BLG 354E

SIGNALS AND SYSTEMS FOR COMPUTER ENGINEERING

HOMEWORK I

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Question 1

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1) Taylor Series Expansion:

$$f(x) = e^x = \sum_{n=0}^{\infty} 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \dots = \frac{x^n}{n!}$$

$$e^{j\theta} = \sum_{n=0}^{\infty} \frac{(j\theta)^n}{n!} = 1 + j\theta - \frac{\theta^2}{2!} - \frac{\theta^3 j}{3!} \dots$$

Seperate Reel & Im parts:

$$re\{e^{j\theta}\} \Rightarrow \left\{ 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} \dots \right\}$$ Maclaurin exp of $\Rightarrow \cos x$

$$im\{e^{j\theta}\} \Rightarrow \left\{ \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \dots \right\}$$ Maclaurin exp of $\Rightarrow \sin x$

Recall $\frac{d}{d\theta} \cos\theta = -\sin\theta \quad \frac{d}{d\theta} \sin\theta = \cos\theta$

$$\cos\theta = \sum_{n=0}^{n=\infty} \frac{f^{(n)}(0)\, \theta^n}{n!} = \frac{(-1)^{n/2}}{n!} \theta^n$$

$$\cos\theta + j\sin\theta = 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} \dots = e^{jx}$$

Question 2

a. Cube roots of $z = -8\sqrt{2} + j8\sqrt{2}$

$$r = \sqrt{\left(\left(-8 * \sqrt{(2)}\right)^2 + \left(8 * \sqrt{(2)}\right)^2\right)} = 16$$

$$\tan\theta = -(8\sqrt{2})/(8\sqrt{2}) = -1$$

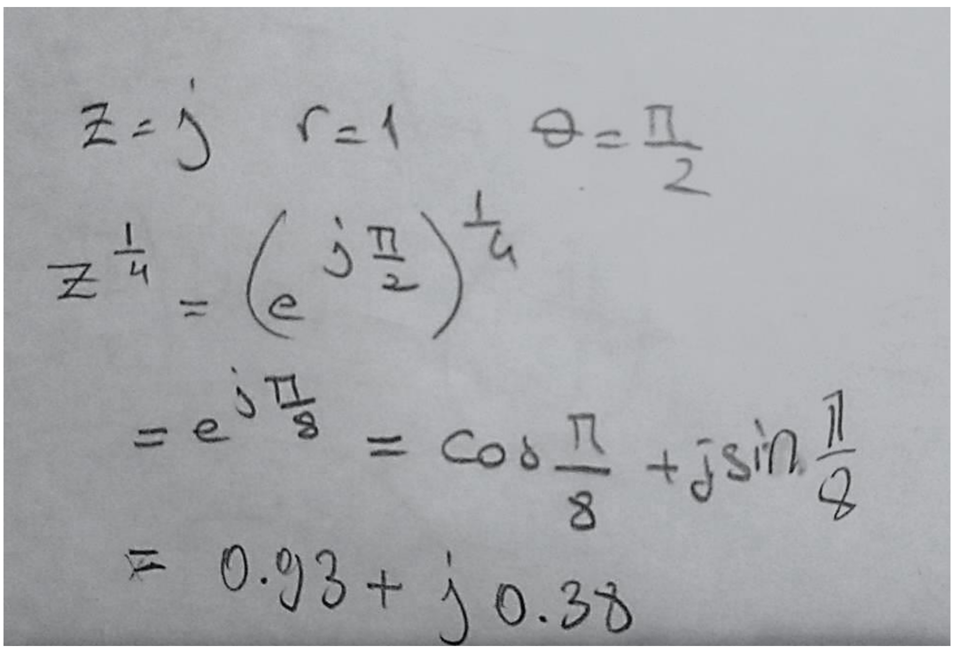
$$\theta = \frac{3 * \pi}{4}$$

$$\sqrt[3]{16} * \sqrt[3]{e^{\frac{j3*\pi}{4}}} = \sqrt[3]{16} e^{j\pi/4}$$

$$= \sqrt[3]{2} * 2 * \left(\cos\left(\frac{\pi}{4}\right) + j * \sin\left(\frac{\pi}{4}\right)\right) = 2 * \sqrt[3]{2} * \left(\frac{1}{\sqrt{2}} + j * \frac{1}{\sqrt{2}}\right)$$

