Agenda-Setting with Legislative Precommitments

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Abstract

I consider an agenda setting environment where the voters commit upfront to the reforms they are willing to pass, and the agenda setter chooses from among the passable reforms or the status quo. I first characterize the outcomes that emerge in subgame perfect equilibria of this game with majoritarian voting rules. Motivated by the weak predictive power of subgame perfection for this game, I consider a refinement to account for the possibility of coalitional deviations. Compared to the predictions of standard models, the agenda setter's power is significantly reduced in this game, especially in the presence of coalitions and with simple majority rule.

1 Introduction

Agenda setting power in politics is of great importance but as to how one sets the agenda is not well understood. For instance, the majority leader is the agenda setter of the U.S. Senate, whereas the Senators are the voters on issues brought to the floor by the majority leader. The exact nature of the agenda setting process in the Senate is not regulated, and how it is pursued depends on the choices of the majority leader.

In standard models, the agenda setter observes the status quo policy and the ideal policies of the voters, and proposes a reform. The voters then vote in favor of or against the reform. If the reform gets the required support from the legislation, it is

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chosen; otherwise, the status quo policy remains in effect. In the equilibrium, voters support a reform if it is preferred to the status quo based on the distance from their ideal policies. The agenda setter can perfectly anticipate the support each possible reform would get, and proposes her favorite policy among the ones that would pass.

Expecting a legislator to approve a reform policy simply because it is preferred to the status quo relative to the ideal policy ignores the commitments that legislators have. A voter might commit to restricting the policies he would approve based on his own views or his promises to his constituents, or he might do so to stay within his party's line or because of a quid pro quo arrangement with another voter.

When the legislators possibly commit to restricting the policies they would approve, the agenda setter would benefit from finding out these restrictions before proposing a reform. Then we can imagine an alternative model where the voters restrict the set of policies they are willing to pass first, and then the agenda setter proposes a reform after observing the restrictions of the voters. My paper investigates such a setting.

In this paper, I consider the interaction of an agenda setter with a group of legislators. There exists a status quo policy, from which each legislator and the agenda setter would accrue payoffs if it were selected. The agenda setter has the option to propose a reform policy. But before she proposes the reform policy, each legislator announces the set of reforms that he is willing to support. Effectively, the agenda setter can pass a reform only if q legislators are willing to support it, and otherwise, the status quo policy is selected.

I study this setting in the canonical framework of spatial politics: each policy is a point in a multidimensional policy space, and each legislator and the agenda setter has an ideal point in that policy space. For simplicity, I assume that when a legislator suggests a set of policies that he is willing to support, he can choose any compact and convex set of policies. A leading example of such sets are restricting himself to policies that form a closed neighborhood around his ideal point. Each legislator suggests his restrictions independently.

The paper maintains the assumption that the voters have commitment power: when a policy is included in their restriction set, the voter is required to accept it when proposed; and when a policy is not included in their restriction set, the voter must reject it when proposed. Given this assumption, an actual voting stage is not modeled explicitly after the agenda setter's proposal. After the voters announce their restriction sets simultaneously, the agenda setter can then choose either the status quo policy or any reform policy that is among the restrictions announced by at least q legislators. I allow q to be between $\frac{n+1}{2}$ and n, and focus on unanimity and simple majority as focal cases when presenting the results.

The model above is a simple dynamic game with complete information. I first characterize the set of subgame perfect equilibria (SPE) and show that this is too permissive. With a unanimity rule, the set of SPE outcomes is those that are better than the status quo for each legislator and the agenda setter. With a simple majority rule, the set of SPE outcomes is those that are better than the status quo for only the agenda setter.

Then I refine the equilibrium concept and characterize the set of subgame perfect coalitional equilibria (SPCE). A strategy profile is an SPCE if (i) there exists no coalition of legislators that can profitably deviate from their restriction sets and (ii) the agenda setter chooses the final reform closest to her ideal point. With a unanimity rule, the SPE outcomes that belong in the convex hull of the ideal points of the legislators form the set of SPCE outcomes. With a simple majority rule, the set of SPCE outcomes is the Condorcet Winner in the set of policies that are better than the status quo for the agenda setter.

2 Model

In this section, I describe the agenda setting environment, define the equilibrium concepts, and describe the equilibrium outcomes under a simplifying assumption. I then define a notion of coalitional stability and characterize the stable outcomes under unanimity and simple majority rules.

There are n voters, where n is an odd number, and an agenda setter. Let $N_P = \{1, \dots n\}$ denote the set of voters, and $N = \{0, 1, \dots n\}$ the set of all players where the agenda setter is indexed as 0. The outcome space is given by $\bar{\mathbb{R}}^d \equiv \{\mathbb{R} \bigcup \{\infty\}\}^d$. A group decision is to be made according to the game described below.

Let $\mathcal{M}$ be the set of all compact and convex subsets of $\bar{\mathbb{R}}^d$. Each voter $i \in N_P$ announces a *commitment set* $C_i \in \mathcal{M}$ simultaneously. Let $\tilde{a} \in \bar{\mathbb{R}}^d$ be an exogenous status quo. A *(majoritarian) voting rule*, indexed by the number of votes it requires $q \leqslant n$, is given exogenously at the beginning of the game, and it is common knowledge. The function of the voting rule is to determine how to aggregate the announcements of voters. The voting rule defines a *winning coalition*, a set of voters that can nominate a policy for consideration, and consequently the set of all winning coalitions, $\mathbb{W}$.

One can easily observe that $q = n$ is unanimity and $q = \frac{n+1}{2}$ is simple majority, and the main text focuses on these two voting rules[1]. Under unanimity rule, the unique winning coalition is the set of all the voters, $\mathbb{W}_U = \{N_P\}$. Under simple majority, a winning coalition is any subset of voters with at least half of the legislature, $\mathbb{W}_M = \{N' \subseteq N_P : |N'| \geqslant \frac{n+1}{2}\}$.

Given the announced commitment sets, $\{C_i\}_{i \in N_P}$, I define a set of *feasible outcomes* as $F(C_1, \ldots, C_n) \equiv \bigcup_{N' \in \mathbb{W}} \bigcap_{i \in N'} C_i$, where $\mathbb{W}$ is the set of winning coalitions of voters determined by the voting rule in effect as explained above, and where $N' \subseteq N_P$. I simply refer to the set of feasible outcomes as F when there is no ambiguity. Given the set of feasible outcomes and the outside option, the agenda setter's strategy is the mapping $a : \mathcal{M} \longrightarrow \bar{\mathbb{R}}^d \bigcup \{\tilde{a}\}$, where $a(F(C_1, \ldots, C_n)) \in F \bigcup \{\tilde{a}\}$.

