The Many Rewards of Putting Absolutely Everything into Introductory Logic

James M. Henle

Smith College
Northampton, Massachusetts, USA
jhenle@smith.edu

Abstract. I co-teach an introductory logic course with philosopher Jay Garfield. The contains an absurd amount of material—formal and informal, theoretical and applied, trivial and profound, sacred and mundane.

The course gives ordinary students important critical skills. For some, it kindles a love of logic that shapes their academic careers.

Keywords: formal logic, teaching, critical reasoning.

I do mean everything.

In this paper I argue for an introductory logic course that does both formal logic and critical reasoning, includes serious philosophy and mathematics, hits topics in linguistics and computing, touches on law and rhetoric, and has time to visit politics, religion, poetry, meteorology and science fiction.

The benefits of a course like this are many.

- It puts logic where it belongs, at the center of the universe.
- It grows logicians.
- It fosters a vibrant logical community.
- It makes you look good to your dean.

And surprisingly, for a course so broadly conceived,

- It reaches great depths.

The contents may seem too much for one course. It is more than one instructor can reasonably do. For this reason I recommend a team. My first co-teacher was the late philosopher Tom Tymoczko. I now teach with philosopher Jay Garfield. Most of the ideas in this paper are his as much as mine.

The origin of the course is embarrassing. We designed it to please ourselves. We made it great fun to teach. We did just about whatever we wanted. Fortunately, the course has turned out to be almost as much fun for the students. It's also good for them.

There is much to explain here. I'll start by describing some of the problems the course is meant to address. Then I'll go into the composition of the course—what it must contain and what it can contain. At the end I'll discuss the consequences of offering such a course.

P. Blackburn et al. (Eds.): TICTTL 2011, LNAI 6680, pp. 109–122, 2011.
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1 What's Wrong Right Now

1.1 Irrelevance

Logic is relevant, but that's not how it's regarded, at least in the United States. Formal logic lives on a pedestal. It's respected, but distantly. Everyone admires it; a few study it; but most ignore it. In general, the public makes no connection between the strange symbols of formal logic and the great issues confronting the world.

Informal logic gets more attention, but it too has trouble. It's relevance is respected abstractly but not practically. Everyone understands that there's something special about arguments that are logical, but for many reasons logic is seldom sought and seldom achieved. Politicians, advertisers and artists prefer appeals to emotion. Critics and commentators respond in kind.

Even in academia, logic is more an ideal than a concrete goal. There are many fields where logic would expose appalling gaps and terrible inconsistencies. Naturally logic is not especially welcome in such places. I am not at liberty to be more specific.

In curricular matters, the same is true. For example, logic should be at the center of any writing course. Most courses give it only cursory attention. Almost never is it the organizing principle.

In sum, the relationship of the professoriate to logic is a bit like its relationship to quantitative reasoning. Faculties will usually vote to require that students be "quantitatively literate." But few in the faculty are themselves quantitatively literate.

1.2 Fear

Many students fear logic. The reasons vary.

Philosophy majors are required to take logic. As a result, logic is regarded by them as an unpleasant hurdle. Many majors put off taking logic until their senior year.

Logic is strongly recommended for pre-law students. The consequence of that is that all the fear that pre-law students have of the LSATs[1] transfers to logic.

"Fear" is too strong a word for most people, but many in the general population have a distinct discomfort with logic. This is true even among mathematicians. In mathematical social gatherings, I often feel like the clergyman who has walked into a party. Everyone suddenly is on their best behavior. They don't want to say the wrong thing.

1.3 Tedium

Many philosophy professors teach logic every year. For some, it is a simple chore. There are students that get it; there are students that don't. The ones that don't

[1] The Law School Admission Test, a lengthy standardized test critical in applying to law school in the United States.

are hopeless. The material doesn't change. The same exercises and the same examples trot across the blackboard term after term.

It's tedious, but tedium is not universally seen as a problem. Some faculty are more than content to teach a course that's not challenging and not surprising. They can relax teaching logic.

But even if tedium is not a problem, I have a solution.

1.4 Indifference

We (the reader and I) are logicians. We're fascinated by its power, its beauty, and the marvelous surprises it offers us time and again. But we're not very successful recruiting followers. Unlike historians, economists, and psychologists, we have few undergraduate accolytes. Certainly not as many as we deserve.

