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Semi-Lagrangian advection scheme with controlled damping: An alternative to nonlinear horizontal diffusion in a numerical weather prediction model

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ABSTRACT: This paper proposes a nonlinear horizontal diffusion scheme for models using semi-Lagrangian formulations. The scheme is made flow dependent and not entirely linked to the model levels. As an extension, the implementation of the scheme to the model Aladin is given. The damping abilities of interpolation are used for the diffusion filtering. The aim is to provide a horizontal diffusion scheme of similar stability and computational efficiency as the existing linear spectral diffusion scheme in Aladin. Preserving such qualities, the new scheme brings beneficial new skills to the model. The differences between the performances of the two diffusion schemes are examined and discussed. Finally, some interesting case-studies simulated with both horizontal diffusion schemes are presented. Copyright © 2008 Royal Meteorological Society

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1. Introduction

The use of horizontal diffusion schemes has been a common feature in numerical models since the beginning of numerical weather prediction (NWP) (e.g. Shuman, 1957). The reasons for incorporating horizontal diffusion inside a model (beside the formal mathematical reason to avoid hyperbolic equations) are at least threefold.

First, we wish to represent the horizontal effects of turbulence and molecular dissipation acting on smaller scales than the ones resolved by the model. The horizontal diffusion is from this point of view some kind of physical parametrization which is introduced in such a way as to be consistent with the dissipation theory for viscous fluids.

Second, we need to avoid the accumulation of energy at the small-scale end of the bounded model spectrum. The cascading energy propagating in the atmosphere from the large scales down to the molecular scales accumulates in any numerical model at the end of its representation spectrum and may then propagate upwards back towards the larger scales, thus contaminating the model results. The aim of horizontal diffusion, from this point of view, is to keep the slope of the model-resolved kinetic energy and enstrophy spectra close to those derived theoretically (Leith, 1970; Chen and Wiin-Nielsen, 1978).

Third, we need to damp as much as acceptable the waves which have lost their predictive skill and thus improve the model forecast scores. These are mainly the short waves, so that diffusion defined from this point of view is preferably acting on the smallest scales resolved by a model. The criterion to calibrate such diffusion is to obtain the smallest possible model errors while keeping a reasonable cascade in the above-mentioned sense.

Traditionally, the design of the horizontal diffusion scheme in atmospheric numerical models makes use of the operator $K\nabla^r$ applied to the diffused fields, the order of diffusion r being an even integer. Although such formulations are based mostly on experience and possess only little theoretical or observational foundation (Koshyk and Boer, 1995), they result in suitable kinetic energy and enstrophy spectra in model simulations. In this approach, it is sufficient for the horizontal diffusion coefficient K to be a pure constant. The same condition for K was proposed by Jakimow *et al*. (1992), when the horizontal diffusion is regarded as a numerical process optimizing forecast results with respect to verification scores.

This is not the case when the diffusion is viewed as a physically based process. Then, the $K\nabla^r$ operator can also be considered as representing turbulence and molecular dissipation (Sadourny and Maynard, 1997), and the coefficient K should be made flow dependent, typically as a function of deformation and divergence of the flow (Smagorinski, 1963; Sadourny and Maynard, 1997). Such operators are nonlinear and the computation

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of the associated source terms is not straightforward in a spectral formalism.

The explicit formulation of diffusion only results in conditional stability, which is sometimes too restrictive regarding the time step (as shown in Jakimow *et al.*, 1992). Implementation of implicit horizontal diffusion schemes, allowing more stable time stepping, turns out to be quite expensive in grid-point models (Gustafsson and McDonald, 1996). On the other hand, the spectral representation of model fields allows an efficient implementation of a spatial smoothing, with a precise control of the scale selectivity, and preservation of the total mass and energy (Sardeshmukh and Hoskins, 1984). The implicit linear diffusion operator is then ideally suited to spectral models (Laursen and Eliasen, 1989). When a nonlinear operator is required to represent the horizontal damping processes, its formulation becomes relatively expensive for any discretization technique. The problem is further complicated by the fact that even implicit nonlinear horizontal diffusion (with an explicit exchange coefficient) is generally just conditionally stable.

For all these reasons, the physical role of horizontal diffusion used to be neglected. This trend is further supported by the fact that the horizontal physical processes of turbulence and molecular exchange start to be comparable with the other horizontal forcings (e.g. Coriolis force, advection and pressure gradient force) at horizontal scales smaller than about 1 km. Consequently, horizontal diffusion used to be represented by a linear operator using all the numerical benefits it gives. This approach is adopted in many numerical models. However it is questionable whether this simplification is realistic, especially when considering that, even at horizontal scales of around 10 km, there can be model features for which the horizontal component of mixing may dominate the vertical one (sloped terrain, cyclones). Since the vertical mixing is typically always represented by the model, the missing horizontal component may be the source of unrealistic consequences in such areas.

This paper proposes an alternative way to introduce a nonlinear horizontal diffusion in models with semi-Lagrangian (SL) transport schemes. Thanks to its highly stable and accurate time stepping with fairly long time steps in comparison to other methods, these schemes are becoming increasingly popular in NWP models (Staniforth and Côté, 1991, 1998). Due to the necessary interpolation at every time step, the SL advection scheme is a source of uncontrolled damping in model simulations. The principle of the proposed diffusion scheme is to turn this accepted weakness into a useful property. The method is based on the control of the degree of damping resulting from the interpolation process of the SL scheme. Thus it will be referred hereafter as 'semi-Lagrangian horizontal diffusion' scheme (SLHD). Since even in spectral models SL computations are performed in physical space, it is relatively easy to locally control the damping characteristics of interpolators through a criterion defined as a function of the local flow. The aim of the proposed scheme is therefore to provide an alternative to the standard linear diffusion, offering nonlinear capabilities at an acceptable computational extra cost and without additional stability constraints.

The scheme presented here has been introduced, tuned and validated in the spectral SL limited-area models Aladin and Arome. The first model, developed through international cooperation at Météo-France (Horányi *et al.*, 2006) is currently used as the operational NWP model for about fifteen European and North African countries. In this paper, the hydrostatic dynamical core with the operational physical package used around 2004, which describes relatively simple physics with a focus on high computational efficiency, will be referred to as Aladin. Extending the operational hydrostatic dynamical core of Aladin to fully elastic non-hydrostatic equations, something available through a logical switch (Bubnová *et al.*, 1995; Bénard *et al.*, 2005) and coupling it to the physics of the Méso-NH model (Lafore *et al.*, 1998), which is much more complete and allows an accurate description of processes occurring at meso-γ scales, forms the main basis of the future operational forecast model named Arome. The SLHD scheme has been implemented for both hydrostatic and non-hydrostatic dynamics, and thus it is available in both model versions. In this paper, some of the results will be presented for the current operational model Aladin, and some for the future Arome one.

The general formulation of the SLHD scheme is given in Section 2. Section 3 describes the concrete implementation of the SLHD into the Aladin spectral limited-area model. Section 4 describes the typical SLHD behaviour when it is activated instead of a linear horizontal diffusion scheme. Some specific case-studies are shown and discussed in Section 5, and finally some conclusions are summarized in Section 6.

2. Formulation of the semi-Lagrangian horizontal diffusion scheme

The general form of the SL forecast equation for any prognostic variable X may be written as

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \mathcal{R},$$

where $\mathcal{R}$ is the total Lagrangian source term for X. The two-time-level extrapolating SL discretization of this equation (Temperton *et al.*, 2001) is written as

$$X_{\mathrm{F}}^{+} = X_{\mathrm{O}}^{-} + \Delta t \mathcal{R}_{M}^{*}, \tag{1}$$

where $X_{\mathrm{F}}^{+} = X(\mathbf{x}, t + \Delta t)$ is the X value at the final point at time $t + \Delta t$, $X_{\mathrm{O}}^{-} = X(\mathbf{x} - \alpha, t)$ is the X value at the origin point at time t, $\mathcal{R}_{M}^{*} = \mathcal{R}(\mathbf{x} - \alpha/2, t + \Delta t/2)$ is the right-hand side (RHS) evaluated at the middle point of the particle trajectory at time $t + \Delta t/2$ (through a simple time extrapolation), and α is the displacement vector, determined by an iterative procedure at every time step.

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In this paper, for simplicity, all time-discrete aspects about diffusion will be illustrated only for two-time-level extrapolating time discretizations, but any discussion and result may be easily extended to other types of time discretization (e.g. three-time-level leap-frog schemes).

Since the departure and middle points of the trajectory are generally off the model grid, an interpolation method has to be used for the evaluation of all terms in the RHS of (1). The interpolator is chosen in such a way as to be sufficiently accurate while offering acceptable computational efficiency. Typically a cubic polynomial interpolator is found to be best suited for numerical weather simulations (Staniforth and Côté, 1991).

