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Classical Syllogisms in Logic Teaching

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Abstract. This paper focuses on the challenges of introducing classical syllogisms in university courses in elementary logic and human reasoning. Using a program written in Prolog+CG, some empirical studies have been carried out involving three groups of students in Denmark; one group of philosophy students and two groups of students of informatics. The skills of the students in syllogistic reasoning before and after the logic courses have been studied and are discussed. The empirical observations made with the program make it possible to identify syllogisms which are found difficult by the students, and to identify others which the students find easier to handle. It is discussed why certain syllogisms are more difficult than others to assess correctly with respect to validity. The results are compared with findings from earlier studies in the literature. As in other studies, it is shown that the test persons have a tendency correctly to assess valid syllogisms as such more often than correctly assessing invalid syllogisms as such. It is also investigated to what extent the students have improved their skills in practical reasoning by attending the logic courses. Finally, some open questions regarding syllogistic reasoning are discussed.

1 Introduction

For centuries the Aristotelian syllogisms have been a crucial part of university courses introducing basic logic and human reasoning. In the medieval universities, syllogistics was regarded as an essential component of basic academic learning. At many modern universities this is still the view.

There is, obviously, a close relation between the ontological primitives (e.g., SubclassOf) and the categorical statements which are used in classical syllogistics. In fact, a number of syllogistic arguments can be inferred from the hierarchical structures used in formal ontology (see [4]). It appears to be evident that a proper understanding of conceptual structures in many cases will depend on the ability to handle basic syllogistic arguments correctly. Based on such observations syllogistic reasoning should still be considered to be an important prerequisite for the understanding of conceptual structures and indeed for science in general. For mathematics, syllogistics can be said to form part of the foundation for mathematics as such. For engineering, a basic knowledge of syllogistics could enhance the potential for systematic reasoning about the artefacts being constructed. For this reason, it is our view that students working with science, technology, engineering, and mathematics – as well as students in the humanities – should be introduced to these basic forms of logic and reasoning.

From a modern point of view classical syllogistics may be seen as a fragment of first order predicate calculus. A classical syllogism corresponds to an implication of the following kind:

$$(p \wedge q) \supset r$$

where each of the propositions p, q, and r matches one of the following four forms

$\mathrm{a}(X, Y)$	(read: "All X are Y")
$\mathrm{i}(X, Y)$	(read: "Some X are Y")
$\mathrm{e}(X, Y)$	(read: "No X are Y")
$\mathrm{o}(X, Y)$	(read: "Some X are not Y")

These four functors were suggested by the medieval logicians referring to the vowels in the words "affirmo" (Latin for "I confirm") and "nego" (Latin for "I deny"), respectively. The classical syllogisms occur in four different figures:

$(x(M,P) \wedge y(S,M)) \supset z(S,P)$	(1st figure)
$(x(P,M) \wedge y(S,M)) \supset z(S,P)$	(2nd figure)
$(x(M,P) \wedge y(M,S)) \supset z(S,P)$	(3rd figure)
$(x(P,M) \wedge y(M,S)) \supset z(S,P)$	(4th figure)

where x, y, $z \in \{\mathrm{a, i, e, o}\}$ and where M, S, P are variables corresponding to "the middle term", "the subject" and "the predicate". In this way 256 different syllogisms can be constructed. According to classical (Aristotelian) syllogistics, however, only 24 of them are valid. The medieval logicians named the valid syllogisms according to the vowels, $\{\mathrm{a, i, e, o}\}$, involved. In this way the following artificial names were constructed (see [1]):

1st **figure:** barbara, celarent, darii, ferio, <u>barbarix</u>, <u>feraxo</u>
2nd **figure:** cesare, camestres, festino, baroco, <u>camestrop</u>, <u>cesarox</u>
3rd **figure:** <u>darapti</u>, disamis, datisi, <u>felapton</u>, bocardo, ferison
4th **figure:** <u>bramantip</u>, camenes, dimaris, <u>fesapo</u>, fresison, <u>camenop</u>

