## Appendix A. Tables

### Table A.1. Computation expressions

| Name | Expression | $d$（intermediate variable） |
|---|---|---|
| $d(1,2)$ | $r_6 \arccos\left(\frac{L+2b}{2r_6}\right)$ | - |
| $d(1,3)$ | $r_6\left[\arccos\left(\frac{d^2-{r_5}^2+{r_6}^2}{2dr_6}\right)-\arccos\left(\frac{L+2b}{d}\right)\right]$ | $\sqrt{(L+2b)^2+(r_5-L-b+5a)^2}$ |
| $d(1,4)$ | $r_6 \sin^{-1}\left(\frac{b+3a}{r_6}\right)$ | - |
| $d(1,5)$ | $r_6\left[\arccos\left(\frac{d^2-{r_4}^2+{r_6}^2}{2dr_6}\right)-\arccos\left(\frac{L+2b}{d}\right)\right]$ | $\sqrt{(L+2b)^2+(r_4-L-b+5a)^2}$ |
| $d(1,6)$ | $r_6 \arccos\left(\frac{{r_7}^2-{r_6}^2-d^2}{2dr_6}\right)$ | $r_7-5a-b$ |
| $d(1,7)$ | $r_6\left[\pi-\arccos\left(\frac{{r_7}^2-{r_6}^2-d^2}{2dr_6}\right)-\arccos(\frac{L+2b}{d})\right]$ | $\sqrt{(L+2b)^2+(R_7-5a-b)^2}$ |
| $d(1,8)$ | $r_6 \arccos\left(\frac{b+3a}{r_6}\right)$ | - |
| $d(1,9)$ | $r_6 \sin^{-1}\left(\frac{d^2-{r_4}^2+{r_6}^2}{2dr_6}\right)$ | $r_4+b+5a$ |
| $d(1,10)$ | $r_6 \sin^{-1}\left(\frac{{r_5}^2-{r_6}^2-d^2}{2dr_6}\right)$ | $r_5+b+5a$ |
| $d(1,11)$ | $r_6 \sin^{-1}\left(\frac{d}{r_6}\right)$ | $L+2b$ |
| $d(1,12)$ | $\frac{\pi r_6}{2}$ | - |
| $d(1,13)$ | $r_5\left[\arccos(\frac{r_5+5a+b}{d})-\arccos\left(\frac{d^2-{r_7}^2+{r_5}^2}{2dr_5}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_7+a+b)^2}$ |
| $d(1,13)$ | $r_5\left[\arccos(\frac{r_5+5a+b}{d})-\arccos\left(\frac{d^2-{r_7}^2+{r_5}^2}{2dr_5}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_7+a+b)^2}$ |
| $d(1,14)$ | $r_5 \arccos\left(\frac{d^2-{r_6}^2+{r_5}^2}{2dr_5}\right)$ | $r_5+5a+b$ |
| $d(1,15)$ | $r_5\left[\arccos\left(\frac{d}{r_5}\right)-\arccos\left(\frac{r_5+5a+b}{d}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_5-L+5a-b)^2}$ |
| $d(1,16)$ | $r_5 \, arcsin\left(\frac{b+3a}{r_5}\right)$ | - |
| $d(1,17)$ | $r_5\left[\arccos\left(\frac{d^2-{r_4}^2+{r_5}^2}{2dr_5}\right)-\arccos\left(\frac{r_5+5a+b}{d}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_4-L-b+5a)^2}$ |
| $d(1,18)$ | $r_5\left[\pi-\arccos\left(\frac{{r_7}^2-{r_5}^2-d^2}{2dr_5}\right)-\arctan\left(\frac{r_7-5a-b}{r_5+5a+b}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_7-5a-b)^2}$ |
| $d(1,19)$ | $r_5 \arccos\left(\frac{r_5-b-4a}{r_5}\right)$ | - |
| $d(1,20)$ | $r_5\left[\arccos\left(\frac{r_5-L+5a-b}{d}\right)-\arccos\left(\frac{d}{2r_5}\right)\right]$ | $\sqrt{(r_5-L+5a-b)^2+(r_5+5a+b)^2}$ |
