Statistics for Business and Economics
6th Edition

Chapter 3

Describing Data: Numerical

Chapter Goals

After completing this chapter, you should be able to:

- Compute and interpret the mean, median, and mode for a set of data
- Find the range, variance, standard deviation, and coefficient of variation and know what these values mean
- Apply the empirical rule to describe the variation of population values around the mean
- Explain the weighted mean and when to use it
- Explain how a least squares regression line estimates a linear relationship between two variables

Chapter Topics

- Measures of central tendency, variation, and shape
  - Mean, median, mode, geometric mean
  - Quartiles
  - Range, interquartile range, variance and standard deviation, coefficient of variation
  - Symmetric and skewed distributions
- Population summary measures
  - Mean, variance, and standard deviation
  - The empirical rule and Bienaymé-Chebyshev rule

Chapter Topics

(continued)

- Five number summary and box-and-whisker plots
- Covariance and coefficient of correlation
- Pitfalls in numerical descriptive measures and ethical considerations

Describing Data Numerically

- **Describing Data Numerically**
  - **Central Tendency**
    - Arithmetic Mean
    - Median
    - Mode
  - **Variation**
    - Range
    - Interquartile Range
    - Variance
    - Standard Deviation
    - Coefficient of Variation

Measures of Central Tendency

Overview

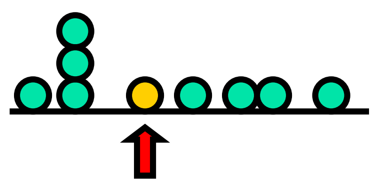
Central Tendency

Mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

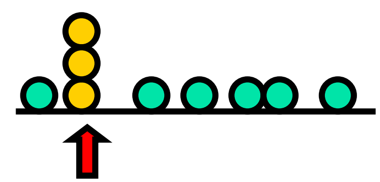
Arithmetic average

Median



Midpoint of ranked values

Mode



Most frequently observed value

Arithmetic Mean

- The arithmetic mean (mean) is the most common measure of central tendency
  - For a population of N values:

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

Population values

Population size

  - For a sample of size n:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

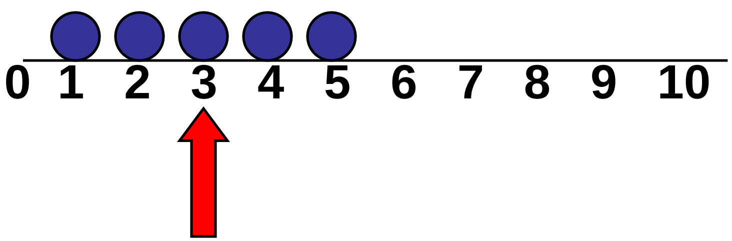
Observed values

Sample size

Arithmetic Mean

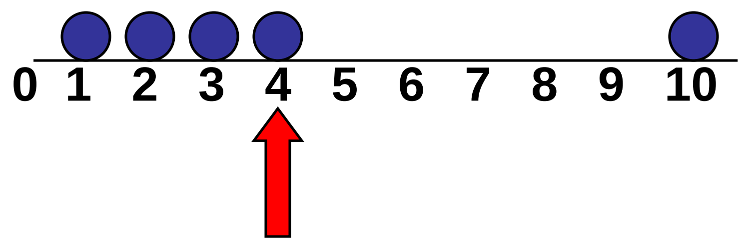
(continued)

- The most common measure of central tendency
- Mean = sum of values divided by the number of values
- Affected by extreme values (outliers)



Mean = 3

$$\frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

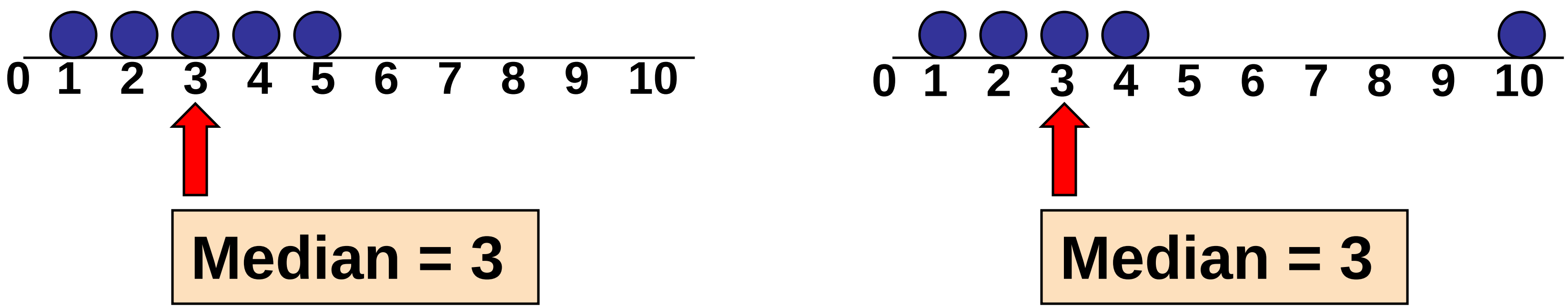


Mean = 4

$$\frac{1+2+3+4+10}{5} = \frac{20}{5} = 4$$

Median

- In an ordered list, the median is the "middle" number (50% above, 50% below)



- Not affected by extreme values

Finding the Median

- The location of the median:

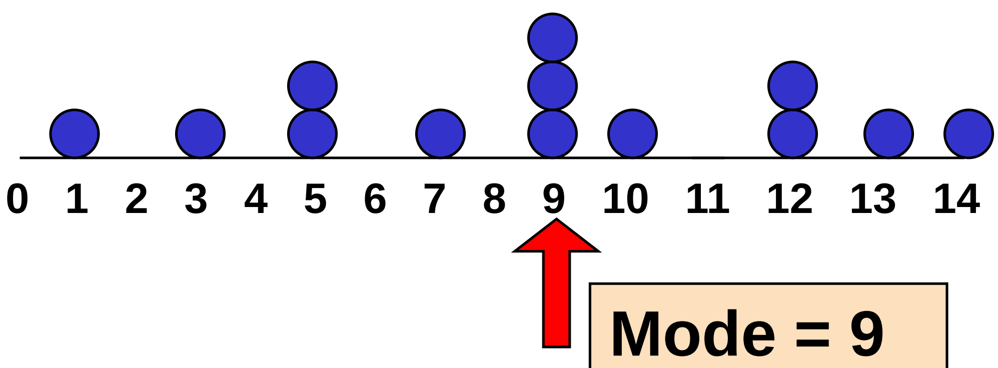
$$\text{Median position} = \frac{n+1}{2} \text{ position in the ordered data}$$

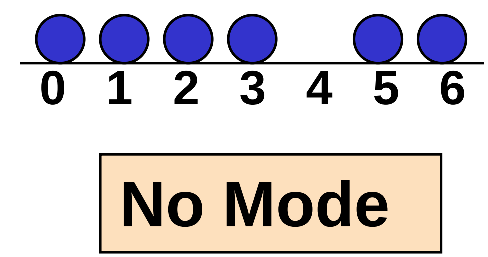
  - If the number of values is odd, the median is the middle number
  - If the number of values is even, the median is the average of the two middle numbers

- Note that $\frac{n+1}{2}$ is not the *value* of the median, only the *position* of the median in the ranked data

Mode

- A measure of central tendency
- Value that occurs most often
- Not affected by extreme values
- Used for either numerical or categorical data
- There may may be no mode
- There may be several modes



