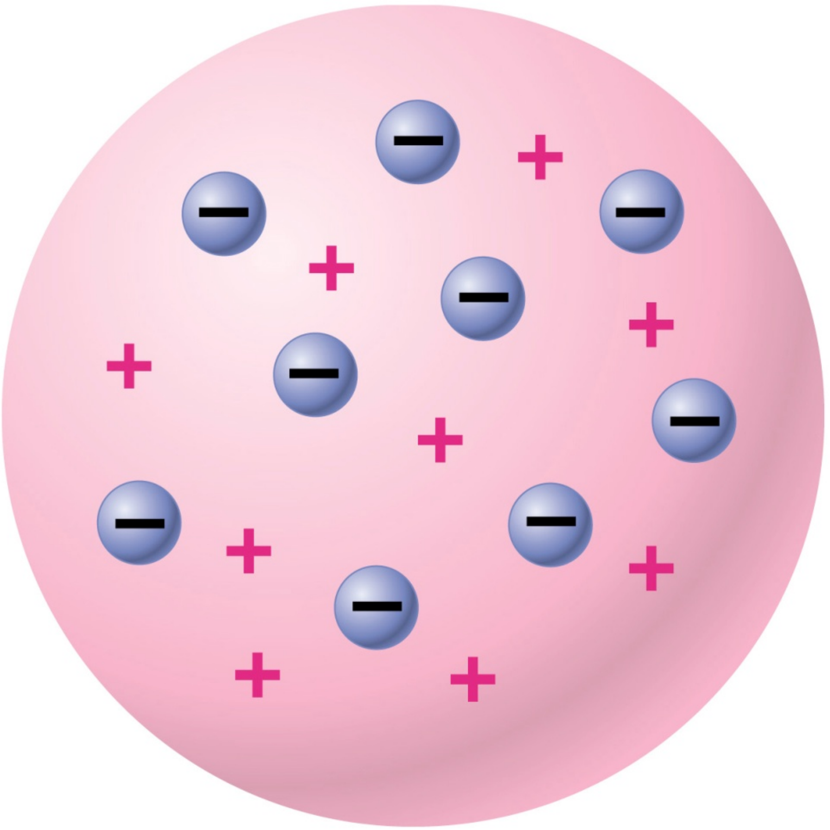
Rutherford Scattering

Eric Chen, MIT Department of Physics, 12/05/2018

History

- 1909: The structure of the atom is still unknown

- Thomson proposes the plum pudding model:



Scattering

- Ernest Rutherford imagines probing atomic structure experimentally by scattering alpha particles

Scattering

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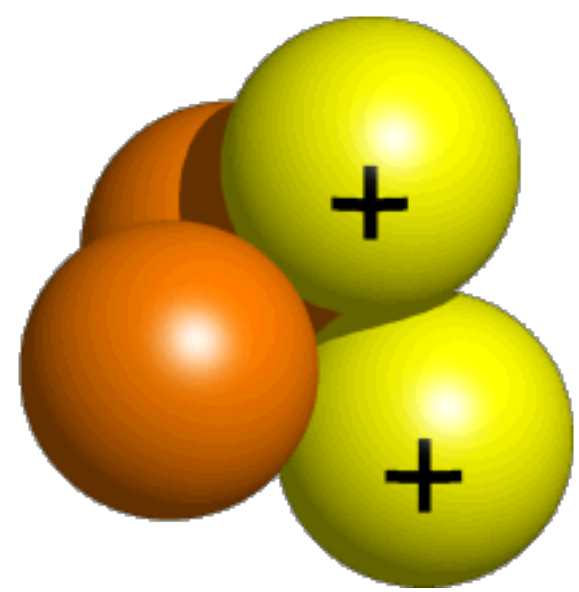
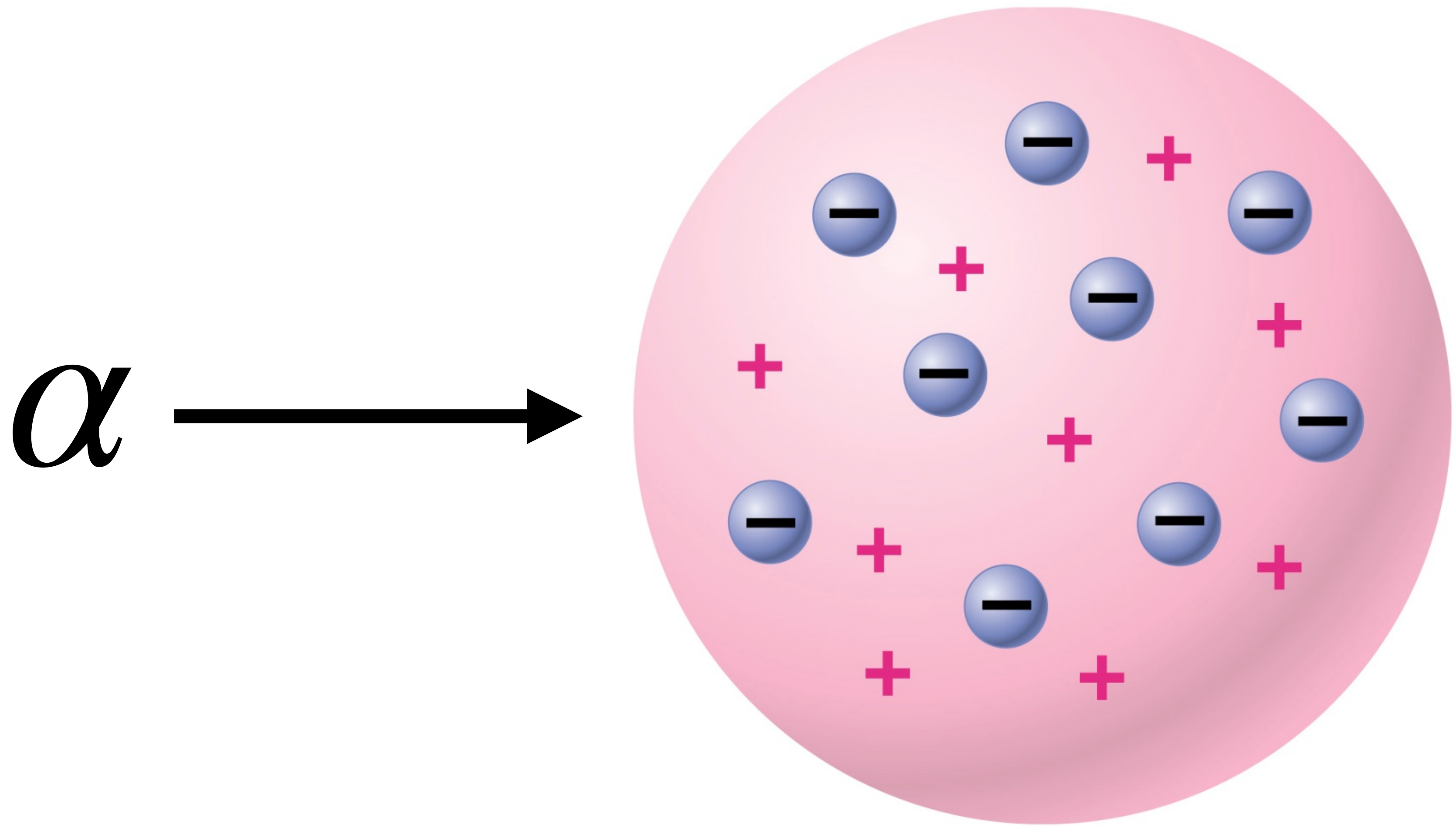
- Alpha particle is a positive helium ion: 2 protons, 2 neutrons:

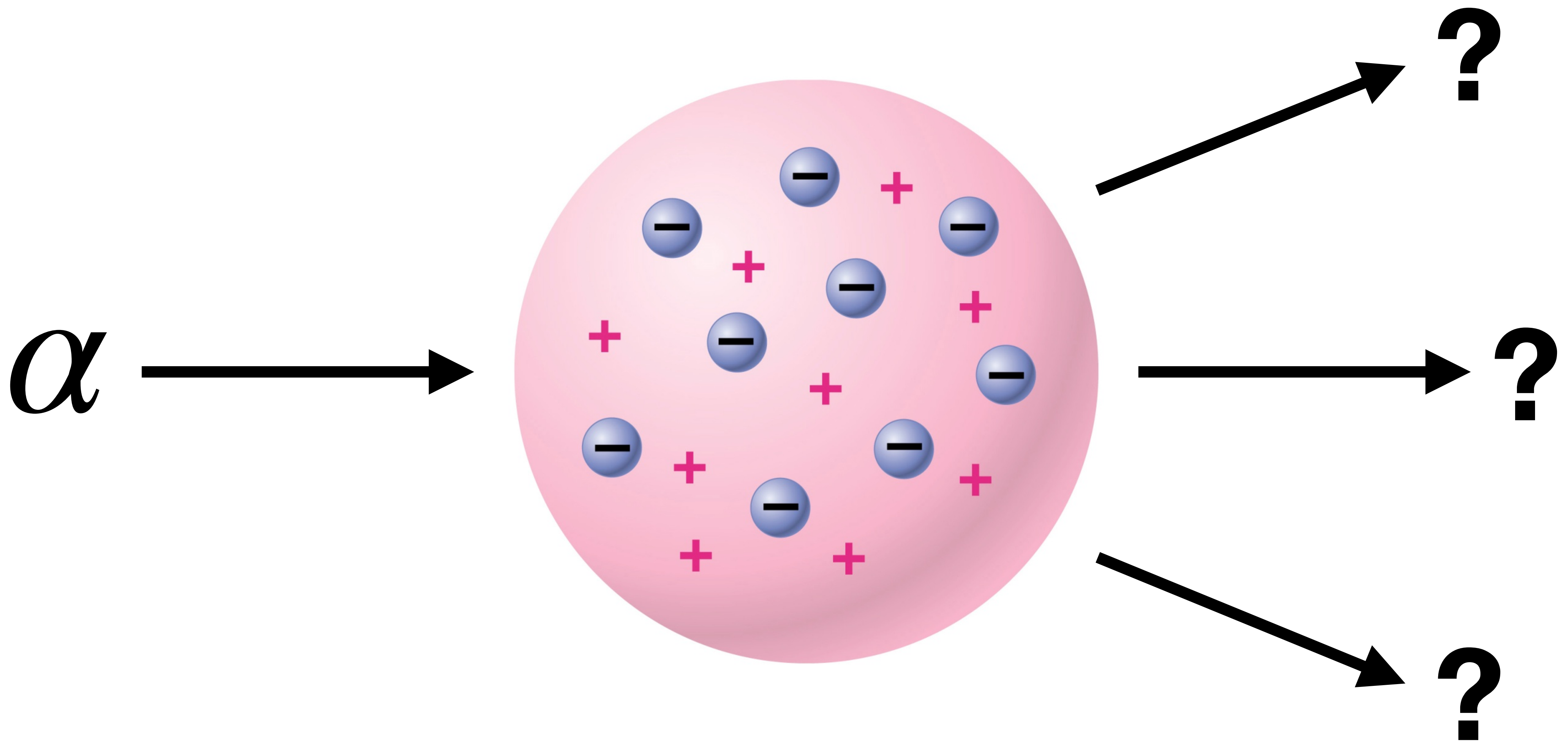
$$\alpha =$$



- Shoot an alpha particle at an atom:

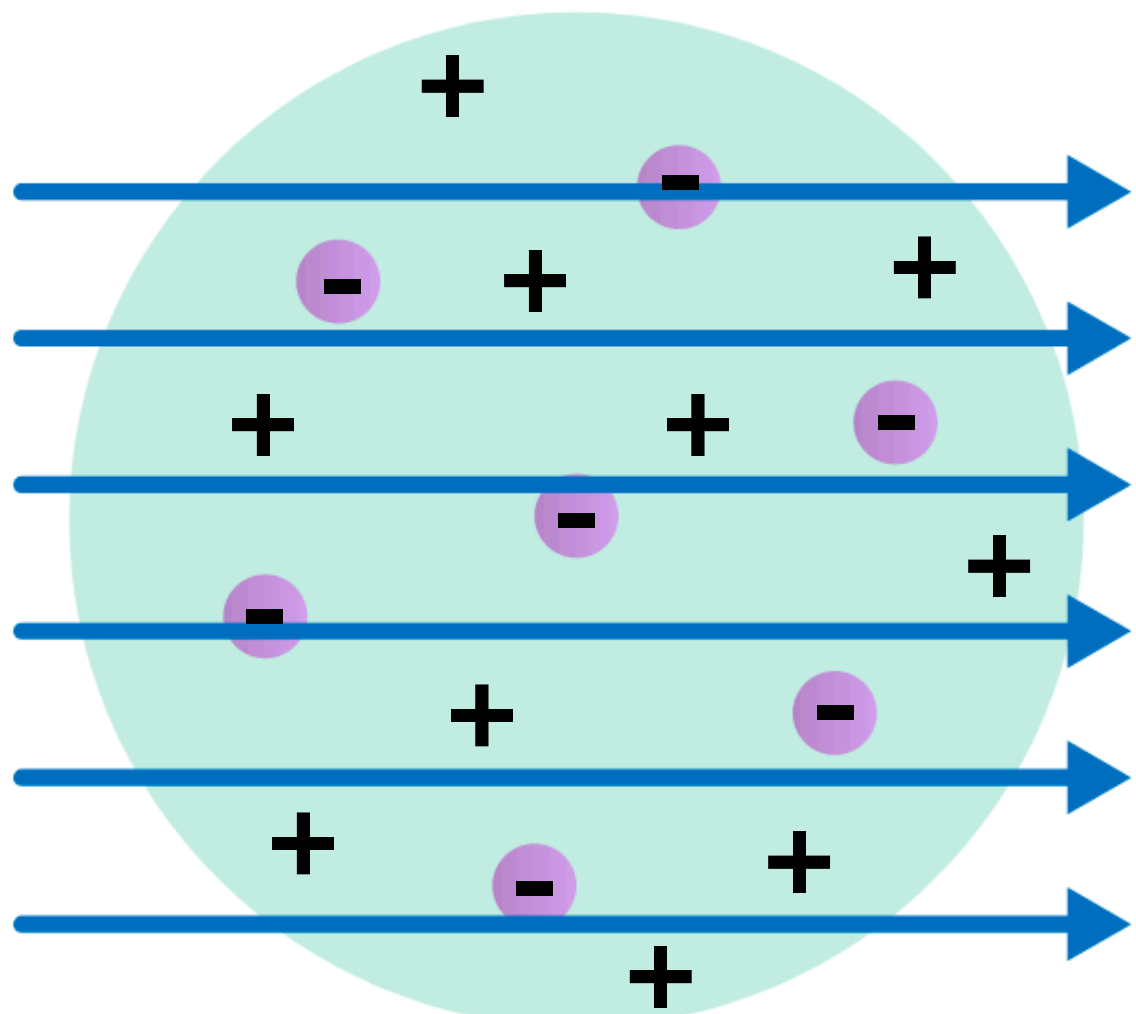


- Observe alpha particle deflection to gain insight on the structure of the atom:



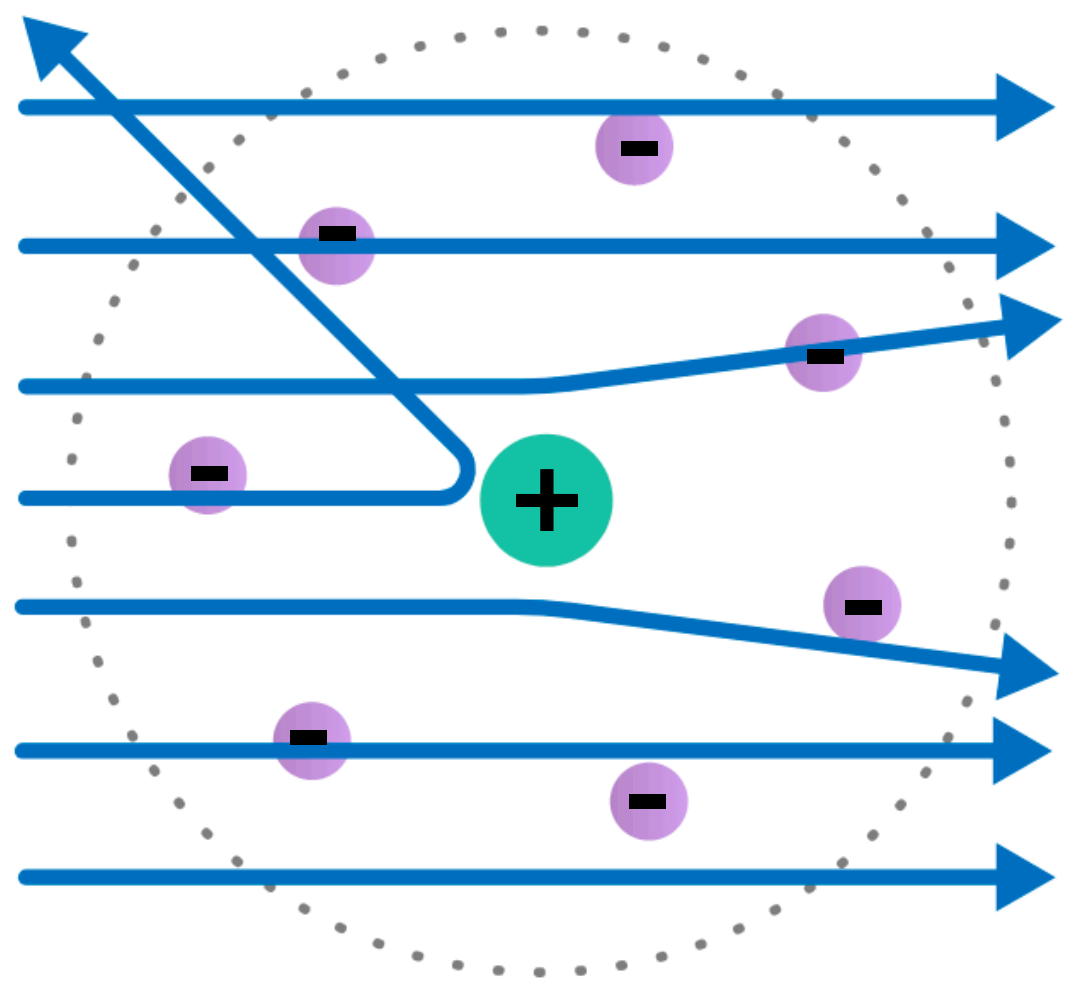
Thomson Prediction

- No deflection: electric field from spread out positive charge is too weak to affect fast moving alpha particles

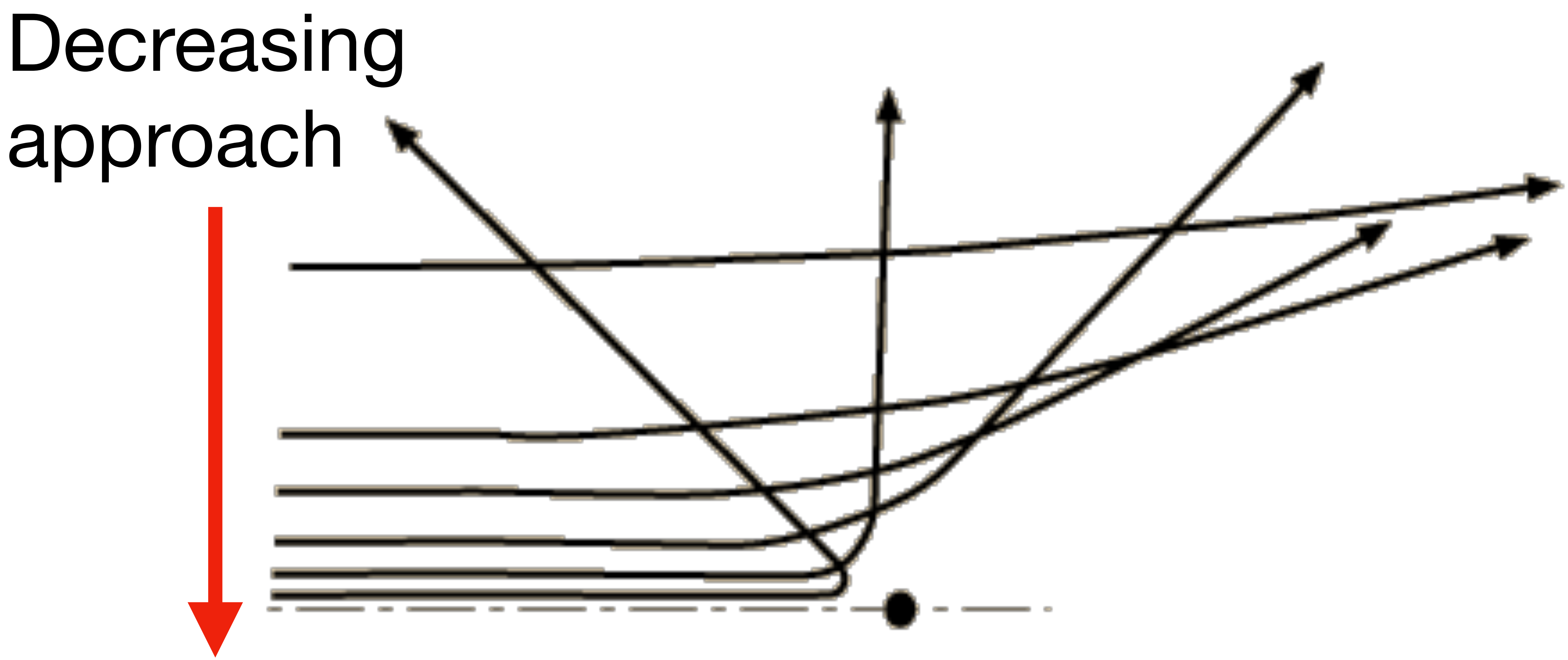


Rutherford Theory

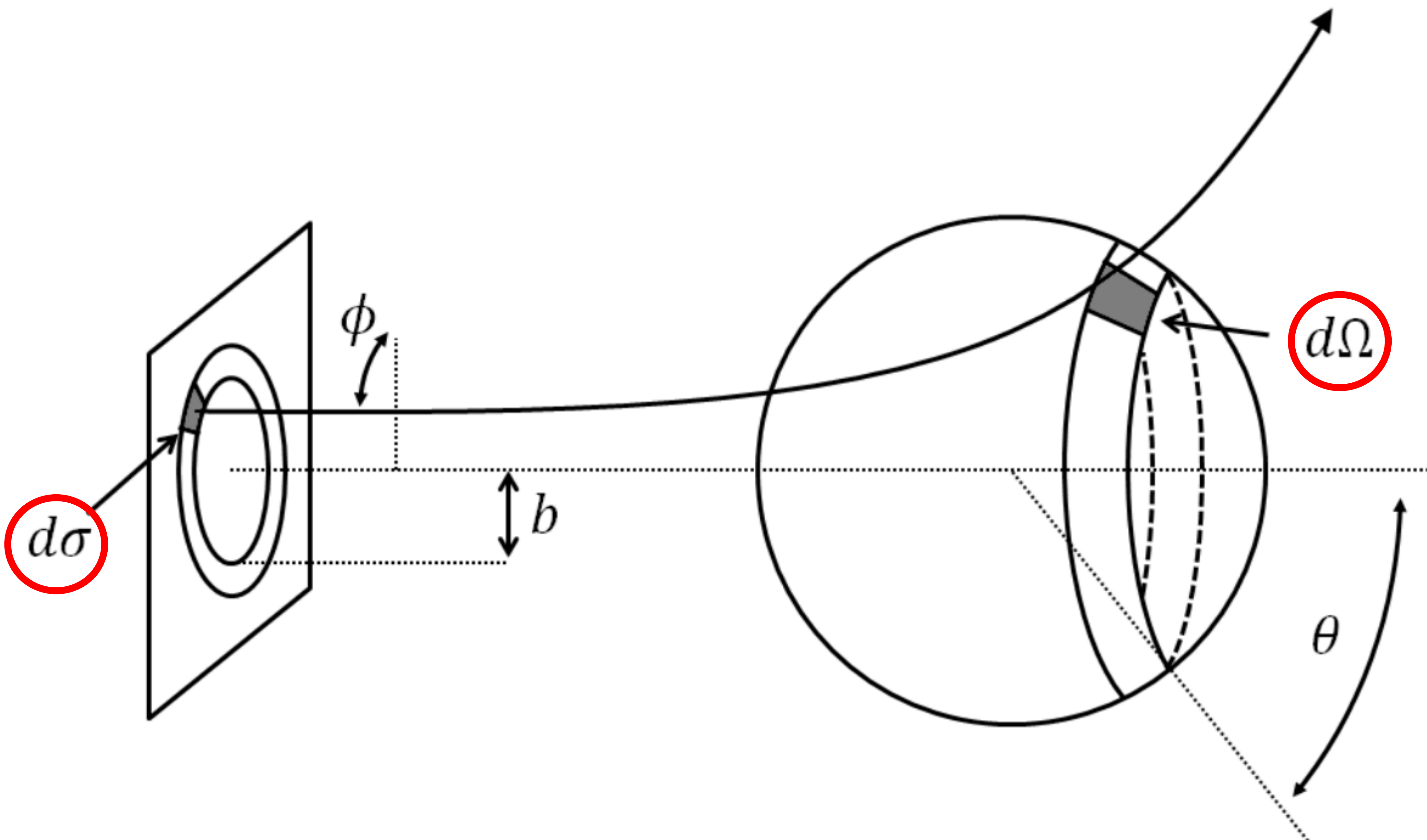
- Deflection: centrally concentrated positive charge produces a strong electric field capable of affecting alpha particle path



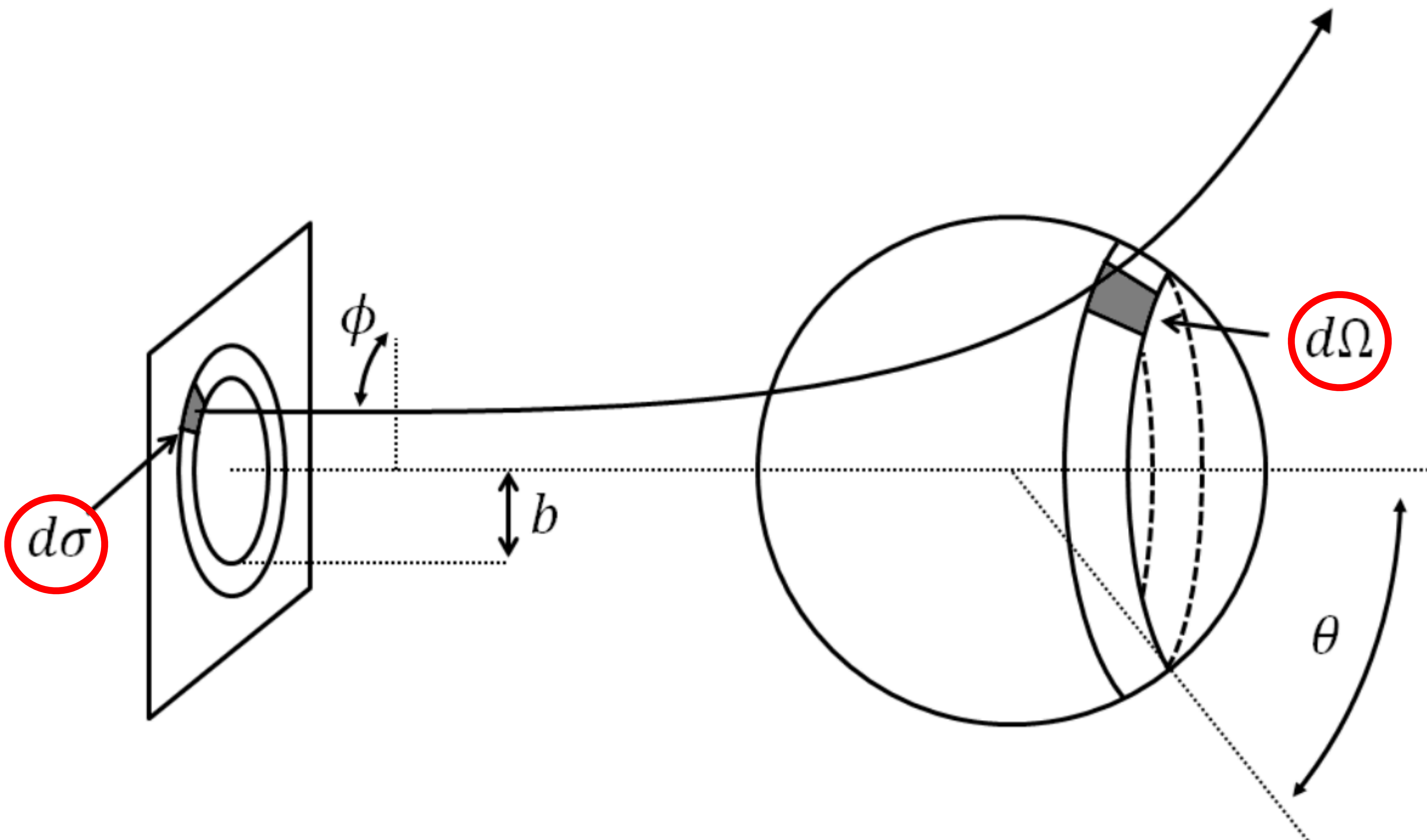
- The closer the distance of approach, the greater the deflection angle:



- A small change in cross section $d\sigma$ yields a small change in scattering angle $d\Omega$



- Differential scattering cross section: $\dfrac{d\sigma}{d\Omega}$



- Rutherford derived a theoretical formula for this cross section in terms of incident alpha particle beam angle θ:

$$\frac{d\sigma}{d\Omega} = (\frac{ZZ'e^2}{4E})^2 \sin^{-4}\frac{\theta}{2}$$

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$$\frac{d\sigma}{d\Omega} = \left(\underline{\frac{ZZ'e^2}{4E}}\right)^2 \sin^{-4}\frac{\theta}{2}$$