Every player has an *ideal point* $b_i \in \bar{\mathbb{R}}^d$, $i \in \{0, 1, \ldots n\}$. Player i's utility function is $u_i(a) = -d(a, b_i)^2$ for $a \in \bar{\mathbb{R}}^d$, where $d(.,.)$ is the Euclidean distance with $d(a, b) = \sqrt{(a_1 - b_1)^2 + \ldots + (a_d - b_d)^2}$. Quadratic loss function is used for simplicity and without loss of generality as any spatial preferences would work for this analysis.

Given the players' ideal points and the outside option $\tilde{a}$, I define the set of *acceptable outcomes* for a player i , denoted as A_i, as outcomes that give the player i higher utility than the outside option, $A_i \equiv \{x \in \bar{\mathbb{R}}^d \colon u_i(x) \geqslant u_i(\tilde{a})\}$.

A strategy profile in this game is a tuple $(C_1, \ldots, C_n, a(F(C_1, \ldots, C_n)))$ where $a(F(C_1, \ldots, C_n)) \in F(C_1, \ldots, C_n)) \bigcup \{\tilde{a}\}$ is the agenda setter's strategy that takes all voters' restrictions as input and gives a feasible outcome or the outside option as the output. The paper characterizes the policies that can arise in Subgame Perfect Nash Equilibria (SPE) and Subgame Perfect Coalitional Equilibria (SPCE) of this game.

I interpret the results in two cases regarding the outside option to illustrate the intuition behind them. In the first one, Nuclear case, the outside option is the worst outcome for every player, $\tilde{a} = (\infty, \ldots, \infty)$. In the second case, conversely called Non-Nuclear case, the outside option belongs to the multi-dimensional real space, $\tilde{a} \in \bar{\mathbb{R}}^d$. Nuclear scenario is merely a special case of the model and it simplifies the initial analysis of equilibrium outcomes.[2]

[1]The results generalize to $\frac{n+1}{2} < q < n$ in a straightforward fashion. I include the formal statements in the Appendix A.5.

[2]We can simply observe that in the case of Nuclear, set of acceptable outcomes is $A_i(\tilde{a}) = \bar{\mathbb{R}}^d$ for all $i \in N$. When the outside option is the least preferred outcome, the entire outcome space is acceptable for every player.

3 Subgame Perfect Equilibrium Outcomes

In this section I characterize the set of SPE outcomes with each of unanimity and majority voting. The first result states that for a policy to be an SPE outcome with unanimity voting, it has to be acceptable for every member of the legislature.

Definition. $(C_1^*, \ldots, C_n^*, a^*(\cdot))$ is a Subgame Perfect Equilibrium if;

1. The agenda setter is sequentially rational.

$$a^*(\cdot) = a^*(F(C_1, \ldots, C_n)) = \underset{a \in F \bigcup \{\tilde{a}\}}{\arg\max}\, u_0(a)$$

2. $u_i(a^*(F(C_i^*, C_{-i}^*))) \geqslant u_i(a^*(F(C_i, C_{-i}^*)))$ for all $i \in N_P$ and all $C_i \in \mathcal{M}$.

Subgame perfect equilibrium requires the agenda setter to choose her most preferred option among the feasible outcomes and the outside option given any restrictions, and the voters to not have a profitable unilateral deviation. I refer to an outcome $a \in \bar{\mathbb{R}}^d$ as a subgame perfect equilibrium outcome if $(C_1^*, \ldots, C_n^*, a^*(\cdot))$ is an SPE where $a^*(\cdot) = a$.

Proposition 1. The set of SPE outcomes is the set of policies that are acceptable for every player, the agenda setter and the voters, with unanimity voting.

This result has a simple logic. Suppose we are in the Nuclear case. Consider any $a \in \bar{\mathbb{R}}^d$, and suppose that every voter i chooses $C_i = \{a\}$. Because the agenda setter necessarily prefers a to $\tilde{a}$, she will choose a. If a voter considers deviating to C_i' there are two possibilities: either $a \in C_i'$ in which case the outcome is unchanged or $a \notin C_i'$ in which case the outside option is selected (which offers a lower payoff to voter i) under unanimity and the outcome is unchanged under majority. Thus, there is no strictly profitable deviation. I then conclude that any point in the policy space can be an SPE outcome in the Nuclear case since any any of those points are acceptable to all the players.

In the general statement, I say the set of SPE outcomes is the intersection of every player's set of acceptable outcomes. There are two points to highlight. The first point is that the player who has the closest ideal point to the outside option has the smallest set of acceptable outcomes. It follows that this player has bargaining power in a sense because the bounds of the set of equilibrium outcomes depend on her ideal point. The second point is that when the outside option belongs to the convex hull of voters' points, the set of SPE outcomes collapses to the outside option. There exists no point in the outcome space that every voter prefers more than the outside option which is a point in the convex hull.[3]

Proposition 2. The set of SPE outcomes is the set of acceptable policies for the agenda setter with majority voting.

Unlike the unanimity, there are more than one winning coalition under majority voting rule. It is reasonable to expect that the SPE outcomes would be those that belong to the set of acceptable outcomes of the members of each winning coalition as well as to the agenda setter's set of acceptable outcomes. However, I show that the set of SPE outcomes coincides with the agenda setter's set of acceptable outcomes. We can easily see why this is the case. Take any point in the agenda setter's set of acceptable outcomes. Suppose every voter announces only that point in their restriction sets. Then, if a voter excludes that point from their restriction set, that point stays in the set of feasible outcomes since there is still a majority including it in their restriction sets. Thus, no voter can unilaterally and profitably deviate to another strategy. Conversely, if a point outside of the agenda setter's set of acceptable outcomes is proposed by the voters, the agenda setter can simply choose the outside option as the final policy.

It is natural to consider whether restricting attention to the undominated strategies in this case might refine the set of SPE outcomes: the answer is negative. We can see that none of the strategies used in proving Proposition 2 weakly dominated. For any $C_i = \{a\} \in A_0$, even if $a \notin A_i$, we can imagine the other voters' announcements such that announcing $C_i = \{a\}$ prevents some b from becoming the final policy where $u_i(b) < u_i(a)$. In fact, through the same argument, we realize that any $C_i = \{a\} \in \bar{\mathbb{R}}^d$ is an undominated strategy. The only way to refine the set of SPE outcomes with the help of undominated strategies is when the announcements are restricted to be singletons. In that case, any policy that is acceptable for the voter and the agenda setter belongs to the voter's set of undominated strategies.

Remark 1. When the restriction sets are singleton for every voter, intersection of a

[3]I formalize the second observation in Proposition 3 when I characterize the set of SPCE outcomes with unanimity voting.

voter's set of acceptable policies and the agenda setter's acceptable policies except the status quo is the set of undominated strategies for each voter. Then, the set of outcomes of SPE in undominated strategies is the set of policies that are acceptable for the agenda setter and at least a majority of the voters.