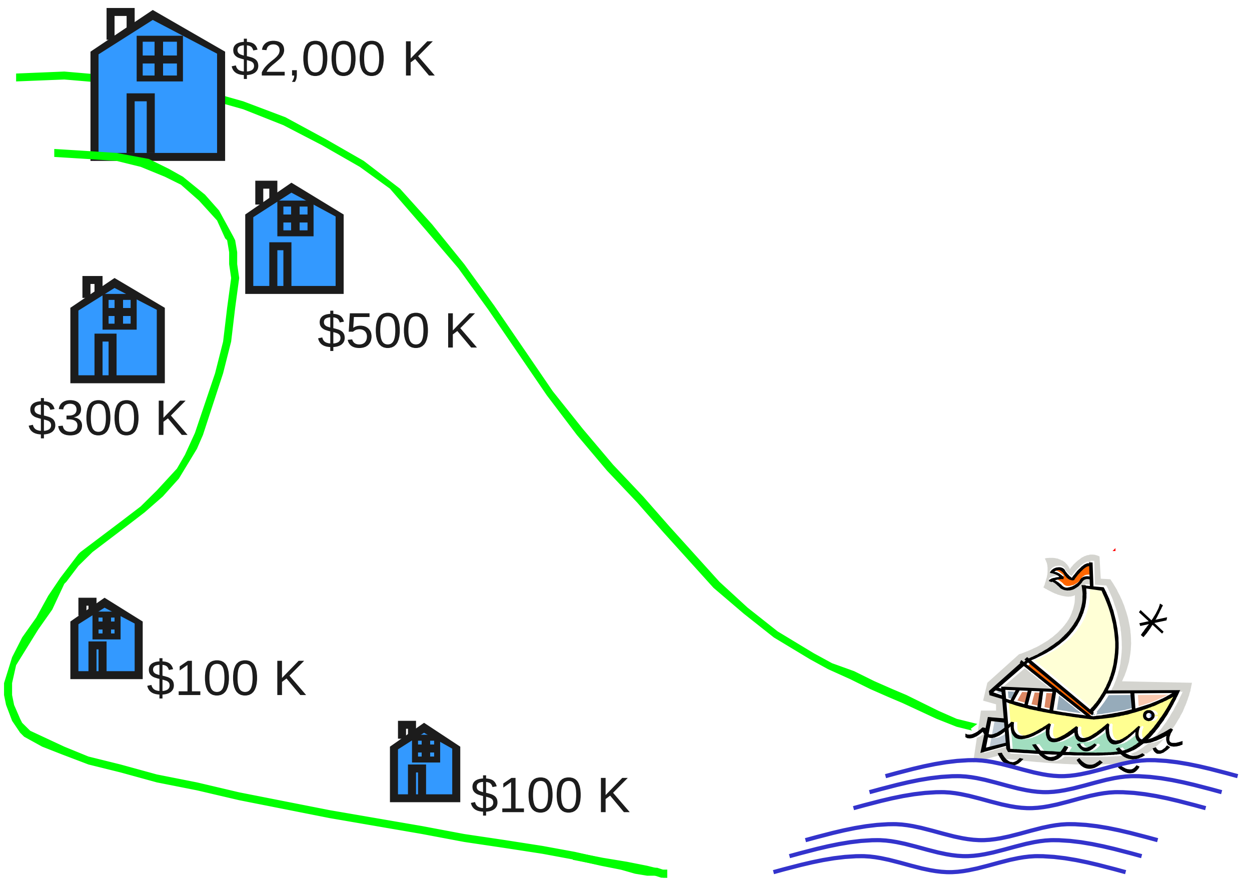


Review Example

- Five houses on a hill by the beach

House Prices:

$2,000,000
500,000
300,000
100,000
100,000



Review Example: Summary Statistics

House Prices:
$2,000,000
500,000
300,000
100,000
100,000
Sum **3,000,000**

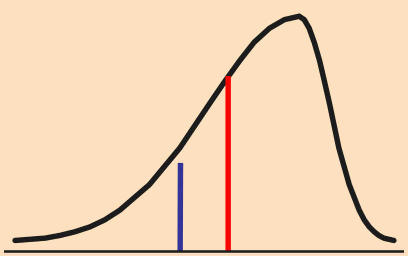
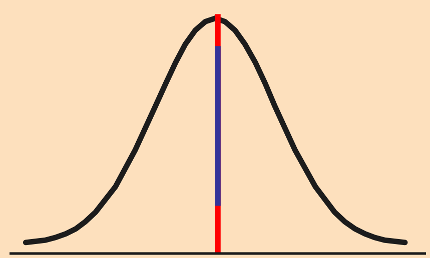
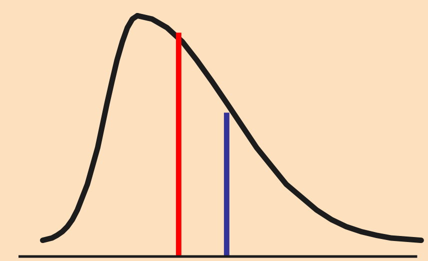
- **Mean:** ($3,000,000/5)
  = **$600,000**
- **Median:** middle value of ranked data
  = **$300,000**
- **Mode:** most frequent value
  = **$100,000**

Which measure of location is the "best"?

- **Mean** is generally used, unless extreme values (outliers) exist
- Then **median** is often used, since the median is not sensitive to extreme values.
  - Example: Median home prices may be reported for a region – less sensitive to outliers

Shape of a Distribution

- Describes how data are distributed
- Measures of shape
  - Symmetric or skewed

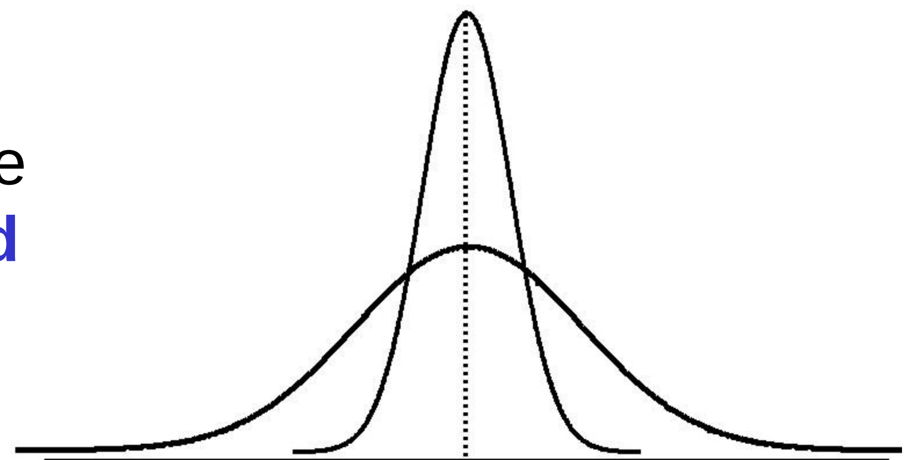
Left-Skewed	**Symmetric**	**Right-Skewed**
Mean < Median	**Mean = Median**	**Median < Mean**
		

Measures of Variability

Variation

- **Range**
- **Interquartile Range**
- **Variance**
- **Standard Deviation**
- **Coefficient of Variation**

- Measures of variation give information on the **spread** or **variability** of the data values.