1.5 Cowardice, or Possibly Lethargy

College is the time and the place where students should challenge everything. It is the period in life which is ideal for intellectual exploration and adventure. Students should wrestle with ideas. They should try on beliefs without preconditions. But on many campuses, this is difficult.

On some issues there is a "politically correct" position. It can take courage to doubt it. Alternative ideas will be attacked. The individual who ventures the alternative idea can suffer socially.

Even in the best environments, it takes energy to tackle an issue. It takes work to achieve the intellectual distance that makes toying with unusual positions possible. It's always easier to accept what you have always accepted.

And then, of course, some widely-accepted ideas are accepted widely because they're correct. Those ideas are especially difficult to challenge. But of course, they should be.

Finally, while everyone believes that ideas should be challenged, and while everyone knows that college students have grown lazy and complacent, just about everyone is confident that they at least challenge ideas frequently and courageously. So maybe it's not a problem.

2 The Course That Will Fix All That

2.1 Two Sides

To begin with, this course needs a team. At Smith, it's taught by a mathematician and a philosopher. We add one or two graduate students from the University of Massachusetts for section help and we have a staff of undergraduate veterans to grade papers, tutor students and cheer them on.

Students take logic for different reasons. If different perspectives are offered, more students have the pleasure of seeing their issues raised. More students find meaning in logic. More students see themselves as future logicians.

There are other benefits from having two different viewpoints in the classroom. Class is more exciting with disputations. Arguments are a signal to students that disagreement is possible, that disagreement is good.

The differences don't have to be extreme; in our case they are not. We disagree, for example, over what constitutes a reasonable resolution of the Liar paradox. Most of our quarrels are about what is interesting or what is significant. Only occasionally do we differ about what is actually true. Students are surprised at first when we fuss over what seem to them trivialities. But the fuss tells them that they don't have to accept any view; they can form their own. That's liberating. And it also tells them that maybe, underneath it all, there are no trivialities.

2.2 Formal Logic and Critical Reasoning

Formal logic and critical reasoning are different sorts of logic. Their differences have kept them apart in the curriculum, to their detriment. For despite the differences there is a close kinship.

Formal logic is symbolic and abstract. Some are attracted to its intellectual challenges. But for many, Ps and Qs seem pointless. For these students, informal arguments can give meaning to formal logic. They can bring the symbols to life. The effect is especially strong when formal structure can be seen in arguments made in the world today, or in the arguments one wants to make.

Many formal logic courses spend little time with argumentation in natural language because it is feared that would take time away from deeper topics. But in fact no time need be lost. The illustration of logical ideas in language facilitates understanding. Our students, yours and mine, have an innate logical sense. That sense comes with the practice of speech. If we use that logical sense, we can move faster. Our students come to us with a foundation for formal logic. Critical reasoning strengthens that foundation; it enhances learning formal logic.

On the other side, critical reasoning lives with the ambiguity and irregularity that goes with natural language. It can be difficult to sort valid argumentation from invalid. But formal logic can reveal the hidden structure. And it's not enough to do this in simple situations. Students won't believe the formal analysis unless they get more than taste of it.

Rich logical structure can be found in the public record. As a challenging exercise, we might ask students to formalize in propositional logic:

"If any qualified retailer fails to provide the notice described in section 4041(n)(3)(A)(ii) to any seller of diesel fuel to such retailer, unless it is shown that such failure is due to reasonable cause and not to willful neglect, there shall be paid, on notice and demand of the Secretary and in the same manner as tax, by such retailer with respect to each sale of diesel fuel to such retailer by such seller

to which section 4041(n)(4) applies an amount equal to 5 percent of the tax imposed by section 4041(n)(1) on such sale by reason of paragraphs (3) and (4)(A) of section 4041(n)."[2]

Critical reasoning is often offered as a soft logic course, a course for students not ready for abstract thinking. There may be students like this, but most students, even high school students can handle formal logic and indeed are not served well by the limited ambition of critical reasoning courses. The fact that many students don't do well in critical reasoning doesn't mean that they would have failed a more rigorous course.

By analogy, consider high school algebra. Many floundering algebra students flounder because they are left with the impression that the rules they are taught are arbitrary. Lacking a formal basis for the subject, they suspect the laws of algebra were hammered out in committee after lengthy negotiations. Random rules (and to clueless algebra students, that's how they appear) are difficult to learn.

Interestingly, when Tom and I first began work on the course I am describing, it was the philosopher who yearned for symbolic logic and the mathematician who sought natural language argumentation. Both of us knew instinctively what we needed to be whole.

