

1. Original idea I proposed was a multi-link pendulum, representing a whip, swinging and hitting a block off a table. This proposal was lacking in rotational inertia, so I changed the whip from a link of pendulums to a link of block chains that will hit a block being launched in the air. The reason why the block is no being launched instead of being static on a table is because I was unsure of how to describe in Mathematica the equations needed to keep the block on the table and fall off after being pushed some distance.

2. See last page of PDF for drawing of system.

Transformation from World (W) to Center of Mass (COM) of Arm: Rotation by  $\Theta_1$  and Translation by half the length of Arm.

Transformation from COM of Arm to the Base of Link 1 (L1): Translation by half the length of Arm and Rotation by  $\Theta_2$ .

Transformation from Base of L1 to COM of L1: Translation by half the length of Link1.

Repeat the process to find the transformations to get to the frames of the COM of Link 2 to 4, with respect to their previous link. Then do matrix multiplication to find the frame of Link 1 to 4 with respect to the World frame.

The frame for the box, with respect from the World frame, can be found with translation in the X and Y direction and Rotation by  $\Theta_6$ .

3. Euler-Lagrange Equations (E-L Eqs.) was calculated by first finding the twist of the arm, links, and box from their COM with respect to the World frame. This was done by matrix multiplication of the inverse of each component's COM frame with respect to the world frame and its derivative. The result of this was then broken up into pieces to find the angular and linear velocities of the components, which leads to finding the KE of each component. The PE can be found by taking the y portion of the transformation matrix of each component and plugging it into the  $PE = m \cdot g \cdot y$  equation.

The only constraint in this system is that Link1, representing the handle of the whip, remains at a 90 degree angle from the Arm. This can be described in the constraint equation as  $\Phi = \Theta_2 - \pi/2$ .

Two external forces were used: one used to move the arm, described in as  $-420 \cdot \sin(\pi \cdot t/7 - \pi/32)$ , and one used to launch the block, described as  $60/(t+0.1)$ . The Sin force function describes the gradual increase of force to rotate the whip. The box launch equation describes a large initial force needed to shoot the ball up and will taper off, leaving gravity to bring the ball back down. The reason why the ball is needed to shoot up so high is because this allows an impact to occur between the swinging whip and block.

After impact, the updated velocities for each component was:

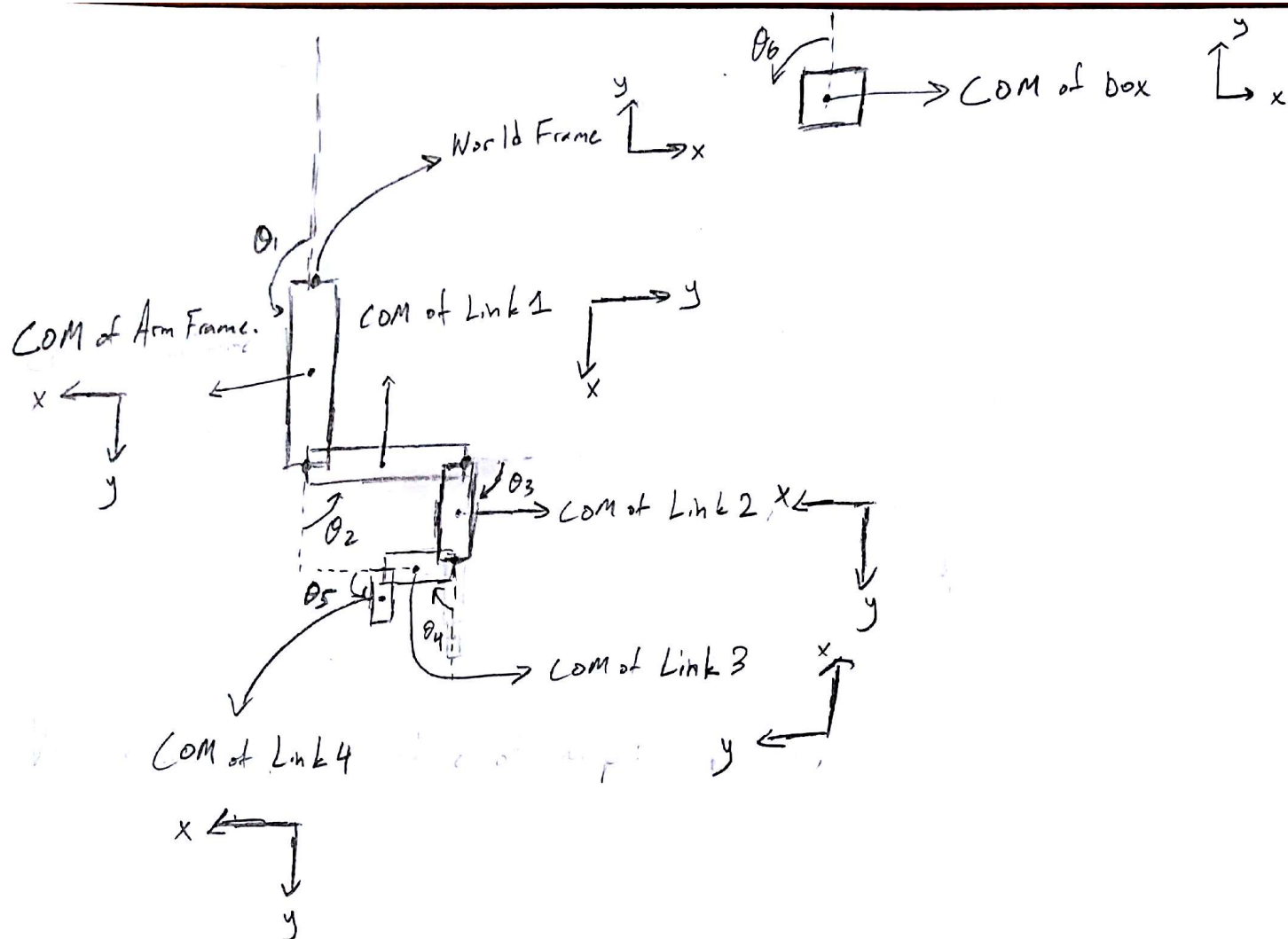
$\Theta_1$ :-3.90501,  $\Theta_2$ :-0.805529,  $\Theta_3$ : -5.52132,  $\Theta_4$ : 7.54729,  $\Theta_5$ :-11.9691,  
 $\Theta_6$ : -30.8968, xBox: 5.14947, yBox: -19.0859

4. In the beginning of the simulation, the whip will sway from left to right as the force needed to rotate the arm from the bottom to the top increases. While this occurs, the box will be launched up and fly past the whip, only moving in the y direction, with no rotation.

As the whip begins to reach the top and go around, the box will begin to fall. At the point of impact, the tip of the whip catches the top left corner of the box. This causes the box to begin rotating and begin motion in the x direction. It also causes the y velocity of the box to increase faster downwards.

The result of this simulation is reasonable, as hitting the top left of the corner of the box will cause the box to begin rotating. It will also cause the box to begin motion in the x direction.

Overall, the Mathematica code will take about 1 – 2 minutes to run.



Link can be described with variables;  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$

Box is described as:  $x, y, \theta_6$