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Homework 6

1) Betweenness centrality:
   - Node 1: 0 → standardized score: 0
   - Node 2: 0 → standardized score: 0
   - Node 3: 16/3 → standardized score: 16/45
   - Node 4: 13/3 → standardized score: 13/45
   - Node 5: 13/3 → standardized score: 13/45
   - Node 6: 0 → standardized score: 0
   - Node 7: 0 → standardized score: 0

   The vector is: $[0,\ 0,\ 16/45,\ 13/45,\ 13/45,\ 0,\ 0]^T$

   Eigenvalue centrality:

   Adjacency Matrix A

Nodes	1	2	3	4	5	6	7
1	0	0	1	0	0	0	0
2	0	0	1	0	0	0	0
3	1	1	0	1	0	0	0
4	0	0	1	0	1	0	0
5	0	0	0	1	0	1	1
6	0	0	0	0	1	0	1
7	0	0	0	0	1	1	0

   λ_1=0, λ_2= -1, λ_3 = 0.23266, λ_4= -1.92747, λ_5= 2.25327, λ_6= -1.20331, λ_7= 1.64485

   The max eigenvalue is λ_5= 2.25327 and the corresponding eigenvector is $[0.122766, 0.122766, 0.276626, 0.377781, 0.574617, 0.458493, 0.458493]^T$. The components of the eigenvector are the eigenvector centralities for the graph.

   Katz centrality (with alpha=0.2):

$$Katz = ((I - \alpha A^T)^{-1} - I)1 =$$

$$I = \begin{bmatrix} 1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0 \\ 0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0 \\ 1&1&0&1&0&0&0 \\ 0&0&1&0&1&0&0 \\ 0&0&0&1&0&1&1 \\ 0&0&0&0&1&0&1 \end{bmatrix} \quad (I - \alpha A^T) = \begin{bmatrix} 1&0&-0.2&0&0&0&0 \\ 0&1&-0.2&0&0&0&0 \\ -0.2&-0.2&1&-0.2&0&0&0 \\ 0&0&-0.2&1&-0.2&0&0 \\ 0&0&0&-0.2&1&-0.2&-0.2 \\ 0&0&0&0&-0.2&1&-0.2 \end{bmatrix}$$

0 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 −0.2 −0.2 1

$(I - \alpha A^T)^{-1} =$

	1	2	3	4	5	6	7
1	1.0455 508474 576271 183	0.0455 508474 576271 18641	0.2277 542372 881355 9318	0.0476 694915 254237 28809	0.0105 932203 389830 50846	0.0026 483050 847457 627113	0.0026 483050 847457 627114
2	0.0455 508474 576271 18641	1.0455 508474 576271 183	0.2277 542372 881355 9318	0.0476 694915 254237 28809	0.0105 932203 389830 50846	0.0026 483050 847457 627113	0.0026 483050 847457 627114
3	0.2277 542372 881355 932	0.2277 542372 881355 932	1.1387 711864 406779 659	0.2383 474576 271186 4403	0.0529 661016 949152 5423	0.0132 415254 237288 13556	0.0132 415254 237288 13556
4	0.0476 694915 254237 28811	0.0476 694915 254237 28811	0.2383 474576 271186 4402	1.0963 983050 847457 625	0.2436 440677 966101 6946	0.0609 110169 491525 42363	0.0609 110169 491525 42364
5	0.0105 932203 389830 50846	0.0105 932203 389830 50846	0.0529 661016 949152 54229	0.2436 440677 966101 6946	1.1652 542372 881355 931	0.2913 135593 220338 9827	0.2913 135593 220338 9828
6	0.0026 483050 847457 627116	0.0026 483050 847457 627116	0.0132 415254 237288 13557	0.0609 110169 491525 42366	0.2913 135593 220338 9828	1.1144 950564 971751 411	0.2811 617231 638418 0789
7	0.0026 483050 847457 627117	0.0026 483050 847457 627117	0.0132 415254 237288 13556	0.0609 110169 491525 42366	0.2913 135593 220338 9829	0.2811 617231 638418 0788	1.1144 950564 971751 412