$$= 2^{\frac{5}{6}} * (1 + j)$$

b. Fourth roots of *z* = *j*

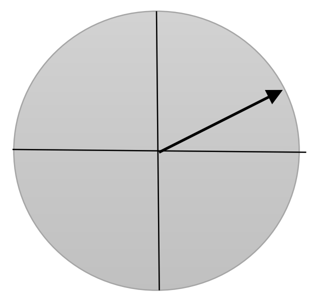


$z = j \quad r = 1 \quad \theta = \frac{\pi}{2}$

$z^{\frac{1}{4}} = \left(e^{j\frac{\pi}{2}}\right)^{\frac{1}{4}}$

$= e^{j\frac{\pi}{8}} = \cos\frac{\pi}{8} + j\sin\frac{\pi}{8}$

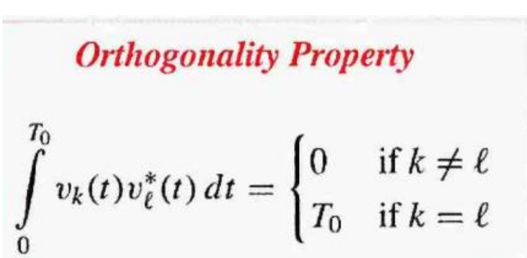
$= 0.93 + j0.38$



Representation of $e^{j*\left(\frac{\pi}{8}\right)}$ is given on unit circle of complex plane, angle between the vector and the x plane is equal to $= \frac{\pi}{8}$

Question 3

For given signals to be orthogonal, dot product of two signals must be equal to zero.



Orthogonality Property

$$\int_0^{T_0} v_k(t) v_\ell^*(t)\, dt = \begin{cases} 0 & \text{if } k \neq \ell \\ T_0 & \text{if } k = \ell \end{cases}$$

(Taken from book, p. 49)

(a) sin(2πnft) and sin(2πmft) on -L ≤ t ≤ L; n ≠m and m; n integer

$< sin(2\pi fnt)\,, \sin(2\pi fmt) > = 0$

$= 1/2 \int_{-L}^{L} (\cos(2\pi f(n-m)t) - \cos(2\pi f(m+n)t))dt$

$= 1/2 \int_{-L}^{L} \cos(2\ \pi f(n-m)t)dt - 1/2 \int_{-L}^{L} \cos(2\ \pi f(m+n)t)dt$

$= \frac{1}{2} * (\frac{\sin(2\pi f(n-m)L - si\,n(2\pi f(m-n)(-L)))}{2\pi f(n-m)} - \frac{1}{2} * (\frac{\sin(2\pi f(n-m)L - si\,n(2\pi f(m+n)(-L)))}{2\pi f(n+m)}$

$\sin(k * \pi) = 0$ for every integer value of k. Second integral becomes 0, since sin(2π(m+n)) = 0.

For the first integral, we know that n ≠ m, and their difference is some integer value, (denominator does not become 0 for first equation) sum of the solution is zero, therefore sin(2πnft) and sin(2πmft) are orthogonal.

(b) *cos(2π*nft*)* and *cos(2π*mft*)* on -L ≤ t ≤ L; n ≠ m and m; n integer

$< cos(2\pi fnt)\,, \cos(2\pi fmt) > = 0$

$= 1/2 \int_{-L}^{L} (\cos(2\pi f(m+n)t) + \cos(2\pi f(n-m)t))dt$

$= 1/2 \int_{-L}^{L} \cos(2\ \pi f(m+n)t)dt + 1/2 \int_{-L}^{L} \cos(2\ \pi f(n-m)t)dt$

$= \frac{1}{2} * (\frac{\sin(2\pi f(n+m)L - si\,n(2\pi f(n+m)(-L)))}{2\pi f(n+m)} + \frac{1}{2} * (\frac{\sin(2\pi f(n-m)L - si\,n(2\pi f(n-m)(-L)))}{2\pi f(n-m)}$

Similarly to part a, integrals become 0, which proves that cos(2πnft) and cos(2πmft) are orthogonal in given conditions.