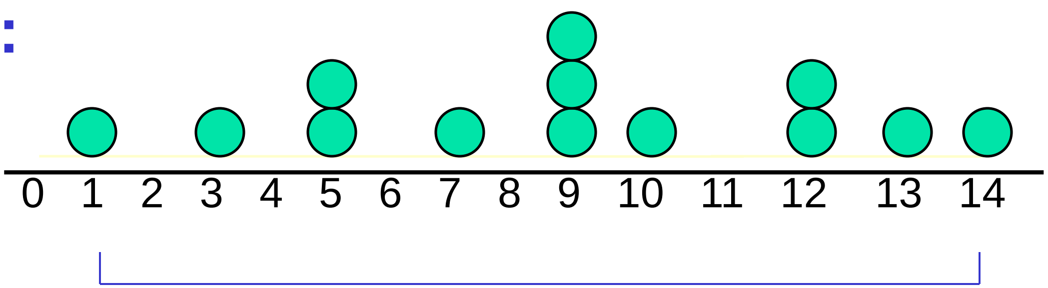
Same center, different variation

Range

- Simplest measure of variation
- Difference between the largest and the smallest observations:

$$\text{Range} = X_{\text{largest}} - X_{\text{smallest}}$$

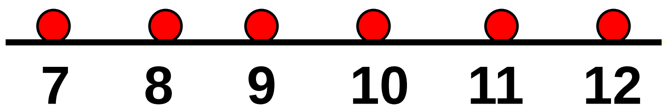
Example:



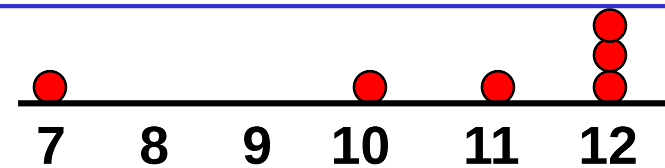
Range = 14 - 1 = 13

Disadvantages of the Range

- Ignores the way in which data are distributed



Range = 12 - 7 = 5



Range = 12 - 7 = 5

- Sensitive to outliers

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,5

Range = 5 - 1 = 4

1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,3,3,3,3,4,120

Range = 120 - 1 = 119

Interquartile Range

- Can eliminate some outlier problems by using the **interquartile range**

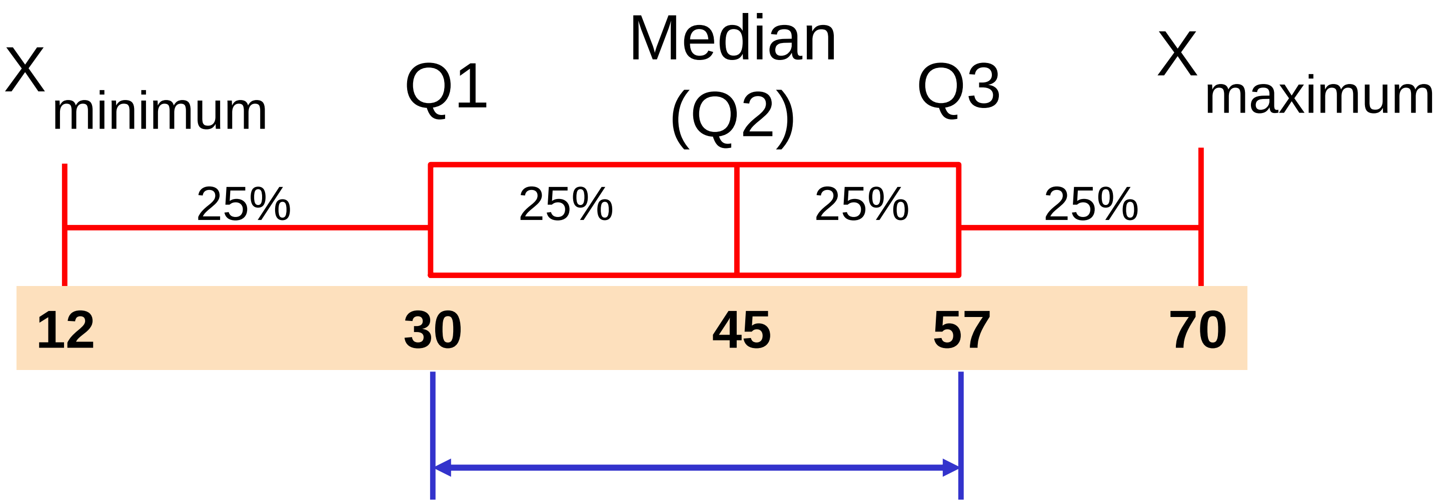
- Eliminate high- and low-valued observations and calculate the range of the middle 50% of the data

- Interquartile range = 3rd quartile – 1st quartile

$$IQR = Q_3 - Q_1$$

Interquartile Range

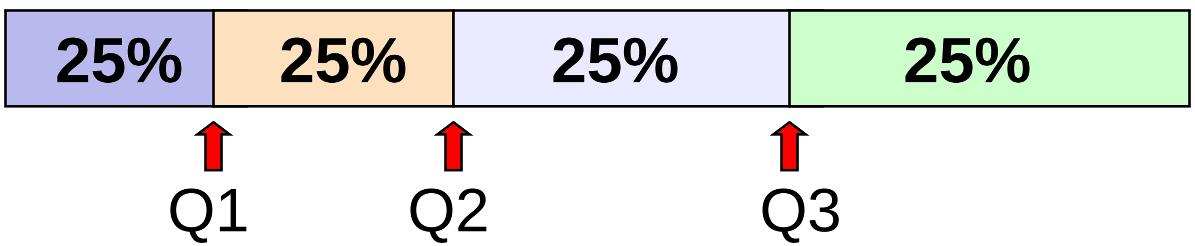
Example:



Interquartile range
= 57 – 30 = 27

Quartiles

- Quartiles split the ranked data into 4 segments with an equal number of values per segment



- The first quartile, Q_1, is the value for which 25% of the observations are smaller and 75% are larger
- Q_2 is the same as the median (50% are smaller, 50% are larger)
- Only 25% of the observations are greater than the third quartile

Quartile Formulas

Find a quartile by determining the value in the appropriate position in the ranked data, where

First quartile position: $Q_1 = 0.25(n+1)$

Second quartile position: $Q_2 = 0.50(n+1)$
(the median position)

Third quartile position: $Q_3 = 0.75(n+1)$

where **n** is the number of observed values

Quartiles

- Example: Find the first quartile

Sample Ranked Data:	**11**	**12**	**13**	**16**	**16**	**17**	**18**	**21**	**22**

(n = 9)

Q_1 = is in the 0.25(9+1) = 2.5 position of the ranked data

so use the value half way between the 2^{nd} and 3^{rd} values,

so **Q_1 = 12.5**

Population Variance

- Average of squared deviations of values from the mean
  - Population variance:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N-1}$$

Where μ = population mean

N = population size

x_i = i^{th} value of the variable x

Sample Variance

- Average (approximately) of squared deviations of values from the mean
  - Sample variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Where $\bar{X}$ = arithmetic mean

n = sample size

X_i = i^{th} value of the variable X

Population Standard Deviation

- Most commonly used measure of variation
- Shows variation about the mean
- Has the same units as the original data

  - Population standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N-1}}$$

Sample Standard Deviation

- Most commonly used measure of variation
- Shows variation about the mean
- Has the same units as the original data

  - Sample standard deviation:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Calculation Example: Sample Standard Deviation

Sample Data (x_i) :