$(I - \alpha A^T)^{-1} - I =$

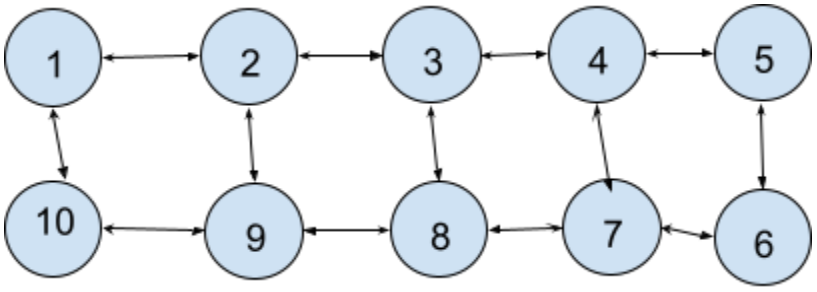
	1	2	3	4	5	6	7

1	0.0455508474576272	0.0455508474576271 15	0.2277542372881356	0.0476694915254237 3	0.0105932203389830 5	0.0026483050847457626	0.0026483050847457626
2	0.0455508474576271 15	0.0455508474576272	0.2277542372881356	0.0476694915254237 3	0.0105932203389830 5	0.0026483050847457626	0.0026483050847457626
3	0.2277542372881356	0.2277542372881356	0.138771186440678	0.2383474576271186 5	0.0529661016949152 5	0.0132415254237288 13	0.0132415254237288 13
4	0.0476694915254237 3	0.0476694915254237 3	0.2383474576271186 5	0.0963983050847457	0.2436440677966101 6	0.0609110169491525 4	0.0609110169491525 4
5	0.0105932203389830 5	0.0105932203389830 5	0.0529661016949152 5	0.2436440677966101 6	0.1652542372881356	0.2913135593220339	0.2913135593220339
6	0.0026483050847457626	0.0026483050847457626	0.0132415254237288 13	0.0609110169491525 4	0.2913135593220339	0.1144950564971752	0.2811617231638418
7	0.0026483050847457626	0.0026483050847457626	0.0132415254237288 13	0.0609110169491525 4	0.2913135593220339	0.2811617231638418	0.1144950564971752

Grabbing the highest values from each column, the Katz Centrality vector is:

$[0.04766949152542373,\ 0.04766949152542373, 0.23834745762711865,\ 0.24364406779661016,$
$0.2913135593220339,\ 0.2913135593220339, 0.2913135593220339]^T$

2)

a) 

Here is a DeGroot Model that represents how much weight its neighbors affect each house's decision to bring out the trash or not.

	1	2	3	4	5	6	7	8	9	10
1	0	1/2	0	0	0	0	0	0	0	1/2
2	1/3	0	1/3	0	0	0	0	0	1/3	0
3	0	1/3	0	1/3	0	0	0	1/3	0	0
4	0	0	1/3	0	1/3	0	1/3	0	0	0
5	0	0	0	1/2	0	1/2	0	0	0	0
6	0	0	0	0	1/2	0	1/2	0	0	0
7	0	0	0	1/3	0	1/3	0	1/3	0	0
8	0	0	1/3	0	0	0	1/3	0	1/3	0
9	0	1/3	0	0	0	0	0	1/3	0	1/3
10	1/2	0	0	0	0	0	0	0	1/2	0

b) Assuming only the first two houses (1 and 10) put their recyclin bins out, then the initial belief vector of each house is represented by: $b(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 1]^T$.

b(1) : [0.5, 0.3333, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.3333, 0.5]
b(2) : [0.4166, 0.2778, 0.1111, 0.0, 0.0, 0.0, 0.0, 0.1111, 0.2778, 0.4166]
b(3) : [0.3472, 0.2685, 0.1296, 0.037, 0.0, 0.0, 0.037, 0.1296, 0.2685, 0.3472]
b(4) : [0.3079, 0.2484, 0.145, 0.0555, 0.0185, 0.0185, 0.0555, 0.145, 0.2484, 0.3079]
b(5) : [0.2782, 0.2338, 0.1496, 0.073, 0.037, 0.037, 0.073, 0.1496, 0.2338, 0.2782]
b(6) : [0.256, 0.2205, 0.1521, 0.0865, 0.055, 0.055, 0.0865, 0.1521, 0.2205, 0.256]
b(7) : [0.2383, 0.2095, 0.153, 0.0979, 0.0707, 0.0707, 0.0979, 0.153, 0.2095, 0.2383]
b(8) : [0.2239, 0.2003, 0.1535, 0.1072, 0.0843, 0.0843, 0.1072, 0.1535, 0.2003, 0.2239]
b(9) : [0.2121, 0.1926, 0.1537, 0.115, 0.0958, 0.0958, 0.115, 0.1537, 0.1926, 0.2121]
b(10) : [0.2024, 0.1861, 0.1538, 0.1215, 0.1054, 0.1054, 0.1215, 0.1538, 0.1861, 0.2024]
b(11) : [0.1942, 0.1808, 0.1538, 0.1269, 0.1134, 0.1134, 0.1269, 0.1538, 0.1808, 0.1942]
b(12) : [0.1875, 0.1763, 0.1538, 0.1314, 0.1202, 0.1202, 0.1314, 0.1538, 0.1763, 0.1875]
b(13) : [0.1819, 0.1725, 0.1538, 0.1351, 0.1258, 0.1258, 0.1351, 0.1538, 0.1725, 0.1819]
b(14) : [0.1772, 0.1694, 0.1538, 0.1382, 0.1305, 0.1305, 0.1382, 0.1538, 0.1694, 0.1772]

