Introduction to Machine Learning
Homework 2: Multiple Linear Regression

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1. Given the following data: $((0,0),1),((0,1),4),((1,0),3),((1,1),7)$

   - create the data matrix X
   - create the design matrix A
   - create the target vector y
   - write out the closed form solution for computing $\hat{\beta}$ that we discussed in class.
   - compute[1] $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$
   - compute RSS
   - compute TSS
   - compute R^2
   - what portion of variance in y explained by x?
   - predict the value of $x = (0.5, 0.5)$

2. For the multiple linear regression assignment on the housing data using that predicts the price of a house using 4 features, after you computed $\hat{\beta}$

   - compute RSS
   - compute TSS
   - compute R^2
   - what portion of variance in y explained by x?

3. Suppose you were interested in crop yields and you had collected data on the amount of rainfall, the amount of fertilizer, the average temperature, and the number of sunny days.

   How could you formalize this a as regression problem?

[1] You can use numpy to compute this value.

4. For the following two functions:

   - $f_1(b_1, b_2) = (b_1 - 4)^2 + (b_2 + 3)^2$
   - $f_2(b_1, b_2) = (4 - b_1)^2 + 34 \cdot ((b_1 + 4) - (b_2 - 4)^2)^2$

   run the gradient descent algorithm (on your computer) where initialy $(b_1, b_2) = (0, 0)$ and learning rate of $\alpha = 0.5$, for `num_iters`$= 4$ iterations. Report the value of b_1, b_2 at the end of each step.

5. (extra credit) For linear regression, on a data set $X = \{(x_1, y_1), \ldots, (x_n, y_n)\}$, if the jth feature for every example is scaled by c and shifted by $s > 1$, does $\hat{\beta}$ change? If it does change, describe how.