MATHEMATICS DEPARTMENT
CALIFORNIA POLYTECHNIC STATE UNIVERSITY SAN LUIS OBISPO

Math 143 **Final Exam** **Winter 2020**

Name:________________________________

Section Number:_____________________

- You have 170 minutes to complete this exam.
- No notes, books, cell phones, or other references are allowed.
- In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.
- There are 13 pages, including this one, in this exam and **nine** numbered problems. **Make sure you have them all before you begin!**
- There are **three** additional blank pages at the end of the exam if you need more space to write down your solutions.
- **You must show all work to receive credit**. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).
- Let me wholeheartedly wish you good luck!!

1.	(10)	__________
2.	(10)	__________
3.	(15)	__________
4.	(10)	__________
5.	(10)	__________
6.	(20)	__________
7.	(10)	__________
8.	(15)	__________
9.	(20)	__________
Total		__________

Perfect Paper $\longrightarrow$ 120 Points.

1. (10 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, (n!)^2}{(2n)!}$$

converges or diverges. Make sure you state which test you use.

2. (a) (7 points) Find the **radius** R and **interval** I of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}.$$

Show all your work and state any tests you used.

(b) (3 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^n}{2^{n^2}}$$

converges or diverges. Make sure you state which test you use.

3. (15 points) Consider the function $f(x) = \cos x$ on the interval $[0, \pi/4]$.

   (a) (5 points) Derive the Maclaurin series of the function $f(x) = \cos x$. Write **separately** $P_k(x)$ and $R_k(x)$.

   (b) (10 points) Estimate the error if $k = 4$ is used to estimate the value of $\cos x$ at $x = \pi/4$.

4. (10 points) Consider a curve with parametrization given by

$$x = t^3 - 3t, \quad y = t^2 - 3, \quad t \in \mathbb{R}.$$

(a) (6 points) Find the tangent line to the curve at the point $(2, 1)$.

(b) (4 points) Find all the points where there is a vertical tangent line on $[0, 2\pi]$.

5. (a) (8 points) Write down the equation of the plane P containing the point $A(2, 1, -1)$ which is orthogonal/perpendicular to the line L determined by the intersection of the planes

$$P_1 : 2x + y - z = 3, \quad P_2 : x + 2y + z = 2.$$

(b) (2 points) What is the distance D from the point $C(-2, 1, 2)$ to the plane P?

6. (20 points) Consider the points $A(-1,1,3)$, $B(2,5,2)$, and $C(1,2,6)$.

   (a) (5 points) Find the cosine of the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

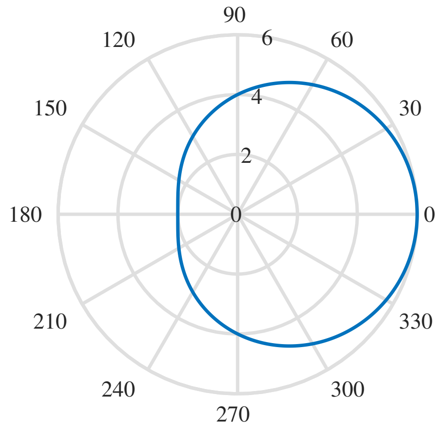
   (b) (5 points) Let L_1 be the line which passes through the points A and B. Find the parametric equations for L_1.

   (c) (5 points) Determine parametric equations for the line L_2 which is **tangent** to the space curve given by $\vec{r}(t) = \langle t, 2t^2, 2t \rangle$ at the point $D(1,2,2)$.

   (d) (5 points) Find the area of the triangle with corners the points $A(-1,1,3)$, $B(2,5,2)$, and $C(1,2,6)$.

7. (10 points) Consider the **oval limaçon** of Figure 7 whose polar equation is given by:

$$r = 4 + 2\cos\theta.$$



Find the area of the region **inside** the **oval limaçon**.

8. (a) (2 points) Suppose a particle moving in space has velocity given by $\vec{r}'(t) = \langle 2t, 2\sqrt{t}, 1\rangle$ for $t \geq 0$. If $\vec{r}(t=0) = \langle -1, 5, 4\rangle$, then determine $\vec{r}(t)$ for all t.

(b) (3 points) Find a **vector** equation of the tangent line to the curve at $t = 4$.

(c) (5 points) Does the particle ever pass through the point $P(80, 41, 13)$?

(d) (5 points) Find the length L of the graph of the vector function $\vec{r}(t)$ on the interval $1 \leq t \leq 2$.

9. (20 points) The position of a space ship is specified by $\vec{r}(t) = \langle t^2, t^2, t^3 \rangle$ for $t \geq 0$.

(a) (6 points) Without finding $\vec{T}$ (unit tangent vector) and $\vec{N}$ (unit normal vector), write the acceleration vector of the space ship $\vec{a}$ as $\vec{a} = a_T\vec{T} + a_N\vec{N}$.

(b) (14 points) Find the unit tangent vector $\vec{T}$, unit normal vector $\vec{N}$ and unit binormal vector $\vec{B}$ of the space ship's trajectory.