In most models, the interpolation operator I used to evaluate the RHS of (1) is constructed as a combination of cubic polynomial interpolation on 3D or 2D stencils. The SLHD interpolation operator extends this situation by the use of an additional interpolation operator with extra damping properties. The resulting operator I then becomes a combination of two interpolators: the 'accurate' one denoted as I_{A} and the 'damping' one I_{D}. Formally the SLHD operator can then be expressed as:

$$\begin{aligned} I &= (1-\kappa)I_{\mathrm{A}} + \kappa I_{\mathrm{D}} \\ &= I_{\mathrm{A}} + \kappa(I_{\mathrm{D}} - I_{\mathrm{A}}). \end{aligned} \tag{2}$$

Interpreting the difference $I_{\mathrm{D}} - I_{\mathrm{A}}$ as the damping contribution of the operator I_{D} with respect to the accurate interpolation I_{A}, the coefficient κ can be seen as a diffusion coefficient; with $\kappa \equiv 0$, no extra damping is present and the original accurate interpolation is restored.

The fact that I is evaluated in grid-point space makes it possible to define κ as a general function of any atmospheric field. Ideally, the definition of κ should be inspired by fluid mechanics theories, in order to introduce a high degree of physical realism in what would then become a sophisticated diffusion scheme. However, by nature, the approach chosen here is clearly a simplified one (e.g. the notion of turbulent flux is simply ignored), and the aim of the scheme is rather to access nonlinear diffusive sources at marginal cost, in view of improving NWP applications. Consistent with this choice, the definition of κ should therefore remain relatively simple and computationally efficient. In their formulation of lateral diffusion for fluid dynamics models, Sadourny and Maynard (1997) express the dissipation of the horizontal velocity as a sum of isotropic and anisotropic parts. The isotropic part, with the physical meaning of dynamical pressure due to the sub-grid scale, plays an important role only in strong incompressibility cases. Hence, for most NWP models, the term of main relevance is the anisotropic part of the dissipation. This term is related to the tensor of the deformation rates. It can be further simplified by setting proportional to the total deformation d along the reference surface a scalar quantity having the dimension of inverse time and being defined as

$$d = \frac{1}{2}\left[\left(\frac{h_2}{h_1}\frac{\partial}{\partial x_1}\frac{u_1}{h_2} - \frac{h_1}{h_2}\frac{\partial}{\partial x_2}\frac{u_2}{h_1}\right)^2 + \left(\frac{h_1}{h_2}\frac{\partial}{\partial x_2}\frac{u_1}{h_1} + \frac{h_2}{h_1}\frac{\partial}{\partial x_1}\frac{u_2}{h_2}\right)^2\right]^{1/2}. \tag{3}$$

Here u_1, u_2 are the x_1, x_2 components of the horizontal velocity field, with metric factors h_1 and h_2. In a way similar to Smagorinski (1963), the coefficient κ in (2) is defined as a monotonic function $f(d)$ of the total deformation d. Regarding the value of I_{D} as an equilibrium value, the second term in the RHS of (2) can be seen as a relaxation source term. Using the standard expression for implicit schemes with explicit relaxation coefficient (following for example the suggestion of Jarraud in Kalnay and Kanamitsu, 1988) κ can be generally expressed as a bounded function of the time step Δt in a following way:

$$\kappa = \frac{f(d^-)\Delta t}{1 + f(d^-)\Delta t}. \tag{4}$$

In this equation, d^- represents the total deformation evaluated at time t. Note that substituting in (2) the definition (4) of κ makes the SLHD scheme equivalent to an implicit relaxation of I_{A} toward the operator I_{D}. This has the advantage of ensuring the stability of the scheme for any time step Δt while keeping a simple evaluation of the function $f(d^-)$ at time t. Consistently with the SL discretization, $f(d^-)$ should be evaluated at the origin point of the trajectory, but in order to avoid an interpolation bringing some degree of smoothing and to save computational cost, $f(d^-)$ is evaluated at the final point instead. The experimental results justify this simplification as the correlation of $d(\mathbf{x}, t)$ and $d(\mathbf{x} - \alpha, t)_{\mathrm{interp}}$ is usually very high, over 90%. Moreover the difference between the simplified and SL consistent discretization consists of meteorologically insignificant noise, even for active storms typically sensitive to horizontal diffusion.

The damping interpolator I_{D} now remains to be defined. Classical interpolators are always prescribed so that they become exact at the 'nodes', where the value of the field is known, and this is true of course even for the most damping ones among them. As a consequence, the damping character of the interpolator in a SL scheme is only 'statistical', because the distribution of origin points is not likely to be correlated with the computational grid. From a purely local point of view, classical interpolators are not damping when the origin point lies on a node. In order to alleviate this non-physical dependency on the actual position of the origin point, a simple solution is to define the damping interpolator as a combination of a classical interpolator, and of a local – pointwise defined – diffusion operator. This is the option retained in the SLHD scheme presented below, where a linear interpolator is combined with a pointwise local second-order diffusion operator. This design, using local information only, can be achieved as a very simple modification of any pre-existing SL scheme.

As will be shown in the next sections, the proposed design for the nonlinear diffusion scheme, in spite of

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the above-mentioned simplifications, results in improved performances in typical NWP applications, at a relatively small increase of computational cost.

3. Implementation of the SLHD into the Aladin model

3.1. General description

As an illustration of the general SLHD scheme presented in the previous section, the particular application to the limited-area spectral model Aladin will be described here.

The transport scheme used for all applications reported in this paper is a two-time-level semi-implicit SL scheme (Temperton *et al.*, 2001) with quasi-cubic interpolations (Ritchie *et al.*, 1995), which makes a space averaging of dynamical source terms along the trajectory (Tanguay *et al.*, 1992). The total Lagrangian source term can be split into an implicitly treated dynamical linear part $\mathcal{L}X$, an explicitly-treated dynamical nonlinear residual $\mathcal{N}$, and a parametrized physical part $\mathcal{F}$. The time- and space-discretization of the Aladin model can therefore be summarized as

$$\frac{X_{\mathrm{F}}^{+} - X_{\mathrm{O}}^{-}}{\Delta t} = \frac{1}{2}\left(\mathcal{L}X_{\mathrm{O}}^{-} + \mathcal{L}X_{\mathrm{F}}^{+}\right) + \frac{1}{2}\left(\mathcal{N}_{\mathrm{O}}^{*} + \mathcal{N}_{\mathrm{F}}^{*}\right) + \mathcal{F}_{\mathrm{O}}^{-}.$$

The subscripts and superscripts have the same meaning as in (1). The Helmholtz equation to be solved for X_{F}^{+} is then:

$$X_{\mathrm{F}}^{+} = \left(1 - \frac{\Delta t}{2}\mathcal{L}\right)^{-1} \times \left[\underbrace{X_{\mathrm{O}}^{-} + \frac{\Delta t}{2}\mathcal{L}X_{\mathrm{O}}^{-} + \Delta t\mathcal{F}_{\mathrm{O}}^{-} + \frac{\Delta t}{2}\mathcal{N}_{\mathrm{O}}^{*}}_{I_{\mathrm{n}}} + \frac{\Delta t}{2}\mathcal{N}_{\mathrm{F}}^{*}\right]. \tag{5}$$

The terms evaluated at the origin point (horizontal brace denoted by I_{n}) are obtained by interpolation. In the original SL scheme of Aladin, the first and third terms in the horizontal brace are computed with a quasi-cubic interpolator (Ritchie *et al.*, 1995), while the others are obtained by simple linear interpolations. The default horizontal diffusion is then a fourth-order spectral linear diffusion scheme (Geleyn, 1998). When the SLHD scheme is activated, the quasi-cubic interpolator is replaced by the operator I defined in (2) where I_{A} represents the original quasi-cubic interpolator. From (5) it is clear that the term

$$\left(1 - \frac{\Delta t}{2}\mathcal{L}\right)^{-1} \frac{\Delta t}{2}\mathcal{N}_{\mathrm{F}}^{*}$$

is not directly affected by the SLHD as this term is not subjected to any interpolation.

For simplicity, the parameter κ is taken as identical for all the prognostic variables to which the horizontal diffusion is applied. The value of κ is given by (4) with the function $f(d^{-})$ defined as

$$f(d) = a\,d\left(\max\left[1,\ \frac{d}{d_0}\right]\right)^{B}, \tag{6}$$

where a is the scaling factor for the SLHD scheme, d_0 is the threshold value of total deformation d above which an enhanced damping is activated and B is a tunable parameter for the control of this enhancement. The presence of the enhancement part allows an increased selectivity by affecting smaller areas with a stronger diffusive effect, as discussed later. In order to make the tuning of the scheme independent of the resolution and domain size, the a and d_0 parameters are expressed as functions of the 'spectral' mesh size $[\Delta x]$ (i.e. the inverse of the maximum wavenumber):

$$a = 2A\left(\frac{[\Delta x]_{\mathrm{ref}}}{[\Delta x]}\right)^{P}, \tag{7}$$

$$d_0 = \frac{D}{2}\left(\frac{[\Delta x]_{\mathrm{ref}}}{[\Delta x]}\right)^{Q}, \tag{8}$$

where A, D are the tunable parameters, and P, Q are empirical constants for maintaining independence of A and D to the domain definition. The $[\Delta x]_{\mathrm{ref}}$ is a reference value for the mesh size.

