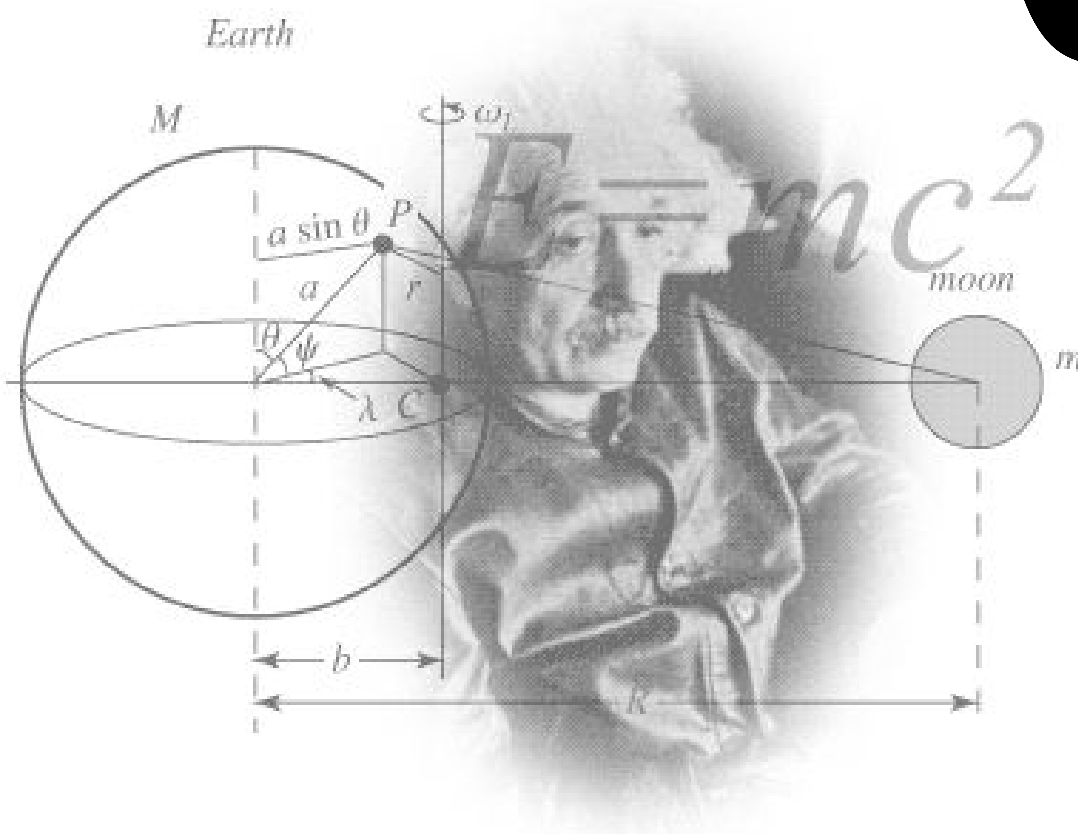
AP Physics C – Practice Workbook – Book 1

Mechanics

C



2012-2013

Table of Contents

This book is a compilation of all the problems published by College Board in AP Physics C organized by topic.

The problems vary in level of difficulty and type and this book represents an invaluable resource for practice and review and should be used… often. Whether you are struggling or confident in a topic, you should be doing these problems as a reinforcement of ideas and concepts on a scale that could never be covered in the class time allotted.

The answers as presented are not the only method to solving many of these problems and physics teachers may present slightly different methods and/or different symbols and variables in each topic, but the underlying physics concepts are the same and we ask you read the solutions with an open mind and use these differences to expand your problem solving skills.

Finally, we *are* fallible and if you find any typographical errors, formatting errors or anything that strikes you as unclear or unreadable, please let us know so we can make the necessary announcements and corrections.

Table of Information and Equation Tables for AP Physics Exams

The accompanying Table of Information and Equation Tables will be provided to students when they take the AP Physics Exams. Therefore, students may NOT bring their own copies of these tables to the exam room, although they may use them throughout the year in their classes in order to become familiar with their content. **Check the Physics course home pages on AP Central for the latest versions of these tables (apcentral.collegeboard.com).**

Table of Information

For both the Physics B and Physics C Exams, the Table of Information is printed near the front cover of the multiple-choice section and on the green insert provided with the free-response section. The tables are identical for both exams except for one convention as noted.

Equation Tables

For both the Physics B and Physics C Exams, the equation tables for each exam are printed only on the green insert provided with the free-response section. The equation tables may be used by students when taking the free-response sections of both exams but NOT when taking the multiple-choice sections.

The equations in the tables express the relationships that are encountered most frequently in AP Physics courses and exams. However, the tables do not include all equations that might possibly be used. For example, they do not include many equations that can be derived by combining other equations in the tables. Nor do they include equations that are simply special cases of any that are in the tables. Students are responsible for understanding the physical principles that underlie each equation and for knowing the conditions for which each equation is applicable.

The equation tables are grouped in sections according to the major content category in which they appear. Within each section, the symbols used for the variables in that section are defined. However, in some cases the same symbol is used to represent different quantities in different tables. It should be noted that there is no uniform convention among textbooks for the symbols used in writing equations. The equation tables follow many common conventions, but in some cases consistency was sacrificed for the sake of clarity.

Some explanations about notation used in the equation tables:

1. The symbols used for physical constants are the same as those in the Table of Information and are defined in the Table of Information rather than in the right-hand columns of the tables.
2. Symbols in bold face represent vector quantities.
3. Subscripts on symbols in the equations are used to represent special cases of the variables defined in the right-hand columns.
4. The symbol Δ before a variable in an equation specifically indicates a change in the variable (i.e., final value minus initial value).
5. Several different symbols (e.g., d, r, s, h, ℓ) are used for linear dimensions such as length. The particular symbol used in an equation is one that is commonly used for that equation in textbooks.

TABLE OF INFORMATION DEVELOPED FOR 2012 (see note on cover page)

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg$\cdot$s^2
Universal gas constant,	$R = 8.31$ J/(mol$\cdot$K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7} \text{ (T}\cdot\text{m)/A}$
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5} \text{ N/m}^2 = 1.0 \times 10^{5} \text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

*IV. For mechanics and thermodynamics equations, W represents the work done on a system.

*Not on the Table of Information for Physics C, since Thermodynamics is not a Physics C topic.

ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

MECHANICS

$\upsilon = \upsilon_0 + at$

$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$

$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \leq \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}m\upsilon^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{\upsilon^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$\upsilon = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$x = x_{\max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Ne\upsilon_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

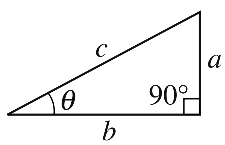
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

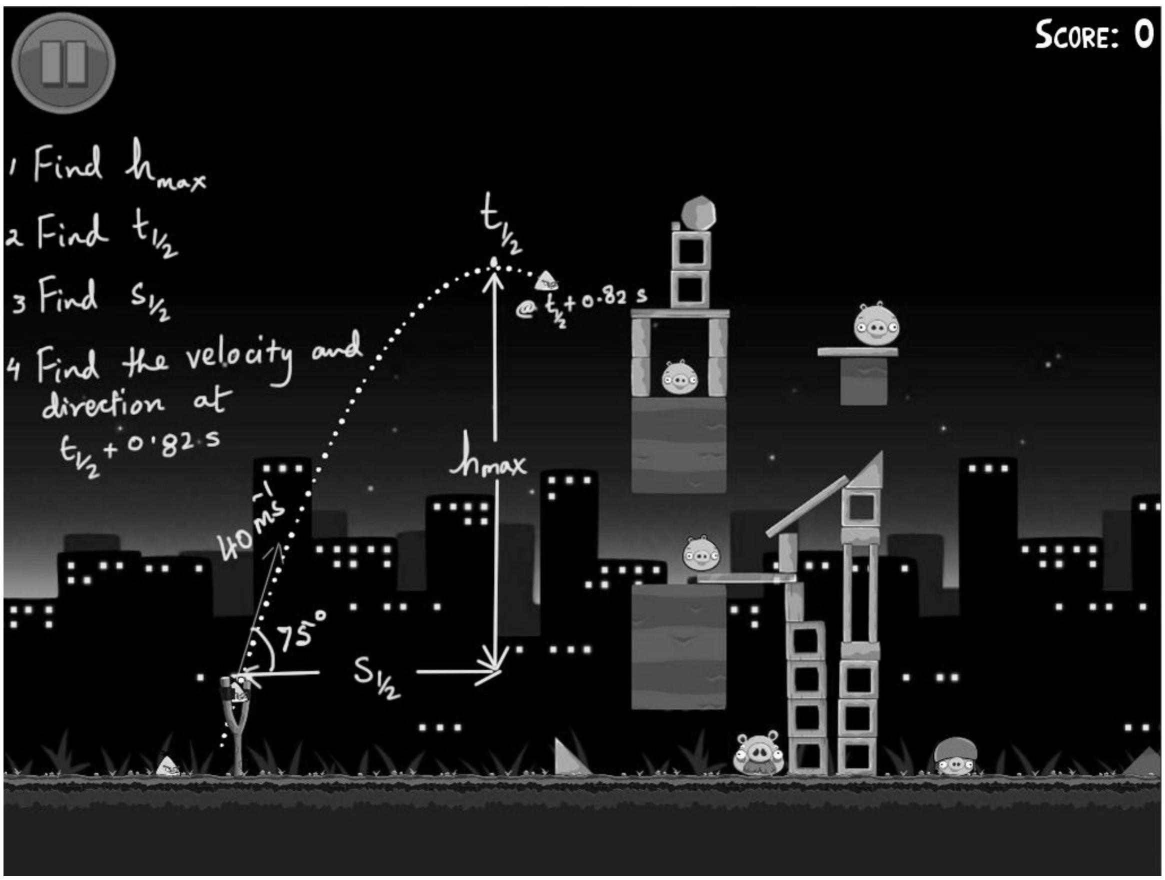
$$\int \frac{dx}{x} = \ln|x|$$

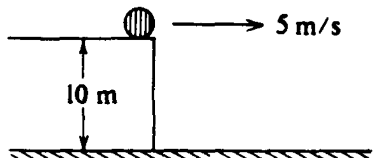
$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

Chapter 1

Kinematics

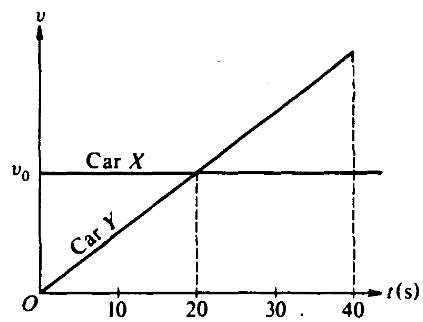




1 An object slides off a roof 10 meters above the ground with an initial horizontal speed of 5 meters per second as shown above. The time between the object's leaving the roof and hitting the ground is most nearly

(A) $\frac{1}{2}$ s (B) $\frac{1}{\sqrt{2}}$ s (C) $\sqrt{2}$ s (D) 2 s (E) $5\sqrt{2}$ s

<u>Questions 2-3</u>



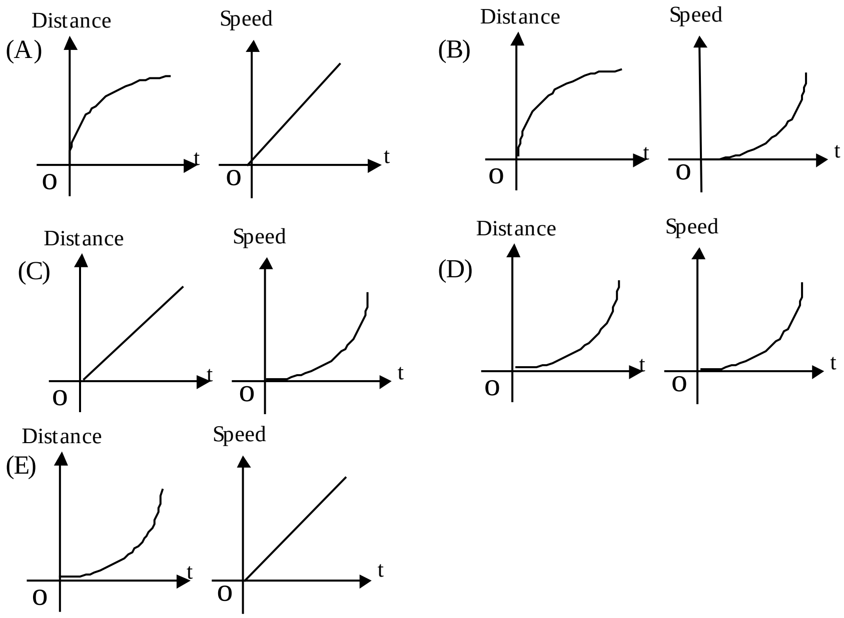
At time t = 0, car X traveling with speed v0 passes car Y. which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above.

2 Which of the following is true at time t = 20 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

3 From time t = 0 to time t = 40 seconds, the areas under both curves are equal. Therefore, which of the following is true at time t = 40 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

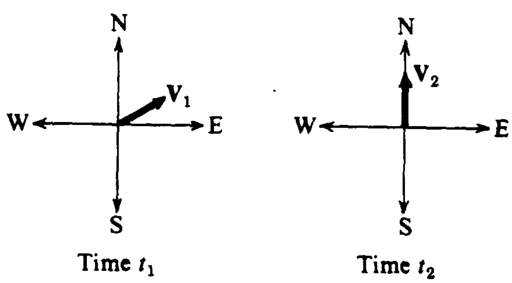
4 A body moving in the positive x direction passes the origin at time t = 0. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At t = 1 second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at t = 11 seconds is
(A) +99 m (B) +36 m (C) -36 m (D) -75 m (E) -99 m

5 Which of the following pairs of graphs shows the distance traveled versus time and the speed versus time for an object uniformly accelerated from rest?



6 An object released from rest at time t = 0 slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time interval from t = 1 second to t = 2 seconds is
(A) 1 m (B) 2 m (C) 3 m (D) 4m (E) 5 m

7 Two people are in a boat that is capable of a maximum speed of 5 kilometers per hour in still water, and wish to cross a river 1 kilometer wide to a point directly across from their starting point. If the speed of the water in the river is 5 kilometers per hour, how much time is required for the crossing?
(A) 0.05 hr (B) 0.1 hr (C) 1 hr (D) 10 hr
(E) The point directly across from the starting point cannot be reached under these conditions.



8 Vectors V_1 and V_2 shown above have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. The average acceleration of the object between time t_1 and t_2 was
(A) zero (B) directed north (C) directed west (D) directed north of east (E) directed north of west

9 A projectile is fired from the surface of the Earth with a speed of 200 meters per second at an angle of 30° above the horizontal. If the ground is level, what is the maximum height reached by the projectile?
(A) 5 m (B) 10 m (C) 500 m (D) 1,000 m (E) 2,000 m

10 A particle moves along the x-axis with a nonconstant acceleration described by a = 12t, where a is in meters per second squared and t is in seconds. If the particle starts from rest so that its speed v and position x are zero when t = 0, where is it located when t = 2 seconds?
(A) x = 12 m (B) x = 16m (C) x = 24 m (D) x = 32 m (E) x = 48 m

Questions 11-12

An object moving in a straight line has a velocity v in meters per second that varies with time t in seconds according to the following function. $\mathbf{v = 4 + 0.5\, t^2}$

11 The instantaneous acceleration of the object at t = 2 seconds is
(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2 (E) 8 m/s^2

12 The displacement of the object between t = 0 and t = 6 seconds is
(A) 22 m (B) 28 m (C) 40 m (D) 42 m (E) 60 m

13 A rock is dropped from the top of a 45-meter tower, and at the same time a ball is thrown from the top of the tower in a horizontal direction. Air resistance is negligible. The ball and the rock hit the level ground a distance of 30 meters apart. The horizontal velocity of the ball thrown was most nearly
(A) 5 m/s (B) 10 m/s (C) 14.1 m/s (D) 20 m/s (E) 28.3 m/s

14 In the absence of air friction, an object dropped near the surface of the Earth experiences a constant acceleration of about 9.8 m/s^2. This means that the
(A) speed of the object increases 9.8 m/s during each second
(B) speed of the object as it falls is 9.8 m/s
(C) object falls 9.8 meters during each second
(D) object falls 9.8 meters during the first second only
(E) derivative of the distance with respect to time for the object equals 9.8 m/s^2

15 A 500-kilogram sports car accelerates uniformly from rest, reaching a speed of 30 meters per second in 6 seconds. During the 6 seconds, the car has traveled a distance of
(A) 15 m (B) 30 m (C) 60 m (D) 90 m (E) 180 m

16 At a particular instant, a stationary observer on the ground sees a package falling with speed v1 at an angle to the vertical. To a pilot flying horizontally at constant speed relative to the ground, the package appears to be falling vertically with a speed v2 at that instant. What is the speed of the pilot relative to the ground?

(A) $v_1 + v_2$ (B) $v_1 - v_2$ (C) $v_2 - v_1$ (D) $\sqrt{v_1^2 - v_2^2}$ (E) $\sqrt{v_1^2 + v_2^2}$

17 An object is shot vertically upward into the air with a positive initial velocity. Which of the following correctly describes the velocity and acceleration of the object at its maximum elevation?

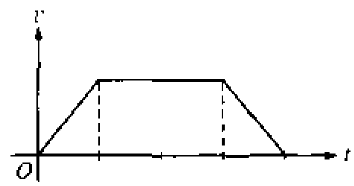
	Velocity	Acceleration
(A)	Positive	Positive
(B)	Zero	Zero
(C)	Negative	Negative
(D)	Zero	Negative
(E)	Positive	Negative

18 A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

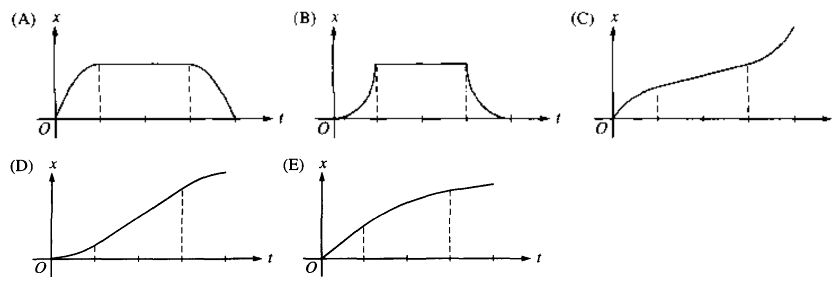
(A) h/4 (B) $\dfrac{h}{2\sqrt{2}}$ (C) h/2 (D) $\dfrac{h}{\sqrt{2}}$ (E) h

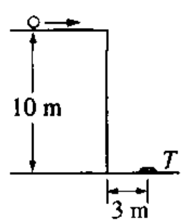
19 The velocity of a projectile at launch has a horizontal component vh and a vertical component vv. Air resistance is negligible. When the projectile is at the highest point of its trajectory, which of the following show the vertical and horizontal components of its velocity and the vertical component of its acceleration?

	Vertical Velocity	Horizontal Velocity	Vertical Acceleration
(A)	v_v	v_h	0
(B)	v_v	0	0
(C)	0	v_h	0
(D)	0	0	g
(E)	0	v_h	g



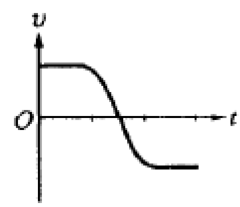
20. The graph above shows the velocity *v* as a function of time *t* for an object moving in a straight line. Which of the following graphs shows the corresponding displacement *x* as a function of time *t* for the same time interval?



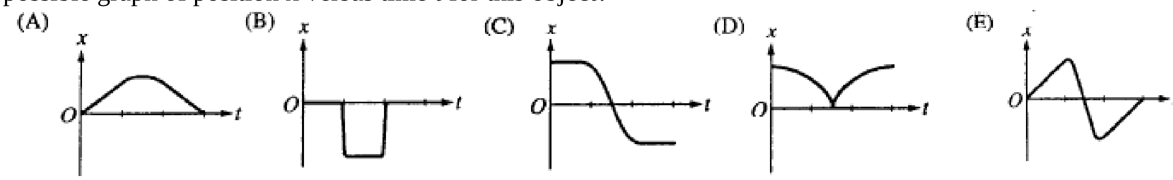


21. A target *T* lies flat on the ground 3 m from the side of a building that is 10 m tall, as shown above. A student rolls a ball off the horizontal roof of the building in the direction of the target. Air resistance is negligible. The horizontal speed with which the ball must leave the roof if it is to strike the target is most nearly

(A) 3/10 m/s (B) $\sqrt{2}$ m/s (C) $\frac{3}{\sqrt{2}}$ m/s (D) 3 m/s (E) $10\sqrt{\frac{5}{3}}$ m/s



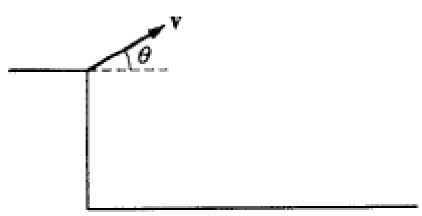
22 The graph above shows velocity v versus time t for an object in linear motion. Which of the following is a possible graph of position x versus time t for this object?



23 An object is dropped from rest from the top of a 400 m cliff on Earth. If air resistance is negligible, what is the distance the object travels during the first 6 s of its fall?
(A) 30 m (B) 60 m (C) 120 m (D) 180 m (E) 360 m

24 The position of an object is given by the equation $x = 3.0\ t^2 + 1.5\ t + 4.5$, where x is in meters and t is in seconds. What is the instantaneous acceleration of the object at t = 3.0 s?
(A) 3.0 m/s^2 (B) 6.0 m/s^2 (C) 9.0 m/s^2 (D) 19.5 m/s^2 (E) 36 m/s^2

25 A student is testing the kinematic equations for uniformly accelerated motion by measuring the time it takes for light-weight plastic balls to fall to the floor from a height of 3 m in the lab. The student predicts the time to fall using g as 9.80 m/s2 but finds the measured time to be 35% greater. Which of the following is the most likely cause of the large percent error?
(A) The acceleration due to gravity is 70% greater than 9.80 m/s2 at this location.
(B) The acceleration due to gravity is 70% less than 9.80 m/s2 at this location.
(C) Air resistance increases the downward acceleration.
(D) The acceleration of the plastic balls is not uniform.
(E) The plastic balls are not truly spherical.



Note: Figure not drawn to scale.

26 An object is thrown with velocity v from the edge of a cliff above level ground. Neglect air resistance. In order for the object to travel a maximum horizontal distance from the cliff before hitting the ground, the throw should be at an angle θ with respect to the horizontal of
(A) greater than 60° above the horizontal
(B) greater than 45° but less than 60° above the horizontal
(C) greater than zero but less than 45° above the horizontal
(D) zero
(E) greater than zero but less than 45° below the horizontal

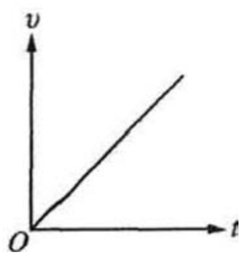
Questions 27-28

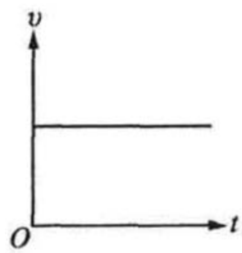
Starting from rest, a vehicle accelerates on a straight level road at the rate of 4.0 m/s2 for 5.0 s.

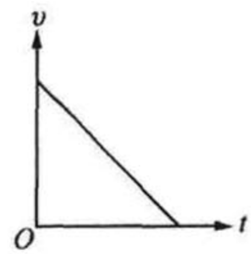
27 What is the speed of the vehicle at the end of this time interval?
(A) 1.3 m/s (B) 10 m/s (C) 20 m/s (D) 80 m/s (E) 100 m/s

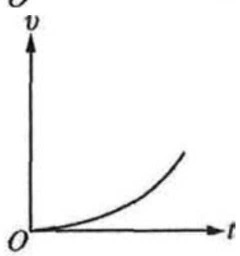
28 What is the total distance the vehicle travels during this time interval?
(A) 10 m (B) 20 m (C) 25 m (D) 40 m (E) 50 m

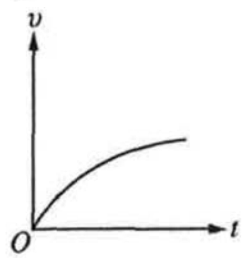
29 An object is thrown vertically upward in a region where g is constant and air resistance is negligible. Its speed is recorded from the moment it leaves the thrower's hand until it reaches its maximum height. Which of the following graphs best represents the object's speed υ versus time t ?

(A) 

(B) 

(C) 

(D) 

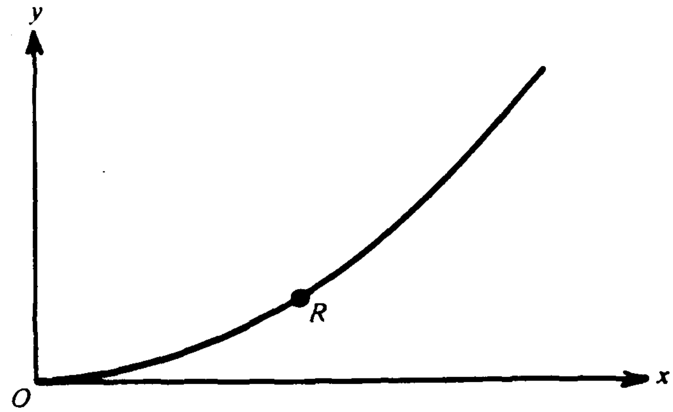
(E) 

30 If air resistance is negligible, the speed of a 2 kg sphere that falls from rest through a vertical displacement of 0.2 m is most nearly
(A) 1 m/s (B) 2m/s (C) 3 m/s (D) 4 m/s (E) 5 m/s

31 A projectile is launched from level ground with an initial speed $\upsilon 0$ at an angle θ with the horizontal. If air resistance is negligible, how long will the projectile remain in the air?
(A) $2v_0/ g$ (B) $2v_0 \cos\theta / g$ (C) $v_0 \cos\theta / g$ (D) $v_0 \sin\theta / g$ (E) $2v_0 \sin\theta / g$

32 An object of unknown mass is initially at rest and dropped from a height h. It reaches the ground with a velocity v_1. The same object is then raised again to the same height h but this time is thrown downward with velocity υ_1 It now reaches the ground with a new velocity υ_2. How is v_2 related to v_1?

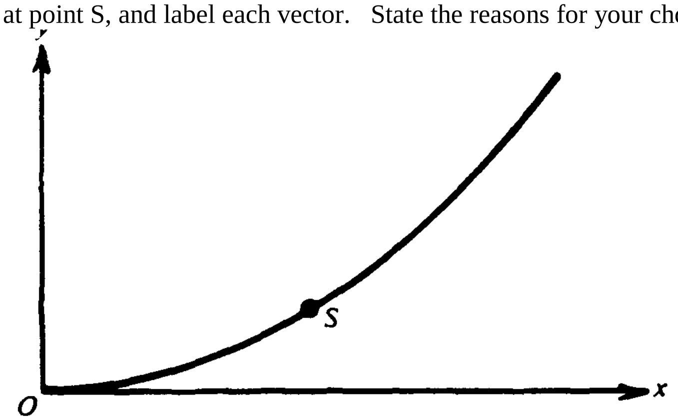
(A) $v_2 = v_1/2$ (B) $v_2 = v_1$ (C) $v_2 = \sqrt{2}\, v_1$ (D) $v_2 = 2v_1$ (E) $v_2 = 4v_1$



1983M1. A particle moves along the parabola with equation $y = \frac{1}{2}x^2$ shown above.

a. Suppose the particle moves so that the x-component of its velocity has the constant value $v_x = C$; that is, $x = Ct$
   i. On the diagram above, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point R, and label each vector.
   ii. Determine the y-component of the particle's velocity as a function of x.
   iii. Determine the y-component of the particle's acceleration.

b. Suppose, instead, that the particle moves along the same parabola with a velocity whose x-component is given by $v_x = C/(1+x^2)^{1/2}$
   i. Show that the particle's speed is constant in this case.
   ii. On the diagram below, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point S, and label each vector. State the reasons for your choices.