In these names the consonants signify the logical relations between the valid syllogisms, and they also indicate which rules of inference should be used in order to obtain the syllogism in question from syllogisms which were considered to be fundamental: barbara, celarent, darii, and ferio. – In fact, the system of syllogisms may in this way be seen as the first axiomatic system ever (see [1] and [3]). According to Aristotle a universal statement concerning an empty term cannot be true. For this reasoning an a-proposition must imply the corresponding i-proposition. This view was rejected, when philosophers and mathematicians began to pay more attention to the idea of an empty set. Without this Aristotelian view the number of valid syllogisms was reduced to 15 (leaving out the 9 syllogisms which have been underlined in the above list, i.e. the syllogisms whose names contain either an x or a p).

The conceptual structures which form the foundation of the experiment conducted in this paper, may be illustrated by reference to the classical hierarchy of categorization depicted in Figure 1.[3]

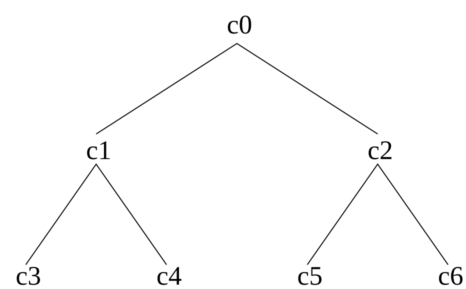


Fig. 1. A classical hierarchy of categorization.

The relation between the concepts in this figure can all be described in terms of the Aristotelian propositions which are used in syllogistics. For example:

- All c5 are c2
- Some c0 are c1
- No c1 is c2
- Some c0 are not c1

Even the properties of the IsA-relation can be said to correspond to the syllogistic logic. One very simple example would be the transitivity of this relation, which turns out to be equivalent with the syllogism called barbara, 1st figure. For example:

All c3 are c1
All c1 are c0
Ergo: All c3 are c0

[3] It should be noted that in this paper, when we capitalize "Figure", we refer to an illustration within the text. When, however, we do not capitalize "figure", we refer to the four figures of the Aristotelian syllogisms.

For this reason, it seems obvious that the proper conception of the hierarchical structure of ontologies depends on a proper understanding of the logic embedded in classical syllogistics.

2 Data and Method

The data analysed in this study have been obtained using the system Syllog (described in detail in [17]), which has been implemented using an extended version of Prolog+CG (see [5–7]). Syllog presents the user with an arbitrary syllogistic argument and asks him or her to evalute the syllogism that appears on the screen as to its Aristotelian validity. The arbitrary arguments are generated by Syllog with S belonging to the set {"swedes","politicians","dentists"}, P belonging to the set {"halffools", "halfminded","redheaded"}, and M belonging to the set {"thieves", "crackpots", "fools"}. Syllog picks an arbitrary figure number (1-4) and an arbitrary number (1-12). The latter number corresponds to a particular syllogism within the chosen figure. The numbers, (1-6), point to the valid syllogisms listed above, whereas the numbers (7-12) stand for invalid syllogisms assumed to be somewhat "tempting":

1st **figure:** aia, oae, iai, ieo, iii, oao
2nd **figure:** oae, aoe, oio, ioo, ieo, oao
3rd **figure:** aaa, iaa, aia, eae, oae, eie
4th **figure:** aaa, aie, iaa, eae, eie, aoe

Three groups of students in Denmark have been involved in the tests:

1. Two groups of second year University students in informatics (one in Copenhagen and one in Aalborg)
2. One group of first year University students in philosophy (in Aalborg).

All three groups were going to attend a basic course in logic – including Aristotelian syllogistics. The course was based on course material in Danish (corresponding to parts of [1] along with elements from [2]). The students were taught in basically the same way, to the same extent, and to a large extent using the same course material. The teacher of the Aalborg Informatics students also taught the course for the Philosophy students.

