1) a) $2n^3 - 2n + 1 \in O(n^3)$ doğrudur.

$$2n^3 - 2n + 1 \leqslant cn^3$$

$$2n^3 - 2n + 1 \leqslant 2n^3 + 2n^3 + n^3 \leqslant 5n^3 \quad \forall n \geq n_0$$

$c = 5$
$n_0 = 1$

b) $2n^2 - n + 1 \in \Theta(n^3)$ Yanlış ✗

$$c_1 g(n) \leq f(n) \leq c_2 g(n) \quad \forall n \geq n_0$$

$$c_1 \cdot n^3 \overset{?}{\leqslant} 2n^2 - n + 1 \overset{✓}{\leqslant} c_2 \cdot n^3 \quad c_2 > 0$$

$c_1 n^3 \leqslant 2n^2 - n + 1$ ✗ Yanlış

↳ Hiçbir sabit sonsuzu sıfırlayamaz.

c) $2n^3 - n + 1 \in O(n^4)$ doğrudur

$$f(n) \leq g(n)$$

$$2n^3 - n + 1 \leqslant cn^4$$

$$2n^3 - n + 7 \leqslant cn^3 \leqslant cn^4 \quad ✓$$

$$2n^3 + n^3 + 7n^3$$

d) $2n^3 - 3n + 1 \in \Omega(n^2)$ doğrudur ✓

$$f(n) \geqslant c g(n)$$

$$2n^3 - 3n + 1 \geqslant cn^2$$

↳ Sonsuza daha hızlı gider

2) a) $x(n) = x(n+1) + 7, \quad n \geqslant 1 \quad x(1) = 0$

$$x(1) = \cancel{x(2)} + 7$$
$$\cancel{x(2)} = \cancel{x(3)} + 7$$
$$\vdots$$
$$\cancel{x(n-2)} = \cancel{x(n-1)} + 7$$
$$+ \quad \cancel{x(n-1)} = x(n) + 7$$

$$x(1) = x(n) + (n-1) \cdot 7 \Rightarrow 0 = x(n) + 7n - 7$$

$$\boxed{x(n) = 7 - 7n}$$

b) $x(n) = 3x(n-1) + 4 \ , \ n > 1 \quad x(1) = 1$

$$x(n-1) = \underbrace{3 \cdot x(n-2) + 4}$$

$$x(n) = 3\left[3x(n-2) + 4\right] + 4$$

$$x(n-2) = 3 \cdot x(n-3) + 4$$

$$x(n) = 3^2\left[3 \cdot x(n-3) + 4\right] + 3^2 \cdot 4 + 3 \cdot 4 + 4$$

$$x(n) = 3^3 \cdot x(n-3) + 3^2 \cdot 4 + 3 \cdot 4 + 4$$

$$x(n) = 3^k \cdot x(n-k) + 3^{k-1} \cdot 4 + 3^{k-2} \cdot 4 + \dots + 3 \cdot 4 + 4$$

$k = n-1$ seçilmeli

$$x(n) = 3^{n-1} \cdot x(1) + \underbrace{3^{n-2} \cdot 4 + 3^{n-3} \cdot 4 + \dots + 3 \cdot 4 + 4}$$

$$= 3^{n-1} + 4 \cdot (\underbrace{3^{n-2} + \dots + 1}_{r=3})$$

$$= 3^{n-1} + \cancel{4}^{\,2} \left[\frac{3^{n-1} - 1}{\cancel{2}}\right] = 3^{n-1} + 2 \cdot 3^{n-1} - 2$$

$$\boxed{x(n) = 3^n - 2}$$

c) $x(n) = x(n/3) + 2 \ , \ n > 1, \quad x(1) = 1 \quad (n = 3^k$ için çözünüz$)$

