Relativistic Dynamics

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Classical Mechanics for high speed objects

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- An objects motion in time can be described with velocity $\overrightarrow{v}$, momentum $\overrightarrow{p}$ and kinetic energy K
- Classical physics fails when predicting the dynamics of high speed objects
- A correction to classical model is needed for objects at high speeds

Relativity

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- No object can move faster than the speed of light c
- Relativity corrects classical mechanical predictions for high speed objects

Newtonian Momentum

$$\overrightarrow{p} = m\overrightarrow{v}$$

- No restriction on how fast objects can move

Relativistic Correction

$$\overrightarrow{p} = \gamma m \overrightarrow{v}$$

where $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

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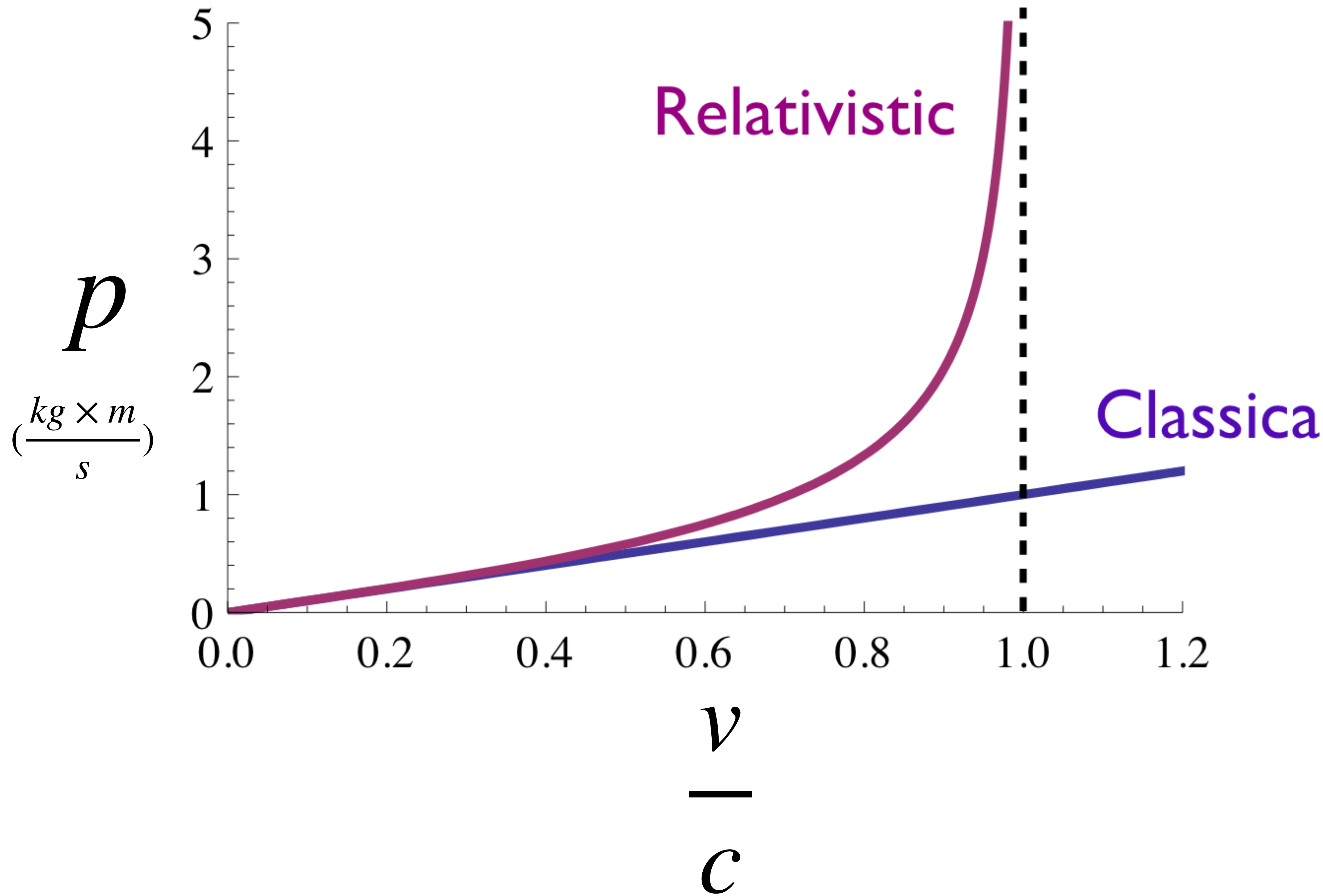
$$\overrightarrow{p} = \gamma m \overrightarrow{v}$$

where $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

- When $v \ll c$:

$\gamma = 1$ and we get back classical momentum

Newtonian vs. Relativistic Momentum



Testing Newtonian vs. Relativistic

- Finding relationships between $\overrightarrow{v}$, $\overrightarrow{p}$, K for a fast object allows us to see which model fits best at high speeds

- We also extract $\dfrac{e}{m}$ (electron charge to mass ratio)

Apparatus Capabilities

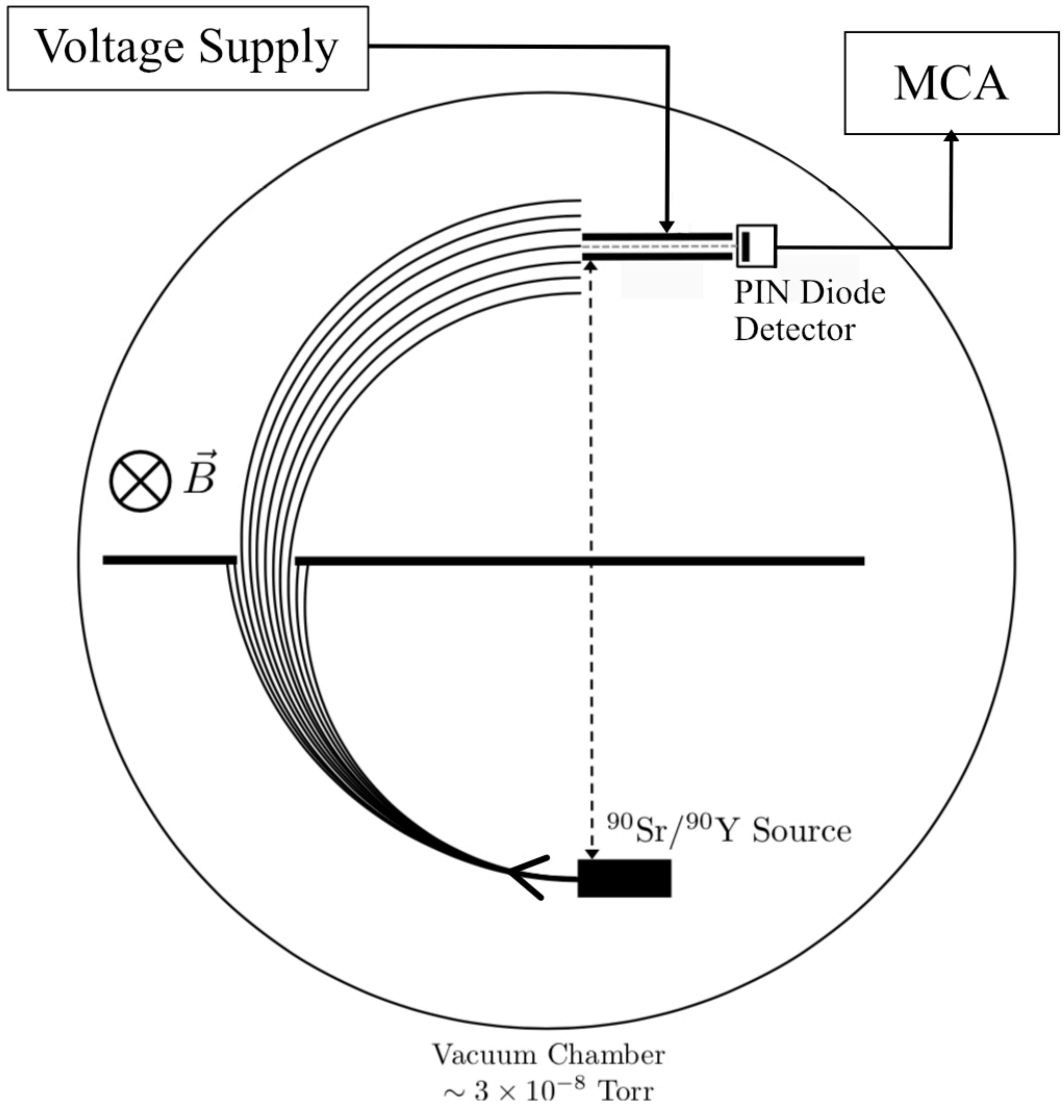
1) Needs to produce objects moving at high speeds (close to speed of light)

2) Needs to be capable of finding $\overrightarrow{v}$, $\overrightarrow{p}$, K for this fast object

Apparatus

Spherical magnet with a vacuum chamber inside

Vacuum Chamber



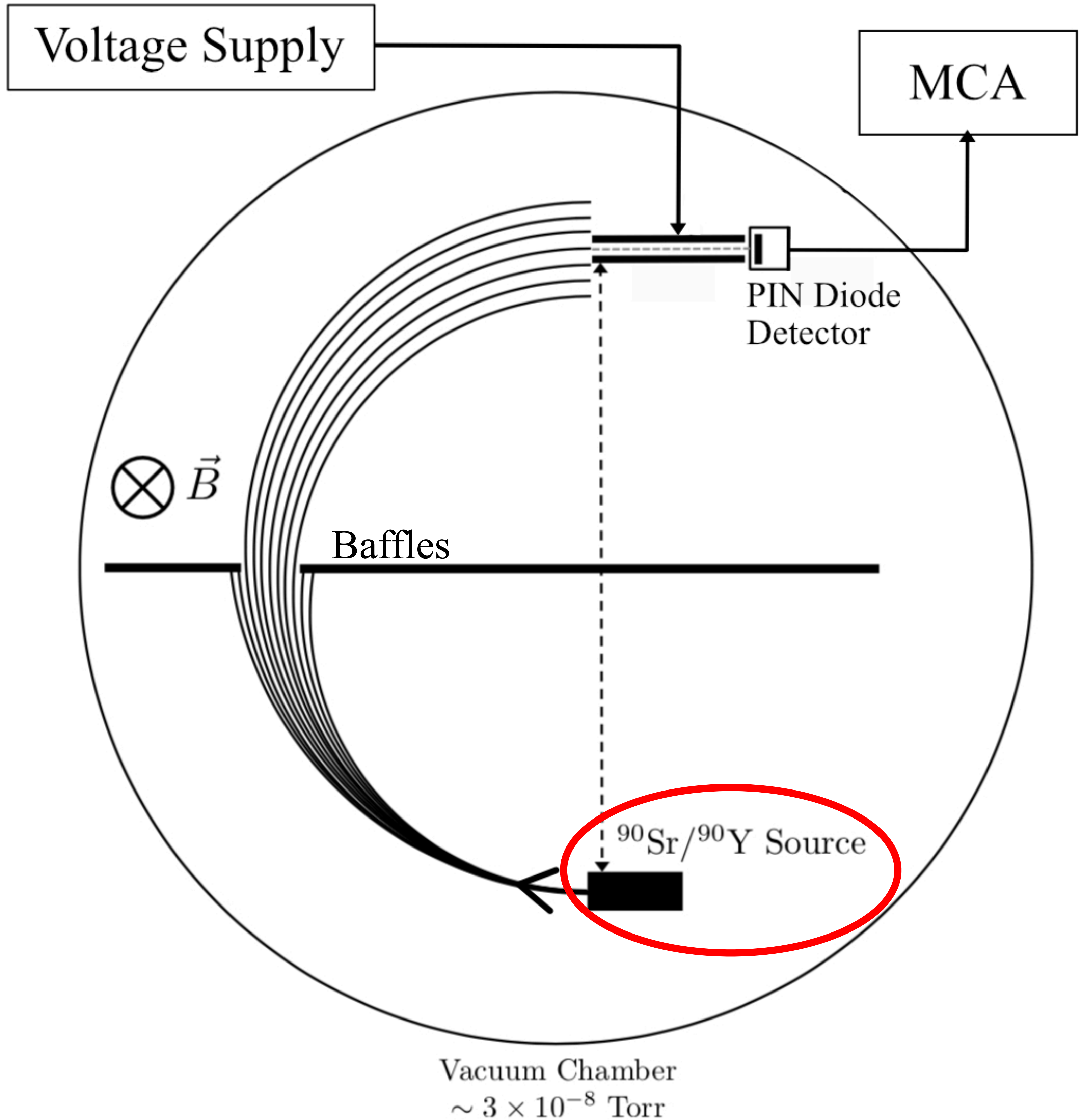
What fast object is used?