ANSWERS - AP Physics C Multiple Choice Practice – Kinematics

	Solution	Answer
1.	$y = \frac{1}{2} at^2$, remember, for horizontal projectiles $v_{oy} = 0$	C
2.	The cars were at the same spot at t = 0; we might as well call this the origin. The distance traveled by each during the next 20 seconds is the integral of v with respect to t; it is the area under the curve. Car X has traveled twice as far as Car Y in this time, because the rectangle has twice the area of the triangle.	A
3.	The areas are the same, so each car has traveled the same distance in the 40 seconds. But Car Y is traveling much faster than Car X (higher slope), so it is passing Car X.	B
4.	At t = 0, the object is at the origin. At t = 1, the object is at (24 m/s)(1 s) = 24 m. The initial speed is 24 m/s. The object is now given an acceleration of $-6\ m/s^2$. At t = 11, this is ten seconds after the acceleration begins. That is, reset the clock to zero at t = 1. $x = v_o t + \frac{1}{2} at^2$	C
5.	The speed vs time graph values should represent the slope of the distance time graph	E
6.	Since under uniform acceleration x is proportional to t^2, if the object travels 1m in 1 second, it should travel 4 m in 2 seconds. Which means from the time 1 to 2 seconds, the object traveled the additional 3 m.	C
7.	In order to travel directly across a river, the boat's velocity must have a component that cnacels the river's current. In order to do this, the boat must point directly upstream. This leaves no way for the boat to have any component of its velocity across the river and hence, cannot make the trip.	E
8.	$\vec{a} = \vec{V}_2 - \vec{V}_1$	E
9.	$v_y^2 = v_{oy}^2 + 2gy$ where $v_{oy} = v_o \sin\theta$	C
10.	v(t) is the integral of a(t) and x(t) is the integral of v(t). Integrating the given function twice and plugging in the initial conditions gives $x = 2t^3$	B
11.	a = dv/dt = t	A
12.	x is the integral of v, which gives $x = 4t + t^3/6$	E
13.	$y = \frac{1}{2} at^2$, remember, for horizontal projectiles $v_{oy} = 0$. Since the cliff is 45 m high, the rocks take 3 seconds to strike the ground. In this time, the rock thrown horizontally travelled 30 m. v = x/t	B
14.	$9.8\ m/s^2$ can also be stated as 9.8 meters per second, **per second**	A
15.	average speed = d/t = $(v_o + v_f)/2$	D
16.	velocity of package relative to observer on ground $v_{pg} = v_1 = \searrow$ velocity of package relative to pilot $v_{pp} = v_2 = \downarrow$ velocity of pilot relative to ground $v_{po} = \longrightarrow$ Putting these together into a right triangle yields $v_{pg}^2 + v_2^2 = v_1^2$	D
17.	While the object momentarily stops at its peak, it never stops accelerating downward.	D

18. Maximum height of a projectile is found from $v_y = 0$ at max height and $v_y{}^2 = v_{iy}{}^2 + 2gh$ and gives $h_{max} = v_{iy}{}^2/2g = (v_i \sin\theta)^2/2g$. Fired straight up, $\theta = 90°$ and we have $v_i = \sqrt{2gh}$ — C

Plugging this initial velocity into the equation for a 45º angle ($\sin 45° = \frac{1}{\sqrt{2}}$) gives

$h_{new} = (\sqrt{2gh}\ \frac{1}{\sqrt{2}})^2/2g = h/2$

19. horizontal velocity v_x remains the same thorughout the flight. g remains the same as well. — E

20. A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows an increasing slope, then a constant slope, then a decreasing slope (to zero) — D

21. For a horizontal projectile, the initial speed does not affect the time in the air. Use $v_{0y} = 0$ with 10 m = ½ gt^2 to get $t = \sqrt{2}$ For the horzontal part of the motion; $v = d/t$ — C

22. A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows a constant slope, then a decreasing slope to zero, becoming negative and increasing, then a constant slope. Note this is an analysis of the *values* of v, not the slope of the graph itself — A

23. For a dropped object: $d = ½ gt^2$ — D

24. Acceleration is the second derivative of position. — B

25. By process of elimination (A and B are unrealistic; C is wrong, air resistance should decrease the acceleration; E is irrelevant) — D

26. The 45° angle gives the maximum horizontal travel to the original elevation, but the smaller angle causes the projectile to have a greater horizontal component of velocity, so given the additional time of travel allows such a trajectory to advance a greater horizontal distance. In other words given enough time the smaller angle of launch gives a parabola which will eventual cross the parabola of the 45° launch. — C

27. $v = v_o + at$ — C

28. $x = v_o t + ½ at^2$ — E

29. It is the line whose slope is equal to -9.8 m/s^2 (neative and constant) — C

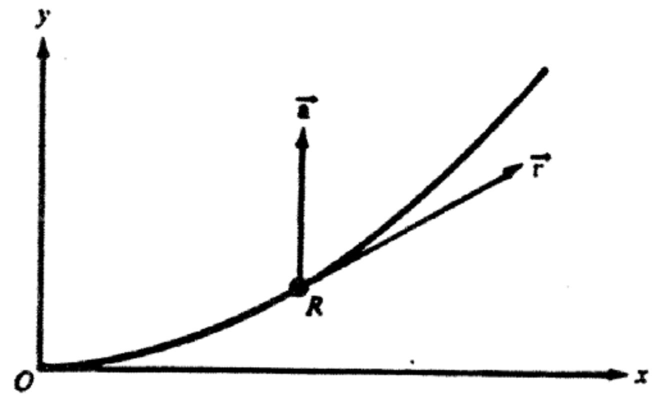
30. $v^2 = v_o{}^2 + 2gy$ (mass is irrelevant) — B

31. Time to reach maximum height can be found from $v_f = 0 = v_{oy} + at = v_o \sin\theta - gt$. Solve for time to maximum height and double it to find the total time in the air — E

32. $v_1{}^2 = 2gh$; $v_2{}^2 = v_1{}^2 + 2gh = 2gh + 2gh = 4gh = 2(v_1{}^2)$ — C

1983M1

a) i. The velocity vector is tangent to the path. Since v_x is constant, $a_x = 0$, and the acceleration vector has only a positive y component.

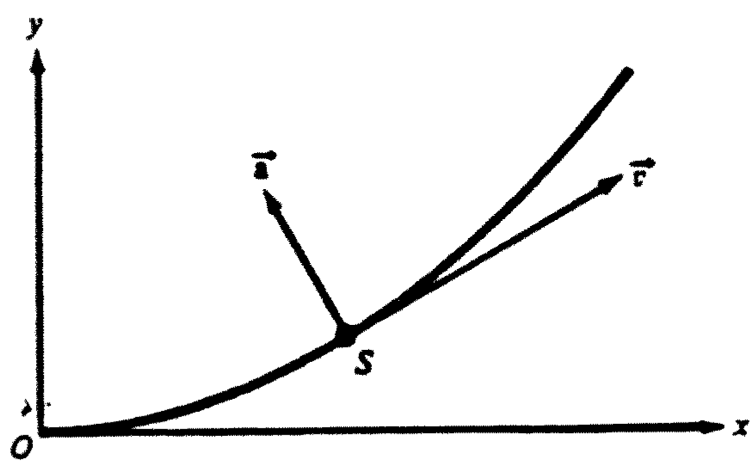


ii. The y component of the velocity is $v_y = dy/dt$, and by the chain rule $dy/dt = (dy/dx)(dx/dt)$ since $y = ½ x^2$, $dy/dt = x$ and $v_y = x(dx/dt) = Cx$

iii. The acceleration is given by $a_y = dv_y/dt$, so $a_y = C(dx/dt) = C^2$

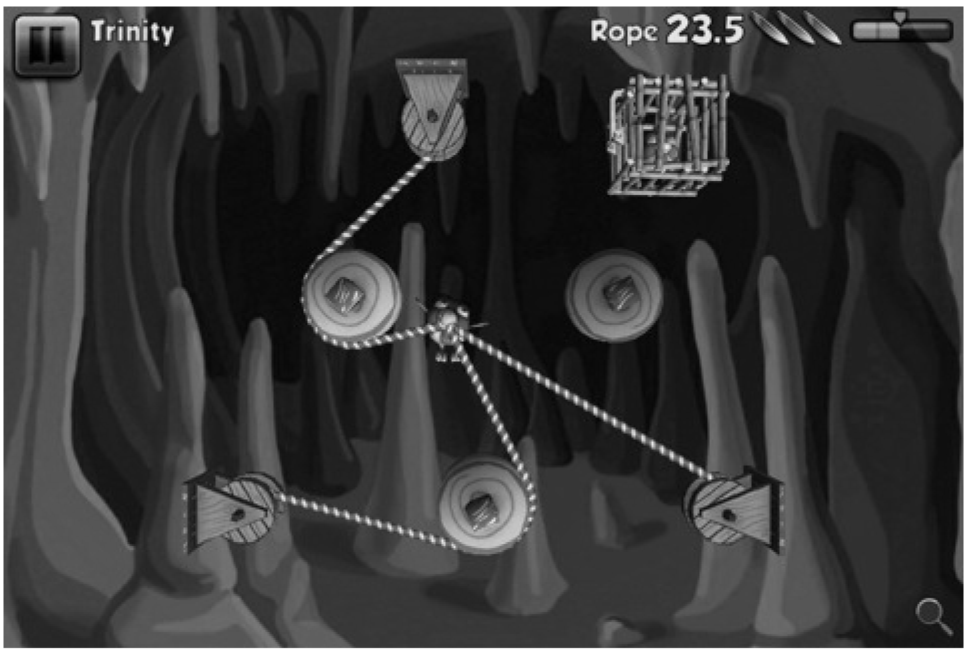
b) i. The speed is given by $v = \sqrt{{v_x}^2 + {v_y}^2}$, by the chain rule, $v_y = dy/dt = (dy/dx)(dx/dt) = xv_x$ so $v = \sqrt{{v_x}^2(1 + x^2)} = \sqrt{\frac{C^2}{1+x^2}(1 + x^2)} = C$

ii. Again the velocity vector is tangent to the path, but since the speed is constant, there is no component of the acceleration along the path, so a is centripetal, perpendicular to v

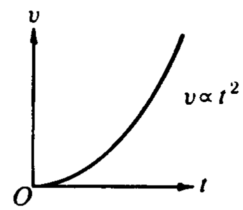


Chapter 2

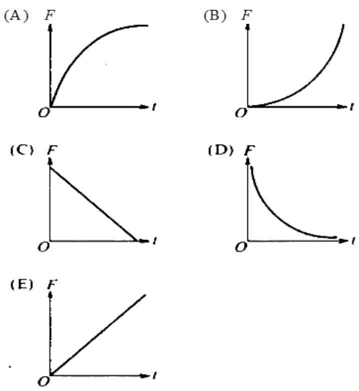
Dynamics

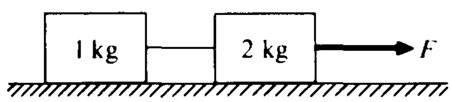


SECTION A – Linear Dynamics

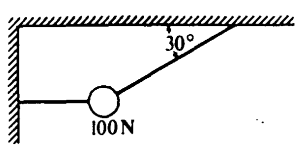


1. The parabola above is a graph of speed v as a function of time t for an object. Which of the following graphs best represents the magnitude F of the net force exerted on the object as a function of time t?





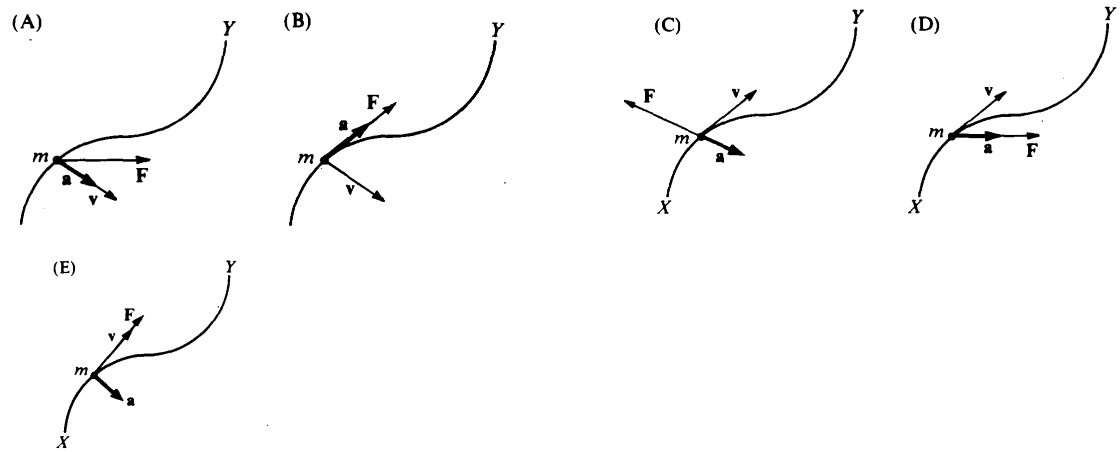
2. When the frictionless system shown above is accelerated by an applied force of magnitude the tension in the string between the blocks is (A) 2F (B) F (C) 2/3 F (D) ½ F (E) 1/3 F

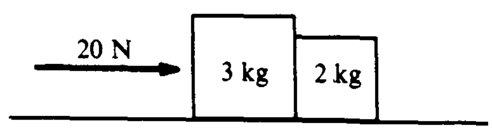


3. A 100-newton weight is suspended by two cords as shown in the figure above. The tension in the slanted cord is (A) 50 N (B) 100 N (C) 150 N (D) 200 N (E) 250 N

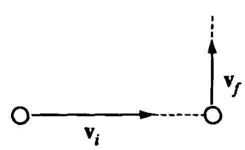
4. A particle of mass m moves along a straight path with a speed v defined by the function $v = bt^2 + c$, where b and c are constants and t is time. What is the magnitude F of the net force on the particle at time $t = t_1$?
(A) $bt_1^2 + c$ (B) $3mbt_1 + 2c$ (C) mbt_1 (D) $mbt_1 + c$ (E) $2mbt_1$

5. A mass m moves on a curved path from point X to point Y. Which of the following diagrams indicates a possible combination of the net force F on the mass, and the velocity v and acceleration a of the mass at the location shown?

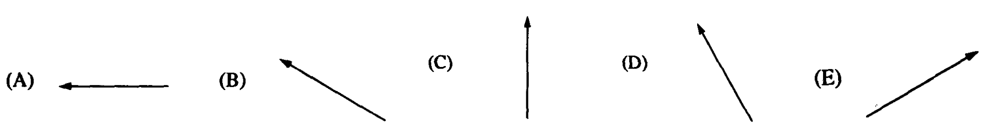




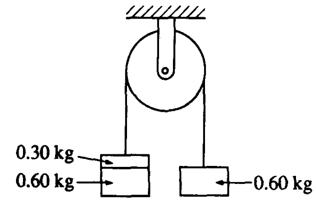
6. Two blocks are pushed along a horizontal frictionless surface by a force of 20 newtons to the right, as shown above. The force that the 2-kilogram block exerts on the 3-kilogram block is
(A) 8 newtons to the left (B) 8 newtons to the right (C) 10 newtons to the left
(D) 12 newtons to the right (E) 20 newtons to the left



7. A ball initially moves horizontally with velocity v_i, as shown above. It is then struck by a stick. After leaving the stick, the ball moves vertically with a velocity v_f, which is smaller in magnitude than v_i. Which of the following vectors best represents the direction of the average force that the stick exerts on the ball?

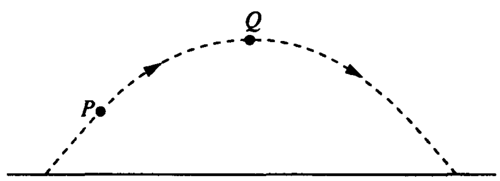


8. If F_1 is the magnitude of the force exerted by the Earth on a satellite in orbit about the Earth and F_2 is the magnitude of the force exerted by the satellite on the Earth, then which of the following is true?
(A) F_1 is much greater than F_2. (B) F_1 is slightly greater than F_2. (C) F_1 is equal to F_2. (D) F_2 is slightly greater than F_1 (E) F_2 is much greater than F_1



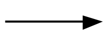
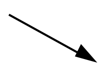
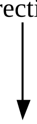
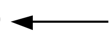
9. Two 0.60-kilogram objects are connected by a thread that passes over a light, frictionless pulley, as shown above. The objects are initially held at rest. If a third object with a mass of 0.30 kilogram is added on top of one of the 0.60-kilogram objects as shown and the objects are released, the magnitude of the acceleration of the 0.30-kilogram object is most nearly
(A) 10.0 m/s^2 (B) 6.0 m/s^2 (C) 3.0 m/s^2 (D) 2.0 m/s^2 (E) 1.0 m/s^2

Questions 10-11

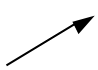
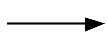
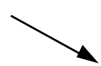
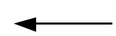


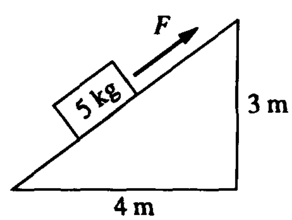
A ball is thrown and follows a parabolic path, as shown above. Air friction is negligible. Point Q is the highest point on the path.

10. Which of the following best indicates the direction of the acceleration, if any, of the ball at point Q ?

(A)  (B)  (C)  (D)  (E) There is no acceleration of the ball at point Q.

11. Which of the following best indicates the direction of the net force on the ball at point P ?

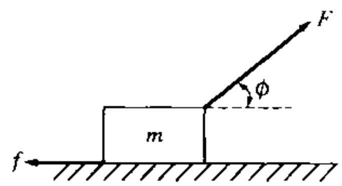
(A)  (B)  (C)  (D)  (E) 



12. A block of mass 5 kilograms lies on an inclined plane, as shown above. The horizontal and vertical supports for the plane have lengths of 4 meters and 3 meters, respectively. The coefficient of friction between the plane and the block is 0.3. The magnitude of the force F necessary to pull the block up the plane with constant speed is most nearly
(A) 30 N (B) 42 N (C) 49 N (D) 50 N (E) 58 N

13. The position of a toy locomotive moving on a straight track along the x-axis is given by the equation $x = t^3 - 6t^2 + 9t$, where x is in meters and t is in seconds. The net force on the locomotive is equal to zero when t is equal to
(A) zero (B) 2 s (C) 3 s (D) 4 s (E) 5 s

Questions 14-15

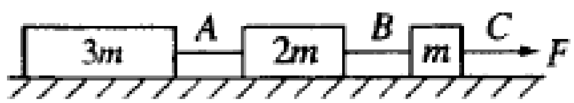


A block of mass m is accelerated across a rough surface by a force of magnitude F that is exerted at an angle ϕ with the horizontal, as shown above. The frictional force on the block exerted by the surface has magnitude f.

14. What is the acceleration of the block?
(A) F/m (B) $(F\cos\phi)/m$ (C) (F-f)/m (D) $(F\cos\phi - f)/m$ (E) $(F\sin\phi - mg)/m$

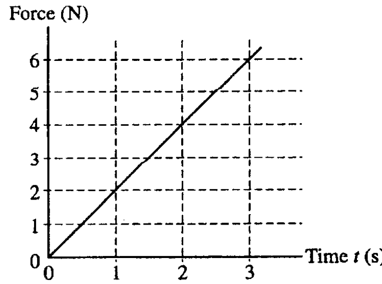
15. What is the coefficient of friction between the block and the surface?
(A) f/mg (B) mg/f (C) $(mg - F\cos\phi)/f$ (D) $f/(mg - F\cos\phi)$ (E) $f/(mg - F\sin\phi)$

16. An object is released from rest at time t = 0 and falls through the air, which exerts a resistive force such that the acceleration a of the object is given by a = g - bv, where v is the object's speed and b is a constant. If limiting cases for large and small values of t are considered, which of the following is a possible expression for the speed of the object as an explicit function of time?
(A) $v = g(1 - e^{-bt})/b$ (B) $v = (ge^{bt})/b$ (C) $v = gt - bt^2$ (D) $v = (g + a)t/b$ (E) $v = v_0 + gt$, $v_0 \neq O$



17. Three blocks of masses 3m, 2m, ands are connected to strings A, B, and C as shown above. The blocks are pulled along a rough surface by a force of magnitude F exerted by string C. The coefficient of friction between each block and the surface is the same. Which string must be the strongest in order not to break?
(A) A (B) B (C) C (D) They must all be the same strength.
(E) It is impossible to determine without knowing the coefficient of friction.

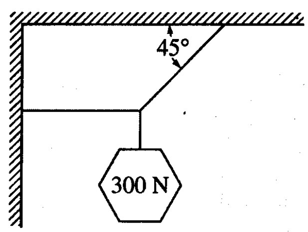
Questions 18-19



A block of mass 3 kg, initially at rest, is pulled along a frictionless, horizontal surface with a force shown as a function of time t by the graph above.

18. The acceleration of the block at t = 2 s is
(A) 3/4 m/s^2 (B) 4/3 m/s^2 (C) 2 m/s^2 (D) 8 m/s^2 (E) 12 m/s^2

19. The speed of the block at t = 2 s is
(A) 4/3 m/s (B) 8/3 m/s (C) 4 m/s (D) 8 m/s (E) 24 m/s



20. An object weighing 300 N is suspended by means of two cords, as shown above. The tension in the horizontal cord is
(A) 0 N (B) 150 N (C) 210 N (D) 300 N (E) 400 N

Questions 21-23

A small box is on a ramp tilted at an angle θ above the horizontal. The box may be subject to the following forces: frictional
(f) ,gravitational (mg), pulling or pushing (F_P) and normal (N). In the following free-body diagrams for the box, the lengths of the vectors are proportional to the magnitudes of the forces.

Figure A

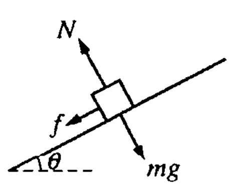


Figure B

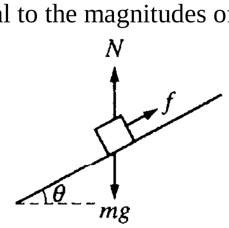


Figure C

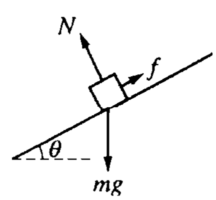


Figure D

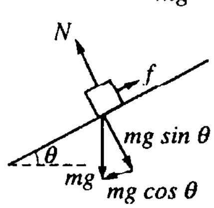
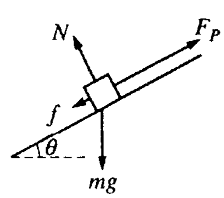


Figure E



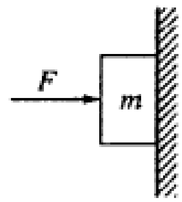
21. Which figure best represents the free-body diagram for the box if it is accelerating up the ramp?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

22. Which figure best represents the free-body diagram for the box if it is at rest on the ramp?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

23. Which figure best represents the free-body diagram for the box if it is sliding down the ramp at constant speed?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

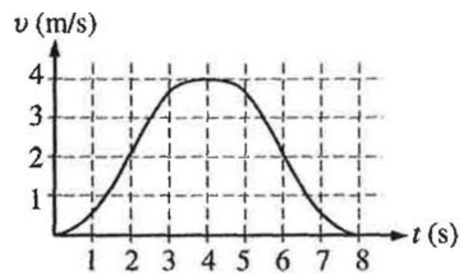
24. Two blocks of masses M and m, with M > m, are connected by a light string. The string passes over a frictionless pulley of negligible mass so that the blocks hang vertically. The blocks are then released from rest. What is the acceleration of the block of mass M?

(A) g (B) $\frac{M-m}{M}g$ (C) $\frac{M+m}{M}g$ (D) $\frac{M+m}{M-m}g$ (E) $\frac{M-m}{M+m}g$

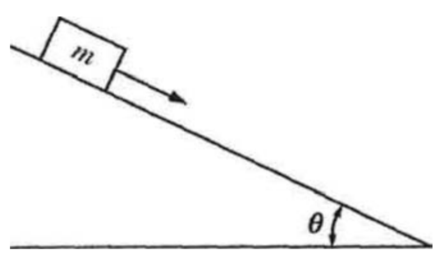


25. A horizontal force F pushes a block of mass m against a vertical wall. The coefficient of friction between the block and the wall is μ. What value of F is necessary to keep the block from slipping down the wall?
(A) mg (B) μ mg (C) mg/μ (D) mg(1 - μ) (E) mg(1 + μ)

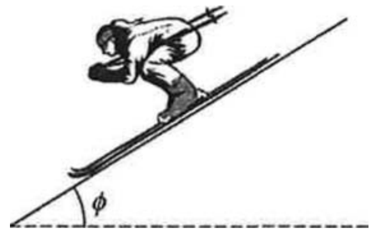
26. A dart gun is used to fire two rubber darts with different but unknown masses, M_1 and M_2. The gun exerts the same constant force on each dart, but its magnitude F is unknown. The magnitudes of the accelerations of both darts, a_1 and a_2, respectively, are measured. Which of the following can be determined from these data?
(A) F only (B) M_1 and M_2 only (C) The ratio of M_1 and M_2 only
(D) F and the ratio of M_1 and M_2 only (E) F, $M_{1;}$and M_2



27. The velocity υ of an elevator moving upward between adjacent floors is shown as a function of time t in the graph above. At which of the following times is the force exerted by the elevator floor on a passenger the <u>least</u>?
(A) 1 s (B) 3 s (C) 4 s (D) 5 s (E) 6 s



28. An object of mass m moves with acceleration a down a frictionless incline that makes an angle with the horizontal, as shown above. If N is the normal force exerted by the plane on the block, which of the following is correct?
(A) N = mg (B) N = ma (C) a = mg sin θ (D) a = g sin θ (E) a = mg cos θ



29. A skier slides at constant speed down a slope inclined at an angle ϕ to the horizontal, as shown above. If air resistance is negligible, the coefficient of friction μ between the skis and the snow is equal to

    (A) $\dfrac{1}{\tan\phi}$ (B) $\dfrac{1}{\cos\phi}$ (C) sin ϕ (D) cos ϕ (E) tan ϕ

30. The object of mass m shown above is dropped from rest near Earth's surface and experiences a resistive force of magnitude kv, where υ is the speed of the object and k is a constant. Which of the following expressions can be used to find υ as a function of time t? (Assume that the direction of the gravitational force is positive.)

    (A) $\displaystyle\int_0^v \frac{dv}{mg-kv} = \int_0^t \frac{dt}{m}$

    (B) $\displaystyle\int_0^t \frac{dv}{mg-kv} = \int_0^v \frac{dt}{m}$

    (C) $\displaystyle\int_0^v \frac{dv}{kv} = \int_0^t \frac{dt}{m}$

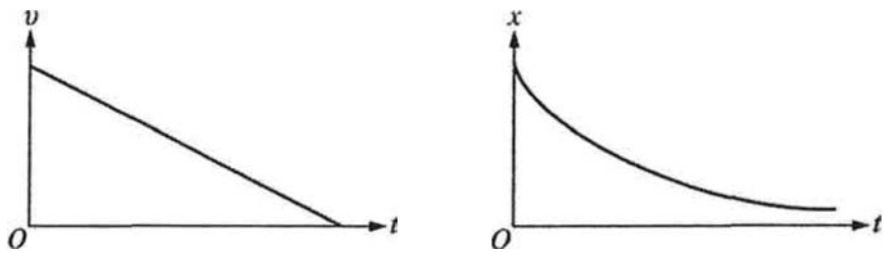
    (D) $\displaystyle\int_0^v (mg-kv)dv = \int_0^t m\,dt$

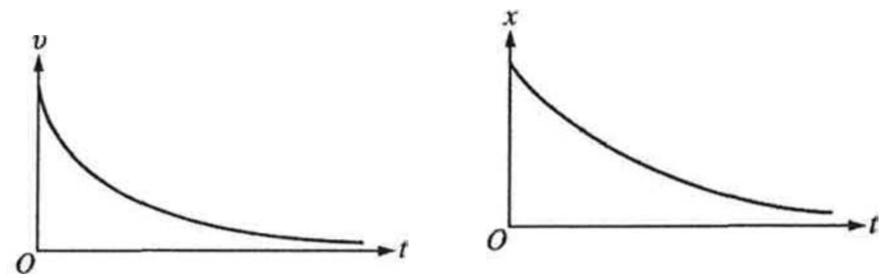
    (E) $\displaystyle\int_0^v (mg-kv)dt = \int_0^t m\,dv$

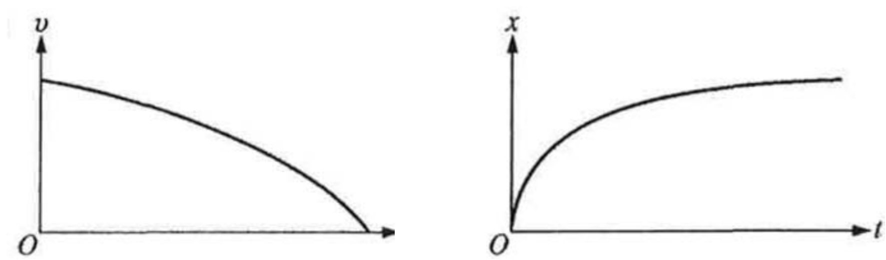
31. A 5 kg object is propelled from rest at time t =0 by a net force F that always acts in the same direction. The magnitude of F in newtons is given as a function of t in seconds by F = 0.5t. What is the speed of the object at t = 4 s?

