

CENG424 - Homework 1

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1. (a)

A	B	$A \rightarrow B$	$\neg(A \wedge \neg B)$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

As truth table shows, $A \rightarrow B \equiv \neg(A \wedge \neg B)$

(b)

A	B	$A \leftrightarrow B$	$(\neg A \vee B) \wedge (\neg B \vee A)$
F	F	T	T
F	T	F	F
T	F	F	F
T	T	T	T

As truth table shows, $A \leftrightarrow B \equiv (\neg A \vee B) \wedge (\neg B \vee A)$

(c)

A	B	$A \rightarrow (\neg A \rightarrow B)$	1
F	F	T	T
F	T	T	T
T	F	T	T
T	T	T	T

As truth table shows, $A \rightarrow (\neg A \rightarrow B) \equiv 1$

(d)

A	B	C	$(A \vee \neg B) \rightarrow C$	$(\neg A \wedge B) \vee C$
F	F	F	F	F
F	F	T	T	T
F	T	F	T	T
F	T	T	T	T
T	F	F	F	F
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

As truth table shows, $(A \vee \neg B) \rightarrow C \equiv (\neg A \wedge B) \vee C$

2. (a) $A \wedge (\neg A \rightarrow A) \equiv A \wedge (\neg \neg A \vee A) \equiv A \wedge (A \vee A) \equiv A \wedge A \equiv A$

(b) $\equiv (A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$

$\equiv (A \rightarrow B) \rightarrow ((\neg A \vee \neg B) \rightarrow \neg A)$

$\equiv (A \rightarrow B) \rightarrow (\neg(\neg A \vee \neg B) \vee \neg A)$

$\equiv (A \rightarrow B) \rightarrow (\neg(\neg A \vee \neg B) \vee \neg A)$

$$\begin{aligned}
&\equiv (A \rightarrow B) \rightarrow (\neg\neg A \wedge \neg\neg B) \vee \neg A) \\
&\equiv (A \rightarrow B) \rightarrow ((A \wedge B) \vee \neg A) \\
&\equiv (A \rightarrow B) \rightarrow ((A \vee \neg A) \wedge (B \vee \neg A)) \\
&\equiv (A \rightarrow B) \rightarrow (T \wedge (B \vee \neg A)) \\
&\equiv (A \rightarrow B) \rightarrow (B \vee \neg A) \equiv (\neg A \vee B) \rightarrow (B \vee \neg A) \equiv \neg(\neg A \vee B) \vee (B \vee \neg A) \\
&\equiv (A \wedge \neg B) \vee (B \vee \neg A) \equiv (A \vee B \vee \neg A) \wedge (\neg B \vee B \vee \neg A) \equiv T \wedge T \equiv T
\end{aligned}$$

$$(c) \equiv (\neg A \vee (B \vee \neg C)) \wedge \neg A \wedge B$$

$$3. F = (\neg A \wedge B) \wedge (\neg(B \wedge C)) \wedge (C \vee D) \wedge (\neg(\neg A \rightarrow D))$$

Since all branches are closed for $F = 1$, F is not satisfiable and the given forms are not mutually consistent.

$$(\neg A \wedge B) \wedge (\neg(B \wedge C)) \wedge (C \vee D) \wedge (\neg(\neg A \rightarrow D)) \equiv 1 \quad a)$$

$$\begin{array}{l}
\downarrow \text{a)} \\
\begin{array}{ll}
(\neg A \wedge B) \equiv 1 & b) \\
(\neg(B \wedge C)) \equiv 1 & c) \\
(C \vee D) \equiv 1 & d) \\
(\neg(\neg A \rightarrow D)) \equiv 1 & e)
\end{array}
\end{array}$$

$$\begin{array}{l}
\downarrow \text{b)} \\
\neg A \equiv 1 \\
B \equiv 1
\end{array}$$

$$\begin{array}{ll}
\swarrow \text{c)} & \searrow \text{c)} \\
\neg C \equiv 1 & \neg B \equiv 1 \\
& \text{closed}
\end{array}$$

$$\begin{array}{ll}
\swarrow \text{d)} & \searrow \text{d)} \\
C \equiv 1 & D \equiv 1 \\
\text{closed} &
\end{array}$$

$$\begin{array}{l}
\downarrow \text{e)} \\
\neg A \equiv 1 \\
\neg D \equiv 1 \\
\text{closed}
\end{array}$$