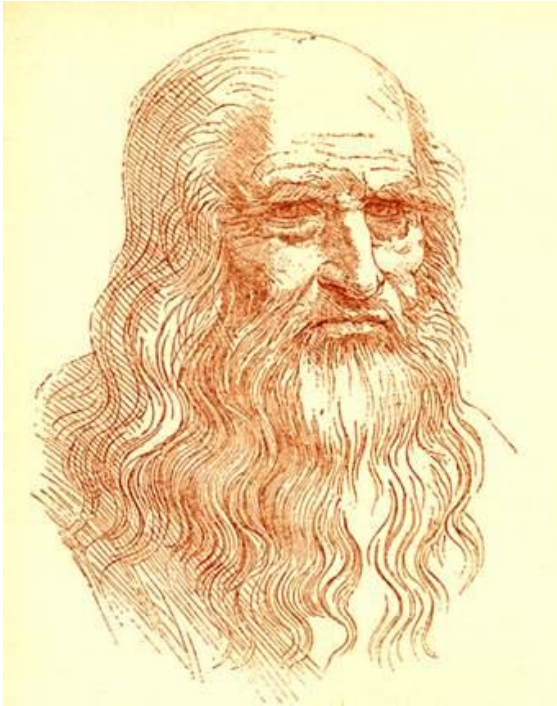


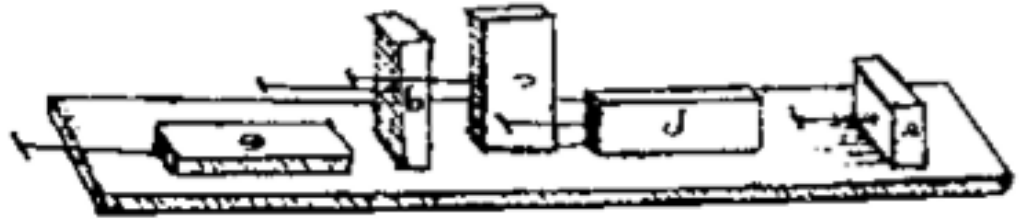
# Brittle deformation and Faulting

Goal: To understand relationships between brittle failure, stresses, and fault orientation.

# Frictional Strength



Question Da Vinci asked: Given that all objects shown below are of equal mass and identical shape, in which case the frictional force is greater?



Leonardo Da Vinci (1452-1519) showed that the friction force is independent of the geometrical area of contact.

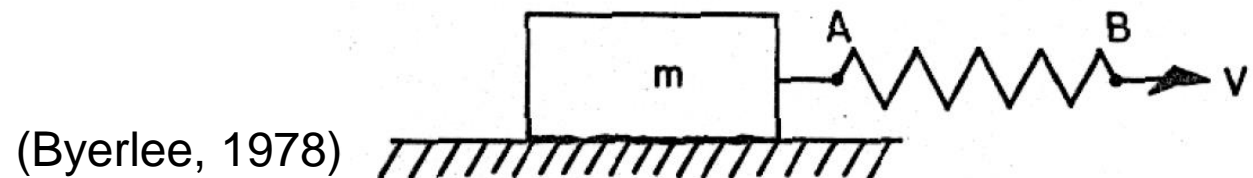
# Frictional Strength

- Amontons' first law: The frictional force is independent of the geometrical contact area.
- Amontons' second law: The friction force,  $F_S$ , is proportional to the normal force,  $F_N$ :

$$F_S = \mu F_N$$

$$\sigma_s = \mu \sigma_n$$

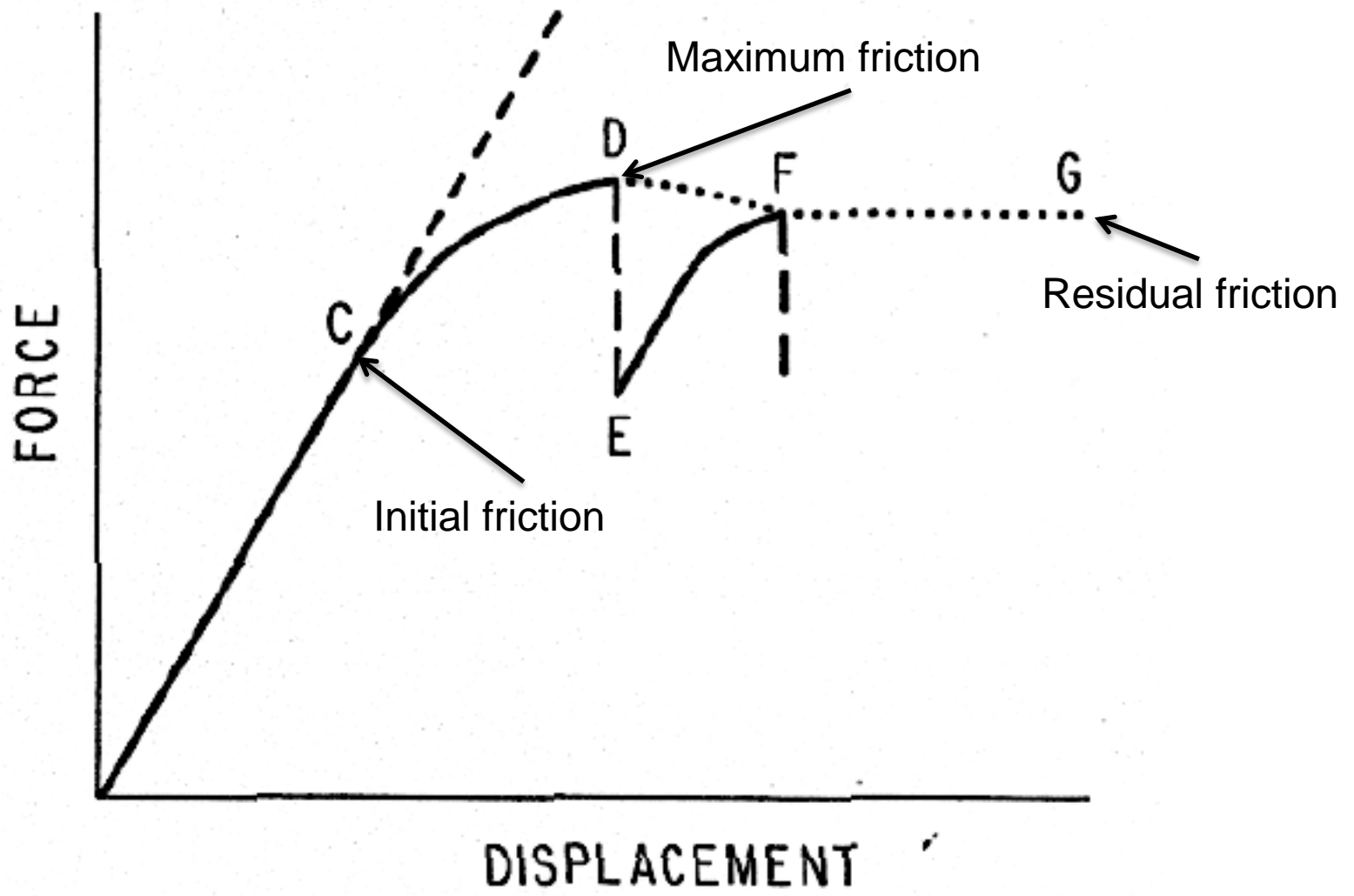
$\mu$ : static coefficient of friction



(Byerlee, 1978)

Figure 1  
Schematic diagram of a typical friction experiment. For explanation see text.

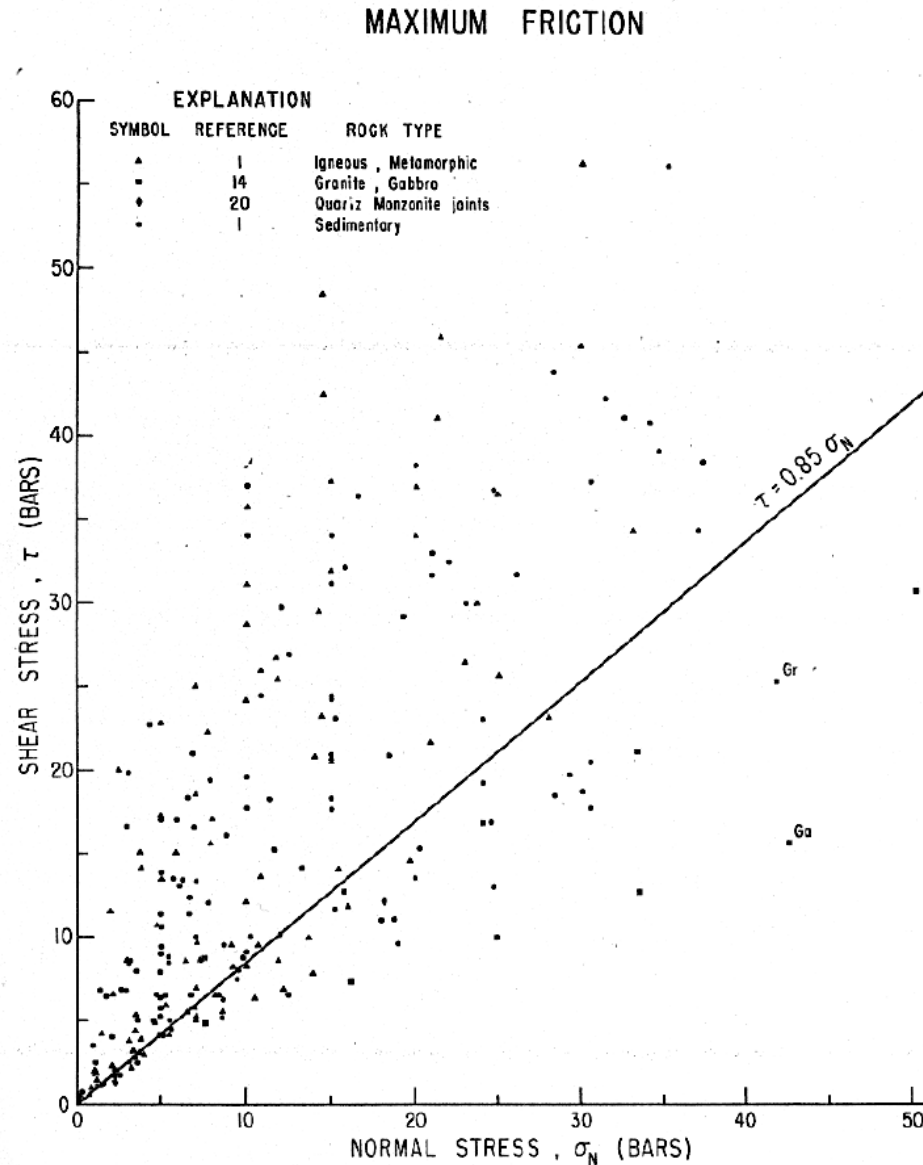
# Frictional Strength



(Byerlee, 1978)

