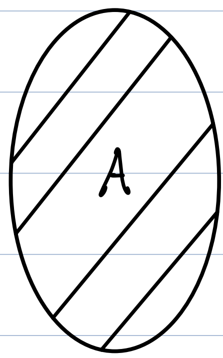
# DC Current

$dq$
$dt$




$\frac{dq}{dt}$

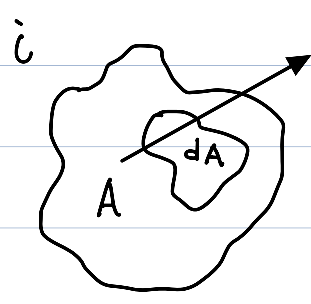
$dt \rightarrow dq$

$1 \rightarrow \frac{dq}{dt}$

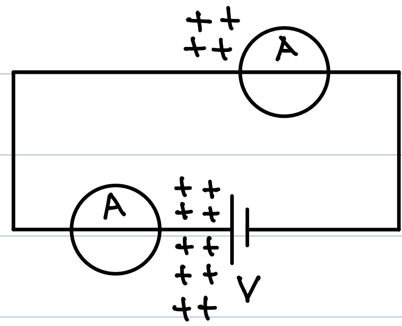
$$i = \frac{dq}{dt}$$

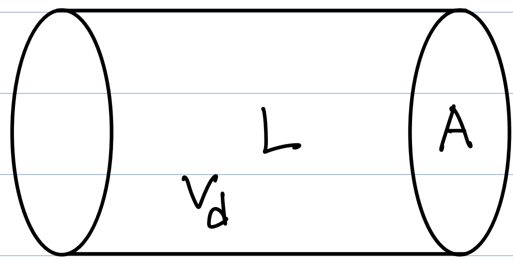
$A \longrightarrow i$

$1 \longrightarrow \frac{i}{A} = j \longrightarrow$ Current density




$$i = \int_A \vec{j} \cdot d\vec{A}$$




$1m^3 \longrightarrow n$

$1m^3$ total charge $= nq$

Total charge $= nqAL$

$dt \longrightarrow nqAL$

$1 \longrightarrow \dfrac{nqAL}{dt}$

$$i = \frac{nqAL}{dt} \qquad L = V_d\,dt$$

$$= \frac{nqAV_d\,dt}{dt}$$

$$= nqAv_d$$

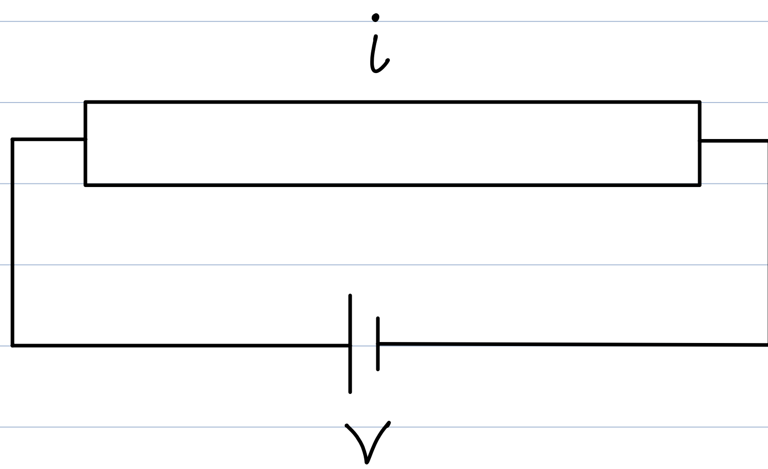
$$s = ut + \cancel{\frac{1}{2}at^2}^{\,0}$$