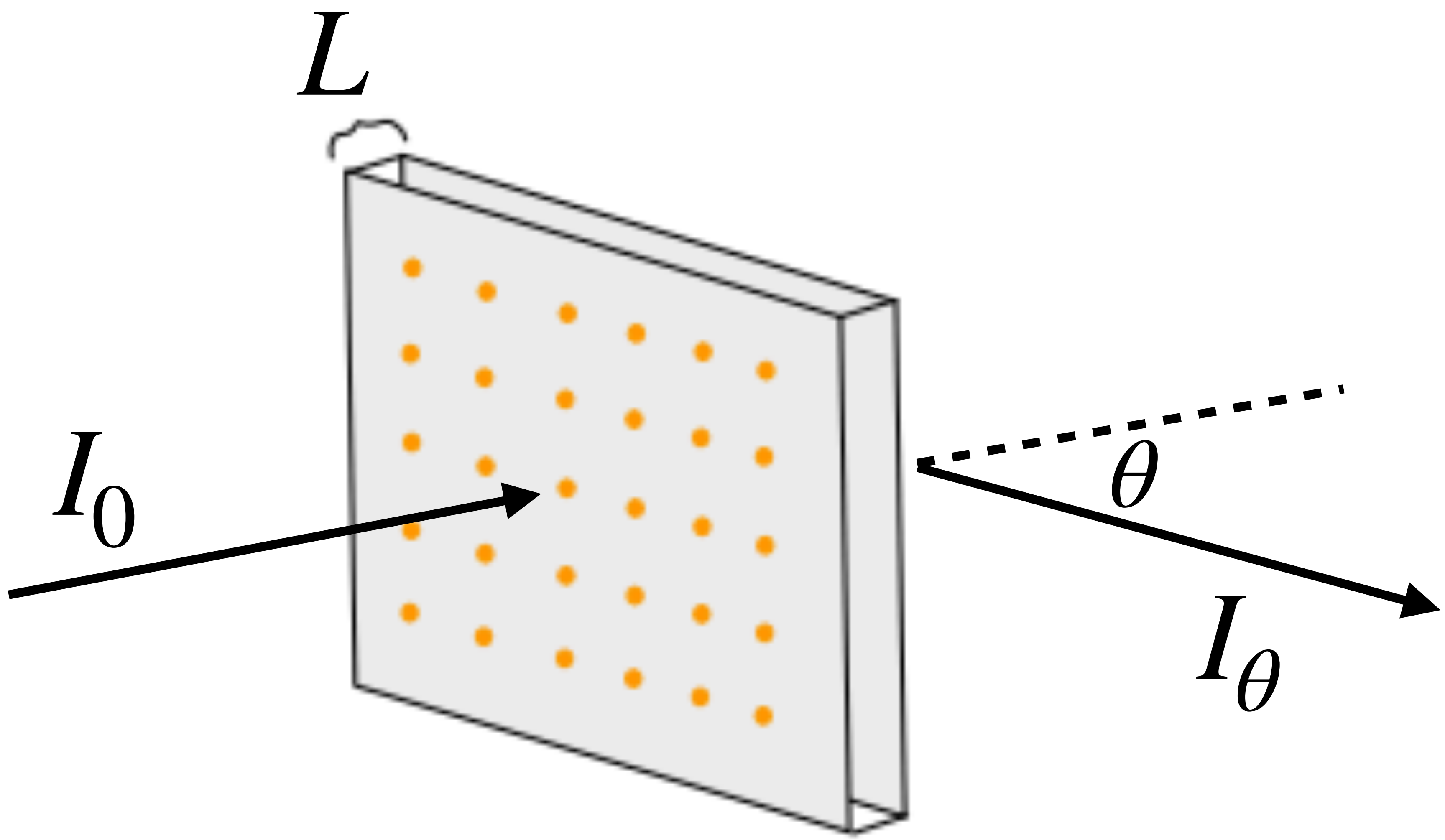
Z : target atom atomic number

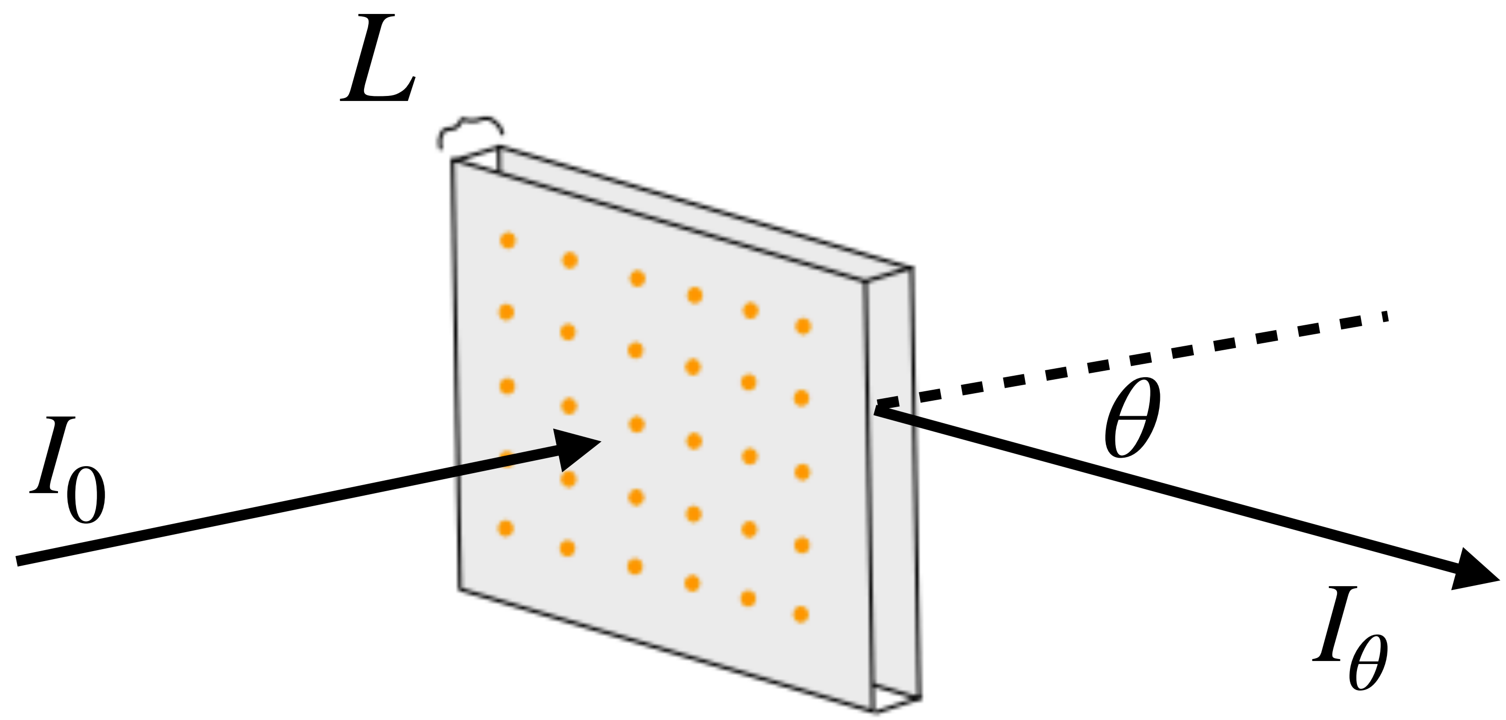
Z' : alpha particle atomic number

e : electron charge

E : alpha particle energy

- Cross section can also be expressed in terms of the measurable rate of particles scattered at an angle θ when passed through a foil of thickness L:





$$\frac{d\sigma}{d\Omega} = \frac{I_\theta A}{I_0 L \rho N_A d\Omega}$$

N_A : avogadro's number

ρ : foil density

A : foil atomic number

$I_0,\ I_\theta$: incident and scattered particle rates

- Comparing the two expressions:

1) $$\frac{d\sigma}{d\Omega} = (\frac{ZZ'e^2}{4E})^2 \sin^{-4}\frac{\theta}{2}$$

2) $$\frac{d\sigma}{d\Omega} = \frac{I_\theta A}{I_0 L \rho N_A d\Omega}$$

- Comparing the two expressions:

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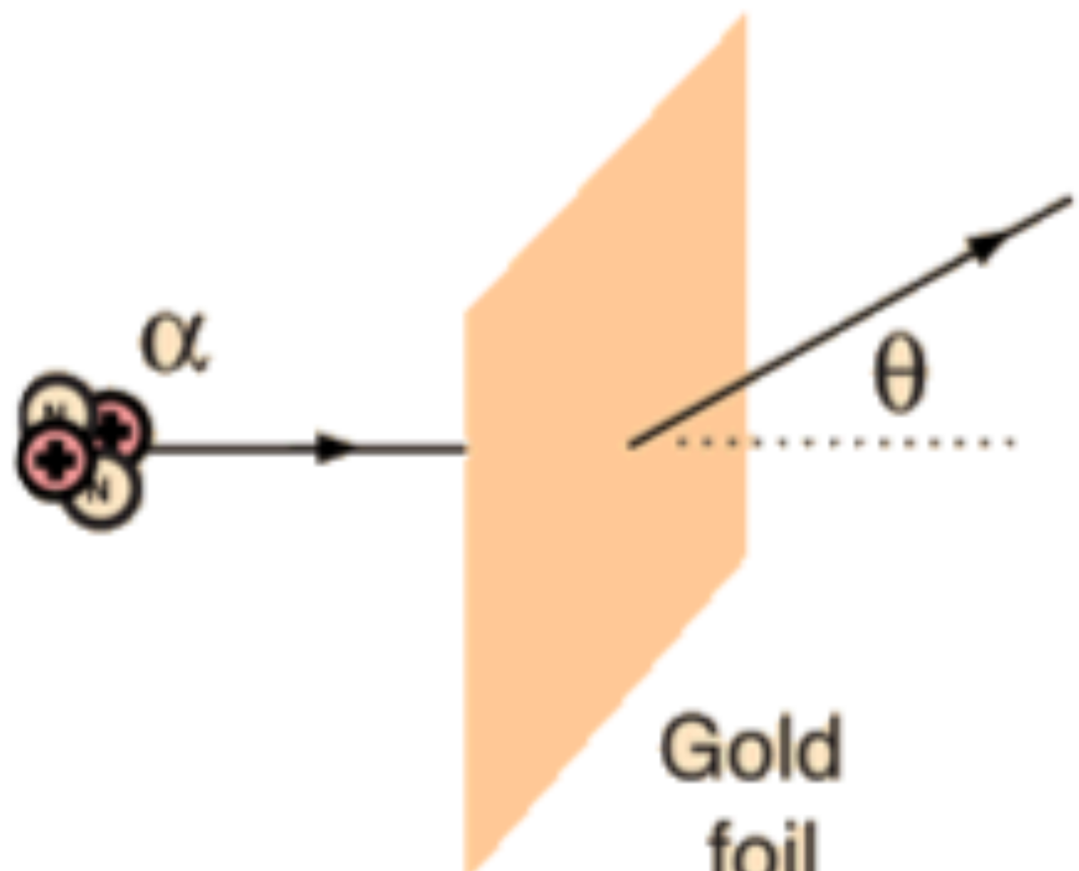
2) $$\frac{d\sigma}{d\Omega} = \frac{\underline{I_\theta} A}{\underline{I_0} L \rho N_A d\Omega}$$

- By measuring I_0, I_θ as a function of howitzer angle θ, the validity of the Rutherford model can be determined

In this Lab:

- We scatter alpha particles off of gold atom targets and measure $I_0,\ I_\theta$ to verify the nuclear model of the atom



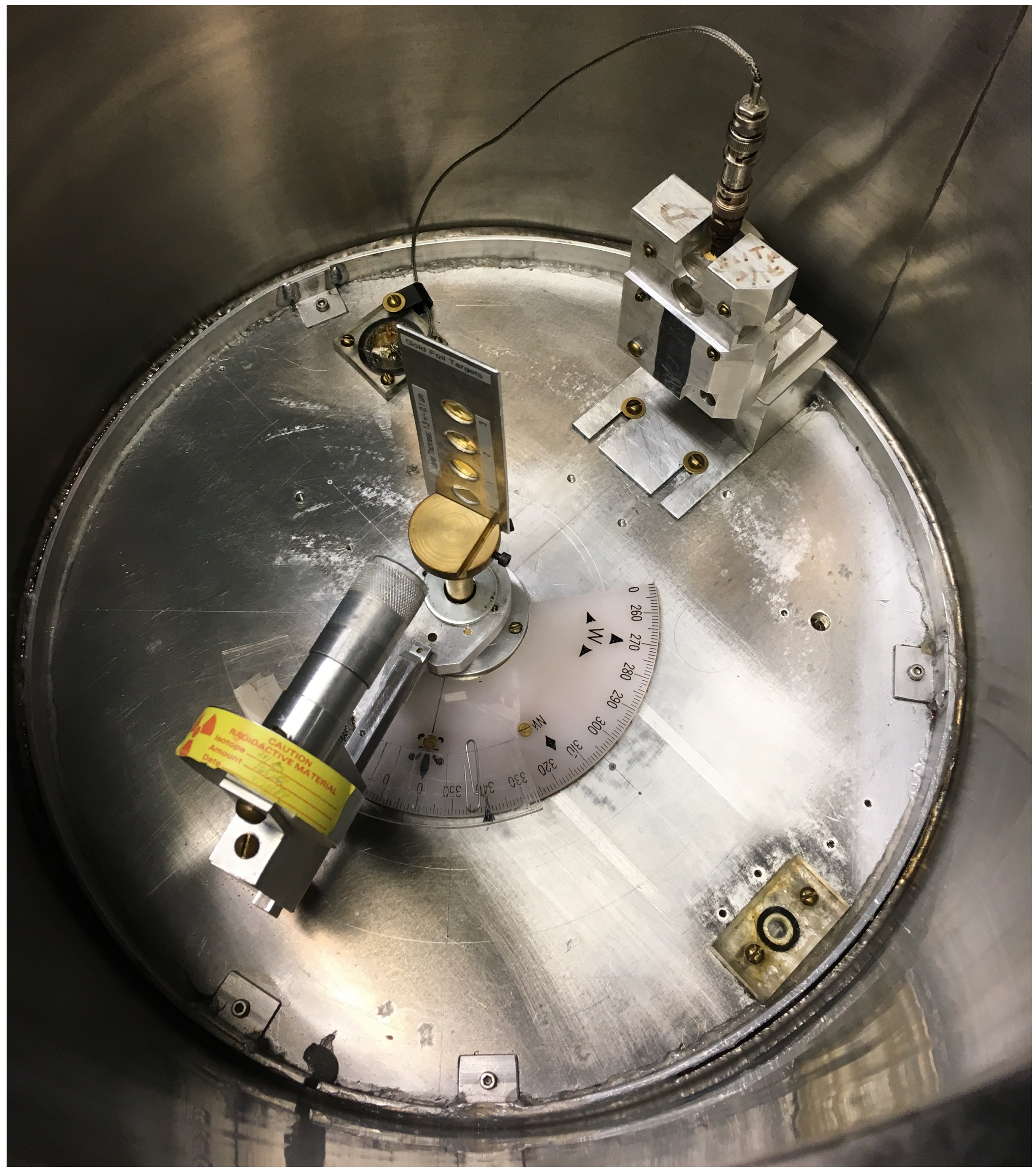
From Measurements:

- The Rutherford cross section prediction is tested

- Extract a value for the differential scattering cross section

- The thicknesses of different gold foils are extracted

Apparatus

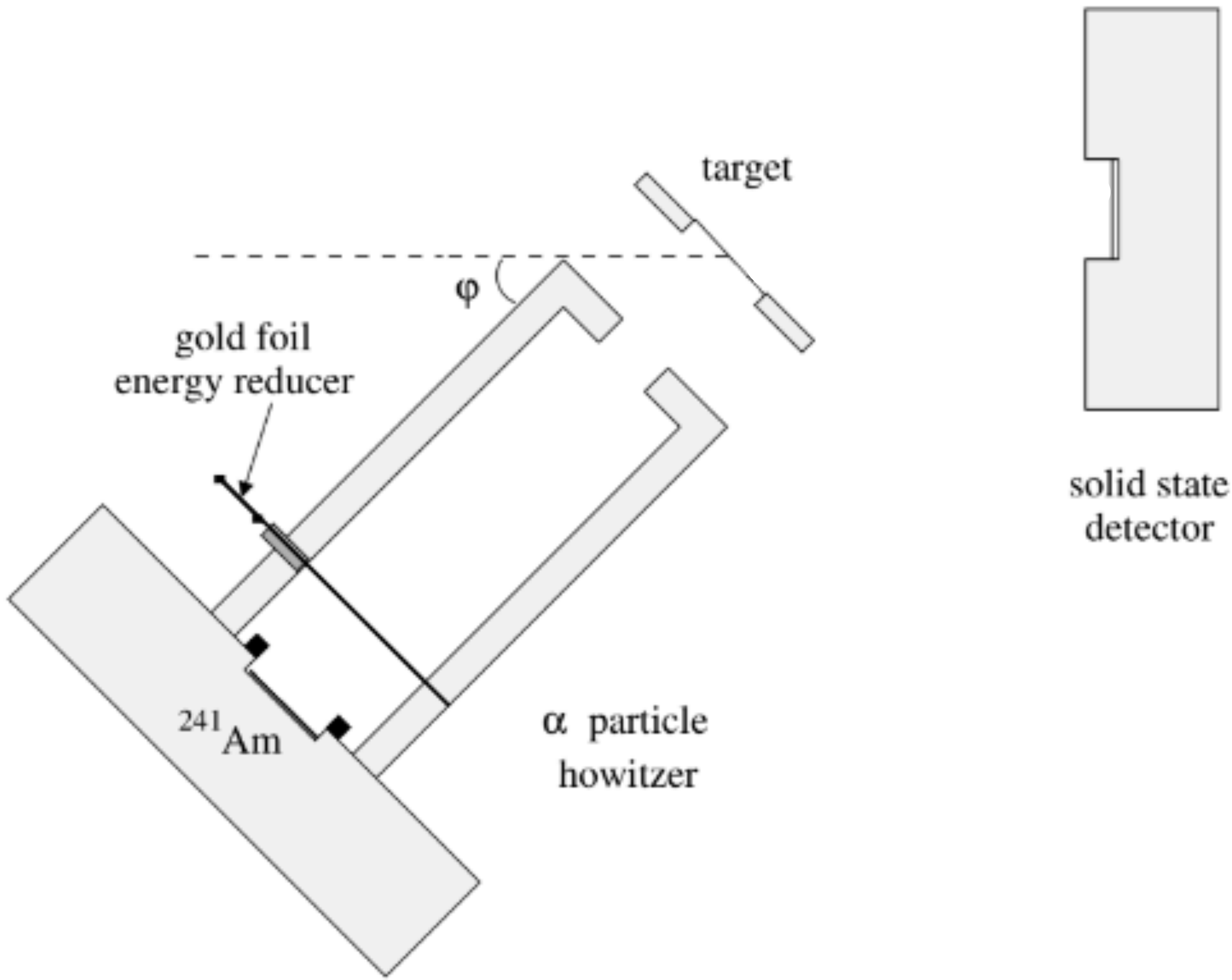


- Americium alpha particle source

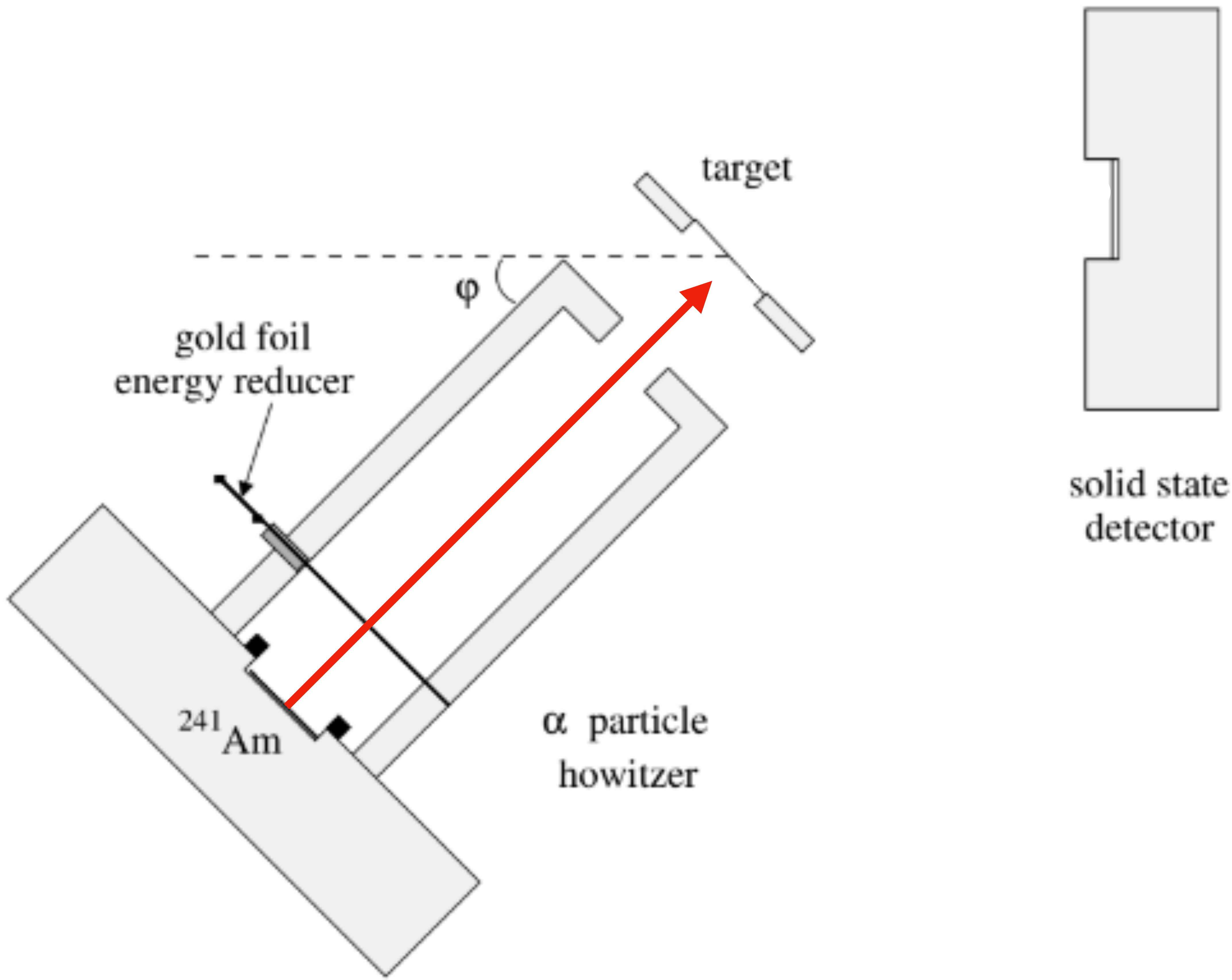
- Americium alpha particle source

- Target element foil

- Americium alpha particle source
- Target element foil
- Solid state detector



target
φ
gold foil
energy reducer
solid state
detector
241Am
α particle
howitzer


target
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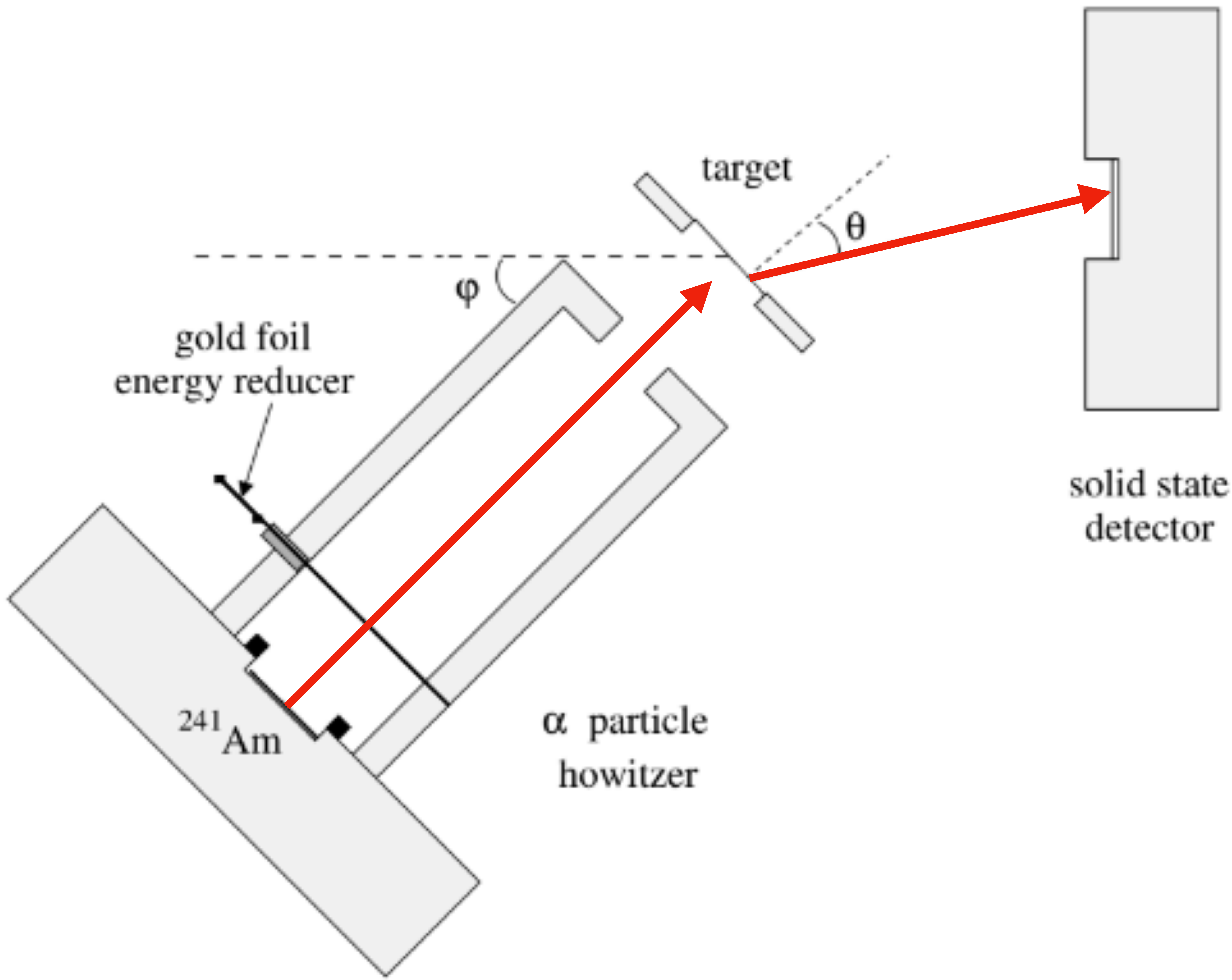
target
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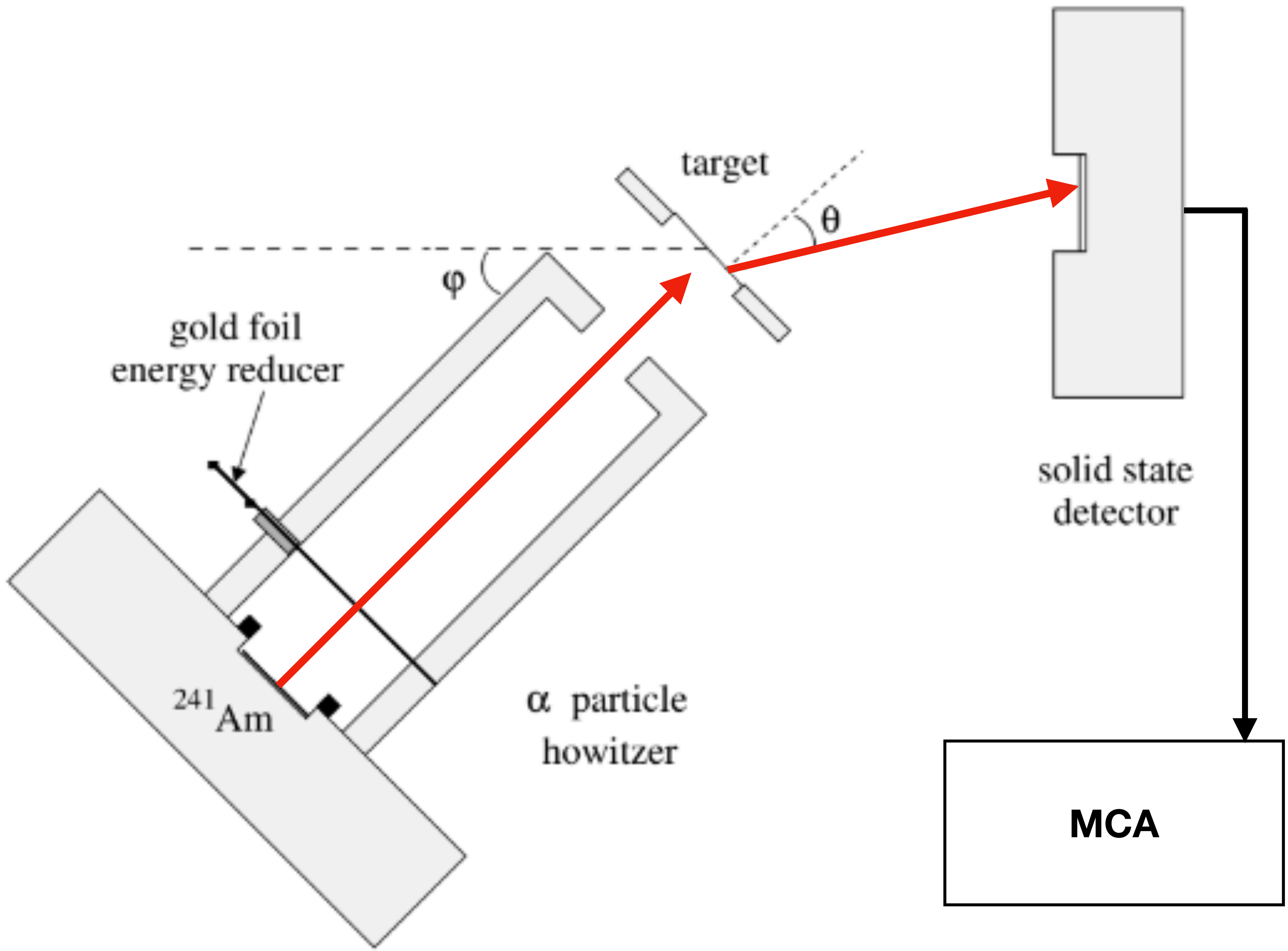


target
θ
φ
gold foil
energy reducer
solid state
detector
241Am
α particle
howitzer
MCA

Procedure

1) Shoot a beam of alpha particles at 2 layers of gold foil

2) Determine the count rate from the MCA spectrum

3) Repeat this at varying howitzer angles, to determine scattering angular dependence

