge y \text{ is } \neg B = 0.7 \wedge 0.6 = \min(0.7, 0.6) = 0.6$$

$$z \text{ is } M = \left\{ \frac{0.1}{25}, \frac{0.8}{50}, \frac{0.9}{75} \right\} \quad (2)$$

$$z \text{ is } \neg M = \left\{ \frac{0.9}{25}, \frac{0.2}{50}, \frac{0.1}{75} \right\}$$

$$\text{If } 0.6 \text{ then } z \text{ is } \neg M = \hat{\mu}_{\neg M}(z) = \left\{ \frac{0.6}{25}, \frac{0.2}{50}, \frac{0.1}{75} \right\}$$

$$\begin{aligned}
&\text{If } x \text{ is } \neg A \vee y \text{ is } B \text{ then } z \text{ is } \neg K. x \text{ is } \neg A = 0.7 \\
&\quad y \text{ is } B = 0.4 \\
&0.7 \vee 0.4 = \max(0.7, 0.4) = 0.7 \\
&z \text{ is } K = \left\{ \frac{0.7}{25}, \frac{0.1}{50}, \frac{0.7}{75} \right\} \\
&z \text{ is } \neg K = \left\{ \frac{0.3}{25}, \frac{0.9}{50}, \frac{0.3}{75} \right\} \\
&\text{If } 0.7 \text{ then } z \text{ is } \neg K = \hat{\mu}_{\neg K}(z) = \left\{ \frac{0.3}{25}, \frac{0.7}{50}, \frac{0.3}{75} \right\}
\end{aligned} \tag{3}$$

$$\begin{aligned}
&\hat{\mu}_{N \neg M \neg K}(z) = \max(\hat{\mu}_N(z), \hat{\mu}_{\neg M}(z), \hat{\mu}_{\neg K}(z)) \\
&\hat{\mu}_{N \neg M \neg K}(z) = \left\{ \frac{0.6}{25}, \frac{0.7}{50}, \frac{0.3}{75} \right\} \\
&z = \frac{0.6 * 25 + 0.7 * 50 + 0.3 * 75}{0.6 + 0.7 + 0.3} \\
&\quad z = \frac{15 + 35 + 22.5}{1.6} \\
&\quad z = \frac{72.5}{1.6} \\
&z \approx 45.3 \text{ Domain of } z = \{25, 50, 75\} \\
&\quad z = 50
\end{aligned} \tag{4}$$