# Frictional Strength

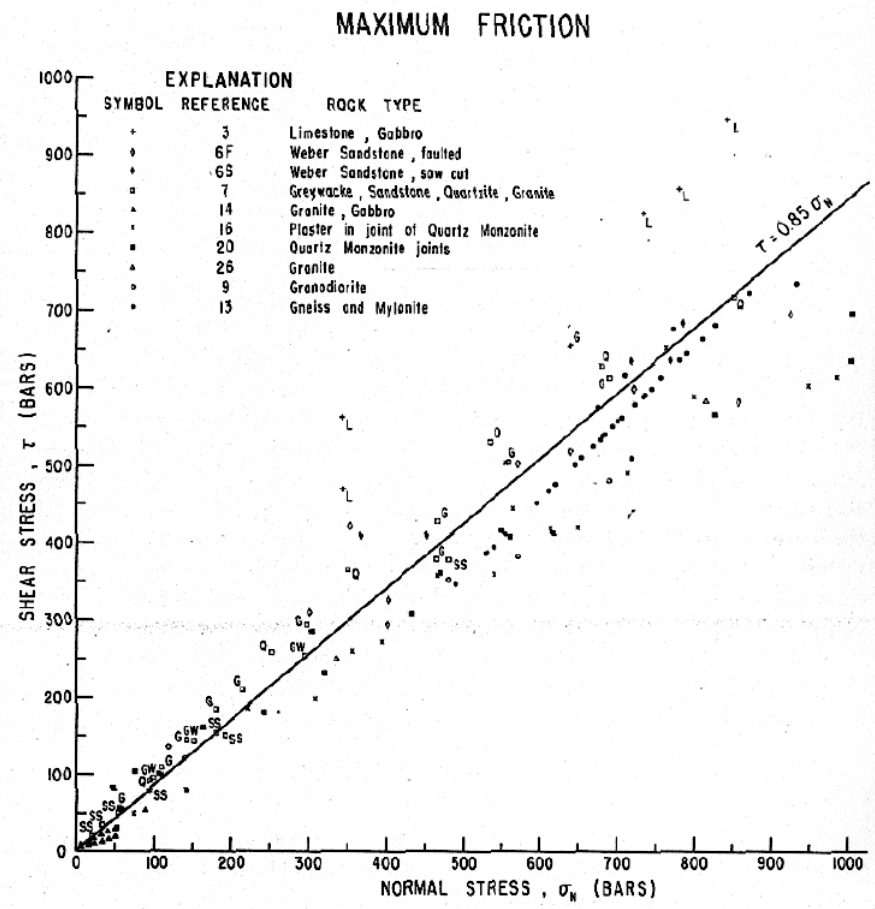
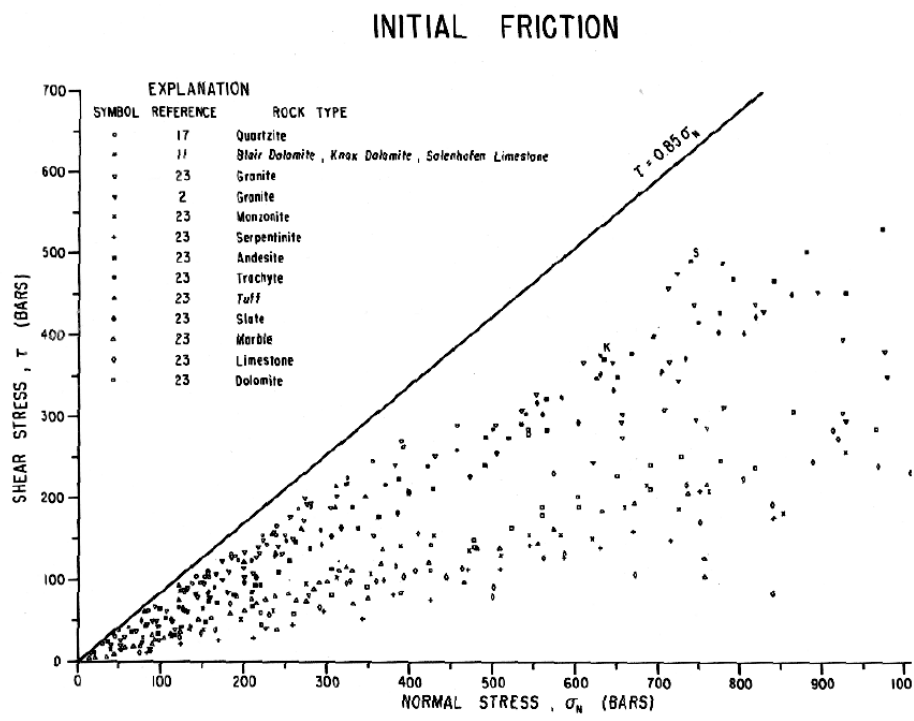
< Maximum friction for normal stress up to 50 bar >



(Byerlee, 1978)

# Frictional Strength

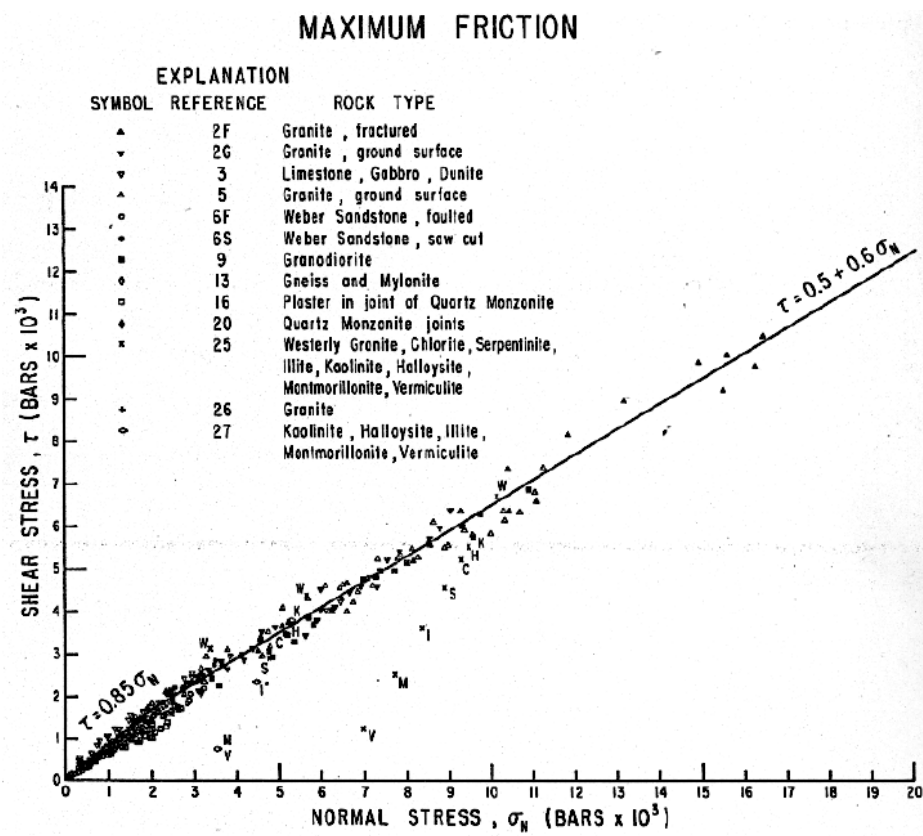
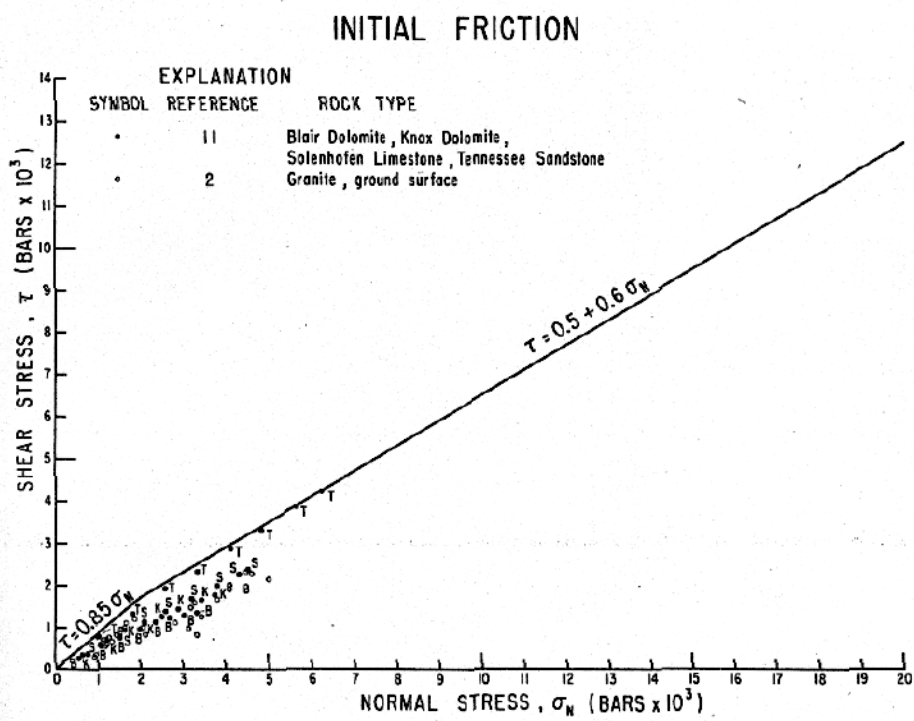
< Initial and maximum friction for normal stress up to 1 kbar >



(Byerlee, 1978)