b(15) : [0.1733, 0.1668, 0.1538, 0.1408, 0.1343, 0.1343, 0.1408, 0.1538, 0.1668, 0.1733]
b(16) : [0.1701, 0.1646, 0.1538, 0.143, 0.1376, 0.1376, 0.143, 0.1538, 0.1646, 0.1701]
b(17) : [0.1673, 0.1628, 0.1538, 0.1448, 0.1403, 0.1403, 0.1448, 0.1538, 0.1628, 0.1673]
b(18) : [0.1651, 0.1613, 0.1538, 0.1463, 0.1426, 0.1426, 0.1463, 0.1538, 0.1613, 0.1651]
b(19) : [0.1632, 0.1601, 0.1538, 0.1476, 0.1445, 0.1445, 0.1476, 0.1538, 0.1601, 0.1632]
b(20) : [0.1617, 0.159, 0.1538, 0.1486, 0.1461, 0.1461, 0.1486, 0.1538, 0.159, 0.1617]
b(21) : [0.1603, 0.1582, 0.1538, 0.1495, 0.1474, 0.1474, 0.1495, 0.1538, 0.1582, 0.1603]
b(22) : [0.1593, 0.1574, 0.1538, 0.1502, 0.1484, 0.1484, 0.1502, 0.1538, 0.1574, 0.1593]
b(23) : [0.1583, 0.1568, 0.1538, 0.1508, 0.1493, 0.1493, 0.1508, 0.1538, 0.1568, 0.1583]
b(24) : [0.1575, 0.1563, 0.1538, 0.1513, 0.15, 0.15, 0.1513, 0.1538, 0.1563, 0.1575]
b(25) : [0.1569, 0.1559, 0.1538, 0.1517, 0.1507, 0.1507, 0.1517, 0.1538, 0.1559, 0.1569]
b(26) : [0.1564, 0.1555, 0.1538, 0.1521, 0.1512, 0.1512, 0.1521, 0.1538, 0.1555, 0.1564]
b(27) : [0.156, 0.1552, 0.1538, 0.1524, 0.1517, 0.1517, 0.1524, 0.1538, 0.1552, 0.156]
b(28) : [0.1556, 0.155, 0.1538, 0.1526, 0.1521, 0.1521, 0.1526, 0.1538, 0.155, 0.1556]
b(29) : [0.1553, 0.1548, 0.1538, 0.1528, 0.1524, 0.1524, 0.1528, 0.1538, 0.1548, 0.1553]
b(30) : [0.155, 0.1546, 0.1538, 0.153, 0.1526, 0.1526, 0.153, 0.1538, 0.1546, 0.155]
b(31) : [0.1548, 0.1545, 0.1538, 0.1531, 0.1528, 0.1528, 0.1531, 0.1538, 0.1545, 0.1548]
b(32) : [0.1547, 0.1544, 0.1538, 0.1532, 0.153, 0.153, 0.1532, 0.1538, 0.1544, 0.1547]
b(33) : [0.1546, 0.1543, 0.1538, 0.1533, 0.1531, 0.1531, 0.1533, 0.1538, 0.1543, 0.1546]
b(34) : [0.1544, 0.1542, 0.1538, 0.1534, 0.1532, 0.1532, 0.1534, 0.1538, 0.1542, 0.1544]
b(35) : [0.1543, 0.1541, 0.1538, 0.1535, 0.1533, 0.1533, 0.1535, 0.1538, 0.1541, 0.1543]
b(36) : [0.1542, 0.1541, 0.1538, 0.1535, 0.1534, 0.1534, 0.1535, 0.1538, 0.1541, 0.1542]
b(37) : [0.1542, 0.154, 0.1538, 0.1536, 0.1535, 0.1535, 0.1536, 0.1538, 0.154, 0.1542]
b(38) : [0.1541, 0.154, 0.1538, 0.1536, 0.1535, 0.1535, 0.1536, 0.1538, 0.154, 0.1541]
b(39) : [0.154, 0.154, 0.1538, 0.1536, 0.1535, 0.1535, 0.1536, 0.1538, 0.154, 0.154]
b(40) : [0.154, 0.1539, 0.1538, 0.1536, 0.1535, 0.1535, 0.1536, 0.1538, 0.1539, 0.154]
b(41) : [0.154, 0.1539, 0.1538, 0.1536, 0.1535, 0.1535, 0.1536, 0.1538, 0.1539, 0.154]
After 41 iterations, we see that it finally convergest. It seems like each house will have roughly a 15% chance of taking the trash out.

