MATHEMATICS DEPARTMENT
CALIFORNIA POLYTECHNIC STATE UNIVERSITY SAN LUIS OBISPO

Math 143 **Exam 2** **Winter 2020**

Name:______________________________

Section Number:____________________

- You have 50 minutes to complete this exam.
- No notes, books, calculators, cell phones, or other references are allowed.
- In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.
- There are 7 pages, including this one, in this exam and 5 numbered problems. **Make sure you have them all before you begin!**
- Use page 7 if you need more space to write down your solutions.
- **You must show all work to receive credit**. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).
- Let me wholeheartedly wish you good luck!!

1.	(15)	
2.	(25)	
3.	(20)	
4.	(20)	
5.	(20)	
Total		

Perfect Paper $\longrightarrow$ 100 Points.

1. (15 points) Find the Taylor Series for the function below. Express your final answer in summation notation. Simplify completely.

$$f(x) = \frac{1}{x} \quad \text{centered at} \quad a = 3.$$

2. (25 points) Consider the curve given by the parametric equations

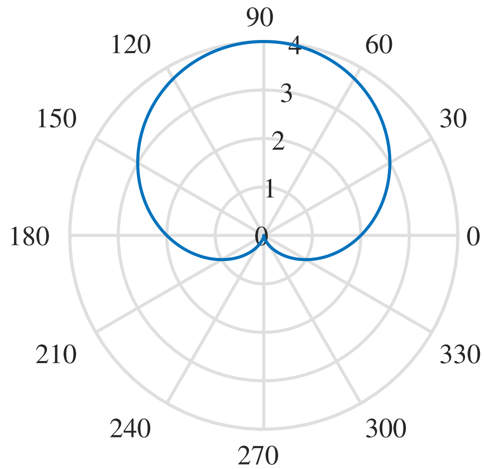
$$x = 9\sin t, \quad y = 9\cos t, \quad t \in [0, \pi].$$

(a) (15 points) Calculate d^2y/dx^2 at $t = 3\pi/4$.

(b) (10 points) Find the **exact length** of the curve on the given interval.

3. (20 points) Consider the **cardioid** shown in the figure below and given by

$$r = 2 + 2\sin(\theta).$$



(a) (5 points) Set up an integral which computes the area enclosed. Expand any algebraic identities and simplify the integrand. Mathematically justify how you find the limits of integration.

(b) (5 points) Set up an integral which computes the length of the curve. Similarly as in part (a), mathematically justify how you find the limits of integration.

(c) (10 points) Find the slope of the tangent line for $r = 2 + 2\sin(\theta)$ at $\theta = \pi/4$.

4. (20 points) Let $\vec{a} = \langle 1, 2, 2 \rangle$ and $\vec{b} = \langle 1, 1, 1 \rangle$.

(a) (5 points) Compute the $\vec{a} \cdot \vec{b}$.

(b) (5 points) Find the vector projection, call it $\vec{c}$, of $\vec{b}$ onto $\vec{a}$.

(c) (5 points) Calculate the vector $\vec{b} - \vec{c}$, call it $\vec{d}$, and then show that this new vector is perpendicular to $\vec{a}$.

(d) (5 points) Find a unit vector in the direction of $\vec{d}$.

5. (20 points) Consider the vectors: $\vec{a} = \langle 2, -4, 0\rangle$ and $\vec{b} = \langle 2, -1, -2\rangle$.

   (a) (15 points) Find $\vec{a} \times \vec{b}$ and its **direction**. Is the vector $\vec{a} \times \vec{b}$ parallel or perpendicular to both $\vec{a}$ and $\vec{b}$?

   (b) (5 points) Find the **sine** (not θ!) of the angle θ between the vectors $\vec{a}$ and $\vec{b}$. Simplify the radical expressions.