FFT Analysis of Sound Wave: Time to Frequency Domain Transformation

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Abstract

This report explores the analysis of sound waves through Fast Fourier Transform (FFT). The experiment included transforming sinusoidal, square, and natural sound waveforms from the time domain to the frequency domain. By examining FFT spectra, we analyzed the fundamental frequencies, harmonics, and beat patterns of different sound sources, including tuning forks, whistles, the human voice, and musical instruments. Through this, we gained insight into the physical and physiological origins of these sounds, and how they can be characterized by varying frequency composition and amplitudes.

I. Introduction

In the physical world, most natural sounds are composed of a blend of multiple frequencies rather than a single, pure tone. These complex longitudinal waveforms can be difficult to interpret when viewed as functions of time. The Fourier Transform offers a solution by converting time-domain signals into the frequency domain, revealing the individual frequency components that make up a sound. This is done using the given equation:

$$A(t) = \int_{-\infty}^{\infty} B(f) e^{i2\pi f \cdot t} df \tag{1}$$

In this experiment, we used the Fast Fourier Transform (FFT), a computationally efficient algorithm for performing the Fourier Transform on discrete data. By recording sound waves from various sources—such as tuning forks, musical instruments, and the human voice—and applying the FFT and rescaling with below equation:

$$\delta f = \frac{1}{\Delta t} \tag{2}$$

we were able to analyze their frequency spectra. This allowed us to identify fundamental frequencies, observe harmonics, and investigate beat phenomena. Through this transformation, we gained deeper insight into the acoustic structure and physiological origins of different types of sound.

II. Apparatus

The apparatus consisted of the following.

- Tektronix TBS 1052-B Digital Oscilloscope
- Instek GFG-8219A Function Generator
- Microphone
- Preamplifier
- Speaker
- Two A-426.7 tuning forks
- Casio handheld piano
- Guitar
- Recorder
- Cello
- MATLAB
- Excel

III. Procedure

1. Basic FFT

To begin the experiment, we generated a pure sine wave described by the equation:

$$y = \sin(2\pi \cdot 100x) \tag{3}$$

The signal was recorded over a time interval from 0 to 0.1 seconds, using 2048 equally spaced data points since FFT software Require that the number of points be equal to N = 2^n. To reduce spectral leakage and other artifacts in the Fast Fourier Transform (FFT), we apply a Hamming filter to the data. This filter tapers the signal smoothly to zero at the edges, shown in *Figure 1*, minimizing discontinuities. This is especially important later when signals aren't ideal, unlike the generated sine wave.

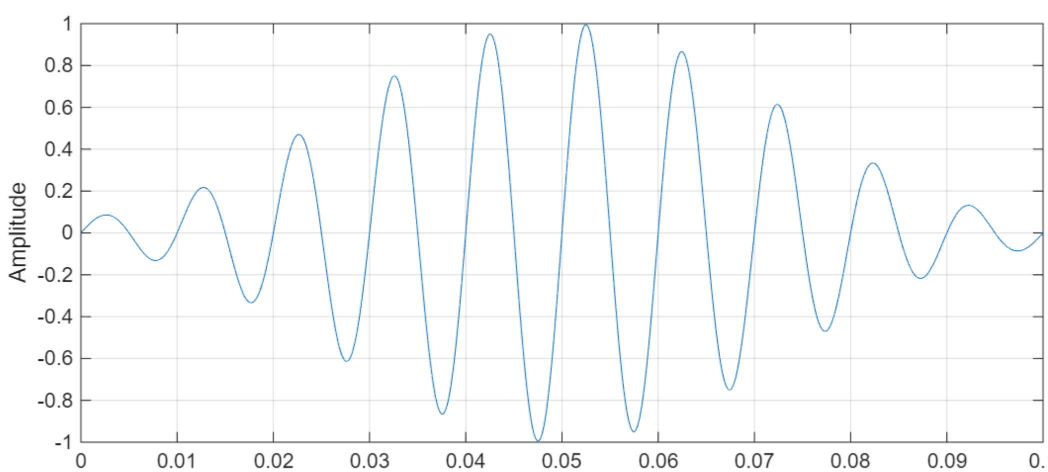

Figure 1. 100Hz sine wave from 0-0.1s with Hamming filter applied

This simulated waveform was then subjected to the FFT and rescaled by equation [2]. This results in the frequency spectrum shown in *Figure 2*. We can see from the FFT, that it peaks at 100Hz. This is the consistent with the fundamental frequency of the waveform, which was designed to oscillate at 100Hz.

