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% Inputs
XIN1, XIN2 : VAR [tick -> real]
EPS        : VAR [tick -> posreal] % Assumption: positive deadband size

% Output
Q          : VAR [tick -> bool]

HYSTERESIS_st_impl (XIN1, XIN2, EPS, Q): bool =
  FORALL t:
    Q(t) =
      IF      init(t)                                THEN False
      ELSIF Q(pre(t)) & XIN1(t) < (XIN2(t) - EPS(t)) THEN False
      ELSIF      XIN1(t) > (XIN2(t) + EPS(t)) THEN True
      ELSE                                           Q(pre(t))
      ENDIF

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HYSTERESIS_tab_req (XIN1, XIN2, EPS, Q): bool =
  FORALL t:
    Q(t) =
      IF init(t) THEN False
      ELSE LET prev = Q(pre(t)) IN
        TABLE
          | XIN1(t) < (XIN2(t) - EPS(t)) | False ||
          | (XIN2(t) - EPS(t)) <= XIN1(t) & XIN1(t) <= (XIN2(t) + EPS(t)) | prev  ||
          | (XIN2(t) + EPS(t)) < XIN1(t) | True  ||
        ENDTABLE
      ENDIF

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Hysteresis_correctness: THEOREM
  FORALL XIN1, XIN2, EPS, Q:
    HYSTERESIS_st_impl(XIN1, XIN2, EPS, Q) IMPLIES
      HYSTERESIS_tab_req(XIN1, XIN2, EPS, Q)

Hysteresis_consistency: THEOREM
  FORALL XIN1, XIN2, EPS:
    EXISTS Q:
      HYSTERESIS_st_impl(XIN1, XIN2, EPS, Q)

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% Assumption: low and high alarm hysteresis regions do not overlap
HIGH_LIMIT: TYPE =
  [ l: [tick -> real], eps: [tick -> posreal] ->
    { h: [tick -> real] | FORALL (t: tick): h(t) - eps(t) > l(t) + eps(t) } ]

% Inputs
X, L      : VAR [tick -> real]
EPS       : VAR [tick -> posreal]
H         : VAR HIGH_LIMIT

% Outputs
QH, Q, QL : VAR [tick -> bool]

LIMITS_ALARM_fbd_impl (X, H, L, EPS, QH, Q, QL): bool =
  EXISTS (w1: [tick -> posreal]), (w2, w3: [tick -> real]):
    DIV(EPS, (LAMBDA (t: tick): 2.0), w1)
    & SUB(H(L, EPS), w1, w2)
    & ADD(L, w1, w3)
    & HYSTERESIS_tab_req(X, w2, w1, QH)
    & HYSTERESIS_tab_req(w3, x, w1, QL)
    & DISJ(QH, QL, Q)

LIMITS_ALARM_correctness: THEOREM
FORALL X, H, L, EPS, QH, Q, QL:
  LIMITS_ALARM_fbd_impl(H, X, L, EPS, QH, Q, QL) IMPLIES
  LIMITS_ALARM_tab_req(H, X, L, EPS, QH, Q, QL)

LIMITS_ALARM_consistency: THEOREM
FORALL H, X, L, EPS:
  EXISTS QH, Q, QL:
    LIMITS_ALARM_fbd_impl(H, X, L, EPS, QH, Q, QL)

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