Rutherford Scattering

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At the start of the 20th century, little was known about the structure of the atom. In order to probe atomic structure, Ernest Rutherford pioneered the experimental technique of α particle scattering and generated evidence supporting the nuclear atomic model [1]. In this experiment, α particles are scattered off of gold nuclei in order to confirm the validity of Rutherford's nuclear atomic model. The model is found to provide an excellent fit to observed measurements with $\chi^2_{pdf} = 0.81$ for 5 degrees of freedom. From these measurements, the differential scattering cross section for $50°$ scattering is extracted to be $(3.89 \pm 0.603)e^{-22}\ cm^2$, and the thicknesses of various gold foils are extracted to within one standard deviation of their true values.

I. INTRODUCTION

In the early 1900's, discovery of the negatively charged electron and its properties launched the challenge of further understanding the neutrally charged atomic structure surrounding it. Although the electron's mass and charge were known with considerable accuracy, the location of the surrounding positive charge required to produce the expected neutral atom was unknown. The most prominent theory on the matter was proposed by J.J. Thomson in 1904. His famous "plum pudding" model held that the majority of atomic volume was filled with a sea of free floating positive charge, with negatively charged electrons occupying distinct equilibrium positions within this same volume.

the Thomson atomic theory predicts little to no deflection of α particles as they travel through the atom at high speeds due to the weak electric fields produced by negative charges interacting with a large volume of spread out positive charge. In 1909 however, experiments scattering α particles off of gold atoms performed by Gieger and Marsden under the supervision of Rutherford [1] demonstrated the possibility of large scattering angles inconsistent with the plum pudding model. This evidence led Rutherford to postulate a nuclear atomic model in which a centrally localized concentration of positive charge known as a nucleus is surrounded by negatively charged electrons in orbit.

II. RUTHERFORD SCATTERING THEORY

Mathematically, the scattering process of an α particle off of an atomic nucleus can be described in terms of the scattering cross section σ. This cross section describes the two dimensional area perpendicular to the particles path through which traversal will yield an angular deflection greater than or equal to a set angle θ. Thus, the differential scattering cross section $\frac{d\sigma}{d\Omega}$ can be thought of as the infinitely small change in cross section $d\sigma$ required to produce an infinitely small change in scattering angle $d\Omega$. If the α particle and the electrons surrounding the nucleus are correctly assumed to have negligible mass in comparison to the nucleus, then the differential scattering cross section under the Rutherford model is as follows:

$$\frac{d\sigma}{d\Omega} = (\frac{ZZ'e^2}{4E})^2 \sin^{-4}(\frac{\theta}{2}) \tag{1}$$

where Z represents the atomic number of the target atom, e is the charge of an electron, and E and Z' represent the kinetic energy and atomic number of the incident scattering particle. This differential scattering cross section can also be shown to be directly proportional to the expected rate of α particles scattered at an angle θ:

$$\frac{d\sigma}{d\Omega} = \frac{I_\theta A}{I_0 L \rho N_A d\Omega} \tag{2}$$

where I_θ and I_0 are the scattered and incident particle rates, respectively. L and ρ represent the thickness and density of the foil scattering target, and N_A and A represent Avogadro's number and the atomic mass number of the target atom. The Rutherford model predicts a gradual drop in scattering rate as scattering angle increases. Under the Thomson model, the differential scattering cross section is modeled by a steep Gaussian peak that predicts a sharp drop in scattering rate at any angle outside the close vicinity of $0°$. By observing α particle scattering rate as a function of angle, the validity of the Rutherford nuclear model can be tested directly.

III. EXPERIMENTAL SETUP

The experimental apparatus consists of a ^{241}Am radioactive source, a gold foil target, and a solid state detector all contained within a cylindrical vacuum chamber as shown in Fig. 1.

III.1. Alpha Particle Source

The cylindrical device containing the α particle source is called the "howitzer". It can be pivoted about varying

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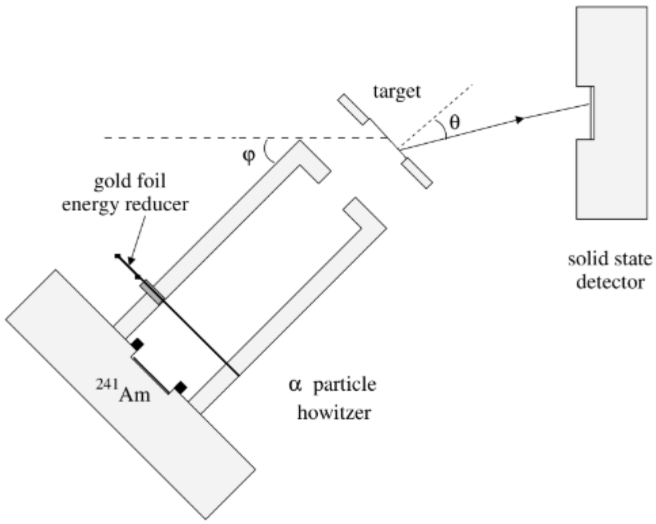