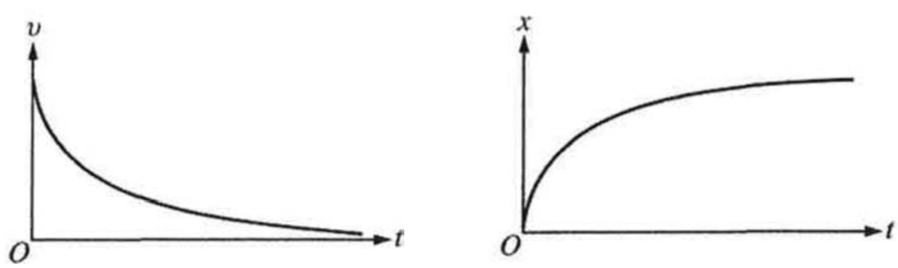
    (A) 0.5 m/s (B) 0.8 m/s (C) 2.0 m/s (D) 4.0 m/s (E) 8.0 m/s

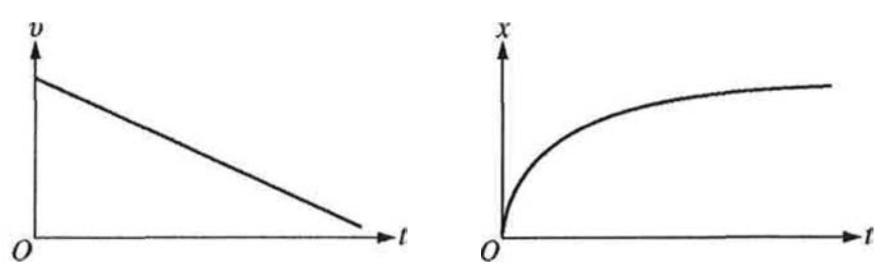
32. A car is traveling along a straight, level road when it runs out of gas at time t = 0. From this time on, the net force on the car is a resistive force of -kv, where v is velocity and k is a constant. Which of the following pairs of graphs best represents the speed υ and position x of the car as functions of time after t = 0?

(A)


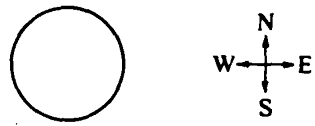
(B)


(C)


(D)


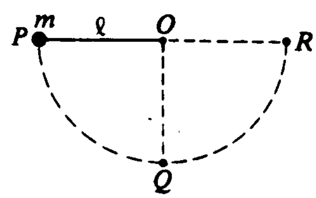
(E)


SECTION B – Circular Motion



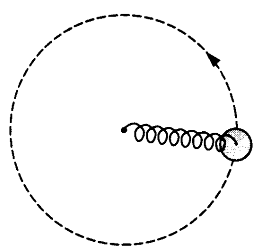
View of Track from Above

1. A racing car is moving around the circular track of radius 300 meters shown above. At the instant when the car's velocity is directed due east, its acceleration is directed due south and has a magnitude of 3 meters per second squared. When viewed from above, the car is moving
   (A) clockwise at 30 m/s (B) clockwise at 10 m/ s (C) counterclockwise at 30 m/s
   (D) counterclockwise at 10 m/s (E) with constant velocity



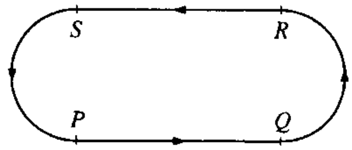
2. A ball of mass m is attached to the end of a string of length Q as shown above. The ball is released from rest from position P. where the string is horizontal. It swings through position Q. where the string is vertical, and then to position R. where the string is again horizontal. What are the directions of the acceleration vectors of the ball at positions Q and R?

	Position Q	Position R
(A)	Downward	Downward
(B)	Downward	To the right
(C)	Upward	Downward
(D)	Upward	To the left
(E)	To the right	To the left

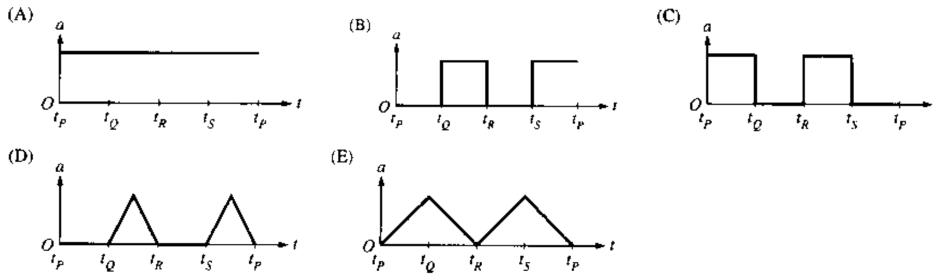


Top View

3. A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view above. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible. What is the centripetal force on the disc?
   (A) 0.3 N (B) 3N (C) 10 N (D) 300 N (E) 1,000 N



4. A figure of a dancer on a music box moves counterclockwise at constant speed around the path shown above. The path is such that the lengths of its segments, PQ, QR, RS, and SP, are equal. Arcs QR and SP are semicircles. Which of the following best represents the magnitude of the dancer's acceleration as a function of time t during one trip around the path, beginning at point P ?



5. One end of a string is fixed. An object attached to the other end moves on a horizontal plane with uniform circular motion of radius R and frequency *f*. The tension in the string is F_s. If both the radius and frequency are doubled, the tension is

(A) (B) $\frac{1}{2}F_s$ (C) $2F_s$ (D) $4\ F_s$ (E) $8\ F_s$

SECTION A – Linear Dynamics

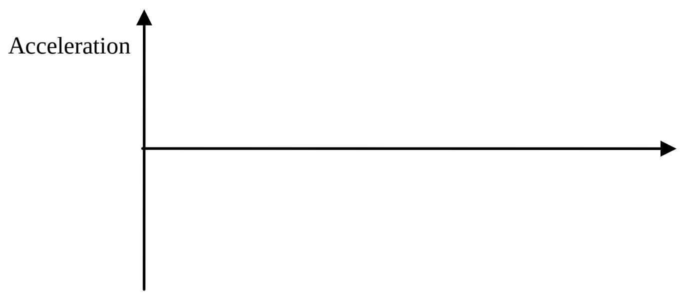
1975M1. A sphere of mass m is released from rest. As it falls, the air exerts a retarding force on the sphere that is proportional to the sphere's velocity ($F_R = -kv$). Neglect the buoyancy force of the air.

a. On the circles below draw vectors representing the forces acting on the sphere
   i. just after it is released and
   ii. after it has been falling- for a long time and reached terminal velocity. Give each vector a descriptive label

(i) (ii)

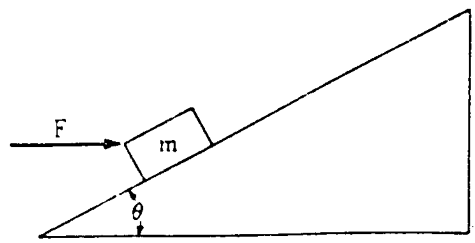
b. Determine the terminal velocity of the sphere

c. Draw the following three graphs for the sphere's motion clearly showing significant features of the motion just after the sphere is released as well as after a long time.
   i. Acceleration as a function of time ii. Velocity as a function of time iii. Position as a function of time.



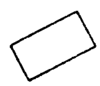
1977M1 (modified) A block of mass m, which has an initial velocity v_o at time t = 0, slides on a horizontal surface. If the sliding friction force f exerted on the block by the surface is directly proportional to its velocity (that is, f = -kv) determine the following:

a. The acceleration a of the block in terms of m, k, and v.
b. The speed v of the block as a function of time t.
c. The total distance the block slides.



1981M1. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.

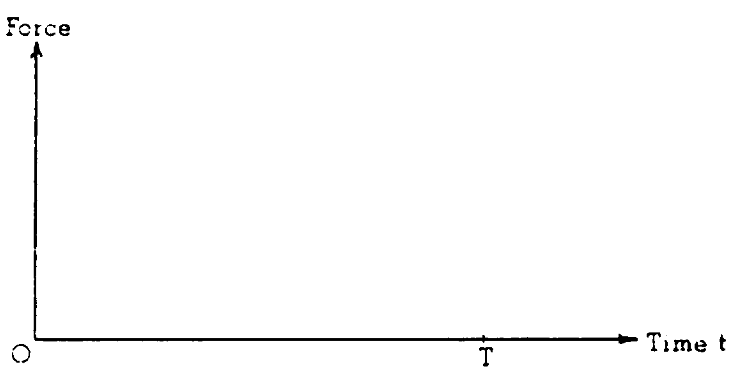
a. On the diagram of the block below, draw and label all the forces that act on the block as it slides up the plane.



b. Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.
c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?

1982M2 (modified) A car of mass M moves with an initial speed v_o on a straight horizontal road. The car is brought to rest by braking in such a way that the speed of the car is given as a function of time t by $v = (v_o^2 - Rt/M)^{1/2}$ where R is a constant.

a. Determine the time it takes to bring the car to a complete stop.
b. Develop an equation for the acceleration of the car as a function of time t.
c. On the axes below, sketch the magnitude of the braking force as a function of time t.



1984M3. A small body of mass m located near the Earth's surface falls from rest in the Earth's gravitational field. Acting on the body is a resistive force of magnitude kmv, where k is a constant and v is the speed of the body.

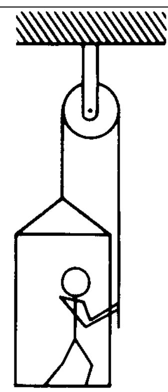
a. On the diagram below, draw and identify all of the forces acting on the body as it falls.

b. Write the differential equation that represents Newton's second law for this situation.

c. Determine the terminal speed v_T of the body.

d. Integrate the differential equation once to obtain an expression for the speed v as a function of time t. Use the condition that v = 0 when t = 0.

e. On the axes provided below, draw a graph of the speed v as a function of time t.



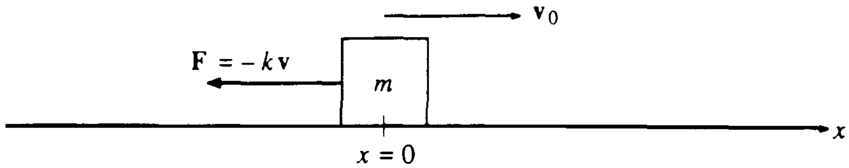
1986M1. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.

a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

b. What is the tension in the rope under these new conditions?

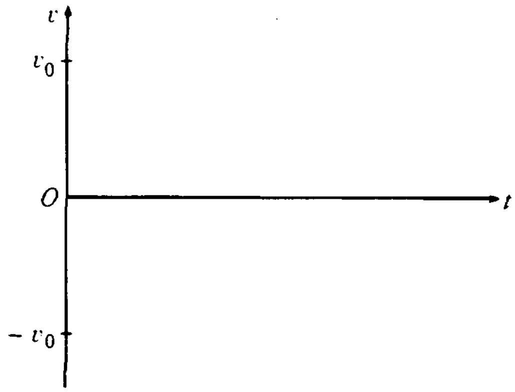
c. Under these conditions, what is the force exerted by the platform on the person?



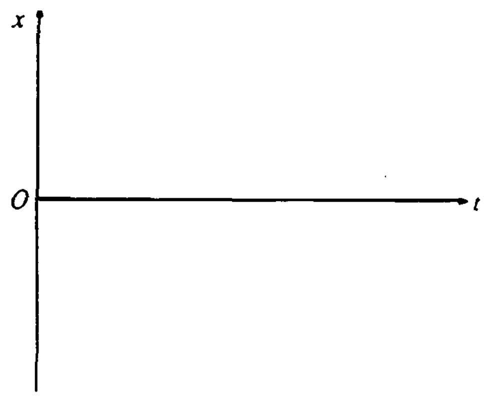
1990M1. An object of mass m moving along the x-axis with velocity v is slowed by a force F = -kv, where k is a constant. At time t = 0, the object has velocity v_o at position x = 0, as shown above.

a. What is the initial acceleration (magnitude and direction) produced by the resistance force?

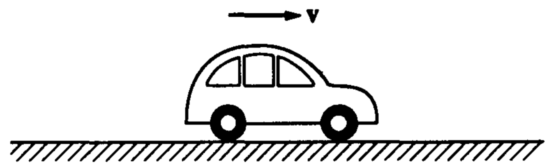
b. Derive an equation for the object's velocity as a function of time t, and sketch this function on the axes below. Let a velocity directed to the right be considered positive.



c. Derive an equation for the distance the object travels as a function of time t and sketch this function on the axes below.



d. Determine the distance the object travels from t = 0 to t = ∞.



1993M2. A car of mass m, initially at rest at time t = 0, is driven to the right, as shown above, along a straight, horizontal road with the engine causing a constant force F_o to be applied. While moving, the car encounters a resistance force equal to -kv, where v is the velocity of the car and k is a positive constant.

a. The dot below represents the center of mass of the car. On this figure, draw and label vectors to represent all the forces acting on the car as it moves with a velocity v to the right.

b. Determine the horizontal acceleration of the car in terms of k, v, F_o, and m.

c. Derive the equation expressing the velocity of the car as a function of time t in terms of k, v, F_o, and m.

d. Sketch a graph of the car's velocity v as a function of time t. Label important values on the vertical axis.

e. Sketch a graph of the car's acceleration a as a function of time t. Label important values on the vertical axis.

1996M2. A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5 m/s^2

a. Determine the upward force exerted by the horizontal platform on the box as it is lowered.

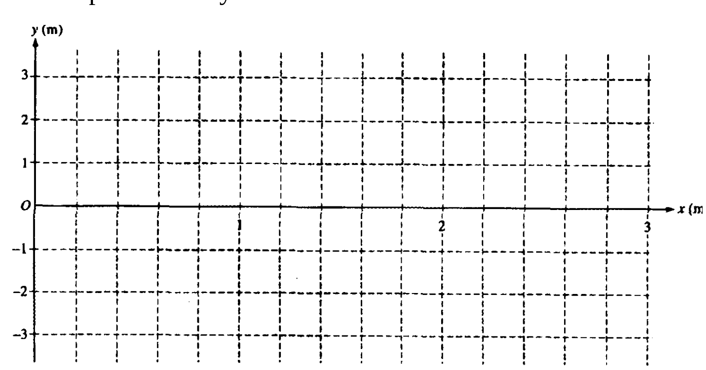
At time t = 0, the forklift also begins to move forward with an acceleration of 2 m/s^2 while lowering the box as described above. The box does not slip or tip over.

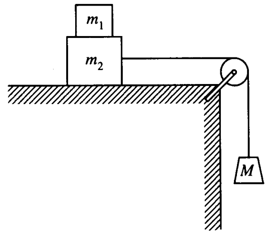
b. Determine the frictional force on the box.

c. Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.

d. Determine an equation for the path of the box that expresses y as a function of x (and <u>not</u> of t), assuming that, at time t = 0, the box has a horizontal position x = 0 and a vertical position y = 2 m above the ground, with zero velocity.

e. On the axes below sketch the path taken by the box





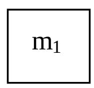
1998M3. Block 1 of mass m_1 is placed on block 2 of mass m_2 which is then placed on a table. A string connecting block 2 to a hanging mass M passes over a pulley attached to one end of the table, as shown above. The mass and friction of the pulley are negligible. The coefficients of friction between blocks 1 and 2 and between block 2 and the tabletop are nonzero and are given in the following table.

	Coefficient Between Blocks 1 and 2	Coefficient Between Block 2 and the Tabletop
Static	μ_{s1}	μ_{s2}
Kinetic	μ_{k1}	μ_{k2}

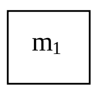
Express your answers in terms of the masses, coefficients of friction, and g, the acceleration due to gravity.

a. Suppose that the value of M is small enough that the blocks remain at rest when released. For each of the following forces, determine the magnitude of the force and draw a vector on the block provided to indicate the direction of the force if it is nonzero.

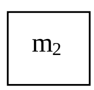
i. The normal force N_1 exerted on block 1 by block 2



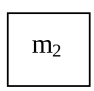
ii. The friction force f_1 exerted on block 1 by block 2



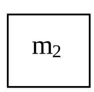
iii. The force T exerted on block 2 by the string



iv. The normal force N_2 exerted on block 2 by the tabletop



v. The friction force f_2 exerted on block 2 by the tabletop



b. Determine the largest value of M for which the blocks can remain at rest.

c. Now suppose that M is large enough that the hanging block descends when the blocks are released. Assume that blocks 1 and 2 are moving as a unit (no slippage). Determine the magnitude a of their acceleration.

d. Now suppose that M is large enough that as the hanging block descends, block 1 is slipping on block 2. Determine each of the following.

i. The magnitude a_1 of the acceleration of block 1

ii. The magnitude a_2 of the acceleration of block 2

2000M2. A rubber ball of mass m is dropped from a cliff. As the ball falls, it is subject to air drag (a resistive force caused by the air). The drag force on the ball has magnitude bv^2, where b is a constant drag coefficient and v is the instantaneous speed of the ball. The drag coefficient b is directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

a. On the figure below, draw and label all the forces on the ball at some instant before it reaches terminal speed.

b. State whether the magnitude of the acceleration of the ball of mass m increases, decreases, or remains the same as the ball approaches terminal speed. Explain.

c. Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t, the given quantities, and fundamental constants.

d. Determine the terminal speed v_t in terms of the given quantities and fundamental constants.

2005M1. A ball of mass M is thrown vertically upward with an initial speed of v_o. It experiences a force of air resistance given by F = -kv, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_o, and fundamental constants.

a. Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases ____ decreases ____ remains the same

Justify your answer.

b. Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t as the ball moves upward.

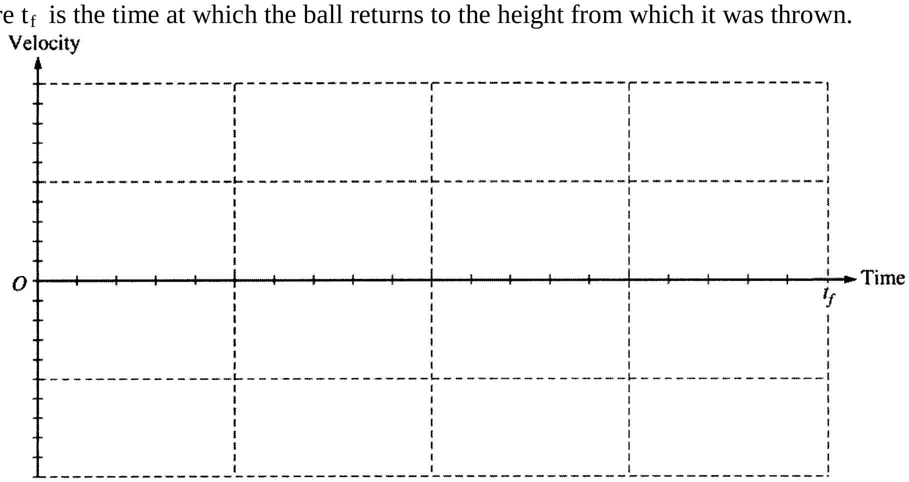
c. Determine the terminal speed of the ball as it moves downward.

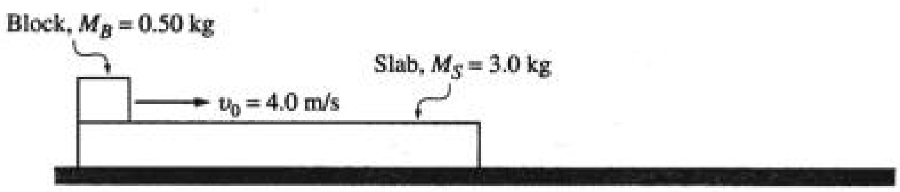
d. Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____ longer to rise ____ longer to fall

Justify your answer.

e. On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.





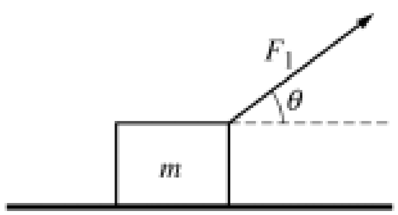
2006M1. A small block of mass M_B = 0.50 kg is placed on a long slab of mass M_S = 3.0 kg as shown above. Initially, the slab is at rest and the block has a speed v_o of 4.0 m/s to the right. The coefficient of kinetic friction between the block and the slab is 0.20, and there is no friction between the slab and the horizontal surface on which it moves.

a. On the dots below that represent the block and the slab, draw and label vectors to represent .the forces acting on each as the block slides on the slab.

At some moment later, before the block reaches the right end of the slab, both the block and the slab attain identical speeds v_f .

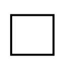
b. Calculate v_f .

c. Calculate the distance the slab has traveled at the moment it reaches v_f.



2007M1. A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1. Express all algebraic answers in terms of m, F_1, θ , a_1, and fundamental constants.

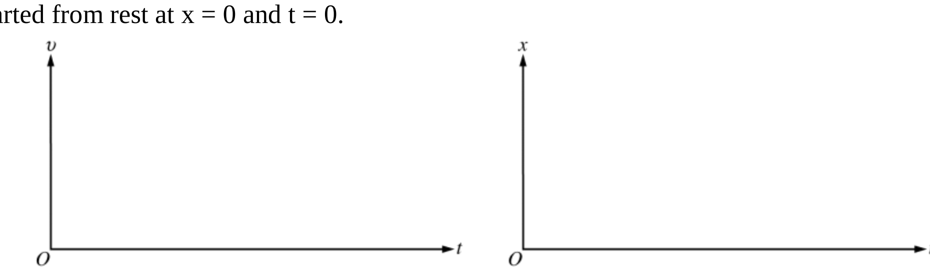
a. On the figure below, draw and label a free-body diagram showing all the forces on the block.



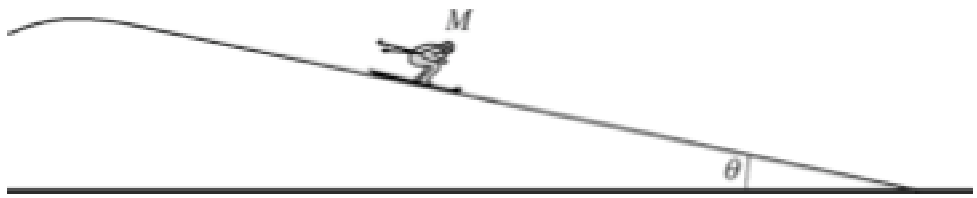
b. Derive an expression for the normal force exerted by the surface on the block.

c. Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

d. On the axes below, sketch graphs of the speed v and displacement x of the block as functions of time t if the block started from rest at x = 0 and t = 0.

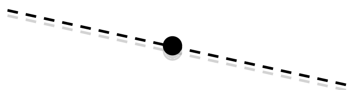


e. If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.



2008M1. A skier of mass M is skiing down a frictionless hill that makes an angle θ with the horizontal, as shown in the diagram. The skier starts from rest at time t = 0 and is subject to a velocity-dependent drag force due to air resistance of the form F = –bv, where v is the velocity of the skier and b is a positive constant. Express all algebraic answers in terms of M, b, θ, and fundamental constants.

a. On the dot below that represents the skier, draw a free-body diagram indicating and labeling all of the forces that act on the skier while the skier descends the hill.

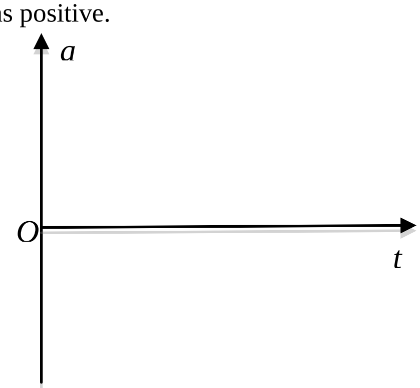


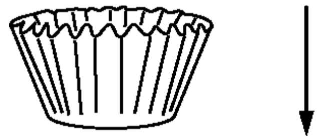
b. Write a differential equation that can be used to solve for the velocity of the skier as a function of time.

c. Determine an expression for the terminal velocity v_T of the skier.

d. Solve the differential equation in part (b) to determine the velocity of the skier as a function of time, <u>showing all your steps.</u>

e. On the axes below, sketch a graph of the acceleration a of the skier as a function of time t, and indicate the initial value of a. Take downhill as positive.





2010M1. Students are to conduct an experiment to investigate the relationship between the terminal speed of a stack of falling paper coffee filters and its mass. Their procedure involves stacking a number of coffee filters, like the one shown in the figure above, and dropping the stack from rest. The students change the number of filters in the stack to vary the mass m while keeping the shape of the stack the same. As a stack of coffee filters falls, there is an air resistance (drag) force acting on the filters.

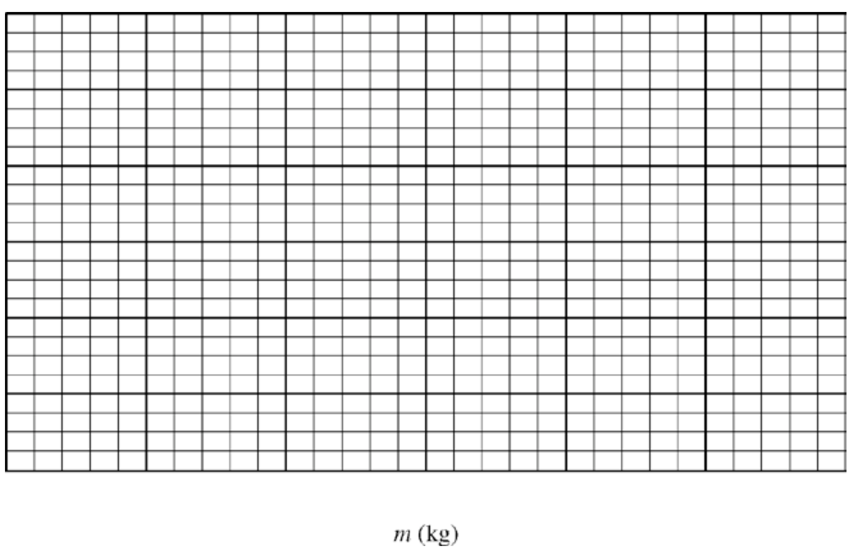
a. The students suspect that the drag force F_D is proportional to the square of the speed v : $F_D = Cv^2$, where C is a constant. Using this relationship, derive an expression relating the terminal speed v_T to the mass m.

The students conduct the experiment and obtain the following data.

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	.51	.62	.82	.92	1.06

b.

i. Assuming the functional relationship for the drag force above, use the grid below to plot a linear graph as a function of m to verify the relationship. Use the empty boxes in the data table, as appropriate, to record any calculated values you are graphing. Label the vertical axis as appropriate, and place numbers on both axes.



ii. Use your graph to calculate C.

2010M1(continued)

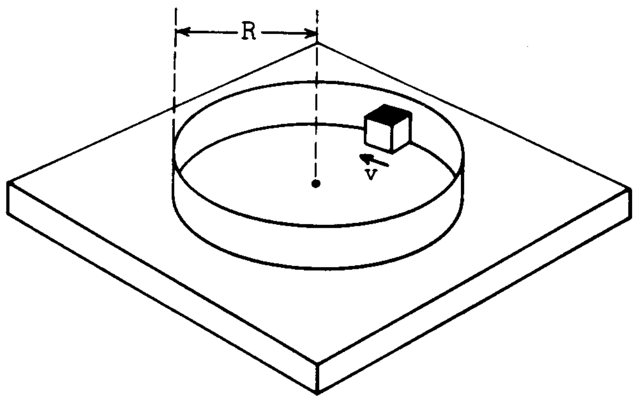
A particular stack of filters with mass m is dropped from rest and reaches a speed very close to terminal speed by the time it has fallen a vertical distance Y.

c.

i. Sketch an approximate graph of speed versus time from the time the filters are released up to the time t=T that the filters have fallen the distance Y. Indicate time t=T and terminal speed $v=v_T$ on the graph.

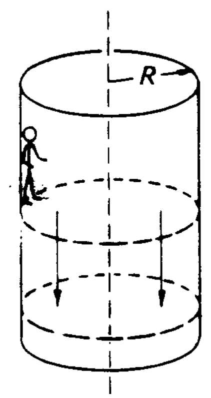
ii. Suppose you had a graph like the one sketched in (c) (i) that had a numerical scale on each axis. Describe how you could use the graph to approximate the distance Y.

SECTION B – Circular Motion



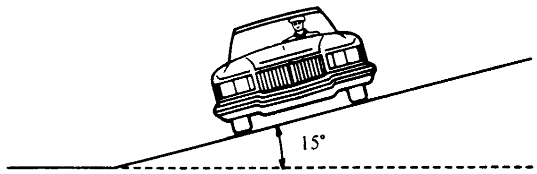
1976M1. A small block of mass m slides on a horizontal frictionless surface as it travels around the inside of a hoop of radius R. The coefficient of friction between the block and the wall is μ; therefore, the speed v of the block decreases. In terms of m, R. μ, and v, find expressions for each of the following.

a. The frictional force on the block
b. The block's tangential acceleration dv/dt
c. The time required to reduce the speed of the block from an initial value v_0 to $v_0/3$



1984M1. An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, the radius R of the cylinder is 5 meters, the angular velocity of the cylinder when rotating is 2 radians per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

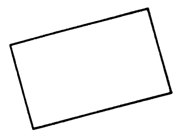
a. On the diagram below. draw and identify the forces on the rider when the system is rotating and the floor has dropped down.
b. Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.
c. Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.
d. At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.



1988M1. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.

a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.



b. On the diagram above, in which the block represents the automobile, draw and label all of the forces on the automobile.

c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.

