Local Search Heuristics

1 Neighborhood relation

A neighborhood relation is defined between solutions of an optimization problem should have the following properties:

- The graph of the relation should be **connected** – a path should exist between any pair of solutions.
- Neighboring solutions should be **similar**.
- The **number of neighbors** of any solution should be small. Preferably $O(n)$ and in all cases $O(n^2)$.
- The **graph diameter** should be small. Preferably $O(n)$ and in all cases $O(n^2)$.

2 Hill-climbing

Start with a random solution. Traverse the graph of the neighborhood relation, moving to the best neighboring solution in every step. When no neighboring solutions are better, end the algorithm.

Questions

Question 1.

Propose a neighborhood relation for the Knapsack Problem. Verify that the relation satisfies the requirements mentioned above.

Consider an instance of the Knapsack Problem with three items. Draw the graph of the neighborhood relation for this instance.

Question 2.

Propose a neighborhood relation for the Travelling Salesman Problem. Verify that the relation satisfies the requirements mentioned above.

Mini-project: Local search

Implement the Hill-climbing algorithm for the Knapsack Problem. Test it using the data in the `knapsack.txt` file. Compare your results to those obtained using the brute force method.

Optionally:

- Implement the Hill-climbing algorithm for the Travelling Salesman Problem. Test it using randomly generated input data and compare the results to optimal solutions obtained using the brute force method.

- Compare the effectiveness of regular Hill-climbing and Simulated Annealing. Test it with different annealing functions.