The stark difference between the unanimity and majority results can be attributed to the difference in the set of veto players in each case. Under unanimity voting, every player is a veto player. The agenda setter can always select the outside option if the set of feasible outcomes does not intersect with her acceptable set. Similarly, any voter can ensure that an unacceptable outcome is not selected by announcing the empty set. By contrast, under majority voting, only the agenda setter can veto a policy by choosing the outside option. In this case, the voters do not have veto power as they cannot always prevent a policy. Motivated by this imbalance between voting rules and the weak predictability power of SPE, next I refine the equilibrium concept to take coalitional deviations into account.

4 What Outcomes are Robust to Coalitional Deviations?

Subgame perfect coalitional equilibrium (SPCE) introduces a notion of coalitional stability. It requires that no possible coalition of voters can profitably deviate to another strategy profile. I consider any possible deviations by any coalition since one voter can change the set of feasible outcomes by simply including or excluding an alternative which might give multiple voters higher payoffs.

Definition. $(A_1^*, \ldots, A_n^*, a^*(\cdot))$ is a Subgame Perfect Coalitional Equilibria if;

1. The agenda setter is sequentially rational.

$$a^*(\cdot) = a^*(F(A_1, \ldots, A_n)) = \underset{a \in F \bigcup \{\tilde{a}\}}{\arg\max}\, u_0(a)$$

2. There does not exist $C \subseteq N_P$ such that for every $i \in C$, there exists $A_i \in \mathcal{M}$ with

   - $u_i(a^*(F(A_C, A_{-C}^*))) \geqslant u_i(a^*(F(A_C^*, A_{-C}^*)))$ for all $i \in C$;

- $u_j(a^*(F(A_C, A^*_{-C}))) > u_j(a^*(F(A^*_C, A^*_{-C})))$ for some $j \in C$,

where $A_C = \{A_i\}_{i \in C}$ and $A^*_{-C} = \{A^*_i\}_{i \in N_P \setminus C}$.

An outcome $a \in \bar{\mathbb{R}}^d$ is a subgame perfect coalitional equilibrium outcome if $(A^*_1, \ldots, A^*_n, a^*(\cdot))$ is an SPCE where $a^*(\cdot) = a$.

We can first consider an interpretation of subgame perfect coalitional equilibrium as a better known object: the core. As described in the agenda setter's strategy, we can see that once the restriction sets are announced, what she will choose as the final policy gets determined. If we reduce the game to the first stage where the voters announce their restriction sets given the agenda setter's choice in the second stage, we can focus on the core of this reduced game, and it corresponds to the SPCE outcomes of the game. This simple observation allows the results to be compared to the core outcomes of agenda setting models, voting games, and common agency models.

Let B denote the set of ideal points of voters, $B = \{b_1, \ldots, b_n\}$. The *convex hull* of voters' ideal points, $co(B)$, is the set of points in the outcome space that can be represented as a convex combination of the ideal points. The first answer to the question this section poses is that the set of SPCE outcomes under unanimity is the intersection of convex hull of voters' ideal points and every player's acceptable sets of outcomes.

Proposition 3. The set of SPCE outcomes with unanimity voting is the set of acceptable policies for every player that are in the convex hull of the voters' ideal points, if such a policy exists. Otherwise, the set of SPCE outcomes is the set of acceptable policies for every player that are closest to the convex hull of the voters' ideal points. Formally, the set of SPCE outcomes with unanimity is equal to

$$\begin{cases} \bigcap_{i=0}^{n} T_i \bigcap co(B) & \text{if } \bigcap_{i=0}^{n} T_i \bigcap co(B) \neq \emptyset \\ \arg\min\limits_{x \in \bigcap_{i=0}^{n} T_i} d(x, co(B)) & \text{otherwise} \end{cases}$$

First part of this result uses two simple insights. First insight is, when we compare two points in the convex hull, one cannot be closer to all the ideal points of voters than the other. Second insight is, for any point outside of the convex hull, we can find a point on the boundary of the convex hull such that it is closer to each voter's ideal point. Second part of the result builds on the first part, and indicates that even if the voters cannot achieve a policy in their convex hull, there is a unique "second-best" alternative they can obtain.

In the case of majority rule, I make a simple observation that the set of SPCE outcomes is equal to the Condorcet winner in T_0 when the set of voters is N_P with ideal points $B = \{b_1, b_2, \dots, b_n\}$. Then I utilize the characterization of Condorcet winner from the multi-dimensional voting games literature. Note that an outcome is a *Condorcet winner* if it is preferred to any other outcome by a majority in a pairwise comparison.

Theorem 1. With majority voting, the set of SPCE outcomes is equal to the set of Condorcet winners from among the agenda setter's acceptable outcomes. [4]

Proof. First of all, when there is no Condorcet winner in the set of acceptable policies for the agenda setter, it is clear that the set of SPCE outcomes is an empty set. For any candidate policy in T_0, there is always a majority of voters who prefer another policy over it and they are able to deviate to that other policy.

Now suppose a Condorcet winner in the agenda setter's acceptable policies exists. Let $x \in \bar{\mathbb{R}}^d$ be a subgame perfect coalitional equilibrium outcome. Let x^* be the Condorcet winner in the agenda setter's set of acceptable policies. Suppose $x \neq x^*$. Then, there is a majority of voters $N' \subseteq N_P$ such that, $u_i(x^*) \geqslant u_i(x)$ for all $i \in N'$ and $u_j(x^*) > u_j(x)$ for some $j \in N'$. But since N' is a majority, it is a winning coalition and its members can deviate to $A'_i = \{x^*\}$ for all $i \in N'$ so that $F = \{x^*\}$. As x^* is an acceptable policy for the agenda setter, by the sequential rationality, $a^*(F(A'_{N'}, A^*_{-N'})) = \{x^*\}$. Thus, x cannot be an SPCE outcome.

Let $x \in \bar{\mathbb{R}}^d$ be the Condorcet winner when the outcomes are restricted to T_0. Then, for any $y \neq x$, there is no $N' \subseteq \mathcal{W}$ such that $u_i(y) \geqslant u_i(x)$ for all $i \in N'$ and $u_j(y) > u_j(x)$ for some $j \in N'$. Consider the strategy profile $A_i = \{x\}$ for all $i \in N_P$. Then, $F(A_1, \dots, A_n) = \{x\}$ and by the sequential rationality of the agenda setter, $a^*(F(A_1, \dots, A_n)) = \{x\}$. Any deviation that changes the outcome is not profitable for any winning coalition. Thus, x is a subgame perfect coalitional equilibrium outcome. $\square$

The idea behind the equivalence of the set of SPCE outcomes and the set of Condorcet winner is simple: If there is no profitable deviation from a policy by

[4] Set of Condorcet winners is either empty or a singleton under this paper's definition.

any coalitions (which is acceptable for the agenda setter), and in particular by any majority of voters, then that policy is a Condorcet Winner; and conversely, if a policy is preferred to every other policy by a majority and is acceptable for the agenda setter, then there cannot be any profitable deviations from it by any coalitions and it is indeed an SPCE outcome.[5] When the agenda setter's ideal point is such that every voter's ideal point lies in her acceptable set of policies, then the standard characterization of Condorcet winner in multidimensional spaces gives us the set of SPCE outcomes. On the other hand, when the agenda setter's acceptable set does not include every voter's ideal point, the Condorcet winner among the policies in that set, which is the set of SPCE outcomes, is not as cleanly characterized.

