İSTANBUL TEKNİK ÜNİVERSİTESİ

KONTROL VE OTOMASYON MÜHENDİSLİĞİ BÖLÜMÜ

KON318E – INTRODUCTION TO ROBOTICS

HOMEWORK #3

Öğretim Üyesi : Prof. Dr. Hakan TEMELTAŞ

Öğrencinin;

Adı Soyadı : Ecem Işıldar

Numarası : 040170450

Bu raporda yer alan tüm içeriğin tamamen şahsıma ait olduğunu beyan ederim.

Tarih: 19/12/2021

İmza: Işıldar

JACOBIAN MATRIX OF KUKA KR5 ARC

The Matlab online code of this pdf can be reached with following link. Especially matrix outputs are completely visible in Matlab online. https://drive.matlab.com/sharing/0492f6be-f0b5-4dd0-9f42-0e3cbf5b33b6

a) Find the Jacobian operator

In second homework DH Table parameters are calculated as follows

```
syms theta1 theta2 theta3 theta4 theta5 theta6;
theta=[theta1 theta2 theta3 theta4 theta5 theta6];%radian joint angle
% theta=[pi pi/2 0 0 0 0];%radian joint angle
a=[180 600 120 0 0 0]*0.001;%meter
alpha=[90 180 -90 90 -90 0];%degree
d=[400 0 0 620 0 115]*0.001;%meter
```

Each homogeneous transformation A is calculated corresponding to the formula below then, forward kinematic matrix T is obtained by multiplication of A matrices

$$A_i = \text{Trans}_{z_{i-1},d_i}\,\text{Rot}_{z_{i-1},\theta_i}\,\text{Trans}_{x_i,a_i}\,\text{Rot}_{x_i,\alpha_i}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A_i **is function of** θ_i **,** d_i**,** α_i **and** a_i

```
z=[];o=[];
T=eye(4);%identity matrix
z=[z,[0;0;1]];%z_0=[0 0 1]
o=[o,[0;0;0]];%o_0=[0 0 0]
for i=1:6%6 DOF
    A=[cos(theta(i)), -sin(theta(i))*cosd(alpha(i)), ...
    sin(theta(i))*sind(alpha(i)), a(i)*cos(theta(i));...
    sin(theta(i)), cos(theta(i))*cosd(alpha(i)), ...
    -cos(theta(i))*sind(alpha(i)), a(i)*sin(theta(i));
    0, sind(alpha(i)), cosd(alpha(i)), d(i);
    0, 0, 0, 1];
    T=T*A
    z=[z,T(1:3,3)];%z_0 to z_6
    o=[o,T(1:3,4)];%o_0 to o_6

end
```

T =

$$
\begin{pmatrix}
\cos(\theta_1) & 0 & \sin(\theta_1) & \frac{9\cos(\theta_1)}{50} \\
\sin(\theta_1) & 0 & -\cos(\theta_1) & \frac{9\sin(\theta_1)}{50} \\
0 & 1 & 0 & \frac{2}{5} \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

T =

$$
\begin{pmatrix}
\cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) & \frac{9\cos(\theta_1)}{50} + \frac{3\cos(\theta_1)\cos(\theta_2)}{5} \\
\cos(\theta_2)\sin(\theta_1) & \sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & \frac{9\sin(\theta_1)}{50} + \frac{3\cos(\theta_2)\sin(\theta_1)}{5} \\
\sin(\theta_2) & -\cos(\theta_2) & 0 & \frac{3\sin(\theta_2)}{5} + \frac{2}{5} \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

T =

$$
\begin{pmatrix}
\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) & \sin(\theta_1) & \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) - \cos(\theta_1)\cos(\theta_2)\,\mathrm{si} \\
\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & -\cos(\theta_1) & \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_1)\,\mathrm{sin} \\
\cos(\theta_3)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_3) & 0 & -\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) \\
0 & 0 & 0
\end{pmatrix}
$$

T =

$$
\begin{pmatrix}
\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\,\sigma_2 & \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) - \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) & \sin(\theta_4)\,\sigma_2 - \cos(\theta_4)\sin(\\
\cos(\theta_4)\,\sigma_1 - \cos(\theta_1)\sin(\theta_4) & \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) & \cos(\theta_1)\cos(\theta_4) + \sin(\theta_4) \\
-\cos(\theta_4)\,\sigma_3 & -\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) & -\sin(\theta_4)\,\sigma_3 \\
0 & 0 & 0
\end{pmatrix}
$$

where

$$\sigma_1 = \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)$$

$$\sigma_2 = \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$\sigma_3 = \cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2)$$