$$L = v_d \, dt$$

$$\frac{i}{A} = nqv_d$$

$$\Rightarrow \vec{j} = nq\vec{v_d}$$

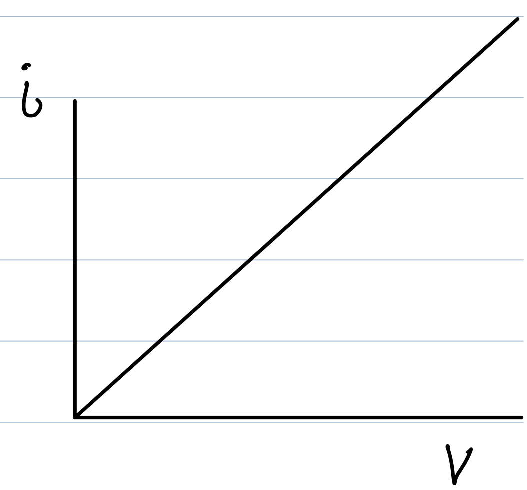
$$\Rightarrow \vec{j} = -ne\vec{v_d}$$

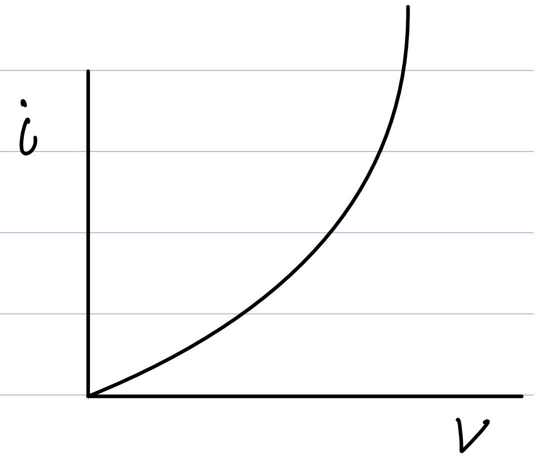


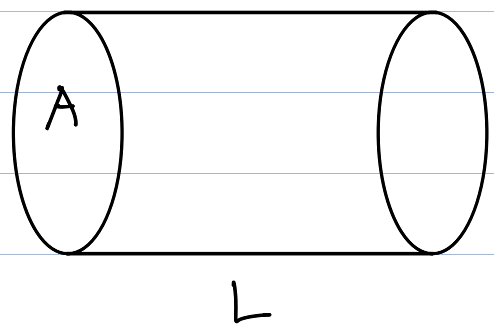

$V \propto i$

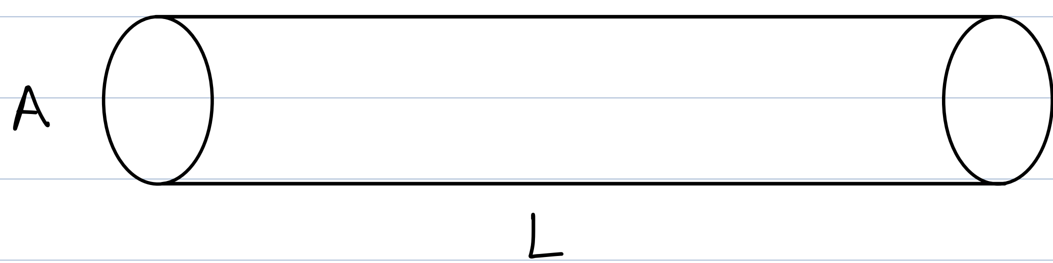
$V = Ri$

↳ Resistance






$R \propto L$

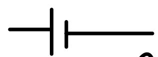
$R \propto \frac{1}{A}$

$R \propto \frac{L}{A}$

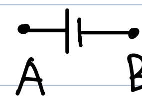
$R = \rho \frac{L}{A}$

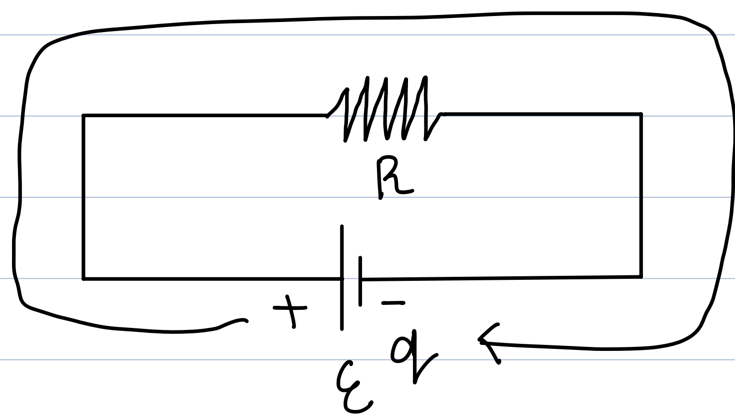
$E_{mf}$ vs $V_{pot}$

↓ Battery




↓ Battery component




$$U = qV = q\varepsilon$$

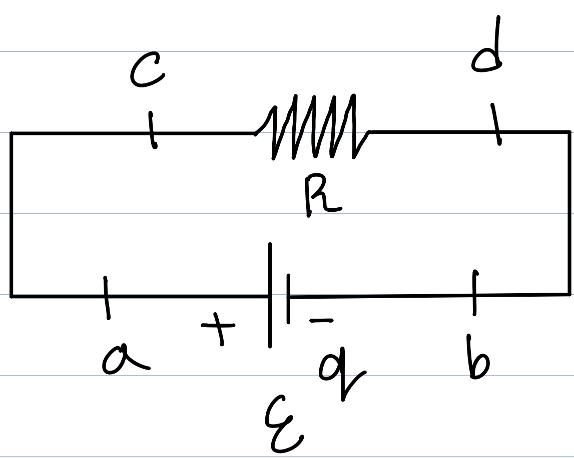
$q \times \varepsilon$

$q \longrightarrow q\varepsilon$

$1 \longrightarrow \dfrac{q\varepsilon}{q}$

$= \varepsilon$

↓ Emf