10	**12**	**14**	**15**	**17**	**18**	**18**	**24**

n = 8 Mean = $\bar{x}$ = 16

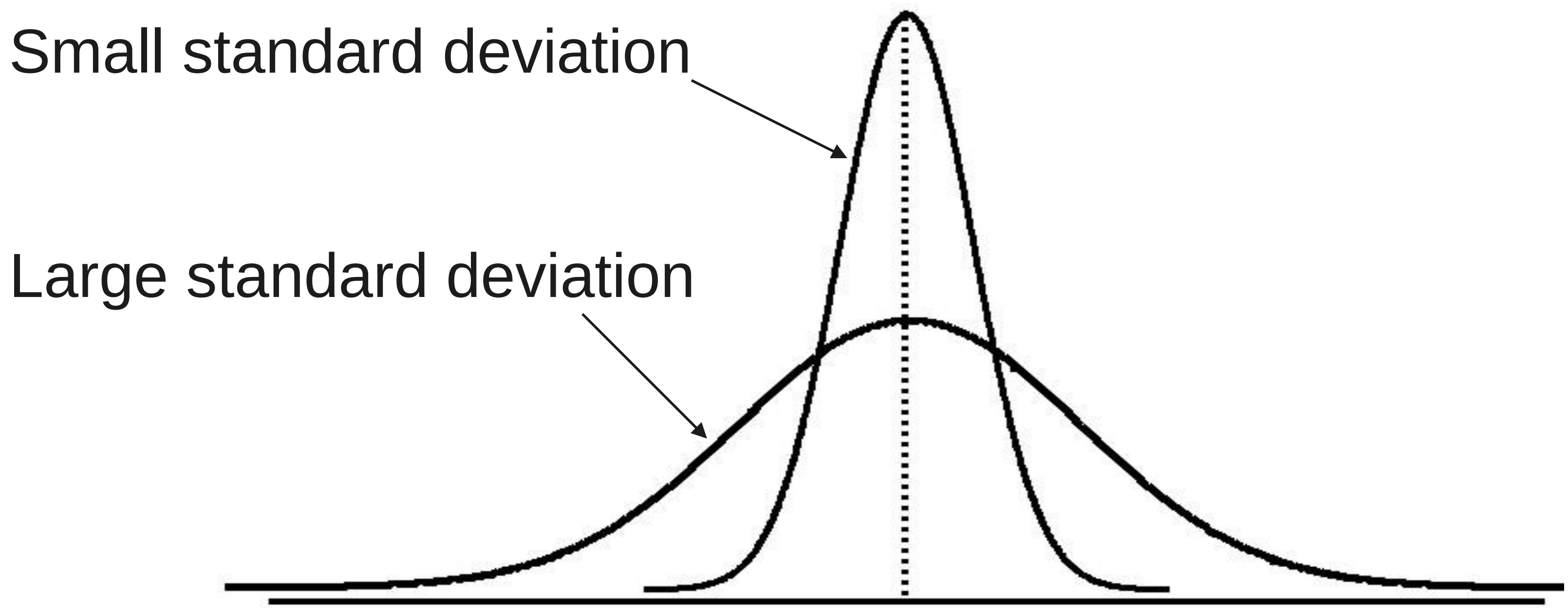
$$s = \sqrt{\frac{(10-\bar{X})^2 + (12-\bar{x})^2 + (14-\bar{x})^2 + \cdots + (24-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(10-16)^2 + (12-16)^2 + (14-16)^2 + \cdots + (24-16)^2}{8-1}}$$

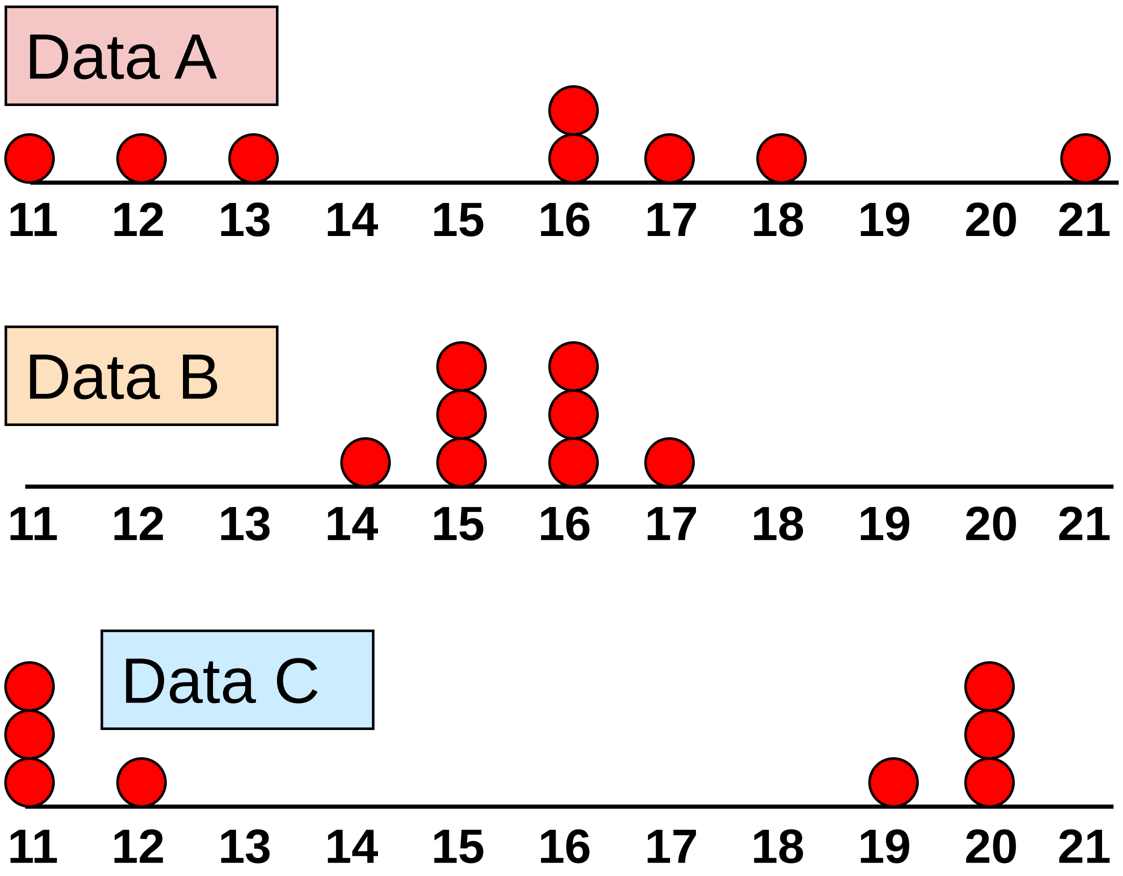
$$= \sqrt{\frac{126}{7}} = \boxed{4.2426}$$

A measure of the "average" scatter around the mean

Measuring variation



Comparing Standard Deviations



Mean = 15.5
s = 3.338

Mean = 15.5
s = 0.926

Mean = 15.5
s = 4.570

Advantages of Variance and Standard Deviation

- Each value in the data set is used in the calculation

- Values far from the mean are given extra weight

  (because deviations from the mean are squared)