T =

$$\left(\begin{array}{cccc}
\cos(\theta_5)\,\sigma_6 - \sin(\theta_5)\,\sigma_3 & \cos(\theta_4)\,\sin(\theta_1) - \sin(\theta_4)\,\sigma_8 & -\sin(\theta_5)\,\sigma_6 - \cos(\theta_5)\,\sigma_3 & \frac{9\cos(\theta}{50} \\
-\cos(\theta_5)\,\sigma_5 - \sin(\theta_5)\,\sigma_2 & -\cos(\theta_1)\,\cos(\theta_4) - \sin(\theta_4)\,\sigma_7 & \sin(\theta_5)\,\sigma_5 - \cos(\theta_5)\,\sigma_2 & \frac{9\sin(}{50} \\
-\sin(\theta_5)\,\sigma_4 - \cos(\theta_4)\,\cos(\theta_5)\,\sigma_1 & \sin(\theta_4)\,\sigma_1 & \cos(\theta_4)\,\sin(\theta_5)\,\sigma_1 - \cos(\theta_5)\,\sigma_4 & \\
0 & 0 & 0 &
\end{array}\right.$$

where

$$\sigma_1 = \cos(\theta_2)\,\sin(\theta_3) - \cos(\theta_3)\,\sin(\theta_2)$$

$$\sigma_2 = \cos(\theta_2)\,\sin(\theta_1)\,\sin(\theta_3) - \cos(\theta_3)\,\sin(\theta_1)\,\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\,\cos(\theta_2)\,\sin(\theta_3) - \cos(\theta_1)\,\cos(\theta_3)\,\sin(\theta_2)$$

$$\sigma_4 = \cos(\theta_2)\,\cos(\theta_3) + \sin(\theta_2)\,\sin(\theta_3)$$

$$\sigma_5 = \cos(\theta_1)\,\sin(\theta_4) - \cos(\theta_4)\,\sigma_7$$

$$\sigma_6 = \sin(\theta_1)\,\sin(\theta_4) + \cos(\theta_4)\,\sigma_8$$

$$\sigma_7 = \sin(\theta_1)\,\sin(\theta_2)\,\sin(\theta_3) + \cos(\theta_2)\,\cos(\theta_3)\,\sin(\theta_1)$$

$$\sigma_8 = \cos(\theta_1)\,\sin(\theta_2)\,\sin(\theta_3) + \cos(\theta_1)\,\cos(\theta_2)\,\cos(\theta_3)$$

T =

$$\left(\begin{array}{cccc}
\sin(\theta_6)\,\sigma_5+\cos(\theta_6)\,\sigma_1 & \cos(\theta_6)\,\sigma_5-\sin(\theta_6)\,\sigma_1 & -\sin(\theta_5)\,\sigma_6-\cos(\theta_5)\,\sigma_7 & \frac{9\cos(}{50} \\
-\sin(\theta_6)\,\sigma_4-\cos(\theta_6)\,\sigma_2 & \sin(\theta_6)\,\sigma_2-\cos(\theta_6)\,\sigma_4 & \sin(\theta_5)\,\sigma_8-\cos(\theta_5)\,\sigma_9 & \frac{9\sin}{5} \\
\sin(\theta_4)\sin(\theta_6)\,\sigma_{10}-\cos(\theta_6)\,\sigma_3 & \sin(\theta_6)\,\sigma_3+\cos(\theta_6)\sin(\theta_4)\,\sigma_{10} & \cos(\theta_4)\sin(\theta_5)\,\sigma_{10}-\cos(\theta_5)\,\sigma_{11} & \\
0 & 0 & 0 &
\end{array}\right.$$

where

$$\sigma_1=\cos(\theta_5)\,\sigma_6-\sin(\theta_5)\,\sigma_7$$

$$\sigma_2=\cos(\theta_5)\,\sigma_8+\sin(\theta_5)\,\sigma_9$$

$$\sigma_3=\sin(\theta_5)\,\sigma_{11}+\cos(\theta_4)\cos(\theta_5)\,\sigma_{10}$$

$$\sigma_4=\cos(\theta_1)\cos(\theta_4)+\sin(\theta_4)\,\sigma_{13}$$

$$\sigma_5=\cos(\theta_4)\sin(\theta_1)-\sin(\theta_4)\,\sigma_{12}$$

$$\sigma_6=\sin(\theta_1)\sin(\theta_4)+\cos(\theta_4)\,\sigma_{12}$$

$$\sigma_7=\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)-\cos(\theta_1)\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_8=\cos(\theta_1)\sin(\theta_4)-\cos(\theta_4)\,\sigma_{13}$$