SECTION A – Linear Dynamics

	Solution	Answer
1.	As v is proportional to t^2 and a is proportional to $\Delta v/t$, this means a should be proportional to t	E
2.	The "diluted" force between objects is the applied force times the ratio of the mass behind the rope to the total mass being pulled. This can be derived from $a = F/m_{total}$ and $F_T = m_{behind\ the\ rope}a$	E
3.	$\Sigma F_y = 0 = T\sin 30^\circ - mg$	D
4.	$F = m(dv/dt)$	E
5.	The net force and the acceleration must point in the same direction. Velocity points tangent to the objects path.	D
6.	$F = ma$ gives 20 N = (5 kg)a or an acceleration of 4 m/s^2. The 2 kg block is accelerating due to the contact force from the 3 kg block $F_{contact} = ma = (2\text{ kg})(4\text{ m/s}^2) = 8$ N. The 2 kg pushes back on the 3 kg block with a force equal in magnitude and opposite in direction.	A
7.	The direction of the force is the same as the direction of the acceleration, which is proportional to $\Delta v = v_f + (-v_i)$	B
8.	Newton's third law	C
9.	$\Sigma F_{external} = m_{total}a$ gives $(0.90\text{ kg} \times 10\text{ m/s}^2) - (0.60\text{ kg} \times 10\text{ m/s}^2) = (1.5\text{ kg})a$	D
10.	g points down in projectile motion. Always.	E
11.	gravity acts downward	D
12.	At constant speed $\Sigma F = 0$; The forces acting parallel to the incline are F (up), F_f (down) and $mg\sin\theta$ (down), which gives $F - F_f - mg\sin\theta = 0$, where $F_f = \mu F_N = \mu mg\cos\theta$ and $\cos\theta = 4/5$	B
13.	$F = m(dv/dt) = m(d^2x/dt^2)$	B
14.	$\Sigma F = ma = F\cos\phi - f$	D
15.	$f = \mu F_N$ where $F_N = mg - F\sin\theta$	E
16.	Considering limiting cases, v = 0 at t = 0 and v should be constant in value as t approaches infinity. Only A satisfies this condition.	A
17.	The string pulling all three masses (total 6*m*) must have the largest tension. String A is only pulling the block of mass 3*m* and string B is pulling a total mass of 5*m*.	C
18.	At t = 2 s the force is 4 N. F = ma	B
19.	This may be done using impulse/momentum, but also consider dividing the y axis by m = 3 kg to get a graph of a vs t. The area under the line is the change in velocity.	A
20.	The upward component of the slanted cord is 300 N to balance the weight of the object. Since the slanted cord is at an angle of 45º, it has an equal horizontal component. The horizontal component of the slanted cord is equal to the tension in the horizontal cord.	D
21.	The normal force must point perpendicular to the surface and the weight must point down. In order to accelerate up the ramp, there must be an applied force up the ramp. If the box is accelerating up the ramp, friction acts down the ramp, opposite the motion.	E

22. The normal force must point perpendicular to the surface and the weight must point down. If the box is at rest on the ramp, friction acts up the ramp, opposing the tendency to slide down — C

23. The normal force must point perpendicular to the surface and the weight must point down. If the box is sliding down at constant speed, friction acts up the ramp, opposing the motion — C

24. $\Sigma F_{external} = m_{total}a$ gives $(Mg) - (mg) = (M + m)a$ — E

25. To keep the box from slipping, friction up the wall must balance the weight of the block, or $F_f = mg$, where $F_f = \mu F_N$ and F_N = the applied force F. This gives $\mu F = mg$ — C

26. By Newton's third law, $M_1a_1 = M_2a_2$, but without the actual value of F, and only a_1 and a_2 known, we can only find the ratio, not the values, of M_1/M_2 — C

27. From elevator physics, the normal force on a person in an elevator can be found from $F_N - mg = ma$, or $F_N = mg + ma$, with a downward acceleration begin negative. F_N is least when the acceleration is at its largest negative value. This is where the slope of the v-t graph has its largest negative value. — E

28. For a block on a frictionless incline: $F_N = mg \cos \theta$ and $mg \sin \theta = ma$ — D

29. For an object on an incline: $F_N = mg \cos \theta$ and $mg \sin \theta - F_f = ma$, at constant speed $a = 0$ so $mg \sin \theta = F_f = \mu F_N = \mu mg \cos \theta$ — E

30. $\Sigma F = ma$; $mg - kv = m(dv/dt)$, separate variables — A

31. $a = F/m$ and v is the integral of (a dt) — B

32. The car continues forward so the position graph must have a positive slope. The velocity time graph has a slope that approaches zero as the acceleration approaches zero — D

SECTION B – Circular Motion

1. With acceleration south the car is at the top (north side) of the track as the acceleration points toward the center of the circular track. Moving east indicates the car is travelling clockwise. The magnitude of the acceleration is found from $a = v^2/r$ — A

2. At Q the ball is in circular motion and the acceleration should point to the center of the circle. At R, the ball comes to rest and is subject to gravity as in free-fall. — C

3. The centripetal force is provided by the spring where $F_C = F_s = kx$ — B

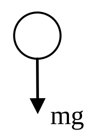
4. In the straight sections there is no acceleration, in the circular sections, there is a centripetal acceleration — B

5. $F_T = mv^2/R$ where $v = 2\pi R f$ so $F_T = 4\pi^2 mRf^2$ — E

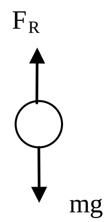
SECTION A – Linear Dynamics

1975M1

a. i.

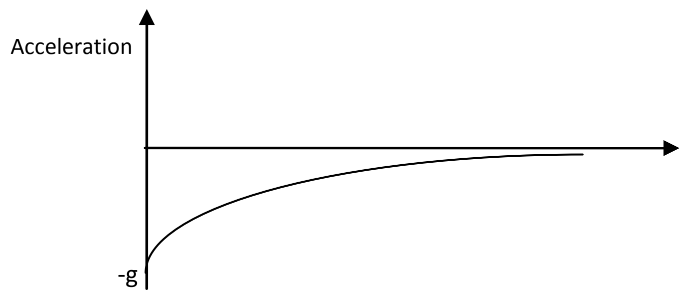


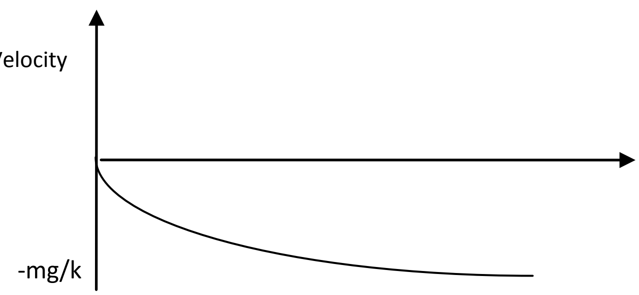
ii.

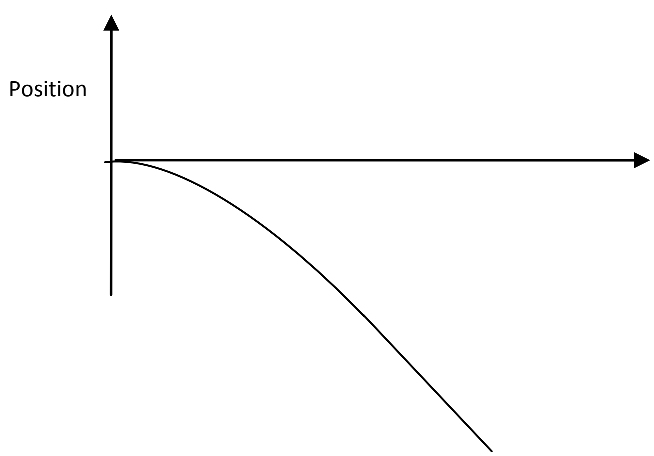


b. Terminal velocity is reached when $\Sigma F = 0$, or $mg = F_R = kv_T$, which gives $v_T = mg/k$

c.







1977M1

a. F = ma; F = –kv = ma; a = –kv/m

b.

$$\frac{dv}{dt} = -\frac{kv}{m}$$

$$\frac{dv}{v} = -\frac{k}{m}dt$$

$$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{k}{m}dt$$

$$ln\left|\frac{v}{v_0}\right| = -\frac{k}{m}t$$

$$\frac{v}{v_0} = e^{-\frac{k}{m}t}$$

$$v = v_0 e^{-\frac{k}{m}t}$$

c.

$$\frac{dx}{dt} = v_0 e^{-\frac{k}{m}t}$$

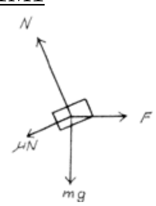
$$\int_0^x dx = \int_0^\infty v_0 e^{-\frac{k}{m}t} dt$$

$$x = -\frac{mv_0}{k} e^{-\frac{k}{m}t}\Big|_0^\infty$$

$$x = \frac{mv_0}{k}$$

1981M1

a.



b. F can be resolved into two components: F sin θ acting into the incline and F cos θ acting up the incline. The normal force is then calculated with ΣF = 0; N – F sin θ – mg cos θ = 0 and f = μN
Putting this together gives ΣF = ma; F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = ma, solve for a

c. for constant velocity, a = 0 in the above equation becomes F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = 0 solving for F gives $F = mg\left(\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right)$ In order that F remain positive (acting to the right), the denominator must remain positive. That is cos θ > μ sin θ, or tan θ < 1/μ

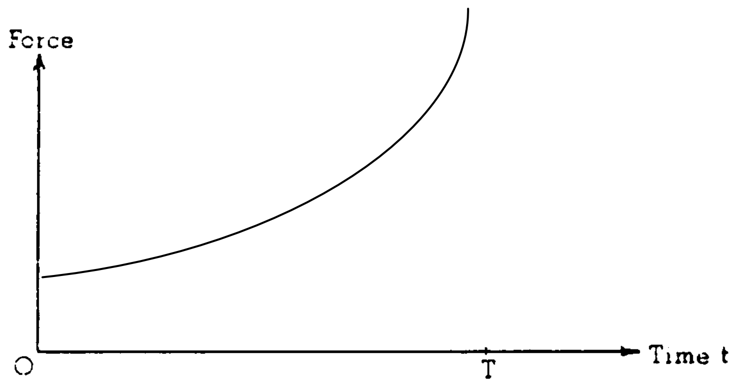
1982M2

a. Setting v = 0 at time T gives $v_0^2 = RT/M$ or $T = Mv_0^2/R$

b. $a = dv/dt = ½ (v_0^2 – Rt/M)^{–½} (–R/M)$

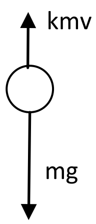
c. F = Ma gives

$$F = \frac{-R}{2\sqrt{{v_0}^2 - Rt/M}}$$



1984M3

a.



b. ΣF = ma; mg – kmv = ma; a = g – kv; dv/dt = g – kv

c. ΣF = 0, which gives $0 = g – kv_T$; $v_T = g/k$

d.

$$\frac{dv}{dt} = g - kv$$

$$\frac{dv}{g - kv} = dt$$

$$\int_0^v \frac{dv}{g - kv} = \int_0^t dt$$

$$-\frac{1}{k} ln|g - kv|\Big|_0^v = t$$

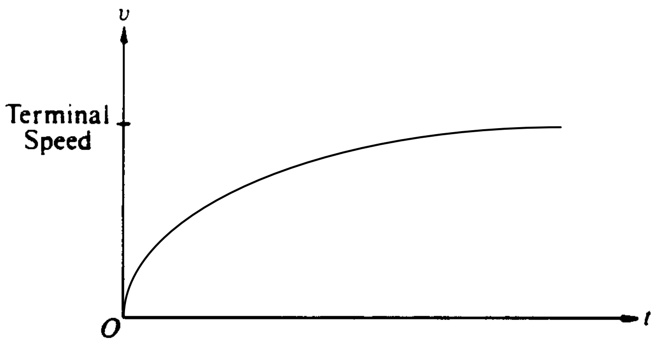
$$\ln(g - kv) - \ln(g) = \ln\left(\frac{g - kv}{g}\right) = -kt$$

$$\frac{g - kv}{g} = e^{-kt}$$

$$g - kv = ge^{-kt}$$

$$v = \frac{g}{k}(1 - e^{-kt})$$

e.



1986M1

a. Combining the person and the platform into one object, held up by two sides of the rope we have ΣF = ma; 2T = (80 kg + 20 kg)g giving T = 500 N
b. Similarly, ΣF = ma; 2T – 1000 N = (100 kg)(2 m/s^2) giving T = 600 N
c. For the person only: ΣF = ma; N + 600 N – mg = ma gives N = 360 N

1990M1

a. F = ma; F = –kv = ma; $a_0 = -kv_0/m$ (negative sign indicates to the left)
b.

$$\frac{dv}{dt} = -\frac{kv}{m}$$

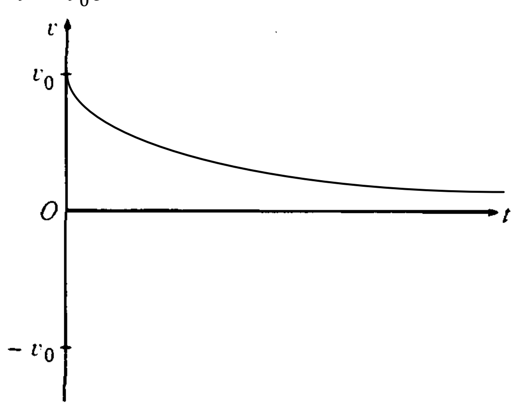
$$\frac{dv}{v} = -\frac{k}{m}dt$$

$$\int_{v_0}^{v}\frac{dv}{v} = \int_0^t -\frac{k}{m}dt$$

$$ln\left|\frac{v}{v_0}\right| = -\frac{k}{m}t$$

$$\frac{v}{v_0} = e^{-\frac{k}{m}t}$$

$$v = v_0 e^{-\frac{k}{m}t}$$



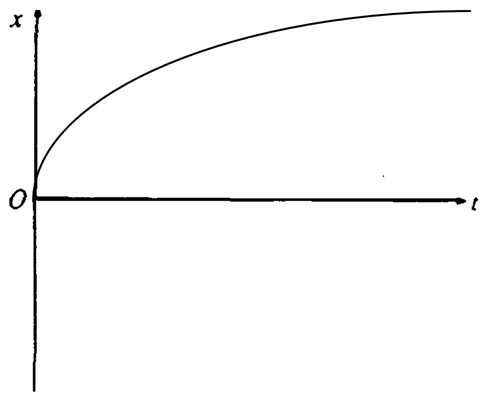
c.

$$\frac{dx}{dt} = v_0 e^{-\frac{k}{m}t}$$

$$\int_0^x dx = \int_0^t v_0 e^{-\frac{k}{m}t} dt$$

$$x = -\frac{mv_0}{k} e^{-\frac{k}{m}t}\Big|_0^t$$

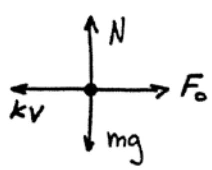
$$x = \frac{mv_0}{k}\left(1 - e^{-\frac{k}{m}t}\right)$$



d. at t = ∝, $x = \frac{mv_0}{k}$

<u>1993M2</u>

a.



b. ΣF = ma; F_0 – kv = ma; a = (F_0 – kv)/m

c.

$$\frac{dv}{dt} = (F_0 - kv)/m$$

$$\frac{dv}{F_0 - kv} = \frac{1}{m} dt$$

$$\int_0^v \frac{dv}{F_0 - kv} = \frac{1}{m}\int_0^t dt$$

$$-\frac{1}{k} ln|F_0 - kv|\Big|_0^v = \frac{t}{m}$$

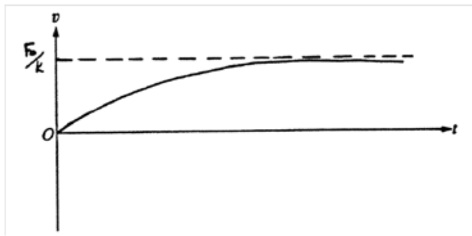
$$\ln(F_0 - kv) - \ln(F_0) = \ln\left(\frac{F_0 - kv}{F_0}\right) = -\frac{kt}{m}$$

$$\frac{F_0 - kv}{F_0} = e^{-\frac{kt}{m}}$$

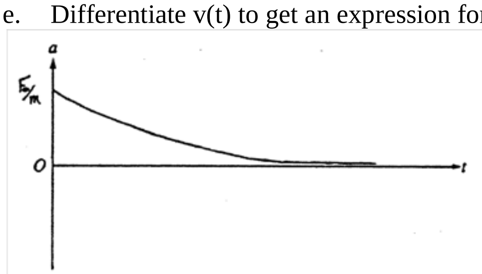
$$F_0 - kv = F_0 e^{-\frac{kt}{m}}$$

$$v = \frac{F_0}{k}(1 - e^{-\frac{kt}{m}})$$

d.



e. Differentiate v(t) to get an expression for a(t)



1996 M2

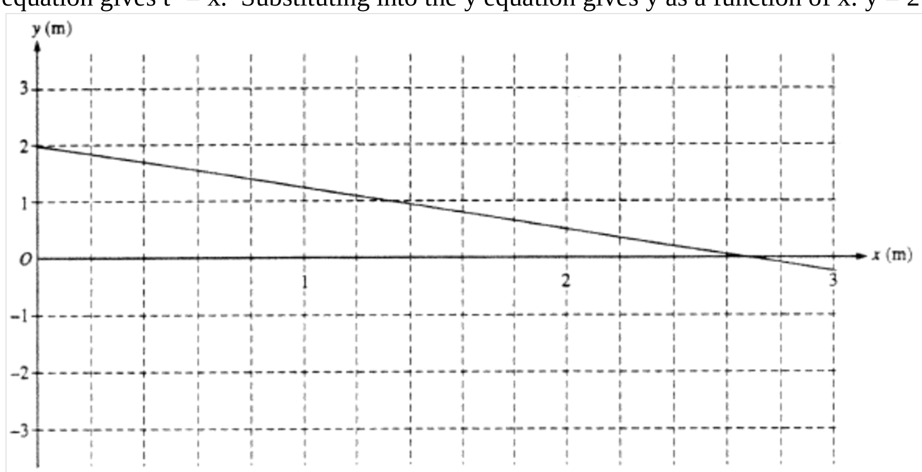
a. ΣF = ma; using downward as the positive direction, $mg - N = ma_y$ gives $N = m(g - a_y) = 2490$ N

b. Friction is the only horizontal force exerted; $\Sigma F = f = ma_x = 600$ N

c. At the minimum coefficient of friction, static friction will be at its maximum value $f = \mu N$, giving $\mu = f/N = (600 \text{ N})/(2490 \text{ N}) = 0.24$

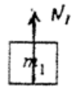
d. $y = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 = 2 \text{ m} + \frac{1}{2} (-1.5 \text{ m/s}^2)t^2$ and $x = x_0 + v_{0x}t + \frac{1}{2} a_x t^2 = \frac{1}{2} (2 \text{ m/s}^2)t^2$, solving for t^2 in the x equation gives $t^2 = x$. Substituting into the y equation gives y as a function of x: $y = 2 - 0.75x$

e.



1998M3

a. i.

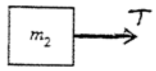


$N_1 = m_1g$

ii.

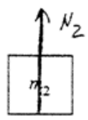
$f_1 = 0$

iii.



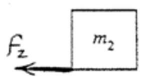
$T = Mg$

iv.



$N_2 = (m_1 + m_2)g$

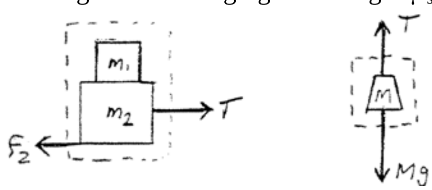
v.



$f_2 = Mg$

b. The maximum friction force on the blocks on the table is $f_{2max} = \mu_{s2}N_2 = \mu_{s2}(m_1 + m_2)g$ which is balanced by the weight of the hanging mass: $Mg = \mu_{s2}(m_1 + m_2)g$ giving $M = \mu_{s2}(m_1 + m_2)$

c.

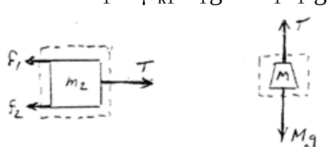


For the hanging block: $Mg – T = Ma$; For the two blocks on the plane: $T – f_2 = (m_1 + m_2)a$

Combining these equations (by adding them to eliminate T) and solving for a gives $a = \left[\frac{M-\mu_{k2}(m_1+m_2)}{M+m_1+m_2}\right]g$

d. i. $f_1 = \mu_{k1}m_1g = m_1a_1$ giving $a_1 = \mu_{k1}g$

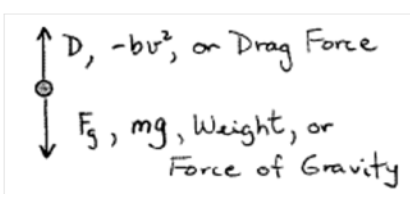
ii.



For the two blocks: $Mg – T = Ma_2$ and $T – f_1 – f_2 = m_2a_2$. Eliminating T and substituting values for friction gives $a_2 = \left[\frac{M-\mu_{k1}m_1-\mu_{k2}(m_1+m_2)}{M+m_2}\right]g$

2000 M2

a.



b. Decreases. As the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

c. $F = mg – bv^2$; $ma = mg – bv^2$; $m(dv/dt) = mg – bv^2$

d. At terminal speed, $a = 0$ so $mg = bv_T^2$; $v_T = (mg/b)^{1/2}$

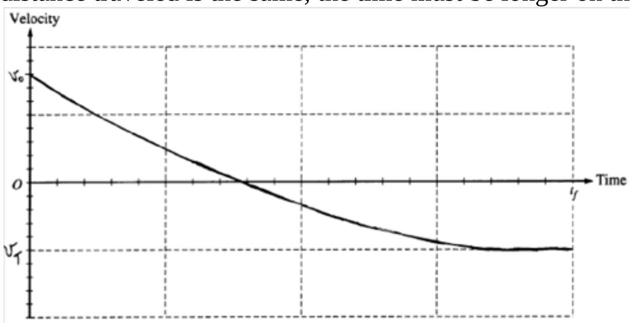
2005M1

a. The magnitude of the acceleration decreases as the ball moves upward. Since the velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

b. At terminal speed $\Sigma F = 0$. $\Sigma F = -Mg + kv_T$ giving $v_T = Mg/k$

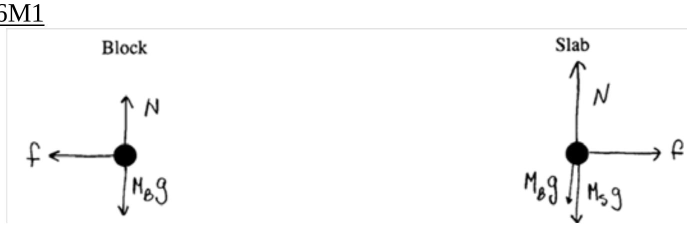
c. It takes longer for the ball to fall. Friction is acting on the ball on the way up and on the way down, where it begins from rest. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.

d.



2006M1

a.

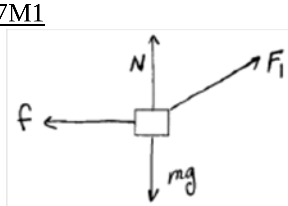


b. $f = \mu M_B g = M_S a_S$ so $a_S = \mu M_B g/M_S = 0.33\ m/s^2$
$f = \mu M_B g = M_B a_B$ so $a_B = 2.0\ m/s^2$
$v_f = v_0 - a_B t$ for the block
$v_f = a_S t$ for the slab
solving for t and plugging back in to find v_f gives $v_f = a_S v_0/(a_S + a_B) = 0.57\ m/s$

c. using $v_f^2 = v_0^2 + 2a_S x$ gives $x = 0.49\ m$

2007M1

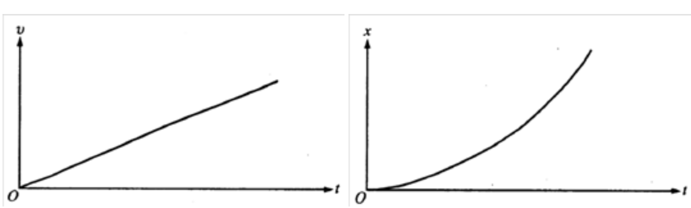
a.



b. $\Sigma F_y = 0$; $N + F_1 \sin\theta - mg = 0$ gives $N = mg - F_1 \sin\theta$

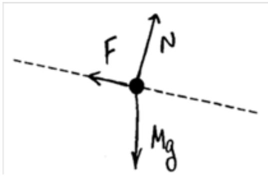
c. $\Sigma F_x = ma$; $F_1 \cos\theta - \mu N = ma_1$. Substituting N from above gives $\mu = (F_1 \cos\theta - ma_1)/(mg - F_1 \sin\theta)$

d.



e. The condition for the block losing contact is when the normal force goes to zero, which means friction is zero as well. $\Sigma F_x = F_{max} \cos\theta = ma_{max}$ and $\Sigma F_y = F_{max} \sin\theta - mg = 0$ giving $F_{max} = mg/(\sin\theta)$ and $a_{max} = (F_{max} \cos\theta)/m$ which results in $a_{max} = g \cot\theta$

2008M1

a. 

b. ΣF = ma

Ma = Mg sin θ – bv

M(dv/dt) = Mg sin θ – bv

c. ΣF = 0 so Mg sin θ = bv_T; v_T = (Mgsinθ)/b

d.

$$\frac{dv}{dt} = (Mgsin\theta - bv)/M$$

$$\frac{dv}{Mgsin\theta - bv} = \frac{1}{M}dt$$

$$\int_0^v \frac{dv}{Mgsin\theta - bv} = \frac{1}{M}\int_0^t dt$$

$$-\frac{1}{b}ln|Mgsin\theta - bv|\Big|_0^v = \frac{t}{M}$$

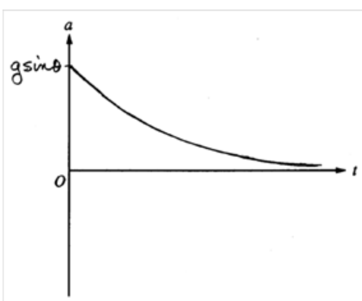
$$\ln(Mgsin\theta - bv) - \ln(Mgsin\theta) = \ln\left(\frac{Mgsin\theta - bv}{Mgsin\theta}\right) = -\frac{bt}{M}$$

$$\frac{Mgsin\theta - bv}{Mgsin\theta} = e^{-\frac{bt}{M}}$$

$$Mgsin\theta - bv = Mgsin\theta e^{-\frac{bt}{M}}$$

$$v = \frac{Mgsin\theta}{b}(1 - e^{-\frac{bt}{M}})$$

e.



2010M1

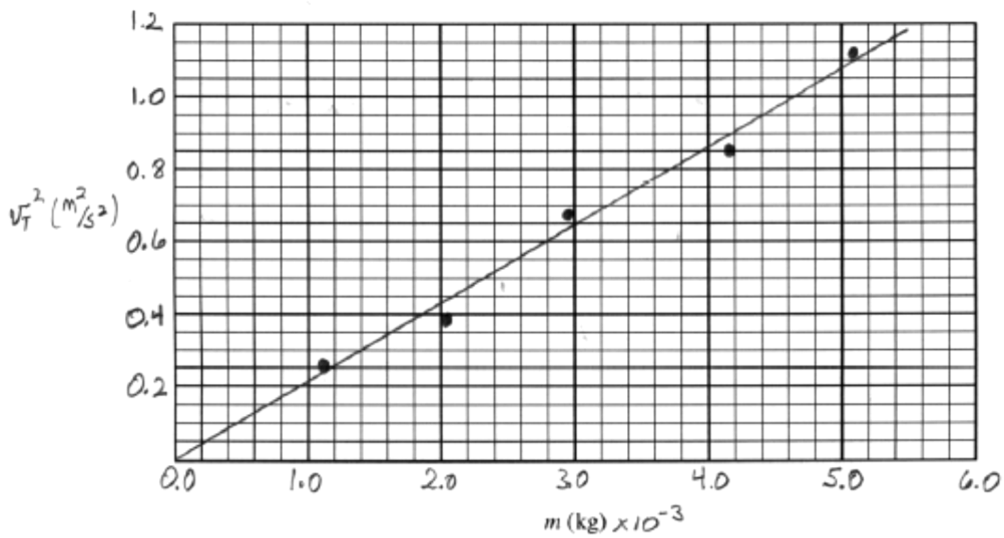
a. $\Sigma F = mg - Cv^2 = ma$, at terminal speed $\Sigma F = 0$

$mg = Cv_T^{\ 2}$

$v_T^{\ 2} = (g/C)m$

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	0.51	0.62	0.82	0.92	1.06
v_T^2 (m^2/s^2)	0.26	0.38	0.67	0.85	1.12

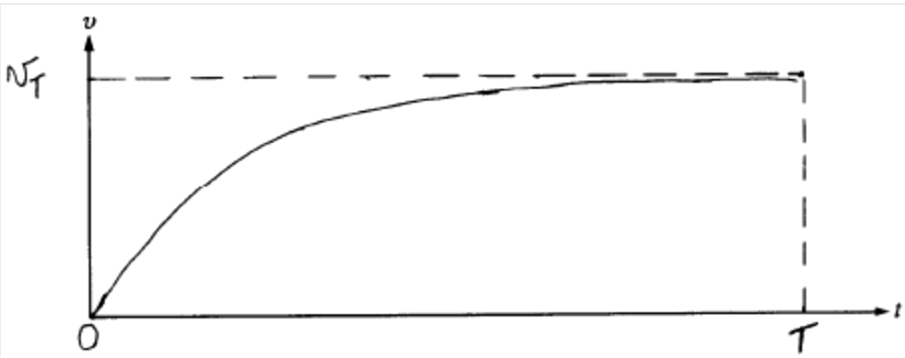
b. i.



b. ii.