The students were asked to run Syllog individually or in groups of 2-4 both before and after their logic course (i.e. pre- and post-test). All these test results have been logged by Syllog.

The statistical analyses of the data were performed using standard methods from descriptive statistics and statistical testing. The chi-square test is applied to detect group differences using frequency (count) data, and also to look for significant differences between results from the pre-test and the post-test. We compared the pre-test results of the informatics-students in Copenhagen and in Aalborg, and found no significant differences between the two groups (p-value = 0.25, data not shown).

Table 1. The three 2x2 tables below summarize counts of how often students replied correctly and incorrectly when presented with valid and invalid syllogisms. The first table is for the pre-test of the whole student group, while the second and third table shows the same data separated in two subgroups. All three tables support strong statistical evidence against the presumption that student will handle valid and invalid syllogism equally well (p-values $< 10^{-5}$ by the two-sided chi-square test).

	Correct reply? (all students, n=174)		Correct reply? (philosophy students, n=33)		Correct reply? (informatics students, n=141)	
Syllogism	Yes	No	Yes	No	Yes	No
Valid:	814	318	307	80	507	238
Invalid:	697	538	283	179	414	357

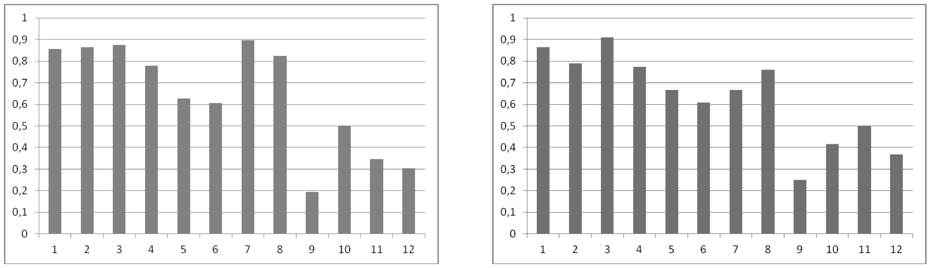
Table 2. Two 2x2 tables summarizing counts of how often students replied correctly and incorrectly in the pre-test and in the post-test. The first table is for the philosophy students, whereas the second table shows the results for the Copenhagen group of informatics students.

	Correct reply? (philosophy students, n=33/n=17)		Correct reply? (informatics students, n=39/n=42)	
Test	Yes	No	Yes	No
Pre:	590	259	382	229
Post:	175	46	486	306

3 Results

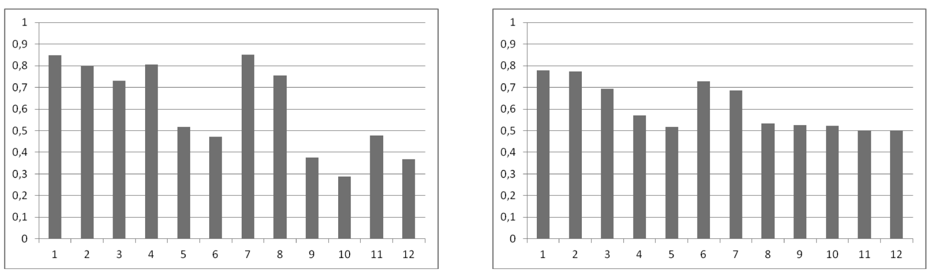
The first online tests were carried out with all the students as a pre-test before their lessons in classical syllogistics. The students answered the exercises individually or in groups of 2-4. If less than 3 exercises were answered by a given individual or group, the record was excluded from the analysis to avoid data that were influenced by technical problems or unserious students. The final data from the pre-test consists of 2365 evaluated syllogisms, from n=174 groups (or individuals), with an average of 13.6 answered exercises. The results of the pre-tests are shown in the (a) parts of Figures 2-5 and in Table 1.