$$\sigma_9=\cos(\theta_2)\sin(\theta_1)\sin(\theta_3)-\cos(\theta_3)\sin(\theta_1)\sin(\theta_2)$$

$$\sigma_{10}=\cos(\theta_2)\sin(\theta_3)-\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_{11}=\cos(\theta_2)\cos(\theta_3)+\sin(\theta_2)\sin(\theta_3)$$

$$\sigma_{12}=\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$\sigma_{13}=\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3)\sin(\theta_1)$$

Unit vectors z and origins of coordinates o matrices are obtained using this information

$$T_i^0 = \left[\begin{array}{ccc:c} n_1 & s_1 & a_1 & p_1 \\ n_2 & s_2 & a_2 & p_2 \\ n_3 & s_3 & a_3 & p_3 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

o_i

z_i

So, Jv and Jw can be easily calculated using the following equations:

$$J_{v_i} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & \text{for } i \text{ revolute} \\ z_{i-1} & \text{for } i \text{ prismatic} \end{cases}$$

$$J_{\omega_i} = \begin{cases} z_{i-1} & \text{for } i \text{ revolute} \\ 0 & \text{for } i \text{ prismatic} \end{cases}$$

All parameters of Jacobian matrix are obtained

```
Jv1=cross(z(:,1),(o(:,7)-o(:,1)))
```

```
Jv1 =
```

$$\left(\begin{array}{l} \frac{23\cos(\theta_5)\ (\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_1)\sin(\theta_2))}{200} - \frac{3\cos(\theta_2)\sin(\theta_1)}{5} - \frac{23\sin(\theta_5)\ (\cos(\theta_1)\ \text{s}}{} \\ \frac{9\cos(\theta_1)}{50} + \frac{3\cos(\theta_1)\cos(\theta_2)}{5} - \frac{23\sin(\theta_5)\ (\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\ \text{cc}}{200} \end{array} \right.$$

```
Jv2=cross(z(:,2),(o(:,7)-o(:,2)))
```

```
Jv2 =
```

$$\left(\cos(\theta_1)\ \left(\frac{3\cos(\theta_1)\cos(\theta_2)}{5} - \frac{23\sin(\theta_5)\ (\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\ \text{cos}}{200} \right. \right.$$

where

$$\sigma_1 = \frac{31\cos(\theta_2)\cos(\theta_3)}{50} - \frac{3\sin(\theta_2)}{5} + \frac{3\cos(\theta_2)\sin(\theta_3)}{25} - \frac{3\cos(\theta_3)\sin(\theta_2)}{25} + \frac{31\sin(\theta_2)\sin(\theta_3)}{50} + \frac{23\cos(\theta_{\cdot}}{}$$

```
Jv3=cross(z(:,3),(o(:,7)-o(:,3)))
```

```
Jv3 =
```

$$\left(\cos(\theta_1) \left(\frac{23 \sin(\theta_5) \; (\sin(\theta_1) \sin(\theta_4) + \cos(\theta_4) \; (\cos(\theta_1) \sin(\theta_2) \sin(\theta_3) + \cos(\theta_1) \cos(\theta_2) \cos(\theta_3)))}{200} + \frac{23 \cdots}{\cdots} \right. \right.$$

where

$$\sigma_1 = \frac{31 \cos(\theta_2) \cos(\theta_3)}{50} + \frac{3 \cos(\theta_2) \sin(\theta_3)}{25} - \frac{3 \cos(\theta_3) \sin(\theta_2)}{25} + \frac{31 \sin(\theta_2) \sin(\theta_3)}{50} + \frac{23 \cos(\theta_5) \; (\cos(\theta_2) \cdots}{\cdots}$$

```
Jv4=cross(z(:,4),(o(:,7)-o(:,4)))
```

```
Jv4 =
```

$$\begin{pmatrix} \sigma_6 \sigma_1 + \sigma_4 \sigma_3 \\ \sigma_6 \sigma_2 - \sigma_5 \sigma_3 \\ -\sigma_5 \sigma_1 - \sigma_4 \sigma_2 \end{pmatrix}$$

where

$$\sigma_1 = \frac{23 \sin(\theta_5) \; (\cos(\theta_1) \sin(\theta_4) - \cos(\theta_4) \; (\sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + \cos(\theta_2) \cos(\theta_3) \sin(\theta_1)))}{200} - \frac{23 \cos(\theta_5 \cdots}{200}$$