$n = 3^k \qquad x(3^k) = x(3^{k-1}) + 2 \qquad\qquad k = \log_3 n$

$$\cancel{x(3^{k-1})} = x(3^{k-2}) + 2$$

$\vdots$

$n = 3 \ (3^1) \quad x(3) = x(1) + 2$

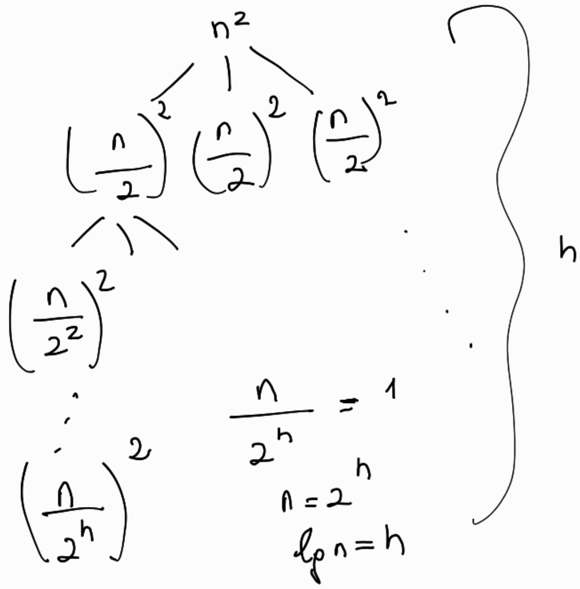
$$x(3^k) = x(1) + 2 \cdot k = 2k + 1$$

$$\boxed{x(n) = 2 \cdot \log_3 n + 1}$$

3) Özyinelemeli Ağaç Yöntemi ile çözünüz

$T(n) = 3T(n/2) + n^2$

$T(1) = 1$



$\frac{n}{2^h} = 1$

$n = 2^h$

$\lg n = h$

$\lg n + 1$

$T(n) = n^2 + 3\left(\frac{n}{2}\right)^2 + 3^2\left(\frac{n}{2^2}\right)^2 + \dots + 3^h\left(\frac{n}{2^h}\right)^2$

$T(n) = n^2\left[1 + 3\cdot\frac{1}{4} + 3^2\cdot\frac{1}{4^2} + \dots 3^h\cdot\frac{n}{2^{2h}}\right]$

$\boxed{r = \frac{3}{4}}$

$T(n) = n^2\cdot\left[\frac{1-\left(\frac{3}{4}\right)^{h+1}}{\frac{1}{4}}\right] = 4\cdot n^2\cdot\left[1-\left(\frac{3}{4}\right)^{h+1}\right]$

$= 4n^2 - 4n^2\cdot\left(\frac{3}{4}\right)^{h+1}$

$= 4n^2 - \frac{\cancel{4n^2}}{\underbrace{4^{h+1}}_{\cancel{4^{\lg n}}\to n^2\cdot 4}}\cdot 3^{h+1} = \boxed{4n^2 - 3^{\lg n+1}}$

4) Master Teoremiyle çözün $(T(1)=1)$

a) $T(n) = 9\cdot T(n/3) + n^2\lg^3 n$

$f(n) = n^2\lg^3 n$

$a = 9$
$b = 3$

$n^{\log_3 9} = n^2$ ile karşılaştır

Durum1

$n^2\lg^3 n = O(n^{2-\varepsilon})$

$\cancel{n^2}\lg^3 n = \cancel{n^2}\cdot\frac{1}{n^\varepsilon}$ $\qquad \lg^3 n > \frac{1}{n^\varepsilon}$

$\boxed{\text{Durum1 olamaz}}$

Durum3

$n^2\lg^3 n \overset{?}{=} \Omega(n^{2+\varepsilon})$

$\cancel{n^2}\lg^3 n = \cancel{n^2}\cdot n^\varepsilon$ $\qquad n^\varepsilon > \lg n$

$\boxed{\text{Durum3 olamaz}}$

→ Alt sınır olamaz

Durum2

$n^2\lg^3 n = \Theta(n^2\cdot\lg^k n)$ $\qquad \boxed{k=3}$

$k+1$

$\boxed{T(n) = \Theta(n^2\lg^4 n)}$

b) $T(n) = 10T(n/3) + n^2 \lg^5 n$

$f(n) = n^2 \lg^5 n$ $\to$ $a = 10$, $b = 3$ $\quad n^{\log_3 10} > n^2$

$n^2 \lg^5 n = O(n^{\log_3 10 - \varepsilon})$ $\quad$ Durum1

$$\boxed{T(n) = \Theta(n^{\log_3 10})}$$

c) $T(n) = 7 \cdot T(n/3) + n^2 \lg^5 n$

$f(n) = n^2 \lg^5 n$ $\to$ $a=7$, $b=3$ $\quad n^{\log_3 7} < n^2 < n^2 \lg^5 n$

$\to$ Durum 1 değil

$f(n) = n^2 \lg^5 n = \Omega(n^{\log_3 7 + \varepsilon})$ Alt sınırı, yani

$\to$ $n^2 \lg^5 n$ daha büyük ✓

Düzgünlük koşulu sağlanıyor mu?

$a \cdot f(\frac{n}{b}) \leq c \cdot f(n)$, $0 < c < 1$ ise

$7 \cdot \left(\frac{n}{3}\right)^2 \cdot \cancel{\lg^5\left(\frac{n}{3}\right)} \leq c \cdot n^2 \cancel{\lg^5 n}$ $\quad <$

$7 \cdot \left(\frac{n}{3}\right)^2 \leq c n^2$

$7 \cdot \frac{n^{\cancel{2}}}{9} \leq c n^{\cancel{2}}$

$\frac{7}{9} \leq c < 1$

$$\boxed{T(n) = \Theta(f(n)) = \Theta(n^2 \lg^5 n)}$$