How far you go, either in formal logic or critical reasoning is negotiable, of course. The formal logic in our course includes predicate logic and spends four weeks on predicate deduction. Our informal logic trains students to read and diagram arguments, to rebut them, and to diagram, outline, and write their own arguments.

2.3 Client Departments

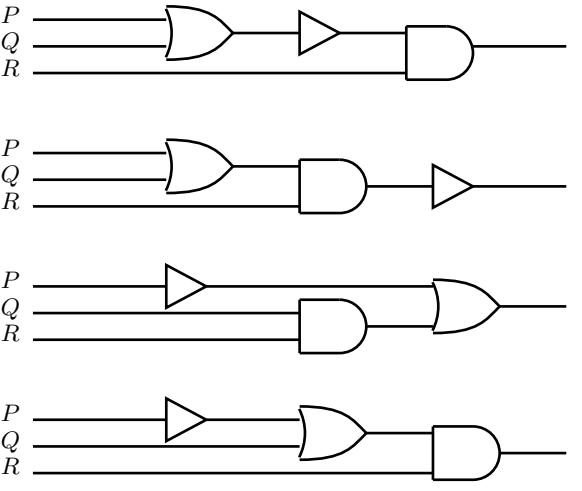
Computer scientists and linguists use logic professionally. It's important for students to understand that. It raises their struggles to a different plane. It shows them that logic is not just a game. It's serious stuff.

At the same time, each of these subjects gives students a different view of the same material, helping their understanding and confidence. We like to give them a matching problem that ties together language, computing, mathematics and logic. We give them four statements,

$$\neg(P \vee Q) \wedge R$$
$$(\neg P \vee Q) \wedge R$$
$$\neg P \vee (Q \wedge R)$$
$$\neg((P \vee Q) \wedge R)$$

to match with four logic circuits,

[2] United States Statutes at Large, Containing the laws and concurrent resolutions enacted during the second session of the ninety-ninth Congress of the United States of America.



to match with four English sentences,

"Either Harold won't come to the party or his aunt will come and it will be a great success."
"It's not true that Harold or his aunt will come to the party and it will be a great success."
"The party will be a great success and either Harold won't come or his aunt will."
"The party will be a great success but its not true that either Harold or his aunt will come."

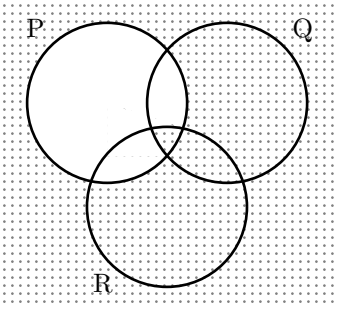
to match with four statements in Polish notation,

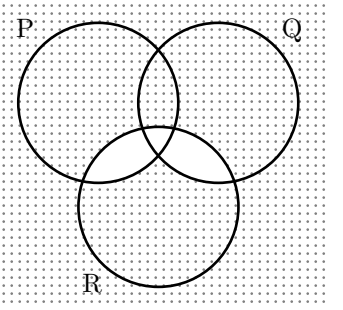
$$\neg \wedge \vee PQR$$
$$\wedge \vee \neg PQR$$
$$\vee \neg P \wedge QR$$
$$\wedge \neg \vee PQR$$

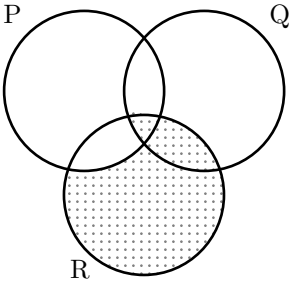
to match with four truth tables,

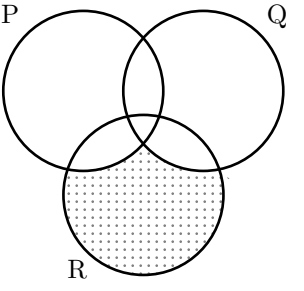
P	Q	R	$\mathcal{A}$	$\mathcal{B}$	$\mathcal{C}$	$\mathcal{D}$
T	T	T	F	T	F	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	F	T	F	T
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	F	T	T	F

to match, finally, with four Venn diagrams.









Students make the matches easily. The exercise convinces them that logical ideas are ubiquitous and recognizable in different contexts. We have a similar matching problem involving different ways to define an artificial language (rewrite rules, recursive definitions, finite state automata, and finite state acceptors).