Optimal values of P and Q have been found empirically as 1.7 and 0.6 respectively within the range of useful resolutions for the Aladin model. (For grid-point resolution varying between 2.2 and 17 km with both quadratic and linear truncations, the corresponding shortest spectral wave varies approximately between 4.5 and 52 km). The value of Q is in good agreement with the conclusions of Leith (1969). There the total deformation d is expressed as proportional to the mesh size to the power 2/3 in models with truncation lying within an inertial range of a $-5/3$ kinetic energy power spectrum. The three tunable parameters affecting the scheme through κ are then A, B and D. In the present scheme they are by default set to 0.25, 4.0 and 6.5×10^{-5}, respectively.

The damping operator I_{D} is defined by

$$I_{\mathrm{D}} = I_{\mathrm{L}}\circ\left[1 + \frac{\Gamma(h)}{2}D_2\right], \tag{9}$$

where I_{L} is the linear interpolator and D_2 is the non-dimensional finite-difference Laplacian operator, classically computed on a three-point stencil

$$(D_2\,X)_i = X_{i+1} - 2X_i + X_{i-1}.$$

In this definition, $\Gamma(h)$ is a function which serves to reduce the dependency of the smoothing effect of I_{D} to the space-shift h between the origin point of the SL trajectory and the computational grid. As mentioned above, I_{L} has no damping effect when the origin point is located at a node of the grid ($h = 0$), but has a maximum damping effect when the origin point is in the middle

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of two nodes ($h = 0.5\Delta x$). Therefore $\Gamma(h)$ is defined in such a way as to have a maximum positive value for $h = 0$ and to reach zero for a given value $h_0 < 0.5\Delta x$. This arrangement provides I_D with a more homogeneous damping with respect to h. The effect of I_D on one-dimensional waves with various wavelengths is illustrated by the response for some constant values of the space-shift h, as shown in Figure 1, where $\Delta x = 1$ is assumed. In Aladin we choose $h_0 = 0.15\Delta x$, defining that 51% of the computational area is affected by the operator D_2. Finally, to ensure time-step independence, Γ is scaled to a reference time step. From experience with real tests, the performance of the SLHD is only weakly sensitive to the presence of D_2, mainly because the sum of the function Γ in [0, 1] is rather small.

In the above, the diffusive properties of the accurate interpolation operator I_A representing the inherent damping of the SL scheme have not been considered. The proposed scheme has no ambition to control this non-excessive diffusivity of the operator I_A. However, as it is known that the role of such diffusion decreases with increasing model resolution (Jakimow *et al.*, 1992), the empirical values P and Q of the SLHD are surely also affected by this feature.

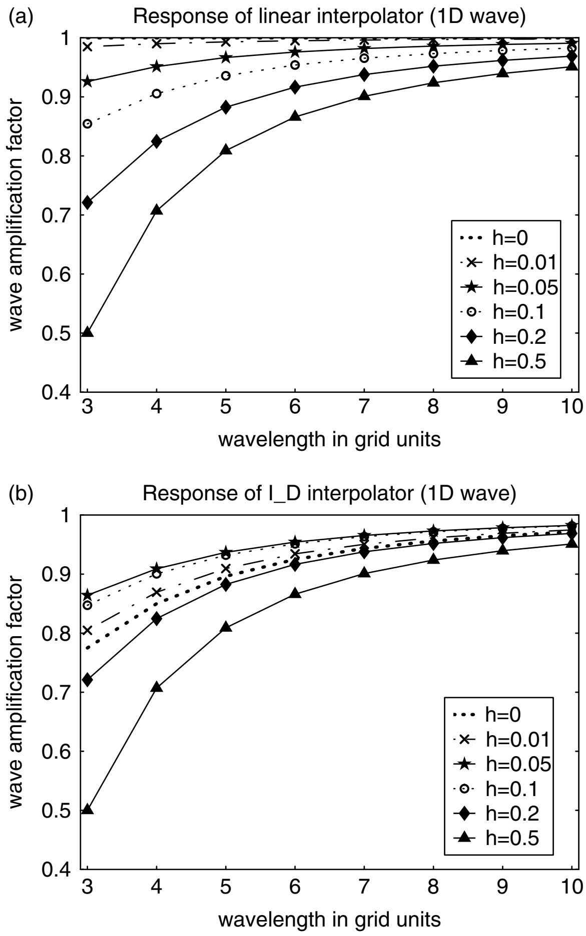

Figure 1. Response of a sinusoidal wave caused by (a) linear interpolation and (b) the I_D operator used in Aladin, as a factor of the distance h from the closest grid-point on a unitary grid (mesh size equal to one).

3.2. Specific implementation features

This definition of the SLHD scheme has to cope with two specific problems requiring further attention. As already mentioned, the SL interpolation does not affect the whole RHS of (5), and consequently, the SLHD scheme cannot control directly all the source terms of the model. Generally, this has been found in experiments not to result in serious problems, since the non-interpolated part remains indirectly affected by the SLHD scheme during the evolution. The only exception arises when this non-interpolated part contains a forcing term which is constant in time, as e.g. the orographic forcing (which acts on horizontal and vertical velocity variables in the momentum equations). With the pure SLHD scheme, the spectrum slope for these variables near the surface is found to closely follow the one of the orography, indicating that the SLHD scheme does not offer an efficient mean to control the scale-selectivity at the end of the spectrum for these variables. This feature is illustrated in Figure 2 showing kinetic energy spectra at the lowest level of the model domain in adiabatic mode after 3 hours of forecast time; in the control experiment (cubic interpolator and fourth-order numerical diffusion), the energy spectrum slope is found to depart significantly from the orography spectrum slope near the largest resolved wavenumbers, thereby confirming the ability of the numerical diffusion to efficiently control the scale-selectivity. In contrast, experiments using either the I_A and the I_D interpolators (and no numerical diffusion) show that in this case, the spectrum slope is entirely controlled by the orography spectrum. Since the SLHD scheme is a combination of these two interpolators, it clearly does not allow a proper control of the selectivity near the end of the resolved spectrum.

Various solutions to this problem may be proposed. A natural one would be to impose interpolations to the whole RHS of (5), but this would imply interpolations at the middle points of SL trajectories for the $\mathcal{N}$ term, thereby losing the advantage of spatial averaging of the source terms along the trajectory (Tanguay *et al.*, 1992). Another solution could be to filter the model orography itself, in such a way as to impose a predefined slope for the tail of the resolved spectrum, but this would imply a deterioration of other fields (e.g. low-level temperatures). The solution finally used in Aladin was to combine the SLHD scheme with a weak and very selective sixth-order numerical (spectral) diffusion applied to momentum variables.

The recourse to some numerical spectral diffusion is also justified by the second specific problem raised by the implementation of the SLHD, i.e. its limited control of the overall selectivity. As examined in the next section, the control of the SLHD scheme's selectivity is not straightforward. On one hand, the range of selectivity is almost entirely prescribed once the interpolators I_A and I_D have been chosen, and on the other hand, the selectivity varies when the scheme is tuned (by modifying the relative

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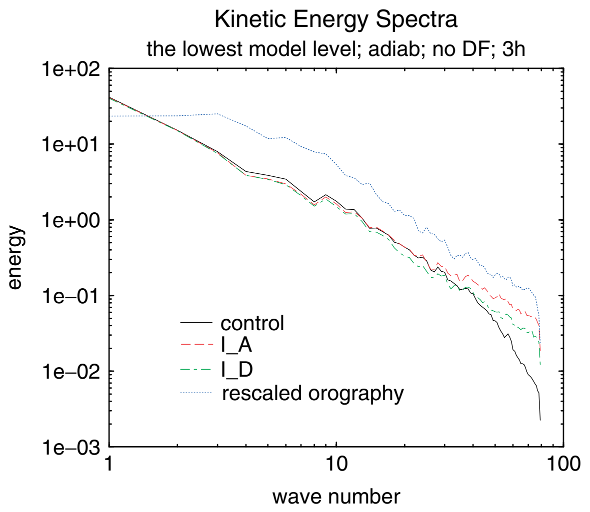

Figure 2. The kinetic energy spectra obtained by the 3-hour adiabatic model simulations. The I_A curve represents results of accurate interpolation I_A only. The I_D curve represents the case when just the damping operator was used for the semi-Lagrangian advection. The reference case using the I_A interpolation and spectral linear diffusion is marked as 'control'. The curve marked as 'rescaled orography' illustrates the spectral representation of the model orography scaled by a constant. This figure is available in colour online at www.interscience.wiley.com/qj

amount of I_A and I_D interpolations). Moreover, the possibility of enhanced diffusion near the top of the domain is not easy to implement in the framework of the SLHD scheme, due to the limited diffusivity of the I_D operator. In Aladin, this increasing diffusion near the top of the domain, acting as a kind of progressive sponge layer, is required in order to prevent excessive wave reflections at the model top boundary. Due to the limited capabilities of the SLHD scheme to become increasingly more active with height while keeping the same selectivity, this arrangement would therefore be difficult to reach within the sole SLHD frame. Hence, another (fourth-order) linear diffusion operator is applied in addition to the SLHD scheme to remove the reflected energy at the domain top, and to allow better control of the selectivity. This additional (spectral) diffusion is implemented and tuned in order to have a minimal impact in the bottom part of the domain, while nearly dominating near the top of the domain.