Chebyshev's Theorem

- For any population with mean μ and standard deviation σ , and $k > 1$, the percentage of observations that fall within the interval

$$[\mu + k\sigma]$$

Is *at least*

$$\boxed{100[1-(1/k^2)]\%}$$

Chebyshev's Theorem

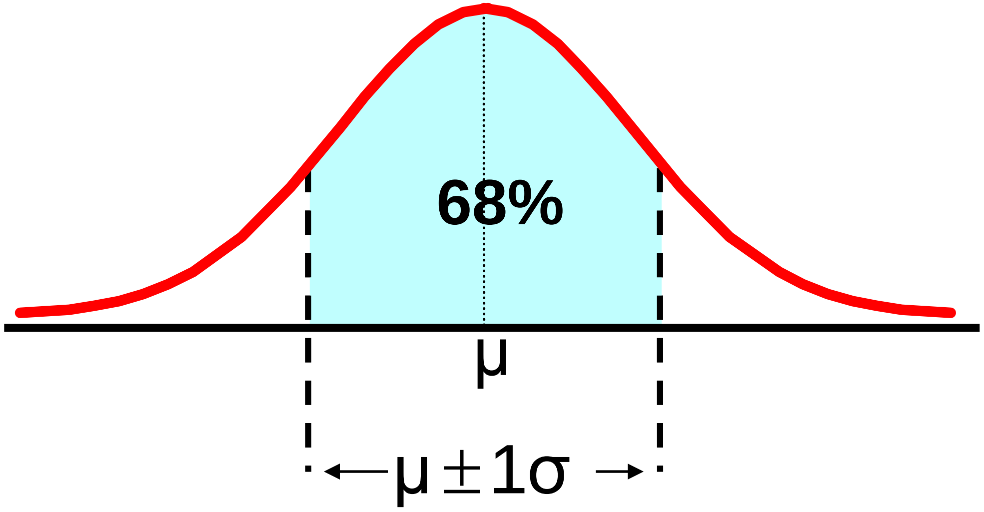
(continued)

- Regardless of how the data are distributed, at least $(1 - 1/k^2)$ of the values will fall within k standard deviations of the mean (for $k > 1$)
  - Examples:

At least	within
$(1 - 1/1^2)$ = 0%	k=1 $(\mu \pm 1\sigma)$
$(1 - 1/2^2)$ = 75%	k=2 $(\mu \pm 2\sigma)$
$(1 - 1/3^2)$ = 89%	k=3 $(\mu \pm 3\sigma)$

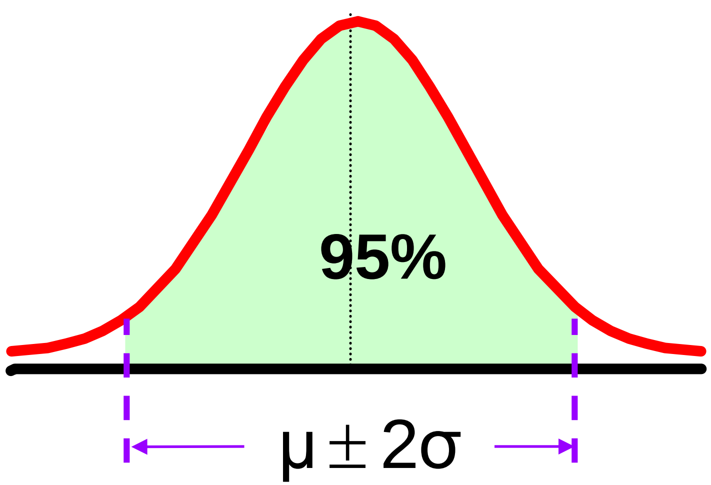
The Empirical Rule

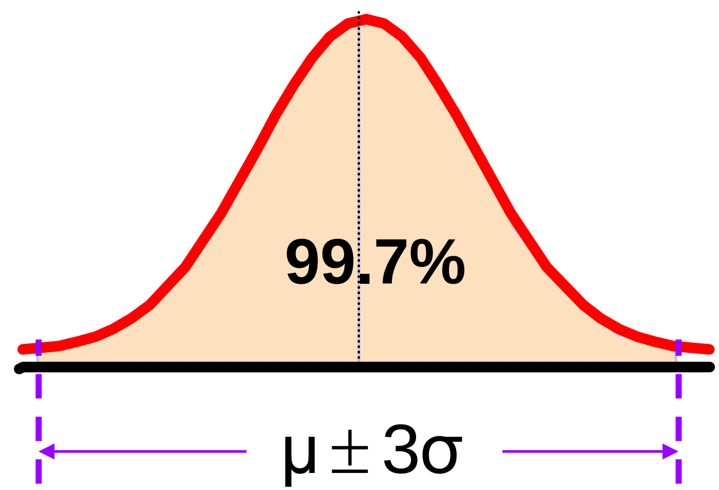
- If the data distribution is bell-shaped, then the interval:
- $\mu \pm 1\sigma$ contains about 68% of the values in the population or the sample



The Empirical Rule

- $\mu \pm 2\sigma$ contains about 95% of the values in the population or the sample
- $\mu \pm 3\sigma$ contains about 99.7% of the values in the population or the sample





Coefficient of Variation

- Measures relative variation
- Always in percentage (%)
- Shows variation relative to mean
- Can be used to compare two or more sets of data measured in different units

$$CV = \left(\frac{s}{\bar{x}} \right) \cdot 100\%$$

Comparing Coefficient of Variation

- Stock A:
  - Average price last year = \$50
  - Standard deviation = \$5

$$CV_A = \left(\frac{s}{\bar{X}}\right) \cdot 100\% = \frac{\$5}{\$50} \cdot 100\% = 10\%$$

- Stock B:
  - Average price last year = \$100
  - Standard deviation = \$5

$$CV_B = \left(\frac{s}{\bar{X}}\right) \cdot 100\% = \frac{\$5}{\$100} \cdot 100\% = 5\%$$

Both stocks have the same standard deviation, but stock B is less variable relative to its price

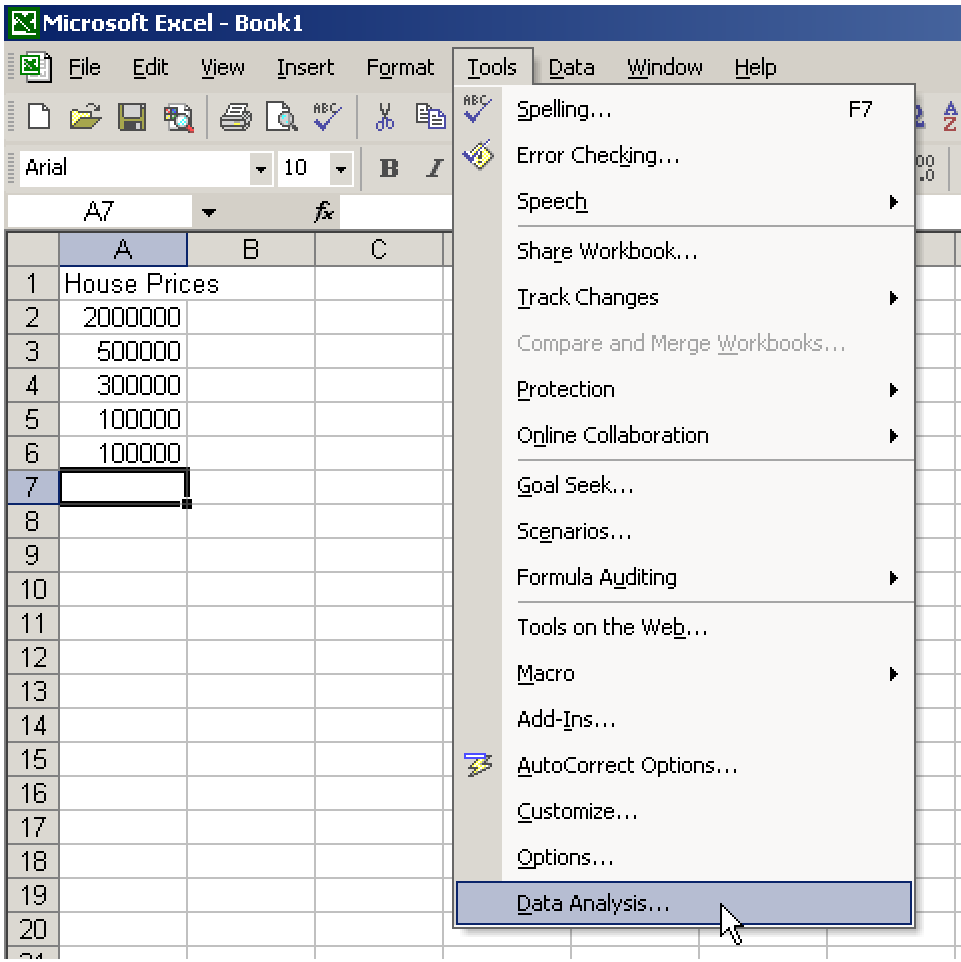
Using Microsoft Excel

- Descriptive Statistics can be obtained from Microsoft® Excel
  - Use menu choice:

    tools / data analysis / descriptive statistics

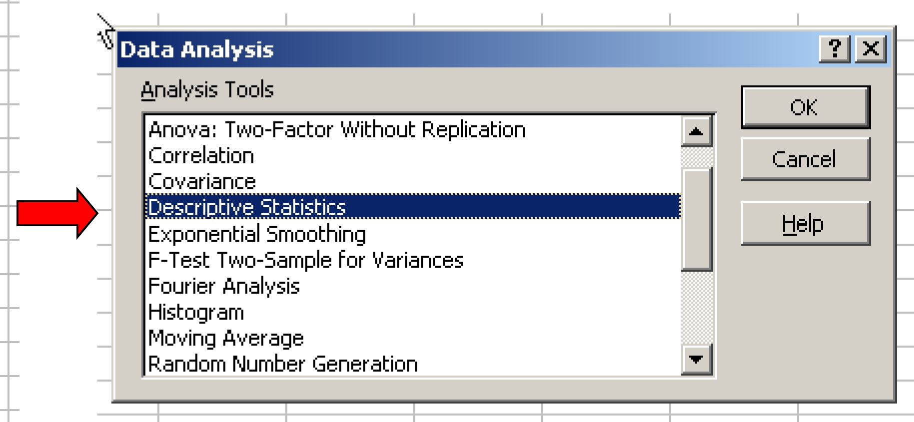
  - Enter details in dialog box

Using Excel



- Use menu choice:

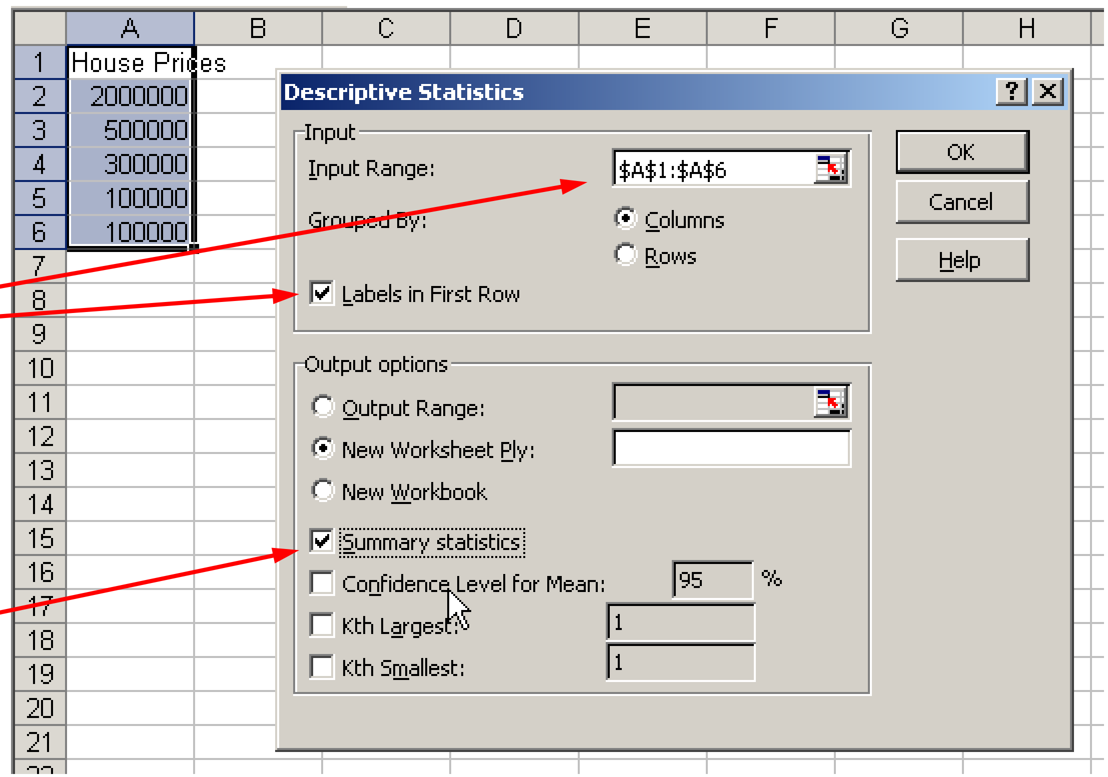
tools / data analysis / descriptive statistics



Using Excel

(continued)

- Enter dialog box details
- Check box for summary statistics
- Click OK



Excel output

Microsoft Excel descriptive statistics output, using the house price data:

House Prices:

$2,000,000
500,000
300,000
100,000
100,000

	A	B
1	*House Prices*	
2		
3	Mean	600000
4	Standard Error	357770.8764
5	Median	300000
6	Mode	100000
7	Standard Deviation	800000
8	Sample Variance	6.4E+11
9	Kurtosis	4.130126953
10	Skewness	2.006835938
11	Range	1900000
12	Minimum	100000
13	Maximum	2000000
14	Sum	3000000
15	Count	5
16		
17		

Weighted Mean

- The weighted mean of a set of data is

$$\bar{X} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum w} = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{\sum w_i}$$

  - Where w_i is the weight of the i^{th} observation

- Use when data is already grouped into n classes, with w_i values in the i^{th} class

Approximations for Grouped Data

Suppose a data set contains values $m_1, m_2, \ldots, m_k$, occurring with frequencies $f_1, f_2, \ldots f_K$

- For a population of N observations the mean is

$$\boxed{\mu = \frac{\sum_{i=1}^{K} f_i m_i}{N}}$$

where $N = \sum_{i=1}^{K} f_i$

- For a sample of n observations, the mean is

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{K} f_i m_i}{n}}$$

where $n = \sum_{i=1}^{K} f_i$

Approximations for Grouped Data

Suppose a data set contains values $m_1, m_2, \ldots, m_k$, occurring with frequencies $f_1, f_2, \ldots f_K$

- For a population of N observations the variance is

$$\sigma^2 = \frac{\sum_{i=1}^{K} f_i (m_i - \mu)^2}{N}$$

- For a sample of n observations, the variance is

$$s^2 = \frac{\sum_{i=1}^{K} f_i (m_i - \bar{x})^2}{n-1}$$

The Sample Covariance

- The covariance measures the strength of the linear relationship between **two variables**
- The population covariance:

$$\mathrm{Cov}(x,y) = \sigma_{xy} = \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{N}$$

- The sample covariance:

$$\mathrm{Cov}(x,y) = s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

  - Only concerned with the strength of the relationship
  - No causal effect is implied

Interpreting Covariance

- **Covariance** between two variables:

Cov(x,y) > 0 $\longrightarrow$ x and y tend to move in the same direction

Cov(x,y) < 0 $\longrightarrow$ x and y tend to move in opposite directions

Cov(x,y) = 0 $\longrightarrow$ x and y are independent

Coefficient of Correlation

- Measures the relative strength of the linear relationship between two variables

- Population correlation coefficient:

$$\rho = \frac{\mathrm{Cov}(x, y)}{\sigma_X \sigma_Y}$$

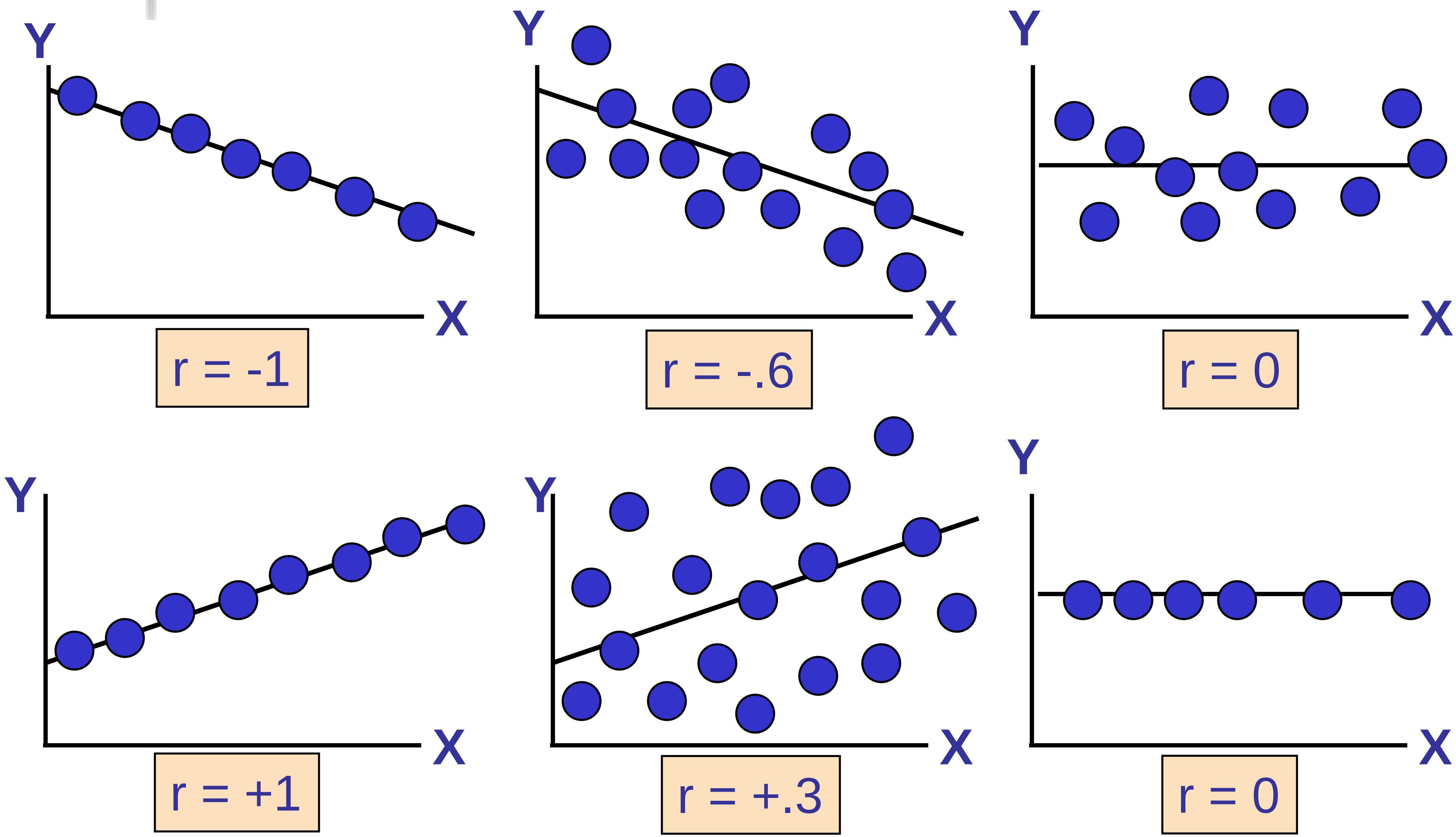
- Sample correlation coefficient:

$$r = \frac{\mathrm{Cov}(x, y)}{s_X s_Y}$$

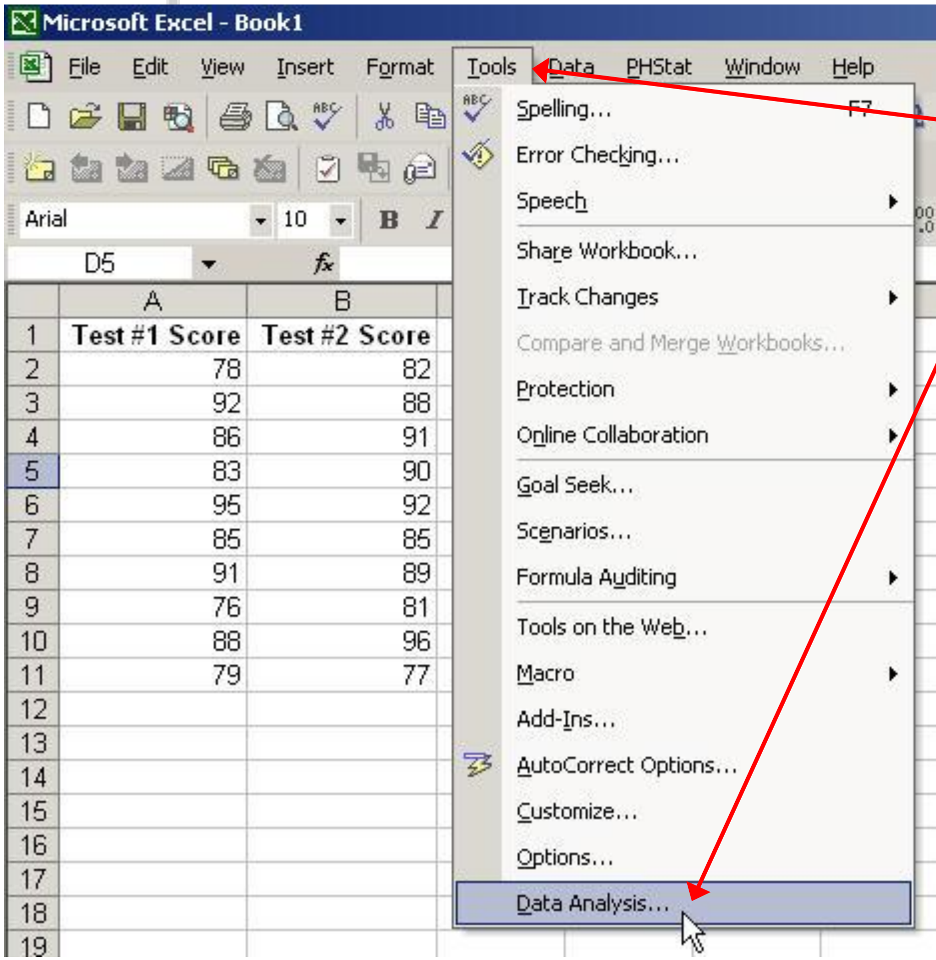
Features of Correlation Coefficient, r

- Unit free
- Ranges between –1 and 1
- The closer to –1, the stronger the negative linear relationship
- The closer to 1, the stronger the positive linear relationship
- The closer to 0, the weaker any positive linear relationship