c) If initially, the two houses at the end (5 and 6) took out their trash, then the initial belief will be $b(0) = [0, 0, 0, 0, 1, 1, 0, 0, 0, 0]^T$.
b(1) : [0.0, 0.0, 0.0, 0.3333, 0.5, 0.5, 0.3333, 0.0, 0.0, 0.0]
b(2) : [0.0, 0.0, 0.1111, 0.2778, 0.4166, 0.4166, 0.2778, 0.1111, 0.0, 0.0]
b(3) : [0.0, 0.037, 0.1296, 0.2685, 0.3472, 0.3472, 0.2685, 0.1296, 0.037, 0.0]
b(4) : [0.0185, 0.0555, 0.145, 0.2484, 0.3079, 0.3079, 0.2484, 0.145, 0.0555, 0.0185]
b(5) : [0.037, 0.073, 0.1496, 0.2338, 0.2782, 0.2782, 0.2338, 0.1496, 0.073, 0.037]
b(6) : [0.055, 0.0865, 0.1521, 0.2205, 0.256, 0.256, 0.2205, 0.1521, 0.0865, 0.055]
b(7) : [0.0707, 0.0979, 0.153, 0.2095, 0.2383, 0.2383, 0.2095, 0.153, 0.0979, 0.0707]
b(8) : [0.0843, 0.1072, 0.1535, 0.2003, 0.2239, 0.2239, 0.2003, 0.1535, 0.1072, 0.0843]
b(9) : [0.0958, 0.115, 0.1537, 0.1926, 0.2121, 0.2121, 0.1926, 0.1537, 0.115, 0.0958]
b(10) : [0.1054, 0.1215, 0.1538, 0.1861, 0.2024, 0.2024, 0.1861, 0.1538, 0.1215, 0.1054]
b(11) : [0.1134, 0.1269, 0.1538, 0.1808, 0.1942, 0.1942, 0.1808, 0.1538, 0.1269, 0.1134]

