**Name:** ______________________

1. (30 points) Consider the data points $\{(0,1), (1,1), (2,5)\}$. Then:

   (a) (10 points) Find the **piecewise linear** interpolating function for the data.

   (b) (10 points) Find the **quadratic** interpolating polynomial.

   (c) (10 points) Find the **natural cubic spline** that interpolates the data.

2. (20 points) Is the following function a cubic spline on the interval $0 \leq x \leq 2$?

$$s(x) = \begin{cases} (x-1)^3, & x \in [0,1], \\ 2(x-1)^3, & x \in [1,2]. \end{cases}$$

3. (30 points) Consider the function $f(x) = \sin(x)$ of Question 1 of HW4.

   (a) (15 points) Interpolate the function $f(x)$ at 5 Chebyshev points over the interval $[0, \pi/2]$ and compare your results with those of Question 1 of HW4, i.e., evaluate the interpolation error magnitude $|p(1.2) - \sin(1.2)|$. Also, plot your data points and the underlying interpolating polynomial for $x \in [0, \pi/2]$ in the same figure.

   (b) (15 points) Repeat the interpolation, although use 5 Chebyshev points over the interval $[0, \pi]$ this time. Plot $f(x)$ at the Chebyshev points as well as the interpolant for $x \in [0, \pi]$. What are your conclusions?

   *Hints:*

   - Recall the definition of Chebyshev points! Also there exists a transformation, mapping $x \in [-1,1]$ onto $\widetilde{x} \in [a,b]$ according to

$$\widetilde{x} = \frac{a+b}{2} + \frac{b-a}{2}x.$$

   - Use the m-files **divdif.m** and **evalnewt.m** to construct the interpolant.

4. (20 points) Find the **linear** least squares approximation to $f(x) = e^x$ on $[-1, 1]$.

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