| $d(1,21)$ | $r_5\left[\arccos\left(\frac{r_5-L+5a-b}{d}\right)-\arccos\left(\frac{d^2-{r_6}^2+{r_5}^2}{2d^2}\right)\right]$ | $\sqrt{(r_5-L+5a-b)^2+(r_6+5a+b)^2}$ |
| $d(1,21)$ | $r_5 \arccos\left(\frac{d^2-{r_7}^2+{r_5}^2}{2dr_5}\right)$ | $r_5-\mathrm{L}+6\mathrm{a}+r_7$ |
| $d(1,23)$ | $r_5\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_3}^2+{r_5}^2}{2dr_5}\right)+\arctan\left(\frac{r_3-6a-r_5+L}{L+2b}\right)\right]$ | $\sqrt{(L+2b)^2+(r_3-r_5+L-6a)^2}$ |
| $d(1,24)$ | $r_5 \arccos\left(\frac{r_5-L+5a-b}{r_5}\right)$ | - |
| $d(1,25)$ | $r_4\left[\arccos\left(\frac{r_4+5a+b}{d}\right)-\arccos\left(\frac{d^2-{r_7}^2+{r_4}^2}{2dr_5}\right)\right]*$ | $\sqrt{(r_4+5a+b)^2+(r_7+a+b)^2}$ |
| $d(1,26)$ | $r_4 \arccos\left(\frac{d^2-{r_6}^2+{r_4}^2}{2dr_5}\right)$ | $r_4+5a+b$ |
| $d(1,27)$ | $r_4\left[\arccos\left(\frac{d^2-{r_5}^2+{r_4}^2}{2dr_5}\right)-\arccos\left(\frac{r_4+5a+b}{d}\right)\right]$ | $\sqrt{(r_4+5a+b)^2+(r_5-L-b+5a)^2}$ |
| $d(1,28)$ | $r_4\left[\arccos\left(\frac{d}{r_5}\right)-\arccos\left(\frac{r_4+5a+b}{d}\right)\right]$ | $\sqrt{(r_4+5a+b)^2+(r_4-L-b+5a)^2}$ |
| $d(1,29)$ | $r_4\left[\pi-\arccos\left(\frac{{r_7}^2-{r_4}^2-d^2}{2dr_4}\right)-\arctan\left(\frac{r_7-5a-b}{r_4+5a+b}\right)\right]$ | $\sqrt{(r_4+5a+b)^2+(r_7-5a-b)^2}$ |
| $d(1,30)$ | $r_4 \arccos\left(\frac{{r_7}^2-{r_4}^2-d^2}{2dr_4}\right)$ | $(r_7-5a-b)-(r_4-L+5a-b)$ |
| $d(1,31)$ | $r_4\left[\arccos\left(\frac{r_4-L+5a-b}{d}\right)-\arccos\left(\frac{d}{2r_4}\right)\right]$ | $\sqrt{(r_4-L+5a-b)^2+(r_4+5a+b)^2}$ |
| $d(1,32)$ | $r_4\left[\arccos\left(\frac{r_4-L+5a-b}{d}\right)-\arccos\left(\frac{d^2-{r_5}^2+{r_4}^2}{2dr_4}\right)\right]$ | $\sqrt{(r_4-L+5a-b)^2+(r_5+5a+b)^2}$ |
| $d(1,33)$ | $r_4\left[\arccos\left(\frac{r_4-L+5a-b}{d}\right)-\arccos\left(\frac{d^2-{r_6}^2+{r_4}^2}{2d^2}\right)\right]$ | $\sqrt{(r_4-L+5a-b)^2+(r_6+5a+b)^2}$ |
| $d(1,34)$ | $r_4 \arccos\left(\frac{r_4-L+5a+3b}{r_4}\right)$ | - |
| $d(1,35)$ | $r_4 \arccos\left(\frac{d^2-{r_7}^2+{r_4}^2}{2d^2}\right)$ | $r_4-L+6a+r_7$ |
| $d(1,36)$ | $r_5\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_3}^2+{r_4}^2}{2dr_4}\right)+\arctan\left(\frac{r_3-6a-r_4+L}{L+2b}\right)\right]$ | $\sqrt{(L+2b)^2+(r_3-6a-r_4+L)^2}$ |
| $d(1,37)$ | $r_4\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_2}^2+{r_4}^2}{2dr_4}\right)-\arctan\left(\frac{r_4-L+6a-r_2}{L+2b}\right)\right]$ | $\sqrt{(L+2b)^2+(r_4-L+6a-r_2)^2}$ |
| $d(1,38)$ | $r_4 \arccos\left(\frac{r_4-L+5a-b}{r_4}\right)$ | - |

