İSTANBUL TEKNİK ÜNİVERSİTESİ

KONTROL VE OTOMASYON MÜHENDİSLİĞİ BÖLÜMÜ

KON318E – INTRODUCTION TO ROBOTICS

HOMEWORK #4

Öğretim Üyesi : Prof. Dr. Hakan TEMELTAŞ

Öğrencinin;

Adı Soyadı : Ecem Işıldar

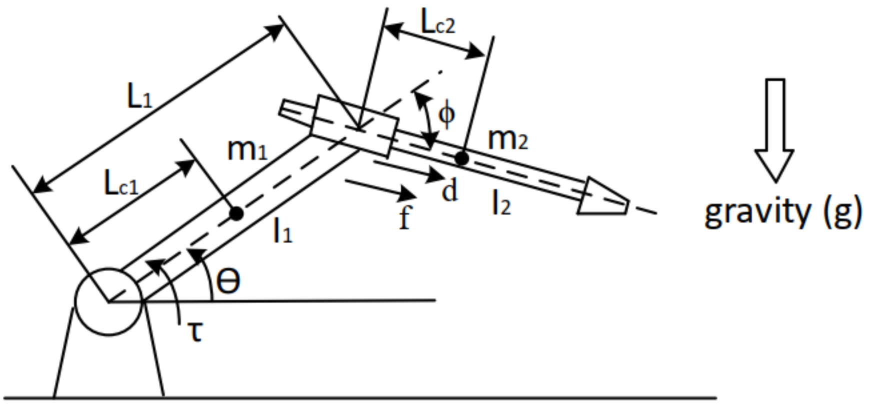
Numarası : 040170450

Bu raporda yer alan tüm içeriğin tamamen şahsıma ait olduğunu beyan ederim.

Tarih: 02/01/2021

İmza: Işıldar

Question 1: A 2-DOF manipulator with one revolute and one prismatic joint is given. Note that prismatic joint is in the direction of angle ϕ from centerline of the first link (angle ϕ is constant). As shown in the figure, θ and d are the joint variables. The first joint actuator produces torque τ, while the second joint actuator generates linear force *f*. Using the given parameters, answer the following questions:

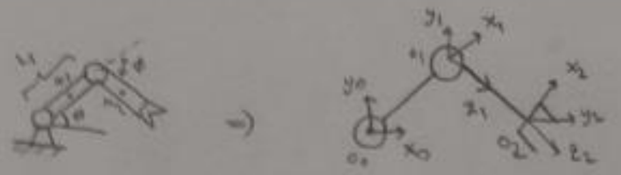


a) Write total Kinetic and Potential Energy of the manipulator parametrically.

DH table is calculated.Then, z and o vectors are found. Jacobian matrix for each joint is calculated. Velocity and angulat velocity is found. Finally, kinetic energy equaitons is obtained. Inertia matrix and corialis terms are found. At last, potential energy and gravity matrces are calcualted.

These steps shown below:

1)



In order to calculate potential and kinetic energies, first DH table should written

a: distance between o_{i-1} and o_i in x_i direction
d: distance between o_{i-1} and o_i in z_{i-1} direction
θ: angle between x_{i-1} and x_i regarding to z_{i-1}
α: angle between z_{i-1} and z_i regarding to x_1

	a	α	d	θ
1	L_1	$90°$	0	$\theta+\phi$
2	0	0	d	0

$$A_1=\begin{bmatrix}\cos(\theta+\phi) & 0 & \sin(\theta+\phi) & L_1\cos(\theta+\phi)\\ \sin(\theta+\phi) & 0 & -\cos(\theta+\phi) & L_1\sin(\theta+\phi)\\ 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix} \quad A_2=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d\\0&0&0&1\end{bmatrix}$$

Then Jacobian matrices for 1st and 2 joint are calculated.