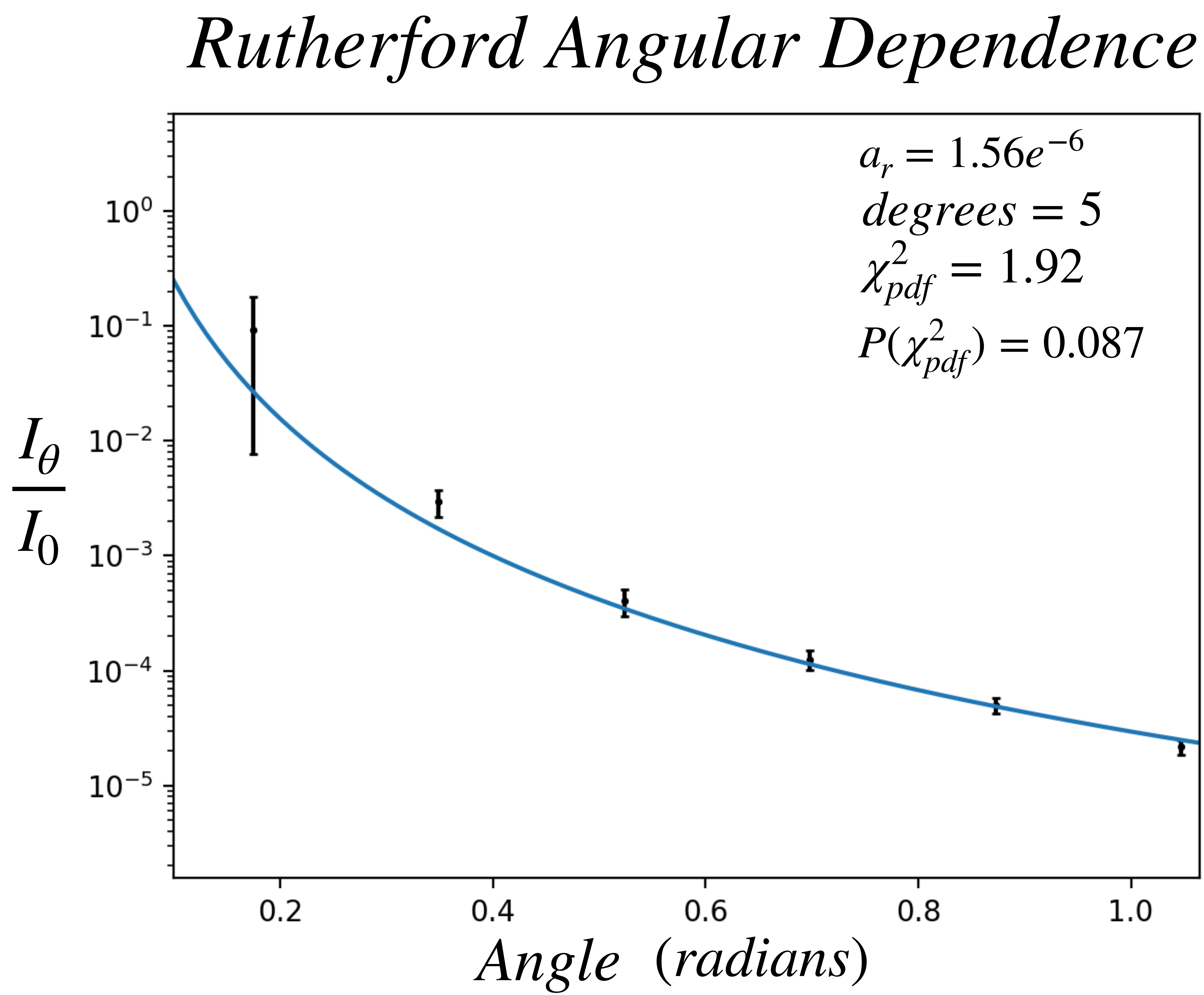
4) Normalize the scattering rate with a daily calibration taken with no foil at 0 degrees

Determination of Fit

- Normalized count rates are plotted against angle

- To determine the validity of the Rutherford model, the following functional form is fit using parameter a_r:

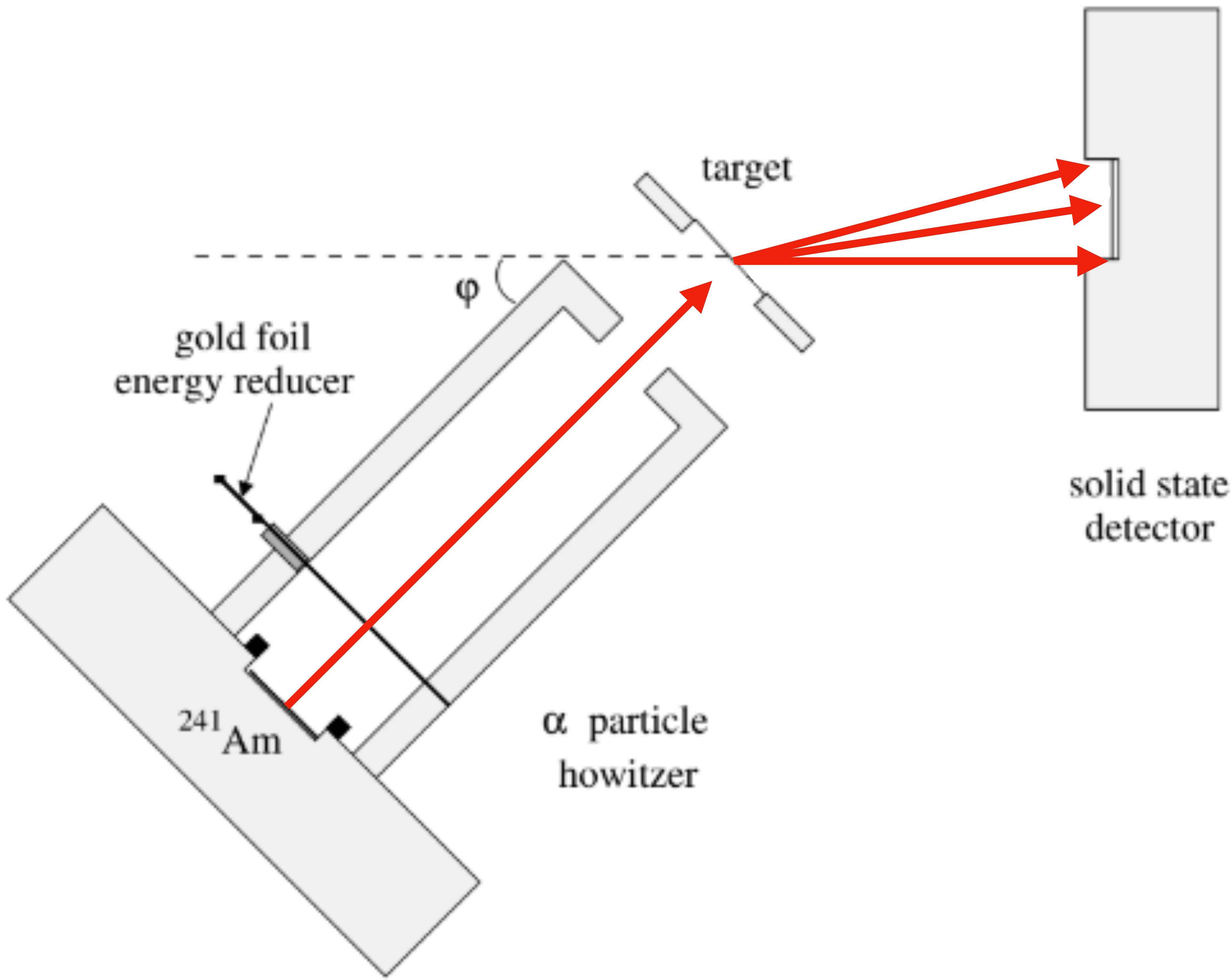
$$f(\theta) = \frac{a_r}{\sin^4 \frac{\theta}{2}}$$



Rutherford Angular Dependence
$a_r = 1.56e^{-6}$
$degrees = 5$
$\chi^2_{pdf} = 1.92$
$P(\chi^2_{pdf}) = 0.087$
$\frac{I_\theta}{I_0}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.2
0.4
0.6
0.8
1.0
$Angle \ (radians)$

Angular Resolution

- The detector has a wide angular resolution



target
φ
gold foil
energy reducer
solid state
detector
^{241}Am
α particle
howitzer

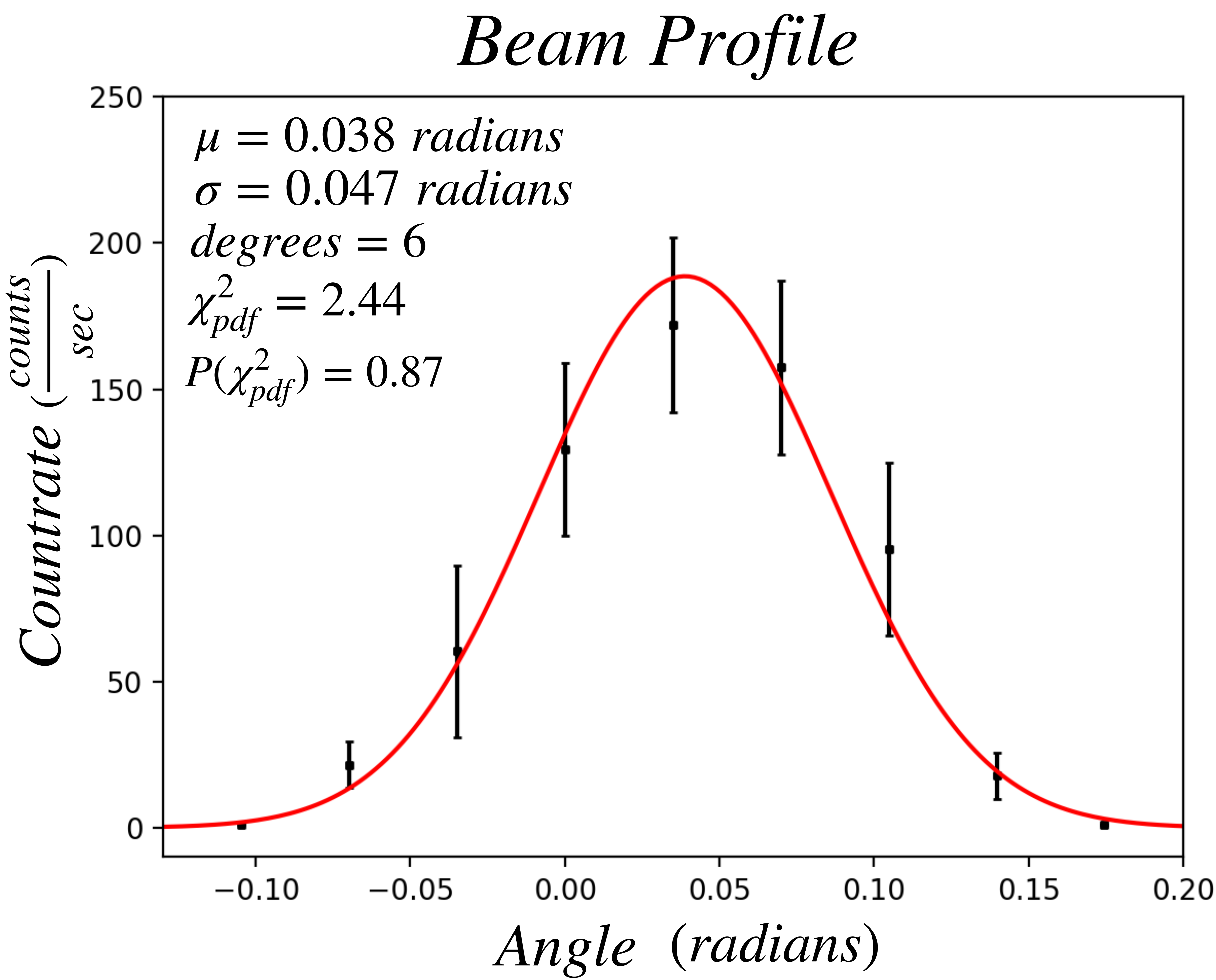
Angular Resolution

- The detector has a wide angular resolution
- The rutherford functional form is accurate for a specific angle, and must be modified to account for the wide angular resolution of the apparatus

Beam Profile

- The angular resolution of the apparatus is characterized by the beam profile

- Measured by pivoting the howitzer in small increments about the 0 degree point, and plotting count rates



Beam Profile
$\mu = 0.038$ radians
$\sigma = 0.047$ radians
degrees = 6
$\chi^2_{pdf} = 2.44$
$P(\chi^2_{pdf}) = 0.87$
Countrate ($\frac{counts}{sec}$)
Angle (radians)
250
200
150
100
50
0
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

Beam Profile

- The beam is highly collimated, and the detector is sensitive - the resolution of both can be seen as square functions

Beam Profile

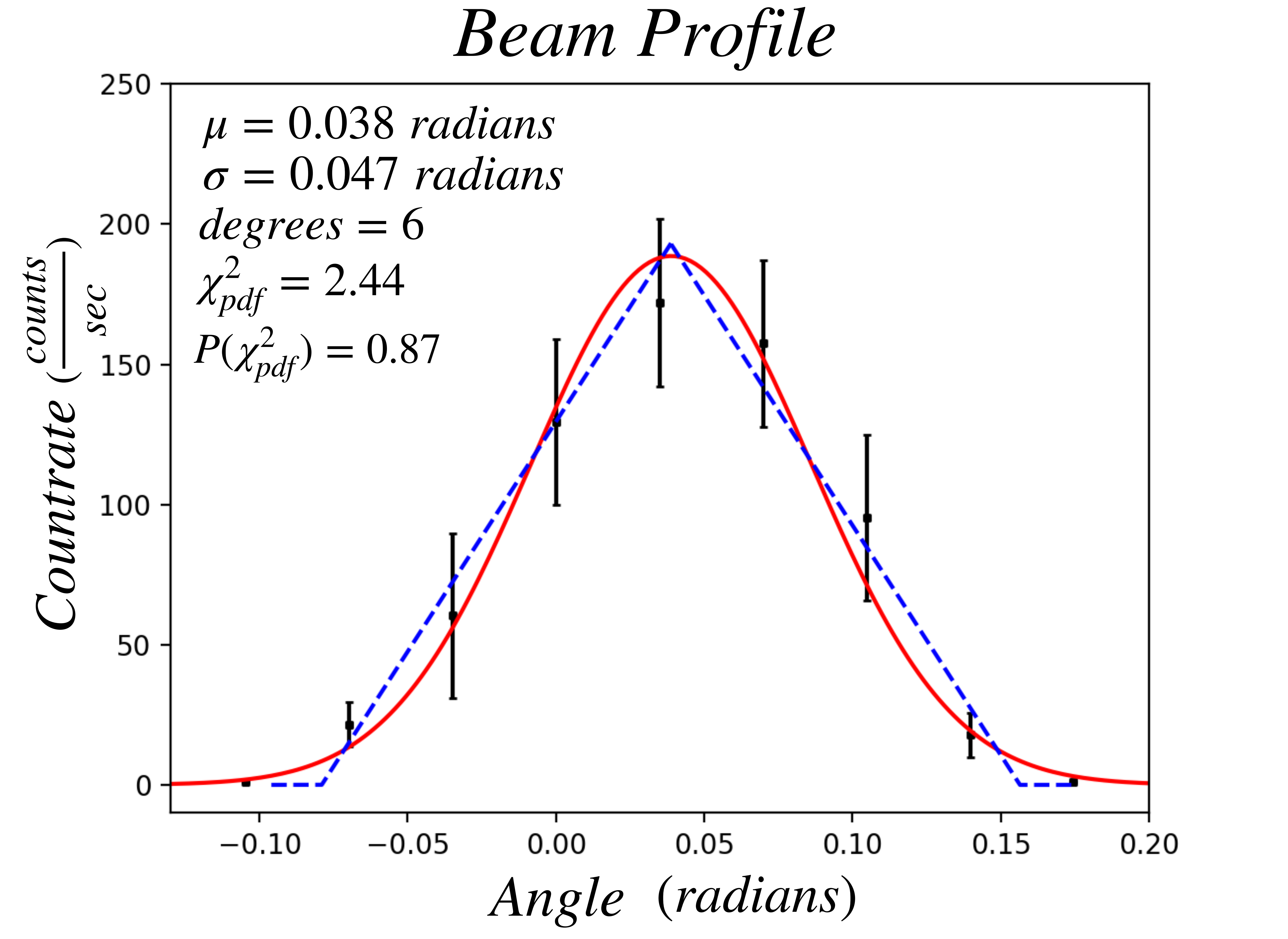
- The convolution of the beam upon the detector produces a triangular function dependent on howitzer angle ϕ and scattering angle θ:

Beam Profile

- The convolution of the beam upon the detector produces a triangular function dependent on howitzer angle ϕ and scattering angle θ:

$$g(\phi - \theta; \theta_0)$$

where θ_0 represents the half width of the triangular function



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Angle (radians)
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150
100
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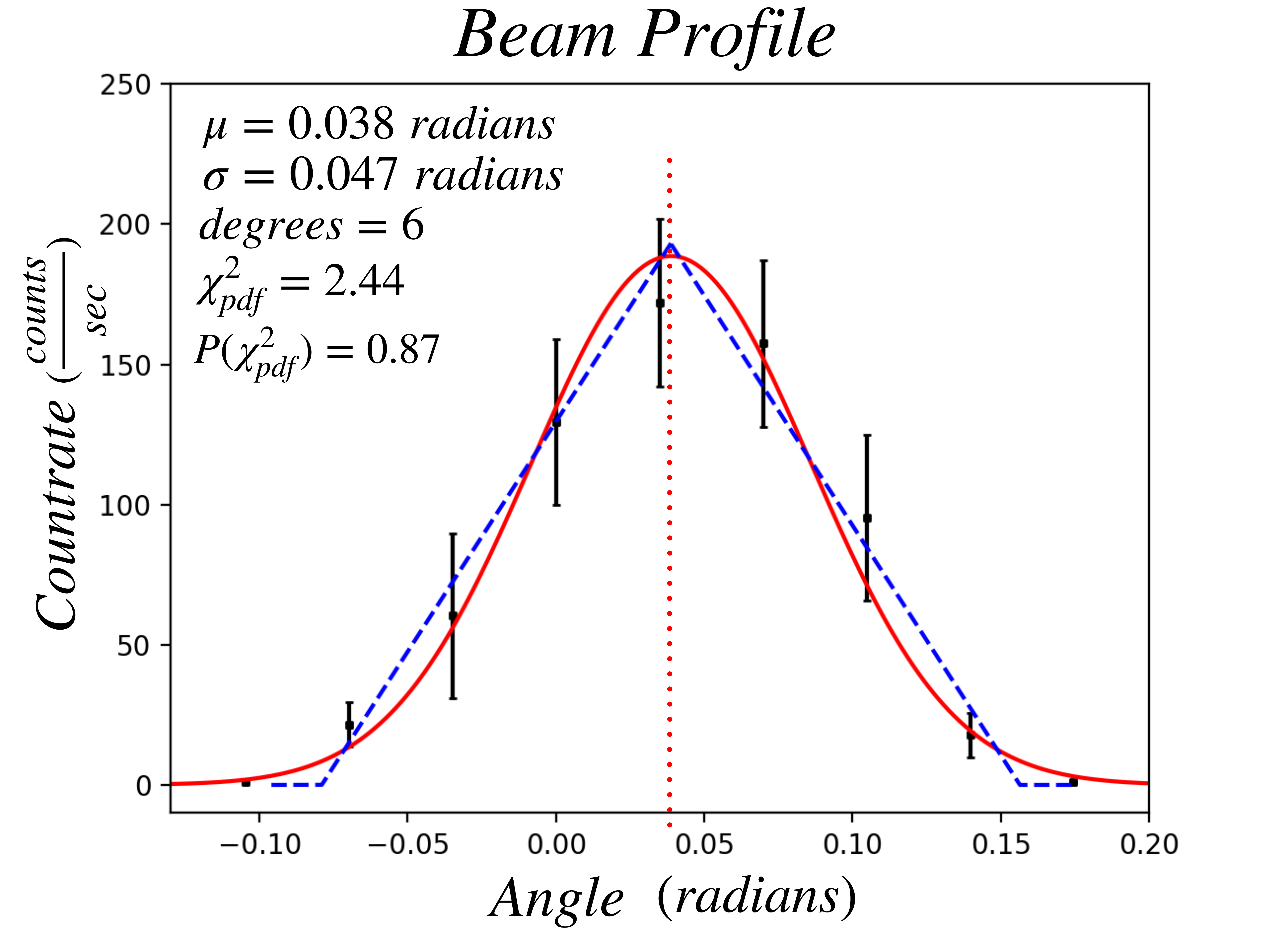
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20



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250
200
150
100
50
0
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

Profile Uncertainty

- Uncertainty in the beam profile half width θ_0 can be approximated using the fit covariance

- The half width value of the beam profile was extracted as:

$$\theta_0 = (0.117 \pm 0.039) \ radians$$

- 0 degree position offset is:

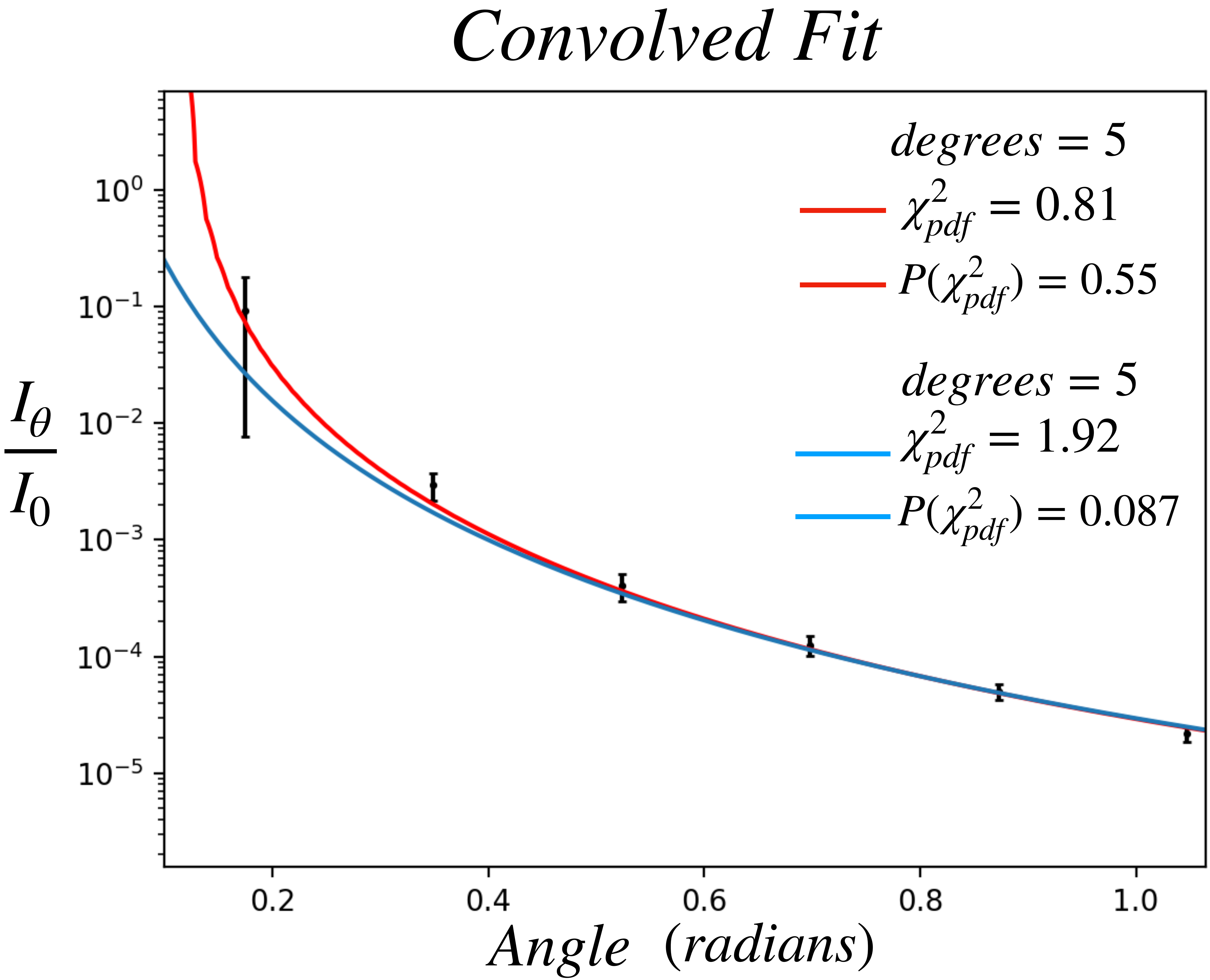
$$\mu \approx 0.038 \ radians$$

Modified Rutherford Model

- In order to model the angular response of our apparatus in our functional form, we convolve the two:

$$C(\phi) = C_0 \int_0^{\pi} g(\phi - \theta; \theta_0) \sin^{-4}(\frac{\theta}{2}) d\theta$$

- We fit the convolved $C(\phi)$ for parameter C_0



Convolved Fit
degrees = 5
$\chi^2_{pdf} = 0.81$
$P(\chi^2_{pdf}) = 0.55$
degrees = 5
$\chi^2_{pdf} = 1.92$
$P(\chi^2_{pdf}) = 0.087$
$\frac{I_\theta}{I_0}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.2
0.4
0.6
0.8
1.0
Angle (radians)

Statistical Uncertainty

- Poisson count error on $\frac{I_\theta}{I_0}$ from uncertainty in MCA counts

$$\sigma_{I_\theta} = \frac{\sqrt{counts}}{measurement\ duration}$$

$$\sigma_{I_0} = \frac{\sqrt{counts}}{120\ secs}$$

- Propagation in quadrature:

$$\sigma_{\frac{I_\theta}{I_0},\ poisson} = \frac{I_\theta}{I_0}\sqrt{(\frac{\sigma_{I_\theta}}{I_\theta})^2 + (\frac{\sigma_{I_0}}{I_0})^2}$$

Statistical Uncertainty

$$\sigma_{\frac{I_\theta}{I_0},\ poisson} = \frac{I_\theta}{I_0}\sqrt{(\frac{\sigma_{I_\theta}}{I_\theta})^2 + (\frac{\sigma_{I_0}}{I_0})^2}$$

- Varies from point to point, but ranges from $\approx \pm 1\,\%$ to $\approx \pm 7.9\,\%$

Systematic Uncertainty

- Howitzer angle: $\sigma_{\frac{I_\theta}{I_0}}, \theta$
- Fairly inaccurate, and includes a 0 position offset of $\mu \approx 0.038\ radians$

Howitzer Angle

- Systematic uncertainty in howitzer angle position: $$\sigma_{\theta} = \pm 1^{\circ}$$

- Propagate vertically using the slope of the convolved fit:

$$\sigma_{\frac{I_{\theta}}{I_0},\ \theta} = \frac{dC(\phi)}{d\theta}\sigma_{theta}$$

- Varies point to point, but ranges from $\approx \pm 6.6\,\%$ to $\approx \pm 38\,\%$

Percent Uncertainty

- Angular uncertainty dominates at lower angles:

  10 degrees: $99\,\%$ of error is angular

- At higher angles, poisson statistical uncertainty and angular systematic uncertainty contributions even out

Differential Cross Section

- For $50°$, using the measured $\dfrac{I_\theta}{I_0}$ and total uncertainty $\sigma_{\frac{I_\theta}{I_0}}$:

$$\frac{d\sigma}{d\Omega} = \frac{I_\theta A}{I_0 L \rho N_A d\Omega} = \underline{(3.89 \pm 0.603)e^{-22}\ (cm^2)}$$

- The theoretical value given is:

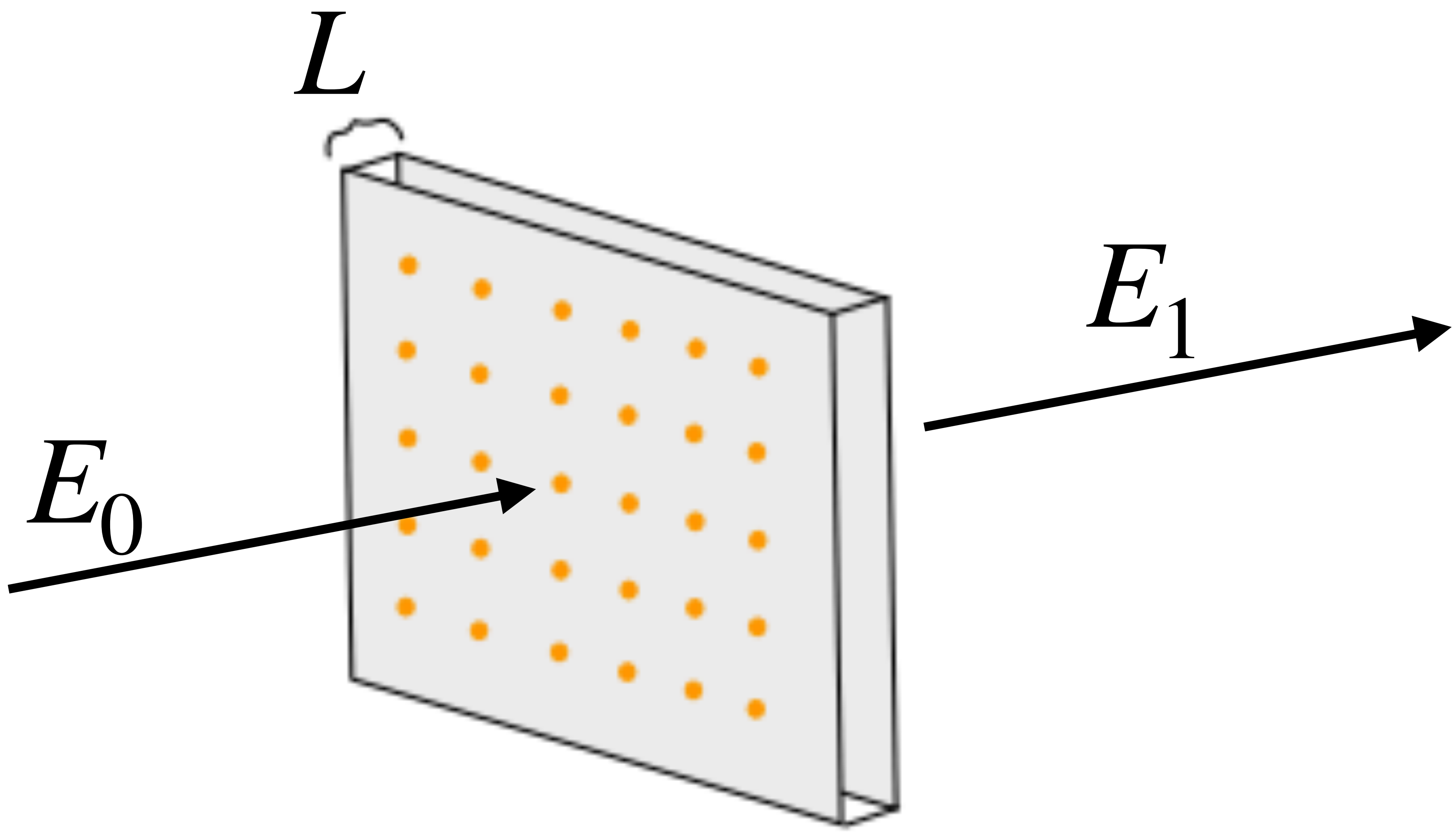
$$\frac{d\sigma}{d\Omega} = (\frac{ZZ'e^2}{4E})^2 \sin^{-4}\frac{\theta}{2} = \underline{3.204e^{-23}\ (cm^2)}$$