- Electrons:

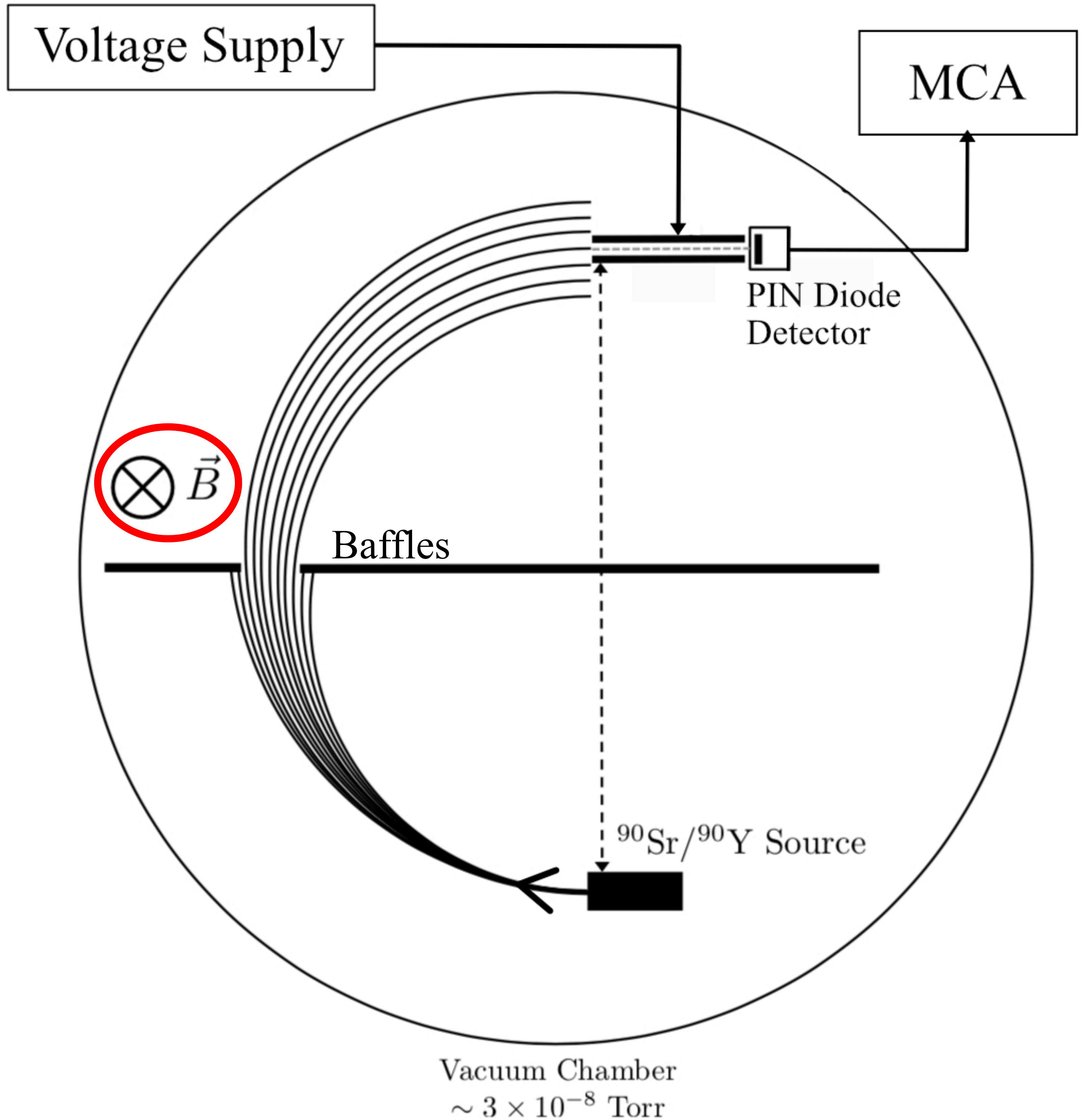
  1) Light enough to travel very fast if given high enough energy

  2) ^{90}Y emits electrons with enough energy to travel at $\approx 70\,\%$ the speed of light

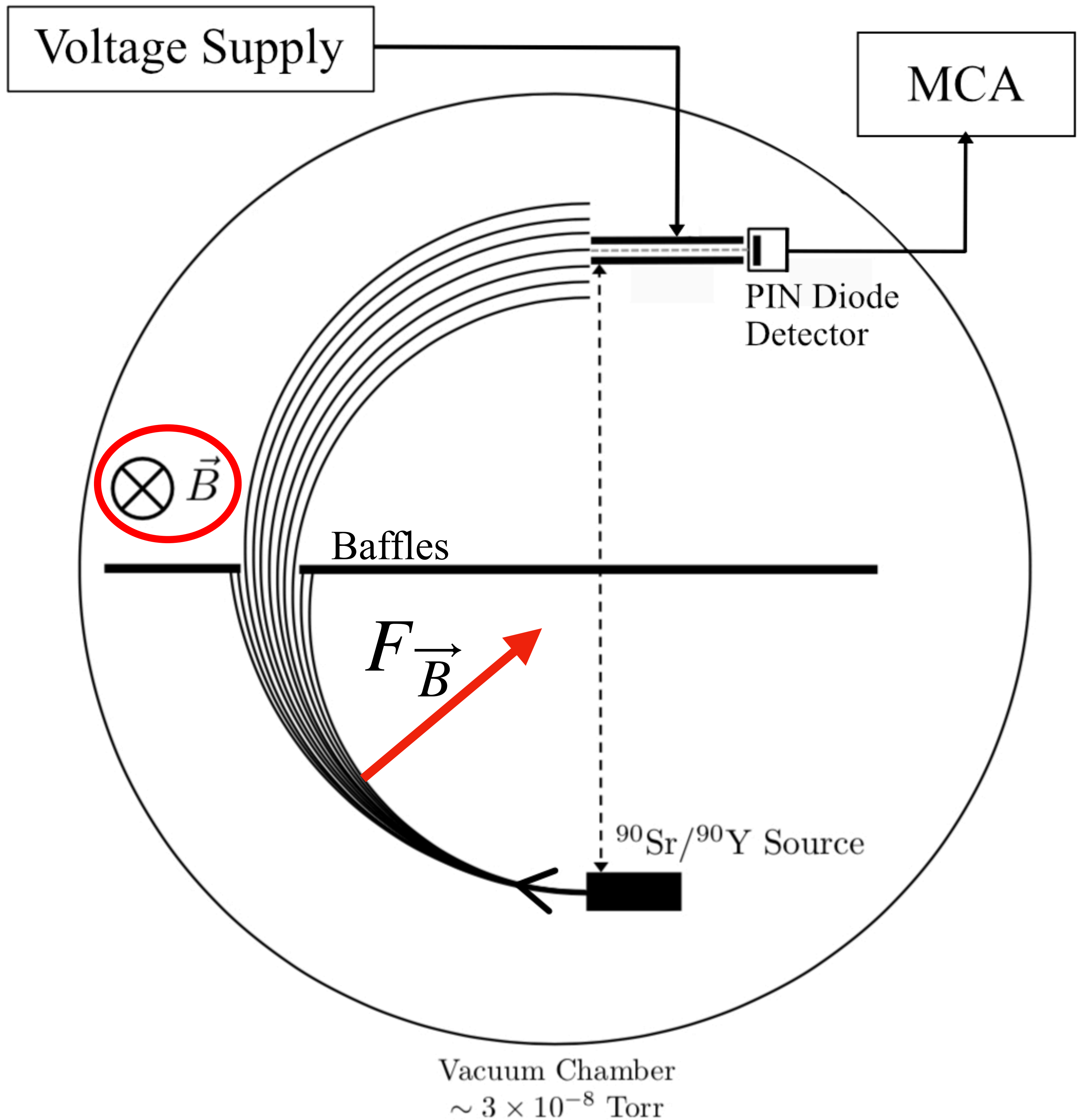
Electron Source



Electron Source

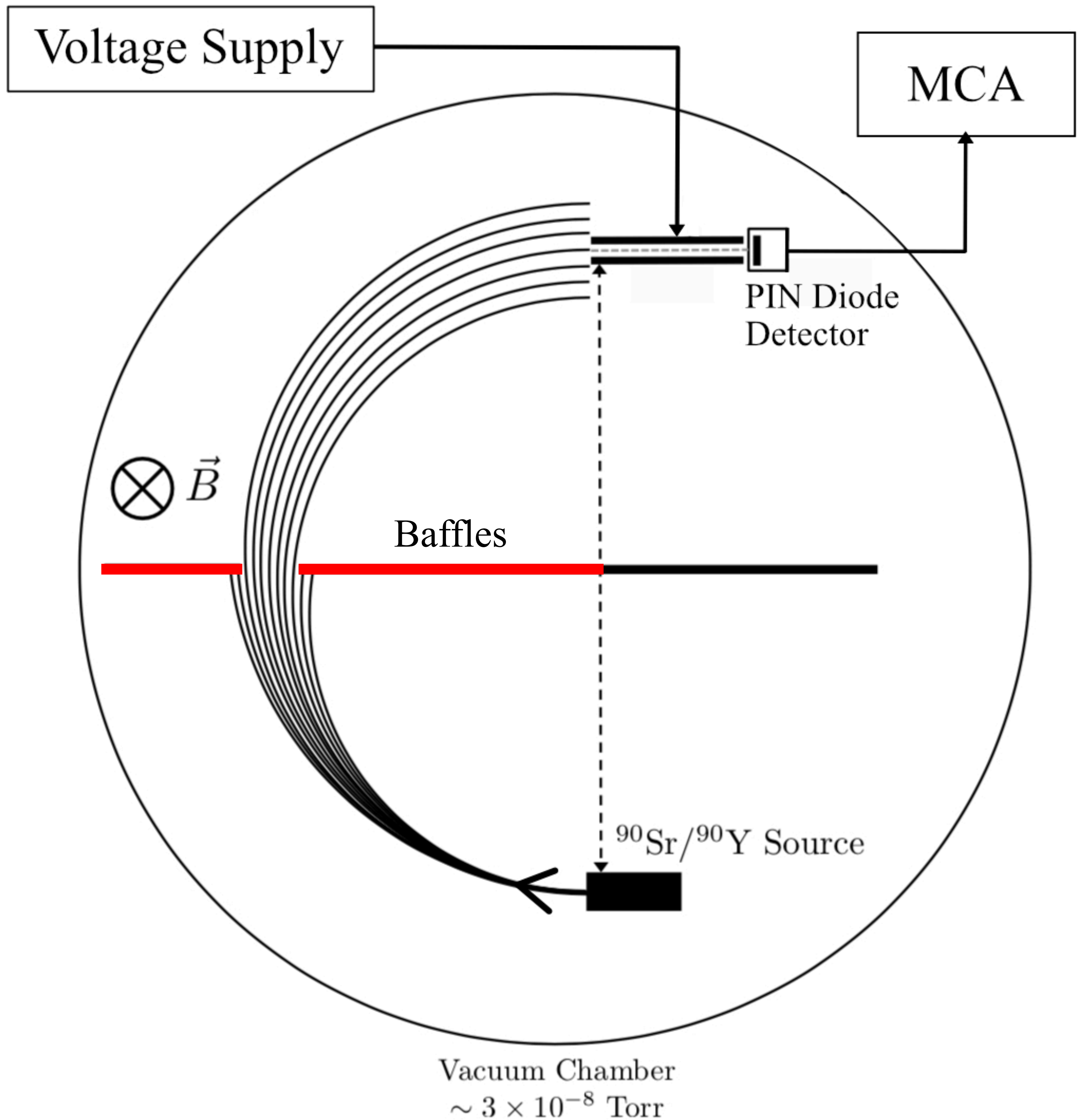


Electron Source



How are $\overrightarrow{v}$, $\overrightarrow{p}$, K found for detected electrons?

Baffles fix $\overrightarrow{p}$



Lorentz Force Law

$$\overrightarrow{F} = q(\overrightarrow{E} + (\frac{\overrightarrow{v}}{c}) \times \overrightarrow{B})$$

