MATHEMATICS DEPARTMENT
CALIFORNIA POLYTECHNIC STATE UNIVERSITY SAN LUIS OBISPO

Math 143   <u>**Exam 1**</u>   **Fall 2019**

Name:

Section Number:

- You have 50 minutes to complete this exam.

- No notes, books, calculators, cell phones, or other references are allowed.

- In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You **must explain** how you arrived at your answers, and show your algebraic calculations.

- There are 7 pages, including this one, in this exam and five numbered problems. **Make sure you have them all before you begin!**

- There is an additional blank page at the end of the exam if you need more space to write down your solutions.

- **You must show all work to receive credit**. Answers for which no work is shown will receive no credit (unless specifically stated otherwise).

- Let me wholeheartedly wish you good luck!!

  1. (16) _____
  2. (14) _____
  3. (20) _____
  4. (20) _____
  5. (30) _____

Perfect Paper $\longrightarrow$ 100 Points.

1. (16 points) Determine whether each of the following statements is true or false. No explanation is required for these problems.

   (a) (2 points) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) (2 points) Let $\sum_{n=1}^{\infty} (-1)^n u_n$. The series converges if $u_n > 0$, $u_{n+1} \geq u_n$ and $u_n \to 0$.

   (c) (2 points) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$.

   (d) (2 points) The series $\sum_{n=1}^{\infty} (-1)^{n+1} n$ diverges.

(e) (2 points) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum (a_n + b_n)$ converges.

(f) (2 points) The series $\sum\limits_{n=0}^{\infty} \cos(n\pi)$ diverges.

(g) (2 points) The sum of the series $\sum\limits_{n=0}^{\infty} \dfrac{7}{4^n}$ is $7/3$.

(h) (2 points) If $0 < a_n < \frac{1}{n}$ $\forall n$, then $\sum\limits_{n=1}^{\infty} a_n$ diverges.

2. (14 points) Let the sequence $\left\{\dfrac{2^n + 3^n}{4^n}\right\}$.

   (a) (6 points) Does it converge or diverge? State the name of any tests you use and check any necessary conditions.

   (b) (8 points) Does the series

$$\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$$

converge or diverge? If it does converge, find its sum. Similarly, as in part (a), make sure to state the name of any tests you use and check any necessary conditions.

3. (20 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n-1}{n^4+2}$$

converges or diverges. Make sure you state which test you use.

4. (20 points) Determine whether the series

$$\sum_{n=1}^{\infty} \left[\ln \left(e^2 + \frac{1}{n} \right) \right]^{n+1}$$

converges or diverges. Make sure you state which test you use.

5. (30 points) Find the **radius** R and **interval** I of convergence of the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{(5x)^n}{3\sqrt{n}+2}.$$

Show all your work and state any tests you used.