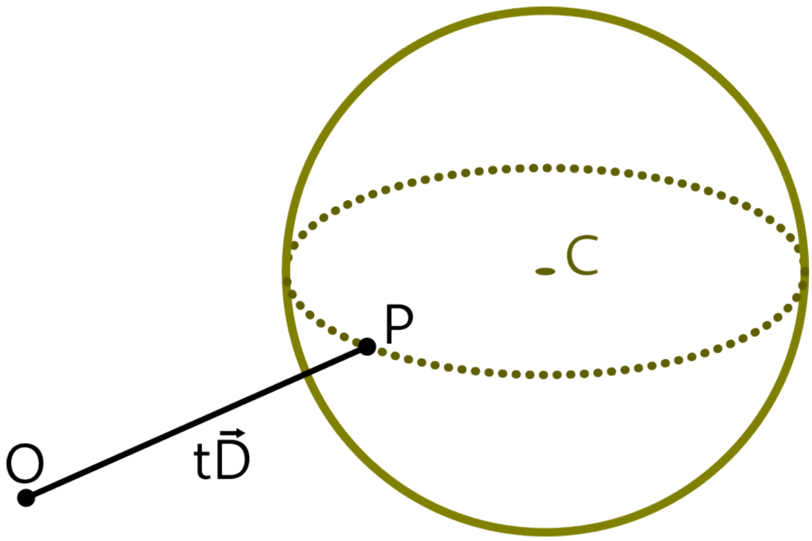
Sphere Intersection in Raytracing

Timur Çakmakoğlu *

April 2023

Ray-sphere intersection at point P with a sphere that is centered at C and has radius r satisfies that the distance between P and C is equal to r,

$$
\begin{gathered}
\|P - C\| = r \\
(P - C) \bullet (P - C) = r^2
\end{gathered}
\tag{1}
$$



In raytracing, we define a ray with origin point O and a unit direction vector $\vec{D}$ multiplied by the length coefficient t. So that, P can be written as a function of t,

$$P(t) = O + t\vec{D}$$

Solve **Equation 1** for $P(t)$,

$$(O + t\vec{D} - C) \bullet (O + t\vec{D} - C) = r^2$$

For the ease of representation we define,

$$\vec{X} = O - C$$

So that,

$$
\begin{gathered}
(t\vec{D} + \vec{X}) \bullet (t\vec{D} + \vec{X}) = r^2 \\
(\vec{D} \bullet \vec{D}) \cdot t^2 + 2(\vec{D} \bullet \vec{X})t + (\vec{X} \bullet \vec{X}) - r^2 = 0
\end{gathered}
\tag{2}
$$

*The solution for sphere intersection is adapted from the first book of Ray Tracing in One Weekend series

Solve the quadratic equation for t where $a = (\vec{D} \bullet \vec{D})$, $b = 2(\vec{D} \bullet \vec{X})$, and $c = (\vec{X} \bullet \vec{X}) - r^2$,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice that $b = 2k$ for some constant $k = (\vec{D} \bullet \vec{X})$, and for optimization purposes we can simplify the equation to,

$$t = \frac{-k \pm \sqrt{k^2 - ac}}{a} \tag{3}$$