It doesn't take much computer science or linguistics to have an effect. We always include both but each year we key on one of them.

2.4 Other Stuff and Lots of It

In the same spirit we include samples from the LSATs. We discuss advertisements. And we analyze presidential debates. It's surprising how many logical issues play out in debates.

"Now, Barack Obama, of course, he's pretty much only voted along his party lines. In fact, 96 percent of his votes have been solely along party line, not having that proof for the American people to know that his commitment, too, is, you know, put the partisanship, put the special interests aside, and get down to getting business done for the people of America."[3]

We discuss the scientific method. We discuss statistical reasoning. And we explore the use of logic in poetry.

[3] Sarah Palin, Vice-presidential debate, 2008.

"If this be error and upon me proved,
I never writ, nor no man ever loved."[4]

We use logic to clarify theological disputes. We use, for example, different sorts of mathematical infinity to illustrate that God is supreme, that God is everywhere, that God and Satan are polar opposites, that there is no god, that there are many gods, that God is omniscient, that God is omnipotent, and that God is so great as to be beyond human understanding.

All of this enhances student experience. It makes logic memorable. It tells them they are studying something vital.

2.5 Serious Fun

We tease our students. It's fun. It's legal.

Over the semester we toss them one paradox after another. Paradoxes amuse—but they also disturb. We write on the board,

Exactly one statement on this board is true.
$2 + 2 = 4$.
Jim Henle and Jay Garfield are the finest professors
in the history of the world.

and they lose sleep worrying about it. Some decide it is more likely that the second statement is false than that the third statement is true.

Paradoxes raise important philosophical questions, questions we pursue in class when we have the logical tools. For some students, and for us, paradoxes drive the course.

We also give the class pretty ordinary jokes and puzzles. By the end of the course it isn't easy for students to tell when joke is just a joke. That's part of the message.

One of our favorite events is to bring to life a character from our book. This is Cathy, an obnoxious junior who intimidates students with arguments for extreme positions, arguments that are difficult to rebut. Toward the end of the semester we start rumors that she has been seen on campus. Then one day she bursts in, abuses us liberally, and engages the class in an outrageous debate. It's a great scene.

2.6 Logic Outreach

Having given our students some powerful but elementary logical skills, we send them out to challenge and irritate their friends. Several times during the semester we take a statement, say, "This college should require all students to take a course in quantitative reasoning." We then ask each student to find someone outside the course to argue the point with. Our student must first ascertain their opponent's view on the issue and then take the opposite position.

[4] William Shakespeare, Sonnet 116.

We don't grade this exercise on our student's success, only on whether the argument takes place. We start with issues of local interest (as above) but later move to national, international or ethical questions. The last usually has some emotional content, which of course, students must manage to transcend.

At some time in the term we combine Logic Outreach with Serious Fun by playing a practical joke on the campus. We invent an issue. One year it was about selling paintings in the art collection to support scholarships. Of course, nothing of the sort was contemplated by the college.[5] The issues we choose are wholly fictitious.

For the practical joke, students are sent to chalk arguments on the sidewalks and scatter angry bursts on the internet. We start with one team opposing the chosen wild idea. The following day a different team rebuts the first. Two more teams follow in subsequent days with counter-proposals and counter-rebuttals.

Students have a lot of fun with this. They learn that no issue is beyond debate. They discover the pleasure of advocacy for its own sake. They experience the thrill of arguing for something ridiculous. All this is healthy.

As a class exercise, the chalking always works. Our record with the rest of the campus, however, is mixed. Sometimes we are gloriously successful in starting arguments. The proposition, "There shouldn't be male professors at a women's college" was one of our best. More recently the campus has caught on. A recent chalking campaign was dismissed with "It's just Logic 100 again." Last year we used a decoy argument.

2.7 Deep Thoughts

After all the above, there doesn't seem to be room for more. But we always save time for something special at the end. It's an important ingredient in the course. It motivates us and it motivates the students. The fact that at the close of the semester we are in a position to discuss ideas and issues at the frontier of logic is exciting. And when at last we take up advanced topics, it's a ratification of the class's progress. We're telling our students that they have achieved something significant.

The advanced topic depends intimately on the taste of the instructor. I'm a mathematician with a research interest in infinite sets. We tease the students throughout the semester with paradoxes of infinity. At the end we give them Cantor's diagonalization proof, construct infinite ordinals, and even discuss Hugh Woodin's latest work on the continuum problem.