The group of informatics students in Copenhagen and the philosophy students in Aalborg also took part in a post-test under conditions similar to the pre-test. For the informatics students the post-test was performed four months after the pre-test, but before the exam, and for the philosophy students the interval between the tests was five weeks. The (b) parts of Figures 2-5 show the results obtained by the post tests, and Table 2 summarizes the results obtained by the pre- and post-tests. For the group of informatics students in Aalborg, the post-test were supposed to be done voluntarily at home, and only 6 students completed it, hence their results are left out.



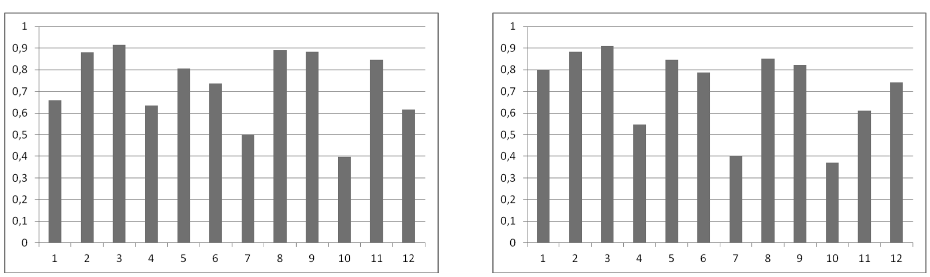
(a) (b)

Fig. 2. Pre-test score (a) and post-test score (b) of syllogisms in figure 1. The first 6 syllogisms are valid according to Aristotelian syllogistics, whereas the 6 other syllogisms are invalid. It should be noted that the scores of syllogism no. 9 are very low (see the discussion in Section 4).



(a) (b)

Fig. 3. Pre-test score (a) and post-test score (b) of syllogisms in figure 2. The first 6 syllogisms are valid according to Aristotelian syllogistics, whereas the 6 other syllogisms are invalid.



(a) (b)

Fig. 4. Pre-test score (a) and post-test score (b) of syllogisms in figure 3. The first 6 syllogisms are valid according to Aristotelian syllogistics, whereas the 6 other syllogisms are invalid. Note that syllogism no. 3 has got very high scores (see the comments in Section 4).

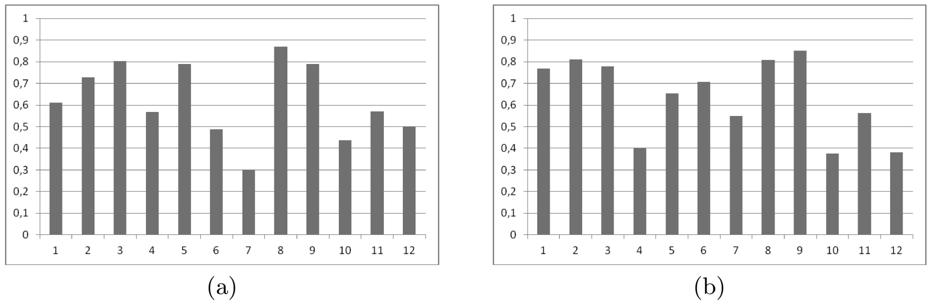


Fig. 5. Pre-test score (a) and post-test score (b) of syllogisms in figure 4. The first 6 syllogisms are valid according to Aristotelian syllogistics, whereas the 6 other syllogisms are invalid. Because of a minor programming error the results regarding syllogism 12 cannot be expected to be correct.

The student dropout is more than 50% in the post-test of the left part of Table 2, and hence the results are only indicative with p-value = 0.004 by the two-sided chi-square test.

The right part of Table 2 provides no significant evidence against the hypothesis that informatics students did not obtain better skills in syllogistic reasoning during the logic course (p-value = 0.66).

