HW-3
Written Assignment Solutions

Ans 1) a) Linear Model. There is no under-fitting.
True parameters is $\beta = (1, 2, 0)$

b) The model is linear. There is under-fitting as the model class doesn't have x^3 term.

c) $f(x) = (x_1 - x_2)^2 = x_1^2 - 2x_1x_2 + x_2^2$. The model is linear. There is underfitting because the model class doesn't have x_1x_2 term.

Ans 2) a) Let x_1 be the cancer volume, x_2 be patients age & x_3 be cancer type:

$$x_3 = \begin{cases} 0 & \text{cancer is Type I} \\ 1 & \text{cancer is Type II} \end{cases}$$

Model 1: $\hat{y} = \beta_0 + \beta_1 x_1$

Model 2: $\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Model 3: $\hat{y} = \beta_0 + \beta_1 x_1 x_3 + \beta_2 x_1 (1 - x_3) + \beta_3 x_2$

In Model 3, we have used 1-hot coding on the slope of x_1. Specifically, when $x_3 = 0$ (Type I cancer), the slope of x_1 is β_1; when $x_3 = 1$ (Type II cancer), the slope of x_1 is β_2.

b) Number of parameters in model 1 is 2

Number of parameters in model 2 is 3

Model 2 is more complex than model 1.

c) Model 1:

$$A = \begin{bmatrix} 1 & 0.7 \\ 1 & 1.3 \\ 1 & 1.6 \\ \vdots & \vdots \end{bmatrix} = \begin{bmatrix} 1 & x_{11} \\ 1 & x_{21} \\ 1 & x_{31} \\ \vdots & \end{bmatrix}$$

Model 2:

$$A = \begin{bmatrix} 1 & 0.7 & 55 \\ 1 & 1.3 & 65 \\ 1 & 1.6 & 70 \\ \vdots & \vdots & \vdots \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ 1 & x_{31} & x_{32} \\ \vdots & & \end{bmatrix}$$

Model 3:

$$A = \begin{bmatrix} 1 & x_{11}x_{13} & x_{11}(1-x_{13}) & x_{12} \\ 1 & x_{21}x_{23} & x_{21}(1-x_{23}) & x_{22} \\ 1 & x_{31}x_{33} & x_{31}(1-x_{33}) & x_{32} \\ \vdots & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.7 & 0 & 55 \\ 1 & 0 & 1.3 & 65 \\ 1 & 0 & 1.6 & 70 \\ \vdots & & & \end{bmatrix}$$

d) Since, the training MSE & test MSE is minimum for Model3, we will select Model 3.