Stefanica - 150 quant interviews
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1 homology lesson 1 Introduction

- fundamental group of surf or space
- good for low dimension data bc about loops
- fundamental group: Π_1
- there are high dimension analogues called homotopy groups $\Pi_n I X), n$
- for example Π_2 is loop of loops
- maps of square to space X which has behaviour around boundary
- Problem : homotopy groups are more complicated and hard to compute
- homomotopy group $\Pi_n I X), n$ of k=sphere S^k exsits if $n > k$!!
- homology = commutative alternative to homotopy
- for space X , cook up various homology groups $H_n(C)$
- these $H_n(C)$ measure n-dim holes of X !! (not sure what n-dim hole is)
- introduce ideas via examples !!
- consider X a graph 3 vertices x y z and directed egdes a b c d
- loop a bc startsat X and ends at X
- loop b c a s/e Y
- loops cab Z
- in commutative setting, does not matter where we start
- we prefer to write the operation as + in commutative setting
- loop now a + b + c = b + c + a = ...
- we call this expression a cycle because physically represents going around (close loop)
- another cycle: not necessarily a starting point
- call it c-d
- another cycle: going round the outside : a + b + d
- there is algebraic relation between cycle a+b+c , c-d, a+b+d
- namely: a+b+c=a+b+d+c-d - d cancels
- let C_0 free abelian group generated by **vertices** x y z
- let C_1 free abelian group generated by **edges** a b c d
- then elements of C0 eg 2x + 4y -5 z
- 0-dim chain = = random combinations with integer coefficients
- then elements of C1 are integral linear combs of **edges** a b c d , eg a+b+d (cycle) or 2a -5b (not a cycle)
- these are 1-dim chains ?
- question: what do we mean by a cycle algebraically
- in other words: what is special about **cycles** eg $a + b + d$, which is not shared by some random combinations
- Snswer: **depends on relationship betwen edges and vertices, expressed in terms of bounday of an edge**
- each edge has a boundary δ
- boundary of a := $\delta(a) = Y - X$ (final - intl)
- boundary of c := $\delta(a) = Z - Y$ (final - intl)
- SO there is a mapping δ edges $C_0 \rightarrow$ vertices C_1
- δ is the group homomorphism which estends xo
- dle(ax+by+)a= a d(x)+bd(y)