MATHEMATICS DEPARTMENT
CALIFORNIA POLYTECHNIC STATE UNIVERSITY SAN LUIS OBISPO

Math 143   <u>**Exam 1**</u>   **Winter 2020**

Name:_____

Section Number:_____

- You have 50 minutes to complete this exam.

- No notes, books, calculators, cell phones, or other references are allowed.

- In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.

- There are 7 pages, including this one, in this exam and five numbered problems. **Make sure you have them all before you begin!**

- Page 7 is an additional blank page if you need more space to write down your solutions.

- **You must show all work to receive credit**. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).

- Let me wholeheartedly wish you good luck!!

1. (20) _____
2. (10) _____
3. (20) _____
4. (20) _____
5. (30) _____

Perfect Paper $\longrightarrow$ 100 Points.

1. (20 points) **Circle one** option that answers the questions below. No explanation is required for these problems.

(a) (5 points) Given $\sum\limits_{n=1}^{\infty} a_n$ with $a_n = f(n)$, which of the following is always true?

I. If $\int_1^{\infty} f(x)\, dx = 5$, then $\sum_{n=1}^{\infty} a_n = 5$ by the Integral test.

II. If $\int_1^{\infty} f(x)\, dx = 5$, then $\sum_{n=1}^{\infty} a_n$ is convergent by the Integral test.

III. If $\int_1^{\infty} f(x)\, dx = 0$, then the Integral test is inconclusive.

IV. If $\int_1^{\infty} f(x)\, dx = \infty$, then the Integral test is inconclusive.

(b) (5 points) Given $a_n = \dfrac{\ln(n)}{\sqrt{n}}$, which of the following is true?

I. $\{a_n\}_{n=1}^{\infty}$ and $\sum_{n=3}^{\infty} a_n$ are both convergent.

II. $\{a_n\}_{n=1}^{\infty}$ is convergent, and $\sum_{n=3}^{\infty} a_n$ is divergent.

III. $\{a_n\}_{n=1}^{\infty}$ is divergent, and $\sum_{n=3}^{\infty} a_n$ is convergent.

IV. $\{a_n\}_{n=1}^{\infty}$ and $\sum_{n=3}^{\infty} a_n$ are both divergent.

(c) (5 points) Which of the following is true about $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n+1}{n^3+9}$?

I. The series converges by the nth term test (Test for Divergence).

II. The series converges by the Direct Comparison Test.

III. The series converges by the Alternating Series Test.

IV. All of the above are true.

(d) (5 points) Which of the following series diverge?

$$(\alpha)\ \sum_{n=1}^{\infty} \frac{1}{2n} \qquad (\beta)\ \sum_{n=1}^{\infty} \frac{1}{\sin^2(n)+2} \qquad (\gamma)\ \sum_{n=2}^{\infty} \frac{1}{\ln(n)} \qquad (\delta)\ \sum_{n=1}^{\infty} \frac{3}{\pi^n}$$

I. (α) and (δ)

II. (α), (β), and (γ)

III. (α) and (γ)

IV. (α), (β), (γ), and (δ)

Hint: It is given that the integral $\int_2^{\infty} \frac{1}{\ln(x)}\, dx$ diverges.

2. (10 points) Let the **sequence** $\left\{\dfrac{2^n + 4^n}{3^n + 4^n}\right\}$.

    (a) (6 points) Does it converge or diverge? State the name of any tests you use and check any necessary conditions.

    (b) (4 points) Does the **series**

$$\sum_{n=0}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$$

converge or diverge? If it does converge, find its sum. Similarly, as in part (a), make sure to state the name of any tests you use and check any necessary conditions.

3. (a) (10 points) Determine whether the series

$$\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$$

converges or diverges. Make sure you state which test you use.

(b) (10 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{3^{2n+1}}{2^{3n+1}}$$

converges or diverges. Make sure you state which test you use.

4. (20 points) Determine whether the series

$$\sum_{n=0}^{\infty} \frac{4n^2 - n^3}{10 + 2n^3}$$

converges or diverges. Make sure you state which test you use.

5. (30 points) Find the **radius** R and **interval** I of convergence of the series

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2 + 3}}.$$

Show all your work and state any tests you used.