4 Discussion of the results

The abilities of performing syllogistic reasoning have been studied earlier using other methods (see [9–16]). Our data can to some extent confirm the findings in these earlier studies. In addition, the present study also allows some new conclusions. The results in Table 1 show that students more often wrongly agree with invalid syllogisms than they wrongly disagree with a valid syllogism. The reason may be that the students find it more natural to agree with a difficult argument, than to disagree. In other words, it may be more natural for the human nature to be positive than to be negative. In this way there seems to be "a belief bias" in syllogistic reasoning (see [16]). The results listed in Table 2 may seem somewhat surprising. It seems that the philosophy students have in fact improved their skills in syllogistic reasoning during the course whereas there is no evidence that informatics students have made similar progress during their course. It is not clear how this difference should be explained. However, several comments and possible explanations may be considered. First of all, the philosophy students in general may see it as fascinating to reflect on the Aristotelian ideas and the notion of human reasoning as such, whereas the informatics students may not find the study and elaboration of Aristotelian syllogisms particularly interesting. In addition, it may be important that the philosophy students had their post-test immediately after the logic course, whereas the informatics students had to wait longer for their post-test. Furthermore, it should also be noted that right from the beginning the philosophy students are clearly better when it comes to syllogistics than the informatics students. However, it is obvious that even the informatics students have skills in syllogistic reasoning without having been taught any logic as such.

According to the data in this study, the syllogism with lowest score (in the pre-tests as well as in the post-tests) is syllogism number 9 in figure 1 (see Figure 2, both its (a) and its (b) parts). This is an invalid syllogism of the following form:

$$(i(M, P) \wedge a(S, M)) \supset i(S, P)$$

In Syllog the values of M, S and P are selected arbitrarily from certain fixed sets as mentioned above. The users may, for instance, be presented with the syllogism in the following way:

Some crackpots are redheaded
All Swedes are crackpots
Ergo: Some Swedes are redheaded

A majority of users have mistakenly evaluated this syllogism as valid. This error may have occurred because the students have not fully understood the difference between "All Swedes are crackpots" and "All crackpots are Swedes". The syllogism

Some crackpots are redheaded
All crackpots are Swedes
Ergo: Some Swedes are redheaded

is clearly valid. It is a disamis (in the 3rd figure). The difference between the two syllogisms can be made clear in terms of Euler circles. The point is that the information in the premises of the iai-syllogism in the 1st figure may correspond to the diagram in Figure 6(a). Here it is obvious that $i(S, P)$ cannot be concluded.

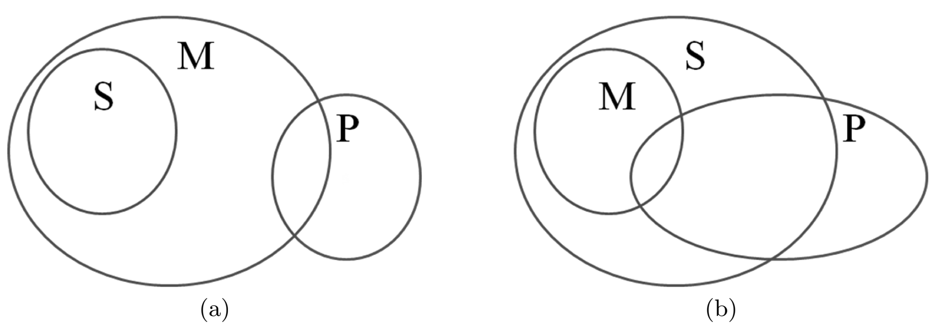


Fig. 6. Euler circles consistent with: (a) the information in the premises of the iai-syllogism in the 1st figure, and (b) the information in the premises of disamis in the 3rd figure.

In disamis (in the 3rd figure), however, the information contained in the premises will have to be represented otherwise, as in Figure 6(b). Here, $\mathrm{i}(S, P)$ obviously follows from the premises. If it is known that M and P have elements in common, and M is a subset of S, then it is evident that S and P must also have elements in common. It seems obvious that if the student immediately had been able to visualize the graphical relation between M and S when being given the premise (i.e. $\mathrm{a}(S, M)$)), then they would not have taken the iai-syllogism in the 1st figure to be valid. This leads to a strong emphasis on the importance of the use of graphical representation in logic courses.