For the first case where the agenda setter's acceptable outcomes include every voter's ideal point, the conditions that characterize the Condorcet winner are well established in the political economy literature (Plott (1967); Davis et al. (1972); McKelvey & Wendell (1976))[6]. Utilizing these well-known results, I present the conditions for when the set of SPCE outcomes is nonempty and for when it is not.

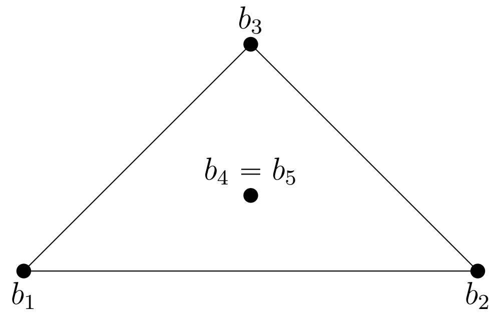
Plott (1967) is among the first to study the core of voting games in multi-dimensional outcome spaces. The notion of core in voting games corresponds to the subgame perfect coalitional equilibrium of this paper. While the analysis for the core is simple in one-dimensional setup, non-emptiness of the core in multi-dimensional policy spaces becomes complex and closely related to relative positions of ideal points of voters.

Set of voters' ideal points satisfies *radial symmetry* around a voter's ideal point b_M if for every line passing through b_M, the number of voters' ideal points on either side of b_M on this line is the same. We say radial symmetry is satisfied if it is satisfied around a voter's ideal point. Radial symmetry is a generalization of a median point to multi-dimensional space. Plott (1967) states that if the ideal points of voters satisfy radial symmetry around a point b_M, then b_M is the Condorcet winner.

While the Plott's condition (or radial symmetry) is an intuitive sufficient condition, it is not necessary for the existence of the Condorcet winner. I illustrate the shortcoming of this condition with a simple example in $\bar{\mathbb{R}}^2$ with five voters. Here, the radial symmetry is not satisfied, however, $b_4 = b_5$ is the Condorcet winner.

[5]It is an abuse of notation to say "any profitable deviation from a policy", which should be read as a profitable deviation to another announcement profile that changes the final policy.

[6]For a broad overview of characterization of Condorcet winner in multidimensional voting games, see Feld & Grofman (1988)



As a necessary counterpart to radial symmetry, there is a weaker condition for the set of ideal points. Set of voters' ideal points satisfy *weak symmetry* if there is a voter's ideal point b_M such that b_M is the median ideal point on every line passing through it. We say weak symmetry is satisfied for b_M, if b_M is the median ideal point on every line passing through it. The difference between having an equal number of ideal points on either side and being the median point on a line makes all the difference for the necessary condition for the Condorcet winner to exist. If a point b_M is the Condorcet winner, then there exists a voter whose ideal point is b_M, and weak symmetry is satisfied for b_M[7].

5 Benchmark

Here I include a brief exposition of the standard agenda setting model and its results. This benchmark model is a minor variation of Romer & Rosenthal (1978) where the agenda setter has an arbitrary ideal point and the policy space is multidimensional.

Consider a legislature composed of an agenda setter and n voters, where n is odd for simplicity. They play a sequential voting game to pick a policy in the multidimensional policy space where there is an exogenous status quo. Both the agenda setter and the voters have ideal points in the policy space and spatial preferences. In the first stage of this game, the agenda setter proposes a policy against the status quo. In the second and final stage, the voters vote either in favor of the proposed policy or against it. The proposed policy passes if q out of n voters vote for it, and the status quo prevails otherwise.

Let the agenda setter's strategy be $a^* \in \bar{\mathbb{R}}^d$. Let a voter's strategy be $a_i \in \{a^*, \tilde{a}\}$, where $\tilde{a}$ is the status quo, so that the each voter faces the same binary choice according

[7]See the proof of Theorem 6' in Feld & Grofman (1988).

to the agenda setter's proposal. If $|i : a_i = a^*| \geqslant q$, the the proposed policy passes, and otherwise, the status quo remains in effect.

Both the agenda setter and the voters have an ideal point $b_i \in \mathbb{R}^d$, $i \in \{0, 1, \ldots n\}$[8]. Player i's utility function is $u_i(a) = -d(a, b_i)^2$ for $a \in \bar{\mathbb{R}}^d$, and again, the set of *acceptable outcomes* for a player i is defined as the outcomes that give the player i higher utility than the outside option, $T_i \equiv \{x \in \bar{\mathbb{R}}^d \colon u_i(x) \geqslant u_i(\tilde{a})\}$.

Next, the definition of the subgame perfect equilibrium needs to be adjusted for this distinct sequential game in order to compare the outcomes that arise in equilibria in both games.

Definition. $(a_1, \ldots, a_n, a^*)$ is a Subgame Perfect Equilibrium if;

1. for each voter, $u_i(a_i) \geqslant u_i(a')$ where $a' = \{a^*, \tilde{a}\} \backslash \{a_i\}$;
2. for the agenda setter, $u_0(a^*) \geqslant u_0(a')$ for any $a' \in \{a \in \bar{\mathbb{R}}^d : |i : u_i(a) \geqslant u_i(\tilde{a})| \geqslant q\}$.

In words, the agenda setter proposes a policy such that it gives her the highest utility compared to all the other policies that can get at least q votes against the status quo; and each voter picks the policy that he prefers between the proposed policy and the status quo. Notice that this definition requires the voters to play undominated strategy of picking the policy they prefer in the binary comparison. In this benchmark model, subgame perfect coalitional equilibrium is equivalent to subgame perfect equilibrium in undominated strategies. As the agenda setter chooses the most preferred policy for herself from the set of passable policies and the voters vote sincerely, there cannot be any coalitional deviations in an SPE.

Theorem. In the standard agenda setting model, the set of SPE outcomes is the best policy for the agenda setter among the policies that q many voters prefer to the status quo.