| | | |
|---|---|---|
| $d(39,40)$ | $r_7\left[\arcsin\left(\frac{L}{r_7}\right)-\arccos\left(\frac{d}{r_7}\right)+\arccos\left(\frac{r_7-5a-b}{d}\right)\right]$ | $\sqrt{(r_7-5a-b)^2+(r_7-5a-b)^2}$ |
| $d(39,41)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arcsin\left(\frac{L-3a+b}{r_7}\right)\right]$ | - |
| $d(39,42)$ | $r_7\left[\arccos\left(\frac{{r_7}^2-{r_6}^2+d^2}{2dr_7}\right)-\arcsin\left(\frac{L+2b}{d}\right)+\arcsin\left(\frac{L+2b}{r_7}\right)\right]$ | $\sqrt{(L+2b)^2+(r_7-5a-b)^2}$ |
| $d(39,43)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arcsin\left(\frac{r_5+5a+b}{d}\right)+\arccos\left(\frac{{r_7}^2-{r_5}^2+d^2}{2dr_7}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_7-5a-b)^2}$ |
| $d(39,44)$ | $r_7\left[\arccos\left(\frac{{r_7}^2-{r_4}^2+d^2}{2dr_7}\right)-\arcsin\left(\frac{r_4+5a+b}{d}\right)+\arcsin\left(\frac{L+2b}{r_7}\right)\right]$ | $\sqrt{(r_4+5a+b)^2+(r_7-5a-b)^2}$ |
| $d(39,45)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{{r_7}^2-{r_4}^2+d^2}{2dr_7}\right)\right]$ | $r_7-r_4+L-10a$ |
| $d(39,46)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{{r_7}^2-{r_5}^2+d^2}{2dr_7}\right)\right]$ | $r_7-r_5+L-10a$ |
| $d(39,47)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{{r_7}^2-{r_6}^2+d^2}{2dr_7}\right)\right]$ | $r_7-5a-b$ |
| $d(39,48)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arcsin\left(\frac{3a+b}{r_7}\right)\right]$ | - |
| $d(39,49)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d^2-{r_5}^2+{r_7}^2}{2dr_7}\right)-\arctan(\frac{r_5+5a-L-b}{r_7-5a+L+b})\right]$ | $\sqrt{(r_5-L+5a-b)^2+(r_7-5a+L+b)^2}$ |
| $d(39,50)$ | $r_7\arcsin\left(\frac{L+2b}{r_7}\right)$ | - |
| $d(39,51)$ | $r_7+a+b-\sqrt{{r_7}^2-(3a+b)^2}$ | - |
| $d(39,52)$ | $3a+b$ | - |
| $d(39,53)$ | $\sqrt{{r_7}^2-(3a+b)^2}-r_7+5a+b$ | - |
| $d(39,54)$ | $\sqrt{{r_6}^2-(3a+b)^2}$ | - |
| $d(39,55)$ | $\sqrt{{r_5}^2-(3a+b)^2}-r_5+L-5a+b$ | - |
| $d(39,56)$ | $\sqrt{{r_4}^2-(3a+b)^2}-r_4+L-5a+b$ | - |
| $d(39,57)$ | $L+2b-\sqrt{{r_6}^2-(3a+b)^2}$ | - |
| $d(39,58)$ | $r_7+L-5a+b-\sqrt{{r_7}^2-(L-3a+b)^2}$ | - |
| $d(39,59)$ | $L+b-3a$ | - |
| $d(39,60)$ | $\sqrt{{r_7}^2-(L-3a+b)^2}-r_7+b+L-a$ | - |
| $d(39,61)$ | $L+2b-[\sqrt{{r_4}^2-(L-3a+b)^2}-r_4+b+L-5a]$ | - |
| $d(39,62)$ | $L+2\mathrm{b}-[\sqrt{{r_3}^2-(3a+b)^2}-r_3+a+b]$ | - |
| $d(39,63)$ | $L+2b$ | - |
| $d(39,64)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-arccos\left(\frac{d}{r_7}\right)+\arctan\left(\frac{r_7+a-L-b}{r_7+a+b}\right)\right]$ | $\sqrt{(r_7+a-L-b)^2+(r_7+a+b)^2}$ |
| $d(39,65)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arcsin\left(\frac{L-3a+b}{r_7}\right)\right]$ | - |