b(12) : [0.1202, 0.1314, 0.1538, 0.1763, 0.1875, 0.1875, 0.1763, 0.1538, 0.1314, 0.1202]
b(13) : [0.1258, 0.1351, 0.1538, 0.1725, 0.1819, 0.1819, 0.1725, 0.1538, 0.1351, 0.1258]
b(14) : [0.1305, 0.1382, 0.1538, 0.1694, 0.1772, 0.1772, 0.1694, 0.1538, 0.1382, 0.1305]
b(15) : [0.1343, 0.1408, 0.1538, 0.1668, 0.1733, 0.1733, 0.1668, 0.1538, 0.1408, 0.1343]
b(16) : [0.1376, 0.143, 0.1538, 0.1646, 0.1701, 0.1701, 0.1646, 0.1538, 0.143, 0.1376]
b(17) : [0.1403, 0.1448, 0.1538, 0.1628, 0.1673, 0.1673, 0.1628, 0.1538, 0.1448, 0.1403]
b(18) : [0.1426, 0.1463, 0.1538, 0.1613, 0.1651, 0.1651, 0.1613, 0.1538, 0.1463, 0.1426]
b(19) : [0.1445, 0.1476, 0.1538, 0.1601, 0.1632, 0.1632, 0.1601, 0.1538, 0.1476, 0.1445]
b(20) : [0.1461, 0.1486, 0.1538, 0.159, 0.1617, 0.1617, 0.159, 0.1538, 0.1486, 0.1461]
b(21) : [0.1474, 0.1495, 0.1538, 0.1582, 0.1603, 0.1603, 0.1582, 0.1538, 0.1495, 0.1474]
b(22) : [0.1484, 0.1502, 0.1538, 0.1574, 0.1593, 0.1593, 0.1574, 0.1538, 0.1502, 0.1484]
b(23) : [0.1493, 0.1508, 0.1538, 0.1568, 0.1583, 0.1583, 0.1568, 0.1538, 0.1508, 0.1493]
b(24) : [0.15, 0.1513, 0.1538, 0.1563, 0.1575, 0.1575, 0.1563, 0.1538, 0.1513, 0.15]
b(25) : [0.1507, 0.1517, 0.1538, 0.1559, 0.1569, 0.1569, 0.1559, 0.1538, 0.1517, 0.1507]
b(26) : [0.1512, 0.1521, 0.1538, 0.1555, 0.1564, 0.1564, 0.1555, 0.1538, 0.1521, 0.1512]
b(27) : [0.1517, 0.1524, 0.1538, 0.1552, 0.156, 0.156, 0.1552, 0.1538, 0.1524, 0.1517]
b(28) : [0.1521, 0.1526, 0.1538, 0.155, 0.1556, 0.1556, 0.155, 0.1538, 0.1526, 0.1521]
b(29) : [0.1524, 0.1528, 0.1538, 0.1548, 0.1553, 0.1553, 0.1548, 0.1538, 0.1528, 0.1524]
b(30) : [0.1526, 0.153, 0.1538, 0.1546, 0.155, 0.155, 0.1546, 0.1538, 0.153, 0.1526]
b(31) : [0.1528, 0.1531, 0.1538, 0.1545, 0.1548, 0.1548, 0.1545, 0.1538, 0.1531, 0.1528]
b(32) : [0.153, 0.1532, 0.1538, 0.1544, 0.1547, 0.1547, 0.1544, 0.1538, 0.1532, 0.153]
b(33) : [0.1531, 0.1533, 0.1538, 0.1543, 0.1546, 0.1546, 0.1543, 0.1538, 0.1533, 0.1531]
b(34) : [0.1532, 0.1534, 0.1538, 0.1542, 0.1544, 0.1544, 0.1542, 0.1538, 0.1534, 0.1532]
b(35) : [0.1533, 0.1535, 0.1538, 0.1541, 0.1543, 0.1543, 0.1541, 0.1538, 0.1535, 0.1533]
b(36) : [0.1534, 0.1535, 0.1538, 0.1541, 0.1542, 0.1542, 0.1541, 0.1538, 0.1535, 0.1534]
b(37) : [0.1535, 0.1536, 0.1538, 0.154, 0.1542, 0.1542, 0.154, 0.1538, 0.1536, 0.1535]
b(38) : [0.1535, 0.1536, 0.1538, 0.154, 0.1541, 0.1541, 0.154, 0.1538, 0.1536, 0.1535]
b(39) : [0.1535, 0.1536, 0.1538, 0.154, 0.154, 0.154, 0.154, 0.1538, 0.1536, 0.1535]
b(40) : [0.1535, 0.1536, 0.1538, 0.1539, 0.154, 0.154, 0.1539, 0.1538, 0.1536, 0.1535]
b(41) : [0.1535, 0.1536, 0.1538, 0.1539, 0.154, 0.154, 0.1539, 0.1538, 0.1536, 0.1535]
We see that we'll have somewhat a similar result to part b; Each house will have roughly a 15% chance of taking the trash out.

d) If initially, every second house on one side (2 and 4) took out their trash, then the initial belief will be $b(0) = [0, 1, 0, 1, 0, 0, 0, 0, 0, 0]^T$.
b(1) : [0.0, 0.0, 0.0, 0.3333, 0.5, 0.5, 0.3333, 0.0, 0.0, 0.0]
b(2) : [0.0, 0.0, 0.1111, 0.2778, 0.4166, 0.4166, 0.2778, 0.1111, 0.0, 0.0]
b(3) : [0.0, 0.037, 0.1296, 0.2685, 0.3472, 0.3472, 0.2685, 0.1296, 0.037, 0.0]
b(4) : [0.0185, 0.0555, 0.145, 0.2484, 0.3079, 0.3079, 0.2484, 0.145, 0.0555, 0.0185]
b(5) : [0.037, 0.073, 0.1496, 0.2338, 0.2782, 0.2782, 0.2338, 0.1496, 0.073, 0.037]
b(6) : [0.055, 0.0865, 0.1521, 0.2205, 0.256, 0.256, 0.2205, 0.1521, 0.0865, 0.055]
b(7) : [0.0707, 0.0979, 0.153, 0.2095, 0.2383, 0.2383, 0.2095, 0.153, 0.0979, 0.0707]
b(8) : [0.0843, 0.1072, 0.1535, 0.2003, 0.2239, 0.2239, 0.2003, 0.1535, 0.1072, 0.0843]
b(9) : [0.0958, 0.115, 0.1537, 0.1926, 0.2121, 0.2121, 0.1926, 0.1537, 0.115, 0.0958]