Slope = g/C; C = g/slope = (9.8 m/s^2)/(217 m^2/kg-s^2) = 0.045 kg/m

c. i.



ii. Distance Y is the area under the curve between 0 and T

SECTION B – Circular Motion

1976M1

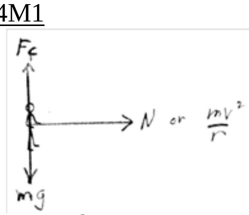
a. $f = \mu F_N$ where F_N is the radial force acting inward on the mass from the wall
$F_N = ma_c = mv^2/R$; $f = \mu mv^2/R$

b. the tangential acceleration comes from the frictional force along the wall
$f = -ma_t$ (negative as it opposes the motion of the block)
$\mu mv^2/R = -ma_t$; $a_t = dv/dt = -\mu v^2/R$

c.
$$\frac{dv}{dt} = -\frac{\mu v^2}{R}$$
$$\frac{dv}{v^2} = -\frac{\mu}{R}dt$$
$$\int_{v_0}^{\frac{v_0}{3}} \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^T dt$$
$$-\frac{1}{v}\Big|_{v_0}^{\frac{v_0}{3}} = -\frac{\mu T}{R}$$
$$-\frac{1}{\frac{v_0}{3}} - \left(-\frac{1}{v_0}\right) = -\frac{2}{v_0} = -\frac{\mu T}{R}$$
$$T = \frac{2R}{\mu v_0}$$

1984M1

a.



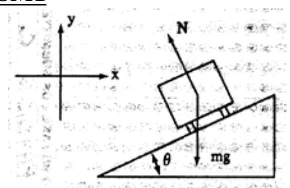
b. $F = mv^2/r$ where $v = 2\pi rf = 2\pi r(1/\pi) = 2r = 10$ m/s giving $F = 1000$ N provided by the normal force

c. $\Sigma F_y = 0$ so the upward force provided by friction equals the weight of the rider = mg = 490 N

d. Since the frictional force is proportional to the normal force and equal to the weight of the rider, m will cancel from the equation, meaning a rider with twice the mass, or any different mass, will not slide down the wall as mass is irrelevant for this condition.

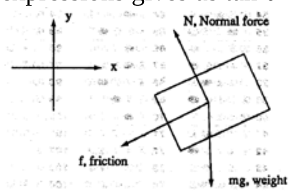
1988M1

a.



Toward the center of the turn we have $\Sigma F = N \sin\theta = mv^2/r$ and vertically $N \cos\theta = mg$. Dividing the two expressions gives us $\tan\theta = v^2/rg$ and $v = 16$ m/s

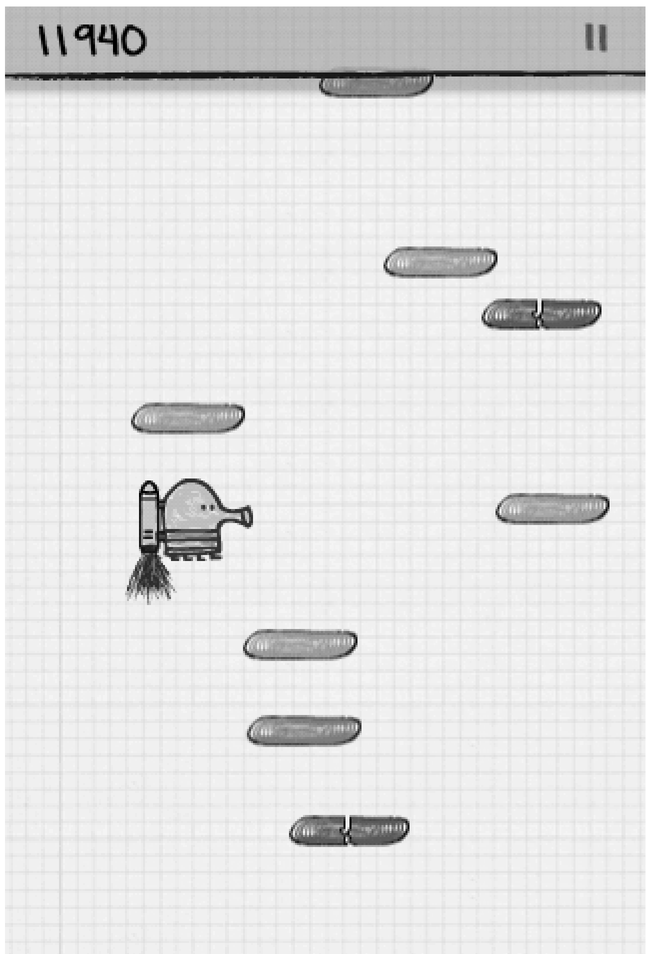
b.



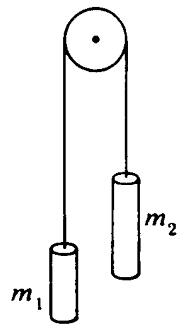
c. $\Sigma F_y = N \cos\theta - f \sin\theta - mg = 0$ and $\Sigma F_x = N \sin\theta + f \cos\theta = mv^2/r$ solve for N and f and substitute into $f = \mu N$ gives $\mu_{min} = 0.32$

Chapter 3

Work and Energy

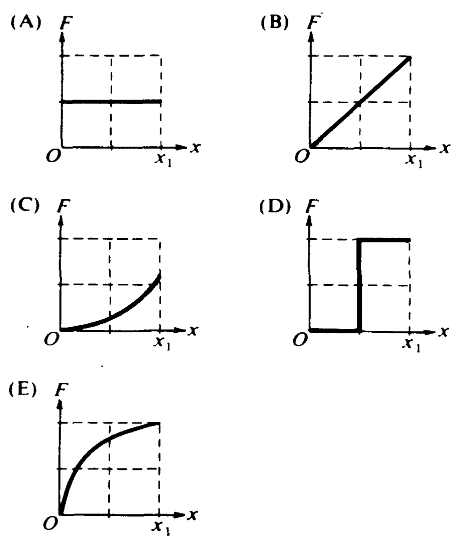


1 An object of mass m is lifted at constant velocity a vertical distance H in time T. The power supplied by the lifting force is (A) mgHT (B) mgH/T (C) mg/HT (D) mgT/H (E) zero



2. A system consists of two objects having masses m_1 and m_2 ($m_1 < m_2$). The objects are connected by a massless string, hung over a pulley as shown above, and then released. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by
(A) $(m_2 - m_1)gh$ (B) m_2gh (C) $(m_1 + m_2)gh$ (D) $\frac{1}{2}(m_1 + m_2)gh$ (E) 0

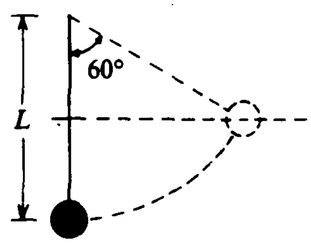
3. The following graphs, all drawn to the same scale, represent the net force F as a function of displacement x for an object that moves along a straight line. Which graph represents the force that will cause the greatest change in the kinetic energy of the object from x = 0 to x = x_1?



4. A person pushes a box across a horizontal surface at a constant speed of 0.5 meter per second. The box has a mass of 40 kilograms, and the coefficient of sliding friction is 0.25. The power supplied to the box by the person is (A) 0.2 W (B) 5 W (C) 50 W (D) 100 W (E) 200 W

5. If a particle moves in such a way that its position x is described as a function of time t by $x = t^{3/2}$, then its kinetic energy is proportional to
(A) t^2 (B) $t^{3/2}$ (C) t (D) $t^{1/2}$ (E) t° (i.e., kinetic energy is constant)

6. From the top of a 70-meter-high building, a l-kilogram ball is thrown directly downward with an initial speed of 10 meters per second. If the ball reaches the ground with a speed of 30 meters per second, the energy lost to friction is most nearly
(A) 0J (B) 100 J (C) 300 J (D) 400 J (E) 700 J



7. A pendulum consists of a ball of mass m suspended at the end of a massless cord of length L as shown above. The pendulum is drawn aside through an angle of 60° with the vertical and released. At the low point of its swing, the speed of the pendulum ball is
   (A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) gL/2 (D) gL (E) 2gL

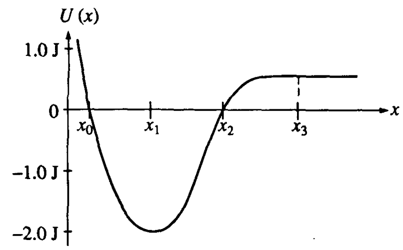
8. A rock is lifted for a certain time by a force F that is greater in magnitude than the rock's weight W. The change in kinetic energy of the rock during this time is equal to the
   (A) work done by the net force (F - W)
   (B) work done by F alone
   (C) work done by W alone
   (D) difference in the momentum of the rock before and after this time
   (E) difference in the potential energy of the rock before and after this time.

9. A 10-kilogram body is constrained to move along the x-axis. The potential energy U of the body in joules is given as a function of its position x in meters by
   $U(x) = 6x^2 - 4x + 3$
   The force on the particle at x = 3 meters is
   (A) 32 N in +x direction (B) 32N in -x direction (C) 45 N in +x direction
   (D) 45 N in -x direction (E) 98 N in +x direction

10. A ball is thrown upward. At a height of 10 meters above the ground, the ball has a potential energy of 50 joules (with the potential energy equal to zero at ground level) and is moving upward with a kinetic energy of 50 joules. Air friction is negligible. The maximum height reached by the ball is most nearly
    (A) 10 m (B) 20 m (C) 30 m (D) 40 m (E) 50 m

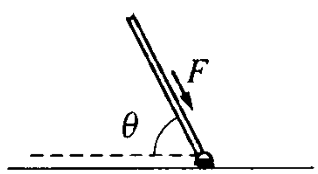
11. During a certain time interval, a constant force delivers an average power of 4 watts to an object. If the object has an average speed of 2 meters per second and the force acts in the direction of motion of the object, the magnitude of the force is
    (A) 16 N (B) 8 N (C) 6 N (D) 4N (E) 2N

12. A weight lifter lifts a mass *m* at constant speed to a height *h* in time *t*. How much work is done by the weight lifter? (A) *mg* (B) *mh* (C) *mgh* (D) *mght* (E) *mgh/t*

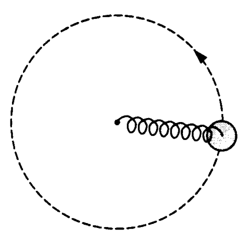


13. A conservative force has the potential energy function U(x), shown by the graph above. A particle moving in one dimension under the influence of this force has kinetic energy 1.0 joule when it is at position x_1 Which of the following is a correct statement about the motion of the particle?
    (A) It oscillates with maximum position x_2 and minimum position x_0.
    (B) It moves to the right of x_3 and does not return.
    (C) It moves to the left of x_0 and does not return.
    (D) It comes to rest at either x_0 or x_2.
    (E) It cannot reach either x_0 or x_2.

14. When an object is moved from rest at point A to rest at point B in a gravitational field, the net work done by the field depends on the mass of the object and
    (A) the positions of A and B only
    (B) the path taken between A and B only
    (C) both the positions of A and B and the path taken between them
    (D) the velocity of the object as it moves between A and B
    (E) the nature of the external force moving the object from A to B

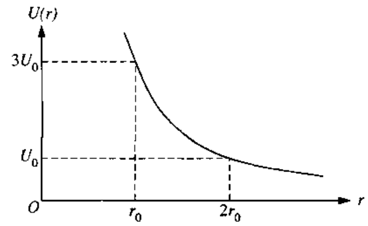


15. A force F is exerted by a broom handle on the head of the broom, which has a mass m. The handle is at an angle θ to the horizontal, as shown above. The work done by the force on the head of the broom as it moves a distance d across a horizontal floor is
    (A) Fd sin θ (B) Fd cosθ (C) Fm cosθ (D) Fm tanθ (E) Fmd sinθ


Top View

16. A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view above. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible. What is the work done on the disc by the spring during one full circle?
    (A) 0 J (B) 94 J (C) 186 J (D) 314 J (E) 628 J

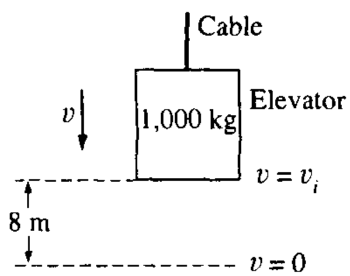
Questions 17-18 refer to the following graph, which represents a hypothetical potential energy curve for a particle of mass *m*.



17. If the particle is released from rest at position r_o, its speed at position $2r_o$ is most nearly
(A) $\sqrt{\frac{8U_0}{m}}$ (B) $\sqrt{\frac{6U_0}{m}}$ (C) $\sqrt{\frac{4U_0}{m}}$ (D) $\sqrt{\frac{2U_0}{m}}$ (E) $\sqrt{\frac{U_0}{m}}$

18. If the potential energy function is given by $U(r) = br^{-3/2} + c$, where b and c are constants, which of the following is an expression for the force on the particle?
(A) $\frac{3b}{2}r^{-5/2}$ (B) $\frac{3b}{2}r^{-1/2}$ (C) $\frac{3}{2}r^{-1/2}$ (D) $2br^{-1/2} + cr$ (E) $\frac{2b}{5}r^{-5/2} + cr$

19. A frictionless pendulum of length 3 m swings with an amplitude of 10°. At its maximum displacement, the potential energy of the pendulum is 10 J. What is the kinetic energy of the pendulum when its potential energy is 5J?
(A) 3.3 J (B) 5 J (C) 6.7 J (D) 10 J (E) 15 J



20. A descending elevator of mass 1,000 kg is uniformly decelerated to rest over a distance of 8 m by a cable in which the tension is 11,000 N. The speed v_i of the elevator at the beginning of the 8 m descent is most nearly
(A) 4 m/s (B) 10 m/s (C) 13 m/s (D) 16 m/s (E) 21 m/s

21. To stretch a certain nonlinear spring by an amount x requires a force F given by $F = 40x - 6x^2$, where F is in newtons and x is in meters. What is the change in potential energy when the spring is stretched 2 meters from its equilibrium position?
(A) 16 J (B) 28 J (C) 56 J (D) 64 J (E) 80 J

22. When a block slides a certain distance down an incline, the work done by gravity is 300 J. What is the work done by gravity if this block slides the same distance up the incline?
(A) 300 J (B) Zero (C) -300 J
(D) It cannot be determined without knowing the distance the block slides.
(E) It cannot be determined without knowing the coefficient of friction.

23. A student holds one end of a string in a fixed position. A ball of mass 0.2 kg attached to the other end of the string moves in a horizontal circle of radius 0.5 m with a constant speed of 5 m/s. How much work is done on the ball by the string during each revolution?
(A) 0 J (B) 0.5 J (C) 1.0 J (D) 2π J (E) 5π J

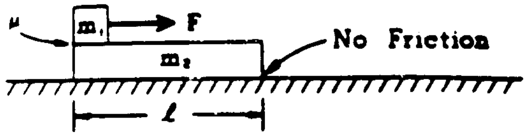
24. For a particular nonlinear spring, the relationship between the magnitude of the applied force F and the resultant displacement x from equilibrium is given by the equation $F = kx^2$. What is the amount of work done by stretching the spring a distance x_o ?
(A) kx_o^3 (B) $½\, kx_o$ (C) $½\, kx_o^3$ (D) $1/3\, kx_o^2$ (E) $1/3\, kx_o^3$

25. A 1000 W electric motor lifts a 100 kg safe at constant velocity. The vertical distance through which the motor can raise the safe in 10 s is most nearly
(A) 1 m (B) 3 m (C) 10 m (D) 32 m (E) 100 m

26. All of the following are units of power EXCEPT
(A) watts
(B) joules per second
(C) electron volts per second
(D) newton meters per second
(E) kilogram meters per second

27. A disc of mass m slides with negligible friction along a flat surface with a velocity υ. The disc strikes a wall head-on and bounces back in the opposite direction with a kinetic energy one- fourth of its initial kinetic energy. What is the final velocity of the disc?
(A) $\upsilon/4$ (B) $\upsilon/2$ (C) $-\upsilon$ (D) $-\upsilon/2$ (E) $-\upsilon/4$

28. A 2000 kg car, initially at rest, is accelerated along a horizontal roadway at 3 m/s^2. What is the average power required to bring the car to a final speed of 20 m/s?
(A) 6×10^3 W
(B) 2×10^4 W
(C) 3×10^4 W
(D) 4×10^4 W
(E) 6×10^4 W



1973M1. A horizontal force F is applied to a small block of mass m_1 to make it slide along the top of a larger block of mass m_2 and length *l*. The coefficient of friction between the blocks is μ. The larger block slides without friction along a horizontal surface. The blocks start from rest with the small block at one end of the larger block, as shown.

a. On the diagrams below draw all of the forces acting on each block. Identify each force.

b. Find the acceleration of each block. a_1 and a_2, relative to the horizontal surface.

c. In terms of ℓ, a_1, and a_2, find the time t needed for the small block to slide off the end of the larger block.

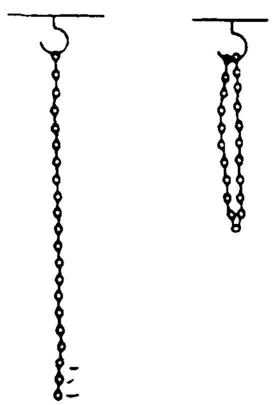
d. Find an expression for the energy dissipated as heat because of the friction between the two blocks.

1973M2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.

a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.

b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?

c. Suppose, instead, that the stopping force increases from zero as the bullet penetrates. Discuss the motion in comparison to the case for a constant deceleration.



1975M3. A uniform chain of mass M and length *l* hangs from a hook in the ceiling. The bottom link is now raised vertically and hung on the hook as shown above on the right.

a. Determine the increase in gravitational potential energy of the chain by considering the change in position of the center of mass of the chain.

b. Write an equation for the upward external force F(y) required to lift the chain slowly as a function of the vertical distance y.

c. Find the work done on the chain by direct integration of $\int F dy$.

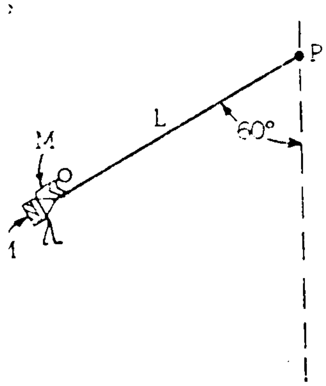


Figure I

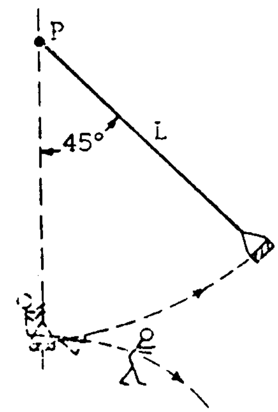
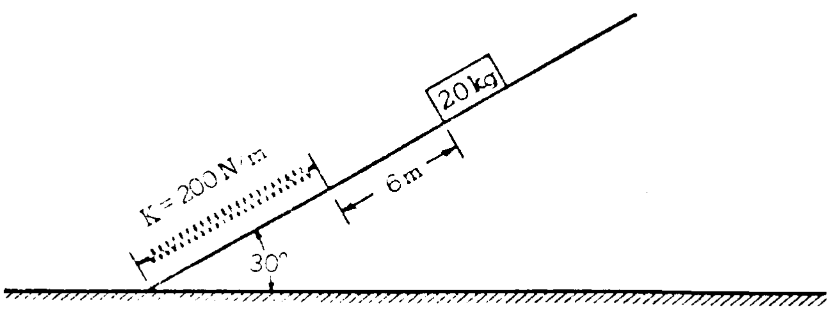


Figure II

1981M2. A swing seat of mass M is connected to a fixed point P by a massless cord of length L. A child also of mass M sits on the seat and begins to swing with zero velocity at a position at which the cord makes a 60° angle with the vertical is shown in Figure I. The swing continues down until the cord is exactly vertical at which time the child jumps off in a horizontal direction. The swing continues in the same direction until its cord makes a 45° angle with the vertical as shown in Figure II: at that point it begins to swing in the reverse direction. With what velocity relative to the ground did the child leave the swing?



1982M1. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30º with the horizontal and strikes a spring of spring constant K = 200 newtons/meter as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved. Use $g = 10\ m/s^2$, ($\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = 0.866$)

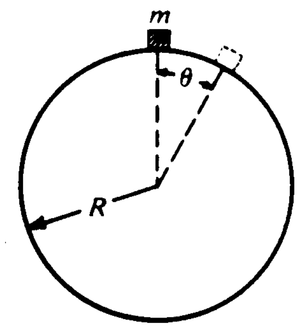
a. Determine the speed of the block just before it hits the spring.

b. Determine the distance the spring has been compressed when the block comes to rest.

c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

1982M2.(modified) A car of mass M moves with an initial speed v_o on a straight horizontal road. The car is brought to rest by braking in such a way that the speed of the car is given as a function of time t by $v = (v_o^2 - Rt/M)^{1/2}$ where R is a constant.

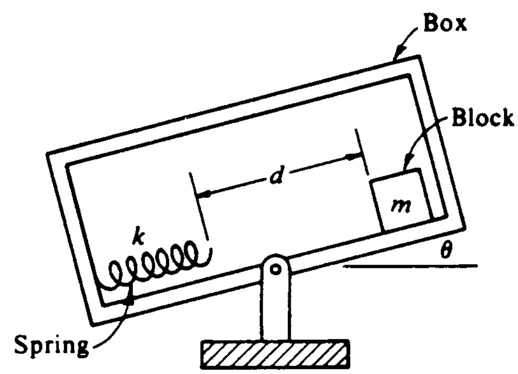
a. Develop an equation that expresses the time rate of change of kinetic energy.



1983M3. A particle of mass m slides down a fixed, frictionless sphere of radius *R*. starting from rest at the top.

a. In terms of m, g, *R*. and θ, determine each of the following for the particle while it is sliding on the sphere.
   i. The kinetic energy of the particle
   ii. The centripetal acceleration of the mass
   iii. The tangential acceleration of the mass

b. Determine the value of θ at which the particle leaves the sphere.

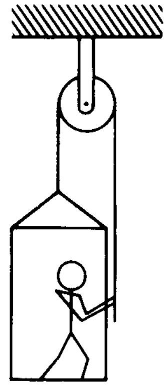


1985M2. An apparatus to determine coefficients of friction is shown above. The box is slowly rotated counterclockwise. When the box makes an angle θ with the horizontal, the block of mass m just starts to slide, and at this instant the box is stopped from rotating. Thus at angle θ, the block slides a distance d, hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities, derive an expression for each of the following.

a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

c. μ_k, the coefficient of kinetic friction.



1986M1. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.

a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

b. What is the tension in the rope under these new conditions?

c. Under these conditions, what is the force exerted by the platform on the person?

After a short time, the person and the platform reach and sustain an upward velocity of 0.4 m/ s.

d. Determine the power output of the person required to sustain this velocity.

1986M3 (modified) A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the *cube of the displacement;* i.e., $F = -kx^3$

This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

a. The potential energy in the spring at the instant the mass is released
b. The maximum speed of the mass
c. The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

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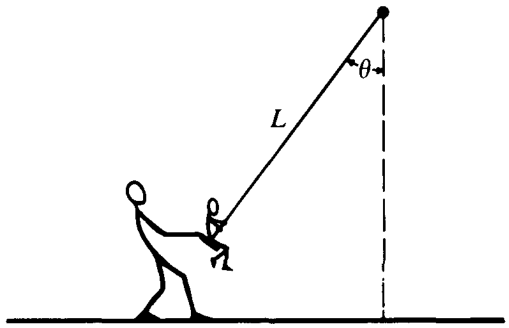
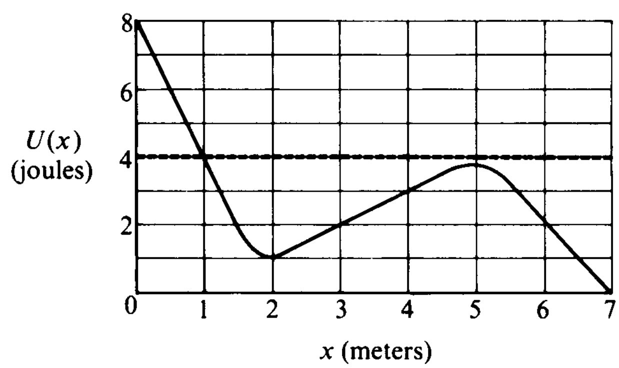


1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

a. the tension in the rope;
b. the horizontal force exerted by the adult.

The adult releases the swing from rest. In terms of W and θ determine

c. the tension in the rope just after the release (the swing is instantaneously at rest);
d. the tension in the rope as the swing passes through its lowest point.

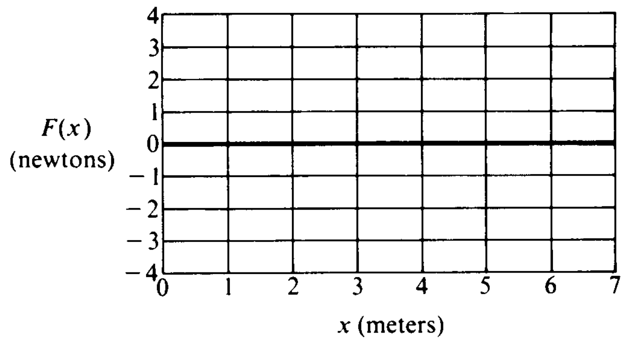


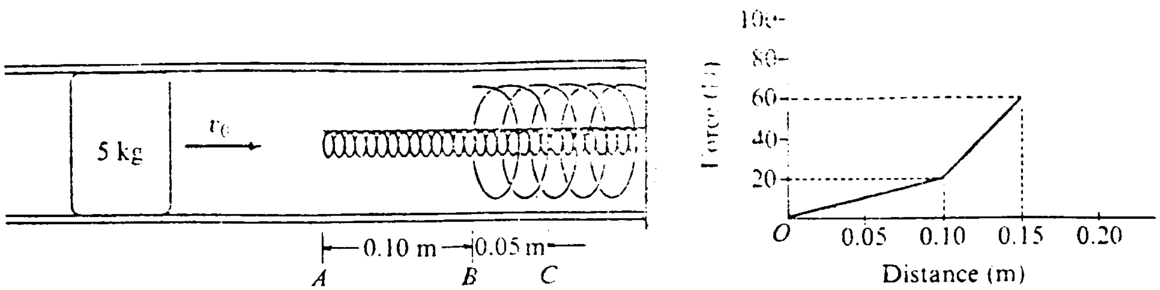
1987M2. The above graph shows the potential energy U(x) of a particle as a function of its position x.

a. Identify all points of equilibrium for this particle.

Suppose the particle has a constant total energy of 4.0 joules, as shown by the dashed line on the graph.

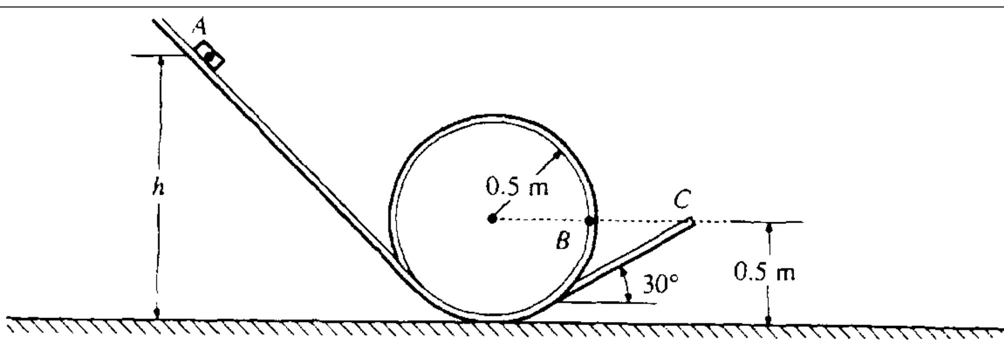
b. Determine the kinetic energy of the particle at the following positions
   i. x = 2.0 m
   ii. x = 4.0 m
c. Can the particle reach the position x = 0.5 m? Explain.
d. Can the particle reach the position x = 5.0 m? Explain.
e. On the grid below, carefully draw a graph of the conservative force acting on the particle as a function of x, for 0 < x < 7 meters.