| $d(39,66)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d^2-{r_4}^2+{r_7}^2}{2dr_7}\right)\right]$ | $r_4-L+r_7+6a$ |
| $d(39,67)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d^2-{r_5}^2+{r_7}^2}{2dr_7}\right)\right]$ | $r_5-L+r_7+6a$ |
| $d(39,68)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d^2-{r_6}^2+{r_7}^2}{2dr_7}\right)\right]$ | $r_6-L+6a+r_7$ |
| $d(39,69)$ | $r_7\left[\arccos\left(\frac{d^2-{r_5}^2+{r_7}^2}{2dr_7}\right)-\arcsin\left(\frac{r_5+5a+b}{d}\right)+\arcsin\left(\frac{L+2b}{r_7}\right)\right]$ | $\sqrt{(r_5+5a+b)^2+(r_7+a+b)^2}$ |
| $d(39,70)$ | $r_7\left[\arccos\left(\frac{d^2-{r_4}^2+{r_7}^2}{2dr_7}\right)-\arcsin\left(\frac{r_4+5a+b}{d}\right)+\arcsin\left(\frac{L+2b}{r_7}\right)\right]$ | $\sqrt{(r_4+5a+b)^2+(r_7+a+b)^2}$ |
| $d(39,71)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d}{r_7}\right)+\arccos\left(\frac{r_7-L+a-b}{d}\right)\right]$ | $\sqrt{(r_7-L+a-b)^2+(r_7-5a-b)^2}$ |
| $d(39,72)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arcsin\left(\frac{3a+b}{r_7}\right)\right]$ | - |
| $d(39,73)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{r_7-L+a-b}{d}\right)+\arccos\left(\frac{d}{r_7}\right)\right]$ | $\sqrt{(r_7-L+a-b)^2+(r_7+a+b)^2}$ |
| $d(39,74)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{d^2-{r_3}^2+{r_7}^2}{2dr_7}\right)+\arccos\left(\frac{r_7+a+b}{d}\right)\right]$ | $\sqrt{(r_3-a-b)^2+(r_7+a+b)^2}$ |
| $d(39,75)$ | $r_7\left[\arcsin\left(\frac{L+2b}{r_7}\right)-\arccos\left(\frac{{r_2}^2-{r_7}^2+d^2}{2dr_7}\right)+\arccos\left(\frac{r_7+a+b}{d}\right)\right]$ | $\sqrt{(r_2-a-b)^2+(r_7+a+b)^2}$ |
| $d(39,76)$ | $r_7\arcsin\left(\frac{L+2b}{r_7}\right)$ | - |
| $d(77,78)$ | $r_3\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_7}^2+{r_3}^2}{2dr_3}\right)-\arcsin\left(\frac{r_3-a-b}{d}\right)\right]$ | $\sqrt{(r_3-a-b)^2+(r_7+a+b)^2}$ |
| $d(77,79)$ | $r_3\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_4}^2+{r_3}^2}{2dR_3}\right)-\arccos\left(\frac{L+2b}{d}\right)\right]$ | $\sqrt{((r_3-r_4+L-6a))^2+(L+2b)^2}$ |
| $d(77,80)$ | $r_3\arcsin\left(\frac{3a+b}{r_3}\right)$ | - |
| $d(77,81)$ | $r_3\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_5}^2+{r_3}^2}{2dr_3}\right)-\arccos\left(\frac{L+2b}{d}\right)\right]$ | $\sqrt{(r_3-r_5+L-6a)^2+(L+2b)^2}$ |
| $d(77,82)$ | $r_3\arcsin\left(\frac{5a+b}{r_3}\right)$ | - |
| $d(77,83)$ | $r_2\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-{r_7}^2+{r_2}^2}{2dr_2}\right)-\arccos\left(\frac{r_7+a+b}{d}\right)\right]$ | $\sqrt{(r_2-a-b)^2+(r_7+a+b)^2}$ |