To summarize this discussion, when the SLHD scheme is used, three diffusions operators are active: the grid-point nonlinear diffusion, a uniform sixth-order diffusion acting on momentum variables only, and a fourth-order diffusion increasing with height applied to all prognostic variables.

4. Comparison of the SLHD properties with respect to the linear spectral diffusion

Being introduced as an alternative to the spectral linear diffusion in Aladin, the SLHD is tuned to produce a similar slope of the kinetic energy spectra, which ensures that the two diffusion schemes act with a similar overall damping effect.

At scales typical of the operational Aladin model (about 10 km), the two diffusion schemes perform almost identically in terms of synoptic forecast. The objective comparison of forecast scores for the two diffusion schemes over a random period (in forecast mode) confirms that they perform similarly on average. The main difference is seen in the temperature field, where SLHD brings an improvement in terms of root-mean-square error. On the other hand, an existing positive small bias in the tendency of the surface pressure is increased by about 15–18% when SLHD is activated. A possible explanation for this is that as SL interpolations tend to create a positive bias in the surface pressure tendency due to their lack of mass conservation (Gravel and Staniforth, 1994), the SLHD scheme may enhance this defect through the use of even less conservative interpolators. This has not been investigated further, because this trend was not seen to be cumulative, at least in limited-area models. Nevertheless, even in long climate simulations, this positive bias of tendency reaches a saturated state, and then the surface pressure field evolution becomes parallel to the one observed in experiments without SLHD. (Farda, 2006, personal communication). Combining the impact of SLHD on temperature and surface pressure fields, the geopotential field tends to be also slightly worsened near the surface. Above around 700 hPa, the integral temperature takes over the surface pressure bias and the geopotential is better represented with the SLHD. The other conventional scores at around 10 km horizontal mesh show neutral impact of SLHD (e.g. on the wind and moisture fields). In Aladin we have decided to accept this slightly worsened conservation caused by SLHD, keeping in mind the other beneficial aspects of this scheme.

Another feature of the SLHD scheme is the possibility of easily making the diffusive operator act in the three spatial dimensions. This has been specifically examined by switching on/off the diffusive interpolation along the vertical direction during the three-dimensional SL interpolations. As expected, this led to a reduction of the surface pressure tendency bias. But surprisingly, the benefit on the temperature field quality, especially near the model surface, was also decreased in this case. This seems to indicate that the presence of the vertical smoothing in the SLHD operator has a positive impact on the scheme performance. This leads to the conclusion that the 3D character of the SLHD operator behaves more physically than any diffusion operator, which acts only on arbitrary defined terrain-following model levels.

Important characteristics of the SLHD scheme are related to the triggering mechanism. Contrary to the linear diffusion, where the activity is related to the spatial variability of the diffused field itself, the SLHD activity is mainly linked to the characteristics of the flow field. In Aladin, the spectral linear diffusion is applied to all prognostic fields including moisture, which potentially leads to unphysical diffusive sources in mountain areas, due to the vertically stratified nature of this field and the sloped model levels, along which the diffusion effectively acts. This especially creates a problem in calm conditions, where the moisture field is not expected to be well-mixed, and where spurious diffusive sources mechanically result in erroneous vertical transfers of moisture. With the SLHD scheme, this problem is alleviated at least in conditions of weak flow deformation, since the diffusive source then almost vanishes. A positive impact of the SLHD scheme can therefore be expected in mountaneous areas of the forecast model, for moisture-related phenomena.

Figure 3 shows an illustration of this improved physical realism in the predicted cloudiness in the Danube valley, for 15 December 2004, 06 UTC. The observation for this case was a generalized low-level cloud cover in the Danube valley (in the centre of the figure). In the forecast made with linear diffusion, most of the small-scale details, including valley clouds, are blindly eliminated by the spectral operator whereas, in agreement with observations, the SLHD scheme tends to better preserve these clouds since the flow is close to stagnation there.

However, beside these potential advantages, a more problematic issue is that the tuning of the SLHD scheme is not straightforward, partly due to its nonlinear character, and partly due to the fact that the damping properties are mostly determined by the choice of the interpolator I_D. The selectivity of the linear interpolation is very comparable with second-order diffusion (Váňa, 2003). A higher selectivity of the scheme can be achieved by further tuning of the (6) parameters, but at the price of less active diffusion. This indicates that the diffusion activity and selectivity are in tight relation, which limits the scheme's tuning potential.

Being designed in a physically inspired manner, SLHD offers the possibility of being tuned independently from the model domain definition, unlike the spectral diffusion, for which the activity at higher wavenumbers is controlled only by the model resolution. However for practical reasons it is useful to increase the diffusion activity for the smallest scales seen by model to remove the accumulated noise at the end of model spectra. As a compromise solution, the tuning of the SLHD is designed to be scale-independent for the part controlled by the parameter A from (6) and (7), while the diffusion activity is increased for the tail of the model spectra by the enhancement controlled by the parameters B and D of (6) and (8). The latter enhancement part cannot be scale-independent as it represents a feedback via the intrinsic model resolution-dependency of d values. Still this arrangement is found significantly more consistent with the requirement that horizontal diffusion should affect mostly middle and small scales of the model when compared to the response of the spectral diffusion.

Figure 4 illustrates the damping responses of the kinetic energy spectra as a function of the model scale (represented by wavenumber relative to the reference domain) for grid-point SLHD diffusion and spectral diffusion. The damping response is computed for each wave as the difference between the wave energy of the control experiment (without any horizontal diffusion) and the same from the experiment with active diffusion

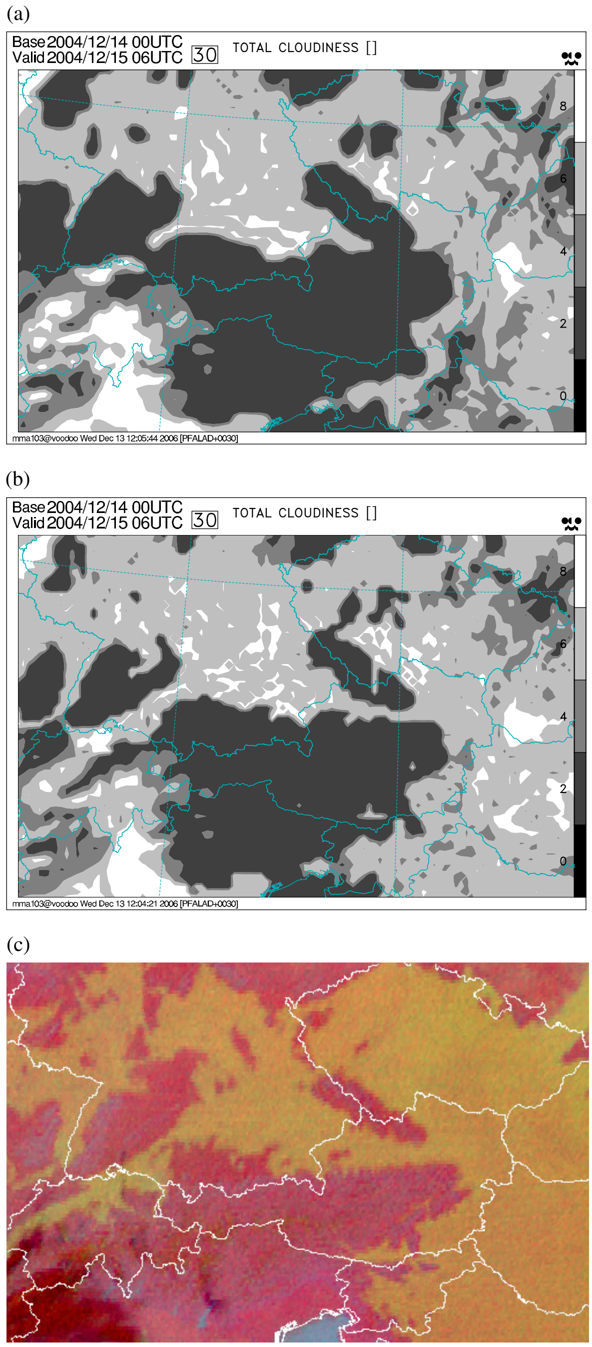

Figure 3. Total cloudiness (30-hour forecast) for 15 December 2004, 06 UTC, over central Europe predicted by the Aladin model using (a) spectral diffusion, and (b) the SLHD scheme. Contour shadings: 0, 2, 4, 6 and 8 octas (from black to white). (c) is the '24-hour Cloud Microphysics' picture composed from three IR channels of MSG/Seviri (converted to grey scale with enhanced contrast) valid for the same time. Here the light grey, grey and black indicate low cloud (and water surfaces (orange, blue, light and dark red in online version), cloud-free land and high-cloud areas, respectively. This figure is available in colour online at www.interscience.wiley.com/qj

normalized by the control experiment energy. If the damping response is nearly zero, the tested diffusion scheme makes almost no impact at the considered scale. The values of damping response around one relate to the situation where the wave energy from the tested scheme is not comparable to the reference scheme, and