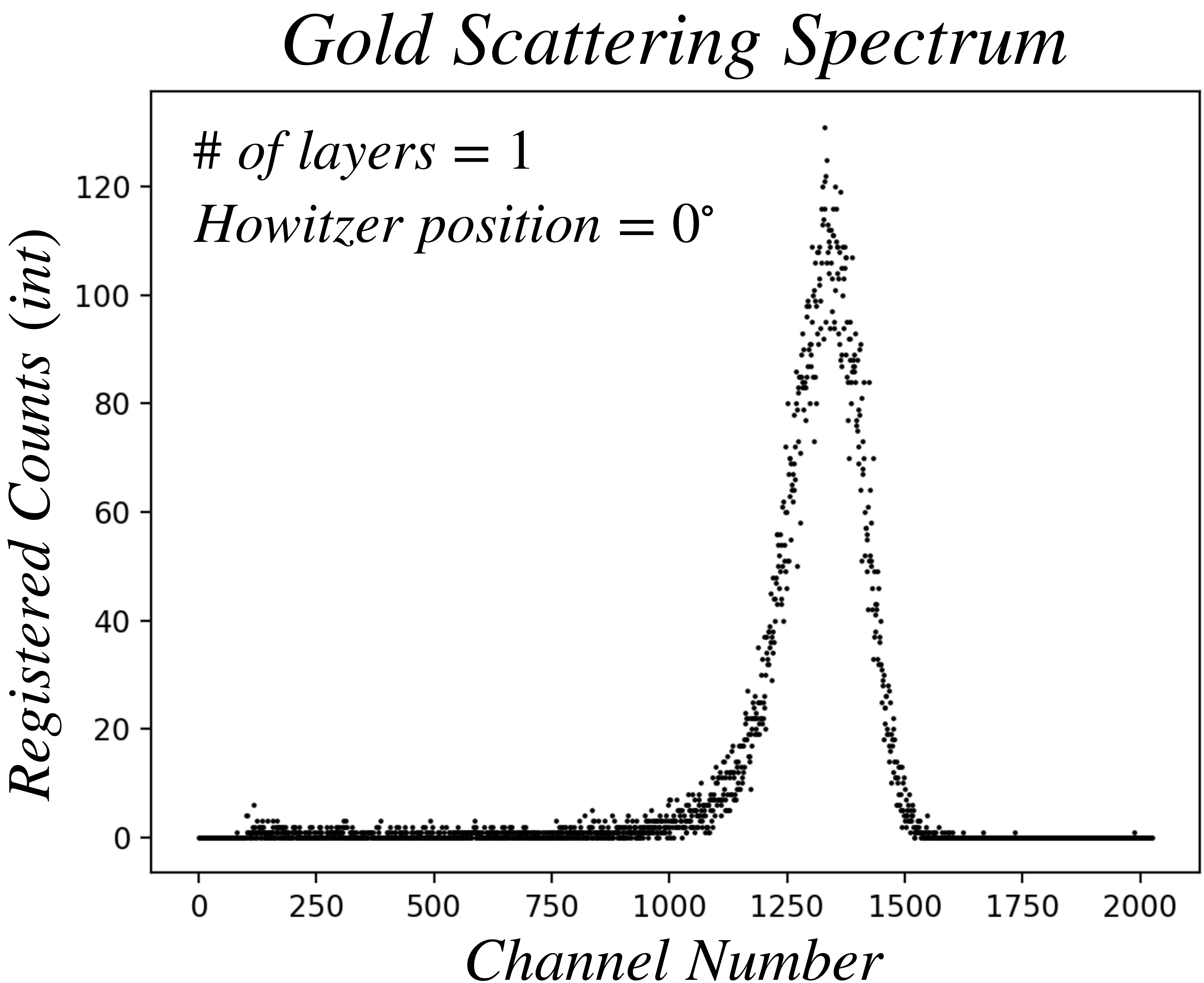
Gold Foil Thickness

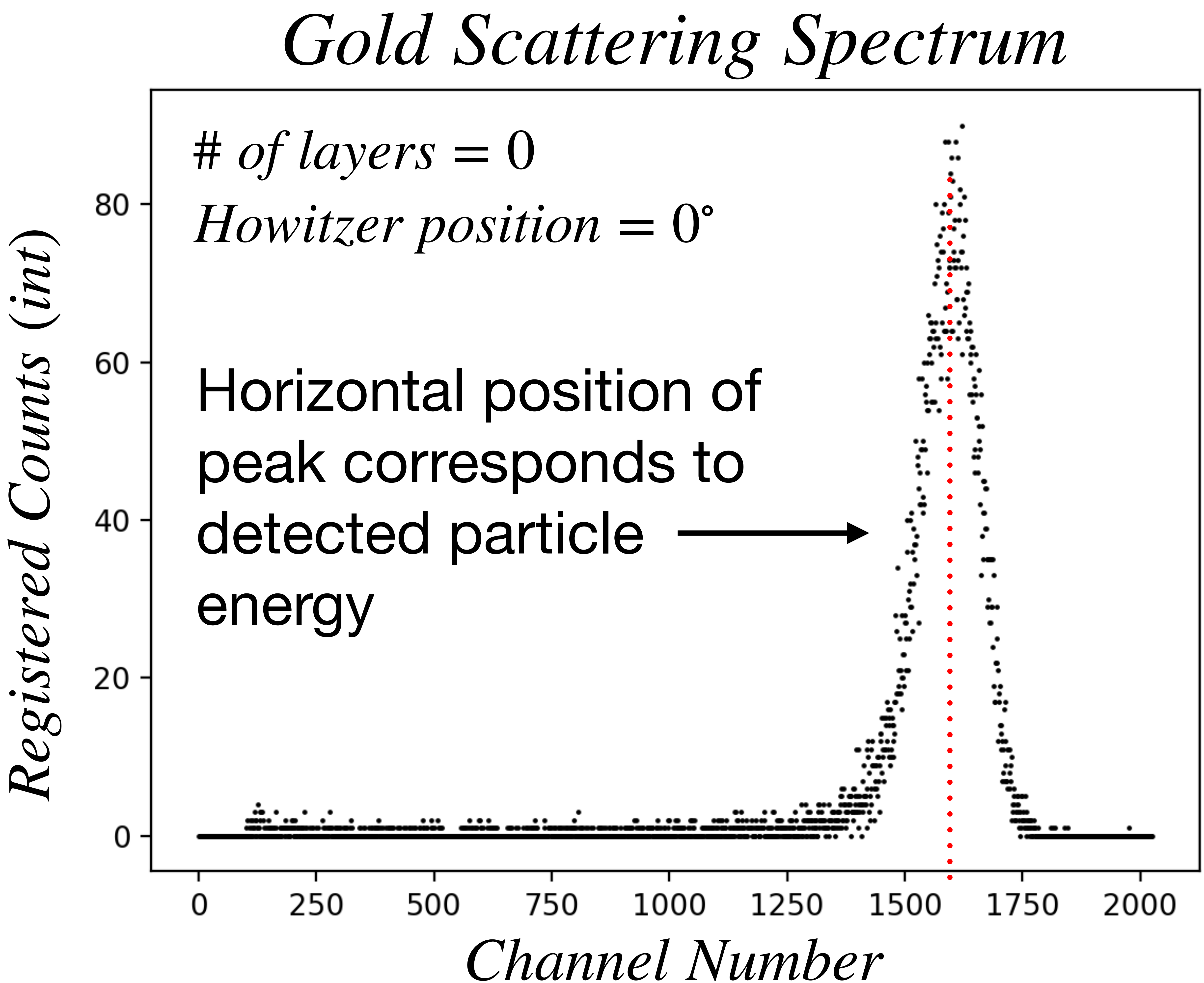
- By scattering with the howitzer at $0°$ it is possible to extract the thickness L of the gold foil target

- Can be done by measuring the energy attenuation affect of various thicknesses of gold foil on the incident beam

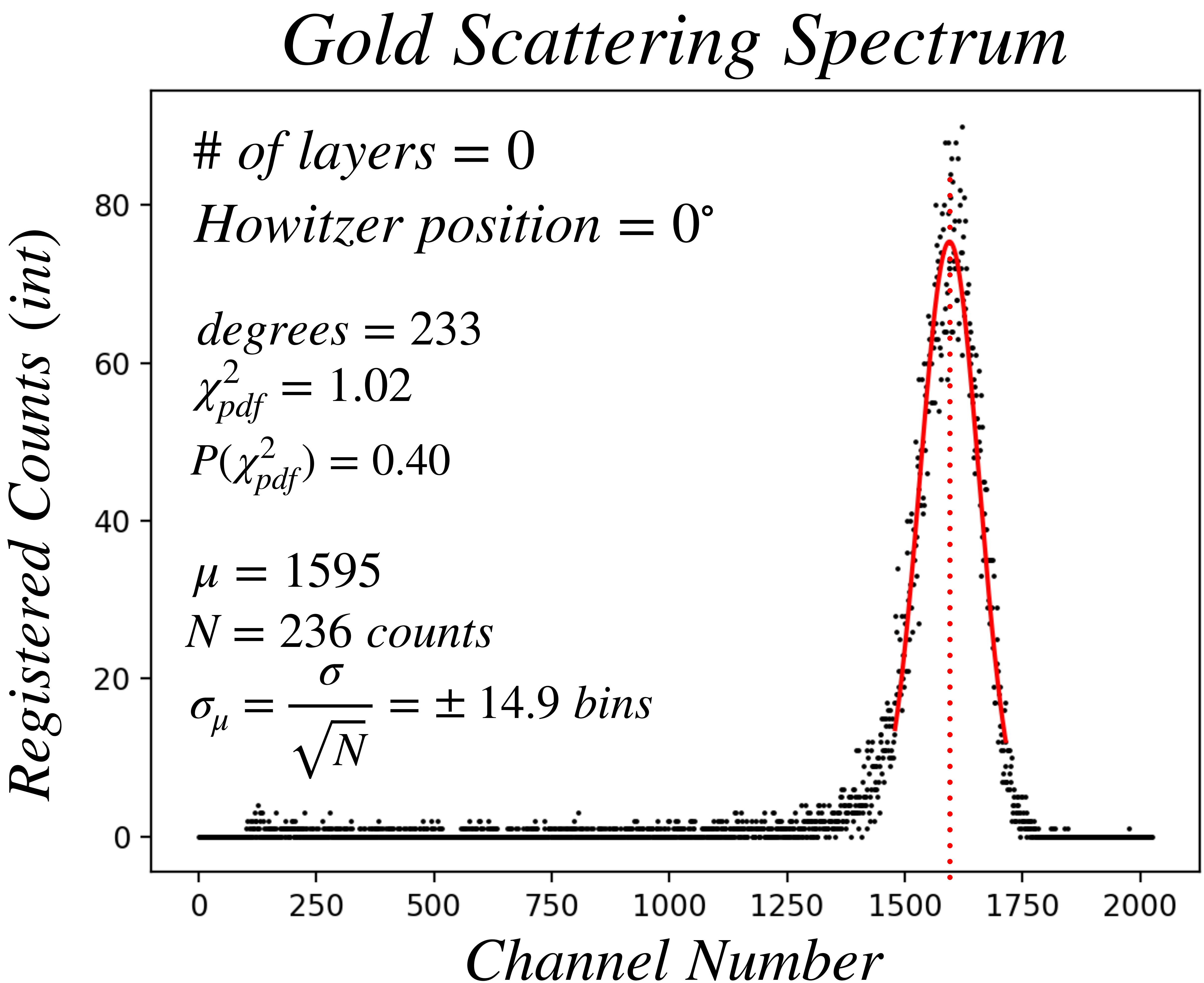
- Incident energy E_0 is attenuated to E_1 after traveling through thickness L of gold




Gold Scattering Spectrum
# of layers = 1
Howitzer position = 0°
Registered Counts (int)
Channel Number
0
20
40
60
80
100
120
0
250
500
750
1000
1250
1500
1750
2000



Gold Scattering Spectrum
# of layers = 0
Howitzer position = 0°
Horizontal position of peak corresponds to detected particle energy
Registered Counts (int)
Channel Number
0
20
40
60
80
0
250
500
750
1000
1250
1500
1750
2000


Gold Scattering Spectrum
# of layers = 0
Howitzer position = 0°
degrees = 233
$\chi^2_{pdf} = 1.02$
$P(\chi^2_{pdf}) = 0.40$
$\mu = 1595$
$N = 236$ counts
$\sigma_\mu = \frac{\sigma}{\sqrt{N}} = \pm 14.9$ bins
Registered Counts (int)
Channel Number
0
20
40
60
80
0
250
500
750
1000
1250
1500
1750
2000

- By finding the MCA channels $c_0,\ c_1$ corresponding to energy peaks, and knowing E_0, we can extract the attenuated energy E_1:

$$E_1 = \frac{c_1}{c_0} E_0$$

- The uncertainty σ_{E_1} is given by propagating the uncertainty on each mean bin:

$$\sigma_{E_1} = E_1 \sqrt{(\frac{\sigma_{c_0}}{c_0})^2 + (\frac{\sigma_{c_1}}{c_1})^2}$$

- Americium emits alpha particles with these three most prominent energies:

    - 86%: $E_\alpha = 5.486\ MeV$

    - 12.7%: $E_\alpha = 5.433\ MeV$

    - 1.4%: $E_\alpha = 5.391\ MeV$

- So we take: $E_0 = 5.486\ MeV$

- Alpha particle energy at the detector for each foil thickness:

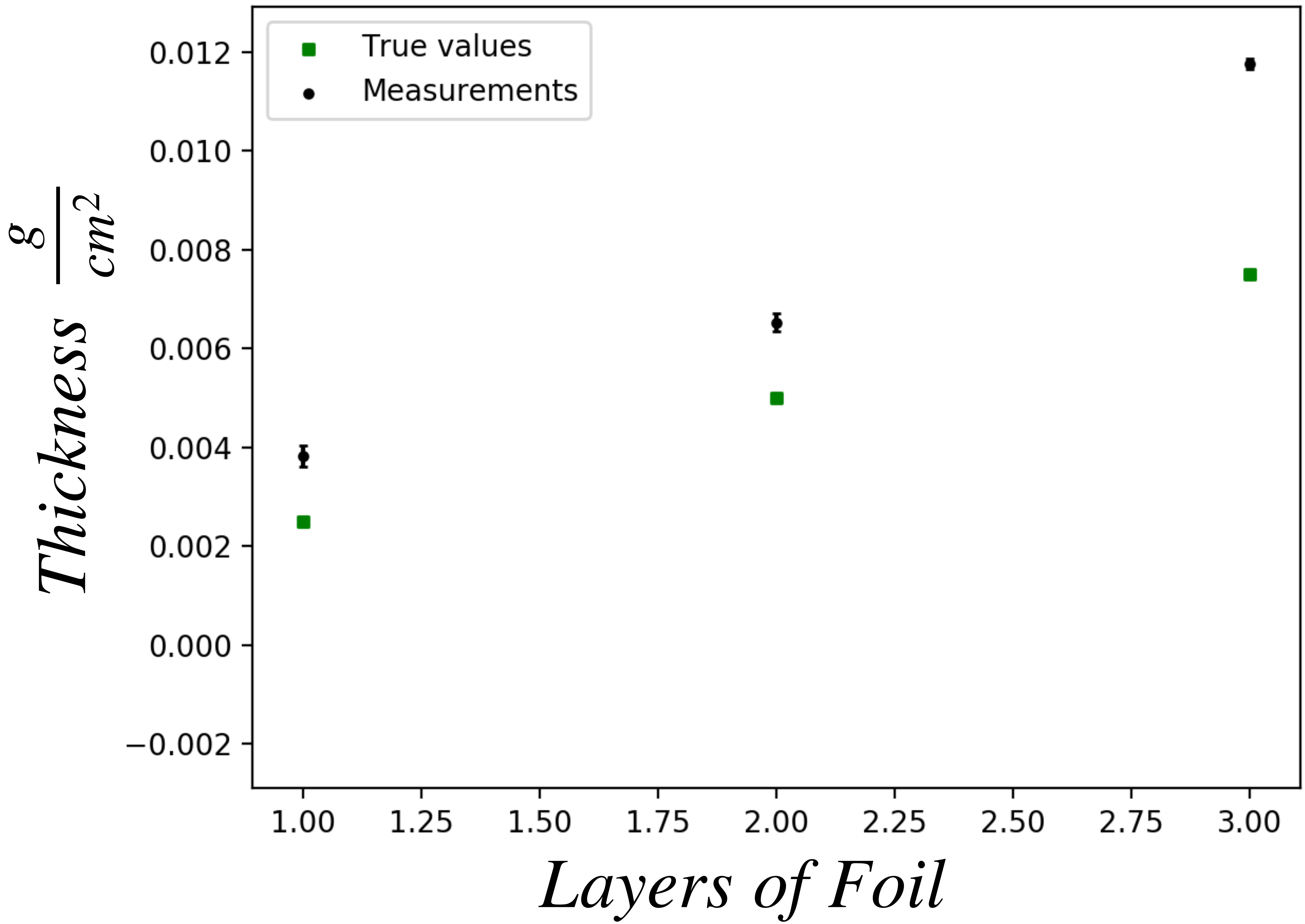
0 layers: $E_\alpha = 5.486\ MeV$

1 layers: $E_\alpha = (4.579 \pm 0.054)\ MeV$

2 layers: $E_\alpha = (3.883 \pm 0.048)\ MeV$

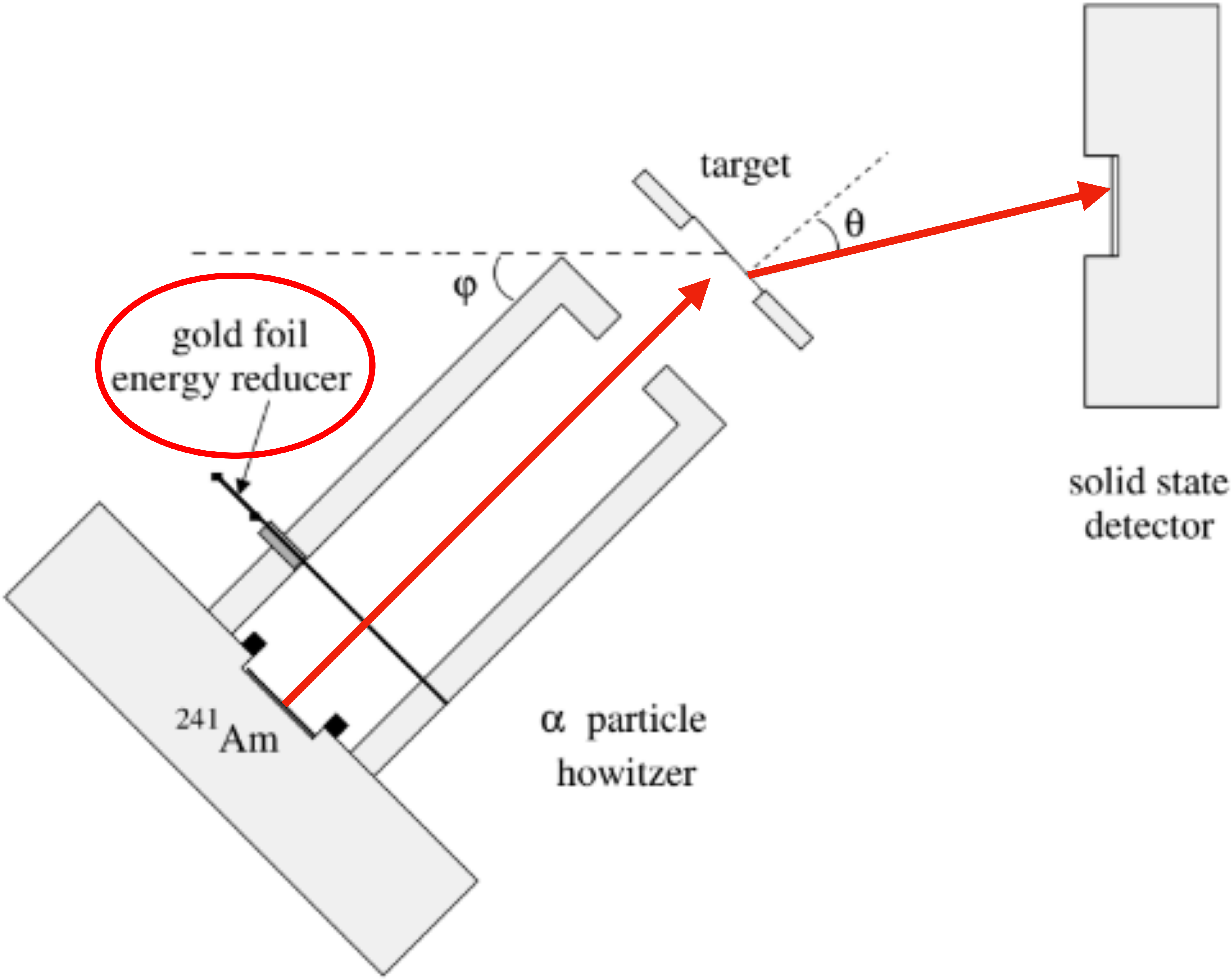
3 layers: $E_\alpha = (2.329 \pm 0.036)\ MeV$

- We use known attenuation data from NIST to calculate thicknesses:



Initial Energy Error

- For safety reasons, there is a thin gold coating in front of the Americium source - our $E_0 = 5.486\ MeV$ is not correct


target
θ
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gold foil
energy reducer
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α particle
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241Am

Initial Energy Error

- For safety reasons, there is a thin gold coating in front of the Americium source - our $E_0 = 5.486\ MeV$ is not correct

- Coating is 1.5 microns thick, so using the attenuation energy for 1 layer of gold foil we take $E_0 = 4.580\ MeV$

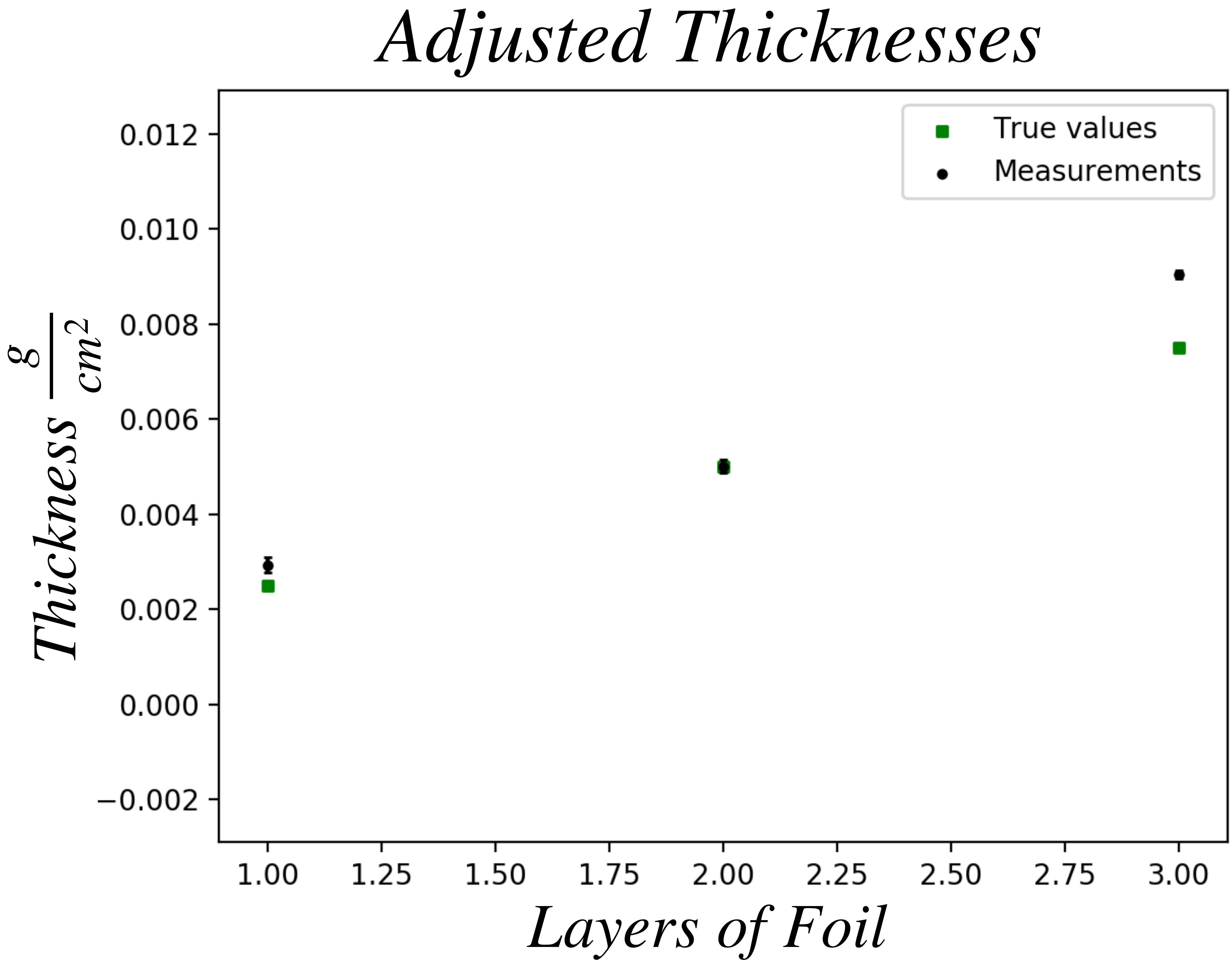
- Adjusted alpha particle energy at the detector for each foil thickness:

0 layers: $E_\alpha = 4.580\ MeV$

1 layers: $E_\alpha = (3.823 \pm 0.045)\ MeV$

2 layers: $E_\alpha = (3.242 \pm 0.041)\ MeV$

3 layers: $E_\alpha = (1.945 \pm 0.030)\ MeV$



Adjusted Thicknesses
True values
Measurements
0.012
0.010
0.008
0.006
0.004
0.002
0.000
−0.002
$Thickness\ \frac{g}{cm^2}$
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
Layers of Foil

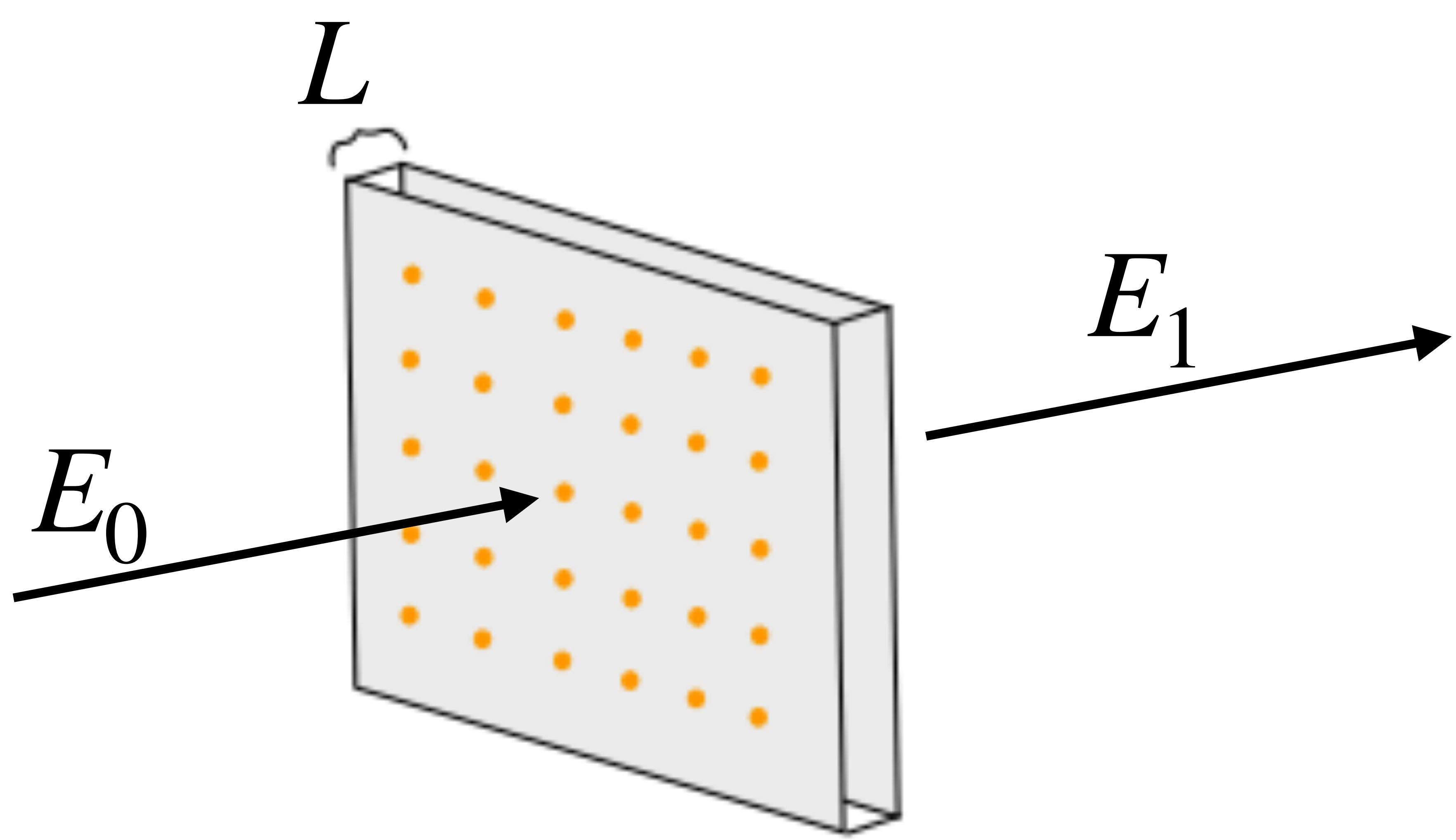
Summary

- The rutherford cross section correctly predicts nuclear atomic structure, with $\chi^2_{pdf} = 0.81$ for 5 degrees of freedom

- The differential cross section extracted from measurements is $(3.89 \pm 0.603)e^{-22}\ (cm^2)$ compared to the theoretical prediction of $3.204e^{-23}\ (cm^2)$

Summary

- Alpha particle scattering allows us to extract foil thicknesses fairly accurately



Extracted:	True values:
$T_1 = (0.0029 \pm 5.63\%)\ (\frac{g}{cm^2})$	$T_1 = 0.0025\ (\frac{g}{cm^2})$
$T_2 = (0.0050 \pm 2.77\%)\ (\frac{g}{cm^2})$	$T_2 = 0.0050\ (\frac{g}{cm^2})$
$T_3 = (0.0090 \pm 0.93\%)\ (\frac{g}{cm^2})$	$T_3 = 0.0075\ (\frac{g}{cm^2})$

Consequences

- The discovery of nuclear atomic structure was a monumental scientific achievement

- The scattering technique pioneered by Rutherford is still used in physics to probe microscopic structure

Thank you!