# Frictional Strength

< Initial and maximum friction for normal stress up to 20 kbar >

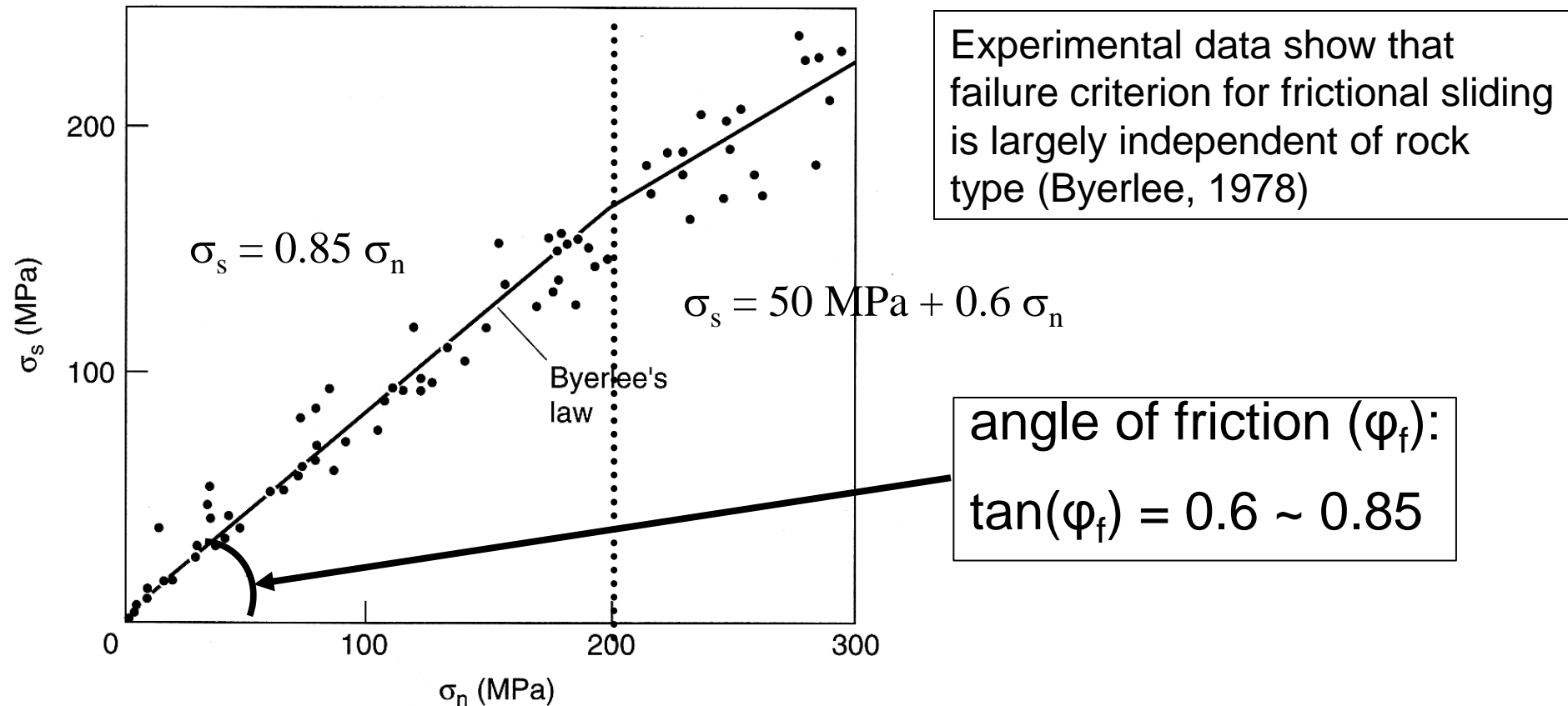


(Byerlee, 1978)

# Frictional Strength

## Frictional sliding criterion for most ROCKS is simple

Because of friction, certain critical shear stress is required before sliding initiates on preexisting fracture



But, besides friction we have to think about how to break a surface.



# Fracture Strength

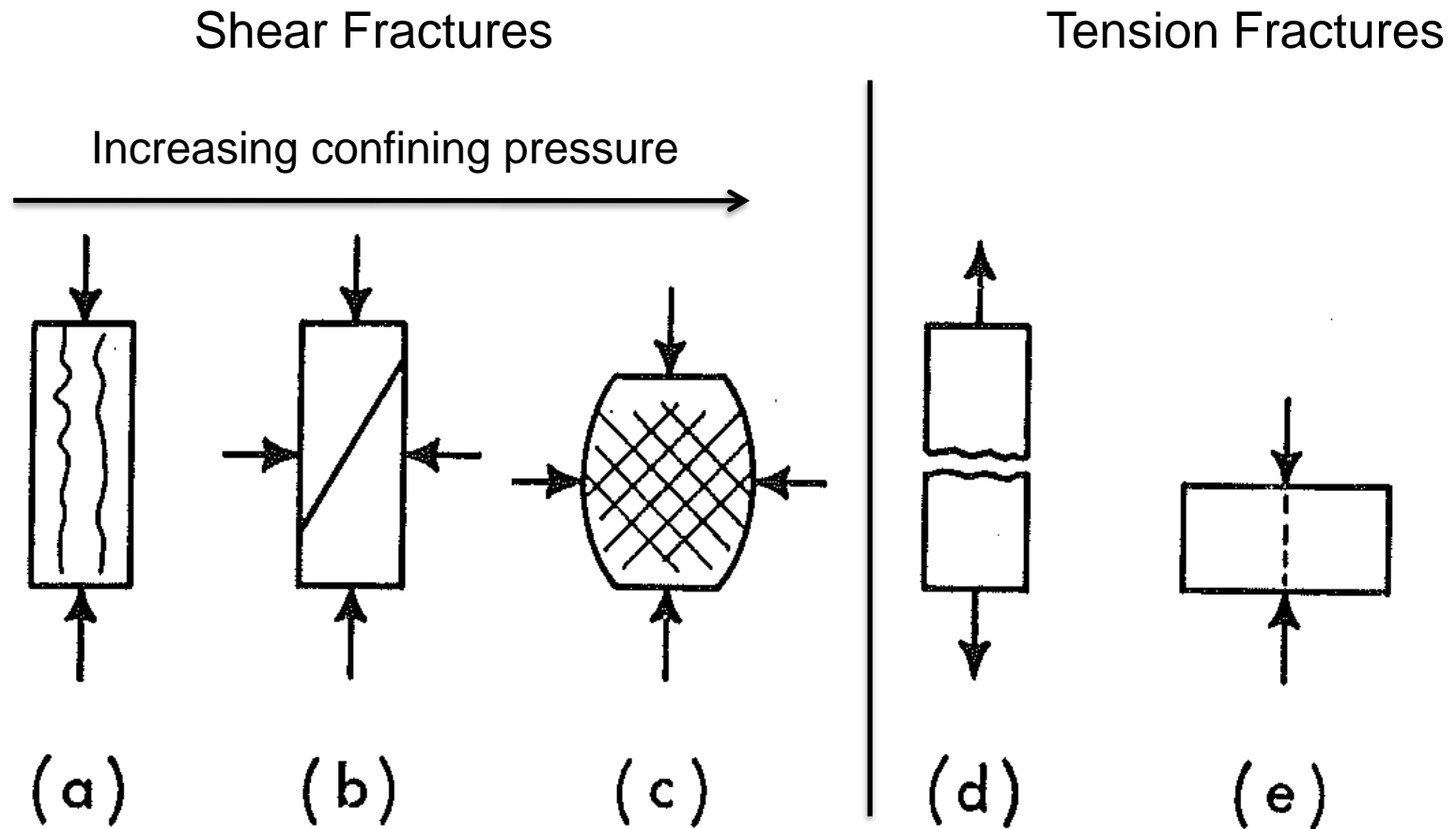
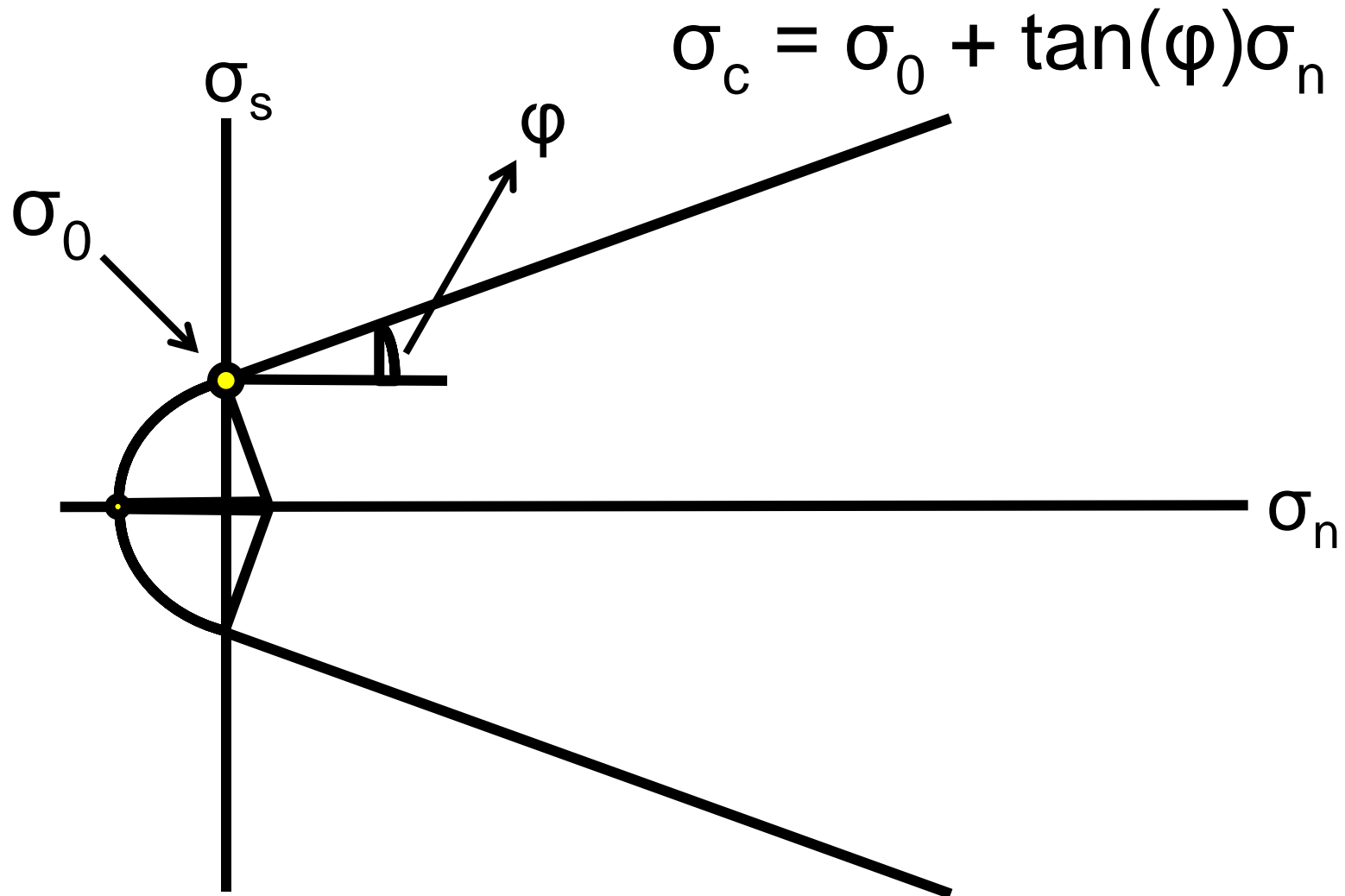


Fig. 4.5.1 (a) Longitudinal splitting in uniaxial compression. (b) Shear fracture. (c) Multiple shear fractures. (d) Extension fracture. (e) Extension fracture produced by line loads.