1988M2. A 5-kilogram object initially slides with speed v_o in a hollow frictionless pipe. The end of the pipe contains two springs. one nested inside the other, as shown above. The object makes contact with the inner spring at point A, moves 0.1 meter to make contact with the outer spring at point B, and then moves an additional 0.05 meter before coming to rest at point C. The graph shows the magnitude of the force exerted on the object by the springs as a function of the objects distance from point A.

a. Calculate the spring constant for the inner spring.
b. Calculate the decrease in kinetic energy of the object as it moves from point A to point B.
c. Calculate the additional decrease in kinetic energy of the object as it moves from point B to point C.
d. Calculate the initial speed v_o of the object
e. Calculate the spring constant of the outer spring



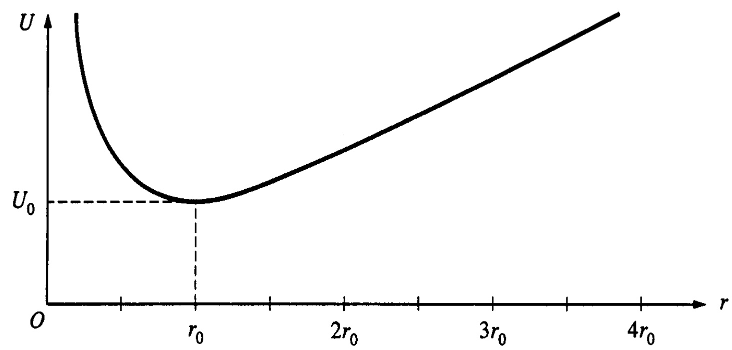
1989M1. A 0.1-kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 m/s at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

a. Determine the height h .

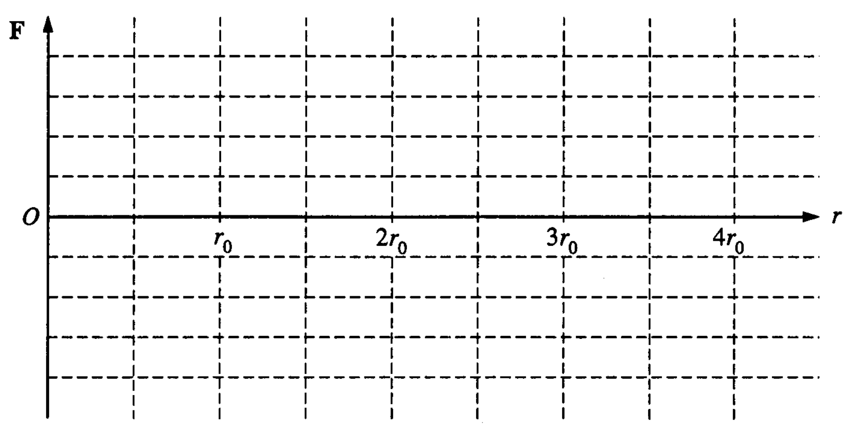
b. On the figure below, draw and label all the forces acting on the block when it is at point B. which is 0.5 meter above the ground.

c. Determine the magnitude of the force exerted by the track on the block when it is at point B.
d. Determine the maximum height above the ground attained by the block after it leaves the track.
e. Another track that has the same configuration, but is **NOT** frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 m/s, the height h must be 2 meters. Determine the work done by the frictional force.

1995M2. A particle of mass m moves in a conservative force field described by the potential energy function $U(r) = a(r/b + b/r)$, where a and b are positive constants and r is the distance from the origin. The graph of U(r) has the following shape.

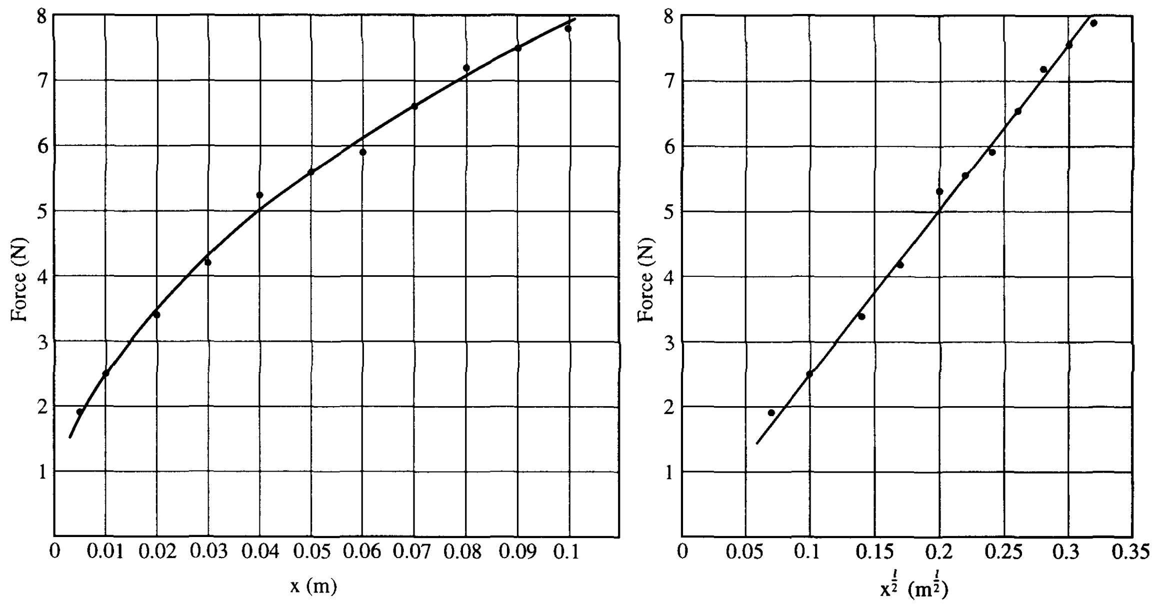


a. In terms of the constants a and b, determine the following.
   i. The position r_o at which the potential energy is a minimum
   ii. The minimum potential energy U_o
b. Sketch the net force on the particle as a function of r on the graph below, considering a force directed away from the origin to be positive, and a force directed toward the origin to be negative.



The particle is released from rest at $r = r_o/2$
c. In terms of U_o and m, determine the speed of the particle when it is at $r = r_o$.
d. Write the equation or equations that could be used to determine where, if ever, the particle will again come to rest. It is not necessary to solve for this position.
e. Briefly and qualitatively describe the motion of the particle over a long period of time.

1997M1. A nonlinear spring is compressed horizontally. The spring exerts a force that obeys the equation $F(x) = Ax^{½}$, where x is the distance from equilibrium that the spring is compressed and A is a constant. A physics student records data on the force exerted by the spring as it is compressed and plots the two graphs below, which include the data and the student's best-fit curves.



a. From one or both of the given graphs, determine A. Be sure to show your work and specify the units.

b. i. Determine an expression for the work done in compressing the spring a distance x.
ii. Explain in a few sentences how you could use one or both of the graphs to estimate a numerical answer to part (b)i for a given value of x.

c. The spring is mounted horizontally on a countertop that is 1.3 m high so that its equilibrium position is just at the edge of the countertop. The spring is compressed so that it stores 0.2 J of energy and is then used to launch a ball of mass 0.10 kg horizontally from the countertop. Neglecting friction, determine the horizontal distance d from the edge of the countertop to the point where the hall strikes the floor

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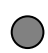
2000M2. A rubber ball of mass m is dropped from a cliff. As the ball falls, it is subject to air drag (a resistive force caused by the air). The drag force on the ball has magnitude bv^2, where *b is* a constant drag coefficient and v is the instantaneous speed of the ball. The drag coefficient *b is* directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

a. On the figure below, draw and label all the forces on the ball at some instant before it reaches terminal speed.



b. State whether the magnitude of the acceleration of the ball of mass m increases, decreases, or remains the same as the ball approaches terminal speed. Explain.

c. Write, but do NOT solve, a differential equation for the instantaneous speed *v* of the ball in terms of time *t*, the given quantities, and fundamental constants.

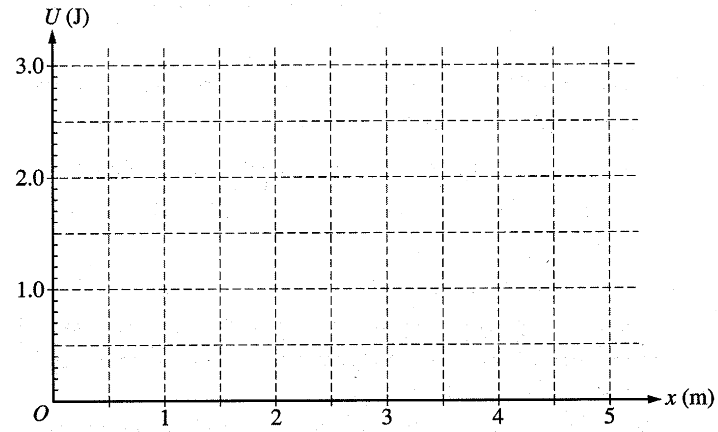
d. Determine the terminal speed v_t in terms of the given quantities and fundamental constants.

e. Determine the energy dissipated by the drag force during the fall if the ball is released at height h and reaches its terminal speed before hitting the ground, in terms of the given quantities and fundamental constants.

2002M3. An object of mass 0.5 kg experiences a force that is associated with the potential energy function

$U(x) = \dfrac{4.0}{2.0 + x}$, where U is in joules and x is in meters.

a. On the axes below, sketch the graph of U(x) versus x.



b. Determine the force associated with the potential energy function given above.

c. Suppose that the object is released from rest at the origin. Determine the speed of the particle at x = 2 m.

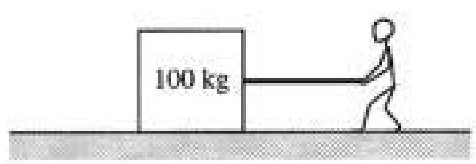
In the laboratory, you are given a glider of mass 0.50 kg on an air track. The glider is acted on by the force determined in part b. Your goal is to determine experimentally the validity of your theoretical calculation in part c.

d. From the list below, select the additional equipment you will need from the laboratory to do your experiment by checking the line to the left of each item. If you need more than one of an item, place the number you need on the line.

______ Meterstick _____ Stopwatch ______ Photogate timer ______String ______Spring

_____ Balance _____ Wood block ______ Set of objects of different masses

e. Briefly outline the procedure you will use, being explicit about what measurements you need to make in order to determine the speed. You may include a labeled diagram of your setup if it will clarify your procedure.



2003M1. The 100 kg box shown above is being pulled along the x-axis by a student. The box slides across a rough surface, and its position x varies with time t according to the equation $x = 0.5t^3 + 2t$, where x is in meters and t is in seconds.

a. Determine the speed of the box at time $t = 0$.

b. Determine the following as functions of time t.
   i. The kinetic energy of the box
   ii. The net force acting on the box
   iii. The power being delivered to the box

c. Calculate the net work done on the box in the interval $t = 0$ to $t = 2$s.

d. Indicate below whether the work done on the box by the student in the interval $t = 0$ to $t = 2$s would be greater than, less than, or equal to the answer in part (c).

_____Greater than _____Less than _____Equal to

Justify your answer

2006M2. A nonlinear spring is compressed various distances x, and the force F required to compress it is measured for each distance. The data are shown in the table below.

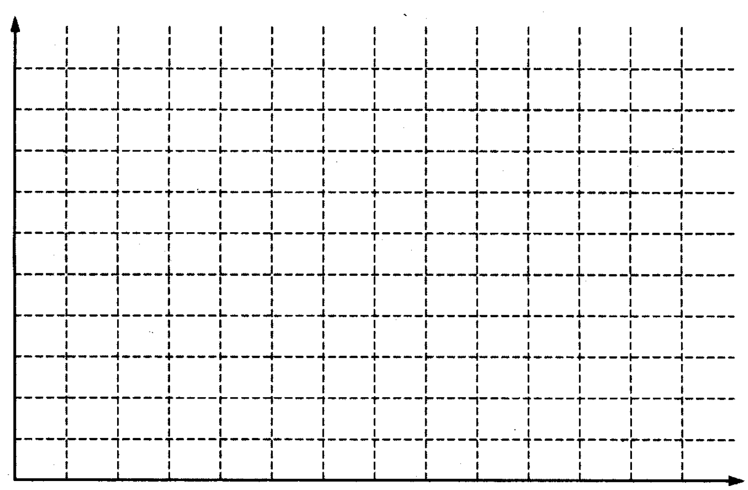
x (m)	F (N)	
0.05	4	
0.10	17	
0.15	38	
0.20	68	
0.25	106	

Assume that the magnitude of the force applied by the spring is of the form $F(x) = Ax^2$.

a. Which quantities should be graphed in order to yield a straight line whose slope could be used to calculate a numerical value for A?

b. Calculate values for any of the quantities identified in a. that are not given in the data, and record these values in the table above. Label the top of the column, including units.

c. On the axes below, plot the quantities you indicated in (a). Label the axes with the variables and appropriate numbers to indicate the scale.

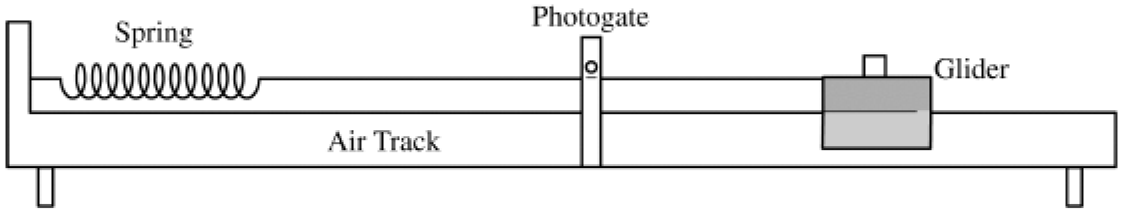


d. Using your graph, calculate A.

The spring is then placed horizontally on the floor. One end of the spring is fixed to a wall. A cart of mass 0.50 kg moves on the floor with negligible friction and collides head-on with the free end of the spring, compressing it a maximum distance of 0.10 m.

e. Calculate the work done by the cart in compressing the spring 0.10 m from its equilibrium length.

f. Calculate the speed of the cart just before it strikes the spring.

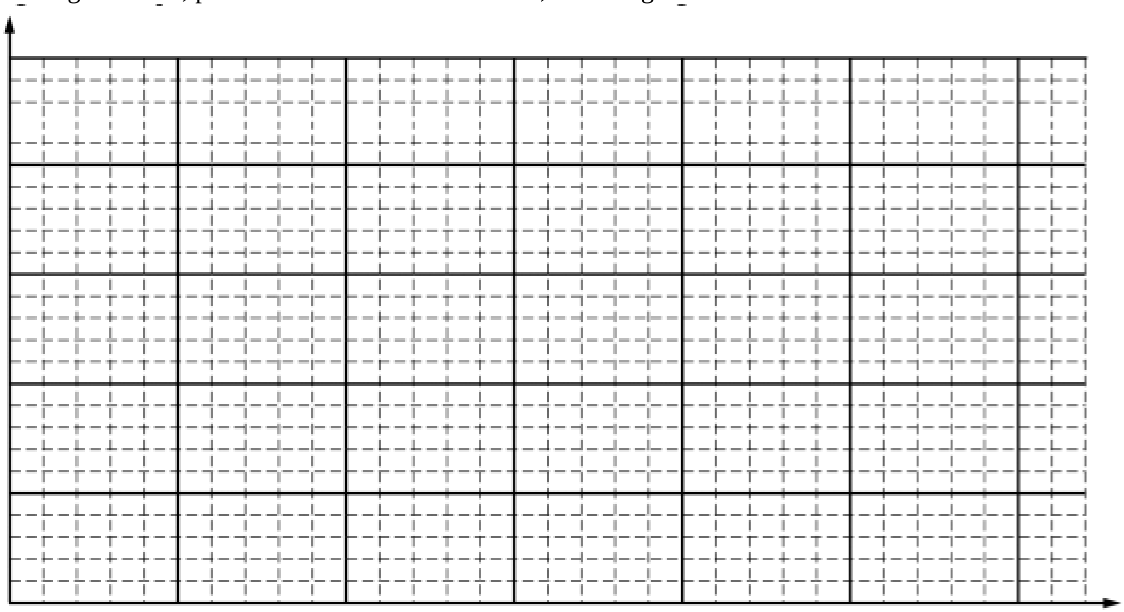


2007M3. The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass *m*. The glider is pulled to stretch the spring an amount *x* from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time *t* that it takes the small block on top of the glider to pass through. Information about the distance *x* and the speed *v* of the glider as it passes through the photogate are given below.

a. Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.

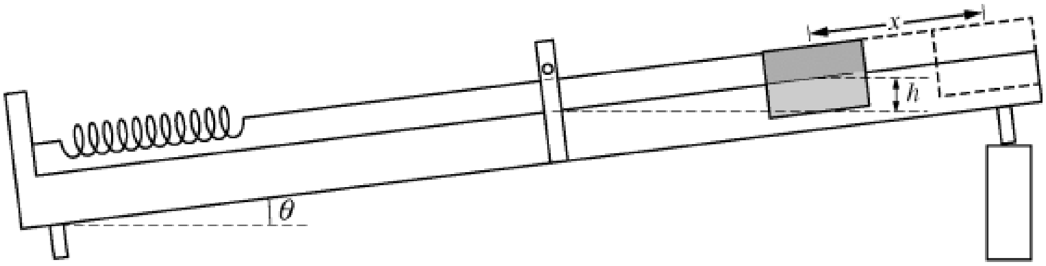
Trial #	Extension of the Spring x (m)	Glider Speed v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

b. On the grid below, plot v^2 versus x^2. Label the axes, including units and scale.



c. i. Draw a best-fit straight line through the data.
ii. Use the best-fit line to obtain the mass *m* of the glider.

d. The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height h above the photogate. The experiment is repeated with a variety of values of x.

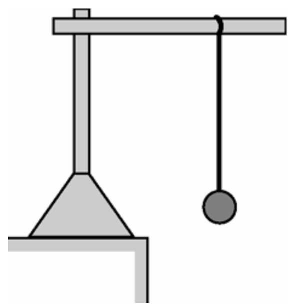


i. Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation.

ii. Will the graph of v^2 versus x^2 for this new experiment be a straight line?

________Yes ________No

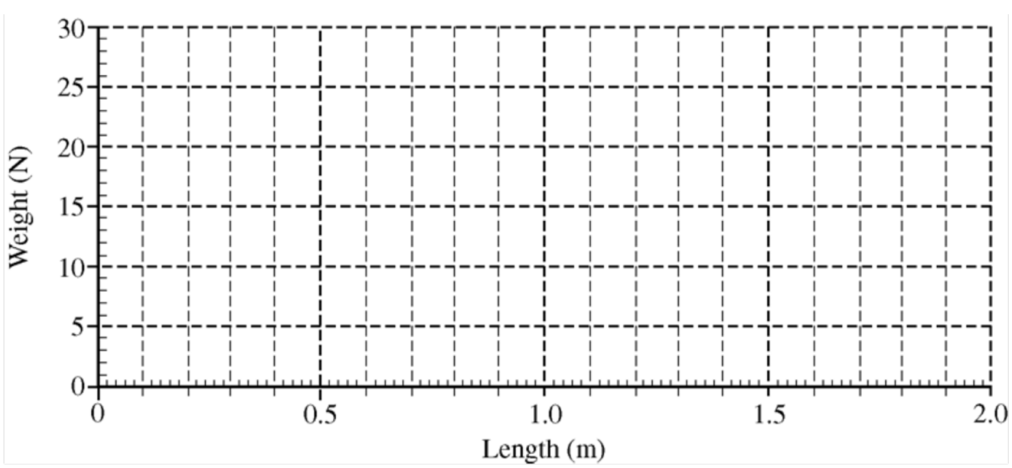
Justify your answer.



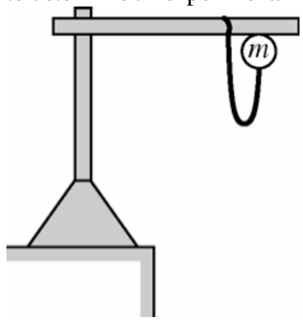
2008M3. In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

a. Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



b. Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as represented above. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

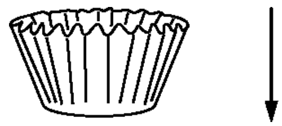
c. Calculate the value of the unknown mass m of the object.
i. Calculate how far down the object has fallen at the moment it attains its maximum speed.
ii. Explain why this is the point at which the object has its maximum speed.
iii. Calculate the maximum speed of the object.

2009M3

A block of mass $M/2$ rests on a frictionless horizontal table, as shown above. It is connected to one end of a string that passes over a massless pulley and has another block of mass $M/2$ hanging from its other end. The apparatus is released from rest.

a. Derive an expression for the speed v_h of the hanging block as a function of the distance d it descends.
Now the block and pulley system is replaced by a uniform rope of length L and mass M, with one end of the rope hanging slightly over the edge of the frictionless table. The rope is released from rest, and at some time later there is a length y of rope hanging over the edge, as shown below. Express your answers to parts (b), (c), and (d) in terms of y, L, M, and fundamental constants.

b. Determine an expression for the force of gravity on the hanging part of the rope as a function of y.
c. Derive an expression for the work done by gravity on the rope as a function of y, assuming y is initially zero.
d. Derive an expression for the speed v_r of the rope as a function of y.
e. The hanging block and the right end of the rope are each allowed to fall a distance L (the length of the rope). The string is long enough that the sliding block does not hit the pulley. Indicate whether v_h from part (a) or v_r from part (d) is greater after the block and the end of the rope have traveled this distance.
____ v_h is greater. ____ v_r is greater. ____ The speeds are equal.
Justify your answer.



2010M1. Students are to conduct an experiment to investigate the relationship between the terminal speed of a stack of falling paper coffee filters and its mass. Their procedure involves stacking a number of coffee filters, like the one shown in the figure above, and dropping the stack from rest. The students change the number of filters in the stack to vary the mass m while keeping the shape of the stack the same. As a stack of coffee filters falls, there is an air resistance (drag) force acting on the filters.

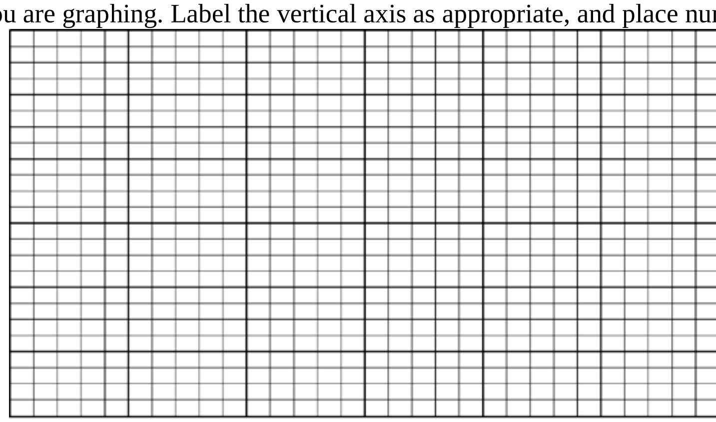
a. The students suspect that the drag force F_D is proportional to the square of the speed v : $F_D = Cv^2$, where C is a constant. Using this relationship, derive an expression relating the terminal speed v_T to the mass m.

The students conduct the experiment and obtain the following data.

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	.51	.62	.82	.92	1.06

b.

i. Assuming the functional relationship for the drag force above, use the grid below to plot a linear graph as a function of m to verify the relationship. Use the empty boxes in the data table, as appropriate, to record any calculated values you are graphing. Label the vertical axis as appropriate, and place numbers on both axes.



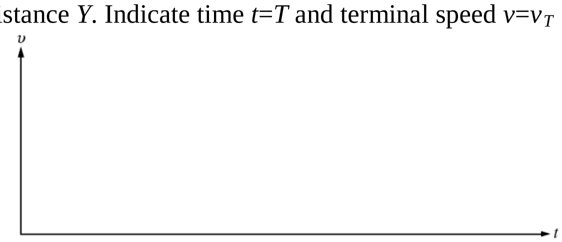
m (kg)

ii. Use your graph to calculate C.

A particular stack of filters with mass m is dropped from rest and reaches a speed very close to terminal speed by the time it has fallen a vertical distance Y.

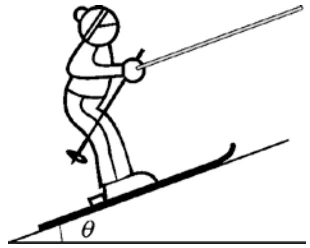
c.

i. Sketch an approximate graph of speed versus time from the time the filters are released up to the time $t=T$ that the filters have fallen the distance Y. Indicate time $t=T$ and terminal speed $v=v_T$ on the graph.



ii. Suppose you had a graph like the one sketched in (c) (i) that had a numerical scale on each axis. Describe how you could use the graph to approximate the distance Y.

d. Determine an expression for the approximate amount of mechanical energy dissipated, ΔE , due to air resistance during the time the stack falls a distance y, where $y > Y$. Express your answer in terms of y , m, v_T , and fundamental constants.



2010M3. A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{max}\sin(\pi t/T) \qquad (0 < t < T)$$
$$= 0 \qquad (t \leq T).$$

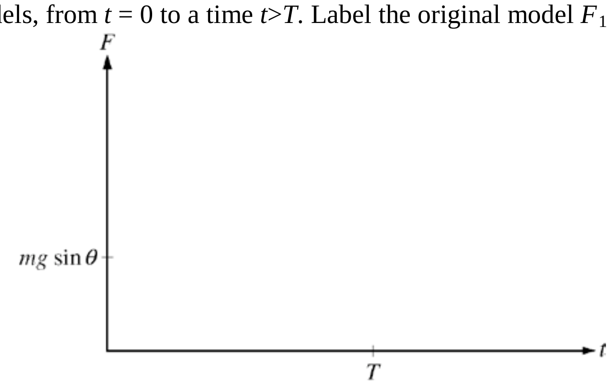
where $a_{\max}$ and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

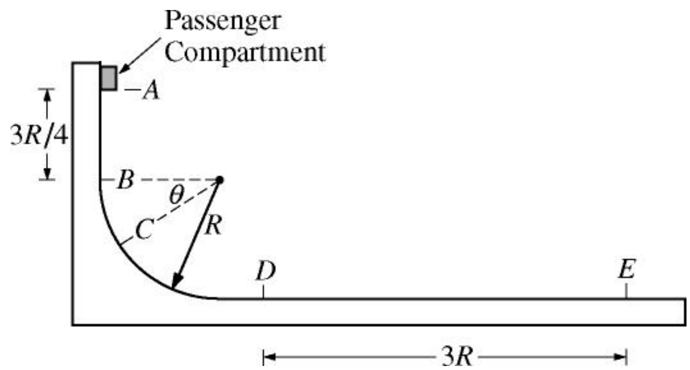
a. Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

b. Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

c. Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

d. Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max}e^{-\pi t/2T}$ for all $t > 0$, where $a_{\max}$ and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2 .

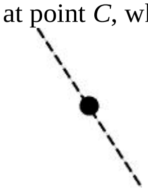




2011M2.

An amusement park ride features a passenger compartment of mass M that is released from rest at point A, as shown in the figure above, and moves along a track to point E. The compartment is in free fall between points A and B, which are a distance of 3R/4 apart, then moves along the circular arc of radius R between points B and D. Assume the track is frictionless from point A to point D and the dimensions of the passenger compartment are negligible compared to R.

a. On the dot below that represents the passenger compartment, draw and label the forces (not components) that act on the passenger compartment when it is at point *C*, which is at an angle θ from point *B*.



b. In terms of θ and the magnitudes of the forces drawn in part (a), determine an expression for the magnitude of the centripetal force acting on the compartment at point *C*. If you need to draw anything besides what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figure in part (a).

c. Derive an expression for the speed v_D of the passenger compartment as it reaches point *D* in terms of *M*, *R*, and fundamental constants, as appropriate.

A force acts on the compartment between points *D* and *E* and brings it to rest at point *E*.

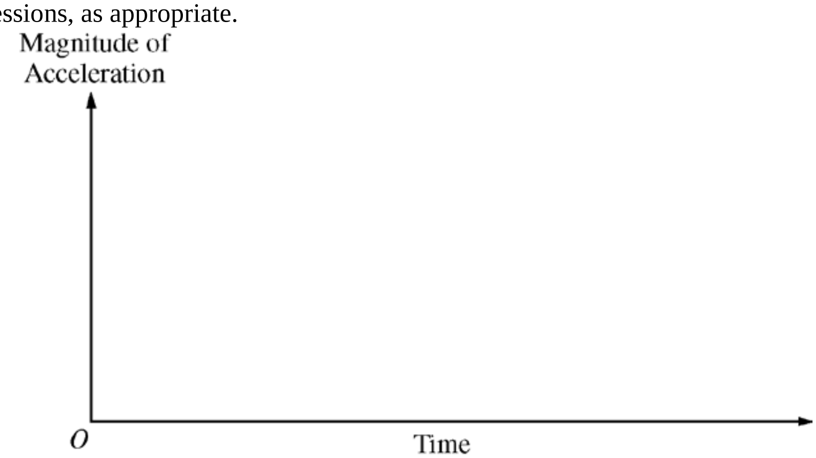
d. If the compartment is brought to rest by friction, calculate the numerical value of the coefficient of friction μ between the compartment and the track.

e. Now consider the case in which there is no friction between the compartment and the track, but instead the compartment is brought to rest by a braking force $-k\mathbf{v}$, where *k* is a constant and **v** is the velocity of the compartment. Express all algebraic answers to the following in terms of *M*, *R*, *k*, v_d, and fundamental constants, as appropriate.

i. Write, but do NOT solve, the differential equation for $v(t)$.

ii. Solve the differential equation you wrote in part i.

iii. On the axes below, sketch a graph of the magnitude of the acceleration of the compartment as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.