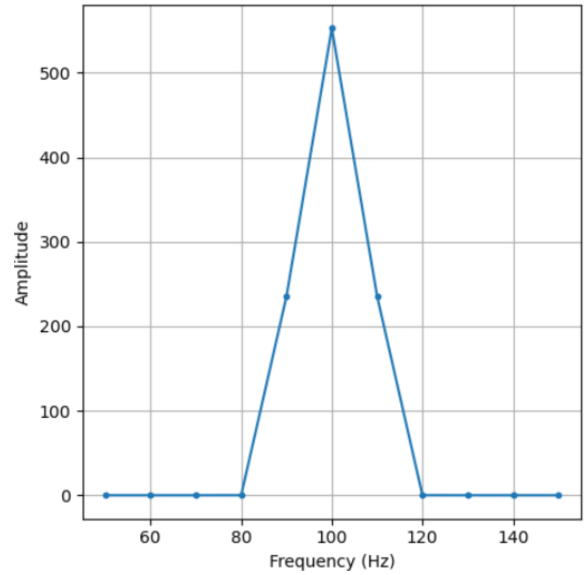

Figure 2. FFT of 100Hz sine wave from 0-0.1s

However, in order to obtain reliable results later on, we must understand what allows us to obtain high precision FFTs. Consequently, we simulated the same waveforem given by equation [3] but from a time of 0 to 1.0s instead. We compared the FFT from *Figure 2* to the new FFT, as shown in *Figure 3*. At first glance, the notable key difference are their full-width at half-maximum (FWHM) linewidths. The increase in the number of recorded cycles led to a narrower linewideth.

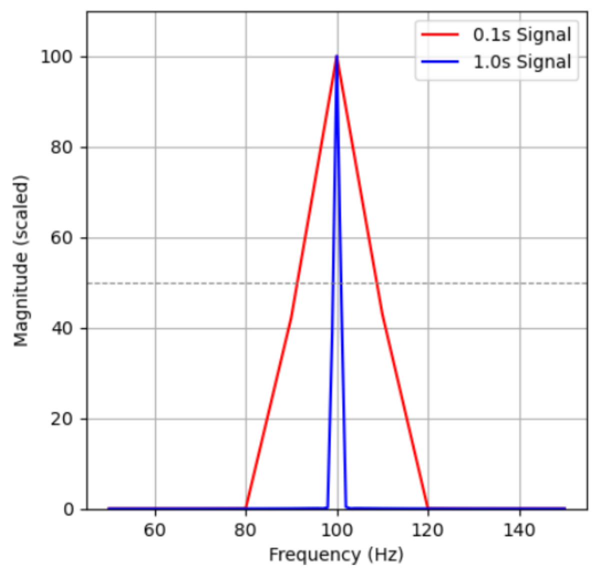

Figure 3. Comparison of FFT of 100Hz sine wave from 0-0.1s and 0-1.0s

The exact results of these metrics for each of these FFTs were placed in *Table 1.* We can see that the full-width half-maximum linewidth (Γ) of the 0 to 1.0s FFT is exactly ten times less than the smaller 0 to 0.1s. Likewise, the same goes for the full range of frequency (Δf), decreasing by ten times with the ten-time increase of cycles. Thus, we can deduce that $\Gamma \propto \frac{1}{Number\ of\ Cycles}$ and that $\Delta f \propto \frac{1}{Number\ of\ Cycles}$.

This demonstrates the frequency resolution, and thus precision, improves with longer observation windows. Additionally, the full frequency range inversely scales with the total time range, reaffirming that high-precision FFTs require many cycles.

	Smaller Number of Cycles (0 to 0.1s)	Larger number of cycles (0 to 1.0s)
Fundamental Frequency (Hz)	100	100
FWHM Linewidth (Hz)	20	2
Full Range Frequency (Hz)	10230	1023

Table 1. Comparison metrics of FFT of 100Hz sine wave from 0-0.1s and 0-1.0s

2. Sine and Square Waves

In this next part of the experiment, we want to start observing real or natural waveform signals that exist in the world. The overall concept will still be the same as before: apply a Hamming filter to the measured signal and then perform the Fourier Transform to analyze the fundamental frequencies.