b(10) : [0.1054, 0.1215, 0.1538, 0.1861, 0.2024, 0.2024, 0.1861, 0.1538, 0.1215, 0.1054]
b(11) : [0.1134, 0.1269, 0.1538, 0.1808, 0.1942, 0.1942, 0.1808, 0.1538, 0.1269, 0.1134]
b(12) : [0.1202, 0.1314, 0.1538, 0.1763, 0.1875, 0.1875, 0.1763, 0.1538, 0.1314, 0.1202]
b(13) : [0.1258, 0.1351, 0.1538, 0.1725, 0.1819, 0.1819, 0.1725, 0.1538, 0.1351, 0.1258]
b(14) : [0.1305, 0.1382, 0.1538, 0.1694, 0.1772, 0.1772, 0.1694, 0.1538, 0.1382, 0.1305]
b(15) : [0.1343, 0.1408, 0.1538, 0.1668, 0.1733, 0.1733, 0.1668, 0.1538, 0.1408, 0.1343]
b(16) : [0.1376, 0.143, 0.1538, 0.1646, 0.1701, 0.1701, 0.1646, 0.1538, 0.143, 0.1376]
b(17) : [0.1403, 0.1448, 0.1538, 0.1628, 0.1673, 0.1673, 0.1628, 0.1538, 0.1448, 0.1403]
b(18) : [0.1426, 0.1463, 0.1538, 0.1613, 0.1651, 0.1651, 0.1613, 0.1538, 0.1463, 0.1426]
b(19) : [0.1445, 0.1476, 0.1538, 0.1601, 0.1632, 0.1632, 0.1601, 0.1538, 0.1476, 0.1445]
b(20) : [0.1461, 0.1486, 0.1538, 0.159, 0.1617, 0.1617, 0.159, 0.1538, 0.1486, 0.1461]
b(21) : [0.1474, 0.1495, 0.1538, 0.1582, 0.1603, 0.1603, 0.1582, 0.1538, 0.1495, 0.1474]
b(22) : [0.1484, 0.1502, 0.1538, 0.1574, 0.1593, 0.1593, 0.1574, 0.1538, 0.1502, 0.1484]
b(23) : [0.1493, 0.1508, 0.1538, 0.1568, 0.1583, 0.1583, 0.1568, 0.1538, 0.1508, 0.1493]
b(24) : [0.15, 0.1513, 0.1538, 0.1563, 0.1575, 0.1575, 0.1563, 0.1538, 0.1513, 0.15]
b(25) : [0.1507, 0.1517, 0.1538, 0.1559, 0.1569, 0.1569, 0.1559, 0.1538, 0.1517, 0.1507]
b(26) : [0.1512, 0.1521, 0.1538, 0.1555, 0.1564, 0.1564, 0.1555, 0.1538, 0.1521, 0.1512]
b(27) : [0.1517, 0.1524, 0.1538, 0.1552, 0.156, 0.156, 0.1552, 0.1538, 0.1524, 0.1517]
b(28) : [0.1521, 0.1526, 0.1538, 0.155, 0.1556, 0.1556, 0.155, 0.1538, 0.1526, 0.1521]
b(29) : [0.1524, 0.1528, 0.1538, 0.1548, 0.1553, 0.1553, 0.1548, 0.1538, 0.1528, 0.1524]
b(30) : [0.1526, 0.153, 0.1538, 0.1546, 0.155, 0.155, 0.1546, 0.1538, 0.153, 0.1526]
b(31) : [0.1528, 0.1531, 0.1538, 0.1545, 0.1548, 0.1548, 0.1545, 0.1538, 0.1531, 0.1528]
b(32) : [0.153, 0.1532, 0.1538, 0.1544, 0.1547, 0.1547, 0.1544, 0.1538, 0.1532, 0.153]
b(33) : [0.1531, 0.1533, 0.1538, 0.1543, 0.1546, 0.1546, 0.1543, 0.1538, 0.1533, 0.1531]
b(34) : [0.1532, 0.1534, 0.1538, 0.1542, 0.1544, 0.1544, 0.1542, 0.1538, 0.1534, 0.1532]
b(35) : [0.1533, 0.1535, 0.1538, 0.1541, 0.1543, 0.1543, 0.1541, 0.1538, 0.1535, 0.1533]
b(36) : [0.1534, 0.1535, 0.1538, 0.1541, 0.1542, 0.1542, 0.1541, 0.1538, 0.1535, 0.1534]
b(37) : [0.1535, 0.1536, 0.1538, 0.154, 0.1542, 0.1542, 0.154, 0.1538, 0.1536, 0.1535]
b(38) : [0.1535, 0.1536, 0.1538, 0.154, 0.1541, 0.1541, 0.154, 0.1538, 0.1536, 0.1535]
b(39) : [0.1535, 0.1536, 0.1538, 0.154, 0.154, 0.154, 0.154, 0.1538, 0.1536, 0.1535]
b(40) : [0.1535, 0.1536, 0.1538, 0.1539, 0.154, 0.154, 0.1539, 0.1538, 0.1536, 0.1535]
b(41) : [0.1535, 0.1536, 0.1538, 0.1539, 0.154, 0.154, 0.1539, 0.1538, 0.1536, 0.1535]