$$J_{v_1}=\left[z_0\times(o_1-o_0)\right] \quad J_{\omega_1}=z_0 \; ; \quad J_{v_2}=\left[z_0\times(o_1-o_0) \;\; z_1\times(o_2-o_1)\right] \quad J_{\omega_2}=\left[z_0 \;\; z_1\right]$$

$$z_0=\begin{bmatrix}0\\0\\1\end{bmatrix}, \; z_1=\begin{bmatrix}s\\-c\\0\end{bmatrix}, \; o_0=\begin{bmatrix}0\\0\\0\end{bmatrix}, \; o_1=\begin{bmatrix}L_1c\\L_1s\\0\end{bmatrix}, \; o_2=\begin{bmatrix}L_1c\\L_1s\\0\end{bmatrix}$$

$$J_{v_1}=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}L_1c/2\\L_1s/2\\0\end{bmatrix}=\begin{bmatrix}-L_1s/2\\L_1c/2\\0\end{bmatrix}, \quad J_{\omega_1}=\begin{bmatrix}0\\0\\1\end{bmatrix} \Rightarrow J_1=\begin{bmatrix}-\frac{L_1s}{2} & 0\\ \frac{L_1c}{2} & 0\\ 0&0\\0&0\\0&0\\1&0\end{bmatrix}$$

$$v_1=\begin{bmatrix}-L_1s/2 & 0\\ L_1c/2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix} \quad \omega_1=\begin{bmatrix}0&0\\0&0\\1&0\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix}$$

$$J_{v_2}=\left[\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}L_1c/2\\L_1s/2\\0\end{bmatrix} \;\; \begin{bmatrix}0&-c&-s\\c&0&0\\s&0&0\end{bmatrix}\begin{bmatrix}0\\0\\d\end{bmatrix}\right]=\begin{bmatrix}-L_1s/2 & -ds\\ L_1c/2 & 0\\ 0 & 0\end{bmatrix} \Rightarrow J_2=\begin{bmatrix}-L_1s/2 & -ds\\ L_1c/2 & 0\\ 0&0\\ 0&s\\ 0&-c\\ 1&0\end{bmatrix}$$

$$v_2=\begin{bmatrix}-L_1s/2 & -ds\\ L_1c/2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix} \quad \omega_2=\begin{bmatrix}0&s\\0&-c\\1&0\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix}$$

Finally kinetic energy can be calculated,

$$D(q)\ddot q + C(q,\dot q)\dot q + g(q) = \tau$$

$$K = \sum_{i=1}^{2} \left(\frac{1}{2} m_i v_i^T v_i + \frac{1}{2} \omega_i^T R_i I_i R_i^T \omega_i \right)$$

1. Equation:

$$\frac{1}{2} m_1 v_1^T v_1 = \frac{m_1}{2}\left[\left(-\frac{L_1}{2} s \dot{\theta}_1\right)^2 + \left(\frac{L_1}{2} c \dot{\theta}_1\right)^2\right] = \frac{m_1}{8} L_1^2 \dot{\theta}_1^2 //$$

2. Equation:

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \quad R_1 = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \frac{1}{2} [0\ 0\ \dot{\theta}_1] \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{xx,1} & 0 & 0 \\ 0 & I_{yy,1} & 0 \\ 0 & 0 & I_{zz,1} \end{bmatrix} \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$$= \frac{1}{2} [0\ 0\ \dot{\theta}_1] \begin{bmatrix} 0 \\ 0 \\ I_{zz,1} \dot{\theta}_1 \end{bmatrix} = \frac{1}{2} I_{zz,1} \dot{\theta}_1^2 //$$

3. Equation:

$$\frac{1}{2} m_2 v_2^T v_2 = \frac{m_2}{2} (-L_1 s \dot{\theta}_1 - d s \dot{\theta}_2)^2 + (L_1 c \dot{\theta}_1)^2 = \frac{m_2}{2} (L_1^2 \dot{\theta}_1^2 + d^2 s^2 \dot{\theta}_2^2 + 2 d L_1 s^2 \dot{\theta}_1 \dot{\theta}_2) //$$