$$V_a - V_b = \mathcal{E} = V_c - V_d$$

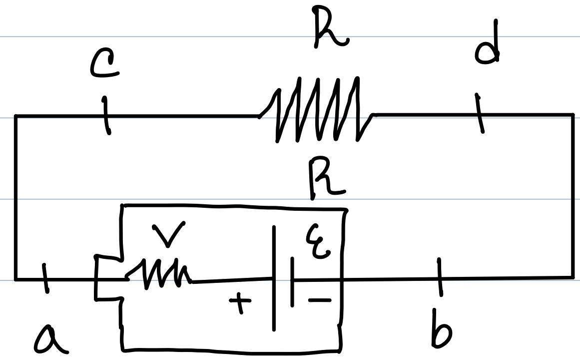
$$V_c - V_d = iR = V_a - V_b$$

$$\boxed{\begin{aligned} V_a &= V_c \\ V_b &= V_d \end{aligned}}$$

$$V_a - V_b = V_c - V_d$$

$$\therefore \mathcal{E} = iR$$

$$\boxed{i = \frac{\mathcal{E}}{R}}$$




$$\mathcal{E} - ir = V_a - V_b$$

$$iR = V_c - V_d$$

$$\Rightarrow \mathcal{E} - ir = iR$$

$$\Rightarrow \mathcal{E} = \underbrace{ir}_{V_i} + \underbrace{iR}_{V_e}$$

$$\mathcal{E} = V_i + V_e$$

$$\Rightarrow q\mathcal{E} = qV_i + qV_e$$

$$\Rightarrow U_{battery} - U_i - U_e = 0$$

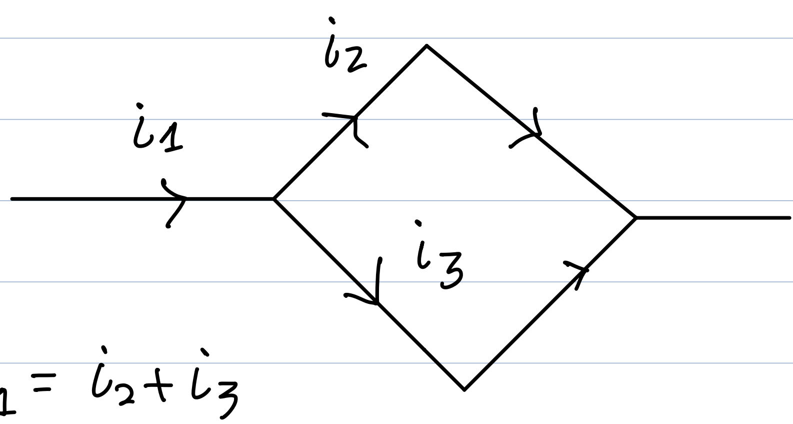
$$\Rightarrow \mathcal{E} - ir = iR$$

$$\Rightarrow \mathcal{E} = ir + iR$$

$$\Rightarrow \mathcal{E} = i(r+R)$$

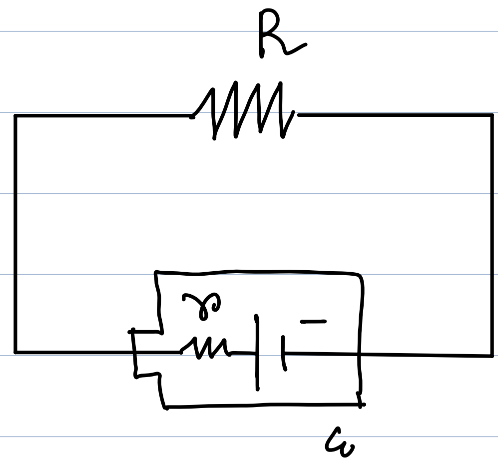
$$\Rightarrow i = \frac{\mathcal{E}}{r+R}$$