Jay is a philosopher with a research interest in paraconsistent logic. We tease the students thoughout the semester with paradoxes of self-reference. At the end we give them a logic in which statements can be true, false, or both true and false. We find time at the end to discuss the latest work of Graham Priest and others on paraconsistent logics.

[5] We won't use this issue again—it's not a joke anymore. Brandeis University seriously proposed selling their collection.

2.8 Syllabus

This is what we cover on the formal side:

1. Sentential or propositional logic, truth tables and logic circuits
2. Validity and inference in sentential logic
3. Defining Sentential language recursively
4. Predicate logic, universes, validity, and inference
5. Defining Predicate recursively; syllogisms
6. Deduction in sentential and predicate logic with identity

On the informal side we cover:

7. Analyzing arguments, identifying conclusions, diagramming logical structure
8. Rebutting premises, rebutting inferences
9. Diagramming your own argument
10. Outlining and writing your argument
11. Argument in debate

In a typical semester we will also cover either the following from linguistics,

12. Rewrite rules
13. Finite state automata
14. Regular languages
15. Finite state acceptors

or the following from computer science,

16. Binary
17. Stacks
18. Prolog
19. The busy beaver problem
20. The halting problem

but in any case we will definitely take up these from set theory,

21. Countability
22. Uncountability
23. Defining numbers from sets
24. Ordinals and ordinal arithmetic

and these on alternative logics:

25. Polish notation
26. Many-valued logic
27. Probability and inductive logic
28. Modal logic
29. Paraconsistent logic.

2.9 Too Much Material

It appears that there is too much in our course. It covers the equivalent of two, perhaps three ordinary courses: a critical reasoning course, a course in formal logic, a survey course in the logic of computer science, linguistics, philosophy, and mathematics, with topics in cardinality, modal logic, possible worlds, and paraconsistent logic. Students can't possibly master it all. Students will certainly forget almost all they learn. There seems to be no point to it.

But there is a point. I'll explain.

3 What This Course Does

This is not so much a course *in* logic but a course *of* logic. The goal is not for students to master content; it's for them to become logical. We want students to recognize logic when they see it, and to note its absence when it's missing. The course is an experience. Every topic, every puzzle, every game, every paradox adds to the experience. In the course, logic happens. It changes the students.

Students do forget the material. Even the best, the ones who become teaching assistants, lose their grasp of some details. But the students grow, logically. They emerge ready for the next course in logic. What they don't recall they can retrieve quickly. And with their logical reflexes they can deal with new logical challenges.

The students are better prepared not just for logic courses, but for math courses, philosophy courses, economic courses—courses in all fields. Students who have passed through logic are quicker, tougher, more resilient. They're smart and they know it.

They're also better writers. They know they have to have something to say. They (some of them) will start their history papers by diagramming and outlining their arguments.

And they're better readers. They know that the English language has traps. They're alert. They're wary.

Isn't this really what we want for our students? These are the true basic skills. Today's employers want smart, knowledgeable people. But smart is far more important than knowledgeable. Smart people can be trained, they can learn what they need to know. But it's challenging to make your employees smart.

Jobs disappear. Industries collapse or move abroad. Paradigms change. Our students can only prepare for this by getting smartness. They get it by studying Logic.

3.1 It Meets Expectations

We tell our students that they will learn a ton of stuff, that they will get smart, that they will join an elite group of students. That sort of happens. Nearly all complete the course. They don't learn everything but they end up smarter and feeling special.

One reviewer for our book looked at the material (some but not all of what I am suggesting here) and said that it was far too much, that maybe Harvard students could handle half of it.

We cover all of the book and additional material we put on the web. And Smith is not Harvard. Our students are not special. Some are good, but about half of them are in their first semester at college. Others are senior philosophy majors who, fearing logic, have put it off as long as they could. A few are women of non-traditional age, returning to finish their degree. Some students struggle, but the teaching assistants keep them going.

Students respond to high expectations. Ambitious goals are critical to the course's success. We set out to move our students a great distance. It seems to work.

3.2 It Grows Logicians

At Smith and in the Five Colleges Consortium (Smith, University of Massachusetts, Amherst College, Hampshire College, and Mount Holyoke College), this course has been the engine of a strong logical community. The best of our introductory students are asked to be teaching assistants the following year. The most interested of our students take follow-up courses in incompleteness, relevance logic and set theory. The most committed students elect to minor in logic. The most obsessed major in it.