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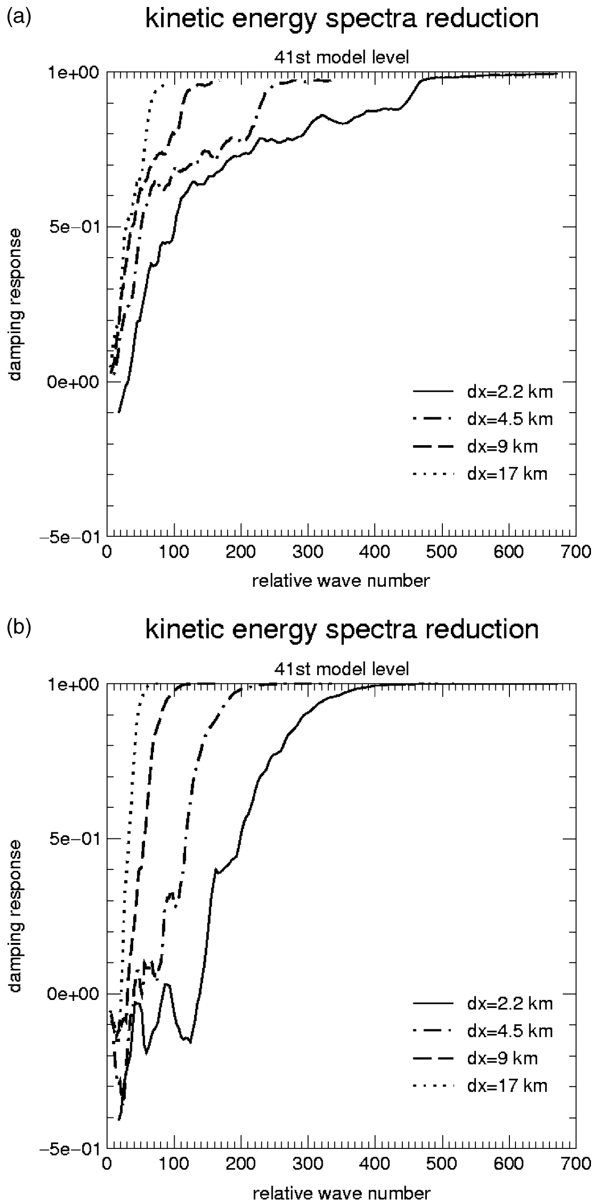

Figure 4. The relative damping response $(\zeta_{\mathrm{cnt}} - \zeta)/\zeta_{\mathrm{cnt}}$ of kinetic energy spectra as a function of the scale (represented by wave number relative to the reference domain) obtained as a result of 6 hours adiabatic simulation of four different domains with various resolution (dx) and size characterized by: dx $= 2.2$ km on 400×320 points, dx $= 4.5$ km on 360×300 points, dx $= 9$ km on 320×288 points and dx $= 17$ km on 180×160 points. The energies ζ represent results from the experiments, and ζ_{cnt} represent the same obtained from the control run without any horizontal diffusion. (a) shows the results of the grid-point SLHD diffusion, and (b) the same obtained with the spectral diffusion. All curves are regularized by 9-point running averages.

so the diffusion has a very strong impact at such scales. A negative damping response arises from the situation when diffusion amplifies the energy for a given scale with respect to the reference. SLHD apparently damps the long waves over a wider range of scales than the linear spectral diffusion experiments. Surprisingly the spectral diffusion amplifies the long waves. This phenomenon contradicts the design of the spectral diffusion and is not completely understood. It probably originates from the quasi-horizontal character of the spectral diffusion acting along the terrain-following model levels. This theory is further supported by the slight amplification of long waves by SLHD at high resolution. There the model slopes become so steep that even the local horizontal diffusion operator (triggered along the terrain-following model levels like the one represented by SLHD) introduces spurious circulation (e.g. Bougeault and Lacarrère, 1989). Another explanation could be linked to the information brought from the extension zone of the limited-area model (an arbitrary extension of the computational area to achieve horizontal periodicity for the model fields) through the spectral representation of the diffused fields.

Finally some comment should be made about the extra cost related to the use of SLHD. As mentioned above, the use of the SLHD scheme implies an extra cost compared to the original diffusion, since two spectral diffusion operators are needed, as well as the computation of κ and of two interpolated values (by I_{A} and I_{D}) for each prognostic variable. With the current Aladin operational configuration where the diffused prognostic fields are horizontal wind components, temperature and moisture, this represents an increase of about 5% on the total model cost. When the number of diffused fields is further extended by additional non-hydrostatic variables plus prognostic cloud and precipitation water phases and turbulent kinetic energy (TKE), as in the case in Arome, the cost for SLHD is increased by almost 8%. This can still be considered as very cheap for a nonlinear diffusion scheme. Moreover there are ways to further optimize the SLHD cost. For example, the weights for accurate interpolation and damping interpolation can be precomputed and combined in advance before the interpolation is performed. Although this technique restricts the choice of the diffusive operator, it allows just one interpolation instead of two to be performed. Another possibility for optimization can be to define the diffusive interpolation I_{D} in (9) as a combination of I_{A} and the smoother, leaving just the finite-difference Laplacian operator to do all the diffusion. In this case (2) becomes

$$I = I_{\mathrm{A}} \circ [1 + \kappa D_2],$$

again saving one interpolation.

5. Real case-studies

The proposed SLHD scheme has been intensively tested in various Aladin and Arome implementations. Since September 2004 this scheme has become an active component for some of the local operational applications of the Aladin model. In this section, some interesting case-studies demonstrating the impact of SLHD are presented in comparison with the reference spectral linear diffusion. As described above, in all the cases the SLHD diffusion is tuned to produce an impact on the model kinetic energy spectra comparable to the original diffusion, thereby ensuring a clean comparison of the two schemes.

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Although the cases presented here show some beneficial (and sometimes significant) impacts, it should be stated that generally the differences between the forecasts produced by the two diffusion schemes are rather subtle on randomly chosen cases. Moreover, there are very few examples of a degradation with the SLHD scheme, and when this is the case, the differences remain quite weak. No evidence of a SLHD forecast significantly worse than its reference counterpart was found in the set of comparisons performed so far.

5.1. Intensive cyclogenesis

A proper description of extratropical cyclonic developments is of great interest for NWP over Europe, since, typically, intense cyclogenesis is accompanied by severe and high-impact weather events. In the Aladin model, a special effort has been made to capture any possible cyclonic development. As a consequence, the frequency and magnitude of predicted cyclones is sometimes exaggerated, with a risk of degradated confidence in forecasters' subjective evaluation. Application of SLHD was shown to reduce these false alarms and to force atmospheric developments closer to the observations.

5.1.1. Adriatic storm, 21 July 2001.

The first case presented here covers phenomena occurring in the Adriatic Sea. The synoptic field of mean sea level pressure over Europe at 21 July, 00 UTC showed a high pressure ridge approaching from the southwest towards Spain and France and a low pressure trough extending from the eastern Mediterranean towards the Balkan peninsula. At 500 hPa the trough was situated over the central part of Europe with an axis extending between Poland and Italy. The model analysis indicates the presence of a shallow mesoscale cyclone over the central Adriatic. However, the 24-hour forecast from the Aladin/LACE (Limited-Area model in Central Europe) operational configuration predicted a deep symmetric meso-cyclone showing large pressure gradients close to its centre (Figure 5(a)).

Compared to the verifying analysis, the simulated cyclone was rather misplaced and too deep by 16 hPa. Any attempt to improve this result by tuning physics or diffusion was not really successful. The cyclone was either too deep or completely removed. The replacement of the linear by the SLHD scheme led to a much more realistic forecast, in which the underestimation of the minimum MSL pressure was reduced to about 2 hPa (Figure 5(b)). Since surface observational data coverage is not very dense over the sea, this can be considered as a very good simulation.