4. Equation:

$$\omega_2 = \begin{bmatrix} s \dot{\theta}_2 \\ -c \dot{\theta}_2 \\ \dot{\theta}_1 \end{bmatrix} \quad R_1 = R_2 \Rightarrow \frac{1}{2} [s \dot{\theta}_2\ \ -c \dot{\theta}_2\ \ \dot{\theta}_1] \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{xx,2} & 0 & 0 \\ 0 & I_{yy,2} & 0 \\ 0 & 0 & I_{zz,2} \end{bmatrix} \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s \dot{\theta}_2 \\ -c \dot{\theta}_2 \\ \dot{\theta}_1 \end{bmatrix}$$

$$= \frac{1}{2} [0\ \ -\dot{\theta}_2\ \ \dot{\theta}_1] \begin{bmatrix} 0 \\ -I_{yy,2} \dot{\theta}_2 \\ I_{zz,2} \dot{\theta}_1 \end{bmatrix} = \frac{1}{2} (I_{yy,2} \dot{\theta}_2^2 + I_{zz,2} \dot{\theta}_1^2) //$$

$$K = \frac{1}{2} \dot{q}^T \left[\sum_{i=1}^{2} m_i v_i^T v_i + \omega_i^T R_i I_i R_i^T \omega_i \right] \dot{q}$$

$$K = \frac{m_1}{8} L_1^2 \underline{\underline{\dot{\theta}_1^2}} + \frac{1}{2} I_{zz,1} \underline{\underline{\dot{\theta}_1^2}} + \frac{1}{2} I_{zz,2} \underline{\underline{\dot{\theta}_1^2}} + \frac{m_2}{2} L_1^2 \underline{\underline{\dot{\theta}_1^2}} + \frac{d^2 s^2 m_2}{2} \underline{\underline{\dot{\theta}_2^2}} + \frac{1}{2} I_{yy,2} \underline{\underline{\dot{\theta}_2^2}} + m_2 d L_1 s^2 \underline{\underline{\dot{\theta}_1 \dot{\theta}_2}}$$

$$D(q) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \Rightarrow \left. \begin{aligned} d_{11} &= \frac{m_1}{8} L_1^2 + \frac{1}{2} I_{zz,1} + \frac{1}{2} I_{zz,2} + \frac{m_2}{2} L_1^2 \\ d_{12} &= d_{21} = m_2 d L_1 s^2 \\ d_{22} &= \frac{d^2 s^2 m_2}{2} + \frac{1}{2} I_{yy,2} \end{aligned} \right\} \text{Inertia Matrix}$$

$$C_{ijk} = \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right) \Rightarrow \text{Coriolis terms}$$

$k = 1$;

$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial \theta_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial \theta_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial \theta_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial \theta_1} = -\frac{1}{2} m_2 d^2 c$$

$k = 2$;

$$c_{112} = \frac{\partial d_{21}}{\partial \theta_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial \theta_1} = 2 m_2 d L_1 c$$

$$c_{122} = c_{212} = \frac{1}{2} \frac{\partial d_{22}}{\partial \theta_1} = m_2 d^2 c$$

$$c_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial \theta_2} = 0$$

Potential Energy:

$$P = \sum_{i=1}^{2} m_i g^T r_{ci} \Rightarrow P_1 = m_1 g L_{c_1} S \quad \text{and} \quad P_2 = m_2 g (L_1 S + L_{c_2} S)$$

$$P_{total} = P_1 + P_2 = m_1 g L_{c_1} S + m_2 g (L_1 S + L_{c_2} S)$$

$$g_1 = \frac{\partial P}{\partial q_1} = (m_1 L_{c_1} + m_2 L_1) g \cos(\Theta_1 + \Phi) + m_2 L_{c_2} g \cos(\Theta_1 + \Phi)$$

$$g_2 = \frac{\partial P}{\partial q_2} = m_2 g L_{c_2} \cos(\Theta_1 + \Phi)$$

b) Find inverse dynamic model of the manipulator parametrically.

