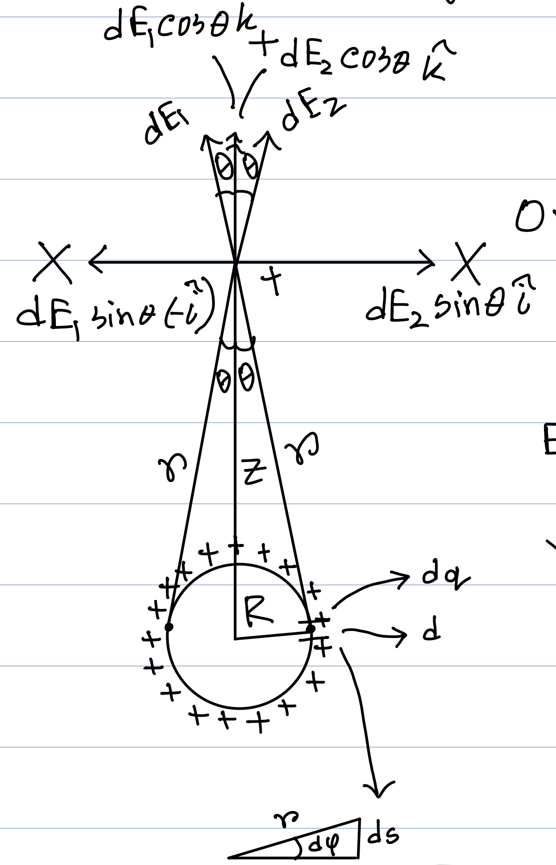
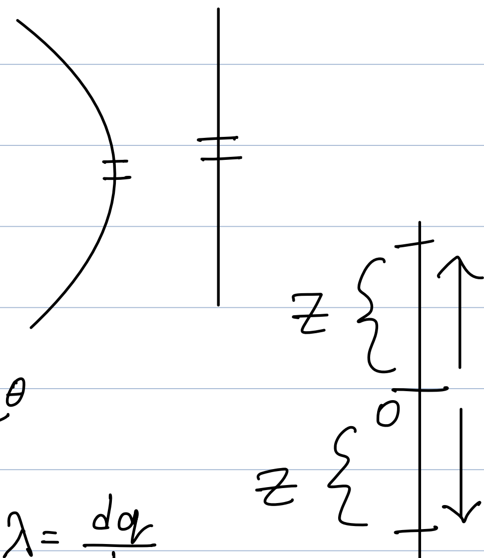
Ring of charge



One dimention

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$



$s = r\theta$

$ds = r\,d\varphi$

$$\lambda = \frac{dq}{ds}$$

$$\lambda = \frac{dq}{r\,d\varphi}$$

$$\therefore \; dq = \lambda \cdot r\,d\varphi$$

$$dE_1 = \frac{1}{4\pi\epsilon_0} \times \frac{dq}{r^2}$$

$$dE_1 = dE_2 = dE$$

$$dE_1 = \frac{1}{4\pi\epsilon_0} \times \frac{dq}{r^2}$$

$$\vec{dE}_{total} = \vec{dE_1} + \vec{dE_2}$$

$$= dE_1 \cos\theta\, (\hat{k}) + dE_2 \cos\theta\, (\hat{k})$$

$$\vec{dE}_{total} = 2 dE \cos\theta\, (\hat{k})$$

$$dE_{total} = 2 \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \cos\theta$$

$$= 2 \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\varphi}{r^2} \frac{z}{r}$$

$$= 2 \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\varphi\, z}{r^3}$$

$$\cos\theta = \frac{z}{r}$$

$$r^2 = z^2 + R^2$$

$$E = \int dE = 2 \int_0^{\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda R d\varphi\, z}{r^3}$$