FIG. 1. The experimental setup used to scatter alpha particles off of gold nuclei. Note that the wide angular acceptance of the detector allows detected particle scattering angles θ that are slightly different than the howitzer position ϕ. Also note the gold foil energy reducer placed in front of the ^{241}Am source. Adapted from [2].

angles in relation to the fixed orientation detector. At the 0° marker, the howitzer is pointing in a straight line towards the solid state detector. The radioactive ^{241}Am source contained within the metal howitzer produces α particles with an energy of about 5.486 MeV. These alpha particles travel through the cavity and exit the howitzer snout in a highly collimated beam.

III.2. Target Foil

The apparatus chamber is subject to a vacuum strength of ≈ 100 *microns* and the α particle beam can be assumed to travel in a straight line at a stable intensity until it hits a target fixed by a metal target holder. The target holder within the apparatus supports a wide range of different targets including an empty target option. Over the course of this experiment, gold foil targets with thicknesses varying from 1 to 3 layers of foil are used. When the α particle beam hits the target, gold atoms within the target foil scatter each particle and they exit the foil traveling in various directions.

III.3. Solid State Detector

A silicon barrier solid state detector is positioned 14 cm opposite the howitzer inside the vacuum chamber. When an α particle is detected, a count is registered in the appropriate channel on a multi-channel analyzer (MCA) spectrum. An important feature of the specific detector used is that it has a detection surface of 1.61 cm^2, and has a broad angular acceptance in relation to the howitzer angle ϕ.

IV. PROCEDURE

In order to determine the validity of the Rutherford model, α particles are scattered off of gold atom target foils and the scattering rate is measured as a function of howitzer angle. These measurements yield a relationship that can be used to examine how well the Rutherford cross section 4 fits.

The angular dependence of scattering rates is measured by positioning the howitzer at different angles in a range of 0° to 60°. For each measurement, the 2 layer gold foil target is used as opposed to the 1 or 3 layer targets in order to achieve a compromise between maximizing count rate and detected particle energy. Once set, the apparatus is left running for a variety of durations in order to achieve acceptable count numbers. These durations fluctuate based on angle, with 600 *secs* for smaller angles, and up to 60,000 *secs* for the larger angle measurements. Because this experiment was conducted over the course of 3 weeks and count rates fluctuated slightly day to day, a normalization run was conducted at the start of each laboratory session. This was done by positioning the howitzer at 0° and measuring counts with no target foil. Count rates were obtained by dividing the total number of counts obtained by the duration of the measurement. Normalized count rates were then obtained by dividing the scattering count rates with the corresponding normalization count rate - $\frac{I_\theta}{I_0}$.

V. RESULTS AND ANALYSIS

V.1. Beam Profile

As mentioned earlier, the solid state detector has a wide angular acceptance. For a single howitzer angular position ϕ, α particles detected by the silicon barrier detector could have been scattered at a range of angles around ϕ. In order to characterize the nature of this angular acceptance, an angular response function $g(\phi, \theta)$ is needed. This function quantifies the probability that a particle will be scattered at an angle θ when the howitzer is positioned at angle ϕ. In order to determine this function experimentally, α particles are projected through the open target aperture containing no foil with the howitzer positioned at various angles ranging from $-6°$ to $10°$ in $2°$ increments. Because the α particle beam is highly collimated and the detector is highly sensitive, the result of passing the beam over the surface of the detector can be approximated by the convolution of two square functions. This leads to the belief that the angular response function of the apparatus might best be characterized by a triangular function $g(\phi-\theta;\theta_0)$, where θ_0 represents the base half width of the triangle. In order to determine a triangular fit, a Gaussian is fit the measured count rates. Two lines are then fit to this Gaussian in an attempt to approximate a symmetric triangular fit as shown in Fig. 2.