There are also interesting questions to ask concerning the data of the 24 syllogisms which are valid from an Aristotelian point of view. First of all one may focus on the syllogism which has got the highest score taking both pre-tests and post-tests into account, i.e. datisi in the 3rd figure:

$$(\mathrm{a}(M, P) \wedge \mathrm{i}(M, S)) \supset \mathrm{i}(S, P)$$

Intuitively, it seems obvious that this structure is a valid syllogism: If all M's are P's, and if some M's are S's, the clearly some S's must be P's. However, disamis in the 3rd figure seems just as obvious:

$$(\mathrm{i}(M, P) \wedge \mathrm{a}(M, S)) \supset \mathrm{i}(S, P)$$

In fact, datisi and disamis in the 3rd figure have got also the same score.

Another interesting question regarding the valid syllogisms has to do with the possible difference between the two subgroups of 15 and 9 syllogisms mentioned above. Given that the empty set is an integrated idea in modern thinking, it seems to be straight forward to expect that the 15 syllogisms would have a higher score the 9 Aristotelian syllogisms which have been questioned in modern logic. For this reason we have compared the scores for the 15 syllogisms which all modern logicians accept (i.e. figure 1 no. 1,2,3,4 & figure 2 no. 1,2,3,4 & figure 3 no. 2,3,5,6 & figure 4 no. 2,3,5) with the 9 syllogisms whose validity some modern logicians would question (i.e. figure 1 no. 5,6 & figure 2 no. 5,6 & figure 3 no. 1,4 & figure 4 no. 1,4,6). We compared the group of these 9 versus the 15 other valued syllogisms and the results appeared as shown in Table 3.

Table 3. Two 2x2 tables summarizing counts of how often students replied correctly to the subgroup of 15 and 9 valid syllogisms. The first table is for the pre-test, whereas the second table shows the results for the post-test.

	Correct reply? (Pre-test n=174)		Correct reply? (Post-test n=65)	
Syllogism	Yes	No	Yes	No
Valid in modern syllogistics	552	126	260	71
Questions in modern syllogistics	262	192	125	70

Table 4. Two 2x2 tables summarizing counts of how often students replied correctly to the subgroup of 15 and 9 valid syllogisms. The first table is for the pre-test, whereas the second table shows the results for the post-test.

	Correct reply? (Valid n=174)		Correct reply? (Invalid n=174)	
Syllogism	Yes	No	Yes	No
Asymmetric (figures 1 and 4)	401	160	338	288
Symmetric (figures 2 and 3)	413	158	359	248

Table 3 shows data obtained in the pre-test of the whole student group, and also shows the data from the post-test. Both parts of the table support strong statistical evidence against the presumption that student will handle the two subgroups of valid syllogism equally well (p-value $< 10^{-5}$ and p-value $= 2 \times 10^{-4}$, respectively, by the two-sided chi-square test). However, we may also observe that the replies given to the subgroup of questioned syllogisms are still significantly different from random where we would expect equally many correct and not-correct answers (p-values 0.02 and 0.007 in the pre-test and post-test, respectively).

The results by Johnson-Laird and Bara [15] suggest that test persons will obtain a higher score for asymmetric syllogisms (figure 1 and 4) than for the symmetric syllogisms (figure 2 and 3). Our results based on the pre-test replies and listed in Table 4 do not confirm this suggestion. None of these tables support significant evidence against the presumption that students will handle the two types of syllogisms equally well (p-value = 0.75 and p-value = 0.068, respectively, by the two-sided chi-square test). – It should, however, be mentioned that our setup differs from the setup used in Johnson-Laird's and Bara's experiment in several respects, in particular regarding the temporal setup. So although their test seems to be analogous to ours, there may also be significant differences in the setup and methods used in the two experiments that can explain the deviation in the results. It is, however, an open question exactly how the discrepancy between their results and ours should be explained.