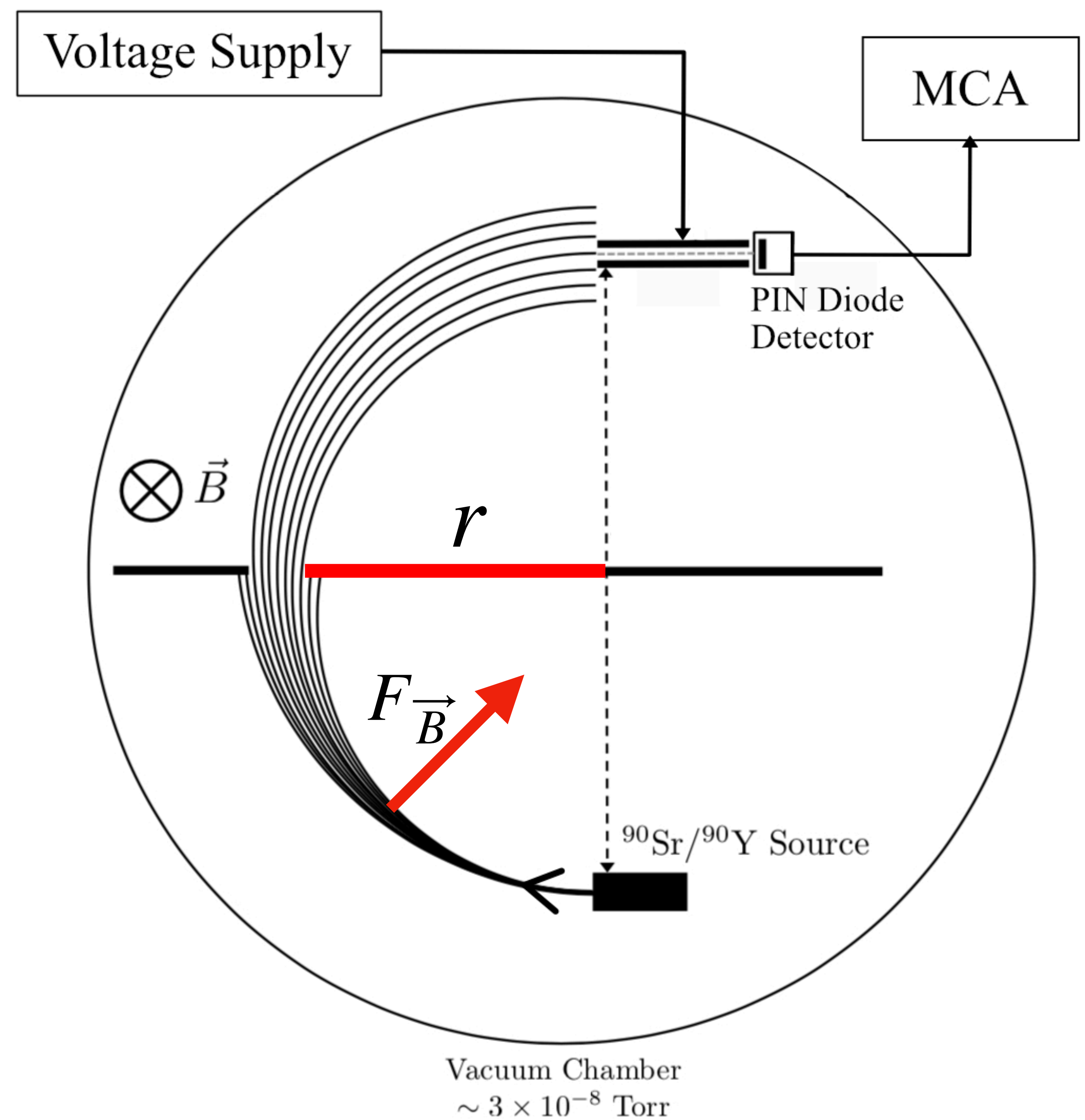
Electric Field $\overrightarrow{E}$

Magnetic Field $\overrightarrow{B}$

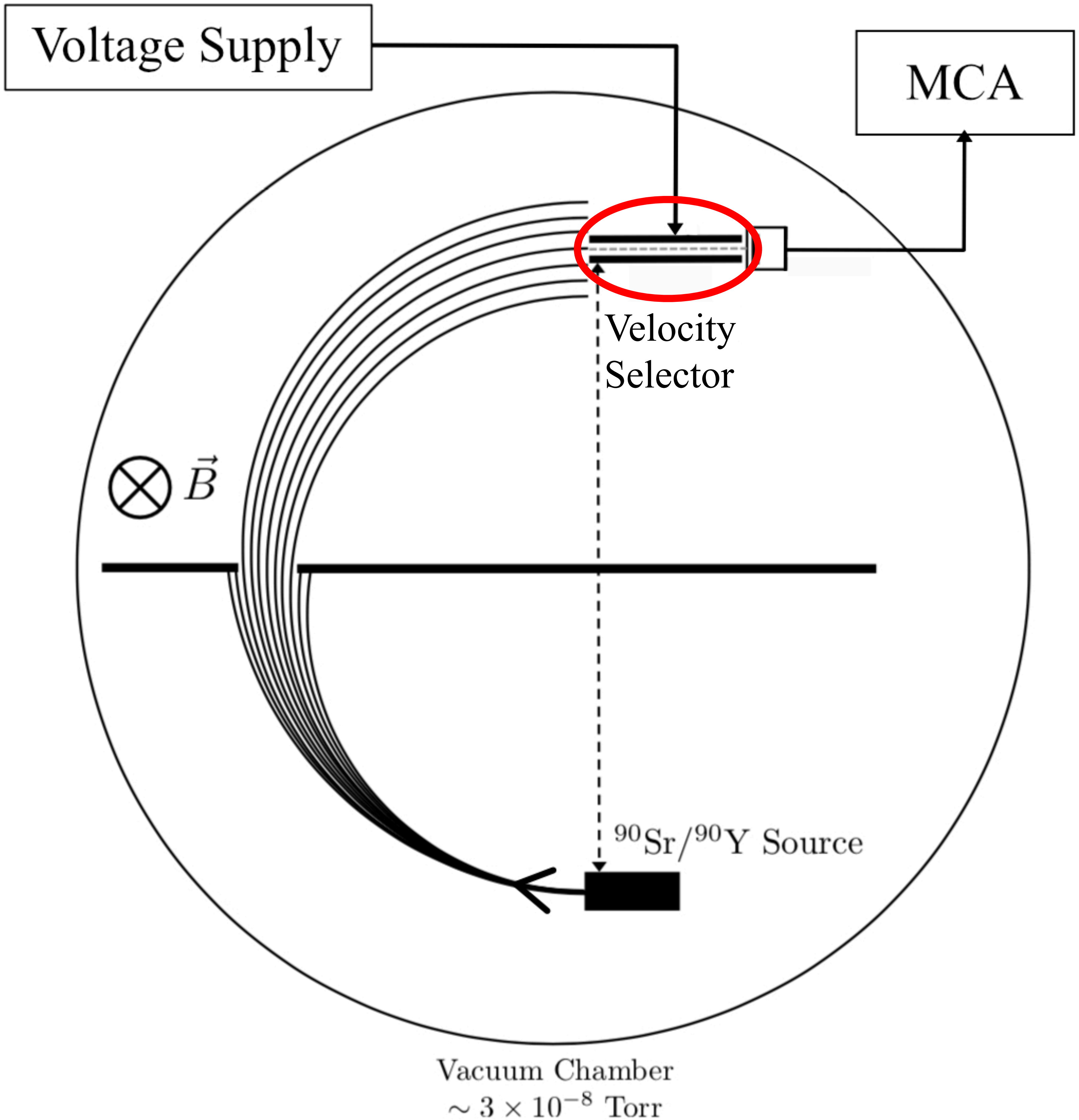
Baffles fix $\vec{p}$

So, momentum of detected particles is:

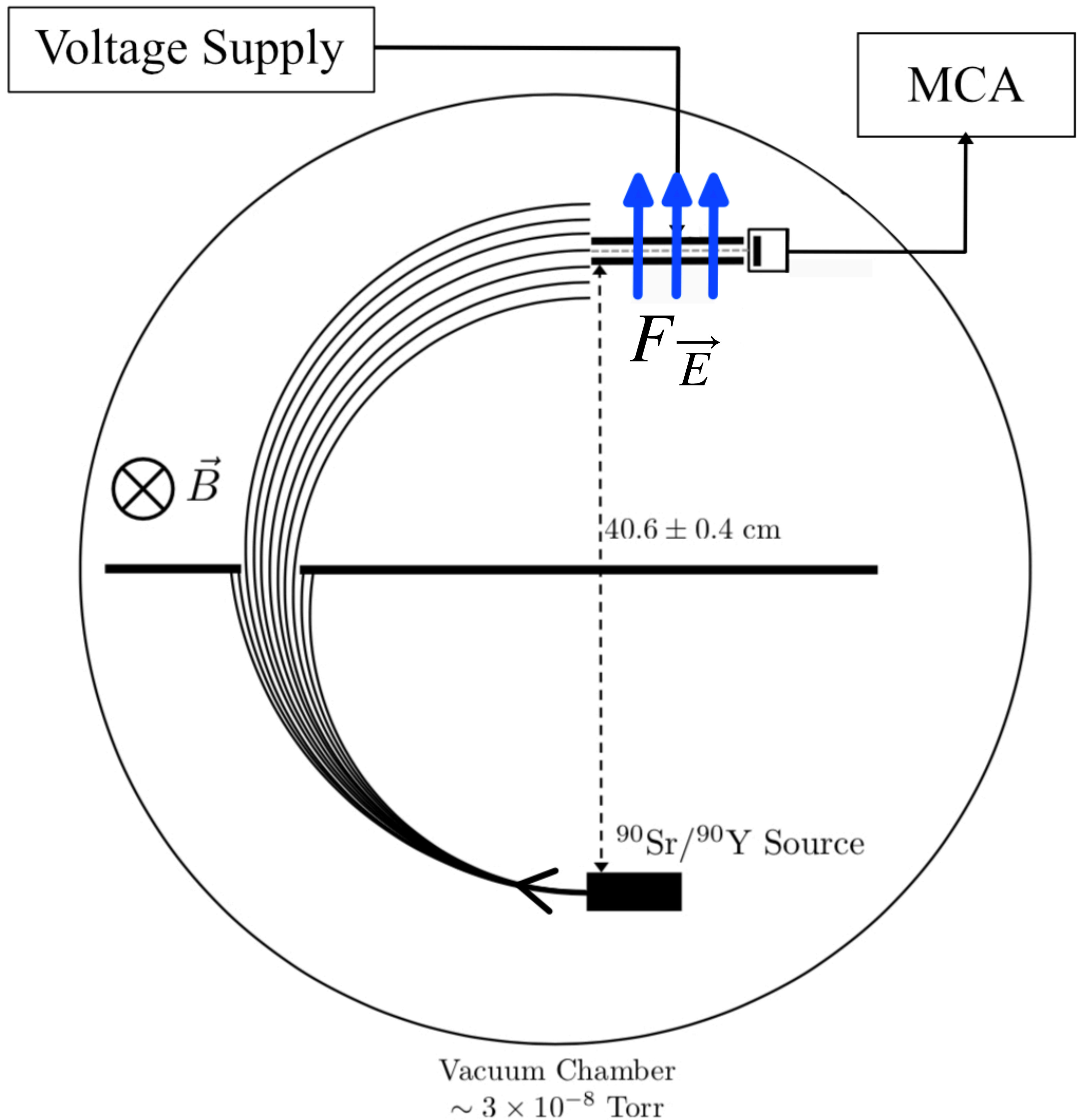
$$p = \frac{erB}{c}$$

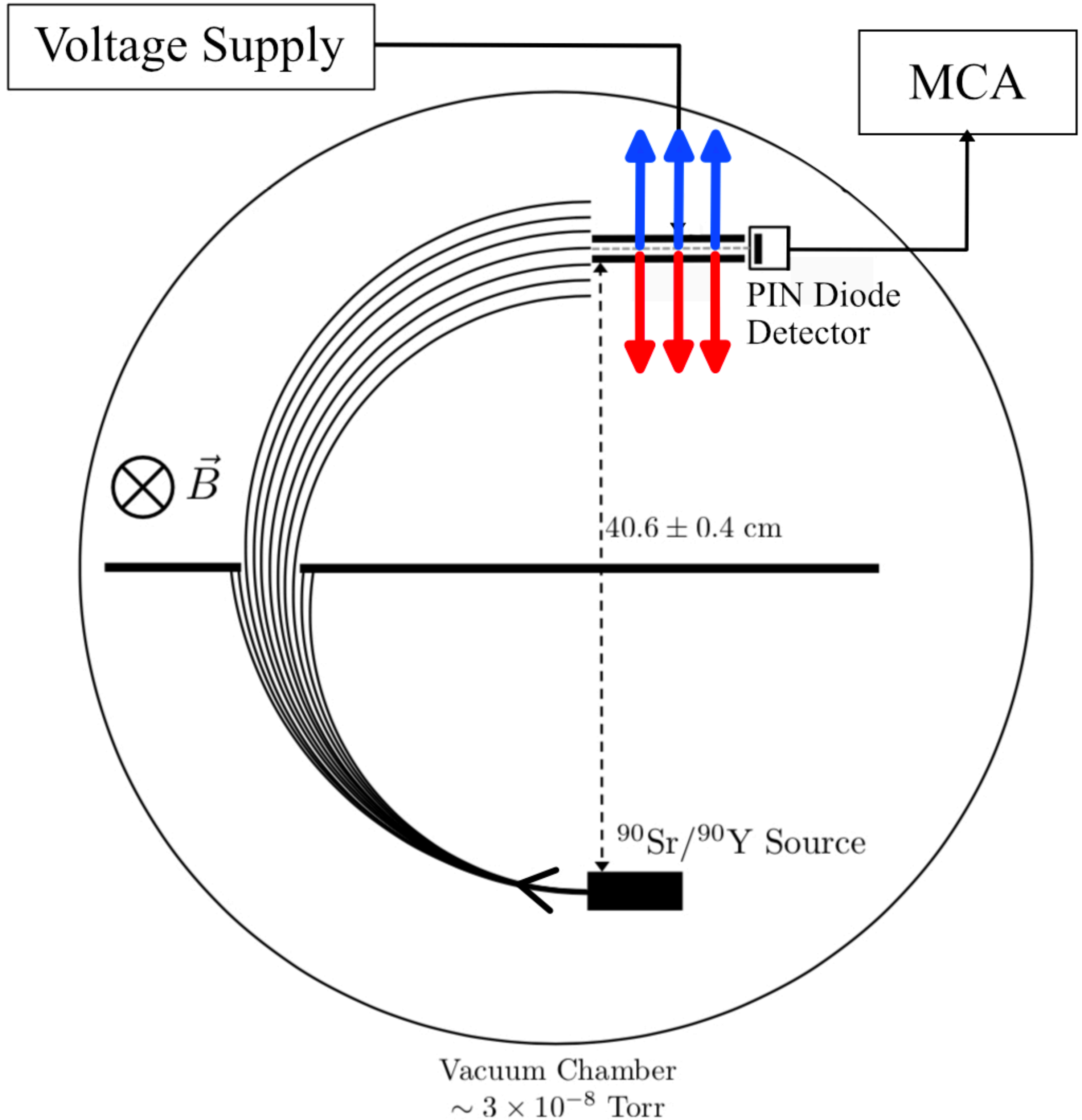


Velocity selector fixes $\overrightarrow{v}$



Velocity selector fixes $\overrightarrow{v}$





$F_{\vec{E}}$ **must cancel** $F_{\vec{B}}$

- For the electric field to cancel magnetic field:

$$e(E - \frac{vB}{c}) = 0$$

- So, only particles with this velocity can get to the detector:

$$v = \frac{cE}{B}$$

Voltage determination

- What voltage applied to velocity selector yields an $F_{\overrightarrow{E}}$ that best cancels $F_{\overrightarrow{B}}$?

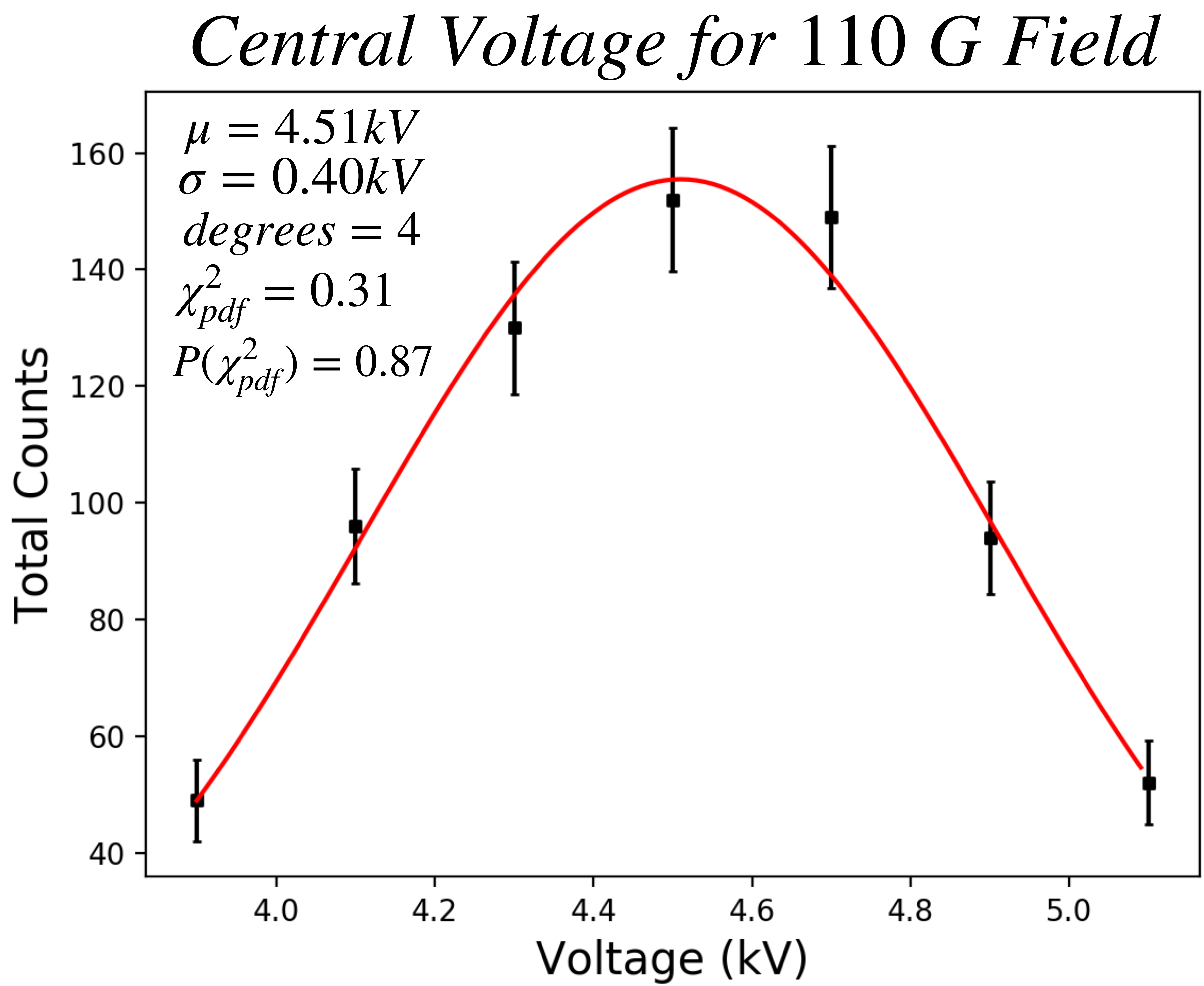
- We want to find this “central voltage” for different magnetic fields