The two signals being analyzed in this section will be a sine wave and a square wave. We set the generator to output a 1k Hz sine wave followed by a 1k Hz Square wave which were both fed to the oscilloscope. The data was then put through a hamming filter and then the FFT to find their fundamental frequencies, shown in *Figure 4 and 5.* Both FFTs have a leading frequency at 1000.34 Hz. However, the square wave consists of many other frequencies that decrease in magnitude, the higher it is the spectrum.

To explore the audible implications of these differences, we connected the function generator to a speaker. The sine wave produced a smooth, pure tone, while the square wave generated a buzzy, harsher, and even louder sound. This auditory contrast reflects the increased harmonic complexity of the square wave.

This contrast is due to the way that the square wave is constructed. The square wave is made of multiple different sine waves at different frequencies and at varying amplitudes to produce the square shape, specifically odd-valued harmonics (3 kHz, 5kHz, etc.). This is the reason why the square wave's FFT had a vast number of different frequencies in it compared to the pure 1 kHz sine wave. So, while both have the same fundamental frequency, their spectral content, and thus their perceived sound, is drastically different

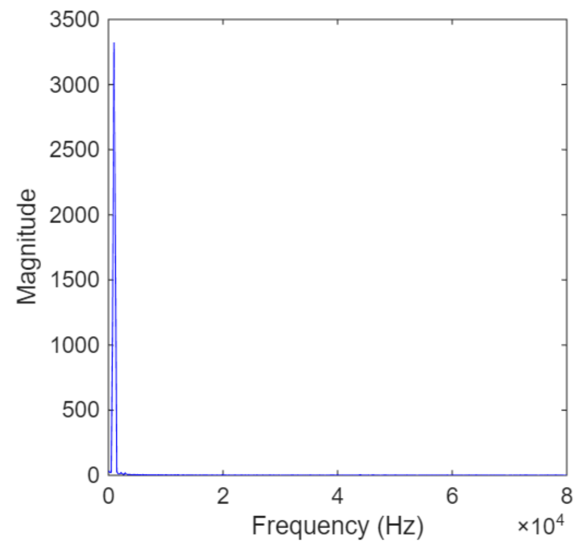

Figure 4. FFT of 1k Hz Sine Wave

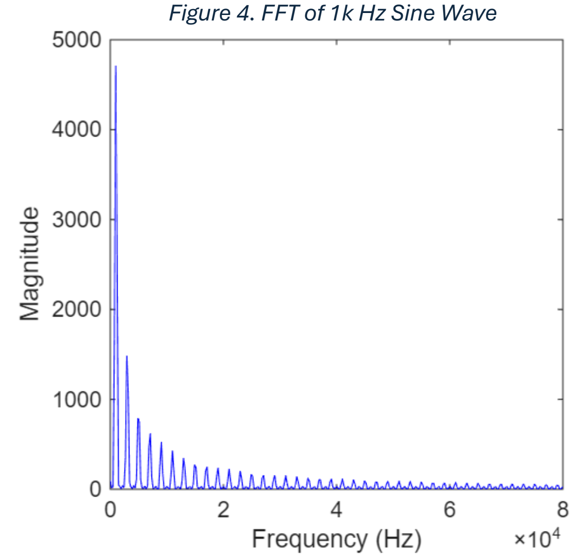

Figure 5. FFT of 1K Hz Square Wave

3. Tuning Forks

In this segment of the experiment, we explore the phenomenon of beat frequencies by using two A 426.7 tuning forks. When struck simultaneously and placed near the microphone, the forks produced a combined sound wave that was recorded over a time base of approximately 0.5 seconds per division on the oscilloscope, shown in *Figure 6.* The captured signal displayed periodic modulations in amplitude —known as beats — resulting from the slight difference in the actual resonant frequencies of the two forks, despite their nominal rating.

