CENG424 - Homework 2

Elif Ecem Ümütlü

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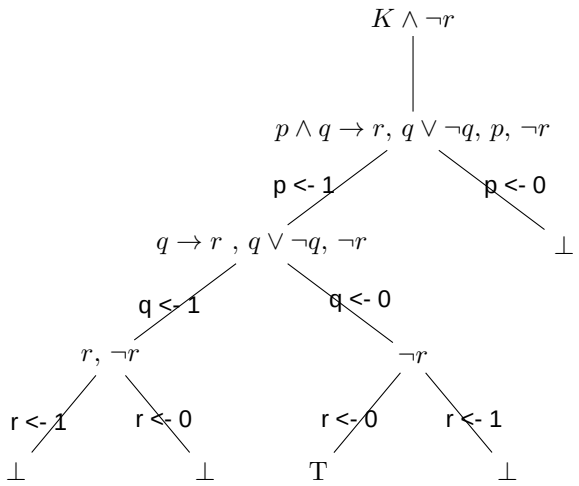
1. K (the set of the given premises) entails the sentence r if and only if
   $K \to r$ is satisfiable.
   $\equiv \neg K \vee r$ is satisfiable. (T)
   $\equiv \neg(\neg K \vee r)$ is unsatisfiable. ($\perp$)
   $\equiv K \wedge \neg r$ is unsatisfiable. ($\perp$)

   If there are no satisfied end, then $K \wedge \neg r$ is unsatisfied. It means K entails r. But in the below case, there is such a satisfied case. Hence, K does not entail r for the following interpretation I $= \{p \leftarrow 1, q \leftarrow 0, r \leftarrow 0\}$



2. 1. $p \to q$ *premise*
   2. $q \to r$ *premise*
   3. $(q \to r) \to (p \to (q \to r)$ II
   4. $p \to (q \to r)$ $MP : 3, 2$
   5. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$ ID
   6. $(p \to q) \to (p \to r)$ $MP : 5, 4$
   7. $p \to r$ $MP : 6, 1$
   8. $(p \to r) \to ((p \to \neg r) \to \neg p)$ CR
   9. $(p \to \neg r) \to \neg p$ $MP : 8, 7$

3. 1. $\neg\neg p$ *premise*
   2. $\neg\neg p \to (\neg p \to \neg\neg p)$ II
   3. $\neg p \to \neg\neg p$ $MP : 2, 1$
   4. $(\neg p \to \neg p) \to ((\neg p \to \neg\neg p) \to p)$ CR
   5. $(\neg p \to (\neg\neg p \to \neg p)) \to (\neg p \to \neg\neg p) \to (\neg p \to \neg p)$ ID

6. $\neg p \to (\neg\neg p \to \neg p)$ II
7. $(\neg p \to \neg\neg p) \to (\neg p \to \neg p)$ $MP : 6, 5$
8. $\neg p \to \neg p$ $MP : 7, 3$
9. $(\neg p \to \neg\neg p) \to p$ $MP : 4, 8$
10. p MP:9,3

4. If negation of the sentence results in {}, then the sentence is valid.

$\neg((p \vee q \to r) \to (p \to (q \to r)))$

I	$\neg(\neg(\neg(p \vee q) \vee r) \vee (\neg p \vee (\neg q \vee r)))$	
N	$\neg(\neg\neg(p \vee q) \wedge \neg r) \vee (\neg p \vee (\neg q \vee r)))$	
N	$\neg((p \vee q) \wedge \neg r) \vee (\neg p \vee (\neg q \vee r)))$	
N	$(\neg(p \vee q) \vee \neg\neg r) \wedge \neg(\neg p \vee (\neg q \vee r))$	
N	$((\neg p \wedge \neg q) \vee r) \wedge (\neg\neg p \wedge \neg(\neg q \vee r))$	
N	$((\neg p \wedge \neg q) \vee r) \wedge (p \wedge (q \wedge \neg r))$	
D	$((r \vee \neg p) \wedge (r \vee \neg q)) \wedge p \wedge q \wedge \neg r$	
O	1. $\{r, \neg p\}$	
-	2. $\{r, \neg q\}$	
-	3. $\{p\}$	
-	4. $\{q\}$	
-	5. $\{\neg r\}$	
-	6. $\{r\}$	1, 3
-	7. $\{\}$	5,6

5. The set of clauses $\Delta =$
$\{\{\neg p, q, s\},$
$\{p, s, t\},$
$\{p, s, \neg t\},$
$\{p, \neg s, \neg t\},$
$\{p, \neg s, t\},$
$\{p, q, \neg s\},$
$\{p, \neg q, s\}\}$

Interpretation I = {} at the beginning.

a. Split on p

$\Delta = \{\{q, s\}\}$
$I = \{p\}$

b. Split on q
$\Delta = \{\}$ SATISFIED
$I = \{p, q\}$

This set of clauses is satisfiable.