Looking more deeply into both simulations, it is evident that the case with the reference diffusion allows the diabatic processes to locally dominate the dynamics for the entire cyclonic area. The subsequent development was then completely artificial having no relevance to the real atmosphere. This can be illustrated in vertical 2D east–west cross-sections through the centre of the cyclone, for potential temperature and potential vorticity fields (Figure 5(c,d)). The structure of the potential temperature field shows a strong non-adiabatic area, around 80 km wide. The troposphere around the centre of the cyclone is extremely stable, as seen from very high values of low-level potential vorticity (over 10 PVU). According to Hoskins *et al.* (1985) and Morgan and Nielsen-Gammon (1998), the typical tropospheric values of potential vorticity should not exceed 2 PVU. The areas with negative values of potential vorticity also denote the presence of dry symmetric instability at around 8 km above. The same analysis for the case with SLHD is clearly more realistic. The dominance of the diabatic processes in the case with spectral diffusion can be also illustrated by the maximum accumulated precipitation over 6 hours of 56.7 mm close to the cyclone centre. The SLHD forecast produced a maximum precipitation of 28.8 mm over the same period. Precipitation measurements are not available for the sea area, but the 6-hour blending assimilation cycle operational at that time for the Aladin/LACE NWP application (Brožková, 2001) with linear spectral horizontal diffusion, indicated maximum accumulated precipitation of 22.9 mm for the cyclone area – much closer to the SLHD simulation. It is also worth mentioning here that, despite its positive smoothing role for the cyclone area, the SLHD is still able to preserve sharp gradients, e.g. over northern Italy or southern Turkey.

5.1.2. Adriatic storm, 6 May 2004.

The previous example might support the impression that the strong impact of the SLHD scheme in high-deformation areas could result in a systematic and exaggerated filling of small-scale pressure lows. This would be detrimental for storms actually occurring, since spurious false alarms would simply be replaced by excessive non-detections. To demonstrate that this is not the case, another situation of cyclonic development over the Adriatic Sea is presented here. For this case, the meso-cyclone was present in a westerly flow over the lee of the Appennines, and rapidly reached the Dalmatian cost, causing strong winds with gusts locally exceeding 40 m s^{-1}. This time, the simulation from 00 UTC with spectral linear diffusion captured the cyclonic development quite well, with a low MSL pressure value of about 991 hPa. The same forecast with the SLHD produced a nearly identical pressure field with the low less than 1 hPa shallower. Also the subsequent development of the cyclone was for this case very similar for both diffusion schemes. Figure 6(a) shows the Aladin/LACE mesoscale analysis using optimal interpolation of SYNOP measurements for surface fields and digital filter initialization (DFI) blending of upper-atmospheric fields (to enrich the assimilated large-scale fields of the global driving model by a small-scale information from the previous Aladin/LACE guess; Brožková *et al.*, 2001; Guidard *et al.*, 2006). Figure 6(b) shows the difference between the MSL pressure fields obtained with the two horizontal diffusion schemes.

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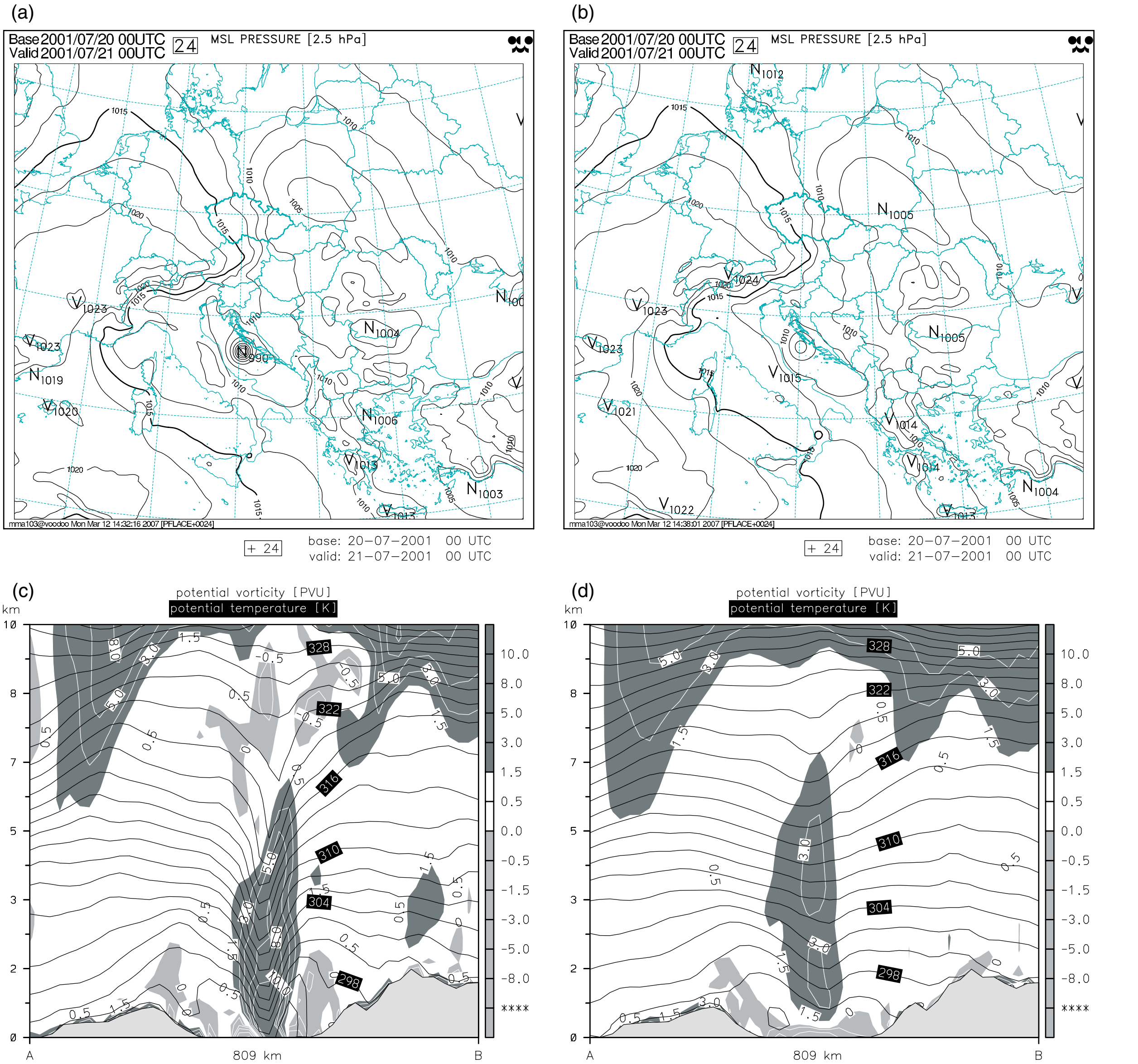

Figure 5. MSL pressure field (24-hour forecast) by Aladin/LACE from 20 July 2001, 00 UTC, with (a) spectral diffusion and (b) SLHD. The contour interval is 2.5 hPa. (c) and (d) are corresponding east–west cross-sections (cutting the cyclone centre) of potential temperature (black contours with spacing 2 K) and potential vorticity (shading and white contours). The dark shading highlights the stable areas above 1.5 PVU, and the light shading represents areas with PVU < 0 with dry symmetric instability. The spacing of the white contours follows the key on the right. This figure is available in colour online at www.interscience.wiley.com/qj

The pressure low analyzed by Aladin/LACE is around 989 hPa, while in the linear diffusion and SLHD simulations, they are 991 hPa and 992 hPa, respectively. The global model assimilation using the same SYNOP data indicated a minimum pressure of 994 hPa (not shown). In this case, one can conclude that both diffusion schemes produced very good and reliable forecasts making forecasters confident with wind predictions near the Dalmatian coast. The good results obtained with spectral diffusion were preserved with the SLHD scheme in this case.

5.2. Wind storms

Experience has shown that the most appropriate weather events for demonstrating the skills of the proposed diffusion are wind storms. Being directly influenced by the wind field, the SLHD scheme is very efficient for such situations. The simulation presented here shows a storm in the lee of the Carpathian mountains, as simulated by the Aladin/Slovakia model (operational application of Aladin in the Slovak weather service) on 17 May 2006. This model, using spectral linear horizontal diffusion, forecast a strong low-level wind jet accompanied by intense gustiness in the area of southern Slovakia and northern Hungary. According to the operational forecast from 16 May, 00 UTC, the maximum predicted winds were about 15–20 m s^{-1} with gustiness between 30 and 45 m s^{-1} during the whole day. However, the observed winds were in fact much weaker, the observed 10m wind hardly exceeding 10 m s^{-1} with a maximum measured