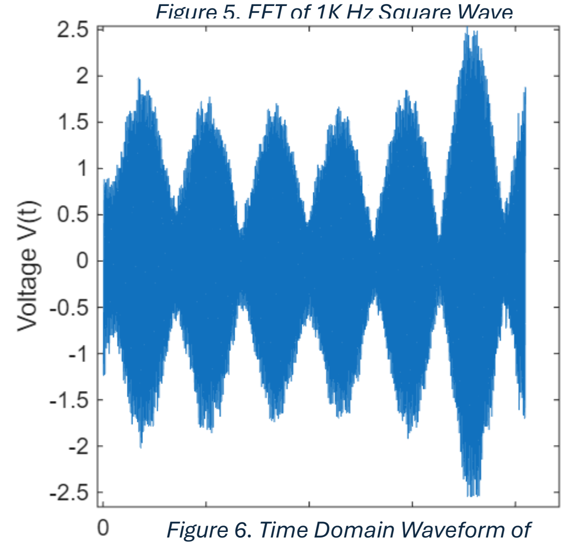

Figure 6. Time Domain Waveform of two A 426.7 Tuning Forks

Looking at the signal, we can estimate that the beat frequency was around 1.2 Hz, indicating that one fork was vibrating slightly faster than the other due to manufacturing tolerance or wear. Also if we were to strike one of the forks at a different time than the other, our measured wave would differ as well as the beat frequency.

4. Whistles

We move to analyzing the spectral characteristics of a human whistle. Since I was unable to whistle, my partner whistles a sustained note which was picked up by the microphone and recorded in the oscilloscope, *Figure* 7, with a time base of approximately 0.025 seconds per division. Appropriately, a Hamming filter was applied and an FFT was conducted on the data, *Figure 8*.

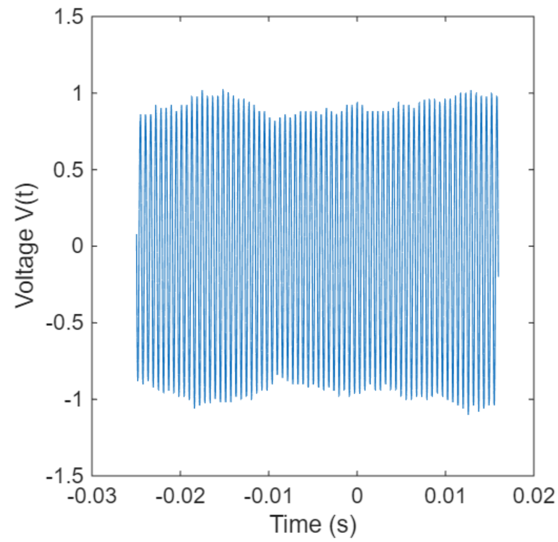

Figure 7. Time Domain Graph of Human Whistle

From the FFT, it's clear that there is a single dominate fundamental frequency of 1709.82 Hz and surprisingly no other significant harmonics. If there were other harmonics, those frequencies would have contributed to the timbre, or tonal quality, of the whistle.

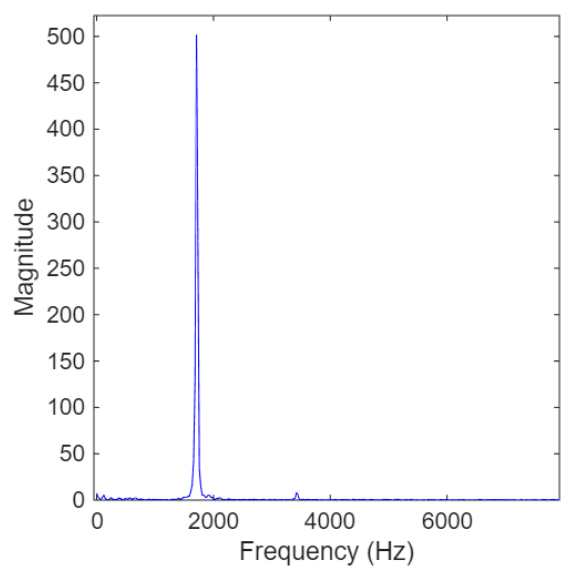

Figure 8. FFT of Human Whistle

We then asked a neighbor to whistle at the same tone as my lab partner. Surprisingly you could make out the two whistlers. These variations would be due to subtle physiological differences lip shape, tongue position and air pressure control, which influence the resonance and overtones produced during whistling (Azola et al., 2017). We didn't get to measure and accurately observe the neighbor's whistling, but it can be inferred that their whistling's FFT would include various other harmonics accompanying the 1709.82 Hz fundamental frequency which would differ from *Figure 8*.