$$\sigma_2 = \frac{23 \sin(\theta_5) \; (\sin(\theta_1) \sin(\theta_4) + \cos(\theta_4) \; (\cos(\theta_1) \sin(\theta_2) \sin(\theta_3) + \cos(\theta_1) \cos(\theta_2) \cos(\theta_3)))}{200} + \frac{23 \cos(\theta_5 \cdots}{200}$$

$$\sigma_3 = \frac{31 \cos(\theta_2) \cos(\theta_3)}{50} + \frac{31 \sin(\theta_2) \sin(\theta_3)}{50} + \frac{23 \cos(\theta_5) \, \sigma_6}{200} - \frac{23 \cos(\theta_4) \sin(\theta_5) \; (\cos(\theta_2) \sin(\theta_3) - \cos(\theta \cdots}{200}$$

$$\sigma_4 = \cos(\theta_2) \sin(\theta_1) \sin(\theta_3) - \cos(\theta_3) \sin(\theta_1) \sin(\theta_2)$$

$$\sigma_5 = \cos(\theta_1) \cos(\theta_2) \sin(\theta_3) - \cos(\theta_1) \cos(\theta_3) \sin(\theta_2)$$

$$\sigma_6 = \cos(\theta_2) \cos(\theta_3) + \sin(\theta_2) \sin(\theta_3)$$

```
Jv5=cross(z(:,5),(o(:,7)-o(:,5)))
```

```
Jv5 =
```

$$\begin{pmatrix} \sin(\theta_4)\,\sigma_1\,\sigma_6 - \sigma_3\,\sigma_5 \\ \sin(\theta_4)\,\sigma_2\,\sigma_6 - \sigma_4\,\sigma_5 \\ \sigma_3\,\sigma_2 - \sigma_4\,\sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{23\sin(\theta_5)\ (\cos(\theta_1)\sin(\theta_4) - \cos(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)))}{200} - \frac{23\cos(\theta_5}{}$$

$$\sigma_2 = \frac{23\sin(\theta_5)\ (\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)))}{200} + \frac{23\cos(\theta_5}{}$$

$$\sigma_3 = \cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))$$

$$\sigma_4 = \cos(\theta_4)\sin(\theta_1) - \sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))$$

$$\sigma_5 = \frac{23\cos(\theta_5)\ (\cos(\theta_2)\cos(\theta_3) + \sin(\theta_2)\sin(\theta_3))}{200} - \frac{23\cos(\theta_4)\sin(\theta_5)\,\sigma_6}{200}$$

$$\sigma_6 = \cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2)$$

```
Jv6=cross(z(:,6),(o(:,7)-o(:,6)))
```

```
Jv6 =
```

$$\begin{pmatrix} \sigma_1\,\sigma_6 - \sigma_4\,\sigma_5 \\ \sigma_2\,\sigma_6 - \sigma_3\,\sigma_5 \\ \sigma_2\,\sigma_4 - \sigma_3\,\sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \frac{23\sin(\theta_5)\;(\cos(\theta_1)\sin(\theta_4) - \cos(\theta_4)\;(\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)))}{200} - \frac{23\cos(\theta_5}{}$$

$$\sigma_2 = \frac{23\sin(\theta_5)\;(\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\;(\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)))}{200} + \frac{23\cos(\theta_5}{}$$

$$\sigma_3 = \sin(\theta_5)\;(\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\;(\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))) + \cos(\theta_5)\;(\cos$$

$$\sigma_4 = \sin(\theta_5)\;(\cos(\theta_1)\sin(\theta_4) - \cos(\theta_4)\;(\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))) - \cos(\theta_5)\;(\cos$$

$$\sigma_5 = \frac{23\cos(\theta_5)\;(\cos(\theta_2)\cos(\theta_3) + \sin(\theta_2)\sin(\theta_3))}{200} - \frac{23\cos(\theta_4)\sin(\theta_5)\;(\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2)}{200}$$

$$\sigma_6 = \cos(\theta_5)\;(\cos(\theta_2)\cos(\theta_3) + \sin(\theta_2)\sin(\theta_3)) - \cos(\theta_4)\sin(\theta_5)\;(\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2))$$

```
Jw1=z(:,1)
```

Jw1 =

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

```
Jw2=z(:,2)
```

Jw2 =

$$\begin{pmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{pmatrix}$$

```
Jw3=z(:,3)
```