	Solution	Answer
1.	The force needed to lift something at a constant speed is equal to the object weight F=mg. The power is then found by P = Fd / t = mgh / t	B
2.	As the system moves, m_2 loses energy over distance h and m_1 gains energy over the same distance h but some of this energy is converted to KE so there is a net loss of U. Simply subtract the $U_2 – U_1$ to find this loss	A
3.	In a force vs. displacement graph, the area under the line gives the work done by the force and the work done will be the change in the K so the largest area is the most K change	E
4.	Since the speed is constant, the pushing force F must equal the friction force $f_k = \mu F_n = \mu mg$. The power is then given by the formula P = Fv = μmgv	C
5.	velocity is the derivative of position therefore is proportional to $t^{1/2}$ and sonce KE is propotional to v^2, KE is proportional to t	C
6.	Compare the U+K (mgh + ½ mv^2) at the top, to the K (½ mv^2) at the bottom and subtract them to get the loss.	C
7.	Use energy conservation, U top = K bottom. As in problem #6 (in this document), the initial height is given by L – Lcos θ, with cos 60 = .5 so the initial height is ½ L.	A
8.	Use application of the net work energy theorem which says … $W_{net} = \Delta K$. The net work is the work done by the net force which gives you the answer	A
9.	F = –dU/dx = –12x + 4	B
10.	First use the given location (h=10m) and the U there (50J) to find the mass. U=mgh, 50=m(10)(10), so m = 0.5 kg. The total mechanical energy is given in the problem as U+K = 100 J. The max height is achieved when all of this energy is potential. So set 100J = mgh and solve for h	B
11.	Simple P = Fv to solve	E
12.	The force needed to lift something at a constant speed is equal to the object weight F=mg. The power is then found by P = Fd / t = mgh / t	E
13.	$U(x_1) = –2$ J and $K(x_1) = 1$ J so E = K + U = – 1J. The horizontal line drawn at E = –1 J has turning points greater than x_0 and less than x_2	E
14.	The work done by a field is path independent and only depends upon the initial and final positions	A
15.	To find work we use the parallel component of the force to the distance, this gives Fcos θ d	B
16.	In a circle at constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle	A
17.	$K(r_0) = 0$ and $U(r_0) = 3U_0$ so $E(r_0) = K + U = 3U_0 = U(2r_0) + K(2r_0) = U_0 + ½ mv^2$	C
18.	F = –dU/dr	A
19.	At the maximum displacement the K=0 so the 10J of potential energy at this spot is equal to the total amount of mechanical energy for the problem. Since energy is conserved in this situation, the situation listed must have U+K add up to 10J	B

20. Using the work-energy theorem. $W_{nc} = \Delta$ ME, — A
$W_{Ft} = \Delta U + \Delta K$,
$- Fd = (mgh_f - mgh_i) + (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$,
$-(11000)(8) = (0 - (1000)(10)(8)) + (0 - \frac{1}{2}(1000)(v_i^2))$ … solve for v_i

21. $\Delta U = -\int F_s dx = -\int -(40x - 6x^2)dx = 20x^2 - 2x^3$ — D

22. $W_g(A \rightarrow B) = -W_g(B \rightarrow A)$ (independent of path) — C

23. First, the speed is constant, so you expect (by the Work-Energy Theorem) that there isn't any work. Beyond that, the definition of work for a constant force is $F \bullet \Delta r = F\Delta r \cos\theta$. Since the force of the string is always radial, and the displacement tangential, the angle between these is 90º; the string can do no work. (It's basically the same story as with magnetic fields acting on free charges; no work can be done by the magnetic field because the force is at right angles to the displacement.) All the given numbers are merely distracters. — A

24. $W = \int F dx = \int k{x_0}^2 dx = \frac{1}{3}k{x_0}^3$ — E

25. Power, in Watts, is work divided by time; power times time gives work. So the work performed by the motor is 1000W×10 s = 10^4 J. This is equal to the potential energy mgh gained by the safe = 100 kg × 10 m/s2 × h, so h = 10^4 J/1000 N = 10 m. — C

26. kg m/s is a unit of momentum — E

27. K is a scalar. $K_f = \frac{1}{4} K_i$; $\frac{1}{2} mv_f^2 = \frac{1}{4}(\frac{1}{2} mv_i^2)$ which gives $v_f = \frac{1}{2} v_i$ but since it is traveling in the opposite direction v_f (a vector) will be negative — D

28. $P_{avg} = F_{net}v_{avg} = mav_{avg}$ = (2000 kg)×(3 m/s^2)×(10 m/s) — E

1973M1

a. Free-Body Diagrams
- free-body diagram for m_1 contains
  - i. friction force F_f directed to the left
  - ii. pulling force F directed to the right
  - iii. gravitational force m_1g directed downwards
  - iv. normal force of 2 on 1 $F_{N(2\text{ on }1)}$ directed upwards
- free-body diagram for m_2 contains
  - i. friction force F_f directed to the right
  - ii. gravitational force m_2g directed downwards
  - iii. normal force of 1 on 2 $F_{N(1\text{ on }2)}$ directed downwards
  - iv. normal force of surface on 2 $F_{N(S\text{ on }2)}$ directed upwards

b. Using Newton's Second Law and the free-body diagrams we get:
$\Sigma F = m_1a_1$; $F - F_f = m_1a_1$; $F - \mu m_1 g = m_1a_1$
$\Sigma F = m_2a_2$; $F_f = m_2a_2$; $\mu m_1 g = m_2a_2$
$a_2 = \mu(m_1/m_2)g$ and $a_1 = (F/m_1) - \mu g$

c. Applying kinematics to m_1: $x_1 = v_it + ½ (a_{rel})t^2$, where $x = l$, $v_i = 0$ and $a_{rel} = a_1 - a_2$: the acceleration of m_1 relative to m_2. Solving for t gives:

$$t = \sqrt{\frac{2l}{\left(\frac{F}{m_1}\right) - \mu g\left(1 + \frac{m_1}{m_2}\right)}}$$

d. $\Delta E = W_f = -F_f d = -\mu m_1 g \ell$

1973M2

a. Apply work-energy theorem
$W_{nc} = \Delta ME$
$W_{fk} = \Delta K \quad (K_f - K_i) \qquad K_f = 0$
$-f_k\, d = -½\, mv_i^2 \qquad -f_k\,(0.12) = -½\,(0.030)\,(500)^2 \qquad f_k = 31250$ N

b. Find find acceleration $\qquad -f_k = ma \qquad -(31250) = (0.03)\,a \qquad a = -1.04\text{x}10^6$ m/s^2

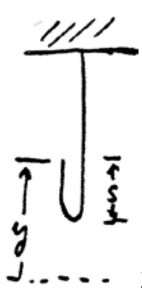
Then use kinematics $\qquad v_f = v_i + at \qquad 0 = 500 + (-1.04\text{x}10^6)\,t \qquad t = 4.8\text{x}10^{-4}$ sec

c. The absolute value of the acceleration increases slowing in size causing the speed to decrease quadratically instead of linearly to zero. This causes the depth of penetration to increase more slowly.

1975M3

a. using the ceiling as a reference point: $U_i = -½$ mgl; $U_f = -¼$ mgl
$\Delta U = U_f - U_i = ¼$ mgl

b.



$\lambda = m/l$, the weight of the piece being held is $\lambda sg = mgs/l$; $s = y/2$ so $F(y) = mgy/2l$

c.

$$\int_0^l F dy = \frac{mg}{2l}\int_0^l y dy = \frac{mg}{2l}\frac{l^2}{2} = \frac{1}{4}mgl$$

1981M2

a. $(2M)gh = ½\,(2M)v^2$

$h = L/2$ so $v = (gL)^{1/2}$ at the bottom

During the upswing, $½\,Mv_{swing}^2 = MgH$ where $H = L(1 - \sqrt{2}/2)$

$$v_s = \sqrt{gL}\left(\sqrt{2-\sqrt{2}}\right)$$

During the jump momentum is conserved

$2Mv = Mv_c + Mv_s$

$$v_c = \sqrt{gL}\left(2-\sqrt{2-\sqrt{2}}\right)$$

1982M1

a. Apply energy conservation, set the top of the spring as h=0, therefore H at start = Lsin θ = 6 sin30 = 3 m

$U_{top} = K_{bot}$ $\quad mgh = ½\, mv^2$ $\quad (9.8)(3) = ½\,(v^2)$ $\quad v = 7.67$ m/s

b. Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.

$h = L \sin\theta + \Delta x \sin\theta$

$U_{top} = U_{sp}(bot)$

$mgh = ½\, k\, \Delta x^2$

$mg(L \sin\theta + \Delta x \sin\theta) = ½\, k\, \Delta x^2$

$(20)(9.8)(6 \sin 30 + 3 \sin 30) = ½\, k\, (3)^2$ $\quad k = 196$ N/m

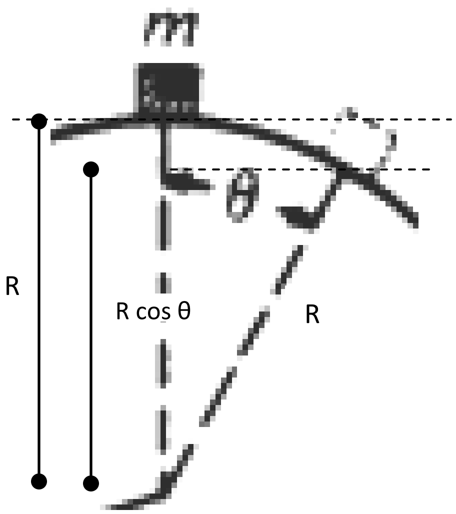
c. The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

1982M2

a. $K = ½ Mv^2 = ½ M(v_0{}^2 – Rt/M)$

(the rest of this question is in the dynamics chapter)

1983M3



$h = R – R \cos\theta = R (1 – \cos \theta)$

i. $K_2 = U_{top}$
   $K_2 = mg(R (1 – \cos \theta))$

ii. From, $K = ½ m v^2 = mgR (1 – \cos \theta)$ …
   $v^2 = 2gR (1 – \cos \theta)$
   Then $a_c = v^2 / R = 2g (1 – \cos \theta)$

iii. The onlt force with a tangential component is the gravitational force mg. It's tangential component is mg sin θ. By Newton's Second Law $\Sigma F_{tan} = ma_{tan}$; $mg \sin \theta = ma_{tan}$; $a_{tan} = g \sin \theta$

b. The particle leaves the sphere when the normal force has decreased to zero. At that point the radial component of the weight provides the centripetal acceleration so $mg \cos \theta = ma_c = m(2g(1–\cos \theta))$ and $\cos \theta = 2/3$; $\theta = 48^\circ$

C1985M2

a. We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} – f_k = 0$ $mg \sin \theta = \mu_s(F_n)$
   $mg \sin \theta = \mu_s mg \cos \theta$ $\mu_s = \tan \theta$

b. Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so h= (d+x)sin θ
   $U_f – U_i + U_{sp(F)} – U_{sp(i)}$
   $0 – mgh + ½ k\Delta x^2 – 0$ $½ kx^2 – mg(d+x)\sin \theta$

c. To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k
   $W_{NC} = ½ kx^2 – mg(d+x)\sin \theta$ $– f_k(d+x) = ½ kx^2 – mg(d+x)\sin \theta$
   $– \mu_k mg \cos \theta (d+x) = ½ kx^2 – mg(d+x)\sin \theta$

   $\mu_k = [mg(d+x)\sin \theta – ½ kx^2] / [mg (d+x)\cos \theta]$

1986M1

a. Combining the person and the platform into one object, held up by two sides of the rope we have ΣF = ma;
   2T = (80 kg + 20 kg)g giving T = 500 N
b. Similarly, ΣF = ma; 2T – 1000 N = (100 kg)(2 m/s^2) giving T = 600 N
c. For the person only: ΣF = ma; N + 600 N – mg = ma gives N = 360 N
d. P = Fv = mgv = 400 W

1986M3

a. $\Delta U = -\int F dx$

$U = -\int_0^A -kx^3 dx = \frac{kA^4}{4}$

b. $\frac{1}{2} M v_{max}^2 = \frac{1}{4} kA^4$

$v_{max} = A^2 (k/2M)^{1/2}$

c. $E_{total} = K + U$; When $K = U$ then $E_{tot} = U + U = 2U$

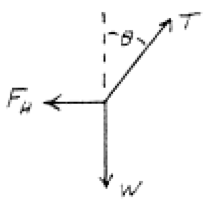
$U = \frac{1}{2} E_{tot}$

$\frac{1}{4}(kx^4) = \frac{1}{2}(kA^4/4)$

$x = A(2^{-1/4})$

1987M1

a.



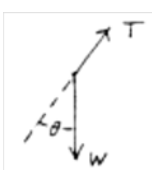
$F_{net(y)} = 0$

$T \cos\theta - W = 0$ $\quad$ $T = W / \cos\theta$

b. Apply SIMULTANEOUS EQUATIONS

$F_{net(y)} = 0$	$F_{net(x)} = 0$
$T \cos\theta - W = 0$	$T \sin\theta - F_h = 0$
Sub T into X equation to get F_h	$F_h = W \tan\theta$

c.



At rest $\Sigma F_{radial} = 0$; $T = W \cos\theta$

d. $U_{top} = K_{bot}$

$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos\theta)}$$

$$v = \sqrt{2gL(1 - \cos\theta)}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(T - W) = m(2gL(1 - \cos\theta)) / L$

$T = W + 2mg - 2mg \cos\theta$

$T = W + 2W - 2W \cos\theta = W(3 - 2\cos\theta)$

1987M2

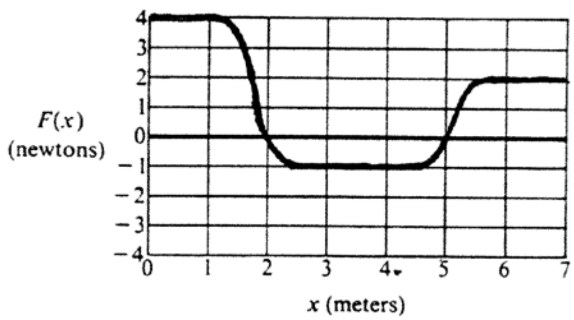
a. Equlibrium is where F = 0 and since F = –dU/dx, F = 0 where the slope is zero, this occurs at x = 2m & x = 5 m

b. $E_{tot} = K + U = 4$ J
   i. at x = 2 m, K = 4 J – U(2 m) = 4 J – 1 J = 3 J
   ii. at x = 4 m, K = 4 J – U(4 m) = 4 J – 3 J = 1 J

c. No, the particle cannot reach x = 0.5 m, the energy is not sufficient

d. Yes, the particle can reach x = 5.0 m, the energy is sufficient

e.



1988M2

a. For linear force relation: F = kx, from the graph (20 N) = k_1(0.10 m); k_1 = 200 N/m

b. $|\Delta K| = W_s$ = area under graph = 1 J

c. $|\Delta K|$ = area under graph from B to C = 2 J

d. $K_i = 3\text{ J} = \frac{1}{2} m v_0^2$; v_0 = 1.1 m/s

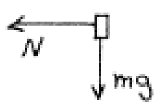
e. $\frac{1}{2} m v_0^2 = \frac{1}{2} k_1 (x_{AC})^2 + \frac{1}{2} k_2 (x_{BC})^2$ OR k_{net} (B to C) = $k_1 + k_2$ = slope
   these methods give k_2 = 600 N/m

1989M1

a. Apply energy conservation from point A to point C setting point C as h=0 location
   (note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

   $U_A = K_C$ $\quad mgh_a = \frac{1}{2} m v_c^2$ $\quad (0.1)(9.8)(h_a) = \frac{1}{2}(0.1)(4)^2$ $\quad h_a = 0.816$m
   $h = h_a + 0.5\text{ m} = 1.32$ m

b.



c. Since the height at B and the height at C are the same, they would have to have the same velocities v_b = 4 m/s

   $F_{net(c)} = mv^2 / r$ $\quad F_n = (0.1)(4)^2/(0.5) = 3.2$ N

d. Using projectile methods … V_{iy} = 4sin30 = 2 m/s
   Then $v_{fy}^2 = v_{iy}^2 + 2\,a\,d_y$
   $(0) = (2)^2 + 2(-9.8)(d_y)$ $\quad d_y = 0.2$
   $h_{max} = d_y$ + initial height = 0.7 m

   Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

e. Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $\quad U_{new} - U_{old} = mgh_{net} - mgh_{old}$ $\quad$ (0.1)(9.8)(2-1.32) = 0.67 J lost.

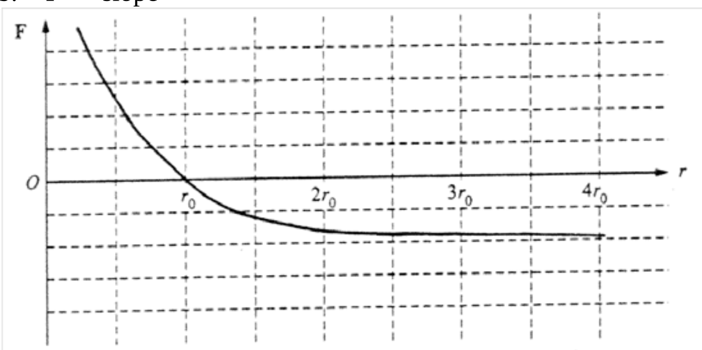
1995M2

a. i. Minimum occurs where $dU/dr = 0$

$dU/dr = a/b - ab/r_0^2 = 0$ gives $r_0 = b$

ii. $U_0 = U(r_0) = U(b) = a(b/b + b/b) = 2a$

b. $F = -$slope



c. $E = K + U =$ constant

$U(r_0/2) = U(r_0) + \frac{1}{2} mv^2$

$U(r_0/2) = a(b/2b + 2b/b) = 5a/2$

$5a/2 = 2a + \frac{1}{2} mv^2$ which gives $v = (U_0/2m)^{1/2}$

d. By conservation of energy the particle will again come to rest at a point r_1 where it has the same potential energy as when at $r_0/2$

$U(r_1) = U(r_0/2)$

e. The particle will oscillate with the end points of the motion at r_1 and $r_0/2$

1997M1

a. One may use the left graph by substituting a point on the line into the force equation or the right graph by substituting a point or calculating the slope. A = 25.5 N/m$^{1/2}$

b. i.

$$W = \int F dx$$

$$W = \int_0^x Ax^{1/2} dx = \frac{2}{3} Ax^{3/2}$$

ii. The work is equal to the area under the left graph

c. U = K

0.2 J = $\frac{1}{2} mv^2$ gives v = 2 m/s

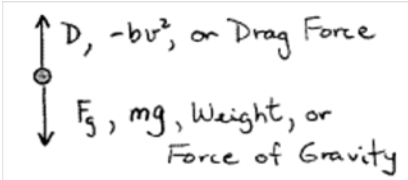
time to hit the ground $t = (2h/g)^{1/2} = 0.52$ s

d = vt = 1 m

2000M2

a. through d. also in dynamics chapter

a.



b. Decreases. As the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

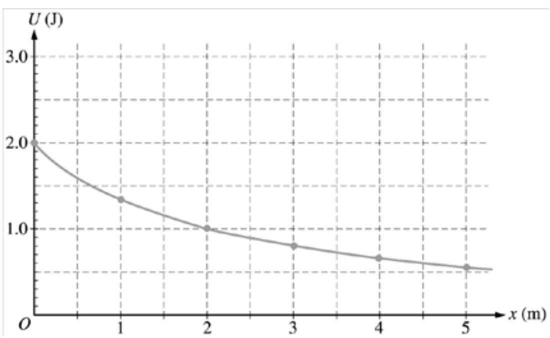
c. $F = mg - bv^2$; $ma = mg - bv^2$; $m(dv/dt) = mg - bv^2$

d. At terminal speed, $a = 0$ so $mg = bv_T^{\,2}$; $v_T = (mg/b)^{1/2}$

e. $\Delta E = U_i - K_f = mgh - \frac{1}{2} mv_f^{\,2} = mgh - \frac{1}{2} m(mg/b) = mg(h - m/2b)$

2002M3

a.



b.

$$F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{4.0}{2.0 + x}\right) = \frac{4.0}{(x + 2.0)^2}$$

c. $E_{tot} = K + U$
$U(0) = U(2) + \frac{1}{2} mv^2$
$2\text{ J} = 1\text{ J} + \frac{1}{2}(0.5\text{ kg})v^2$, which gives $v = 2$ m/s

d./e. The following were common examples. Other examples, though rarely cited, could receive partial or full credit.

1. Using photogates
Place the photogates near $x = 2$ m and a small distance apart (such as a glider length). Measure the distance between the photogates. Measure the time the glider takes to travel between the photogates. Obtain the speed from distance/time. Note: No points were given if the distance measured was from 0 to 2 m and the time to travel 2 m was used.

2. Using a spring
The spring constant k of the spring must be known, or if not, then measured. Set up the spring at $x = 2$ m so that it is compressed when struck by the glider. Measure the distance of maximum compression x_m . The velocity can then be determined from the equation $\frac{1}{2} kx_m^{\,2} = \frac{1}{2} mv^2$

3. Treating the glider as a projectile
Adjust the starting point so that $x = 2$ m is at end of the track. Thus the glider leaves the track at this point and becomes a projectile. The height of the track determines the time interval t that the glider is in the air. The horizontal distance x from the end of the track to the point where the glider hits the ground is measured and then the velocity is computed from x/t.

2003M1

a. $v = dx/dt = 1.5t^2 + 2$
   $v_0 = 2$ m/s

b. i. $K = ½ mv^2 = 50(1.5t^2 + 2)^2$
   ii. $F_{net} = ma = m(dv/dt) = m(3 t) = 300 t$
   iii. $P = Fv = (300 t)(1.5t^2 + 2) = 450t^3 + 600t$ OR $P = dK/dt$

c. $W = \Delta K$; $v(2) = 8$ m/s, $v(0) = 2$ m/s; $W = ½ m(8 \text{ m/s})^2 – ½ m(2 \text{ m/s})^2 = 3000$ J
   Alternately, $W = \int P dt$

d. Greater, the student had to perform work against friction

2006M2

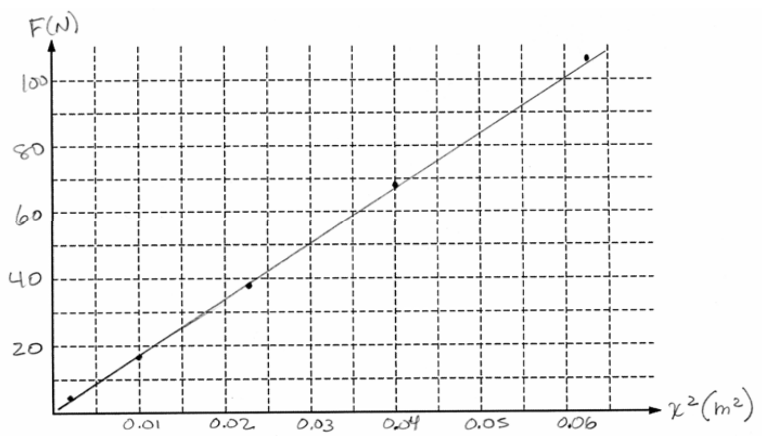
a. F vs. x^2 or $\sqrt{F}$ vs. x

b.

Example using F vs. x^2

x (m)	F (N)	x^2 (m^2)
0.05	4	0.0025
0.10	17	0.010
0.15	38	0.023
0.20	68	0.040
0.25	106	0.063

c.



d. $A = \text{slope} = \Delta F/\Delta(x^2) = 1.7 \times 10^3$ N/m²

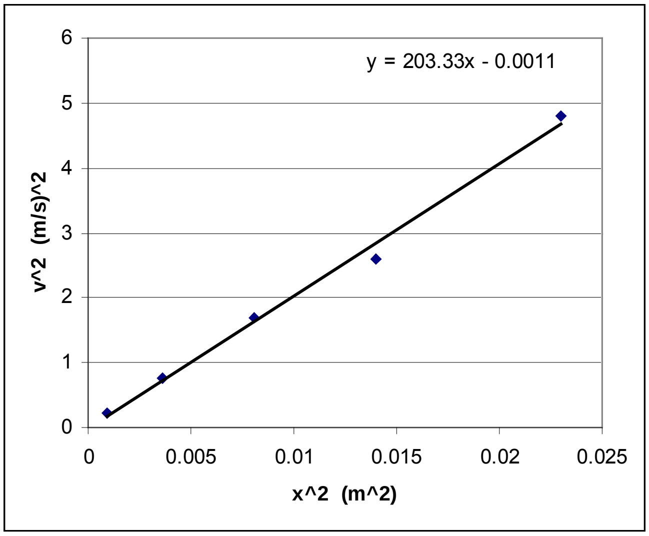
e.
$$W = \int F dx = \int_0^{0.1\,m} A x^2 dx = \frac{1}{3} A(0.10\,m)^3 = 0.57\,J$$

f. $W = \Delta K = ½ mv^2$ gives $v = 1.5$ m/s

2007M3

a. Spring potential energy is converted into kinetic energy $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

b./c. i.



ii. using the equation above and rearrange to the form y = m x with v^2 as y and x^2 as x

y = m x

$v^2 = (k/m)\, x^2$

Slope = 200 = k /m

200 = (40) / m

m = 0.2 kg

d. i. Now you start with spring potential and gravitational potential and convert to kinetic. Note that at position A the height of the glider is given by h + the y component of the stretch distance x.

$h_{initial} = h + x \sin\theta$

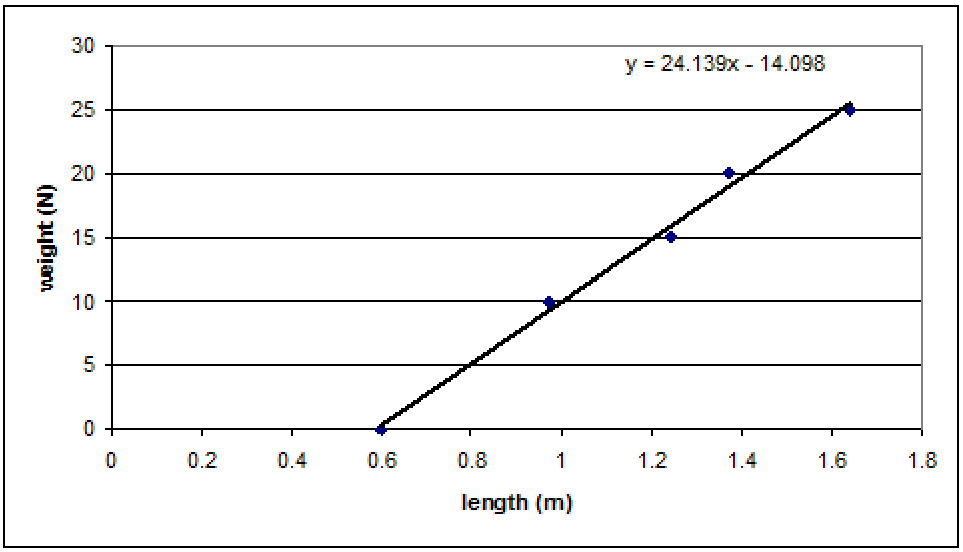
$U + U_{sp} = K$

$mgh + \frac{1}{2} k x^2 = \frac{1}{2} m v^2$ $mg(h + x \sin\theta) + \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

ii. No. v^2 now varies with x^2 and x.