Now the comparison can be made between the result of the standard agenda setting model and the results of agenda setting with legislative pre-commitments model this paper proposes. Consider the following simple example on $\mathbb{R}$ with three voters where b_i denotes the ideal points, T_i denotes the set of acceptable policies, and the agenda setter is labeled as $i = 0$. In Figure 1, SPE outcomes of the standard

[8] Recall that the agenda setter is labeled as player 0.

model are marked above the line, and both SPE and SPCE outcomes of this paper's model are marked below the line.

The standard model makes precise predictions about the SPE outcome: if only a majority of the votes is required, the agenda setter is able to implement her ideal point since voters 2 and 3 prefer it to the status quo; and if unanimity is required to pass a policy, then the best agenda setter can do is to propose the policy closest to her ideal point that all the voters find acceptable, and the preferences of voter 1 becomes binding.

This paper, on the other hand, characterizes the sets of possible SPE and SPCE outcomes. When a simple majority is required, any point acceptable to the agenda setter can be an SPE outcome since she is the only veto player; and the median voter's ideal point is the unique SPCE outcome as it is the only policy to survive coalitional deviations. When unanimity is required, any policy that is acceptable for every voter and the agenda setter can be an SPE outcome; and the intersection of the set of SPE outcomes and the convex hull of the voters' ideal point emerge as the set of SPCE outcomes as they are robust to coalitional deviations under unanimity.

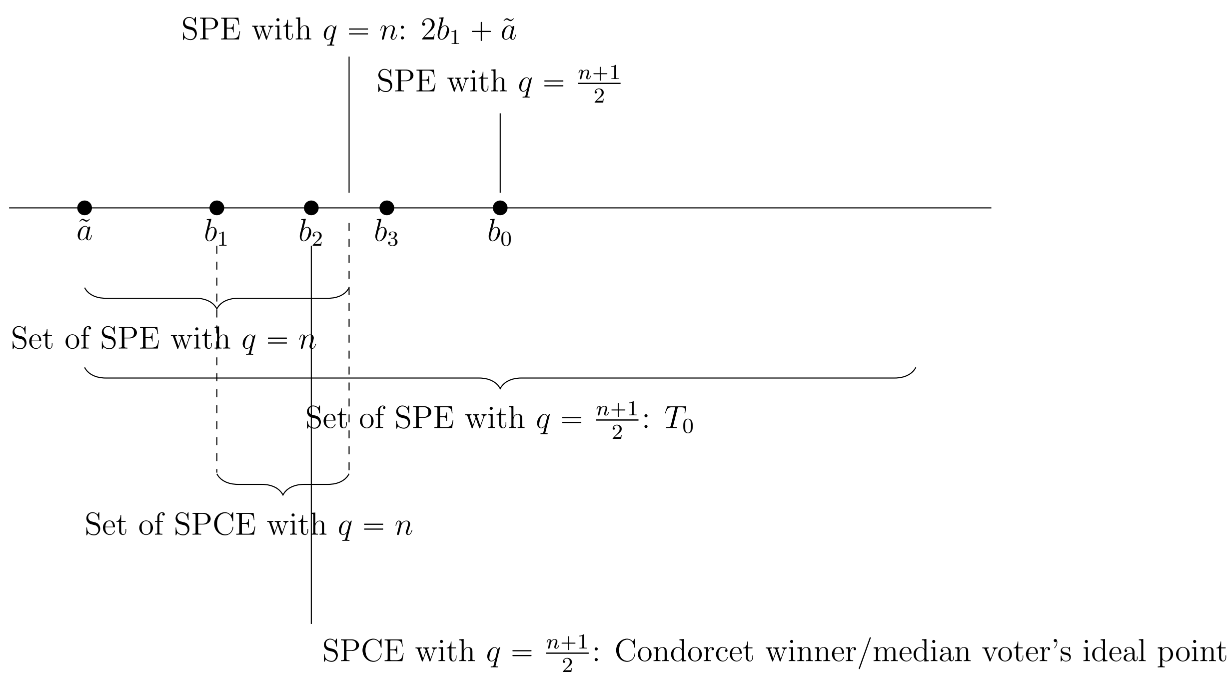


Figure 1: The comparison between the results of Romer & Rosenthal (1978) and this paper.

While this paper's model does not make exact predictions, it captures the agenda setting and policy making as the complex processes they are in the real life institutions. The simplifying idea in the standard model that the agenda setter operates with the assumption that she can pass any policy that is preferred to the status quo by the required number of voters ignores several significant aspects of politics such as communication, reputation building, logrolling, and lobbying efforts.

6 Conclusion

In this paper, I study an agenda setting problem where the voters precommit to policies. A group of voters and an agenda setter seek to choose a policy where they each has an ideal point and there is an outside option. First the voters announce their restriction sets simultaneously, the policies they are willing to vote for, and then the agenda setter chooses a final outcome. I consider this problem with unanimity and majority voting, and I look at the sets of subgame perfect equilibrium (SPE) and subgame perfect coalitional equilibrium (SPCE) outcomes.

I find that the SPE outcome with unanimity voting is any outcome in the set $\bigcap_{i=0}^{n} T_i$, the outcomes that are acceptable for each player, whereas with majority voting it is any outcome in T_0, the outcomes that are acceptable to the agenda setter. In order to make a starker prediction, I then turn to the SPCE outcomes so that the final policies are robust to coalitional deviations. I find that the SPCE outcome with unanimity voting belongs to $\bigcap_{i=0}^{n} T_i \bigcap co(B)$, the SPE outcomes that are in the convex hull of the voters' ideal points. The SPCE outcome with majority voting is the Condorcet winner in T_0, the agenda setter's acceptable outcomes.

Observations from the agenda setting procedures in practice motivate this study of an alternative timing of the moves. Rules of thumb about which bills to bring to the floor (such as the Hastert Rule from the House), statistics about the fraction of bills that are discarded, and news about how some agenda setters regularly investigate the bills that they can pass point out to an initial stage where the voters announce the policies that they would vote in favor of. In order to capture this stage of the agenda setting process, voters in my model commit to and announce the policies they are in favor of, and then the agenda setter chooses the final policy given these restrictions.

In contrast to the standard models of agenda setting, in my setup, the agenda setter loses power over the final policy significantly. In some instances, even when the classical model of Romer & Rosenthal (1978) predicts the agenda setter's ideal point as the unique SPE outcome, my model results in a larger set of possible SPE outcomes. Hence, the agenda setter cannot guarantee her ideal point as the final outcome in my model and loses the great power she holds over the outcome in the classical models.