5. Human Voice

Now, we switch to analyzing the acoustic structure of the Human voice itself. For data, we recorded vocalizing a sustained sound on the oscilloscope at the same time base of 0.025 seconds per division. We tried each of the vowels — a ,e ,i ,o, and u — and once again applied the Hamming filter and FFT.

Figure 9 shows the FFT of the A sound that I sustained.

It had a fundamental frequency of 219.834 Hz as well as another significant, and even dominant, harmonic at 439.668 Hz. There also appear to be other significant harmonics at 659.502 Hz, 1759.67 Hz, and 1979.51 Hz as well many much smaller ones. Now we will compare the A sound to the E sound in *figure 10.* We can see that it both the A and the E sound share many of the same harmonics, in fact they share the same fundamental frequency. The only key difference are the amplitudes of the different harmonics, which appears to be the key between the sounds. In fact, if we compare each of the five vowels, *figure 11,* we the same phenomina occur. Each sound though different, all contained similar harmonics, just at different magnitudes. This is a very clear visualization at how sounds, though they might have the same frequency, can sound unique due to differying underlying harmonics. These harmonic differences are responsible for what we perceive as timbre or vocal color, allowing us to distinguish different vowel sounds even when they are the same pitch, which was an A 440 in this specific case.

This demonstrates how the human voice functions as a complex resonant system, shaped bythe vibration of the vocal cords and the configuration of the vocal tract—including the tongue, lips, and nasal cavities (SingWise, n.d.). The unique spectral fingerprint of each vowel illustrates how subtle anatomical differences can drastically influence acoustic output.