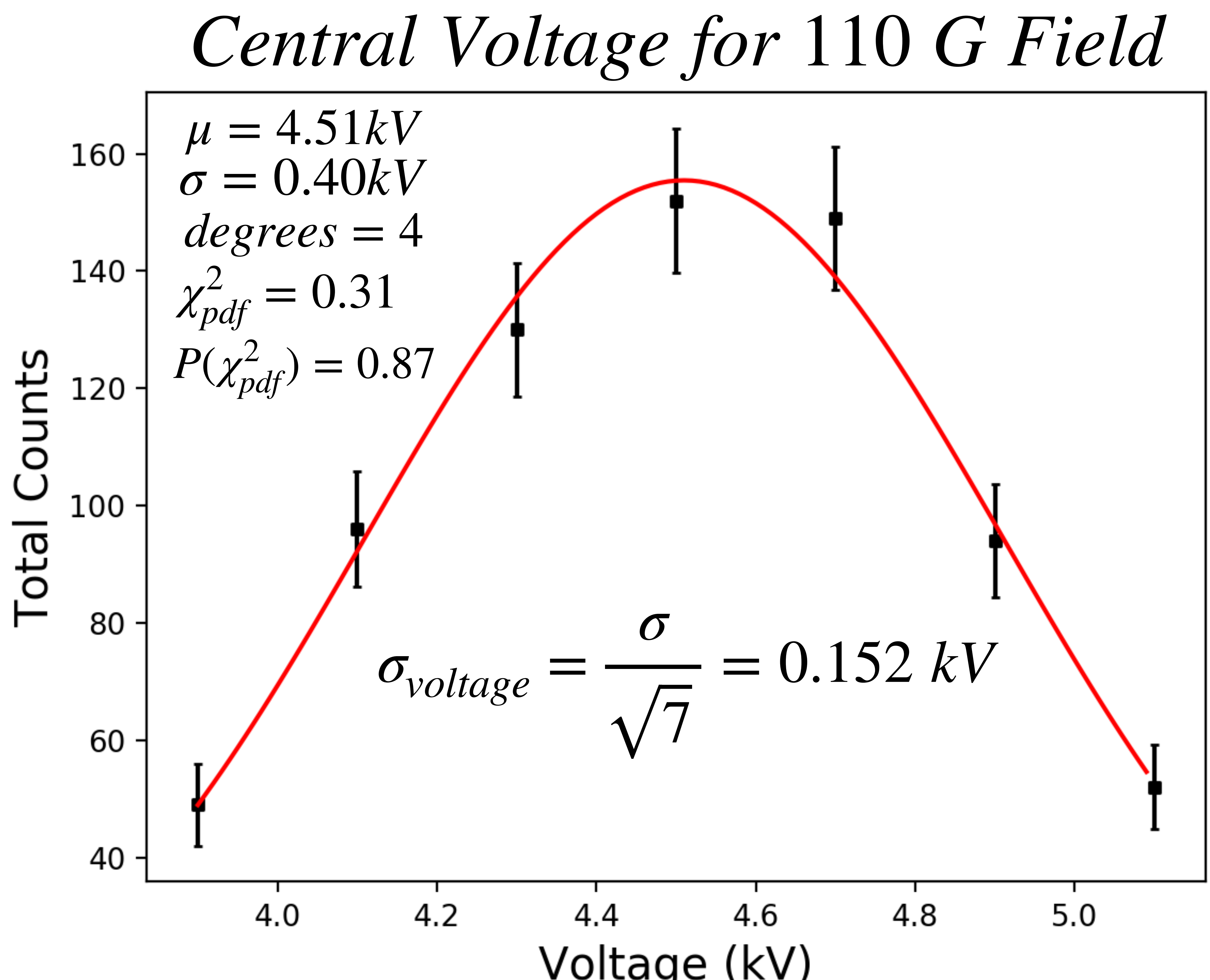
Voltage determination

1) Apply a magnetic field

2) Try applying a range of voltages to velocity selector plates

3) See which voltage lets the most electrons through


Central Voltage for 110 *G Field*
$\mu = 4.51kV$
$\sigma = 0.40kV$
$degrees = 4$
$\chi^2_{pdf} = 0.31$
$P(\chi^2_{pdf}) = 0.87$
Total Counts
Voltage (kV)
160
140
120
100
80
60
40
4.0
4.2
4.4
4.6
4.8
5.0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$\mu = 4.51kV$
$\sigma = 0.40kV$
$degrees = 4$
$\chi^2_{pdf} = 0.31$
$P(\chi^2_{pdf}) = 0.87$
$\sigma_{voltage} = \frac{\sigma}{\sqrt{7}} = 0.152\ kV$
Total Counts
Voltage (kV)
40
60
80
100
120
140
160
4.0
4.2
4.4
4.6
4.8
5.0

Detector measures K

- MCA will have peaks corresponding to detected particle energies:

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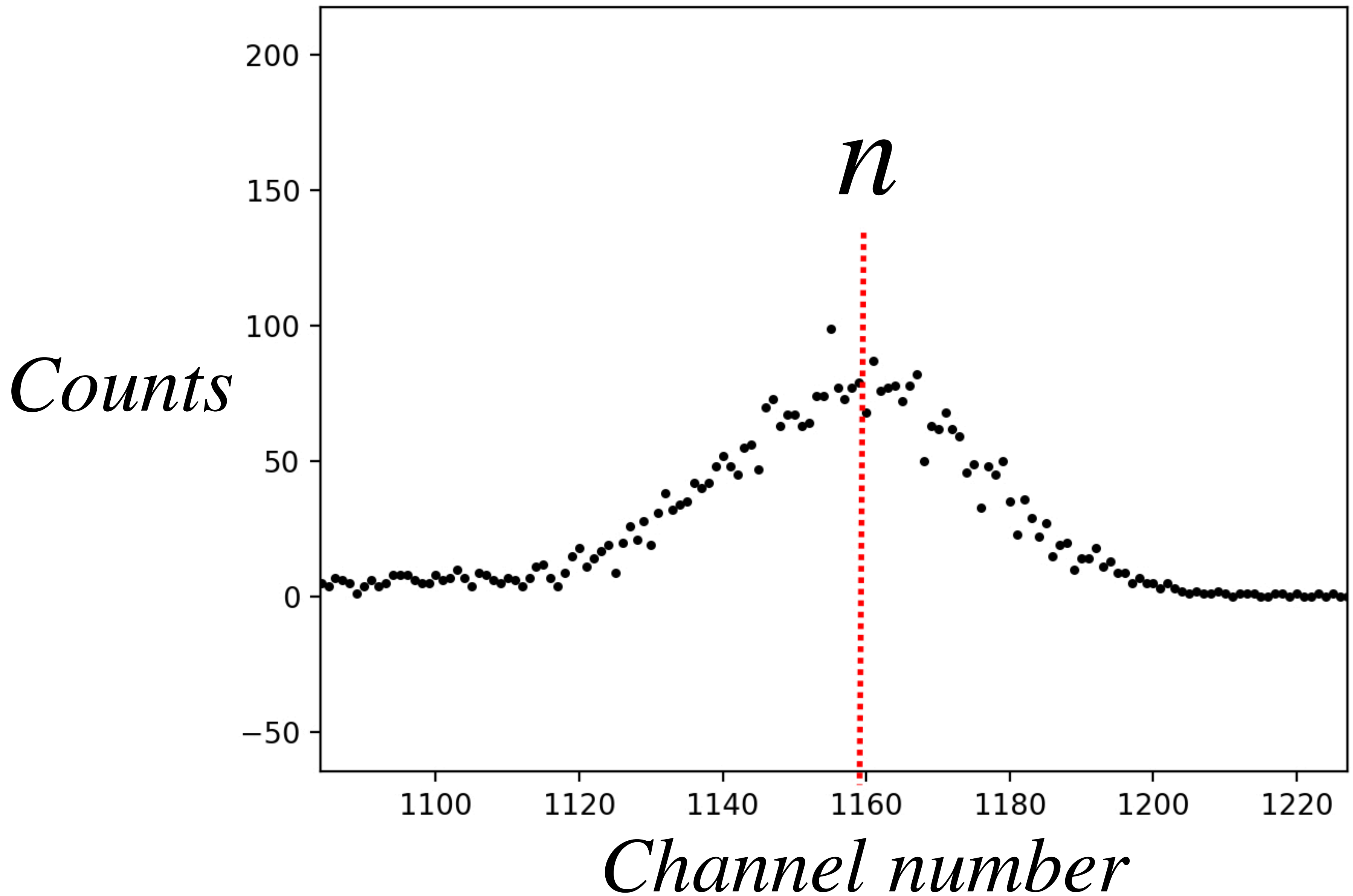
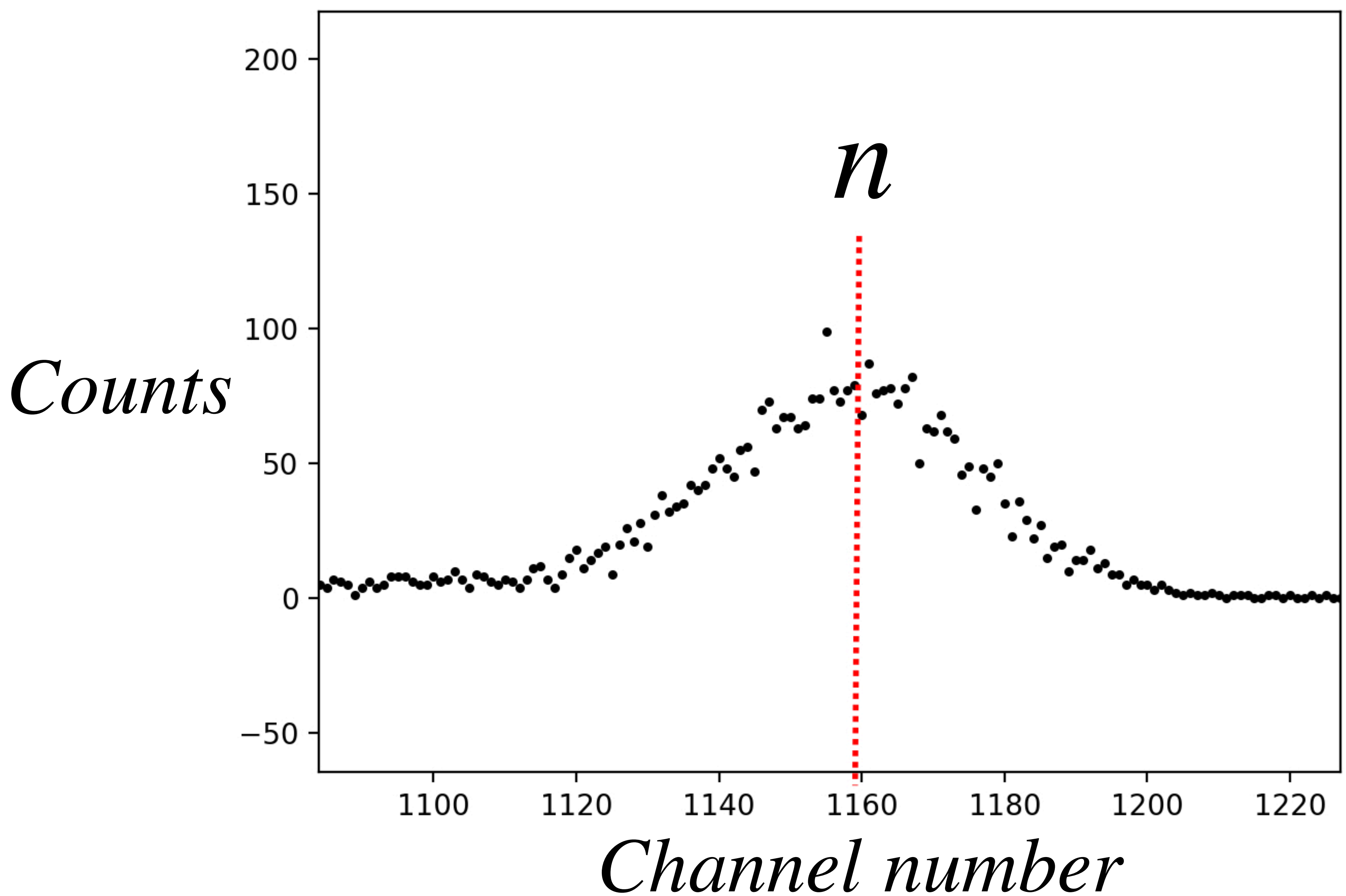


- How do we find energy that corresponds to a peak centered at channel number n?