6.1 Literature Review

A sequential agenda setting problem is considered in Bernheim, Rangel, & Rayo (2006) where each period's agenda setter (a voter recognized endogenously) proposes a policy against an evolving status quo. If the set of agenda setters is sufficiently diverse, the Condorcet winner becomes the final policy if it exists. In environments where the Condorcet winner does not exist, the last proposer implements her ideal point under weak conditions. In this environment, under the Nuclear case, then any policy in the space can be an SPE outcome but the SPCE outcome is the Condorcet winner, whether it exists or not. This implies that the sequential nature of the agenda setting in Bernheim et al. (2006), and the robustness against coalitional deviations in this paper play similar roles in establishing the Condorcet winner as a salient policy.

Equilibrium and core outcomes in multi-dimensional policy space are well-studied objects in spatial voting literature. The core in a multi-dimensional spatial voting game corresponds to the subgame perfect coalitional equilibrium in this paper. Much celebrated Median Voter Theorem is proposed by Black (1948) and more extensively studied in Downs (1957). In one-dimensional policy spaces, the core outcome of the voting game is the median point or the ideal point of the median voter under the majority rule. The idea is generalized to multi-dimensional policy space by Davis et al. (1972). Plott (1967) characterizes the sufficient condition for the existence of the core outcome of the multi-dimensional voting games and the Condorcet winner, known as radial symmetry or the Plott condition. Given the restrictive nature of this condition, the core of the multi-dimensional voting game is often an empty set, as illustrated in McKelvey & Wendell (1976), Schofield (1983), McKelvey & Schofield (1986). Generic non-existence of the core outcome in the voting games is inherited by this paper since the set of SPCE outcomes is equal to the Condorcet winner under majority rule.

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Appendices

A Proofs

Here I include the proofs of the results in the main text, the discussion around the necessary condition for the Condorcet winner, and some formal definitions.

A.1 Proposition 1

Proof. Consider the strategy profile $(A_1^*, \ldots, A_n^*, a^*)$ such that $A_i^* = \{x\}$ and $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for some $x \in \bigcap_{i=0}^n T_i$. After the voters announce their announcement sets A_i^*, the agenda setter chooses $\arg\max_{y \in F \bigcup \tilde{a}} u_0(y)$ where $F(\cdot) = \{x\}$. Since $x \in T_0$, $u_0(x) \geqslant u_0(\tilde{a})$ and x is a best response for the agenda setter, $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$. Now suppose a voter $i \in N_P$ deviates to another strategy A_i'. If $x \in A_i'$, then we still have $F(\cdot) = \{x\}$, and $a^*(F(A_i', A_{-i}^*)) = \{x\}$. If $x \notin A_i'$, $F(\cdot) = \varnothing$ and $a^*(F(A_i', A_{-i}^*)) = \{\tilde{a}\}$. Since $x \in T_i$, $u_i(x) \geqslant u_i(\tilde{a})$, so deviating to $A_i' \not\ni \{x\}$ is not profitable for voter i. We have that $(A_1^*, \ldots, A_n^*, a^*)$ such that $A_i^* = \{x\}$ and $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for $x \in \bigcap_{i=0}^n T_i$ is an SPE. Thus, we can conclude that any point in $\bigcap_{i=0}^n T_i$ is an SPE outcome.

Conversely, suppose there is an equilibrium strategy profile $(A_1^*, \ldots, A_n^*, a^*)$ where $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for some $x \notin \bigcap_{i=0}^n T_i$. If $x \notin T_0$, then $u_0(\tilde{a}) > u_0(x)$ and the agenda setter can profitably deviate to $a^*(F(A_1^*, \ldots, A_n^*)) = \{\tilde{a}\}$. If $x \notin T_i$, for some $i \in N_P$, then $u_i(\tilde{a}) > u_i(x)$. Then the voter can deviate to $A_i' = \varnothing$ so that $F = \varnothing$, and $a^*(F(A_i', A_{-i}^*)) = \{\tilde{a}\}$. Hence, any SPE outcome x must belong to the set $\bigcap_{i=0}^n T_i$. □

A.2 Proposition 2

Proof. Consider the strategy profile $(A_1^*, \ldots, A_n^*, a^*)$ such that $A_i^* = \{x\}$ and $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for some $x \in T_0$. After the voters announce their announcement sets A_i^*, the agenda setter chooses $\arg\max_{y \in F \cup \{\tilde{a}\}} u_0(y)$ where $F = \{x\}$. Since $x \in T_0$, $u_0(x) \geqslant u_0(\tilde{a})$ and x is a best response for the agenda setter, $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$. Now suppose a voter $i \in N_P$ deviates to another strategy $A_i' \in \mathcal{M}$. If $x \in A_i'$, then we still have $F = \{x\}$, and $a^*(F(A_i', A_{-i}^*)) = \{x\}$. If $x \notin A_i'$, we still have $F = \{x\}$ since $n-1$ voters announcing $A_i^* = \{x\}$, which is a majority. Then $a^*(F(A_i', A_{-i}^*)) = \{x\}$. So, deviating to A_i' is not profitable for voter i. Thus, any point in T_0 is an SPE outcome.

Conversely, suppose there is an equilibrium strategy profile $(A_1^*, \ldots, A_n^*, a^*)$ where $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for some $x \notin T_0$. Then, $u_0(\tilde{a}) > u_0(x)$ and the agenda setter can profitably deviate to $a^*(F(A_1^*, \ldots, A_n^*)) = \{\tilde{a}\}$. Hence, any SPE outcome x belongs to the set T_0. □

A.3 Proposition 3

The following lemmas are used to prove Proposition 3.

Lemma 1. Let $V = co(\{b_1, \ldots, b_n\})$. Suppose $x, x' \in V$. Then, if $d(x, b_i) \geqslant d(x', b_i)$ $\forall i = 1, \ldots, n$, it must be the case that $a = a'$.

Proof. A key fact that the proof uses is the following:

$$d(x, b) \geqslant d(x', b) \Rightarrow (b - x) \cdot (x' - x) \geqslant 0.$$

[In geometric terms, this inequality says that the angle between the vectors $(b-x)$ and $(x'-x)$ must be less than 90 degrees whenever $d(x, b) \geqslant d(x', b)$.]