At any time we generally have one or two majors and four or five minors. The minor is on the books but students who wish to major in logic must construct a "self-designed major" to satisfy Smith's graduation requirements. The major they design can focus on philosophy, mathematics, computer science, linguistics, or even law. All logic majors are expected to take courses from several of these fields.

There are logicians at each campus of the Five Colleges. Together we have a logic program that offers a certificate to any undergraduate completing certain course requirements. Earning the certificate requires taking at least six logic courses. We are turning out logicians.

As part of Smith's logic program and in support of the larger logical community, Smith sponsors the annual Alice Ambrose Lazerowitz/Thomas Tymoczko Logic Lecture. This is timed to coincide with the end of the introductory logic course so that students are ready to understand at least half of the talk.

3.3 It Opens the Campus to Debate

Well, perhaps it does. There is no way to measure the success of our logic outreach efforts. We hope students feel more comfortable trying on positions. We have no way of knowing if they do.

3.4 It Is Immeasurably Successful

Or, to put it another way, there is no way to measure its success. Content is not the goal, the goal is smartness. I'm not convinced that smartness can measured. As a mathematician, I have a deep distrust of numerical measures of intelligence.

The only measures I see as meaningful here are the number of students in the course, the number who take a second course, and the number who continue on to major or minor in logic. Students know when they are doing something worthwhile. They may give a course high marks on evaluation sheets, but their decision to continue is more significant and sincere.

Enrollment in the course is high, three to four times higher than it was before Tom and I began teaching together. Students elect the course despite the difficulty and the workload. Many take a second course but we haven't conducted a study. There are three logic majors at the moment with two more at the planning stage. There were none before the program began.

3.5 It Pleases the Administration

The course is highly regarded by our administrators because it addresses so many basic student skills. One could almost describe the course as "reading, writing, and 'rithmetic."

We teach the students to diagram arguments they read. This is, in a sense, advanced reading. It's surprising how difficult it is for students to identify the conclusion of a typical letter to the editor of a paper. Diagramming an argument requires understanding what was in the mind of the writer. That is problematic when the writer is not skillful.

The critical reasoning portion of the course clearly teaches writing skills. We don't spend time on bibliographic concerns, spelling, or grammar (except as it affects the logic of a sentence). We don't ask students to write long papers. Despite that, our course satisfies Smith's writing requirement because it addresses the most critical of writing skills: composing a strong argument and conveying that argument to the reader.

The claim that we teach arithmetic is perhaps harder to justify. The course doesn't address quantitative skills, except for a little time spent on probability. But the course satisfies the college's recommendation for analytic reasoning.[6] The other courses with this distinction are, with the exception of a few philosophy courses, all in mathematics or computer science.

Our dean isn't disturbed by the fact that two professors are being used to teach one course. We attract a large number of students, well more than twice the average number of a course at Smith, and more than five times the size of classes satisfying the writing requirement.

About 15% of Smith students take logic, many of them in their first semester. The course does a lot to teach them how to be a successful student.

3.6 It Pleases Me

In the end, one can never be sure what students will appreciate. Since this is so, my guiding principle has been to fashion a course for my own satisfaction,

[6] Smith has no distribution requirement, but students will not be awarded Latin honors (*cum laude*, *magna cum laude*, etc) unless they take courses in a list of areas, one of which is "analytic reasoning".

hoping that coincidentally it will satisfy my students. I believe this has worked; I believe the course does please our students. But unquestionably the course pleases *me*.

I can do whatever I want. I can justify the inclusion of anything in the course. After all, in every field you find either

a. the presence of logic—in which case it makes sense to discover and analyze that presence,
   or
b. the absence of logic—in which case it makes sense to discover why this is so and how the addition of logic might change things.

The list of topics I have explored in the course is pretty long: economics, politics, law, rhetoric, theology, history, the tax code, comic opera ... The list of stunts I have tried is also pretty long. Some of them I'd rather not discuss here. The attempt to do Monty Python was better than the attempt to do Abbott and Costello. The episodes with the time machine had mixed success.

4 To Sum Up

Logic is special. What makes logic special, what makes it so thrilling and vital, is its cosmic nature. Logic's reach is global. Logic is at the core of all intellectual activity. A new logical truth reverberates up and down the centuries. When you advance logic, you advance understanding in every corner of the intellectual universe.

That being so, the introduction to logic should be special. It should include everything. It should reflect the subject's majesty—and its whimsy.