5 Future Research Agenda

As we have seen some syllogisms (e.g. number 9 in figure 1) have got very low scores in the present study. It is, however, an open question exactly which syllogisms are conceived as the most difficult to handle. Various hypotheses may be considered as potential answers. In order to investigate these hypotheses, more empirical studies will be needed. Looking at the data at hand there are indications of trends worthy of further investigation:

1. Syllogisms consisting of only existential functors, i.e. i- and o-functors (figure 1, syll. 11, figure 2, syll. 9 & 10) are among the syllogisms with the lowest rates of correct answers. Less than 40% of the students answer them correctly. The average rate of students answering these correctly in the pre-test

is 0.49 compared to an average of 0.65 for all syllogisms. Of the syllogisms consisting of only universal functors, i.e. a- and e-functors (figure 1, syll. 1, 2, figure 2, syll. 1, 2, figure 3, syll. 7, 10, figure 4, syll. 2, 7, 10), the students do really well in the first four cases with 80% or more answering them correctly. These figures could indicate that syllogistic reasoning involving universal functors, i.e. a- and e-functors, are less hard than syllogistic reasoning involving existential functors, i.e. i- and o-functors. The current dataset does not allow any definite conclusions on this issue. In future research it would thus be of interest to test more systematically:

   – For each of the figures how well syllogisms consisting of only a- and e-functors or only i- and o-functors are handled by the students.

2. Syllogisms consisting of only affirmative functors, i.e. a- and i-functors (figure 1, syll. 1, 3, 5, 7, 9, 11, figure 3, syll. 1, 2, 7, 8, 9, figure 4, syll. 1, 3, 7, 9) are generally handled well by the students. Only in three cases (figure 1, syll. 9, 11, figure 4, syll. 7) the rate of correct answers drops significantly below 50%. Unfortunately no syllogisms consisting of only negative functors, i.e. e- and o-functors, are included in the current study. In order to more systematically clarify the role of affirmative and negative functors in the reasoning of the students it would be of interest to investigate:

   – For each of the figures how well are syllogisms consisting of only a- and i-functors or only e- and o-functors handled by students?

   Note that research questions (1) and (2) may be combined such that the testing is directed at clarifying the role of quantification and affirmation/negation at the same time. This then requires testing of syllogisms consisting of only a-, e-, i- or o-functors.

3. One of the alleged benefits of formalization is a greater transparency of the logical structure of an argument. In the current study the syllogisms presented for the students are made up of natural kind terms such as "swedes", "politicians", "dentists", "halffools" etc., and the functors are expressed in natural language, e.g. the a-functor as "All ... are ...". The current study does not provide any data that may serve to determine the role of formalization for the ability to reason logically. Syllog may, however, easily be modified to accommodate such research interests. All that is required is for the post-test to be conducted with an on-screen presentation of the syllogisms in their basic form, i.e. with premises and conclusion in the form of e.g. a(S, P) etc. It seems as if the clarification of the role of formalization for the ability to analyse and determine the validity of arguments may have some important implications for the teaching of logic.
4. It would be interesting to measure the effect of not only formalizing the syllogisms being quizzed, but drawing diagrams in terms of Euler circles, Venn Diagrams, or Existential Graphs to support the student in deciding the validity of a certain syllogism. Measurements could be taken with or without the support of the diagrams, and comparisons could be made between groups answering with and without such support. This might lend support to the argument for the use of diagrammatic reasoning in teaching logic.

5. It should be noted that in the present study the students have been free to use as much time as they wanted on each evaluation. Since the evaluations of the syllogisms may depend on the temporal conditions given during the test, it would be interesting to investigate whether the results would be significantly different, if the students were asked to make a fixed number of evaluations within a given time limit. It may even be interesting to study how the results would be under mild stress.
6. Finally, it could also be interesting to carry out an experiment using our technique as closely as possible to the experiment described [15] about symmetric and asymmetric syllogisms, in order to investigate whether we can conform the findings in [15] regarding symmetric and asymmetric syllogisms.

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