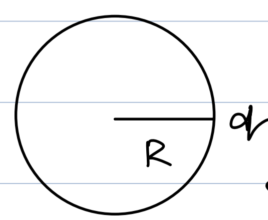
$$= \frac{1}{2\pi\epsilon_0} \frac{\lambda R z}{r^3} \int_0^{\pi} d\varphi$$

$$= \frac{1}{2\pi\varepsilon_0} \frac{\lambda R z}{\left(\sqrt{z^2+R^2}\right)^3} \varphi \Big|_0^{\pi}$$

$$= \frac{\lambda R z \; 2\pi}{4\pi\varepsilon_0 \left(\sqrt{z^2+R^2}\right)^3}$$

$$= \frac{2\pi R \cdot \lambda z}{4\pi\varepsilon_0 \left(\sqrt{z^2+R^2}\right)^3}$$

$$= \frac{q z}{4\pi\varepsilon_0 \left(\sqrt{z^2+R^2}\right)^3}$$



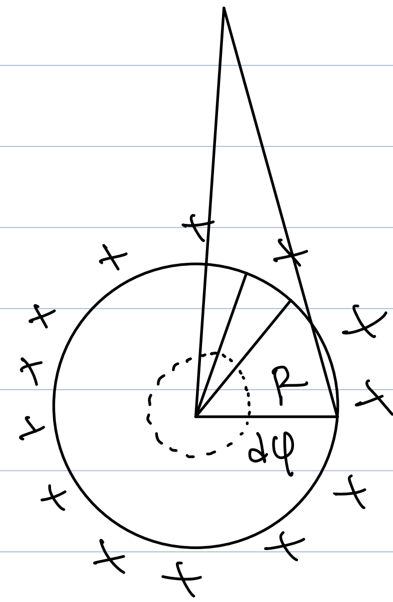
$2\pi R$

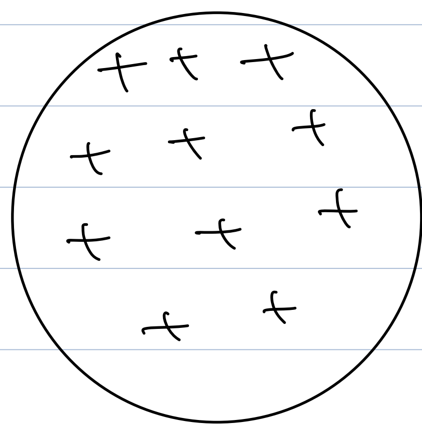
$$\lambda = \frac{dq}{ds}$$

$$\lambda = \frac{q}{2\pi R}$$

$$\therefore q = \lambda \cdot 2\pi R$$

$$\Rightarrow dq = \lambda ds = \lambda R d\varphi$$

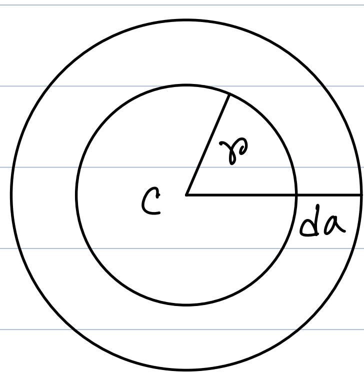




$$z^2 + a^2 = u$$

$$2a\,da = du$$

$$a\,da = \frac{du}{2}$$



dE_{ring}

$$= \frac{1}{4\pi\epsilon_0} \frac{dq\,z}{\left(\sqrt{z^2+a^2}\right)^3}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\sigma \cdot 2\pi a\,da\,z}{\left(\sqrt{z^2+a^2}\right)^3}$$

$$E = \frac{z}{4\pi\epsilon_0}\,\sigma \cdot 2\pi \int_0^R \frac{a\,da}{\left(\sqrt{z^2+a^2}\right)^3}$$

$$\sigma = \frac{dq}{dA} = \frac{dq}{2\pi A\,dA}$$

$$dq = 2\pi A\,dA \cdot \sigma$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$z \ll R$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2\left(\frac{z^2}{R^2}+1\right)}}\right]$$

$$\frac{z}{R} \approx 0$$

$$\frac{z^2}{R^2} \approx 0$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2}}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{R}\right]$$

$$= \frac{\sigma}{2\epsilon_0}$$