Suppose $x, x' \in V$ and let $d(x, b_i) \geqslant d(x', b_i)$ $\forall i = 1, \ldots, n$. Let $x = \lambda_1 b_1 + \ldots \lambda_n b_n$ where $\lambda_1, \ldots, \lambda_n \geqslant 0$ and $\sum_{i=1}^n \lambda_i = 1$. Suppose without loss of generality, achieved by relabeling, that $\lambda_1, \ldots, \lambda_k > 0$ and $\lambda_{k+1}, \ldots, \lambda_n = 0$. Notice that since $d(x, b_i) \geqslant d(x', b_i)$ $\forall i = 1, \ldots, n$ and by the fact above, we must have:

$$(b_1 - x) \cdot (x' - x) \geqslant 0, \ldots, (b_n - x) \cdot (x' - x) \geqslant 0.$$

If $(b_1 - x) \cdot (x' - x) > 0$, then we get:

$$0 = (x - x) \cdot (x' - x) = \sum_{i=1}^n \lambda_i (b_i - x) \cdot (x' - x),$$

which is a contradiction. So we must have:

$$(b_1 - x) \cdot (x' - x) = 0.$$

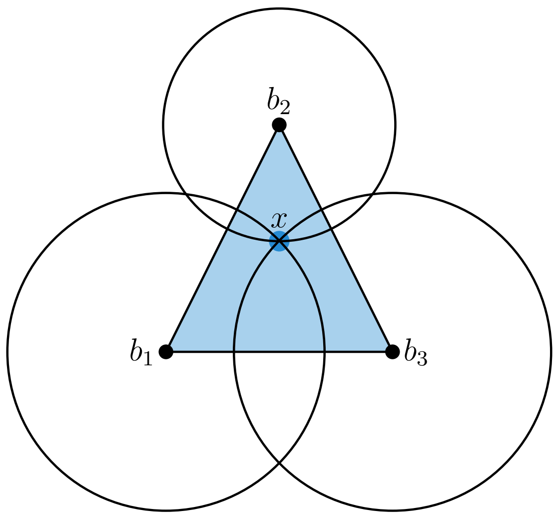
The following holds for $i = 1, \ldots, k$:

$$2(x' - x) \cdot (b_i - x) = d(x', x)^2 + d(x, b_i)^2 - d(x', b_i)^2 \geqslant d(x', x)^2$$

If we sum these inequalities with weights λ_i for $i = 1, \ldots, k$, we get,

$$0 = 2(x' - x)(x - x) \geqslant d(x', x)^2 \Rightarrow x' = x.$$

□

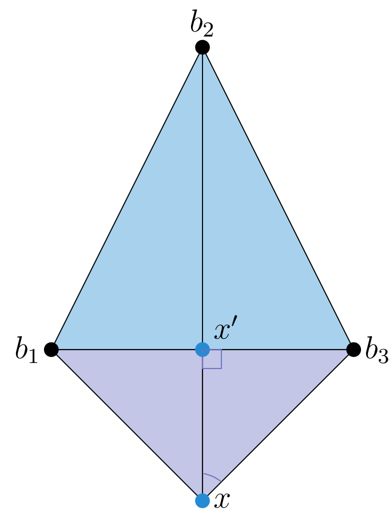


Lemma 2. Let $V = co(\{b_1, \ldots, b_n\})$. For any $x \in \bar{\mathbb{R}}^d \backslash V$ there exists $x' \in V$ such that $d(x', b_i) \leqslant d(x, b_i)$ $\forall i = 1, \ldots, n$.

Proof. Let $x \in \bar{\mathbb{R}}^d \backslash V$. Then, consider the point $x' \in V$ such that $x' = \arg\min_{y \in V} d(x, y)$. In words, x' is the closest point to x among the points in the convex hull.

Now take any b_i for $i = 1, \ldots, n$, and construct the triangle $\triangle(xx'b_i)$. I claim that the angle between the vectors $(b_i - x')$ and $(x' - x)$, α, must be larger than the angle between the vectors $(b_i - x)$ and $(x' - x)$, β, which translates into $d(x, b_i) \geqslant d(x', b_i)$ for all $i = 1, \ldots, n$.

Suppose it's not the case, so $\beta > \alpha$, which implies that $\alpha < \frac{\Pi}{2}$. Then, since $b_i, x' \in V$, the line $\overline{x'b_i}$ belongs to V. Then the point x'' where $x \perp \overline{x'b_i}$ is actually the closest point to x among the points in the convex hull, a contradiction. Hence, for every point outside of the convex hull, there is a point in the convex hull which is closer to every voter's ideal point compared to the point outside the convex hull. $\square$



Using the lemmas above, here I prove Proposition 3.

Proof. We already established that for a policy to arise as an SPE , it has to be acceptable for every player in Proposition 1, and since SPCE

Let $\bigcap_{i=0}^{n} T_i \bigcap co(B) \neq \emptyset$.

Consider the strategy profile $(A_1^*, \dots, A_n^*, a^*)$ such that $A_i^* = \{x\}$ and $a^*(F(A_1^*, \dots, A_n^*)) = \{x\}$ for some $x \in \bigcap_{i=0}^{n} T_i \bigcap co(B)$.

Exactly as in Proposition 1, the agenda setter is best responding and the voters cannot achieve a higher utility through unilateral deviations.

Next, we consider coalitional deviations by the voters. Sinc $x \in co(B)$, by Lemma 1, we know that there is no $x' \in co(B)$ such that $u_i(x') \geqslant u_i(x)$ for all $i \in N_P$. We also know, by Lemma 2, there cannot be any $x'' \notin co(B)$ such that $u_i(x'') \geqslant u_i(x)$ for all $i \in N_P$. Since there are no outcomes that the voters unanimously prefer to x, there does not exist a profile $\{A_i'\}$ such that $u_i(a^*(F(A_1', \dots, A_n'))) \geqslant u_i(a^*(F(A_1^*, \dots, A_n^*)))$ for all $i \in N_P$ with the inequality strict for at least one voter. Thus, x is a SPCE outcome under the strategy profile $(A_1^*, \dots, A_n^*, a^*)$.

Conversely, suppose there is a subgame perfect coalitional equilibrium strategy profile $(A_1^*, \ldots, A_n^*, a^*)$ where $a^*(F(A_1^*, \ldots, A_n^*)) = \{x\}$ for some $x \notin \bigcap_{i=0}^n T_i \bigcap co(B)$. If $x \notin T_0$, then $u_0(\tilde{a}) > u_0(x)$ and the agenda setter can profitably deviate to $a^*(F(A_1^*, \ldots, A_n^*)) = \{\tilde{a}\}$. If $x \notin T_i$, for some $i \in N_P$, then $u_i(\tilde{a}) > u_i(x)$. Then the voter can deviate to $A_i' = \varnothing$ so that $F = \varnothing$, and $a^*(F(A_i', A_{-i}^*)) = \{\tilde{a}\}$. If $x \in \bigcap_{i=0}^n T_i$ but $x \notin co(B)$, then take the point x' such that $x' = \arg\min_{y \in \bigcap_{i=0}^n T_i \bigcap co(B)} d(x,y)$. Then, y is an acceptable outcome for every player, and $u_i(x') \geqslant u_i(x)$ for all $i \in N_P$ and $u_j(x') > u_j(x)$ for some $j \in N_P$ by Lemma 2. Thus, any SPCE outcome is in $\bigcap_{i=0}^n T_i \bigcap co(B)$.

