Solve general five parameter recurrence using repertoire method

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***Abstract*— [1] The repertoire method is an approach to find a closed-form for recurrence relations and sum of series. A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms. Using Repertoire method the recurrence can be solved for sum of series. Here in the five parameter recurrence, repertoire method is applied to solve the sequence of the series.**

***Index Terms*—Repertoire method, Recurrence, Series, Parameter, Sequence,**

I. INTRODUCTION

[2] The repertoire method is a process to help with the intuitive step of figuring out a closed formula for a recurrence equation. It is done by breaking the original problem into smaller parts.

Lets assume we have a system of recurrence equations with parameters, so that the unknown function can be expressed as a linear combination of other (unknown) functions where the coefficients are the parameters:

$$g(1) = b(0, a_1, ..., a_m)$$

$$g(n) = r_n(g_1, ..., g_{n-1}, \alpha_1, ..., \alpha_m)$$

$$= \sum_{i=1}^{m} A_i(n)\alpha_i$$

We can consider g as a specific point in a m-dimensional function space (determined by both the recurrence equations, and the parameters), and because g is a linear combination, we can try to find m base functions (hopefully known or easy to compute)

$$f_k(n) = \sum_{i=1}^{m} A_i(n)\alpha_{ik}$$

with 1¡=k¡=m, expressed in terms of m linearly independent vectors

$$\alpha_{1k}, ..., \alpha_{mk}$$

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The rest of the paper is organized as follows: Section II discuss about the assigned problem for the paper. Section III presents the solution of the problem. Section IV concludes the paper. Section V talks about the overall issues of the paper.

II. PROBLEM

Here this paper deals to solve a general five-parameter recurrence using repertoire method.

$$h(1) = \alpha$$

$$h(2n + j) = 4h(n) + \gamma_j n + \beta_j, \quad \text{for } j = 0, 1 \textit{ and } n >= 1$$

III. SOLUTION

[2]

The general form of h(n) is:

$$h(n) = (n) = \beta_o B_o(n) + \beta_1 B_1(n) + \gamma_o C_o(n) + \gamma_1 C_1(n)$$

We get three of these functions directly by solving:

$$h(1) = \alpha$$

$$h(2n + j) = 4h(n) + \beta_j$$

$$h(2^m + b_m...b_o = (1\beta_{bn}....\beta_{bo})_4$$

So, we have a solution for A(n),B$_o(n) and B_1(n)$.
Setting $h(n) = n$:

$$\alpha = 1$$

$$2n + j = 4n + \gamma_i n + \beta_i$$

$$\beta_i = j$$

$$\gamma_i = -2$$

which gives the equation:

$$n = A(n) + B_1(n) - 2(C_o(n) + C_1(n))$$

Setting h(n)=n^2

$$\alpha(1) = 1$$

$$4n^2 + 4jn + j = 4n^2 + \gamma_i(n) + \beta_i$$

$$\beta_i = j$$

$$\gamma_i = -2$$

which gives the equation:

$$n^2 = A(n) + B(n) + 4C_1(n)$$

The latest gives us

$$C_1(n) = (n^2 - A(n) - B_1(n))/4.$$

To solve for Co one can either replace the value of C1 in the equation for h(n)=n above, or, equivalently, add twice that equation to the one for h(n)=n^2, *which eliminates* $C_1(n)$:

$$2(n) + n^2 = 3A(n) + 3B_1(n) - 4C_o(n)$$

$$C_o(n) = \frac{3A(n) + 3B_1(n) - n^2 - 2(n)}{4}$$

IV. Conclusion

The Repertoire method is based upon two essential ingredients. The first one is the possibility to build linear combinations of already known recurrences. The second ingredient is building a repertoire which can be used for linear combinations. As can be seen above, approaching the problem from both directions (solving for known functions and solving for known parameters) can result in time saved, and simplified expression of the solution.

V. Discussion

In this paper we show that an essential part of classical mathematics is meaningful in the concrete framework. We claim that concrete mathematics is our main mathematical experience determining mathematical truth in a way independent of our axiomatic assumptions. Nevertheless, we do not claim that our mathematics based on set theoretical assumptions give important and successful tools of getting mathematical truths. Let us observe that investigating border-less of concrete mathematics can not be done inside it.

References

[1] DANG-KHOA, “The repertoire method,” p. http://dkhoa.me/post/repertoire/, 12 2017.

[2] F. Dumont, “Concrete mathematics chapter 1 exam problems,” pp. https://blog.wakatta.jp/blog/2012/02/05/concrete–mathematics–chapter–1–exam–problems/, 02 2012.