(Jaeger and Cook, 1976)

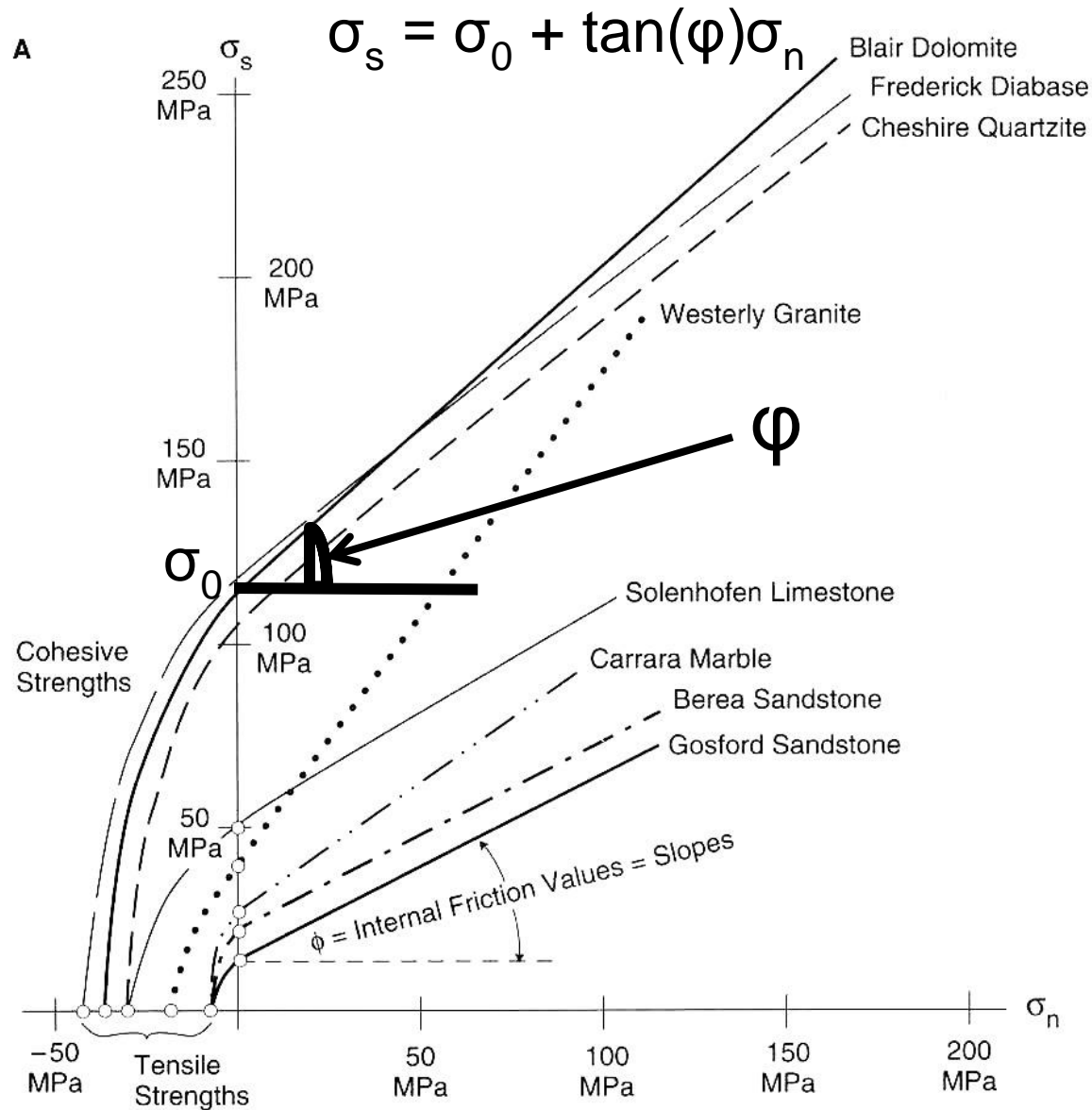
# Fracture Strength

## Mohr-Coulomb failure criterion



# Fracture Strength

< Fracture strength envelopes for different rocks >



# Mohr Circle

Mohr Circle: A device to relate friction and cohesion to fault *orientation*

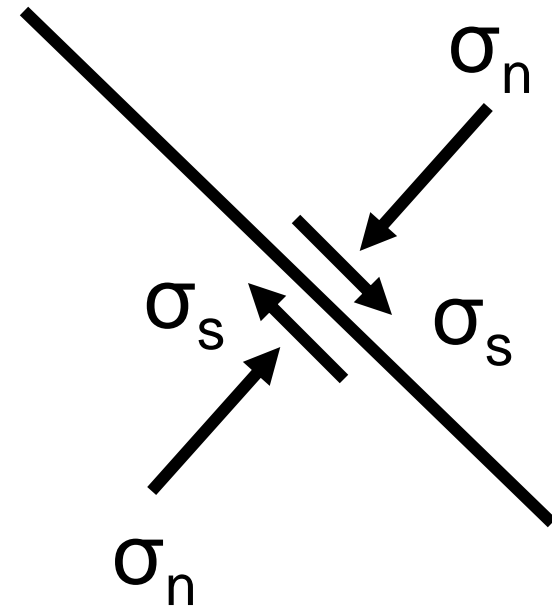
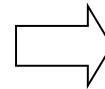
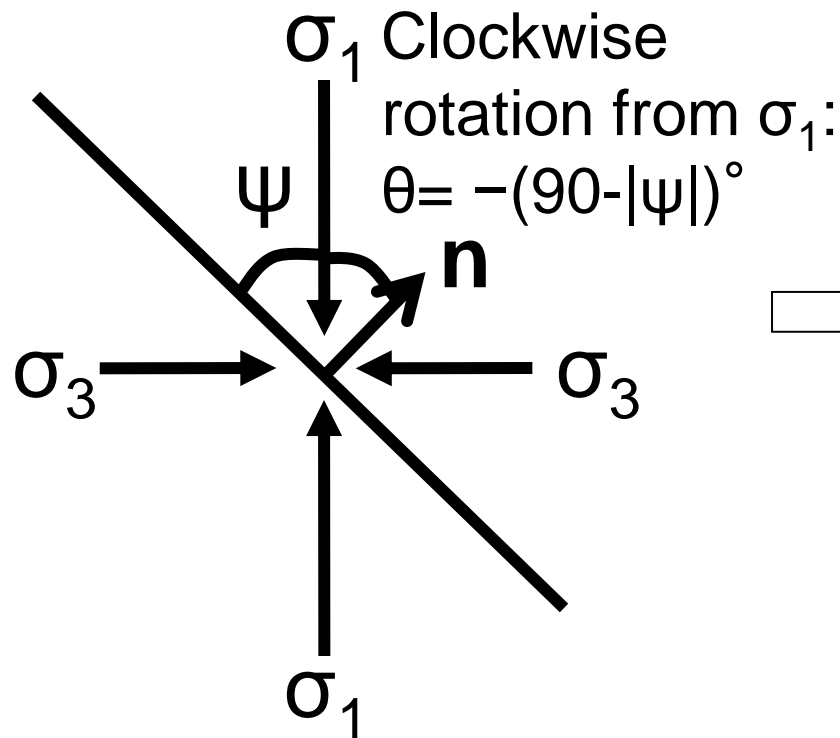
## A stress review for Mohr Circle

- Stress = Force/Area
- 3 principal values,  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ , corresponding to three principal directions.
- $\sigma_1 \geq \sigma_2 \geq \sigma_3$ , and positive when compressional.
- Static stress is when  $\sigma_1 = \sigma_2 = \sigma_3$
- Differential stress ( $\sigma_d$ ) defined as  $(\sigma_1 - \sigma_3)$

# Mohr Circle

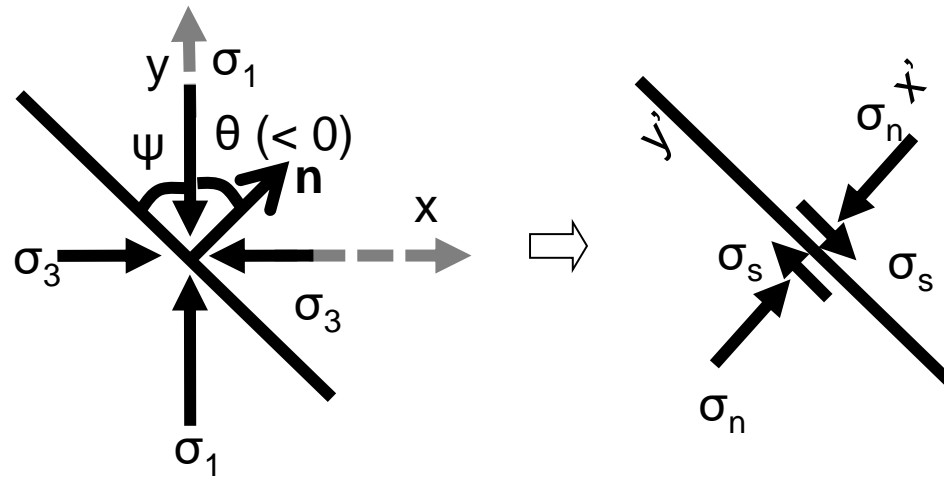
The stress is resolved into 2 components:

1. Shear stress ( $\sigma_s$ ), acting parallel with the plane
2. Normal stress ( $\sigma_n$ ), acting perpendicular to the plane



# Mohr Circle

The stress is resolved into 2 components:



In a 2D case (e.g., plane stress), we get the *Mohr Transformation*:

$$\sigma'_{11} = \sigma_{11} \cos^2 \theta + \sigma_{22} \sin^2 \theta + \sigma_{12} \sin 2\theta$$

$$\sigma'_{22} = \sigma_{11} \sin^2 \theta + \sigma_{22} \cos^2 \theta - \sigma_{12} \sin 2\theta$$