Inverse dynamic model is obtaioned regarding to previous equations.

Equations of Motion

$$d_{11}\ddot{\Theta}_1 + d_{12}\ddot{\Theta}_2 + c_{121}\dot{\Theta}_1\dot{\Theta}_2 + c_{211}\dot{\Theta}_1\dot{\Theta}_2 + c_{221}\dot{\Theta}_2^2 + g_1 = \tau_1$$

$$d_{21}\ddot{\Theta}_1 + d_{22}\ddot{\Theta}_2 + c_{112}\dot{\Theta}_1^2 + (c_{122} + c_{212})\dot{\Theta}_1\dot{\Theta}_2 + c_{222}\dot{\Theta}_2^2 + g_2 = \tau_2$$

c) Assume kv1, kv2 are viscous friction and, ks1, ks2 are static friction coefficients of Joint 1 and Joint 2 respectively. Then extend dynamical equations in part (b) by considering joint frictions.

Frictions

Viscous Friction: $F_V = \begin{bmatrix} k_{v1} & 0 \\ 0 & k_{v2} \end{bmatrix}$

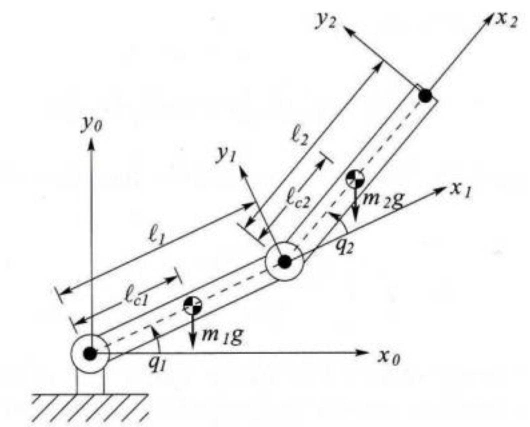
Static Friction: $F_r = \begin{bmatrix} k_{s1} & 0 \\ 0 & k_{s2} \end{bmatrix}$

Extended Dynamic Equations

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \ddot{\Theta}_1 \\ \ddot{\Theta}_2 \end{bmatrix} + \begin{bmatrix} c_{121}\dot{\Theta}_2 & c_{211}\dot{\Theta}_1 + c_{221}\dot{\Theta}_2 \\ c_{112}\dot{\Theta}_1 + c_{122}\dot{\Theta}_2 & c_{212}\dot{\Theta}_1 + c_{222}\dot{\Theta}_2 \end{bmatrix} \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \begin{bmatrix} k_{v1} & 0 \\ 0 & k_{v2} \end{bmatrix} + \begin{bmatrix} k_{s1} & 0 \\ 0 & k_{s2}

Question 2: For 2-link planar arm in the lecture slides, the dynamical model is given as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) = \tau$$

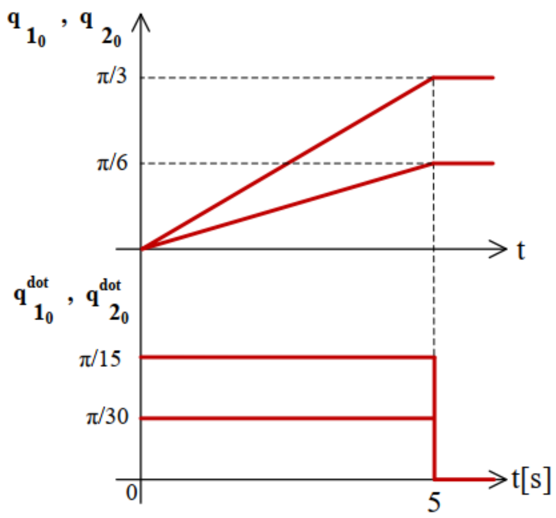


According to the robot manipulator parameters given in the table below, the numerical correspondence of the inverse dynamic model is achieved as