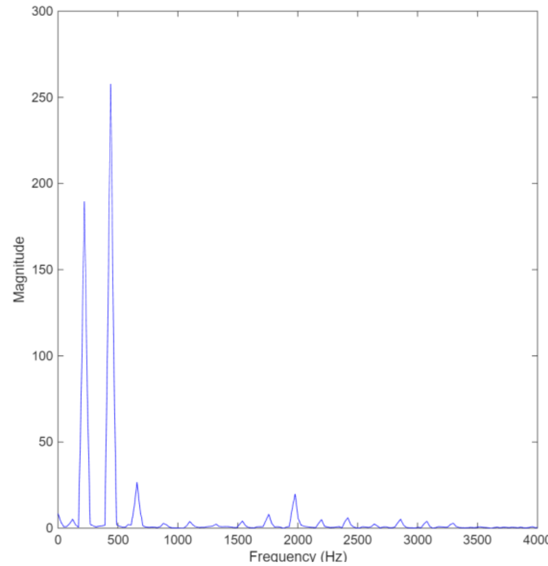

Figure 9. FFT of Human Voice Holding an A Sound

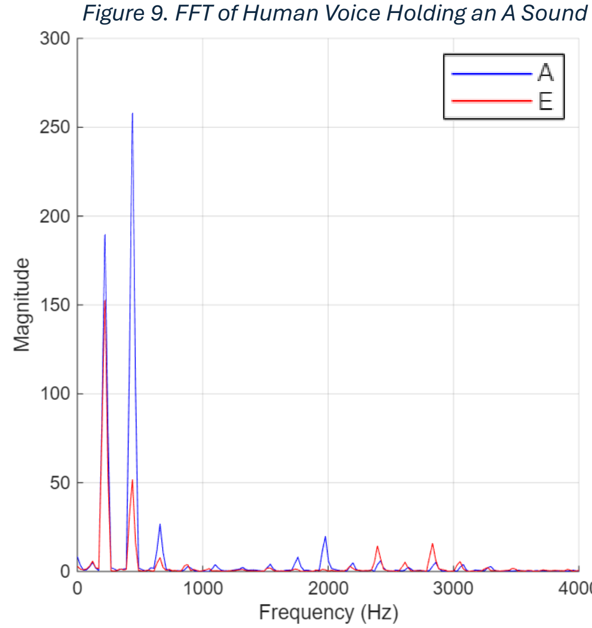

Figure 10. FFT Comparison of Human Voice Holding an A and E Sound

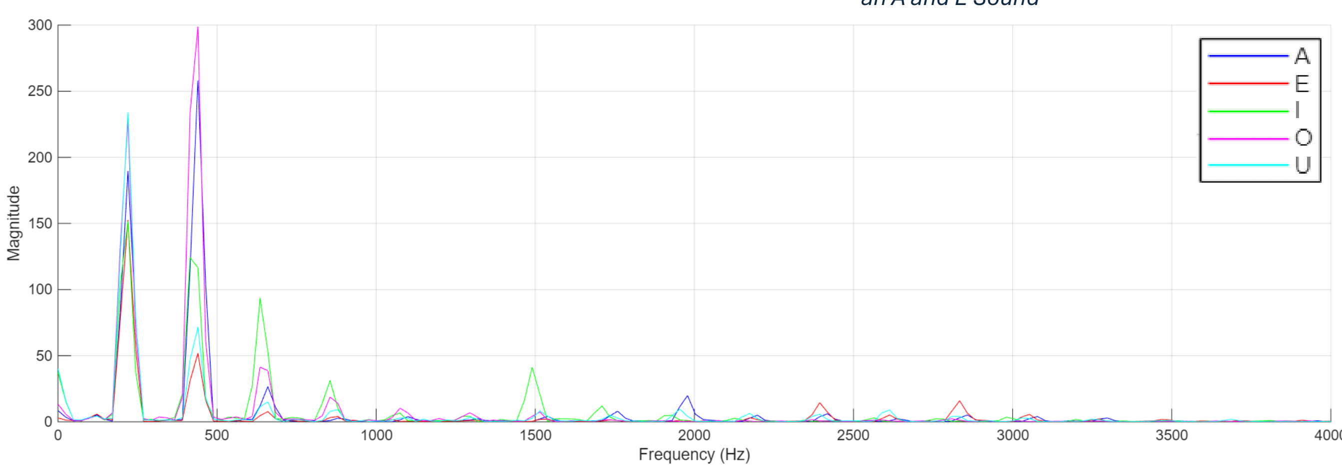

Figure 11. FFT comparison of Human Voice Holding Sound of Each Vowel

6. Musical Harmonics

In this section of the experiment, we further explored the concept of timbre by analyzing the harmonic structure of different musical instruments playing the same note. We chose the note, A-440, to build off the last section. After tuning the instruments, we first recorded the sound of the handheld piano playing an A-440 in the oscilloscope, followed by the guitar playing an A-440. We compared the FFTs of the two in *Figure12.* At first glance, the Guitar has an overall more significant magnitude than the Piano which I found interestingly counter intuitive as the Piano was easily able to sustain its note whereas the Guitar's sound quickly decayed after being plucked. This huge amplitude difference most likely means that the data was recorded right when the guitar was plucked, where the sound was most intense. Aside from that, as expected, both had a fundamental frequency of 439.668 Hz, consistent with the frequency of an A4 note. If the two FFTs were universally scaled, we can see that there are differences between the intensities of their harmonics. From the FFT comparison, we see that the Guitar's third harmonic is more intense than the second whereas the piano's second harmonic is more intense than the third. We can more clearly see these discrepancies by plotting the amplitudes of each harmonic to each harmonic number N (where $f = Nf_0$). After scaling the greatest amplitudes to one hundred, *Figure 13*, we can see the true differences between the guitar and the piano. The guitar's harmonics dropped off more sharply after the third harmonic whereas the piano maintained slightly more energy in the mid-range (harmonics 4-6). This provides further insight into the tonal differences shaped by the instruments' physical properties and methods of sound production.