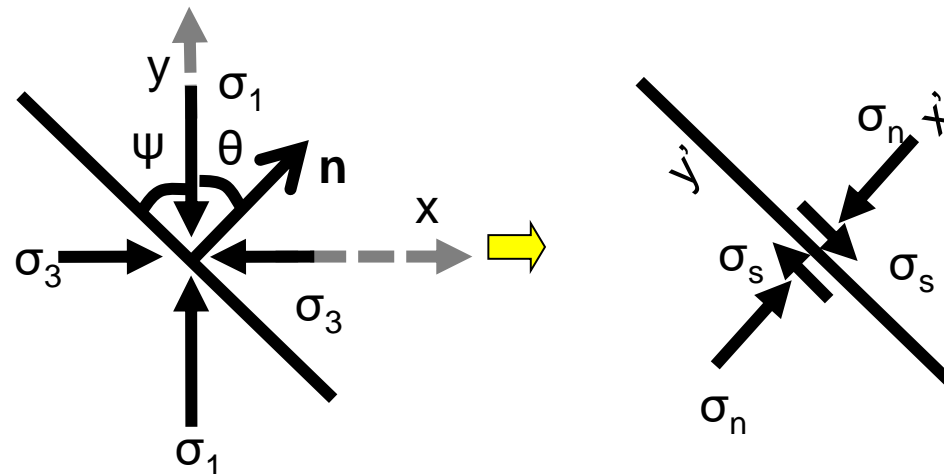
$$\sigma'_{12} = (\sigma_{22} - \sigma_{11}) \sin \theta \cos \theta + \sigma_{12} \cos 2\theta$$

If we start with principal stresses,  $\sigma_{12}=0$ ,  $\sigma_{11}=\sigma_3$  and  $\sigma_{22}=\sigma_1$ .  
Also,  $\sigma_n=\sigma'_{11}$  and  $\sigma_s=\sigma'_{12}$ .

# Mohr Circle

- Stress components are related by:
  - $\sigma_s = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(2\theta)$
  - $\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\theta)$
- From these, we get an equation for a circle:

$$[\sigma_n - \frac{1}{2}(\sigma_1 + \sigma_3)]^2 + \sigma_s^2 = \frac{1}{4}(\sigma_1 - \sigma_3)^2$$

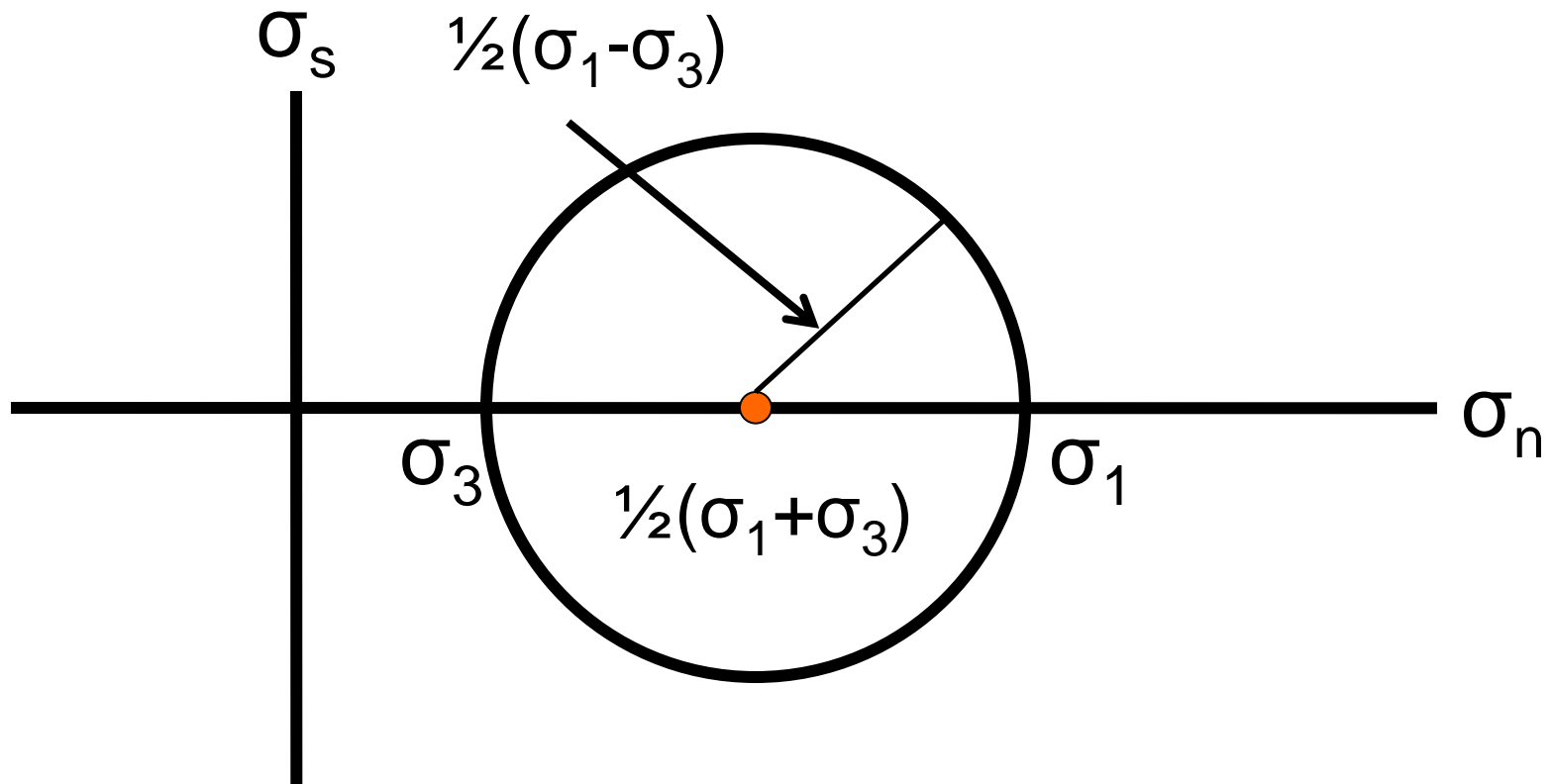


# Mohr Circle

Mohr circle for stress:

Diameter =  $(\sigma_1 - \sigma_3)$ , called “differential stress”.

