Modeling the Color-Maze puzzle as a search problem:

1- States: A state is a tuple (x, y), where x and y are the coordinates of the agent on the NxN grid

2- Successor state function: It generates a set of possible states that can be reached from the current state by performing a move until hitting a wall. This move can be done in 4 directions (up, down, left, right)

3- Initial state: It is defined as the starting position (“S”) of the agent on the grid

4- Goal test: It checks whether all the empty cells have been colored. It is defined as the agent reaching a state where all the cells marked as 0 have been colored

5- Step cost function: It assigns a cost to each move (to reach state s′ from starting state s) made by the agent. In my implementation, this cost is equal to the number of nodes passed while moving from state s to state s’

Extending the search model for Color-Maze to apply A* search, finding a heuristic function, and proving that it is admissible:

We know from the lecture slides that an admissible heuristic satisfies the condition:

- $h(n) \leq h^*(n)$, i.e., h never overestimates the true cost
- Intuition: $h(n) \leq h^*(n)$ means that the search won’t miss any promising paths
  If it really is cheap to get to a goal via n (i.e., both g(n) and h*(n) are low),
  then f(n) = g(n) + h(n) will also be low, and the search
  won't ignore n in favor of more expensive options.

For this problem, I considered the heuristic function as the number of uncolored nodes left.

- h(n) = # of uncolored nodes left
- g(n) = step cost function (# of nodes passed in order to reach the next state)
- f(n) = g(n) + h(n) from the definition

Intuitively, this heuristic makes sense because the fewer uncolored nodes there are, the closer we are to reach the goal. Besides, “the number of uncolored nodes left” can be equal to “the number of uncolored nodes in the beginning” at most. So, it can be said that this heuristic function never overestimates the actual cost to reach the goal test.
In a more formal way, let h*(n) be the actual number of uncolored nodes left on the shortest path from node n to the goal test. We know that h*(n) is always greater than or equal to 0 since we noted that h*(n) = 0 is the condition for the goal test. Then, we can write

- h(n) = # of uncolored nodes left = h*(n) + additional uncolored nodes along other paths

Since h*(n) is the number of uncolored nodes on the shortest path from node n to the goal, we know that additional uncolored nodes along other paths must be at least as many as h*(n). Therefore, we can write:

- $h(n) \geq h^*(n)$

This shows that the heuristic function h(n) never overestimates the actual cost of reaching the goal from node n, and is thus admissible.

Summarizing the results of UCS:

	the number of cells in the maze	**the difficulty level**	**the total distance traveled by the agent**	**the total number of expanded nodes**	**the CPU time (seconds)**	**the memory consumption (MB)**
Maze 1	36	easy	36	49	0.002	11.859375
Maze 2	32	easy	32	36	0.002	12.25
Maze 3	45	easy	44	15536	0.696	18.015625
Maze 4	47	easy	46	16225	0.726	20.828125
Maze 5	50	easy	49	72987	3.345	26.90625
Maze 6	34	medium	35	125	0.012	10.714843
Maze 7	32	medium	35	34641	3.528	21.972656
Maze 8		medium			Not terminated	
Maze 9		medium			Not terminated	
Maze 10	45	medium	63	3124	0.678	13.792968
Maze 11		difficult			Not terminated	
Maze 12		difficult			Not terminated	
Maze 13		difficult			Not terminated	
Maze 14		difficult			Not terminated	

Maze 15		difficult			Not terminated	

Summarizing the results of A* Search:

	the number of cells in the maze	**the difficulty level**	**the total distance traveled by the agent**	**the total number of expanded nodes**	**the CPU time**	**the memory consumption**
Maze 1	36	easy	36	15	0.001	12.0625
Maze 2	32	easy	32	10	0.001	12.265625
Maze 3	45	easy	44	20	0.003	12.359375
Maze 4	47	easy	46	27	0.004	12.296875
Maze 5	50	easy	49	26	0.004	12.4375
Maze 6	34	medium	35	27	0.006	10.546875
Maze 7	32	medium	35	69	0.014	10.7109375
Maze 8	58	medium	67	26820	8.373	25.886718
Maze 9	58	medium	68	4824	1.503	13.390625
Maze 10	45	medium	63	599	0.362	11.574218
Maze 11		difficult				
Maze 12		difficult				
Maze 13		difficult				
Maze 14		difficult				
Maze 15		difficult				