KCL

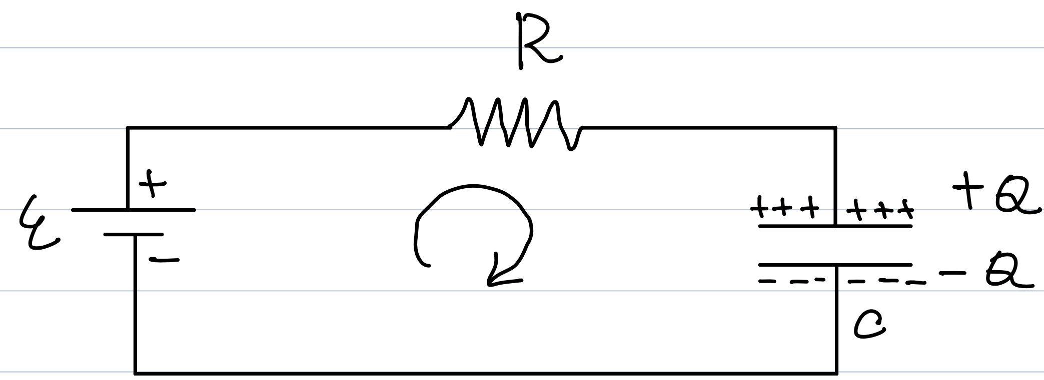



$$i_1 = i_2 + i_3$$

KVL




$$\boxed{\varepsilon - ir - iR = 0}$$




$$\varepsilon - iR - \frac{Q}{C} = 0$$

$$Q = CV$$

$$V = \frac{Q}{C}$$

$$\Rightarrow \varepsilon - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

$$\boxed{i = \frac{Q}{t}}$$

Constant

$$\Rightarrow \varepsilon - \frac{Q}{C} = R\frac{dQ}{dt}$$

$$\Rightarrow \frac{\mathcal{E}C - Q}{RC} = \frac{dQ}{dt} \qquad i = \frac{dq}{dt}$$

$$\Rightarrow \frac{dQ}{\mathcal{E}C - Q} = \frac{dt}{RC}$$

$$\Rightarrow \int_{Q=0}^{Q=Q} \frac{dQ}{\mathcal{E}C - Q} = \int_{t=0}^{t=t} \frac{dt}{RC}$$

$$\Rightarrow \int \frac{-du}{u} = \left[\frac{t}{RC}\right]_0^t$$

$$\mathcal{E}C - Q = u$$

$$\Rightarrow -dQ = du$$

$$\Rightarrow -\ln u = \frac{t}{RC}$$

$$\Rightarrow \left[-\ln|\mathcal{E}C - Q|\right]_0^Q = \frac{t}{RC}$$

$$\Rightarrow -\ln|\mathcal{E}C - Q| + \ln|\mathcal{E}C - 0| = \frac{t}{RC}$$

$$\Rightarrow \ln|\mathcal{E}C - Q| - \ln|\mathcal{E}C| = \frac{-t}{RC}$$

$$\Rightarrow \ln\left|\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right| = \frac{-t}{RC}$$