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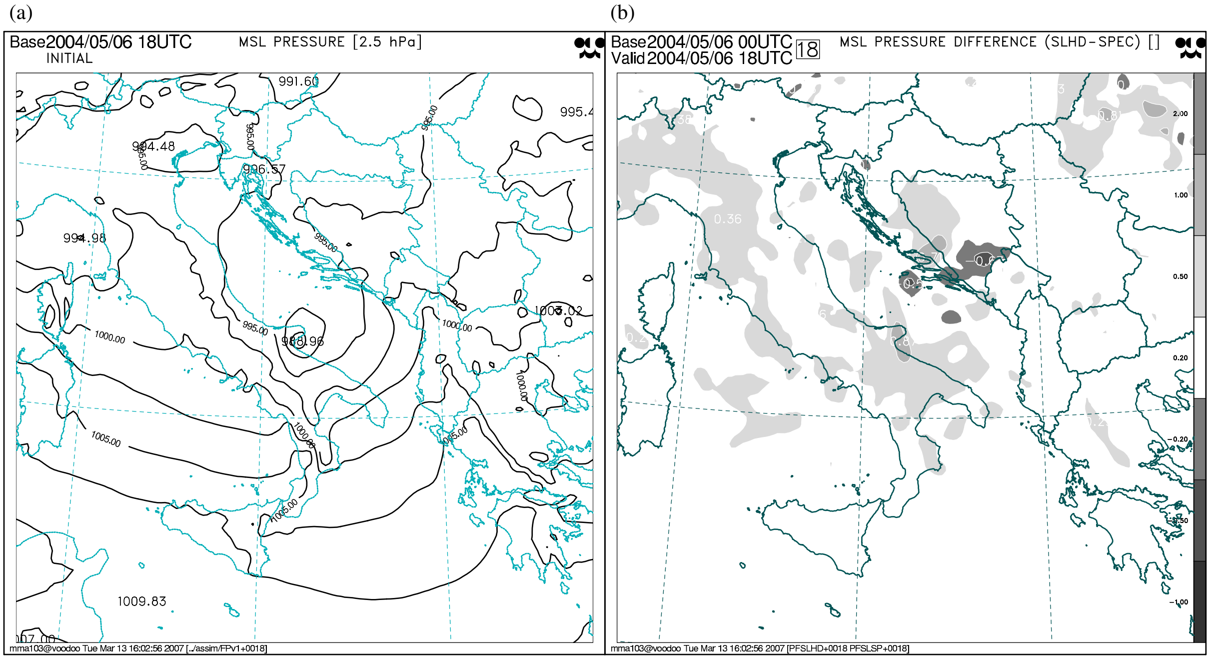

Figure 6. (a) Analysis of the MSL pressure field from Aladin/LACE model (detail) valid for 6 May 2004, 18 UTC, with contour interval 2.5 hPa. (b) MSL pressure difference at the same time between 18-hour simulations of Aladin/LACE with SLHD and with spectral linear diffusion. Shadow thresholds are −1, −0.5 and −0.2 hPa (dark shading) and 0.2, 0.5 and 1 hPa (lighter shading). Areas with differences between −0.2 and 0.2 hPa are not shaded. This figure is available in colour online at www.interscience.wiley.com/qj

gust of 24.9 m s^{-1} (in western Hungary). As illustrated in Figure 7, the use of the SLHD reduces the wind speed by 5–10 m s^{-1}. The forecast gust values were also significantly lowered in this case. Comparing the two simulations with the available SYNOP measurements, it is evident that although the SLHD case still exaggerated the storm, it provided a forecast closer to reality.

5.3. Meso-γ scale simulation

The last presented case is an example of the SLHD behaviour at the kilometric scale. Here, some results obtained with the non-hydrostatic Arome model at 2.5 km horizontal resolution will be presented. For the non-hydrostatic dynamics, the SLHD scheme is advantageous because a major contribution of the diffusion source is evaluated in grid-point space, which then makes possible a better consistency in the formulation of the bottom boundary condition (BBC). Due to the organization of the time step in Arome (or Aladin), the spectral computations are performed after all the grid-point computations, and therefore the spectral diffusion contributions are not yet available when the BBC needs to be evaluated (i.e. in grid-point space). Neglecting this contribution in the specification of the BBC results in an inconsistency which may sometimes lead to spurious effects above orography. (See a similar feature in Klemp *et al.*, 2003.) The replacement of most of the spectral diffusion sources by grid-point SLHD sources offers an attractive solution to this problem, which would otherwise require an expensive and technically difficult rearrangement of the model time-step organization. Moreover, the Arome model has various purely grid-point-defined prognostic variables which are never transformed to spectral space (moisture, cloud-water phases, precipitation species, TKE or optionally, some aerosols and pollutants) and therefore the SLHD scheme offers the possibility of also applying a diffusion operator to these prognostic variables. The presented simulation will then reflect not only the SLHD versus spectral linear diffusion. It would also demonstrate the difference between diffused grid-point-defined prognostic fields in the case of SLHD and no filtering applied to such fields for the case with spectral linear diffusion.

With the 2.5 km resolution of the model, the horizontal mixing starts to be comparable with other forcings. It may then be expected that the nonlinear behaviour of the SLHD scheme leads to some advantage over the reference linear diffusion scheme.

The case described here is a squall line which formed above southern France on 2 May 2006, as illustrated by the radar picture (Figure 8(c)). This is compared with the patterns of instantaneous precipitation predicted by the Arome model at the same time (Figure 8(a,b)). The squall line was very realistically simulated by the Arome model using the SLHD scheme. When this scheme is replaced by spectral linear diffusion, the precipitation pattern becomes quite fragmented and spreads over a much wider area. This is also visible in the cumulative rain picture (not shown) where again the SLHD simulation is closer to reality.

The case presented here illustrates the potential benefit of the SLHD diffusion for the development of mesoscale structures, leading to more organized and more intense atmospheric features, and indicates that the SLHD

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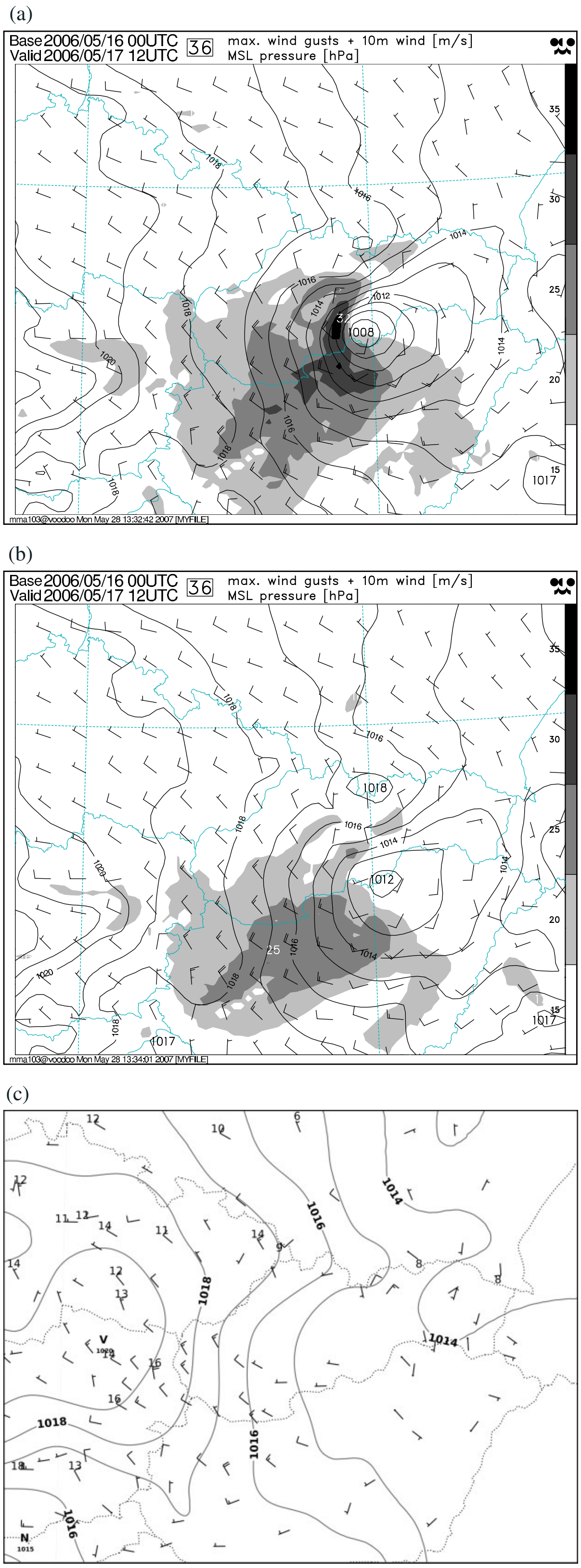

Figure 7. (a) 36-hour forecast of Aladin/Slovakia from 16 May 2005, 00 UTC, performed with spectral diffusion, showing 10 m wind (barbs), gusts (shading) and MSL pressure (isobars). The contour interval for the shading is 5 m s^{-1} starting at 15 m s^{-1}, and 1 hPa for isobars. (b) is as (a), but simulated with the SLHD scheme. (c) MSL pressure field and 10 m wind (barbs) interpolated from the SYNOP observations for 17 May 2005, 12 UTC (contour interval 1 hPa). This figure is available in colour online at www.interscience.wiley.com/qj