Center on the  $\sigma_n$ -axis at point =  $\frac{1}{2}(\sigma_1 + \sigma_3)$

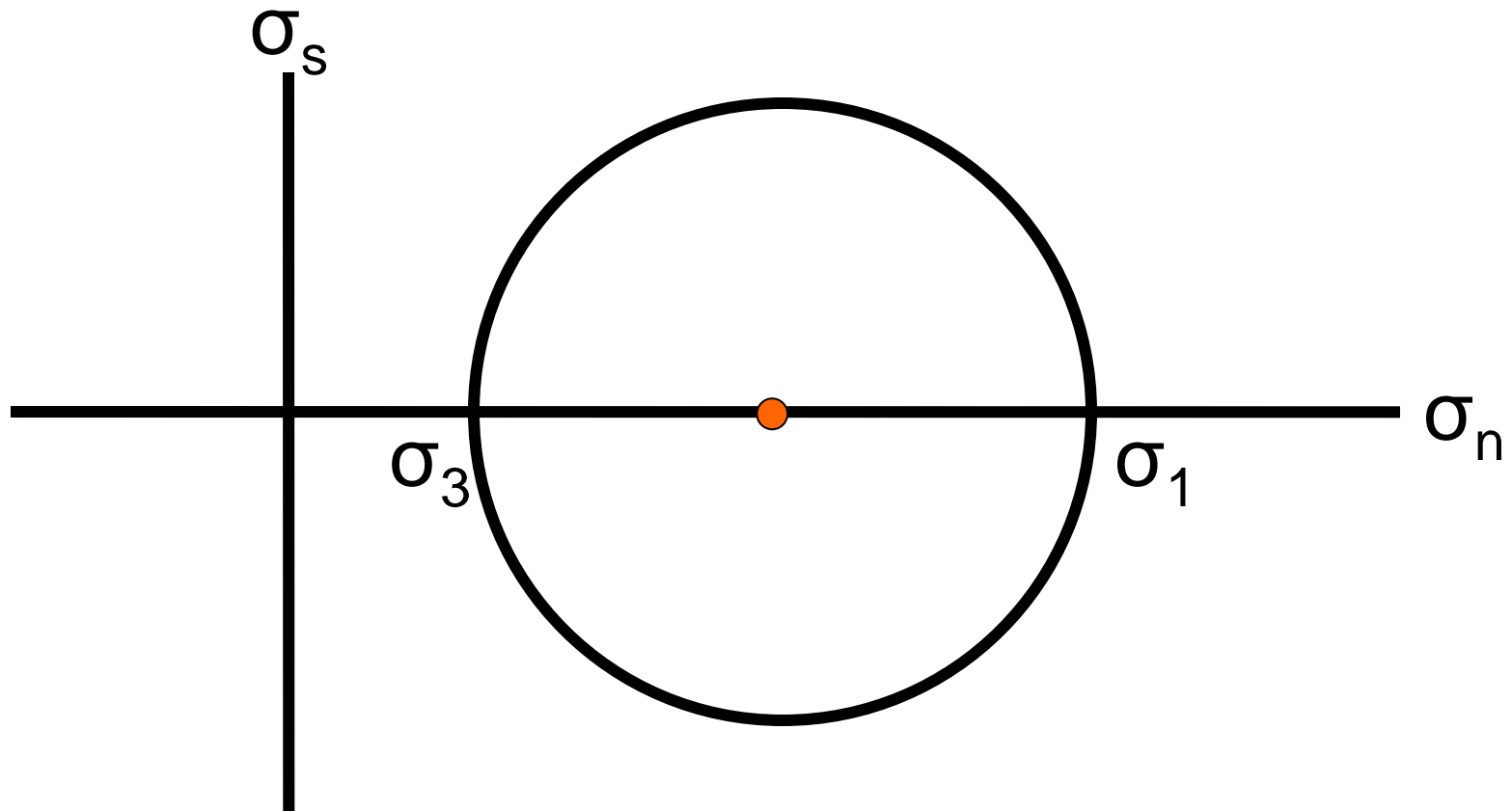




# Mohr Circle

Finding  $\sigma_s$ , and  $\sigma_n$

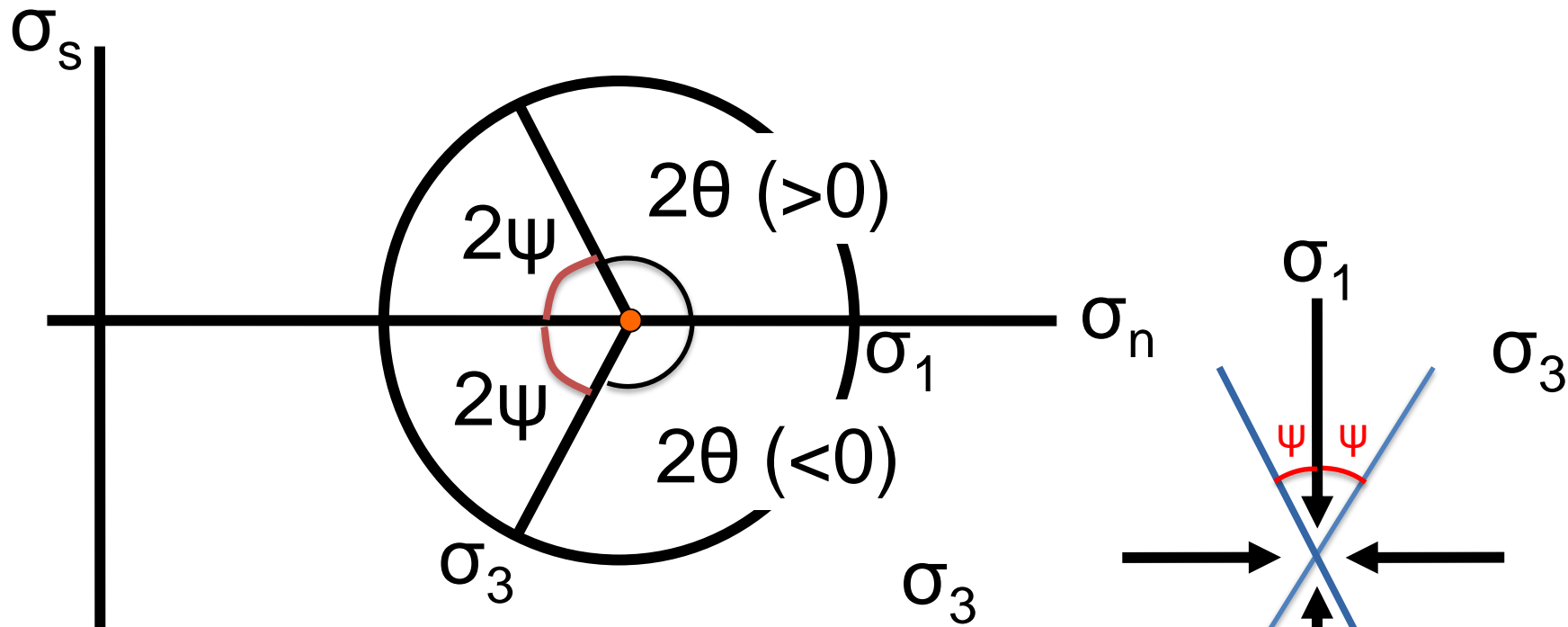
Can use a Mohr circle to find  $\sigma_s$ , and  $\sigma_n$  for any plane



# Mohr Circle

Can use a Mohr circle to find  $\sigma_s$ , and  $\sigma_n$  for any plane.

For instance, plot a line from center to edge of circle at angle  $2\theta = 180 - 2\psi^\circ$  from  $\sigma_1$ .

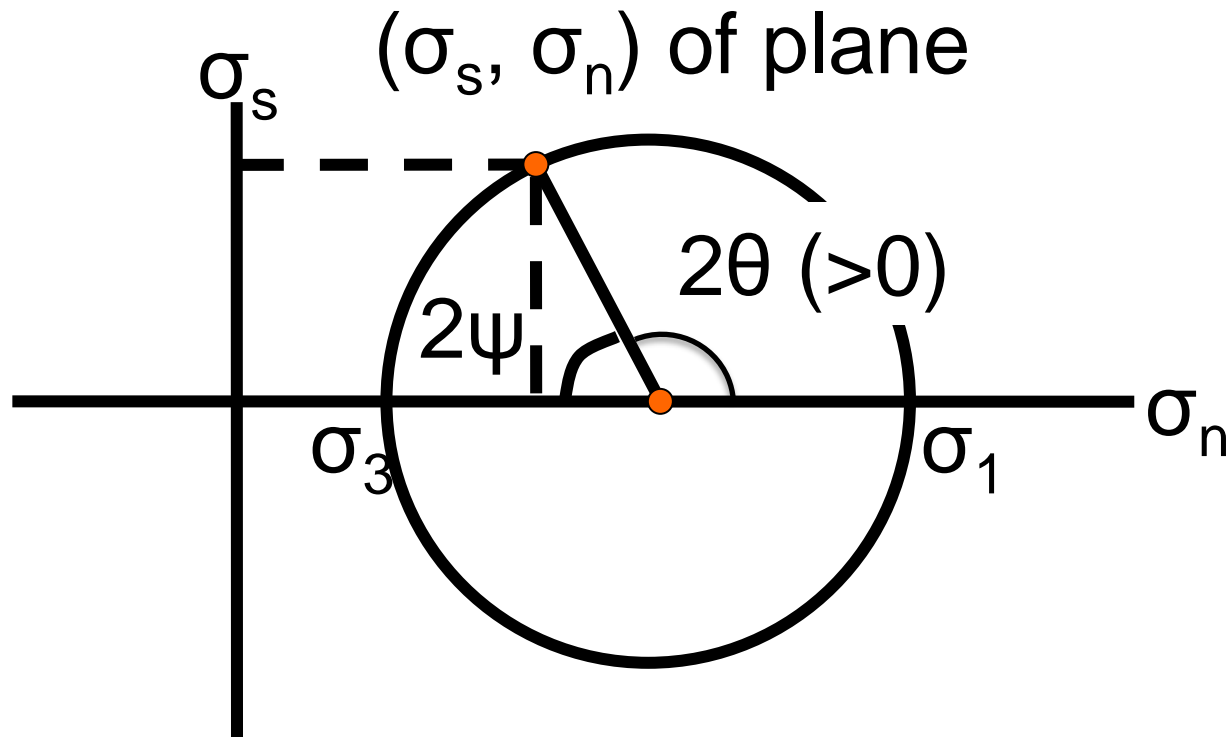


Represents **planes of which normal is  $\pm\theta^\circ$  from  $\sigma_1$** . Dihedral angle of the conjugate faults is  $2\psi^\circ$ .

# Mohr Circle

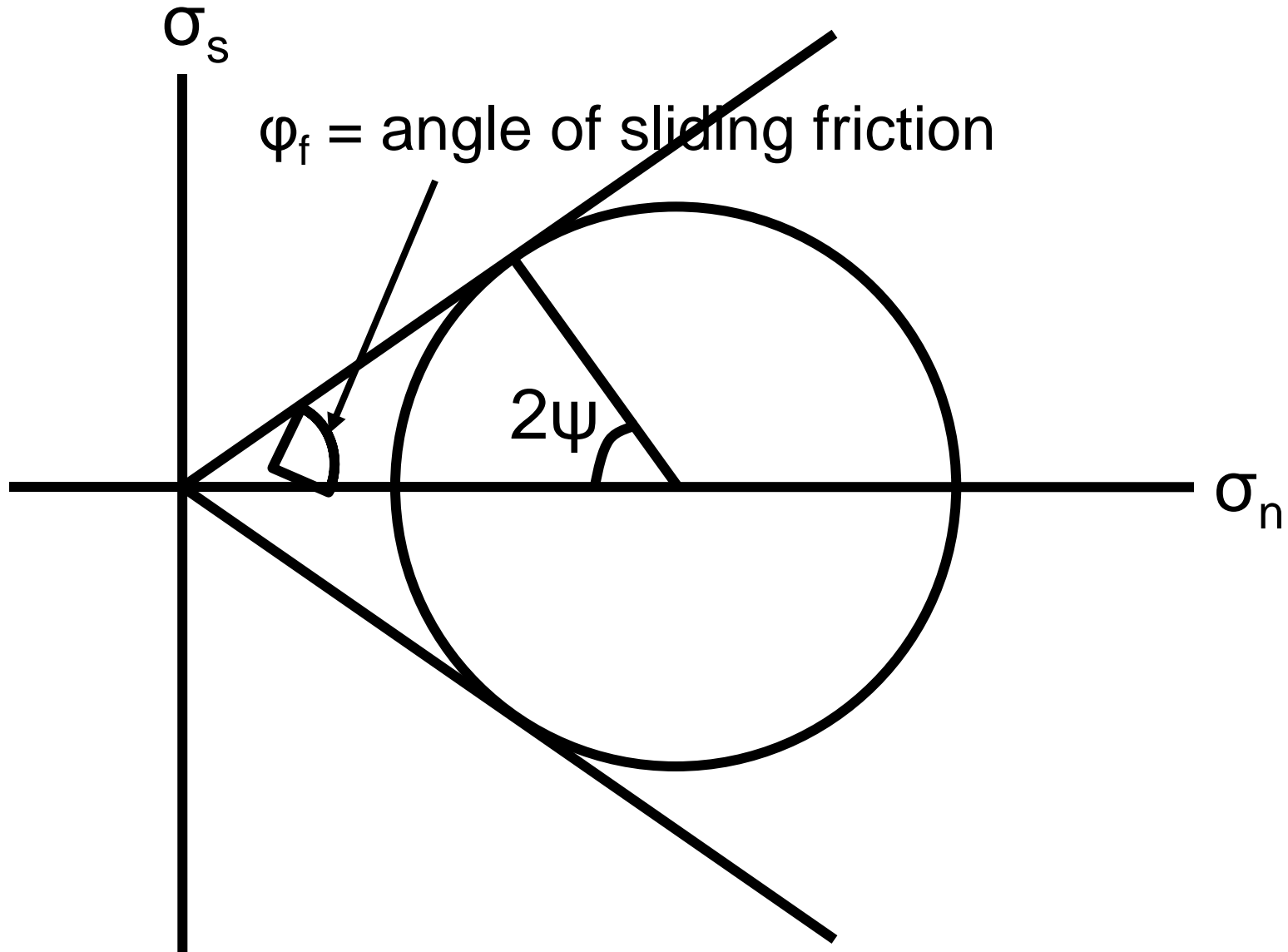
X- and y-coordinates of intersection of line and circle define  $\sigma_s$  and  $\sigma_n$  for the plane

- $\sigma_s = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(2\theta)$
- $\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\theta),$