We see that we'll have somewhat a similar result to part b and c; Each house will have roughly a 15% chance of taking the trash out.

e) If initially two houses at the head of the street, and one of their neighbors, initially put out their trash, then the initial belief will be $b(0) = [1, 1, 0, 0, 0, 0, 0, 0, 1, 1]^T$.

b(1) : [1.0, 0.6667, 0.3333, 0.0, 0.0, 0.0, 0.0, 0.3333, 0.6667, 1.0]
b(2) : [0.8334, 0.6667, 0.3333, 0.1111, 0.0, 0.0, 0.1111, 0.3333, 0.6667, 0.8334]
b(3) : [0.75, 0.6111, 0.3704, 0.1481, 0.0556, 0.0556, 0.1481, 0.3704, 0.6111, 0.75]
b(4) : [0.6805, 0.5772, 0.3765, 0.1914, 0.1018, 0.1018, 0.1914, 0.3765, 0.5772, 0.6805]
b(5) : [0.6289, 0.5447, 0.3817, 0.2232, 0.1466, 0.1466, 0.2232, 0.3817, 0.5447, 0.6289]
b(6) : [0.5868, 0.5184, 0.3832, 0.2505, 0.1849, 0.1849, 0.2505, 0.3832, 0.5184, 0.5868]
b(7) : [0.5526, 0.4961, 0.384, 0.2729, 0.2177, 0.2177, 0.2729, 0.384, 0.4961, 0.5526]