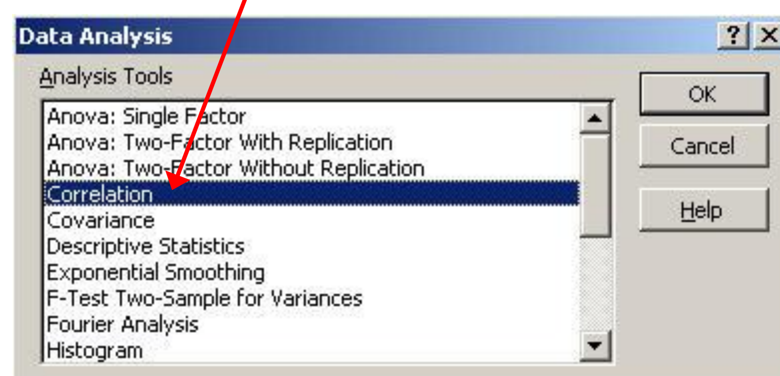
Scatter Plots of Data with Various Correlation Coefficients



Using Excel to Find the Correlation Coefficient

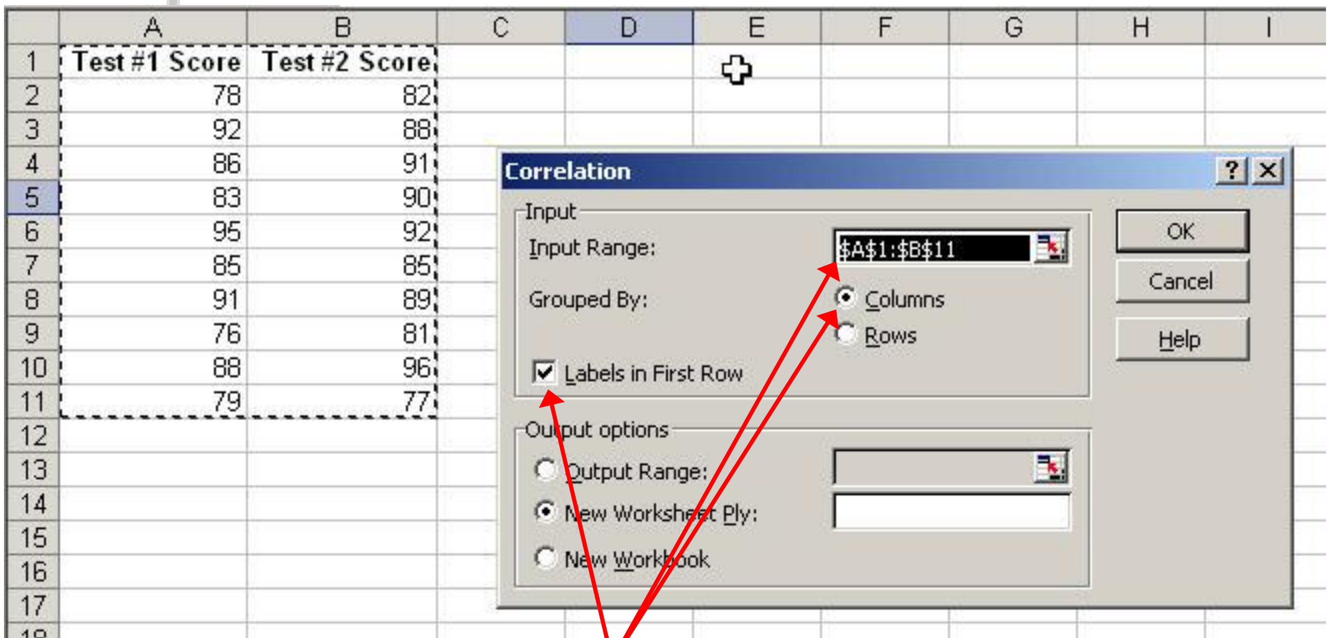


- Select **Tools/Data Analysis**
- Choose **Correlation** from the selection menu
- Click OK . . .

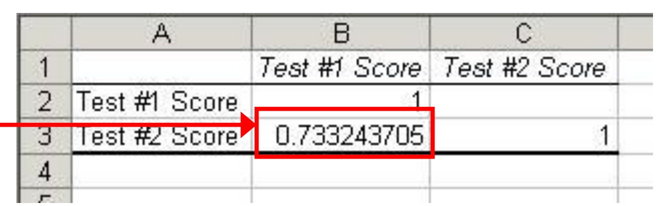


Using Excel to Find the Correlation Coefficient

(continued)

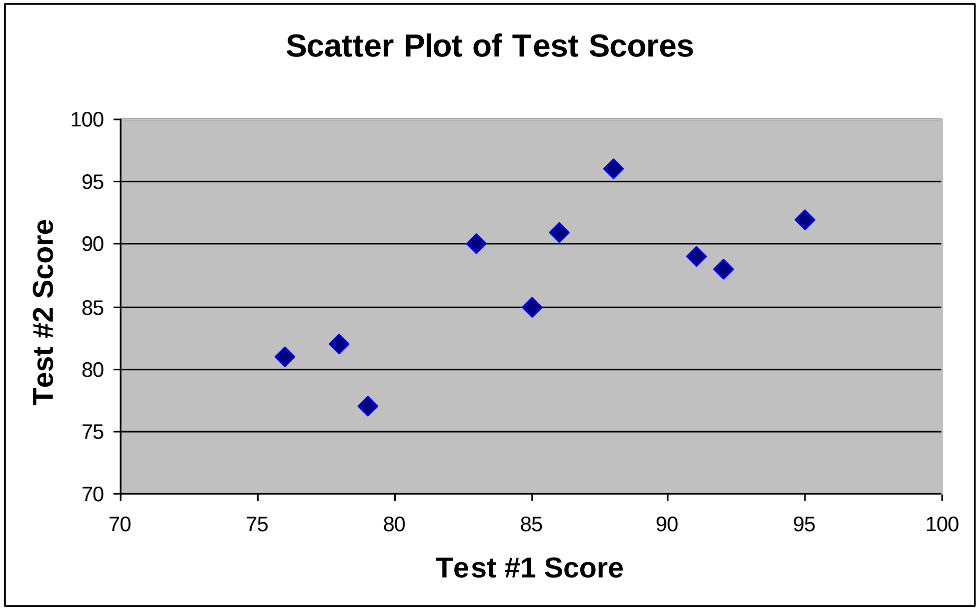


- Input data range and select appropriate options
- Click OK to get output



Interpreting the Result

- r = .733
- There is a relatively strong positive linear relationship between test score #1 and test score #2



- Students who scored high on the first test tended to score high on second test

Obtaining Linear Relationships

- An equation can be fit to show the best linear relationship between two variables:

$$Y = \beta_0 + \beta_1 X$$

Where Y is the dependent variable and X is the independent variable

Least Squares Regression

- Estimates for coefficients β_0 and β_1 are found to minimize the sum of the squared residuals

- The least-squares regression line, based on sample data, is

$$\hat{y} = b_0 + b_1 x$$

- Where b_1 is the slope of the line and b_0 is the y-intercept:

$$b_1 = \frac{\text{Cov}(x,y)}{s_x^2} = r\frac{s_y}{s_x}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

Chapter Summary

- Described measures of central tendency
  - Mean, median, mode
- Illustrated the shape of the distribution
  - Symmetric, skewed
- Described measures of variation
  - Range, interquartile range, variance and standard deviation, coefficient of variation
- Discussed measures of grouped data
- Calculated measures of relationships between variables
  - covariance and correlation coefficient