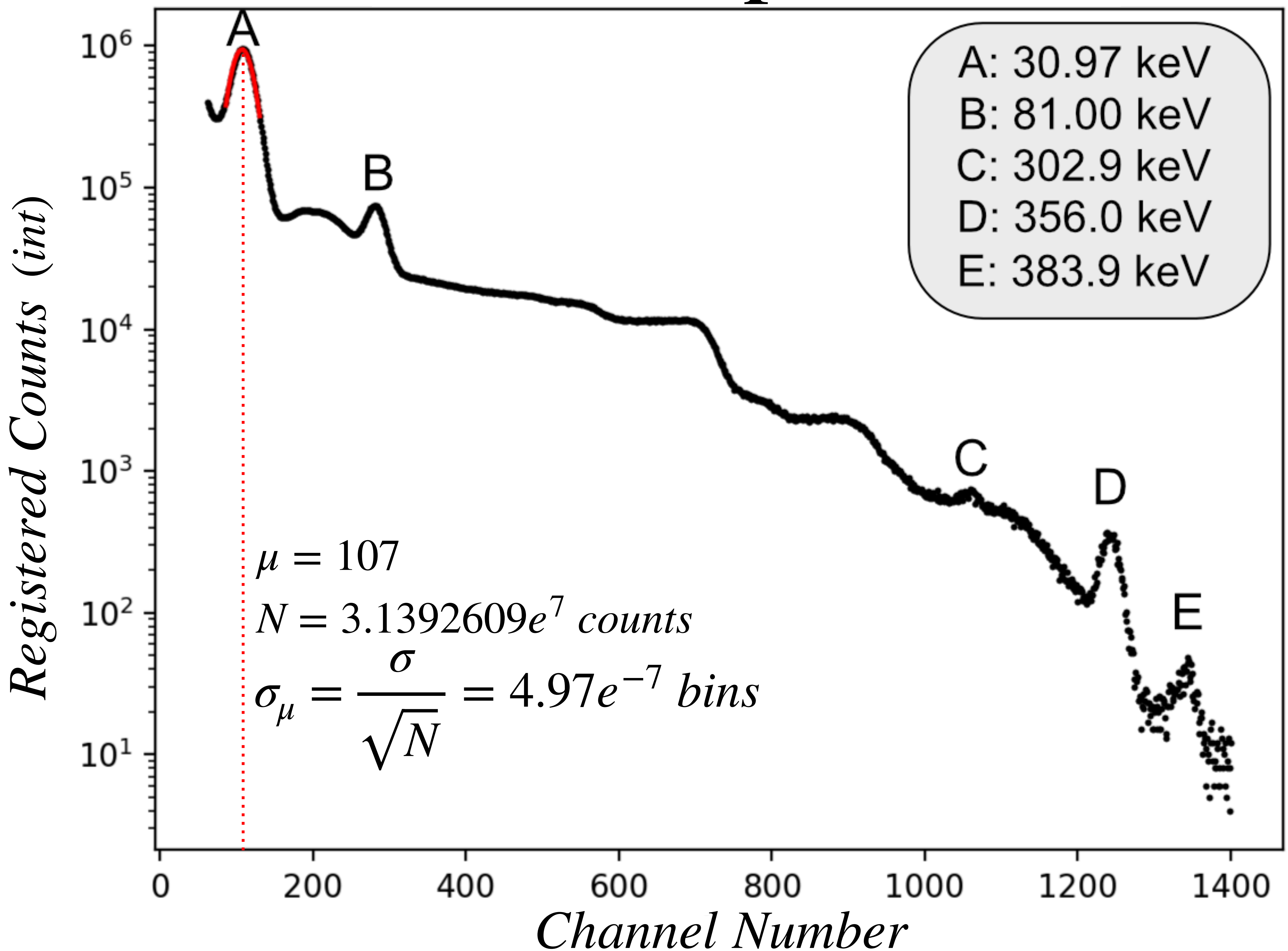
Barium Calibration

- ^{133}Ba has a well known energy spectrum

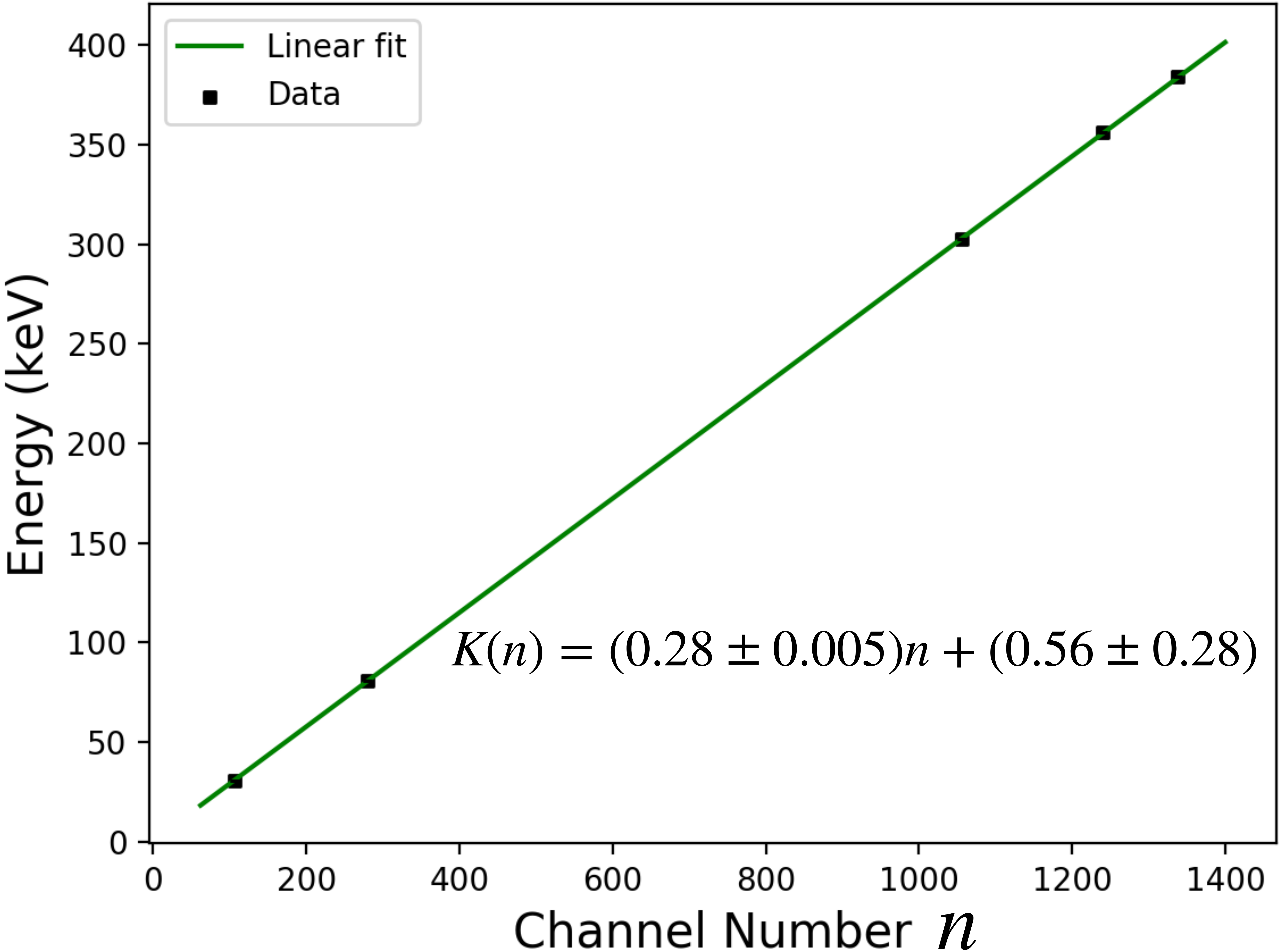
- By measuring barium energy spectrum, we can find:

$$K(n)$$

Barium Spectrum



Energy/Channel Relationship



How are $\overrightarrow{v}$, $\overrightarrow{p}$, K found for detected electrons?

- $\overrightarrow{p}$ is known by fixing the path radius using baffles

- $\overrightarrow{v}$ is known by fixing the electric field between velocity selector plates

- K is known by relating MCA channel to energy using the barium spectrum

Relativistic Beta

$$\beta = \frac{v}{c} = \frac{E}{B}$$

- Ratio of velocity to the speed of light
- Relates the magnetic field exerted on our electron to the electric field between velocity selector plates

For a magnetic field B and velocity selector voltage V_c

$$\beta_{newt} = \frac{erB}{mc^2}$$

$$\beta_{rel} = \frac{erB}{mc^2\sqrt{1 + (\frac{erB}{mc^2})^2}}$$

$$\beta_{data} = \frac{E}{B} = \frac{V_c}{Bd}$$

So which model makes the right prediction?

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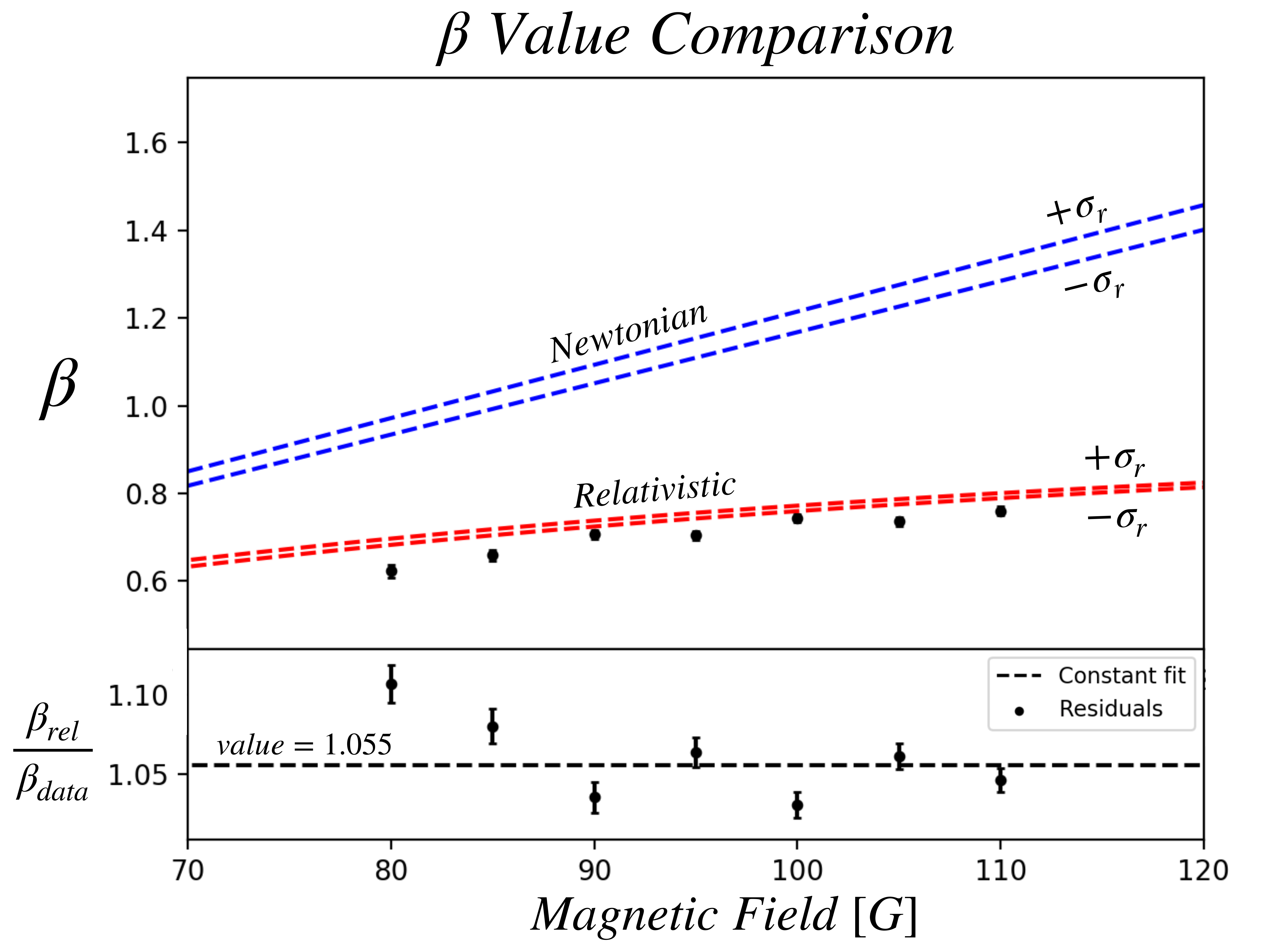
- Relate $\overrightarrow{p}, \ \overrightarrow{v}$ by plotting:

$$\beta = \frac{E}{B} \quad \text{vs.} \quad B$$


β Value Comparison
β
$+\sigma_r$
$-\sigma_r$
Newtonian
$+\sigma_r$
Relativistic
$-\sigma_r$
1.6
1.4
1.2
1.0
0.8
0.6
$\frac{\beta_{rel}}{\beta_{data}}$
1.10
1.05
value = 1.055
Constant fit
Residuals
70
80
90
100
110
120
Magnetic Field [G]

β Systematic Uncertainties:

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Velocity separator distance: $\sigma_d = \pm\, 0.003\ cm$

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Path radius: $\sigma_r = \pm 0.4\ cm$

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Path radius: $\sigma_r = \pm\, 0.4\ cm$

Central voltage: $\sigma_{voltage} = \pm \dfrac{\sigma}{\sqrt{N}}\ kV$

β Systematic Uncertainties:

Velocity separator distance: $\sigma_d = \pm\, 0.003\ cm$

Path radius: $\sigma_r = \pm\, 0.4\ cm$

Central voltage: $\sigma_{voltage} = \pm\, \frac{\sigma}{\sqrt{N}}\ kV$

Magnetic field: $\sigma_B = \pm\, 0.57\ G$

σ_d, σ_r : Correlated Systematics

$$\beta_{newt} = \frac{e\underline{r}B}{mc^2}$$

$$\beta_{rel} = \frac{e\underline{r}B}{mc^2\sqrt{1 + (\frac{e\underline{r}B}{mc^2})^2}}$$

$$\beta_{data} = \frac{V_c}{\underline{B}d}$$

σ_d : Correlated systematic

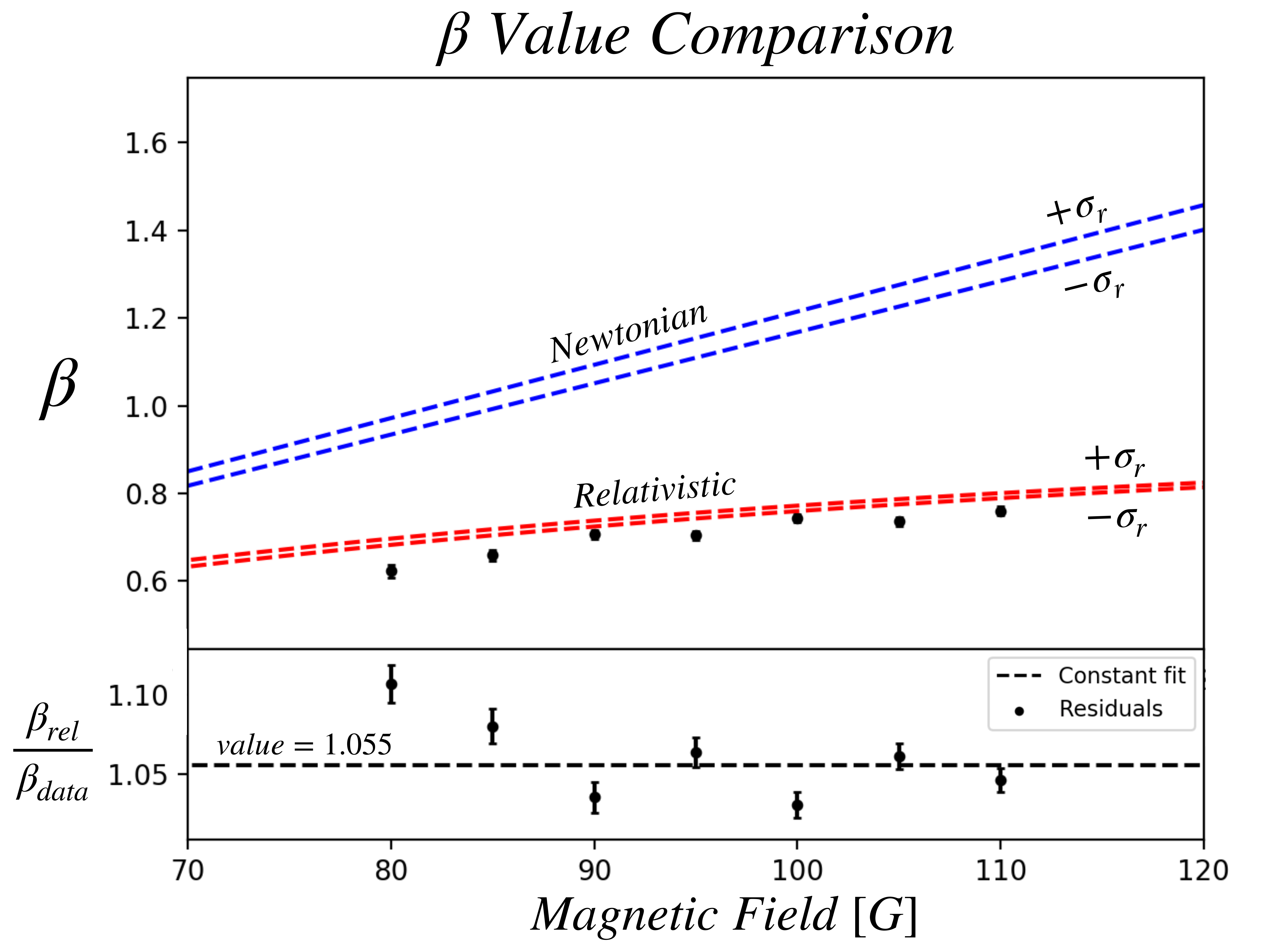
Each data point $\beta_{data} = \frac{V_c}{dB}$

moves up or down by $\pm 1.67\,\%$

σ_r : Correlated systematic

Each prediction curve $\beta_{relativistic}, \ \beta_{newtonian}$

moves up or down by $\pm 1.97\,\%$



β Value Comparison
β
Newtonian
+σr
−σr
Relativistic
+σr
−σr
1.6
1.4
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0.6
βrel / βdata
value = 1.055
Constant fit
Residuals
1.10
1.05
70
80
90
100
110
120
Magnetic Field [G]

$\sigma_{voltage}$: Point to Point Systematic

$$\beta_{data} = \frac{V_c}{dB}$$

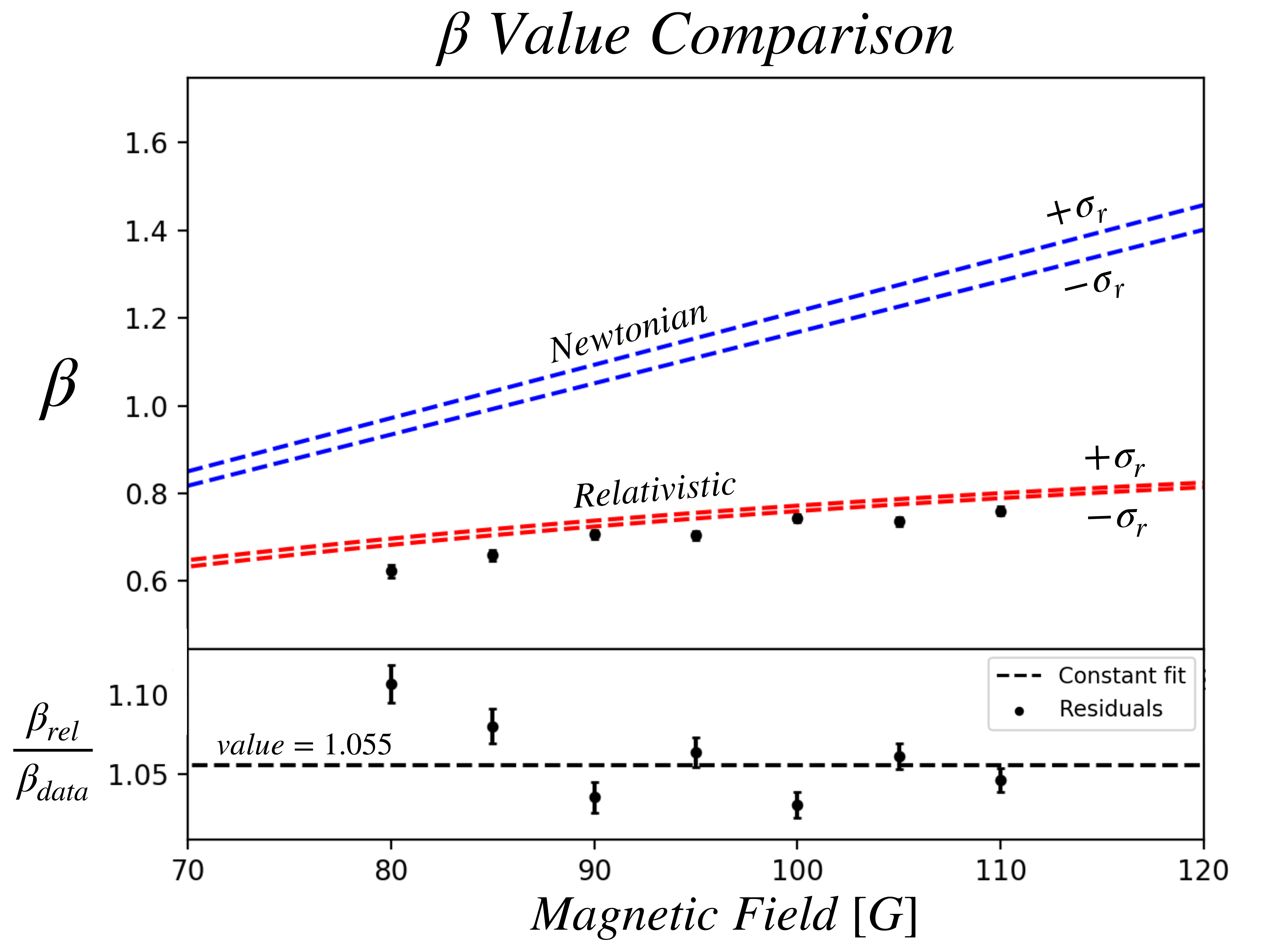
- Varies point to point, but ranges from $\approx \pm 1.0\,\%$ to $\approx \pm 1.50\,\%$

σ_B : A little bit trickier…

$$\beta_{newt} = \frac{erB}{mc^2}$$

$$\beta_{rel} = \frac{erB}{mc^2\sqrt{1 + (\frac{erB}{mc^2})^2}}$$

$$\beta_{data} = \frac{V_c}{Bd}$$



β Value Comparison
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β_rel / β_data
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Constant fit
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70
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Magnetic Field [G]

Normalization Uncertainty

$$f = \frac{\beta_{relativistic}}{\beta_{data}}$$

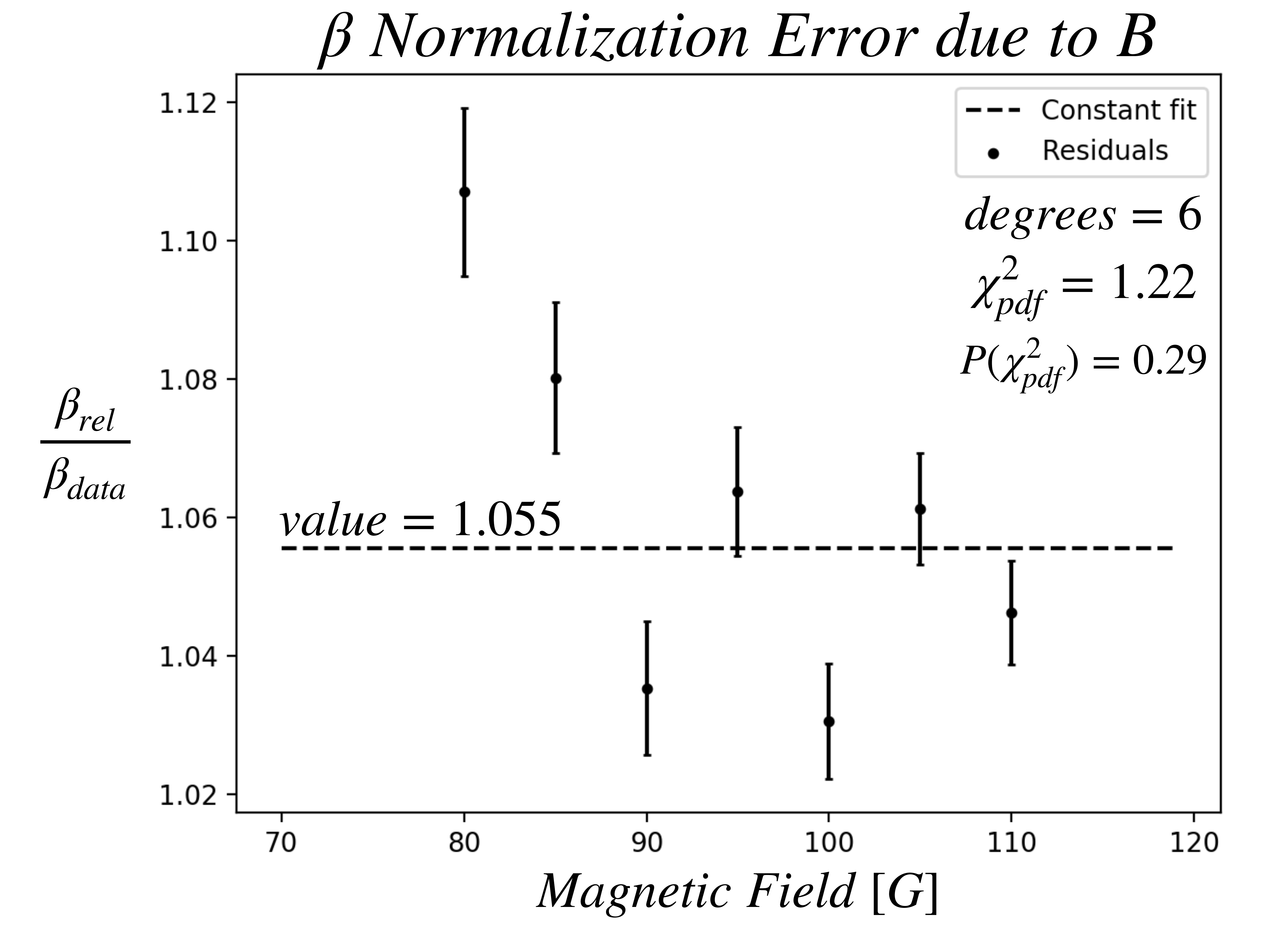
- Uncertainty in f due to B:

$$\sigma_{residual,\ B} = \frac{df}{dB}\sigma_B$$

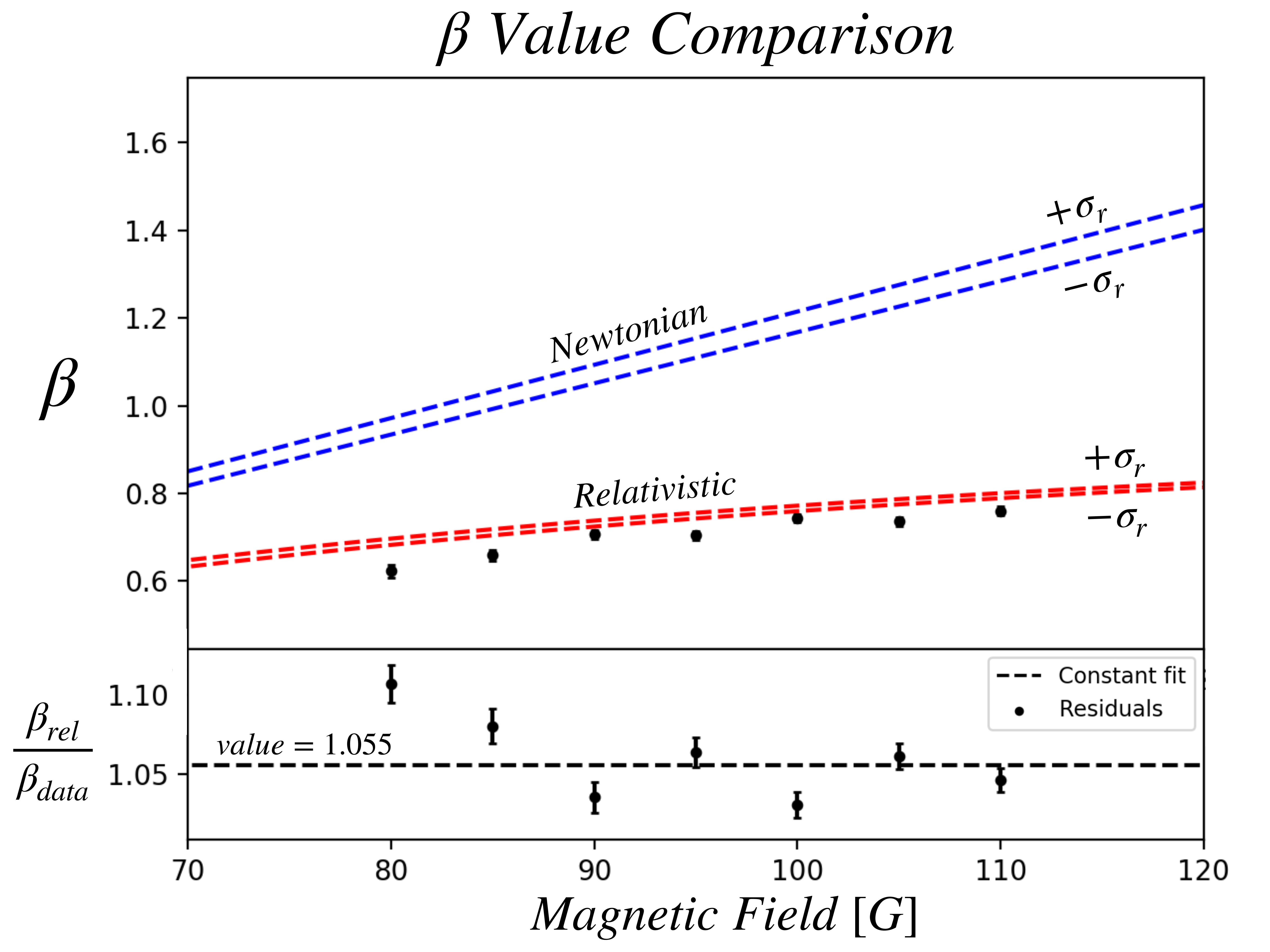
Normalization Uncertainty

$$f = \frac{\beta_{relativistic}}{\beta_{data}}$$

- Varies point to point, but ranges from $\approx \pm 0.80\,\%$ to $\approx \pm 1.50\,\%$



β Normalization Error due to B
Constant fit
Residuals
$degrees = 6$
$\chi^2_{pdf} = 1.22$
$P(\chi^2_{pdf}) = 0.29$
$value = 1.055$
$\frac{\beta_{rel}}{\beta_{data}}$
1.12
1.10
1.08
1.06
1.04
1.02
70
80
90
100
110
120
Magnetic Field [G]



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Magnetic Field [G]

Extracting $\frac{e}{m}$

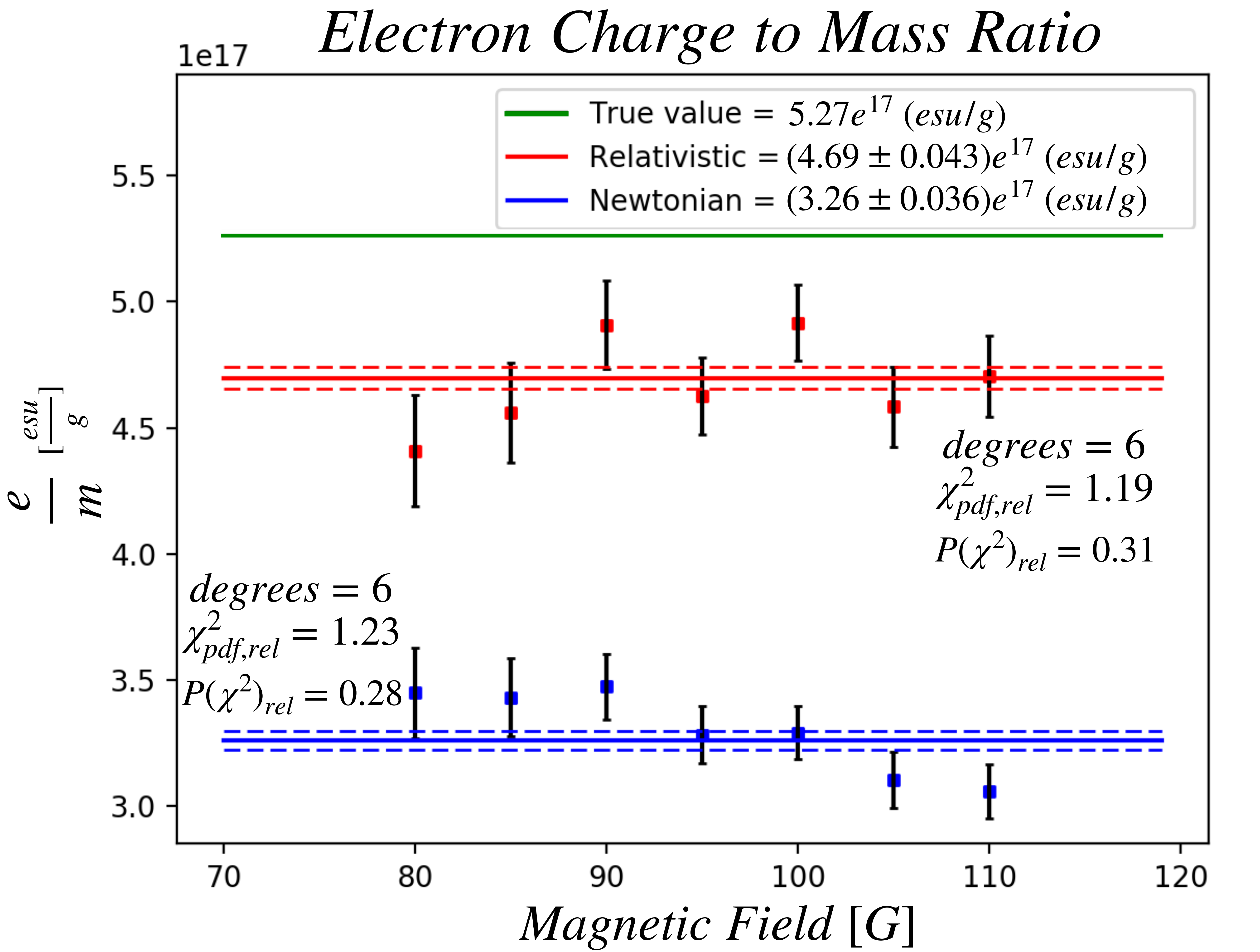
- We can use our calculated values of to find values for the electron charge to mass ratio

- Provides more verification of the relativistic models validity over the classical model for high speed electrons

Rearranging expressions for β :

$$\left(\frac{e}{m}\right)_{newtonian} = \frac{\beta_{data}c^2}{Br}$$

$$\left(\frac{e}{m}\right)_{relativistic} = \frac{\beta_{data}c^2}{Br\sqrt{1-\beta^2}}$$

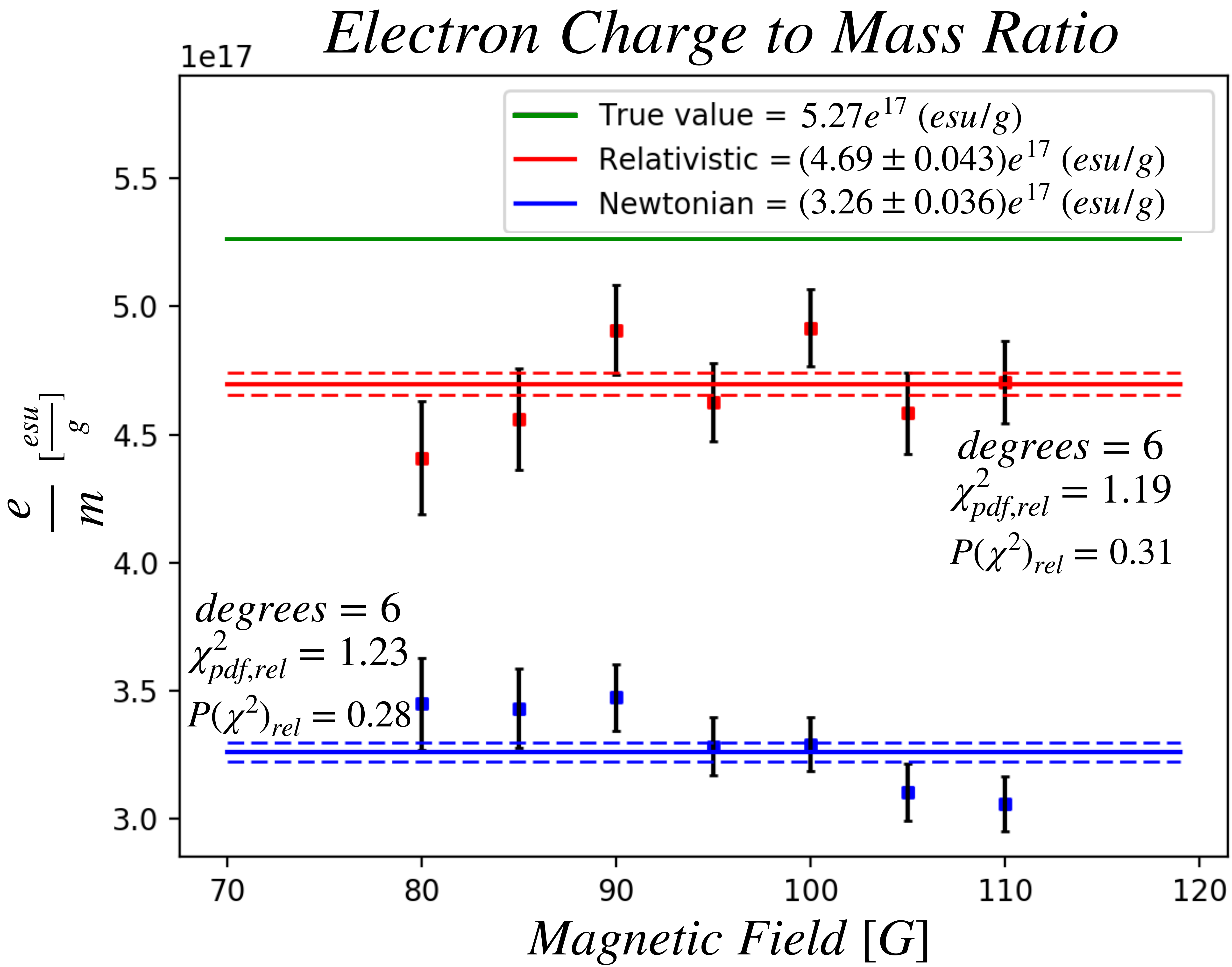


Electron Charge to Mass Ratio
1e17
True value = $5.27e^{17}$ (esu/g)
Relativistic = $(4.69 \pm 0.043)e^{17}$ (esu/g)
Newtonian = $(3.26 \pm 0.036)e^{17}$ (esu/g)
$degrees = 6$
$\chi^2_{pdf,rel} = 1.19$
$P(\chi^2)_{rel} = 0.31$
$degrees = 6$
$\chi^2_{pdf,rel} = 1.23$
$P(\chi^2)_{rel} = 0.28$
$\frac{e}{m}$ $[\frac{esu}{g}]$
Magnetic Field [G]
5.5
5.0
4.5
4.0
3.5
3.0
70
80
90
100
110
120

$\sigma_{\frac{e}{m}}$: Point to Point systematic

$$\sigma_{\frac{e}{m},\ newt} = \left(\sqrt{\sigma_E^2 + 4\sigma_B^2}\right)\left(\frac{e}{m}\right)_{newt}$$

$$\sigma_{\frac{e}{m},\ rel} = \left(\sqrt{\sigma_E^2 + \sigma_B^2}\right)\left(\frac{e}{m}\right)_{rel}$$



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4.0
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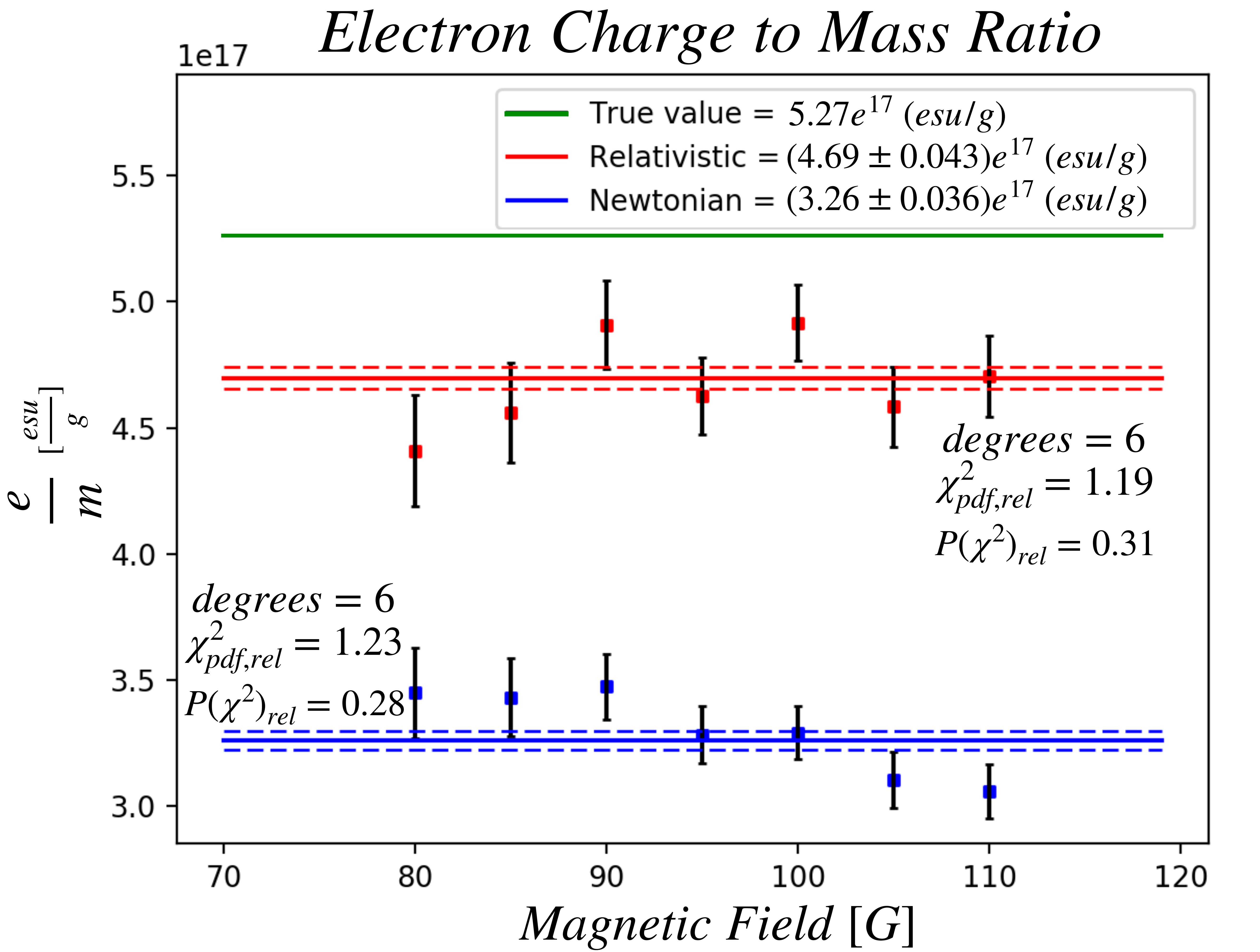
3.0
70
80
90
100
110
120
Magnetic Field [G]