$$D(q)\ddot{q} = \begin{bmatrix} \frac{328}{3}\ddot{q}_1 + \frac{175}{6}\ddot{q}_2 + 50\ddot{q}_1\cos(q_2) + 25\ddot{q}_2\cos(q_2) \\ \frac{175}{6}\ddot{q}_1 + \frac{181}{6}\ddot{q}_2 + 25\ddot{q}_1\cos(q_2) \end{bmatrix} \qquad F(\dot{q}) = \begin{bmatrix} 10.142\dot{q}_1 + 0.1 sign(\dot{q}_1) \\ 10.134\dot{q}_2 + 0.11 sign(\dot{q}_2) \end{bmatrix}$$

$$C(q,\dot{q})\dot{q} = \begin{bmatrix} 50\dot{q}_1\dot{q}_2\sin(q_2) + 25\dot{q}_2^{\,2}\sin(q_2) \\ -25\dot{q}_1^{\,2}\sin(q_2) \end{bmatrix} \qquad G(q) = \begin{bmatrix} 735.75\cos(q_1) + 245.25\cos(q_1+q_2) \\ 245.25\cos(q_1+q_2) \end{bmatrix}$$

For the joints of this manipulator, the following trajectories will be tracked. Draw the required amount of torques with respect to time to follow these trajectories using MATLAB or a similar tool.



Solution 2:

First position and velocity trajectories are plotted after that using given dynamic equation desired torque values are plotted.

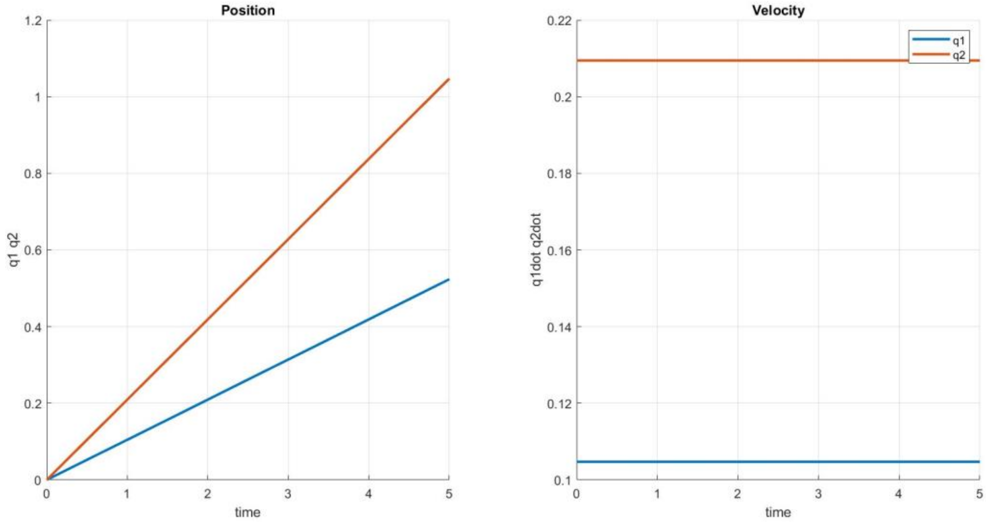


Figure 1: Given trajectories

Torque tracejtory is found by taking zero for acceleration because the derivative of velocity is zero.