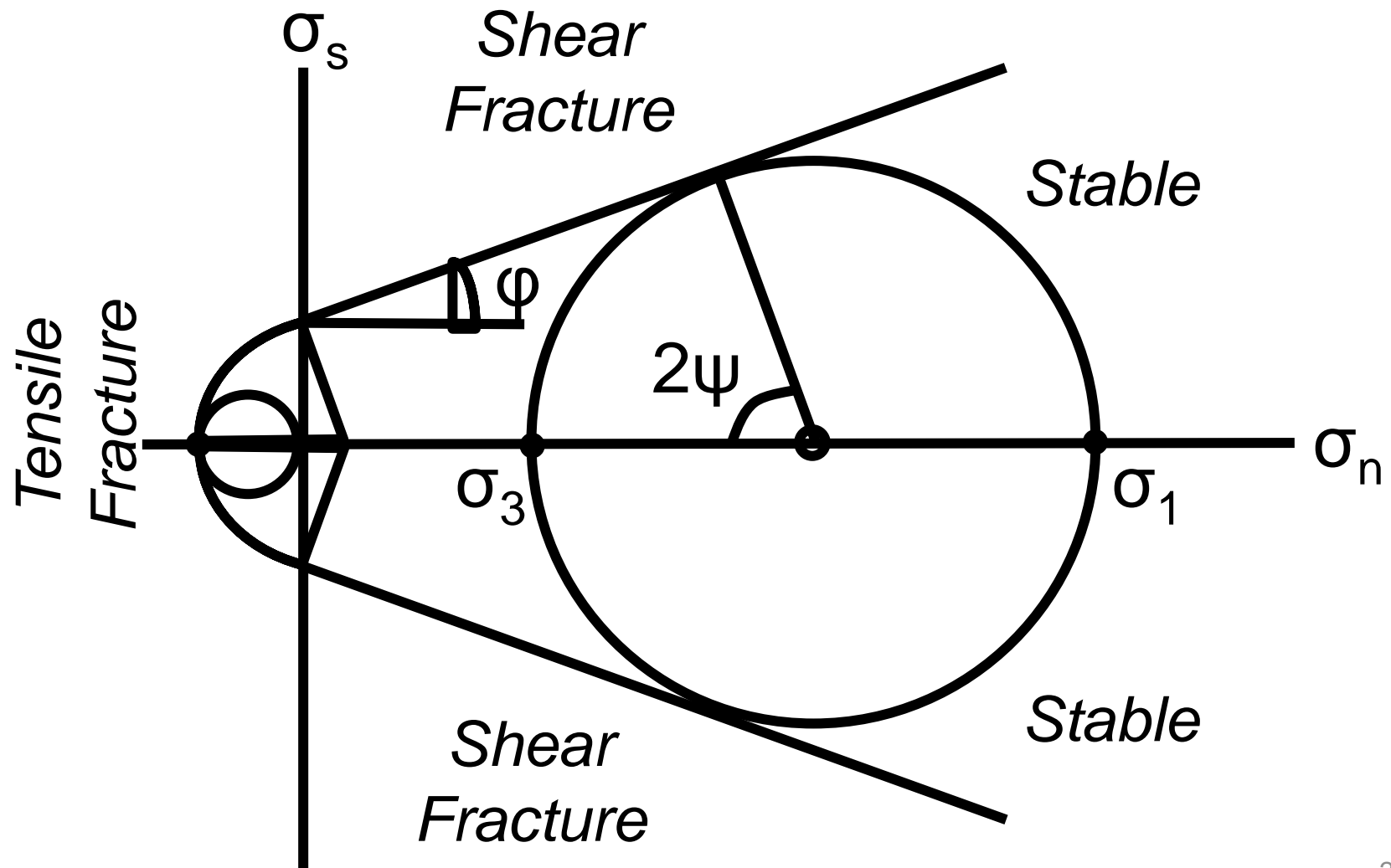
# Mohr Circle and Strength Envelope

Envelope of frictional strength



# Mohr Circle and Strength Envelope

The Coulomb envelope for fracture strength



# Mohr Circle and Strength Envelope

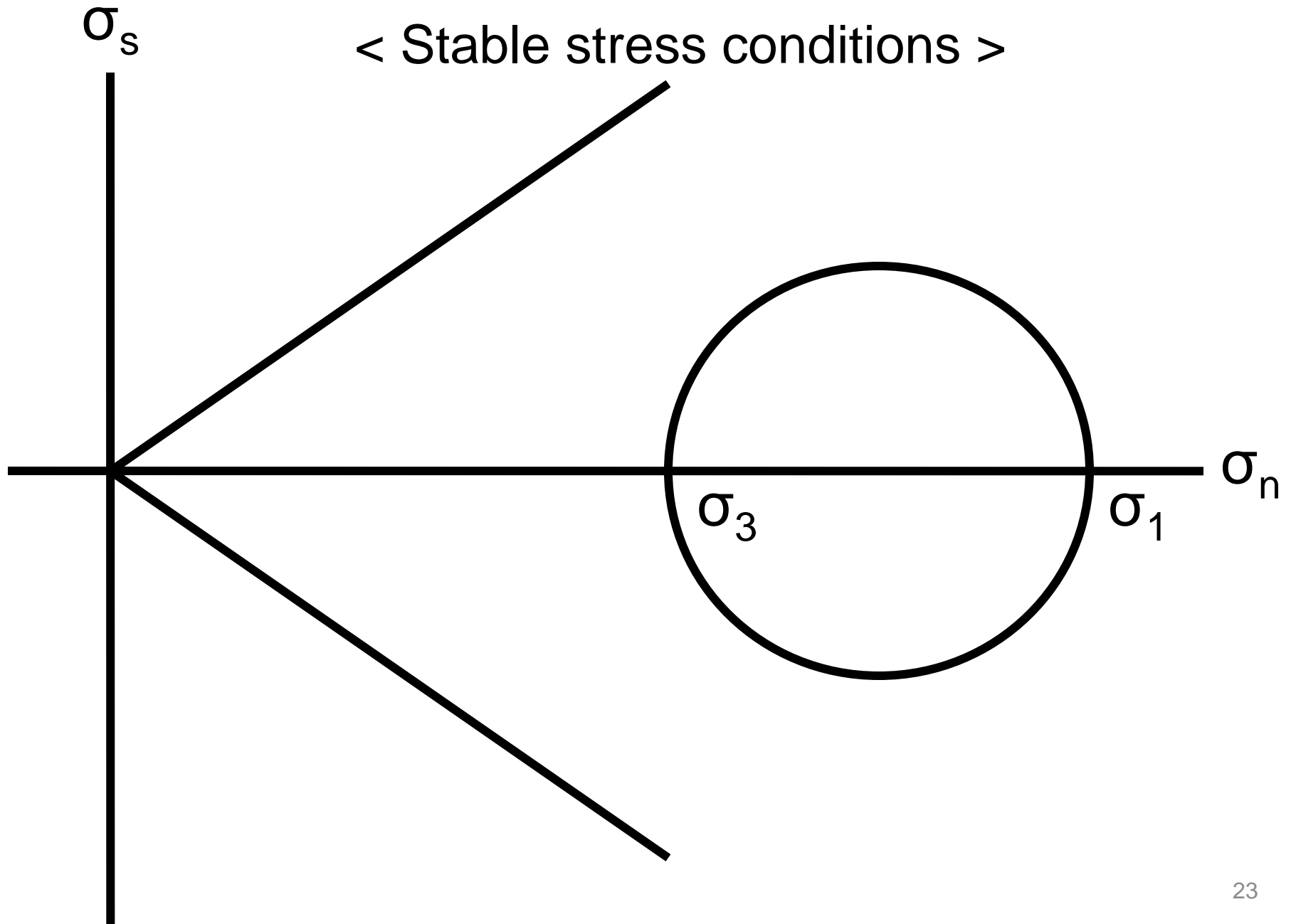
## Effect of pore-fluid pressure

Pore fluid pressure ( $P_f$ ) effectively lowers the stress in all directions

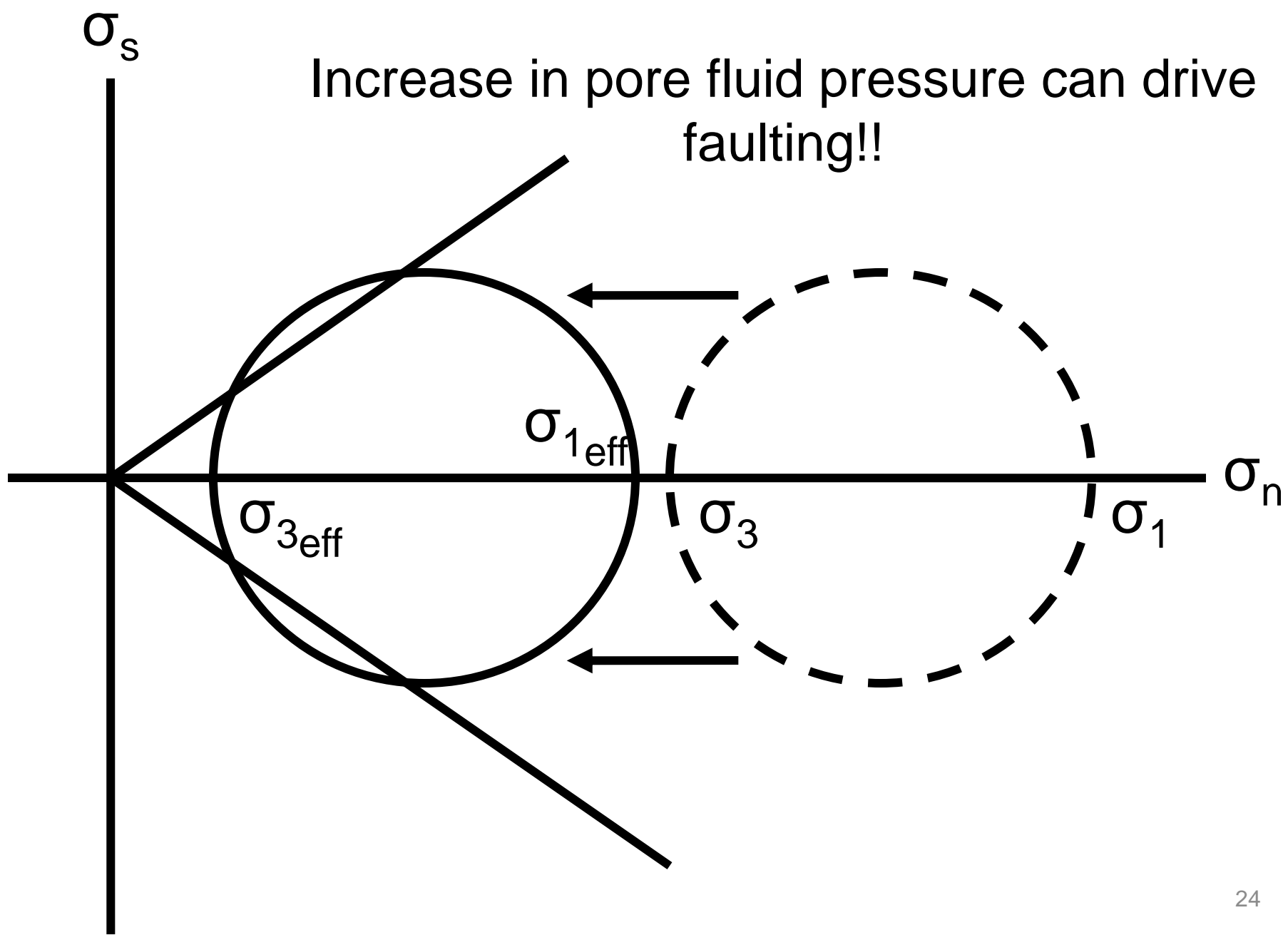
The effective stresses ( $\sigma_{1\text{eff}}$ ,  $\sigma_{2\text{eff}}$ , and  $\sigma_{3\text{eff}}$ ) = principal stresses -  $P_f$