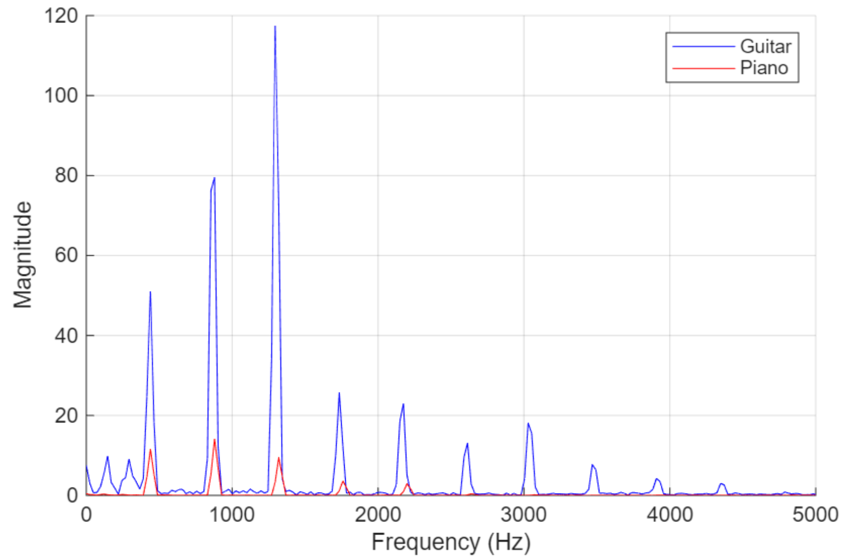

Figure 12. FFT comparison of Guitar and Piano Playing A-440

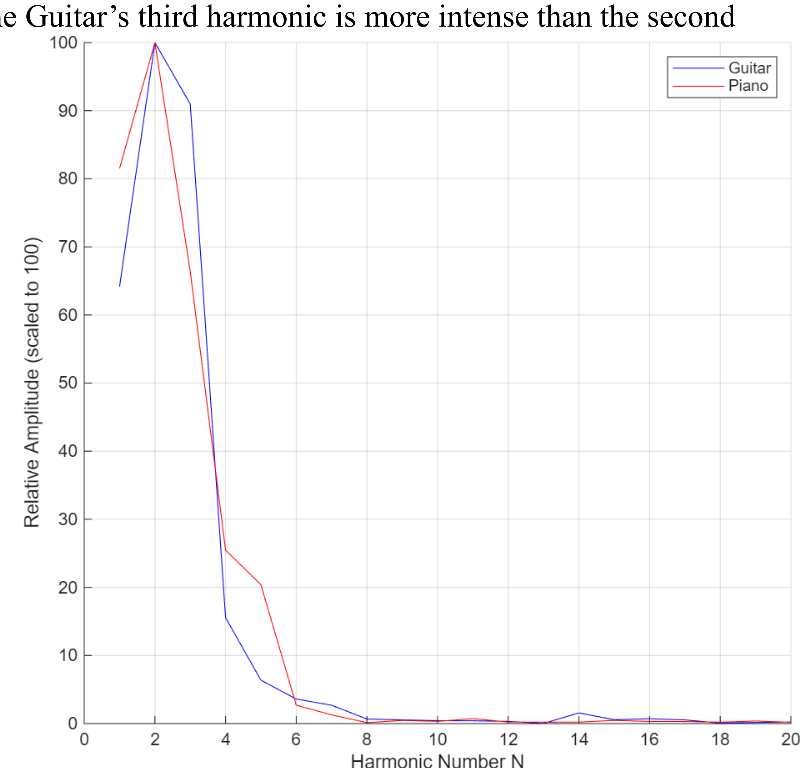

Figure 13. Relative Harmonic Amplitude vs Harmonic Number

7. Cello and Recorder

We decided to measure and observe two additional instruments, a Cello and a Recorder. As before, both were measured with an oscilloscope, playing an A-440. Their FFTs were graphed and compared in *Figure14.* The Cello has a very rich range of harmonics while the Recorder has a singular, but very substantial, peak. What's intriguing, however, is that the recorder's peak is near the second harmonic, at 854.91 Hz, rather than 440 Hz despite playing an A-440. This could be due to the recorder being overblown, a technique or acoustic quirk where the instrument favors high harmonics depending on air pressure and figure positioning (Wikipedia, n.d.). Additionally, the peak being at 854.91 Hz and not closer to 880 Hz means that the A note was being played slightly flatter as well. But as expected, these harmonic characteristics were consistent with what we heard, the cello having a rich sound and unique timbre while the recorder sounded like a very pure A note.