□

A.4 SPE Outcomes in Undominated Strategies

Proposition 4. When the restriction sets are singleton for every voter, intersection of a voter's set of acceptable policies and the agenda setter's acceptable policies except the status quo is the set of undominated strategies for each voter. Then, the set of outcomes of SPE in undominated strategies is the set of policies that are acceptable for the agenda setter and at least a majority of the voters.

Proof. First we will see that any $A_i = \{b\} \subseteq \bar{\mathbb{R}}^d \backslash T_i$ is weakly dominated by some $A_i' = \{a\} \subseteq T_i \bigcap T_0$.[9] We need to look at three cases:

1. Any alternative $x \in \bar{\mathbb{R}}^d$ is announced by at most $\frac{n+1}{2} - 2$ voters.

   In this case, voter i's announcement has no effect on the final outcome, so he is indifferent between announcing a and b.

2. There is one alternative $c \in \bar{\mathbb{R}}^d$ that is announced by $\frac{n+1}{2} - 1$ voters, and any other alternatives are announced by at most $\frac{n+1}{2} - 2$ voters.

   (a) If $c \neq a$ and $c \neq b$, voter i's announcement does not affect the final outcome again, so he is indifferent between announcing a and b.

   (b) If $c = a$, then announcing b leads to the status quo whereas announcing a leads to $a = c$ as the final policy. Since $a \in T_i \bigcap T_0$, $u_i(a) \geqslant u_i(\tilde{a})$.

   (c) If $c = b$, then announcing b leads to $b = c$ as the final outcome whereas announcing a leads to the status quo. Since $b \in \bar{\mathbb{R}}^d \backslash T_i$, $u_i(\tilde{a}) > u_i(b)$.

[9] When there is no confusion, I refer to the singleton set $A_i = \{a\}$ as simply a.

3. There are two alternatives $c, d \in \bar{\mathbb{R}}^d$ that are announced by $\frac{n+1}{2} - 1$ voters, so voter i is the tie breaker.

   (a) If $c \neq a$, $d \neq a$, $c \neq b$ and $d \neq b$: Voter i's announcement does not affect the final outcome again, so he is indifferent between announcing a and b.

   (b) If $c = a$ or $d = a$ and $c \neq b$ and $d \neq b$: Announcing b leads to the status quo whereas announcing a leads to c or d, and since $a \in T_i \bigcap T_0$, $u_i(a) \geqslant u_i(\tilde{a})$.

   (c) If $c \neq a$ and $d \neq a$ and $c = b$ or $d = b$: Announcing b leads to c or d or the status quo whereas announcing a leads to the status quo, and since $b \in \bar{\mathbb{R}}^d \backslash T_i$, $u_i(\tilde{a}) \geqslant u_i(\tilde{a}) > u_i(b)$.

   (d) If $c = a$ and $d = b$ or $d = a$ and $c = b$: Announcing b leads to c (d) or the status quo whereas announcing a leads to d (c), and since $a \in T_i \bigcap T_0$ and $b \in \bar{\mathbb{R}}^d \backslash T_i$, $u_i(a) \geqslant u_i(\tilde{a}) > u_i(b)$.

Next, we see that any $b \in T_i \backslash T_0$ is weakly dominated by some $a \in T_i \bigcap T_0$. We need to check the same three cases again:

1. Any alternative $x \in \bar{\mathbb{R}}^d$ is announced by at most $\frac{n+1}{2} - 2$ voters.

   In this case, voter i's announcement has no effect on the final outcome, so he is indifferent between announcing a and b.

2. There is one alternative $c \in \bar{\mathbb{R}}^d$ that is announced by $\frac{n+1}{2} - 1$ voters, and any other alternatives are announced by at most $\frac{n+1}{2} - 2$ voters.

   (a) If $c \neq a$ and $c \neq b$, voter i's announcement does not affect the final outcome again, so he is indifferent between announcing a and b.

   (b) If $c = a$, then announcing b leads to the status quo whereas announcing a leads to $a = c$ as the final policy. Since $a \in T_i \bigcap T_0$, $u_i(a) \geqslant u_i(\tilde{a})$.

   (c) If $c = b$, then announcing b leads to the status quo since $b \notin T_0$ and announcing a also leads to the status quo, so he is indifferent between announcing a and b.

3. There are two alternatives $c, d \in \bar{\mathbb{R}}^d$ that are announced by $\frac{n+1}{2} - 1$ voters, so voter i is the tie breaker.

(a) If $c \neq a$, $d \neq a$, $c \neq b$ and $d \neq b$: Voter i's announcement does not affect the final outcome again, so he is indifferent between announcing a and b.

(b) If $c = a$ or $d = a$ and $c \neq b$ and $d \neq b$: Announcing b leads to the status quo whereas announcing a leads to c or d, and since $a \in T_i \bigcap T_0$, $u_i(a) \geqslant u_i(\tilde{a})$.

(c) If $c \neq a$ and $d \neq a$ and $c = b$ or $d = b$: Announcing b leads to the status quo and announcing a also leads to the status quo, so the voter is indifferent between announcing a and b.

(d) If $c = a$ and $d = b$ or $d = a$ and $c = b$: Announcing b leads to the status quo in both cases since $b \notin T_0$, and announcing a leads to either d or c, and $u_i(a) \geqslant u_i(\tilde{a}) > u_i(b)$.

Similarly, the status quo $\tilde{a}$ is weakly dominated by some $a \in T_i \bigcap T_0 \backslash \{\tilde{a}\}$.

Finally, observe that any singleton $A_i \subseteq T_i \bigcap T_0 \backslash \{\tilde{a}\}$ is a strict best response to some strategy profile of the remaining voters and the agenda setter. Consider the case where exactly half of the voters are announcing $a \in T_i \bigcap T_0 \backslash \{\tilde{a}\}$ and the other half $b \in \bar{\mathbb{R}}^d \backslash T_i$. Given that the agenda setter will choose a if it is announced by a majority of voters, $A_i = \{a\}$ is the unique best response.

$\square$

A.5 Results for q-Rule

Now let us state the results for q-rule where $\frac{n+1}{2} < q < n$ is the required number of votes to make a policy feasible.

Proposition 5. The set of SPE outcomes is the set of acceptable policies for the agenda setter with q-rule voting.

Proof. The same line of logic as in the proof of Proposition 2 applies. $\square$

The way Theorem 1 works by showing the equivalence of an SPCE outcome and the Condorcet winner suggests that a notion of Condorcet winner corresponding to any q-rule is necessary for characterizing SPCE outcomes for any majoritarian voting rule. Let the *q-rule Condorcet winner* be defined as an alternative which is preferred to any other alternative by q many voters in a pairwise comparison.

Proposition 6. The set of SPCE outcomes is the set of q-rule Condorcet winners from among the agenda setter's acceptable outcomes with q-rule voting.

Proof. The same line of logic as in the proof of Theorem 1 applies. □