$$\sigma_{1\text{eff}} = \sigma_1 - P_f \quad \sigma_{2\text{eff}} = \sigma_2 - P_f \quad \sigma_{3\text{eff}} = \sigma_3 - P_f$$

# Mohr Circle and Strength Envelope

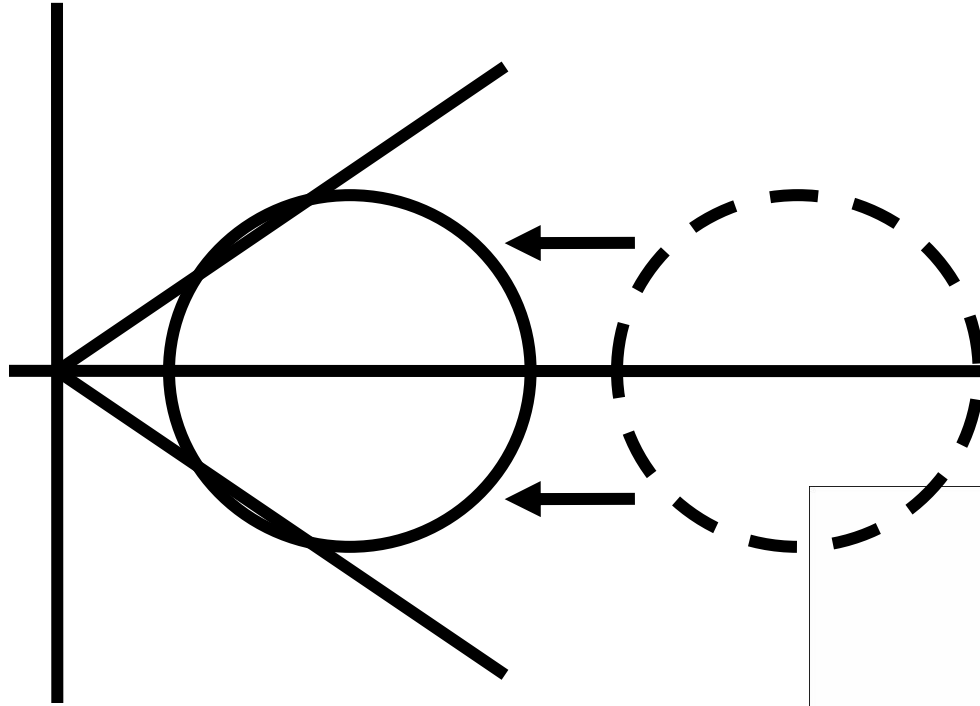


# Mohr Circle and Strength Envelope





# Mohr Circle and Strength Envelope



Analogous to an  
air hockey table



# Anderson's theory of faulting

## Primary assumptions

1. Surface of the earth is not acted on by shear or normal stresses.
  - So 2 of the 3 principal stresses are parallel to the surface.
2. Homogenous rocks
3. Coulomb behavior

# Anderson's theory of faulting

$$\sigma_{xx} = \rho g y + \Delta \sigma_{xx}$$

$$\sigma_{yy} = \rho g y$$

If  $\Delta \sigma_{xx}$  is negative (under extension),  $\sigma_1 = \sigma_{yy}$  and  $\sigma_3 = \sigma_{xx}$ .

Then,  $\sigma_s = \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin(2\theta)$  and

$$\sigma_n = \frac{1}{2}(\sigma_{yy} + \sigma_{xx}) - \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\cos(2\theta)$$

$$\sigma_n = \rho g y + \frac{\Delta \sigma_{xx}}{2}(1 + \cos 2\theta)$$

$$\sigma_s = -\frac{\Delta \sigma_{xx}}{2} \sin 2\theta$$

Plugging  $\sigma_n$  and  $\sigma_s$  into the Coulomb Failure criterion,  $\sigma_s = \sigma_0 + f_s \sigma_n$ , where  $f_s = \tan(\phi)$ , we get

$$\pm \frac{\Delta \sigma_{xx}}{2} \sin 2\theta = f_s \left( \rho g y + \frac{\Delta \sigma_{xx}}{2} (1 + \cos 2\theta) \right)$$

# Anderson's theory of faulting

$$\pm \frac{\Delta\sigma_{xx}}{2} \sin 2\theta = f_s \left( \rho g y + \frac{\Delta\sigma_{xx}}{2} (1 + \cos 2\theta) \right)$$

In the above equation, upper sign applies to  $\Delta\sigma_{xx} > 0$  (i.e., thrust faults) and the lower sign to  $\Delta\sigma_{xx} < 0$  (i.e., normal faults). We can also get an expression for  $\Delta\sigma_{xx}$  from this equation:

$$\Delta\sigma_{xx} = \frac{2f_s \rho g y}{\pm \sin 2\theta - f_s (1 + \cos 2\theta)}.$$

Now we are interested in the smallest possible  $\Delta\sigma_{xx}$  that satisfies the above equation and the corresponding value of  $f_s$  because that value will correspond to the strength.

# Anderson's theory of faulting

$$\tan 2\theta = \mp \frac{1}{f_s}$$

The upper and lower sign corresponds to thrust and normal faults, respectively. This expression simply represents the geometric relation between the failure envelope and the Mohr circle we saw earlier.

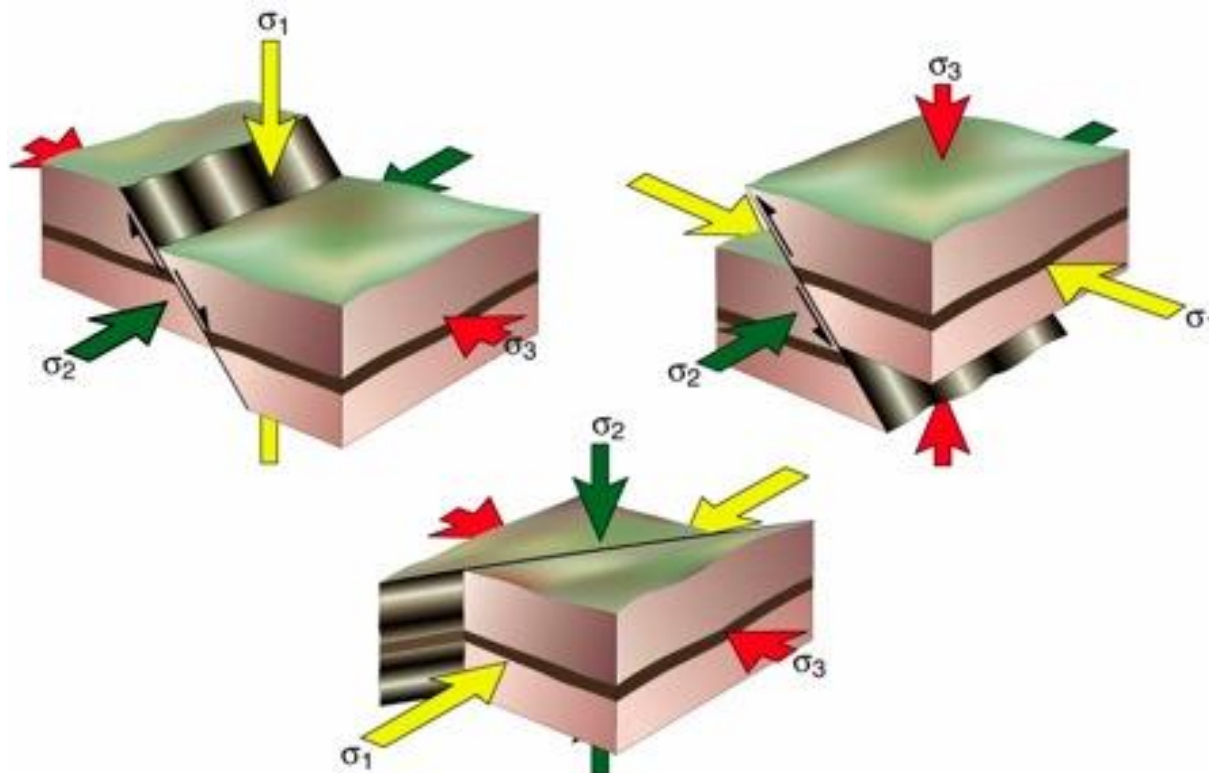
The corresponding differential stress becomes

$$\Delta\sigma_{xx} = \frac{\pm 2f_s \rho g y}{(1 + f_s)^{1/2} \mp f_s}.$$

# Anderson's theory of faulting

Most rocks have an angle of internal friction  $\approx 30^\circ$

- $\sigma_1$  horizontal,  $\sigma_3$  vertical — reverse faults
- $\sigma_1$  vertical,  $\sigma_3$  horizontal — normal faults
- $\sigma_1$  horizontal,  $\sigma_3$  horizontal — strike-slip faults



# Anderson's theory of faulting

- Reverse faults ( $\sigma_1$  horizontal): should form at  $\sim 30^\circ$  dip
- Normal faults ( $\sigma_1$  vertical): should form at  $\sim 60^\circ$  dip
- Strike-slip faults: should form at  $\sim 90^\circ$  dip and  $\sim 60^\circ$  dihedral angle