| | | |
|---|---|---|
| $d(77,84)$ | $r_2\left[\frac{\pi}{2}-\arccos\left(\frac{d^2-r_4{}^2+r_2{}^2}{2dr_2}\right)+\arccos\left(\frac{L+2b}{d}\right)\right]$ | $\sqrt{(r_2-r_4+L-6a)^2+(L+2b)^2}$ |
| $d(77,85)$ | $r_2\arcsin\left(\frac{3a+b}{r_2}\right)$ | - |
| $d(77,86)$ | $\frac{\pi r_2}{2}$ | - |

**Table A.2. Conflict patterns between movements**

| | | S1 | | | S2 | | | S3 | | | E1 | | | E2 | | | E3 | | | N1 | | | N2 | | | N3 | | | W1 | | | W2 | | | W3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | W1 | W2 | W3 | N1 | N2 | N3 | E1 | E2 | E3 | S1 | S2 | S3 | W1 | W2 | W3 | N1 | N2 | N3 | E1 | E2 | E3 | S1 | S2 | S3 | W1 | W2 | W3 | N1 | N2 | N3 | E1 | E2 | E3 | S1 | S2 | S3 |
| S1 | W1 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 |
| | W2 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 |
| | W3 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 |
| S2 | N1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 0 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 |
| | N2 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 |
| | N3 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 7 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 3 | 3 | 3 | 0 | 0 | 0 |
| S3 | E1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 0 | 0 | 0 |
| | E2 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 0 | 0 | 0 |
| | E3 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 |
| E1 | S1 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 0 | 0 |
| | S2 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 0 |
| | S3 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 7 |
| E2 | W1 | 7 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 0 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| | W2 | 0 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| | W3 | 0 | 0 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 7 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| E3 | N1 | 0 | 0 | 0 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| | N2 | 0 | 0 | 0 | 0 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| | N3 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| N1 | E1 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 0 | 0 | 0 | 0 | 0 |
| | E2 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 0 | 0 | 0 | 0 |
| | E3 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 7 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 |
| N2 | S1 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 5 | 5 | 5 | 3 | 3 | 3 | 7 | 0 | 0 |
| | S2 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 5 | 5 | 5 | 3 | 3 | 3 | 7 | 7 | 0 |
| | S3 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 5 | 5 | 5 | 3 | 3 | 3 | 7 | 7 | 7 |
| N3 | W1 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | W2 | 0 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | W3 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| W1 | N1 | 6 | 6 | 6 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 0 | 0 | 6 | 6 | 6 | 4 | 4 | 4 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| | N2 | 6 | 6 | 6 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 0 | 6 | 6 | 6 | 4 | 4 | 4 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| | N3 | 6 | 6 | 6 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 7 | 7 | 7 | 6 | 6 | 6 | 4 | 4 | 4 | 0 | 0 | 0 | 2 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| W2 | E1 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 0 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | E2 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 0 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | E3 | 4 | 4 | 4 | 3 | 3 | 3 | 7 | 7 | 7 | 4 | 4 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| W3 | S1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |
| | S2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |
| | S3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |

**Table A.3. Computation expressions of radius in an unsignalized intersection**

| Name | Expression |
|---|---|
| $r_1$ | $b+a$ |
| $r_2$ | $[(b+a)^2+(b+3a)^2]/(2a+2b)$ |
| $r_3$ | $[(b+a)^2+(b+5a)^2]/(2a+2b)$ |
| $r_4$ | $[(L-5a+b)^2+(L-a+b)^2]/(2L+2b-10a)$ |
| $r_5$ | $[(L-5a+b)^2+(L-3a+b)^2]/(2L+2b-10a)$ |
| $r_6$ | $L+b-5a$ |
| $r_7$ | $a+(L+2b)^2/4a$ |

## Appendix B. Algorithms

**Algorithm B.1. The crossing strategy of an AV**

// $n$ is the amount of preceding vehicles.
// $t_0$ is the initial time.
// $\widehat{ET}$ is the fastest entry time of the vehicle arriving at the intersection without delay.
// $v_0$, $v_E$, $v_C$ are the initial speed, entry speed, and limited speed, respectively

**Begin**
  Calculate the fastest entry time $\widehat{ET}$ of the new incoming CAV by Eq. (9);
  Initialize the permissive entry time $ET \leftarrow \widehat{ET}$;
  **For** $k$ = 1 to $n$ **Do**
    **If** its spatial trajectory is in conflict with that of the preceding vehicle $k$ **Then**
      **If** $ET_k - \varsigma^L < ET_k < ET_k - \varsigma^U$, where $\varsigma^L, \varsigma^U \in \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}$ **Then**
        $ET_k \leftarrow ET_k - \varsigma^U$;
      **End if**
    **End if**
  **End for**
**End**

**Algorithm B.2. Game process of multiple firms for trajectory planning**

//$\delta$ is a small positive constant.
//$S_0$ is the simulated traffic state at time $T_1$.
//Before time $T_1$, $V_n$ is the set of travel requests of firm $n \in N$ during $[T_1, T_2]$.
//At time $T_1$, trajectories for $V_n$, $n \in N$ are planned.