$$\Rightarrow \frac{\varepsilon C - Q}{\varepsilon C} = e^{-\frac{t}{RC}}$$

$$\Rightarrow \varepsilon C - Q = \varepsilon C e^{-\frac{t}{RC}}$$

$$\Rightarrow Q = \varepsilon C - \varepsilon C e^{-\frac{t}{RC}}$$

$$\Rightarrow Q = \varepsilon C \left(1 - e^{-\frac{t}{RC}}\right)$$

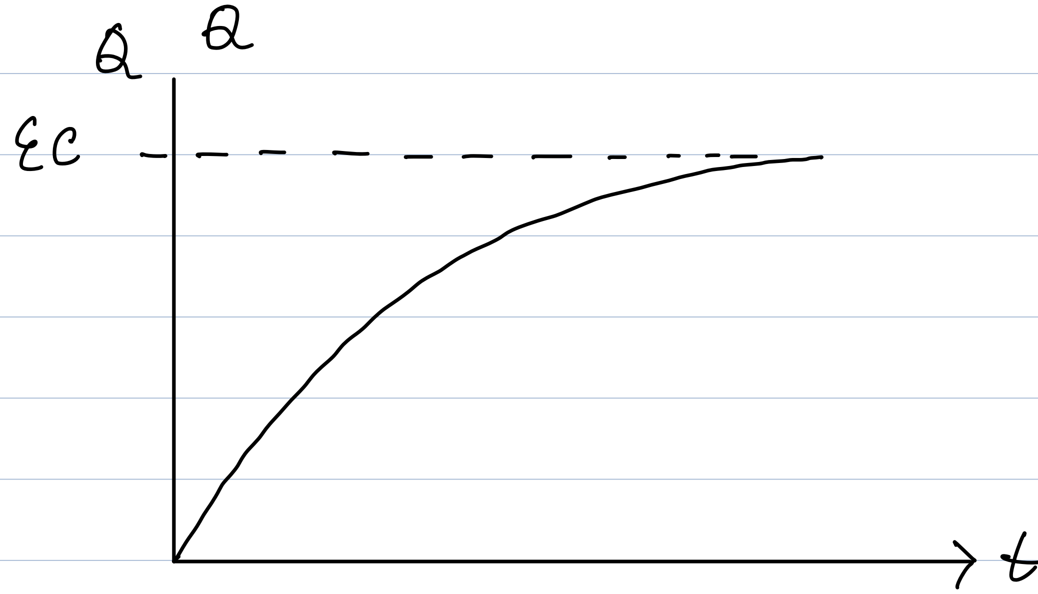
$$t = 0, \; Q = \varepsilon C \left(1 - e^{0}\right)$$
$$= \varepsilon C (1 - 1)$$
$$= 0$$

$$t = \infty, \; Q = \varepsilon C \left(1 - \frac{1}{e^{\infty}}\right)$$
$$= \varepsilon C (1 - 0)$$
$$= \varepsilon C$$

$$Q = CV$$
$$\Rightarrow C = \frac{Q}{V}$$

$$V = IR$$
$$\Rightarrow R = \frac{V}{I}$$

$$\Rightarrow RC = \frac{\cancel{V}}{I} \times \frac{Q}{\cancel{V}}$$
$$= \frac{Q}{I}$$
$$= \frac{Q}{\left(\frac{Q}{t}\right)}$$
$$= \cancel{Q} \times \frac{t}{\cancel{Q}}$$
$$= t$$

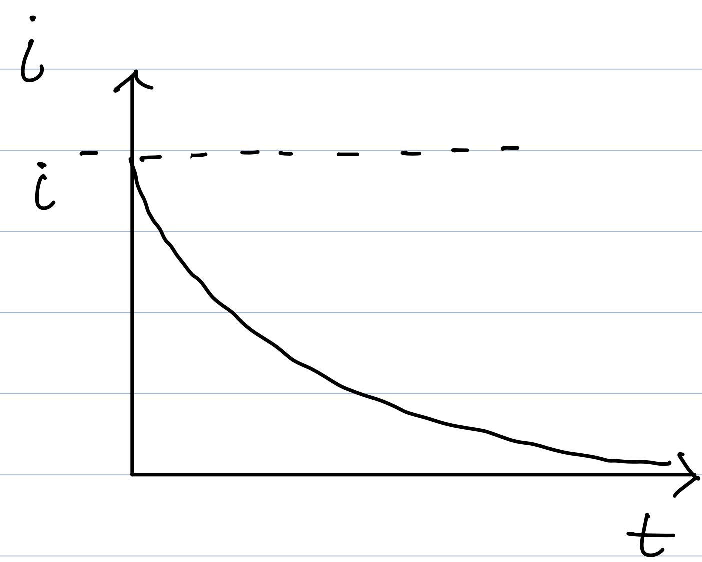



$$Q = \varepsilon C\left(1 - e^{-\frac{RC}{RC}}\right)$$

$$\Rightarrow Q = \varepsilon C\left(1 - \frac{1}{e}\right) = 0.632\, \varepsilon C$$

$$\underbrace{t = RC}_{\tau}$$

constant




$$\Rightarrow i = \frac{dq}{dt}$$

$$\Rightarrow i = ?$$