**Name:** ______________________________

1. (25 points) In class, we saw that loss-of-significance errors can be avoided by rearranging the function evaluated. Consider the following functions $f(x)$ and do something similar assuming that $x \approx 0$ (unless stated otherwise):

   (a) (5 points) $f(x) = \dfrac{1 - \cos x}{x^2}$

   (b) (5 points) $f(x) = \ln{(x+1)} - \ln x$, for $x \gg 1$

   (c) (5 points) $f(x) = \sin{(a+x)} - \sin a$

   (d) (5 points) $f(x) = \sqrt[3]{1+x} - 1$

   (e) (5 points) $f(x) = \dfrac{\sqrt{4+x} - 2}{x}$

2. (15 points) In one of the examples we discussed in class about, concerned with a strategy for avoiding the loss-of-significance errors by using Taylor polynomials. Thus, use Taylor polynomial approximations to avoid such errors in the following functions when $x \approx 0$:

   (a) (5 points) $f(x) = \dfrac{e^x - e^{-x}}{2x}$

   (b) (5 points) $f(x) = \dfrac{\ln{(1-x)} + xe^{x/2}}{x^3}$

   (c) (5 points) $f(x) = \dfrac{x - \sin x}{x^3}$

3. (20 points) Let $f(x)$ be:
$$f(x) = \cosh x + \cos x - \gamma,$$
where $\gamma$ is a parameter and takes the values of $\gamma = 0, 1, 2, 3$. Make a graph of the function $f(x)$ for each value of $\gamma$ on the interval $[-3, 3]$ and determine whether $f(x)$ has a **root**. To do so, you have to check the criteria required by the **Intermediate Value Theorem**. Then, using the m-file "bisect.m", **approximate** the root with absolute tolerance $10^{-10}$ for the value of $\gamma$ that $f(x)$ does have a root. Include a copy of the graph of $f(x)$ for the respective cases and MATLAB output.

4. (10 points) Consider **Newton's method** for finding $+\sqrt{\alpha}$ with $\alpha > 0$ by finding the **positive root** of $f(x) = x^2 - \alpha = 0$. Assuming that $x_0 > 0$, show the following:

   (a) (5 points)
   $$x_{k+1} = \frac{1}{2}\left(x_k + \frac{\alpha}{x_k}\right),$$

   (b) (5 points)
   $$x_{k+1}^2 - \alpha = \left(\frac{x_k^2 - \alpha}{2x_k}\right)^2,$$

   for $k \geq 0$ and therefore $x_k > \sqrt{\alpha}$ for $k \geq 1$. Note that the formula in part (a) refers to as the "Babylonian method" for computing square roots!

5. (30 points) **Implement Newton's method** in MATLAB (or in **any** programming language). In particular, create a function, which utilizes the method, and store it as an m-file. Then, use your function to estimate $\sqrt{7}$ by finding the **positive root** of $f(x) = x^2 - 7$. Try two different initial guesses: (i) $x_0 = 2$ and (ii) $x_0 = 500$ and consider $|x_{k+1} - x_k| < 10^{-10}$ as a convergence criterion.

   Attach your codes and provide MATLAB output for both cases.

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