MATHEMATICS DEPARTMENT
CALIFORNIA POLYTECHNIC STATE UNIVERSITY SAN LUIS OBISPO

Math 143 **Final exam** **Fall 2019**

Name:____________________________________

Section Number:________________________

- You have 170 minutes to complete this exam.
- No notes, books, cell phones, or other references are allowed.
- In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.
- There are 13 pages, including this one, in this exam and **nine** numbered problems. **Make sure you have them all before you begin!**
- There are **three** additional blank pages at the end of the exam if you need more space to write down your solutions.
- **You must show all work to receive credit**. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).
- Let me wholeheartedly wish you good luck!!

1.	(10)	
2.	(10)	
3.	(15)	
4.	(10)	
5.	(15)	
6.	(20)	
7.	(10)	
8.	(15)	
9.	(20)	
Total		

Perfect Paper $\longrightarrow$ 125 Points.

1. (10 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$$

converges or diverges. Make sure you state which test you use.

2. (a) (7 points) Find the **radius** R and **interval** I of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{\sqrt{n}}.$$

Show all your work and state any tests you used.

(b) (3 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$$

converges or diverges. Make sure you state which test you use.

3. (15 points) Consider the function $f(x) = e^x$ on the interval $[0, 1/2]$.

   (a) (5 points) Derive the Maclaurin series of the function $f(x) = e^x$. Write **separately** $P_k(x)$ and $R_k(x)$.

   (b) (10 points) Estimate the error if $k = 4$ is used to estimate the value of e^x at $x = 1/2$. [Hint: You may use the fact that $e^{1/2} \approx 1.65$ and $e \approx 2.7$.]

4. (10 points) Consider a curve with parametrization given by

$$x = e^{\sin t}, \quad y = \cos t + t - \pi, \quad t \in [0, 2\pi].$$

(a) (5 points) Find the tangent line to the curve at the point $(1, -1)$.

(b) (5 points) Find all the points where there is a vertical tangent line on $[0, 2\pi]$.

5. (a) (7 points) Write down the equation of the plane P containing the point $A(1, 2, 3)$ and orthogonal/perpendicular to the line L with parametric equations

$$x = 15 + t, \quad y = 2 + 2t, \quad z = 3 + 3t.$$

(b) (3 points) Find the point, call it B, of intersection of the plane P with the line L. [Hint: First find the time t_0 when the line crosses the plane.]

(c) (2 points) What is the distance from the point $A(1, 2, 3)$ to the line L?

(d) (3 points) What is the distance D from the point $C(-1, 1, 2)$ to the plane P?

6. (20 points) Consider the points $A(2, -1, 1)$, $B(4, 2, 1)$, and $C(1, 2, 3)$.

   (a) (5 points) Find the parametric equations of the line in $\mathbb{R}^3$ which contains the points A and B.

   (b) (5 points) Find the cosine of the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   (c) (10 points) Find the parametric equations for the line L that passes through the point $(2, 4, 6)$ and that is normal to the plane containing the points A, B, and C.

7. (10 points) Consider the points $A(1, 0, 1)$, $B(0, 2, 3)$ and $C(-1, -1, 0)$. Find the area of the triangle with corners the above points.

8. (a) (2 points) Suppose that $\vec{r}'(t) = \langle -\sin 2t, \cos 2t, 0\rangle$ for $t \in [0,1]$. If $\vec{r}(t=0) = \langle 1/2, 0, 1\rangle$, then determine $\vec{r}(t)$ for all t.

(b) (3 points) Find the curve's unit tangent vector.

(c) (5 points) Show that $\vec{r}(t)$ is **orthogonal** to $\vec{r}'(t)$ for all t.

(d) (5 points) Find the length L of the graph of the vector function $\vec{r}(t)$ on the interval $0 \leq t \leq 1$.

9. (20 points) The velocity of a space ship is specified by $\vec{r}\,'(t) = \langle t^2, 0, \cos t \rangle$.

(a) (10 points) Without finding $\vec{T}$ (unit tangent vector) and $\vec{N}$ (unit normal vector), write the acceleration vector of the space ship $\vec{a}$ as $\vec{a} = a_T\vec{T} + a_N\vec{N}$. [Hint: Compute a_N via the formula relating the curvature κ and $|\vec{r}\,'(t)|$.]

(b) (10 points) The space ship approaches the Moon and it has to modify its trajectory. If now its position vector is given by $\vec{r}(t) = \langle 3\sin t, 3\cos t, 4t \rangle$, find the unit tangent vector $\vec{T}$, unit normal vector $\vec{N}$ and unit binormal vector $\vec{B}$ of the space ship's new trajectory.