**Begin**
  Firm $n \in N$ creates a sequence set $\Omega_n$ for $V_n$ based on the departure time sequence;
  Firm $n \in N$ creates an empty sequence set $\Psi_n$ to place the final AV sequence for $V_n$;
  $k \leftarrow 1$;
  **Repeat**
    **If** $k = 1$ **Then**
      **For** firm $n \in N$ **Do**
        Copy a simulated traffic state $S_n$ based on $S_0$;
        **For** AV $a \in \Omega_n$ **Do**
          Plan the trajectory $x_a^k$;
          Update $S_n$ by the trajectory;
        **End for**
      **End for**
    **Else**
      **For firm** $l \in L$ **Do**
        Sort delays of AVs in $\Omega_l$ in descending order;
        Add the AVs with the largest $m < length(V_l)$ delays into $\Psi_l$ and remove them from $\Omega_l$;
        Copy a simulated traffic state $S_l$ based on $S_0$;
        **For** AV $b \in \Psi_l$ **Do**
          Plan the trajectory $x_b^k$;
          Update $S_l$ by the trajectory;
        **End for**
        **For** AV $c \in \Omega_l$ **Do**
          Plan the trajectory $x_c^k$;
          Update $S_l$ by the trajectory;
        **End for**
      **End for**

```
        End if
    The SMC collects all requests X^k and creates a checking sequence set Λ;
    Create an empty set Π to place the AVs who have been checked for safety;
    For AV d ∈ Λ Do
      Modify x_d^k by Appendix B.1 if there is a conflict between d and any AV w ∈ Π;
      Add d into Π;
    End for
    The SMC feeds back feasible trajectories and the average system delay d^k;
    Create an empty firm set L;
    For firm n ∈ N Do
        Calculate the average delay d_n^k;
        If d_n^k − d^k > δ and length(Ω_n^k) > 0 Then
            Add n into L;
        End if
      End for
      k ← k + 1;
    Until length(L) = 0
    End
```

**Algorithm B.3. The fair strategy**

```
    //p is the iteration count.
    //Given the AV sequence set Ψ_n^k and the number of travel requests |V_n| of firm n ∈ N;
1   Begin
2     Create an empty sequence set O to place the checking sequence for all AVs;
3     Find the greatest common divisor θ of |V_1|, … , |V_n|;
4     Round up the quotient |V_n|/θ to the integer r_n, n ∈ N;
5       Find firm a ∈ N who has the most requests;
6     For p = 1 to θ Do
7       Create an empty sequence set P to place the checking sequence for Σ_{i=1}^n |V_i| /θ AVs;
8       Add the first r_a AVs in Ψ_a^k into P in turn and remove them from Ψ_a^k;
9         For firm i = 1 to n Do
10        If i ≠ a Then
11          Randomly generate an index set Γ_i consisting of r_i non-repetitive elements in
12          {1,2, … , |P| − 1};
13          Rearrange the numbers in Γ_i in ascending order;
14            Add the first r_i AVs in Ψ_i^k into P according to Γ_i and remove these AVs from Ψ_a^k;
15          End if
16      End for
17      Add the AVs in P into O in turn;
18    End for
19  End
```

**Algorithm B.4. The partial priority strategy**

```
    //p is the iteration count.
    //Given the AV sequence set Ψ_n^k and the number of travel requests |V_n| of firm n ∈ N;
1   Begin
2     Create an empty sequence set O to place the checking sequence for all AVs;
3     Find the greatest common divisor θ of |V_1|, … , |V_n|;
4     Round up the quotient |V_n|/θ to the integer r_n, n ∈ N;
5       Find firm a ∈ N who has the most requests;
6     For p = 1 to θ Do
7       Create an empty sequence set P to place the checking sequence for Σ_{i=1}^n |V_i| /θ AVs;
8       Add the first r_a AVs in Ψ_a^k into P in turn and remove them from Ψ_a^k;
9         For firm i = 1 to n Do
```

10 **If** $i \neq a$ **Then**
11 Randomly generate an index set $\Gamma_i$ consisting of $r_i$ non-repetitive elements in
12 $\{1,2,\dots,|P|-1\}$;
13 Rearrange the numbers in $\Gamma_i$ in ascending order;
14 Add the first $r_i$ AVs in $\Psi_i^k$ into P according to $\Gamma_i$ and remove these AVs from $\Psi_a^k$;
15 **End if**
16 **End for**
17 Add the AVs in P into O in turn;
18 **End for**
19 **End**