b(8) : [0.5243, 0.4776, 0.3843, 0.2915, 0.2453, 0.2453, 0.2915, 0.3843, 0.4776, 0.5243]
b(9) : [0.501, 0.4621, 0.3845, 0.307, 0.2684, 0.2684, 0.307, 0.3845, 0.4621, 0.501]
b(10) : [0.4816, 0.4492, 0.3845, 0.32, 0.2877, 0.2877, 0.32, 0.3845, 0.4492, 0.4816]
b(11) : [0.4654, 0.4384, 0.3846, 0.3307, 0.3039, 0.3039, 0.3307, 0.3846, 0.4384, 0.4654]
b(12) : [0.4519, 0.4295, 0.3846, 0.3397, 0.3173, 0.3173, 0.3397, 0.3846, 0.4295, 0.4519]
b(13) : [0.4407, 0.422, 0.3846, 0.3472, 0.3285, 0.3285, 0.3472, 0.3846, 0.422, 0.4407]
b(14) : [0.4314, 0.4158, 0.3846, 0.3534, 0.3378, 0.3378, 0.3534, 0.3846, 0.4158, 0.4314]
b(15) : [0.4236, 0.4106, 0.3846, 0.3586, 0.3456, 0.3456, 0.3586, 0.3846, 0.4106, 0.4236]
b(16) : [0.4171, 0.4063, 0.3846, 0.3629, 0.3521, 0.3521, 0.3629, 0.3846, 0.4063, 0.4171]
b(17) : [0.4117, 0.4027, 0.3846, 0.3665, 0.3575, 0.3575, 0.3665, 0.3846, 0.4027, 0.4117]
b(18) : [0.4072, 0.3997, 0.3846, 0.3695, 0.362, 0.362, 0.3695, 0.3846, 0.3997, 0.4072]
b(19) : [0.4034, 0.3972, 0.3846, 0.372, 0.3658, 0.3658, 0.372, 0.3846, 0.3972, 0.4034]
b(20) : [0.4003, 0.3951, 0.3846, 0.3741, 0.3689, 0.3689, 0.3741, 0.3846, 0.3951, 0.4003]
b(21) : [0.3977, 0.3933, 0.3846, 0.3759, 0.3715, 0.3715, 0.3759, 0.3846, 0.3933, 0.3977]
b(22) : [0.3955, 0.3919, 0.3846, 0.3773, 0.3737, 0.3737, 0.3773, 0.3846, 0.3919, 0.3955]
b(23) : [0.3937, 0.3907, 0.3846, 0.3785, 0.3755, 0.3755, 0.3785, 0.3846, 0.3907, 0.3937]
b(24) : [0.3922, 0.3897, 0.3846, 0.3795, 0.377, 0.377, 0.3795, 0.3846, 0.3897, 0.3922]
b(25) : [0.391, 0.3888, 0.3846, 0.3804, 0.3782, 0.3782, 0.3804, 0.3846, 0.3888, 0.391]
b(26) : [0.3899, 0.3881, 0.3846, 0.3811, 0.3793, 0.3793, 0.3811, 0.3846, 0.3881, 0.3899]
b(27) : [0.389, 0.3875, 0.3846, 0.3817, 0.3802, 0.3802, 0.3817, 0.3846, 0.3875, 0.389]
b(28) : [0.3882, 0.387, 0.3846, 0.3822, 0.381, 0.381, 0.3822, 0.3846, 0.387, 0.3882]
b(29) : [0.3876, 0.3866, 0.3846, 0.3826, 0.3816, 0.3816, 0.3826, 0.3846, 0.3866, 0.3876]
b(30) : [0.3871, 0.3863, 0.3846, 0.3829, 0.3821, 0.3821, 0.3829, 0.3846, 0.3863, 0.3871]
b(31) : [0.3867, 0.386, 0.3846, 0.3832, 0.3825, 0.3825, 0.3832, 0.3846, 0.386, 0.3867]
b(32) : [0.3863, 0.3858, 0.3846, 0.3834, 0.3829, 0.3829, 0.3834, 0.3846, 0.3858, 0.3863]
b(33) : [0.3861, 0.3856, 0.3846, 0.3836, 0.3831, 0.3831, 0.3836, 0.3846, 0.3856, 0.3861]
b(34) : [0.3859, 0.3854, 0.3846, 0.3838, 0.3833, 0.3833, 0.3838, 0.3846, 0.3854, 0.3859]
b(35) : [0.3857, 0.3853, 0.3846, 0.3839, 0.3835, 0.3835, 0.3839, 0.3846, 0.3853, 0.3857]
b(36) : [0.3855, 0.3852, 0.3846, 0.384, 0.3837, 0.3837, 0.384, 0.3846, 0.3852, 0.3855]
b(37) : [0.3853, 0.3851, 0.3846, 0.3841, 0.3839, 0.3839, 0.3841, 0.3846, 0.3851, 0.3853]
b(38) : [0.3852, 0.385, 0.3846, 0.3842, 0.384, 0.384, 0.3842, 0.3846, 0.385, 0.3852]
b(39) : [0.3851, 0.3849, 0.3846, 0.3843, 0.3841, 0.3841, 0.3843, 0.3846, 0.3849, 0.3851]
b(40) : [0.385, 0.3849, 0.3846, 0.3843, 0.3842, 0.3842, 0.3843, 0.3846, 0.3849, 0.385]
b(41) : [0.385, 0.3848, 0.3846, 0.3844, 0.3842, 0.3842, 0.3844, 0.3846, 0.3848, 0.385]
b(42) : [0.3849, 0.3848, 0.3846, 0.3844, 0.3843, 0.3843, 0.3844, 0.3846, 0.3848, 0.3849]
b(43) : [0.3849, 0.3848, 0.3846, 0.3844, 0.3843, 0.3843, 0.3844, 0.3846, 0.3848, 0.3849]
We see that each houses will have roughly a 38% chance of putting out the trash.