Jw3 =

$$\begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{pmatrix}$$

```
Jw4=z(:,4)
```

Jw4 =

$$\begin{pmatrix} \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) - \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) \\ \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) \\ -\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) \end{pmatrix}$$

```
Jw5=z(:,5)
```

Jw5 =

$$\begin{pmatrix} \sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)) - \cos(\theta_4)\sin(\theta_1) \\ \cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1)) \\ -\sin(\theta_4)\ (\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2)) \end{pmatrix}$$

```
Jw6=z(:,6)
```

Jw6 =

$$\begin{pmatrix} -\sin(\theta_5)\ (\sin(\theta_1)\sin(\theta_4) + \cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))) - \cos(\theta_5)\ (\cos(\theta \\ \sin(\theta_5)\ (\cos(\theta_1)\sin(\theta_4) - \cos(\theta_4)\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)\sin(\theta_1))) - \cos(\theta_5)\ (\cos(\theta_2 \\ \cos(\theta_4)\sin(\theta_5)\ (\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2)) - \cos(\theta_5)\ (\cos(\theta_2)\cos(\theta_3)\ \cdot \end{pmatrix}$$

At last, all elements of Jacobian matrix is combined in a one matrix called Jacobian Matrix

```
J=[Jv1, Jv2, Jv3, Jv4, Jv5 Jv6; Jw1, Jw2, Jw3, Jw4, Jw5 Jw6]
```

J =

$$\begin{pmatrix} \sigma_{16} - \sigma_9 - \sigma_{12} - \frac{9\sin(\theta_1)}{50} + \sigma_2 - \sigma_1 - \sigma_8 - \sigma_6 & \cos(\theta_1)\,\sigma_{14} \\ \frac{9\cos(\theta_1)}{50} + \sigma_{10} - \sigma_{13} - \sigma_{17} + \sigma_5 + \sigma_7 - \sigma_4 + \sigma_3 & \sin(\theta_1)\,\sigma_{14} \\ 0 & \cos(\theta_1)\,(\sigma_{10} - \sigma_{13} - \sigma_{17} + \sigma_5 + \sigma_7 - \sigma_4 + \sigma_3) + \sin(\theta_1)\,(\\ 0 & \sin(\theta_1) \\ 0 & -\cos(\theta_1) \\ 1 & 0 \end{matrix}$$

where

$$\sigma_1 = \frac{31\cos(\theta_3)\sin(\theta_1)\sin(\theta_2)}{50}$$

$$\sigma_2 = \frac{31\cos(\theta_2)\sin(\theta_1)\sin(\theta_3)}{50}$$

$$\sigma_3 = \frac{31\cos(\theta_1)\cos(\theta_3)\sin(\theta_2)}{50}$$

$$\sigma_4 = \frac{31\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)}{50}$$

$$\sigma_5 = \frac{3\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)}{25}$$

$$\sigma_6 = \frac{3\cos(\theta_2)\cos(\theta_3)\sin(\theta_1)}{25}$$

$$\sigma_7 = \frac{3\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)}{25}$$

$$\sigma_8 = \frac{3\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)}{25}$$

$$\sigma_9 = \frac{3\cos(\theta_2)\sin(\theta_1)}{5}$$

$$\sigma_{10} = \frac{3\cos(\theta_1)\cos(\theta_2)}{5}$$

$$\sigma_{11} = \sin(\theta_5)\,\sigma_{23} - \cos(\theta_5)\,(\sigma_{31} - \sigma_{30})$$

$$\sigma_{12} = \frac{23\sin(\theta_5)\,\sigma_{23}}{200}$$

$$\sigma_{13} = \frac{23\sin(\theta_5)\,\sigma_{32}}{200}$$

b) Discuss the singularities for position and orientation kinematics.

If rank(J) ≤ 6 the manuplator is singular. However, in a 6 DOF manuplator, it is not easy to observe the rank of the matrix. So, there is a way that is called "decoupling singularities". They can be seperated as arm and wrist singularities.

$J = [J_p \; J_o]$

J_p: Position part

J_o: Orientation part

$J_o = \begin{bmatrix} z_3 x(o_6 - o_3) & z_4 x(o_6 - o_4) & z_5 x(o_6 - o_5) \\ z_3 & z_4 & z_5 \end{bmatrix}$ Normally, $o_3 = o_4 = o_5 = o_6 = o$ for the manuplators without offsets.

In our situation, $J_o = J_{12}$ and it is not equal to zero.