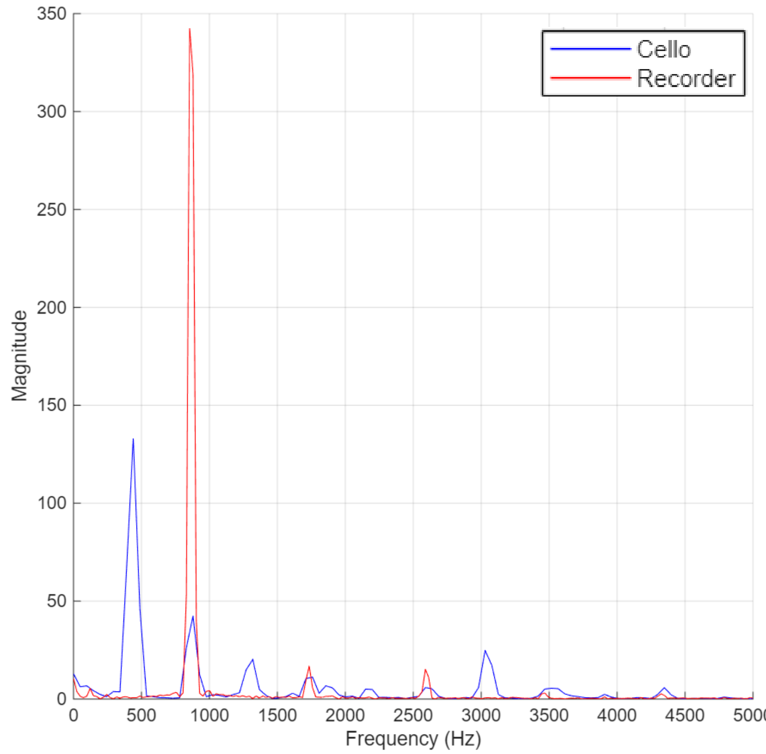

Figure 14. FFT Comparison of Cello and Recorder Playing an A-440

I believe these are a great addition to the previous two instruments, again noting how different these notes can vary even though their fundamental frequency is the same.

IV. Summary

Throughout this experiment, we used Fast Fourier Transform (FFT) techniques to analyze the frequency spectra of various sound sources. By transforming signals from the time domain to into the frequency domain we could isolate and identify fundamental frequencies, harmonics, and phenomena such as beat frequencies.

We observed that increasing the duration of a recorded signal reduced the FFT linewidth, improving frequency resolution. This confirmed the inverse relationship between observation time and spectral width, which is fundamental to Fourier analysis.

In the sine vs. square wave comparison, we found that square waves exhibit rich harmonic content due to their sharp transitions, whereas sine waves remain pure and frequency specific. Listening to both waveforms reinforced this, as the square wave sounded buzzier due to its harmonic richness.

Using two slightly detuned A 426.7 tuning forks, we visualized beat frequencies in the time domain. The waveform revealed amplitude modulations corresponding to the small frequency difference between the forks—an excellent illustration of interference in action.

For natural sound sources like whistling and the human voice, the FFT provided insight into physiological factors. Differences in harmonics between two people whistling the same pitch highlighted how anatomy—such as tongue position and lip shape—influences resonance (Azola et al., 2017). Similarly, vowel sounds showed distinct harmonic profiles that define vocal timbre, consistent with research on vocal tract resonance (SingWise, n.d.).

The analysis of guitar and electric piano revealed that although both played A-440, their harmonic intensities and decay patterns varied significantly. The guitar had stronger initial harmonics but decayed quickly, while the piano showed weaker magnitudes yet a more sustained tone. This highlighted the key to understanding why different instruments are identifiable despite playing the same notes.

An unexpected observation occurred when analyzing a soprano recorder playing A4. The FFT revealed a dominant peak at 880 Hz—suggesting the second harmonic overtook the fundamental, likely due to overblowing or natural resonance favoring higher modes (Wikipedia, n.d.). Other possible areas of error included the instruments not being perfectly in tune and possibly not playing notes in tune just like the recorder data showed. Background noise also had an impact on our FFT data, influencing the amplitudes of various harmonics.

In the future, these errors can be mitigated by recording data in a sound isolated area as well precisely ensuring that instruments are in tune. This also means that notes being played should be checked with a tuner to ensure the data is as precise as possible. Additionally, being able to sample 4096 points of data would also significantly improve the precision of the measured harmonics, as shown in section 1.

V. References

[1] Azola, A., Palmer, J., Matuson, K., Holz, R., Fischbach, F., & Palmeri, T. (2017). The physiology of oral whistling: A combined radiographic and MRI analysis. *Journal of Applied Physiology*, 124(1), 34–39. https://doi.org/10.1152/japplphysiol.00902.2016

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