(c) *sin(2π*nft*)* and *cos(2π*mft*)* on -L ≤ t ≤ L; m and n integer

$< sin(2\pi fnt)\,, \cos(2\pi fmt) > = 0$

$= 1/2 \int_{-L}^{L} (\sin(2\pi f(m+n)t) + \sin(2\pi f(n-m)t))dt$

$= 1/2 \int_{-L}^{L} \sin(2\ \pi f(m+n)t)dt + 1/2 \int_{-L}^{L} \sin(2\ \pi f(n-m)t)dt$

$= \frac{1}{2} * (\frac{\cos(2\pi f(n+m)L - cos(2\pi f(n+m)(-L)))}{2\pi f(n+m)} + \frac{1}{2} * (\frac{\cos(2\pi f(n-m)L - \cos(2\pi f(n-m)(-L)))}{2\pi f(n-m)}$

Cos(x) =cos(-x) , even property of cos function, therefore subtraction in numerator results in zero, as long as n≠0 i7s provided, sin(2πnft) and cos(2πmft) are orthogonal.

(d) ej2πnft/L and ej2πmft/L on 0 ≤ t ≤ L; n ≠m and m; n integer

$$\left\langle e^{\frac{j2\pi nft}{L}}, e^{\frac{j2\pi mft}{L}} \right\rangle = 0$$

$$= \int_0^L e^{\frac{j2\pi nft}{L}} \underbrace{e^{-j\frac{2\pi mft}{L}}}_{\text{complex conjugate of second signal.}} dt$$

$$= \int_0^L e^{\frac{j2\pi f(n-m)t}{L}} dt = \left. \frac{e^{j2\pi(n-m)ft/L}}{j2\pi(n-m)f/L} \right|_0^L = \frac{e^{j2\pi(n-m)f} \overbrace{-1}^{e^0=1}}{j2\pi(n-m)f/L}$$

$m \neq n$ is given, which means that the solution is not equal to the period. According to zero integral property, $e^{j2\pi(n-m)f}$ will be some integer multiple of $e^{j2\pi f}$, $(1)^{n-m} = 1$. $\frac{1-1}{j2\pi(n-m)f/L} = 0$; $e^{j2\pi nft/L}$, $e^{j2\pi mft/L}$ are therefore, orthogonal.

Question 4

a) $\cos^2 x = \dfrac{\cos 2x + 1}{2} \Rightarrow \cos^2(2\pi t) = \dfrac{\cos(4\pi t) + 1}{2}$

cos func is periodic w/ period "2π",

$\cos^2(2\pi(t+T_0)) = \cos^2(2\pi t)$

$$\frac{\cos(4\pi(t+T_0))+1}{2} = \frac{\cos(4\pi t)+1}{2}$$

$\cos 4\pi(t+T_0) = \cos(4\pi t)$ if T_0 is chosen as $\frac{1}{2} \Rightarrow t + 2\pi$.

Also, $\cos(4\pi t) = \cos(2\pi f_0 t)$, $f_0 = 2\,Hz$ $\quad \frac{1}{T_0} = f_0$, $T_0 = \frac{1}{2}$ sec.

b) $x(t) = x(t+T_0)$, $e^{j\pi t} = e^{j\pi t} e^{j\pi T_0}$

$e^{j\pi T_0} = 1 = e^{j2\pi k} \Rightarrow$ (from unit circle in complex plane)

where k is an integer $1, 2, 3, \ldots$

To find fundamental T, set $k=1$, $T_0 = 2\pi$.

c) $1 + 5\cos\left(2257\pi + \frac{\pi}{4}\right) + 2\cos\left(2440\pi + \frac{3\pi}{2}\right)$

$GCD(2257, 2440) = 61$, write cos functions as

$$1 + 5\cos\underbrace{\left(\frac{61 \times 37}{2} 2\pi + \frac{\pi}{4}\right)}_{\text{37th harmonic}} + 2\cos\underbrace{\left(\frac{61 \times 40}{2} 2\pi + \frac{3\pi}{2}\right)}_{\text{40th harmonic.}}$$

$f_0 = \frac{61}{2}\,Hz \quad T_0 = \frac{2}{61}$

All 3 functions are periodic.