$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

$J_{22} = \begin{bmatrix} z_3 & z_4 & z_5 \end{bmatrix}$ To determine wrist singularities, $\det|J_{22}|$ should be examined.

```
J_22=[z(:,3) z(:,4) z(:,5)]
```

J_22 =

$$\begin{pmatrix} -\sin(\theta_1) & \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) - \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) & \sin(\theta_4) \; (\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\,\text{co} \\ \cos(\theta_1) & \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) & \cos(\theta_1)\cos(\theta_4) + \sin(\theta_4) \; (\sin(\theta_1)\sin(\theta_2)\,\text{si} \\ 0 & -\cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) & -\sin(\theta_4) \; (\cos(\theta_2)\sin(\theta_3) - \end{pmatrix}$$

```
rank(J_22)
```

ans = 3

It can be seen that J_{22} has full rank, so there is no wrist singularity. It is also can be proven because in the rist there is 120mm offset in the 3rd joint.

```
J_11=[cross(z(:,1),(o(:,4)-o(:,1))) cross(z(:,2),(o(:,4)-o(:,2))) cross(z(:,3),(o(:,4)
det(J_11);
det_j_11=simplify(det(J_11))
```

det_j_11 =

$$\frac{27\sin(\theta_2)\cos(\theta_3)^2}{3125} - \frac{27\cos(\theta_2)\sin(\theta_3)\cos(\theta_3)}{3125} - \frac{27\sin(\theta_2)}{3125} - \frac{81\sin(\theta_3)}{6250} - \frac{27\cos(\theta_2)\sin(\theta_3)}{625}$$

```
eqn=det_j_11==0;
arm_singularity=solve(eqn,theta);
arm_singularity.theta2
```

ans =

$$\begin{pmatrix} -\log\left(-\frac{3\,e^{2\,i}\sin(1)+\sigma_1}{\sigma_2}\right) i \\ -\log\left(-\frac{3\,e^{2\,i}\sin(1)-\sigma_1}{\sigma_2}\right) i \\ 0 \end{pmatrix}$$

where

$$\sigma_1 = e^{\frac{1}{2}i}\sqrt{26\,e^{i} - 5\,e^{2\,i} - 52\,e^{3\,i} - 5\,e^{4\,i} + 26\,e^{5\,i} + 5\,e^{6\,i} + 9\,e^{3\,i}\sin(1)^2 + 5}$$

$$\sigma_2 = 5\,e^{i}\,i - e^{2\,i}\,i - 5\,e^{3\,i}\,i + i$$

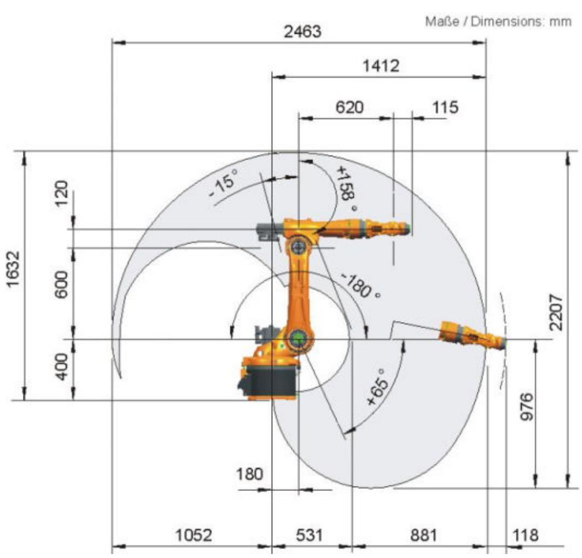
```
arm_singularity.theta3
```

ans =

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

So, it can be seen that there are 4 singularity cases however in real life θ_2 should be a real number so, there is no sinuglarity for that joint and for θ_3 , it is singular 57.3 degree or zero degrees which are not in the range of motion for this manuplator.

Our manuplator has offsets in its 3rd, 4th and 6th joints, it can also be seen in its DH table. As a result, this KUKA manuplator is not singular corresponding to its configurations and offsets are added in order to prevent singularity.




```
det(J)==0
```

ans =

$$3\ \left(\cos(\theta_1)^2+\sin(\theta_1)^2\right)\ \left(\ 186\sin(\theta_5)\cos(\theta_1)^4\cos(\theta_2)^5\cos(\theta_3)^4\cos(\theta_4)^2+186\sin(\theta_5)\cos(\theta_1)^4\cos(\theta_2)^5\cos\right.$$