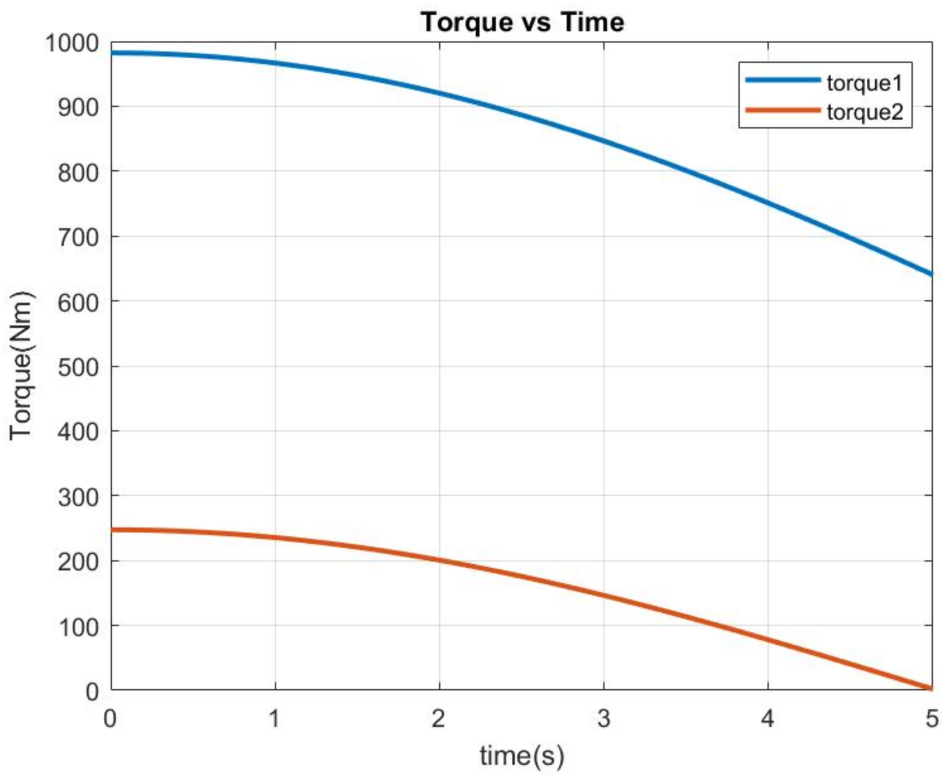


Figure 2: Required torque

Matlab Code is given below:

```
syms q1 q2 q1_dot q2_dot q1_dotdot q2_dotdot;

D_q=[328/3+50*cos(q2), 175/6+25*cos(q2); 175/6+25*cos(q2),
181/6]*[q1_dotdot; q2_dotdot];

C_q=[50*q1_dot.*q2_dot*sin(q2)+25*q2_dot.^2*sin(q2);-
25*q1_dot.^2*sin(q2)];

F_q=[10.142*q1_dot+0.1*sign(q1_dot);
10.134*q2_dot+0.11*sign(q2_dot)];

G_q=[735.75*cos(q1)+245.25*cos(q1+q2);245.25*cos(q1+q2)];

torque(q1,q2,q1_dot,q2_dot,q1_dotdot,q2_dotdot)=D_q+C_q+F_q+G_q;
```

First, I calculated the torque matrix as symbolic, after that I inserted the position, velocity and acceleration values.

```
t=0:0.1:5;
q1_dotdot=0;
q2_dotdot=0;
q1=pi/30*t;
q2=pi/15*t;
q1_dot=(pi/30*ones(length(t),1))';
q2_dot=(pi/15*ones(length(t),1))';
res=torque(q1,q2,q1_dot,q2_dot,q1_dotdot,q2_dotdot);

x=cell2sym(res);

figure(1)
subplot(1,2,1);cla;hold on;grid
on;title('Position');xlabel('time');ylabel('q1 q2');ax1=gca;
plot(ax1,t,q1,'LineWidth',2);hold on;grid on;
plot(ax1,t,q2,'LineWidth',2);hold on;
subplot(1,2,2);cla;hold on;grid
on;title('Velocity');xlabel('time');ylabel('q1dot q2dot');ax2=gca;
plot(ax2,t,q1_dot,'LineWidth',2);hold on;grid on;
plot(ax2,t,q2_dot,'LineWidth',0);hold on;
legend('q1','q2');

figure(2)
plot(t,x,'LineWidth',2);hold on;grid on;title('Torque vs
Time');xlabel('time(s)');ylabel('Torque(Nm)');
legend('torque1','torque2')
```