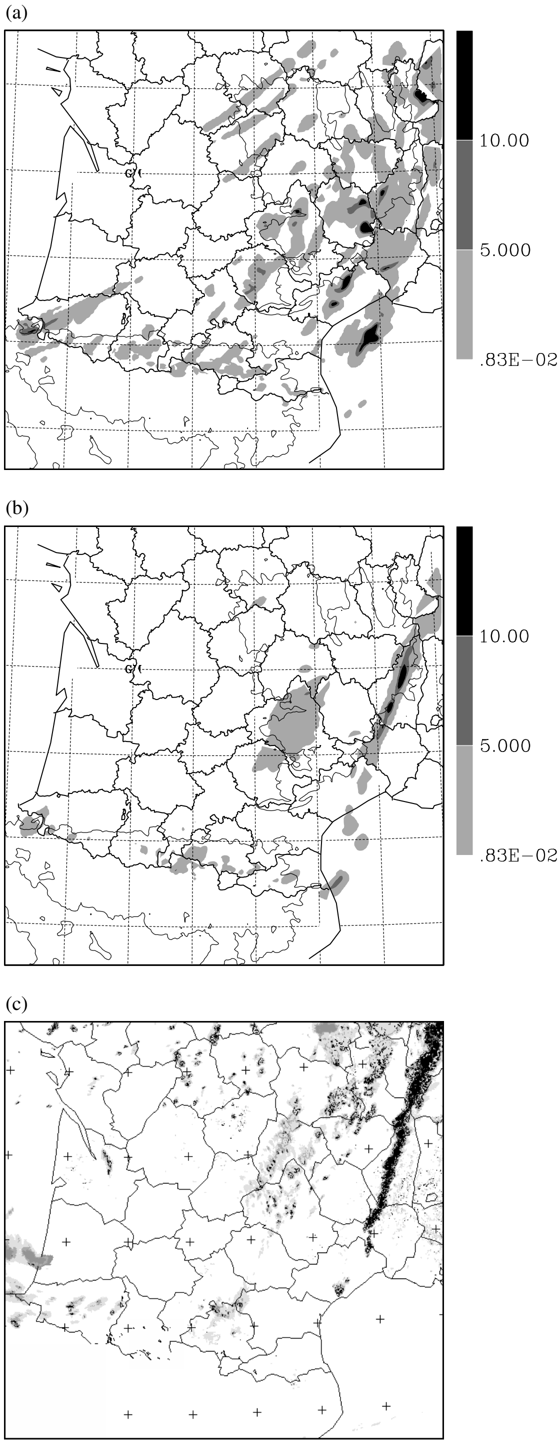

Figure 8. (a) Instantaneous precipitation (mm h^{-1}) as simulated by Arome from 22 May 2006, 00 UTC at the 15th hour of simulation using spectral linear diffusion. (b) is as (a) with the SLHD scheme. (c) is the radar picture for the same situation.

scheme effectively brings some additional capabilities to the model in term of realism, through its area-selective damping effect.

6. Conclusion

An alternative formulation of the horizontal diffusion for SL NWP models has been presented and discussed. The given example of the proposed scheme was designed for typical operational NWP models where the horizontal and vertical damping processes are treated separately. It is the common practice that while the vertical processes are described through a single-column turbulence scheme, the horizontal dissipation is regarded as a numerical process, using viscosity coefficients somewhat stronger than those appropriate to horizontal turbulence and molecular damping processes at a given scale. This artificial increase of horizontal damping seems to nicely compensate the impact of the missing small scales not being explicitly resolved by finitely truncated models. Such numerical horizontal damping then used to be typically represented by linear high-order hyperdiffusion operators. Although this rather empirical solution offers an acceptable simulation of the resolved dynamics, sometimes its linear formulation seems too tight a requirement. Typically it can only be tuned either as too diffusive with respect to desired kinetic energy spectra's slope or as not sufficiently active for severe weather events when the desired kinetic energy and enstrophy spectra are preserved. Seen from this perspective, the nonlinear scheme offering more flexibility seems to be a viable alternative to traditional linear diffusion. As already mentioned, the purely physical approaches to the horizontal mixing known from large-eddy simulation (LES) brings just little impact for the typical NWP scales with horizontal resolution of around several kilometres. Moreover such schemes imply significant increase in the model computational cost. The proposed scheme, in the way implemented here, then seems to be a bridging alternative between the traditional linear schemes and physical approaches. Although it is highly inspired by the LES formulation, it is still able to provide sufficient horizontal damping to represent non-existing scales for acceptable computational costs.

The general principle of the so-called SLHD scheme is to compute the diffusion source terms in the framework of SL computations, thereby taking advantage of the possibility of applying local and flow-dependent grid-point diffusive operators, even in spectral models. Here, this is achieved by the application of purposely damping interpolators whenever required according to the local flow structure. The flow dependence is introduced through a modulation of the diffusive sources' strength by the local flow deformation. For simplicity the presented scheme computes the flow deformation along the terrain-following vertical levels of the model. This computation can be made truly horizontal or even three-dimensional. Since SL interpolators are three-dimensional by nature, vertical flow-dependent diffusion terms can then optionally be introduced, extending the diffusion scheme from a quasi-horizontal one to a three-dimensional one.

As an illustration of the method, the implementation of the SLHD scheme in the dynamical kernel of the Aladin model was presented, and the results obtained with Aladin and Arome applications were discussed for real forecast cases. The main concern was to preserve the existing organization of the data flow, while keeping an acceptable computational cost together with stable and accurate performance. Since the modified SL interpolators resulting from the use of SLHD are damping for any wave, the diffusion scheme does not reduce the stability of the SL scheme, therefore imposing no additional restriction to the length of the model time step.

The limitations and advantages of the proposed scheme compared to classical linear diffusion schemes have been extensively discussed in the paper, and are summarized below.

In the mostly used form of the SL technique, for which interpolations are applied only to the source terms computed at origin points, the SLHD scheme is not able to control the small-scale sources of noise originating from forcing terms computed at arrival points. This problem especially involves orographic forcing terms, hence some additional diffusive process is needed in order to avoid a possible overestimation of the variability at the smallest orographic resolved scales.

Rigorously speaking, the local effect of the SLHD scheme is not totally 'deterministic' but has some 'statistic' aspect, because the local damping response of the scheme (slightly) depends on the position h of the origin point with respect to the computational grid. However, this feature is not a serious source of problem in practice, because of the inherent space and time variability of the modelled flows.

Due to the limited extension of the SL stencil, it is not possible to arbitrarily choose the selectivity of a local diffusive operator defined on this compact stencil. For SL schemes using cubic interpolations, the stencil extension is four points in each direction, and hence it is difficult to achieve a selectivity larger than the second order for the damping operator I_{D}. However, it could be argued that a second-order diffusive operator applied only where and when necessary may have more physical realism than a uniform fourth-order operator.

Using the SL interpolation as a diffusive filter may lead to slightly worse conservation. This topic has not been investigated in more detail since the application considered here is short-range NWP.

In the scheme presented here, a single diagnostic field (the local deformation of the horizontal flow) controls the diffusivity in the same way for all the model variables. This is a very simple approximation of the more complex dependencies which occur in turbulent flows, for which the diffusion for each variable depends on various parameters, including the spatial structure of the field for this particular variable. This illustrates the fact that, under the proposed form, the SLHD scheme, although incorporating some 'flavour' of turbulent effects, remains essentially reminiscent of the traditional concept of simple diffusion.

Finally, a technical restriction of the SLHD is that it can be applied, by nature, only to the prognostic variables which are transported by the SL scheme.

In spite of the above limitations, the SLHD scheme offers various advantages for a numerical model. The main one, and the *raison d'être* of the proposed scheme, is of course the more physical nonlinear resulting diffusivity, since the flow-dependent formulation is closer, in principle, to parametrizations derived from the theory of turbulent fluid mechanics. Moreover, the 3D character of the scheme potentially offers the framework for a better performance with respect to sloping terrain.

Another substantial advantage is for spectral models; the proposed scheme makes possible the application of diffusion also for those prognostic variables which are not transformed to spectral space, where the computation of linear diffusion usually occurs.

Thanks to the possibility of the SLHD scheme being completely nested inside a pre-existing SL framework, the price in terms of code development and computational cost is very low, at least if the definition of the damping coefficients κ and Γ remain reasonably simple. Moreover the scheme is stable as long as the Lipschitz criterion for the SL trajectories is satisfied, as is already the case with linear diffusion.

Thus, the SLHD scheme seems to be a viable alternative to the purely linear diffusion schemes which are almost exclusively used in NWP. Although these two types of diffusion scheme perform very similarly in terms of model scores, the nonlinear diffusion brings some benefits in simulating the meteorological flow, and especially in the case of severe weather events. Another benefit of the proposed scheme is linked to the fact that it reverses the known weakness of the SL advection scheme (the additional damping) to an advantageous feature – a better control of the complex separation between physical signal and noise in numerical models.

The particular implementation of the scheme presented here is an illustration of one possibility among many others for the introduction of some flow dependency in the diffusive sources of the model, using the SLHD concept. As outlined above, the guideline for this implementation was mainly to minimize the model modifications and the additional cost of the scheme, but the general underlying idea of the SLHD scheme offers great flexibility, and more sophisticated schemes could be designed starting from other basic constraints than this one. Some potentially interesting avenues which could be examined from this point of view are (i) the possibility of a wider variety of selectivity/diffusivity combinations, by enlarging the class of damping interpolators involved in the scheme, and (ii) the possibility of making the behaviour of the scheme closer to turbulent behaviour through a more physically based formulation of the local strength of diffusion. These two tracks will be pursued, and evaluated in terms of their benefit for Aladin and Arome NWP applications.

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