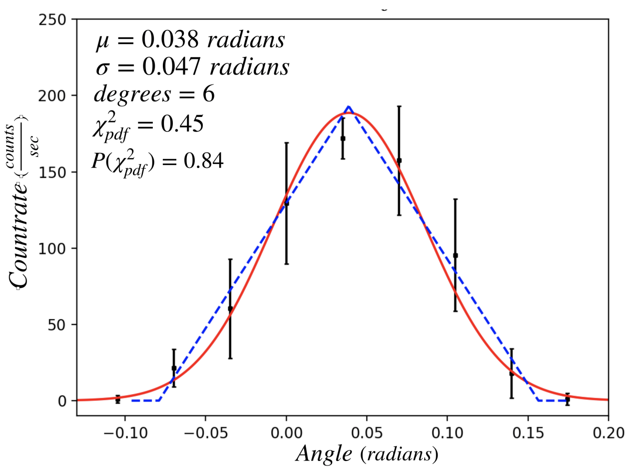

FIG. 2. The beam profile obtained for the specific apparatus used. Note the 0.038 *radian* offset in the mean of the gaussian fit, as well as the base half width of the triangular fit - $\theta_0 = (0.117 \pm 0.039)$ *radians*.

The base half width $\theta_0 = (0.117 \pm 0.039)$ *radians* of the triangular fit in Fig. 2 signifies that at any fixed howitzer angle ϕ, particles scattered at angles within a range $\phi \pm 0.1170.039$ *radians* will be detected by the detector. The uncertainties on the parameters of each linear fit are given by taking the square root of the covariance matrix diagonal. From these values, an upper bound on the uncertainty in θ_0 can be approximated by varying each of the fit parameters within their maximum uncertainty and calculating the subsequent effect on the triangular base half width.

V.2. Scattering Rate Angular Dependence

In order to examine the validity of the Rutherford model, the Rutherford differential cross section functional form expressed in Eq.4 is fit to experimentally obtained scattering rates. The form of this fit function can be expressed as $\frac{a_r}{\sin^4\frac{\theta}{2}}$, where a_r is the fit parameter.

In order to model the effect of our apparatus's wide angular acceptance on the Rutherford differential cross section, the angular response function $g(\phi-\theta;\theta_0)$ extracted from the beam profile measurement is used. The effect is modeled by taking a convolution of the angular response function and the functional fit form as follows:

$$C(\phi) = a_r \int_0^{\pi} g(\phi-\theta;\theta_0)\sin^{-4}(\frac{\theta}{2})d\theta \tag{3}$$

This convolution produces a functional form $C(\phi)$ that predicts count rate as a function of howitzer position ϕ, while accounting for the broad angular acceptance of the apparatus. A fit of this adjusted functional form to the normalized measured count rates is shown in Fig. 3.

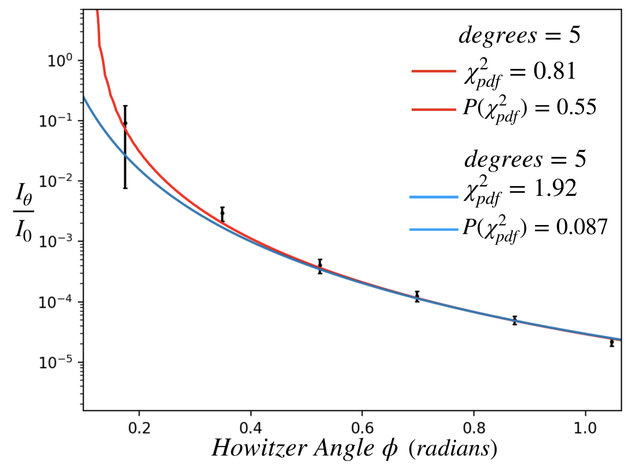

FIG. 3. The fit of the Rutherford differential cross section to the observed scattering rate measurements. Note how the convolved functional form in red provides a better fit than the unconvolved form - especially in the smaller angle regime.

The statistical uncertainty on each normalized count rate is due to the statistical uncertainty on the MCA counts. T he counts registered at each MCA channel can be treated as Poisson random variables, with uncertainties found by taking the square root of the number of counts. The uncertainty on any count rate I is therefore calculated using the expression $\sigma_I = \frac{\sqrt{counts}}{measurement\ duration}$. The uncertainty on the normalized count rates $\sigma_{\frac{I_\theta}{I_0}}$ can be found by propagating σ_{I_0} and σ_{I_θ} in quadrature through the expression $\frac{I_\theta}{I_0}$. The statistical uncertainties on normalized count rates range from $\pm1\%$ in the small angle regime, to $\pm7.9\%$ in the large angle regime.

The largest source of systematic error in our apparatus is the uncertainty in howitzer angle. Howitzer angular position can be determined with an accuracy of $\pm1°$. This uncertainty in howitzer angle was propagated vertically by multiplying the average slope of the functional fit form $C(\phi)$ obtained by convolution of the Rutherford differential cross section and the angular response function. The uncertainty in normalized count rate due to a $\pm1°$ uncertainty in howitzer angle ranged from $\pm6.6\%$ for larger angles, to $\pm38\%$ for smaller angles.

