Name: ______________________________

1. (15 points) Let $f(x) = x^2 - 4x + 3$. Then:
   (a) (5 points) Find $p_1(x)$, $p_2(x)$ and $p_3(x)$ around $x_0 = 0$. How $P_3(x)$ is related to $f(x)$?
   (b) (5 points) Same as part (a) but consider $x_0 = 1$.
   (c) (5 points) In general, given a polynomial $f(x)$ with degree m, what can you say about $f(x) - p_k(x)$ for $k \geq m$?
2. (25 points) Given $f(x) = \cos x$, find both $p_2(x)$ and $p_3(x)$ about $x_0 = 0$, and use them to approximate $\cos(0.1)$. Show that in each case the remainder term provides an upper bound for the true (absolute) error.
3. (30 points) If $f(x) = e^x$, then
   (a) (10 points) derive the Maclaurin series of the function $f(x) = e^x$, i.e., the Taylor series about $x_0 = 0$ (write **separately** $p_k(x)$ and $R_k(x)$),
   (b) (20 points) find a minimum value of k necessary for $p_k(x)$ to approximate $f(x)$ to within 10^{-6} on the interval $[0, 0.5]$ (here, you must use the remainder term).
4. (20 points) Let $f(x) = \sqrt[3]{x}$. Does $f(x)$ have a Taylor polynomial of degree 1 based on expanding about $x = 0$ and $x = 1$? Justify your answers. Include a copy of the graph of $f(x)$ and its associated polynomials (when applicable) on the same figure, as well as your MATLAB script producing the figure.
5. (10 points) Consider the polynomial

$$p(x) = 1 - \frac{x^3}{3!} + \frac{x^6}{6!} - \frac{x^9}{9!} + \frac{x^{12}}{12!} - \frac{x^{15}}{15!}.$$

   Evaluate $p(x)$ as efficiently as possible. How many multiplications are necessary? Assume all coefficients have been computed and stored for later use.

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