Uncertainty in $\frac{e}{m}$ linear fits:

- We use a Monte Carlo simulation to raffle $\frac{e}{m}$ values within a distribution of width $2\sigma_{\frac{e}{m}}$

- We fit a constant to each set of raffled points

- The standard deviation of the differences between raffled point sets is the uncertainty on our fit



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$\frac{e}{m}\,[\frac{esu}{g}]$
5.5
5.0
4.5
4.0
3.5
3.0
70
80
90
100
110
120
Magnetic Field [G]

Summary

Extracted charge to mass ratio:

$$(\frac{e}{m})_{rel} = (4.69 \pm 0.043)e^{17}\ (esu/g)$$

Accepted charge to mass ratio:

$$\frac{e}{m} = 5.27e^{17}\ (esu/g)$$

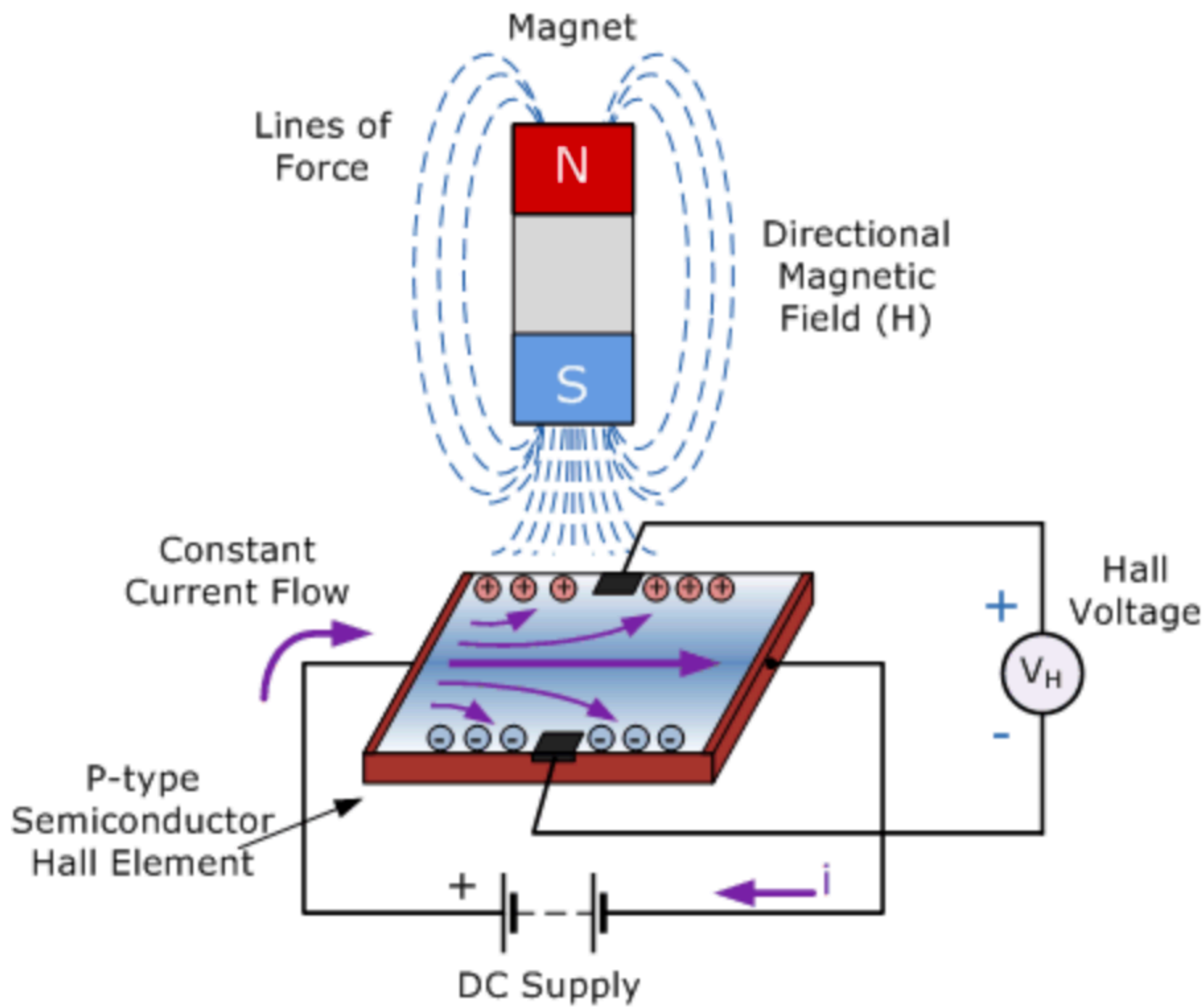
Summary

- The Relativistic model correctly predicts the dynamics for objects moving at high speeds

- Electron charge to mass further validates the correctness of the relativistic approach

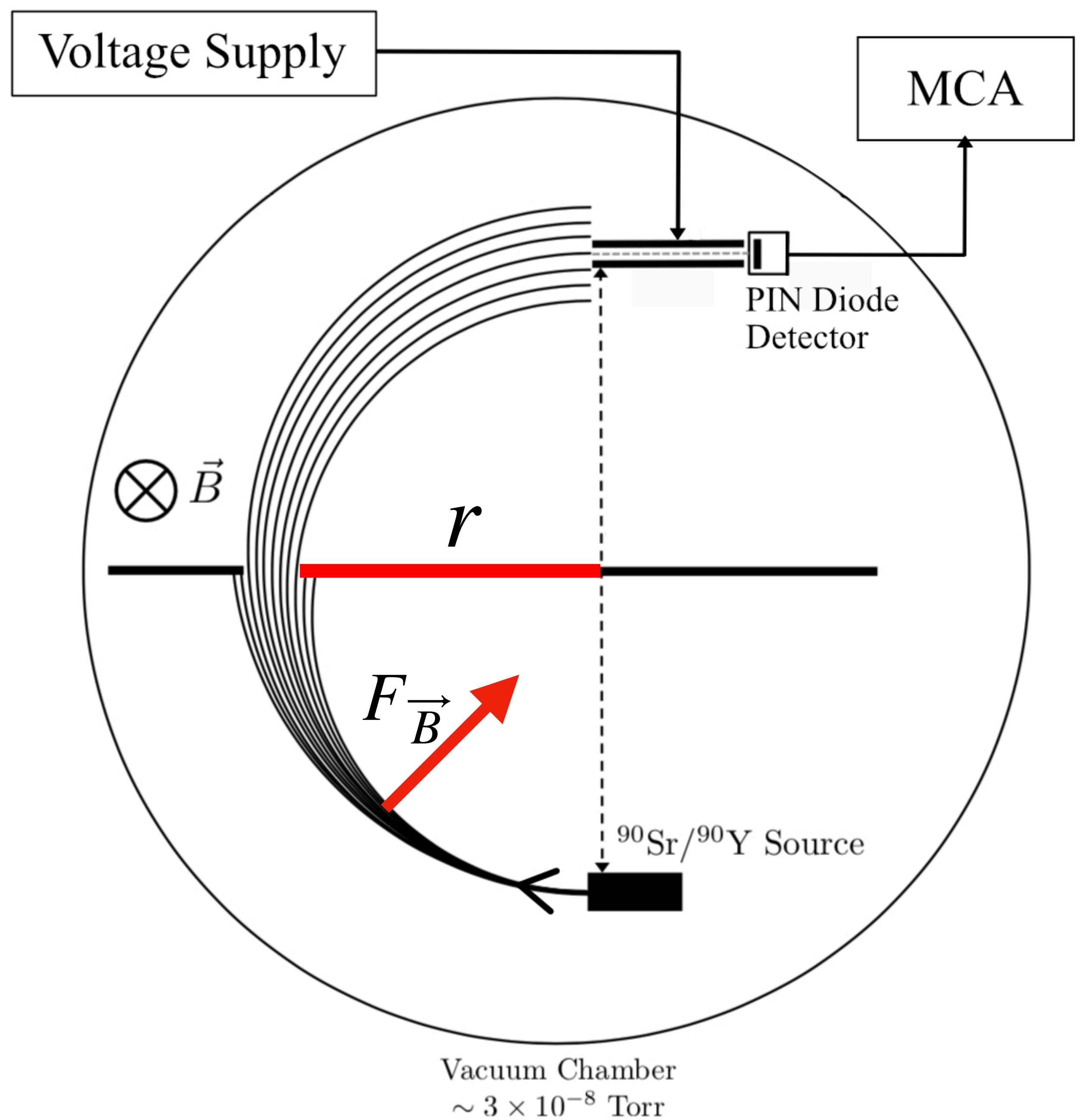
Thank you!

Hall Effect Magnetometer



Baffles fix $\overrightarrow{p}$

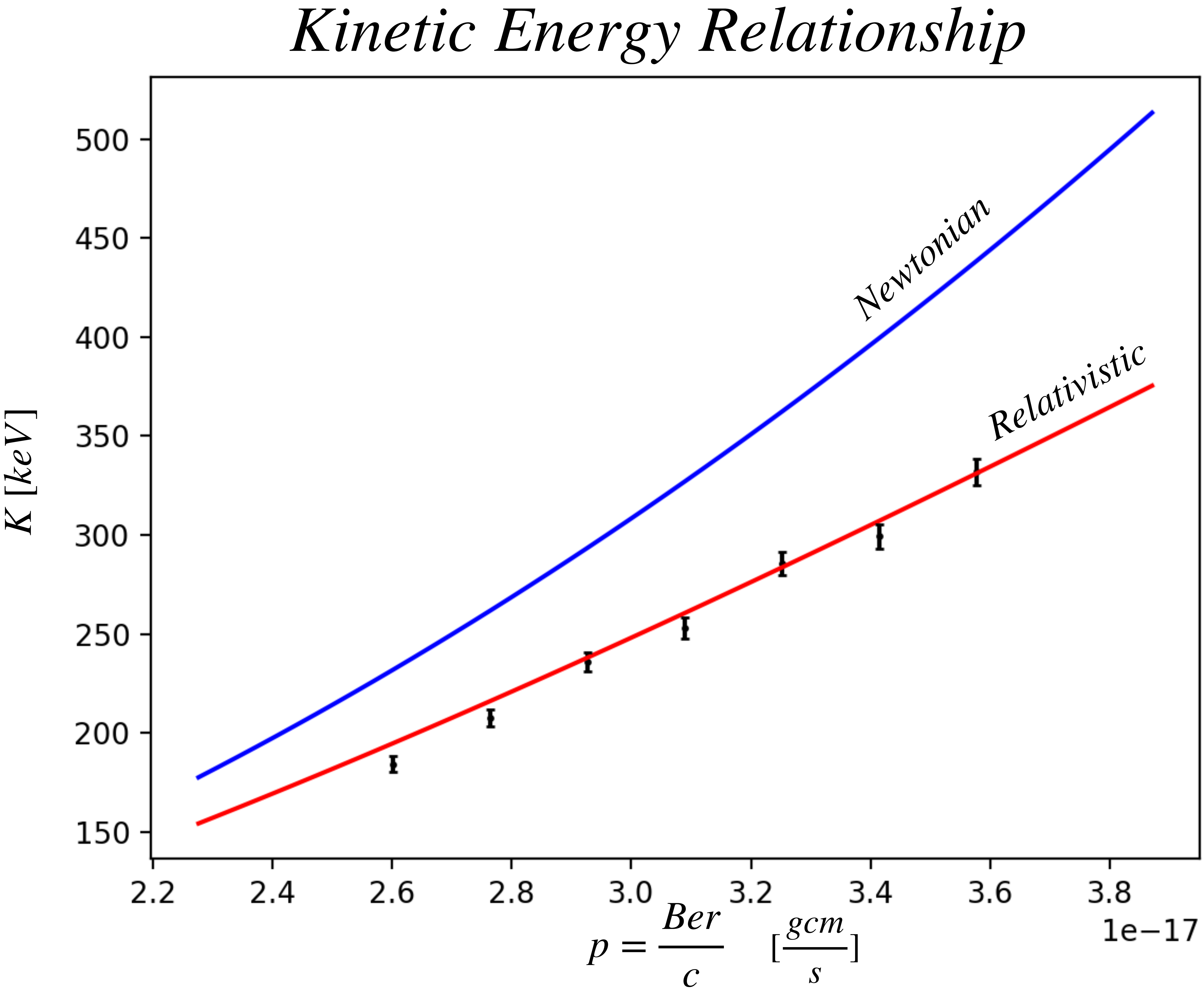
$$F_{\overrightarrow{B}} = \frac{evB}{c} = \frac{pv}{r}$$



Energy Predictions

$$K_{newt} = \frac{p^2}{2m}$$

$$K_{rel} = \sqrt{m^2c^4 + c^2p^2} - mc^2$$



Kinetic Energy Relationship
Newtonian
Relativistic
$K\ [keV]$
500
450
400
350
300
250
200
150
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
1e−17
$p = \frac{Ber}{c} \quad [\frac{gcm}{s}]$