The total error on each point is systematically dominated by an uncertainty in howitzer angle in the small angle regime, which contributes 99% of the total error on the 10° point. As howitzer angle gets larger however, the statistical and systematic error contributions slowly even out. From these results, the Rutherford differential cross section clearly provides a good fit to the observed measurements with a $\chi^2_{pdf} = 1.92$ for 5 degrees of freedom. However, after taking into account the wide angular acceptance of our apparatus, the adjusted convolution $C(\phi)$ produces an excellent fit with $\chi^2_{pdf} = 0.81$ for the same 5 degrees of freedom. Although both fits have similar shapes as scattering angle increases, $C(\phi)$ provides a noticeable improvement in the small angle regime where howitzer angular uncertainty plays a dominating role. Using the normalized count rate $\frac{I_\theta}{I_0}$, the differential cross section can be extracted for any angle measured. For the 50° angle, the extracted differential cross section is $(3.89 \pm 0.603)e^{-22}\ cm^2$. This can be compared to the same 50° differential cross section of $3.204e^{-23}\ cm^2$ obtained using Eq.4.

V.3. Extracting Gold Foil Thickness

Scattering can also be used to useful to obtain measurements for the thicknesses of the gold foil targets used. As the α particle beam travels through a gold foil target, it loses energy to scattering interactions. By comparing the initial energy of the α particle beam with the attenuated energy of the same beam after it has passed through the target foil, it is possible to extract the thickness of the target using α particle stopping range data obtained from NIST [3].

In order to measure the energy of the beam after it has traversed the foil, the howitzer was positioned at 0°, and the α particle beam was scattered through the 1 layer, 2 layer, and 3 layer gold foil targets individually. On each resulting MCA spectrum, a distinct peak corresponding to the energy of the α particle detected after traversing the target foil was observed. A Gaussian was fit to each peak in order to determine the associated MCA channel number. Under the admittedly rough assumption that energy and channel number obey a linear relationship with y-intercept 0, the attenuated energy E_1 was extracted from knowledge of the unattenuated beam energy E_0 and their corresponding associated MCA channel numbers c_0 and c_1 using the following relationship referenced from the lab manual[2]:

$$E_1 = \frac{c_1}{c_0} E_0 \tag{4}$$

Although using 5.486 MeV as the initial energy of α particles E_0 might seem correct, it is actually not quite correct in this case. Due to the 1.5 μm layer of gold coating in front of the ^{241}Am source, the beam is attenuated before reaching the target foil. A corrected value of E_0 is estimated by extracting the attenuated energy of the beam with $E_0 = 5.486\ MeV$ after passing through the 1 layer of gold foil with a given thickness of $(1.3 \pm 0.1)\ \mu m$. From this measurement, the initial energy is taken as $E_0 = 4.58\ MeV$. Using this initial energy and measuring the energy attenuation of the beam as it traverses various layers of foil, the following thicknesses are extracted and tabulated:

	Extracted Value ($\frac{g}{cm^2}$)	True Value ($\frac{g}{cm^2}$)
1 layer	$0.0029 \pm 5.63\%$	0.0025
2 layer	$0.0050 \pm 2.77\%$	0.0050
3 layer	$0.0090 \pm 0.93\%$	0.0075

The statistical uncertainty on the channel number corresponding to each energy peak can is taken as the uncertainty on the mean of the Gaussian fit. These channel number uncertainties were propagated in quadrature to obtain an uncertainty in extracted attenuated energy. An upper bound on the uncertainty in each thickness is then determined by calculating thicknesses from NIST [3] using the upper and lower bounds for each attenuated energy given by their corresponding uncertainties.

VI. CONCLUSIONS

By performing a measurement of scattering count rates, the validity of the nuclear atomic model can be directly verified. The Rutherford differential cross section does an excellent job of fitting to experimental observations, with a $\chi^2_{pdf} = 0.81$ for 5 degrees of freedom. From our measurements, we extract the differential scattering cross section as $(3.89 \pm 0.603)e^{-22}\ cm^2$ compared to the theoretical prediction of $3.204e^{-23}\ cm^2$. The thicknesses of various layers of gold foils were also extracted to within 1 σ of the accepted values, although the assumption of a linear relationship with 0 y-intercept between channel number and energy was slightly inaccurate and could be improved.

ACKNOWLEDGMENTS

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[1] E. Rutherford, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **21**, 669 (1911).

[2] . Staff, “Rutherford scattering lab guide,”.

[3] NIST, "Stopping-power and range tables for helium ions,"
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