2008M3

a.



b. The slope of the line is F / Δx which is the spring constant.   Slope = 24 N/m

c. Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

$mgh = \frac{1}{2} k \Delta x^2$ $\quad m(9.8)(1.5) = \frac{1}{2} (24)(0.9)^2$ $\quad m = 0.66$ kg

d. i. At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $\quad F_{sp} = (0.66)(9.8)$ $\quad F_{sp} = 6.5$ N

ii. $F_{sp} = k \Delta x$ $\quad 6.5 = (24) \Delta x$ $\quad \Delta x = 0.27$ m

iii. $mgy_{v\,max} = \frac{1}{2} kx^2 + \frac{1}{2} mv_{max}^2$; $v_{max}^2 = 2gy_{v\,max} - kx^2/m$; $v_{max} = 3.8$ m/s

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2009M3

a. $mgh = \frac{1}{2} mv^2$
The speed of both blocks is v_h
$(M/2)gd = \frac{1}{2} (M/2 + M/2)v_h^2$
$v_h = (gd)^{1/2}$

b. F_g = the fraction of the rope hanging off the table × the total weight of the rope: $F_g = (Mg)(y/L)$

c.
$$W = \int F dy = \int \frac{Mg}{L} y dy = \frac{Mg}{2L} y^2$$

d. $W = \Delta K$
$Mgy^2/2L = \frac{1}{2} Mv^2$ which gives $v = (g/L)^{1/2} y$

e. The speeds are equal
Substituting L for d and y in the above equations yields $(gL)^{1/2}$ in both cases

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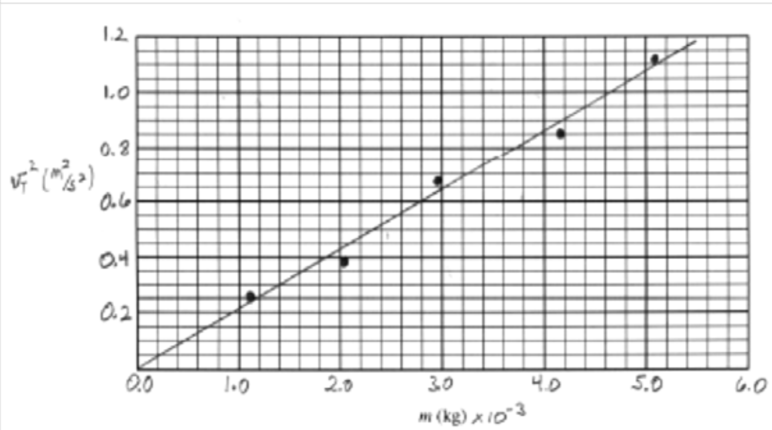
2010M1

a through d is in the dynamics chapter

a. $\Sigma F = mg - Cv^2 = ma$
At terminal velocity v_T, a = 0 so $mg = Cv_T^2$ and $v_T^2 = (g/C)m$

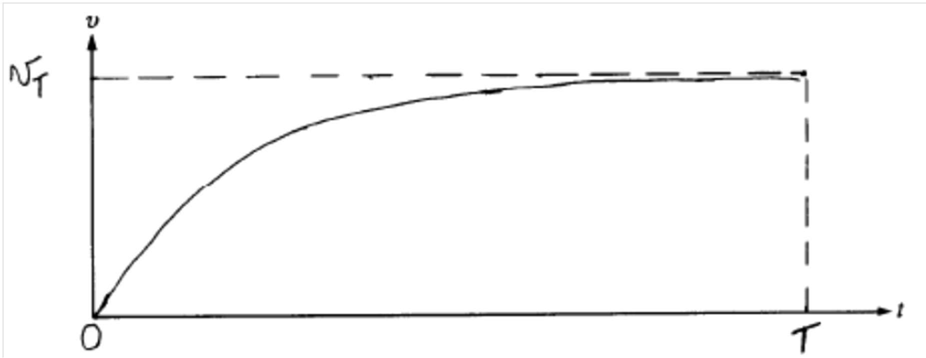
b. i.

Mass of the stack of filters, m (kg)	1.12×10^{-3}	2.04×10^{-3}	2.96×10^{-3}	4.18×10^{-3}	5.10×10^{-3}
Terminal speed, v_T (m/s)	0.51	0.62	0.82	0.92	1.06
v_T^2 (m^2/s^2)	0.26	0.38	0.67	0.85	1.12



ii. slope = g/C = 217 m^2/kg-s^2 giving C = 0.045 kg/m

c. i.



ii. Distance Y is the area under the curve between 0 and T

e. $\Delta E = \Delta U + \Delta K$

$\Delta U = -mgy$ and $\Delta K = \frac{1}{2} mv_T^2$

$\Delta E = \frac{1}{2} mv_T^2 - mgy$

<u>2010M3</u>

a.

$$v = \int a\,dt = \int_0^t a_{max} sin\frac{\pi t}{T}dt = -\frac{a_{max}T}{\pi}cos\frac{\pi t}{T}\bigg|_0^t = \frac{a_{max}T}{\pi}\left(1 - cos\frac{\pi t}{T}\right)$$

b. $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2)$

$v_f = v(T) = 2a_{max}T/\pi$

$v_i = v(0) = 0$

$W = 2ma_{max}^2T^2/\pi^2$

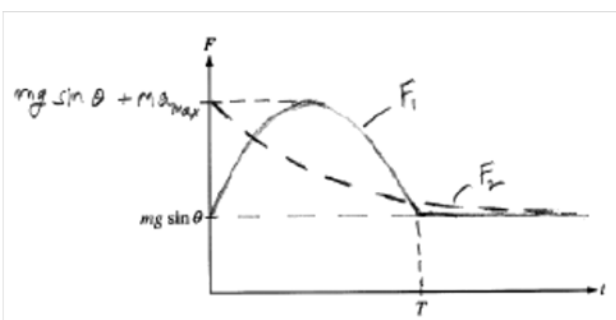
c. $\Sigma F = F_{rope} - mg \sin \theta = ma$ where $a = 0$ at terminal velocity

$F_{rope} = mg \sin \theta$

d.

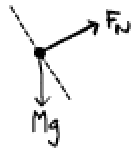
$$F_1 = mg\ sin\ \theta + ma_{max}sin\left(\frac{\pi t}{T}\right)\ \ (0 < t < T)$$

$$F_2 = mg\ sin\ \theta + ma_{max}e^{-\pi t/2T}$$



2011M2

a.



b. $F_c = F_N - mg \sin \theta$

Alternately, one could use conservation of energy $Mg(3R/4 + R \sin \theta) = ½ mv^2$ and $F_c = mv^2/R$ which gives $F_c = 2Mg(3/4 + \sin \theta)$

c. $Mg(3R/4 + R) = ½ Mv^2$ gives $v_D = (7gR/2)^{1/2}$

d. $W = F_f d = \Delta K = - ½ Mv_D{}^2$

$\mu Mgd = ½ M(7gR/2)$

$\mu = 7/12$

e. i. $\Sigma F = ma$; $-kv = M(dv/dt)$

ii.

$$\frac{dv}{dt} = -\frac{kv}{M}$$

$$\frac{dv}{v} = -\frac{k}{M}dt$$

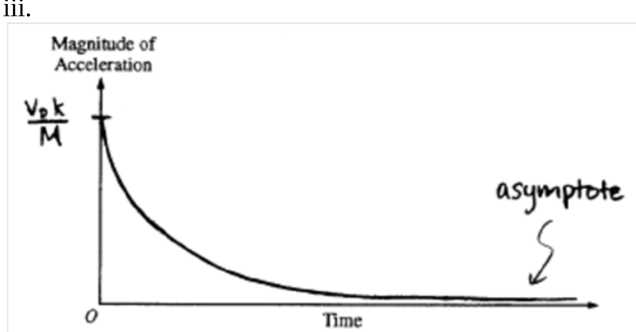
$$\int_{v_D}^{v} \frac{dv}{v} = \int_0^t -\frac{k}{M}dt$$

$$ln\left|\frac{v}{v_D}\right| = -\frac{k}{M}t$$

$$\frac{v}{v_D} = e^{-\frac{k}{M}t}$$

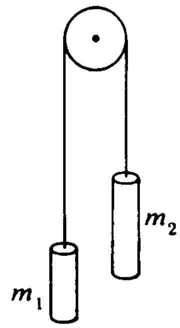
$$v = v_D e^{-\frac{k}{M}t}$$

iii.

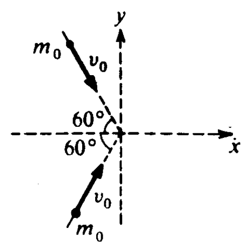


Chapter 4

Center of Mass and Momentum

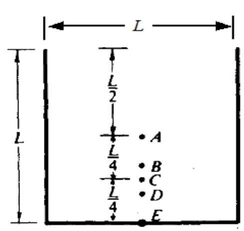


1. A system consists of two objects having masses m_1 and m_2 ($m_1 < m_2$). The objects are connected by a massless string, hung over a pulley as shown above, and then released. When the speed of each object is v, the magnitude of the total linear momentum of the system is
(A) $(m_1 + m_2)$ v (B) $(m_2 - m_1)$ v (C) ½$(m_1 + m_2)$v (D) ½$(m_2 - m_1)$v (E) m_2v



2. Two particles of equal mass m_o, moving with equal speeds v_O along paths inclined at 60° to the x-axis as shown above, collide and stick together. Their velocity after the collision has magnitude

(A) $\frac{v_0}{4}$ (B) $\frac{v_0}{2}$ (C) $\frac{\sqrt{2}v_0}{2}$ (D) $\frac{\sqrt{3}v_0}{2}$ (E) v_o

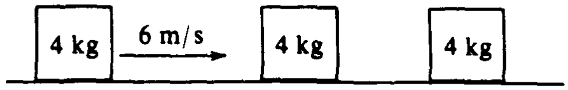


3. The center of mass of a uniform wire, bent in the shape shown above, is located closest to point
(A) A (B) B (C) C (D) D (E) E

4. Mass M_1 is moving with speed v toward stationary mass M_2. The speed of the center of mass of the system is

(A) $\frac{M_1}{M_2}v$ (B) $\left(1+\frac{M_1}{M_2}\right)v$ (C) $\left(1+\frac{M_2}{M_1}\right)v$ (D) $\left(1-\frac{M_1}{M_2}\right)v$ (E) $\left(\frac{M_1}{M_1+M_2}\right)v$

Questions 5-6



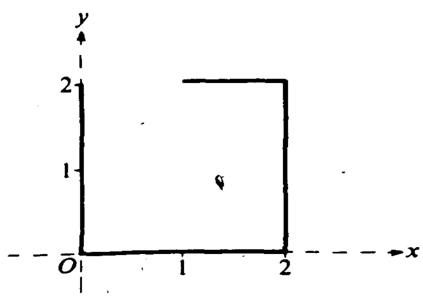
A 4-kilogram mass has a speed of 6 meters per second on a horizontal frictionless surface, as shown above. The mass collides head-on and elastically with an identical 4-kilogram mass initially at rest. The second 4-kilogram mass then collides head-on and sticks to a third 4-kilogram mass initially at rest.

5. The final speed of the first 4-kilogram mass is
(A) 0 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s (E) 6 m/s

6. The final speed of the two 4-kilogram masses that stick together is
(A) 0 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s (E) 6 m/s

7. A projectile of mass M_1 is fired horizontally from a spring gun that is initially at rest on a frictionless surface. The combined mass of the gun and projectile is M_2. If the kinetic energy of the projectile after firing is K, the gun will recoil with a kinetic energy equal to

(A) K (B) $\frac{M_2}{M_1}K$ (C) $\frac{M_1^2}{M_2^2}K$ D) $\frac{M_1}{M_2 - M_1}K$ (E) $\sqrt{\frac{M_1}{M_2 - M_1}}K$



8. A piece of wire of uniform cross section is bent in the shape shown above. What are the coordinates $(\bar{x}, \bar{y})$ of the center of mass?
(A) (15/14, 6/7) (B) (6/7, 6/7) (C) (15/14, 8/7) (D) (1,1) (E) (1,6/7)

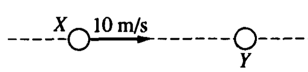


Figure I

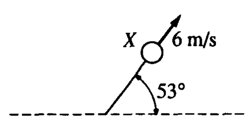
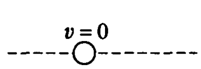
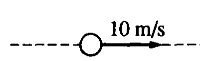
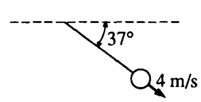


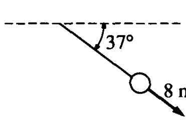
Figure II

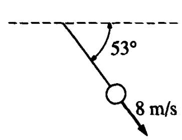
9. Two balls are on a frictionless horizontal tabletop. Ball X initially moves at 10 meters per second, as shown in Figure I above. It then collides elastically with identical ball Y. which is initially at rest. After the collision, ball X moves at 6 meters per second along a path at 53^0 to its original direction, as shown in Figure II above. Which of the following diagrams best represents the motion of ball Y after the collision?

(A)


(B)


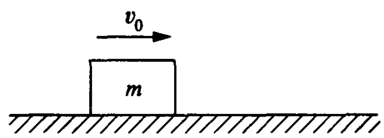
(C)


(D)


(E)


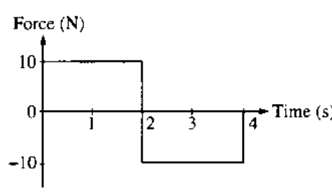
10. A balloon of mass M is floating motionless in the air. A person of mass less than M is on a rope ladder hanging from the balloon. The person begins to climb the ladder at a uniform speed v relative to the ground. How does the balloon move relative to the ground?
(A) Up with speed v
(B) Up with a speed less than v
(C) Down with speed v
(D) Down with a speed less than v
(E) The balloon does not move.

11. If one knows only the constant resultant force acting on an object and the time during which this force acts, one can determine the
(A) change in momentum of the object
(B) change in velocity of the object
(C) change in kinetic energy of the object
(D) mass of the object
(E) acceleration of the object

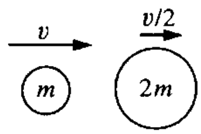


12. An object of mass m is moving with speed v_0 to the right on a horizontal frictionless surface, as shown above, when it explodes into two pieces. Subsequently, one piece of mass 2/5 m moves with a speed $v_0/2$ to the left. The speed of the other piece of the object is
(A) $v_o/2$ (B) $v_o/3$ (C) $7v_o/5$ (D) $3v_o/2$ (E) $2v_o$

13. A 5-kilogram sphere is connected to a 10-kilogram sphere by a rigid rod of negligible mass, as shown above. Which of the five lettered points represents the center of mass of the sphere-rod combination?
(A) A (B) B (C) C (D) D (E) E



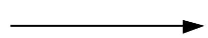
14. The graph above shows the force on an object of mass M as a function of time. For the time interval 0 to 4 s, the total change in the momentum of the object is
(A) 40 kg m/s (B) 20 kg m/s (C) 0 kg m/s (D) -20 kg m/s
(E) indeterminable unless the mass M of the object is known



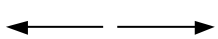
Top View

15. As shown in the top view above, a disc of mass m is moving horizontally to the right with speed v on a table with negligible friction when it collides with a second disc of mass 2m The second disc is moving horizontally to the right with speed v/2 at the moment of impact The two discs stick together upon impact The speed of the composite body immediately after the collision is
(A) v/3 (B) v/2 (C) 2v/3 (D) 3v/2 (E) 2v

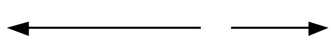
16. Two people are initially standing still on frictionless ice. They push on each other so that one person, of mass 120 kg, moves to the left at 2 m/s, while the other person, of mass 80 kg, moves to the right at 3 m/s. What is the velocity of their center of mass?
(A) Zero (B) 0.5 m/s to the left (C) 1 m/s to the right (D) 2.4 m/s to the left (E) 2.5 m/s to the right

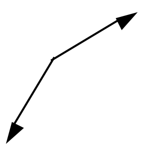
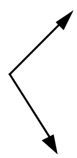


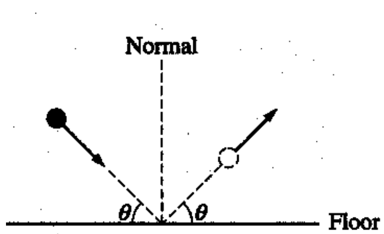
17. An object having an initial momentum that may be represented by the vector above strikes an object that is initially at rest. Which of the following sets of vectors may represent the momenta of the two objects after the collision?

(A) 

(B) 

(C) 

(D)  (E) 

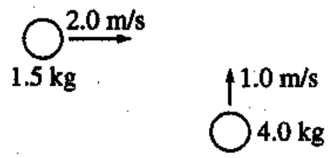


18. A 2 kg ball collides with the floor at an angle θ and rebounds at the same angle and speed as shown above. Which of the following vectors represents the impulse exerted on the ball by the floor?

(A) (B)  (C) (D)  (E) 

19. The momentum p of a moving object as a function of time t is given by the expression $p = kt^3$, where k is a constant. The force causing this motion is given by the expression
(A) $3kt^2$ (B) $3kt^2/2$ (C) $kt^2/3$ (D) kt^4 (E) $kt^4/4$

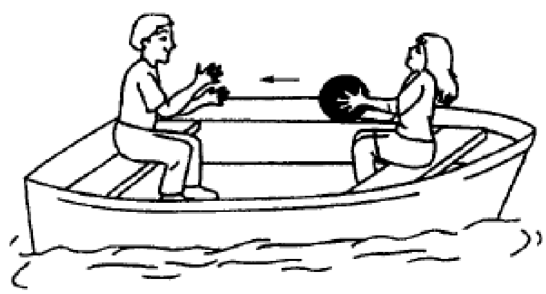
Questions 20-21



Two pucks moving on a frictionless air table are about to collide, as shown above. The 1.5 kg puck is moving directly east at 2.0 m/s. The 4.0 kg puck is moving directly north at 1.0 m/s.

20. What is the total kinetic energy of the two-puck system before the collision?
    (A) $\sqrt{13}$ J (B) 5.0 J (C) 7.0 J (D) 10 J (E) 11 J

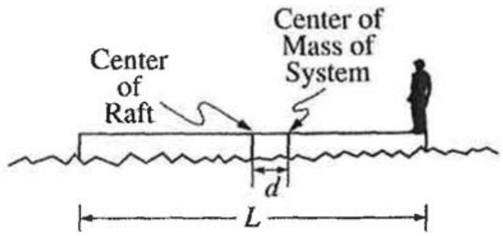
21. What is the magnitude of the total momentum of the two-puck system after the collision?
    (A) 1.0 kg•m/s (B) 3.5 kg•m/s (C) 5.0 kg•m/s (D) 7.0 kg•m/s (E) $5.5\sqrt{5}$ kg•m/s



22. As shown above, two students sit at opposite ends of a boat that is initially at rest. The student in the front throws a heavy ball to the student in the back. What is the motion of the boat at the time immediately after the ball is thrown and, later, after the ball is caught? (Assume that air and water friction are negligible.)

	Immediately After the Throw	After the Catch
(A)	Boat moves forward	Boat moves forward
(B)	Boat moves forward	Boat moves backward
(C)	Boat moves forward	Boat does not move
(D)	Boat moves backward	Boat does not move
(E)	Boat moves backward	Boat moves forward

23. A person holds a portable fire extinguisher that ejects 1.0 kg of water per second horizontally at a speed of 6.0 m/s. What horizontal force in newtons must the person exert on the extinguisher in order to prevent it from accelerating?
    (A) 0 N (B) 6 N (C) 10 N (D) 18 N (E) 36 N



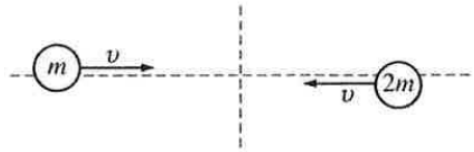
24. A person is standing at one end of a uniform raft of length L that is floating motionless on water, as shown above. The center of mass of the person-raft system is a distance d from the center of the raft. The person then walks to the other end of the raft. If friction between the raft and the water is negligible, how far does the raft move relative to the water?

    (A) $\frac{L}{2}$ (B) L (C) $\frac{d}{2}$ (D) d (E) $2d$

25. Objects 1 and 2 have the same momentum. Object 1 can have more kinetic energy than object 2 if, compared with object 2, it
    (A) has more mass
    (B) has the same mass
    (C) is moving at the same speed
    (D) is moving slower
    (E) is moving faster

26. A 5 kg object is propelled from rest at time $t = 0$ by a net force **F** that always acts in the same direction. The magnitude of **F** in newtons is given as a function of t in seconds by $F = 0.5t$. What is the speed of the object at $t = 4$ s?

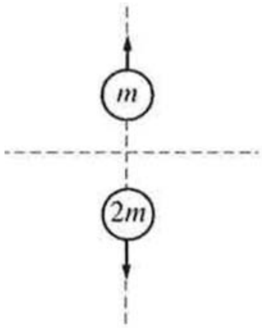
    (A) 0.5 m/s (B) 0.8 m/s (C) 2.0 m/s (D) 4.0 m/s (E) 8.0 m/s



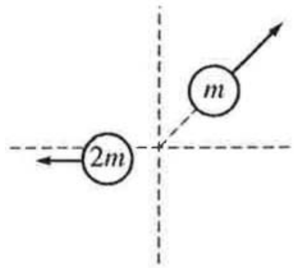
Top View

27. Two balls with masses *m* and *2m* approach each other with equal speeds υ on a horizontal frictionless table, as shown in the top view above. Which of the following shows possible velocities of the balls for a time soon after the balls collide?

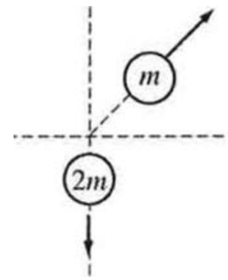
(A)



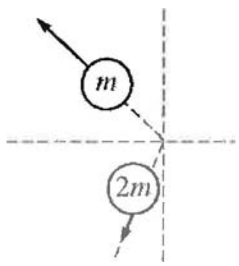
(B)



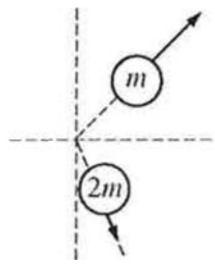
(C)

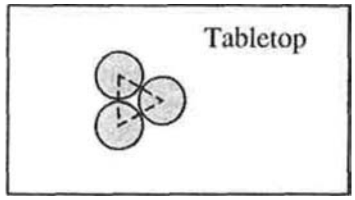


(D)

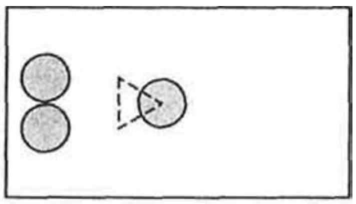


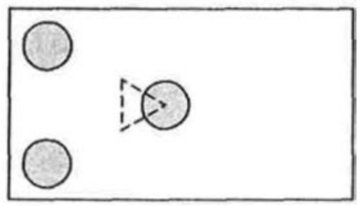
(E)

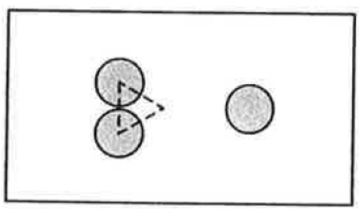


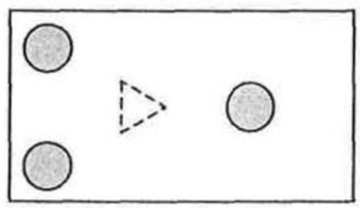


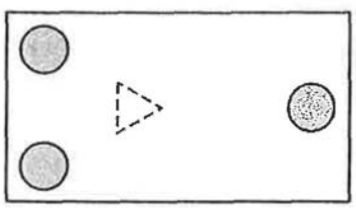
28. Three identical disks are initially at rest on a frictionless, horizontal table with their edges touching to form a triangle, as shown in the top view above. An explosion occurs within the triangle, propelling the disks horizontally along the surface. Which of the following diagrams shows a possible position of the disks at a later time? (In these diagrams, the triangle is shown in its original position.)

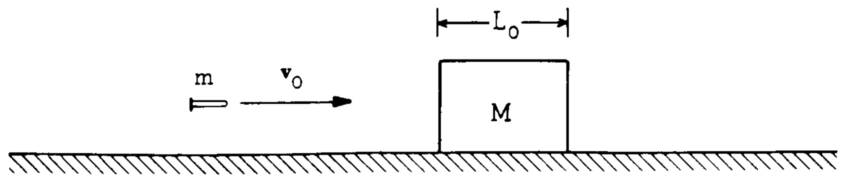
(A) 

(B) 

(C) 

(D) 

(E) 



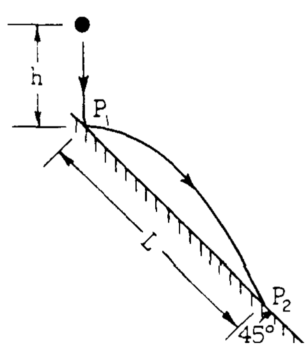
1976M3. A bullet of mass m and velocity v_o is fired toward a block of thickness L_o and mass M. The block is initially at rest on a frictionless surface. The bullet emerges from the block with velocity $v_o/3$.

a. Determine the final speed of block M.

b. If, instead, the block is held fixed and not allowed to slide, the bullet emerges from the block with a speed $v_o/2$. Determine the loss of kinetic energy of the bullet

c. Assume that the retarding force that the block material exerts on the bullet is constant. In terms of L_o, what minimum thickness L should a fixed block of similar material have in order to stop the bullet?

d. When the block is held fixed, the bullet emerges from the block with a greater speed than when the block is free to move. Explain.



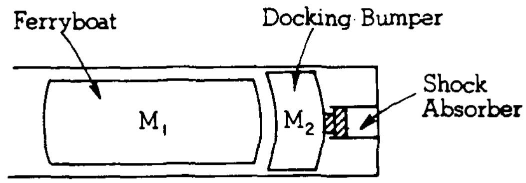
1979M1. A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces elastically off the plane at point P_1 and strikes the plane again at point P_2. In terms of g and h determine each of the following quantities:

a. The velocity (a vector) of the ball just after it first bounces off the plane at P_1.

b. The time the ball is in flight between points P_1 and P_2.

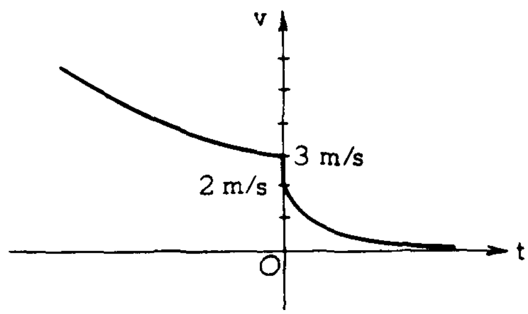
c. The distance L along the plane from P_1 to P_2.

d. The speed of the ball just before it strikes the plane at P_2.



1979M2. A ferryboat of mass $M_1 = 2.0 \times 10^5$ kilograms moves toward a docking bumper of mass M_2 that is attached to a shock absorber. Shown below is a speed v vs. time t graph of the ferryboat from the time it cuts off its engines to the time it first comes to rest after colliding with the bumper.

At the instant it hits the bumper, t = 0 and v = 3 meters per second.



a. After colliding inelastically with the bumper, the ferryboat and bumper move together with an initial speed of 2 meters per second. Calculate the mass of the bumper M_2.

b. After colliding, the ferryboat and bumper move with a speed given by the expression $v = 2e^{-4t}$. Although the boat never comes precisely to rest, it travels only a finite distance. Calculate that distance.

c. While the ferryboat was being slowed by water resistance before hitting the bumper, its speed was given by $1/v = 1/3 + \beta t$, where β is a constant. Find an expression for the retarding force of the water on the boat as a function of speed.

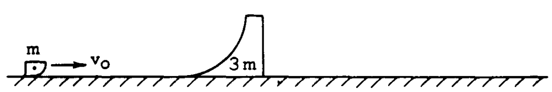


Figure I

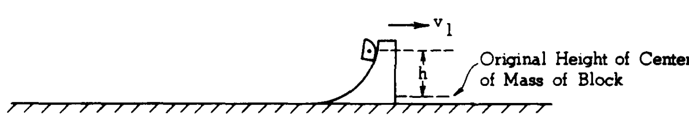


Figure II

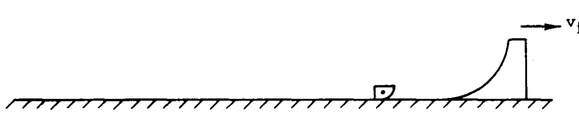


Figure III

1980M2. A block of mass m slides at velocity v_o across a horizontal frictionless surface toward a large curved movable ramp of mass 3m as shown in Figure 1. The ramp, initially at rest, also can move without friction and has a smooth circular frictionless face up which the block can easily slide. When the block slides up the ramp, it momentarily reaches a maximum height as shown in Figure II and then slides back down the frictionless face to the horizontal surface as shown in Figure III.

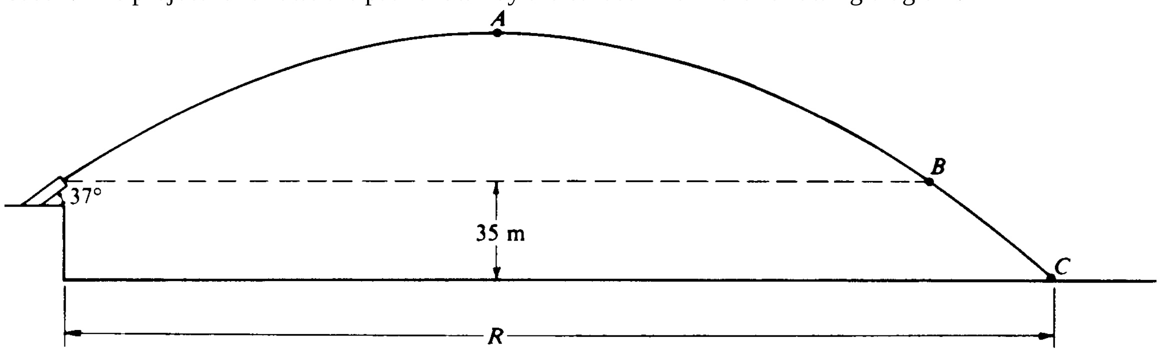
a. Find the velocity v_1 of the moving ramp at the instant the block reaches its maximum height.

b. To what maximum height h does the center of mass of the block rise above its original height?

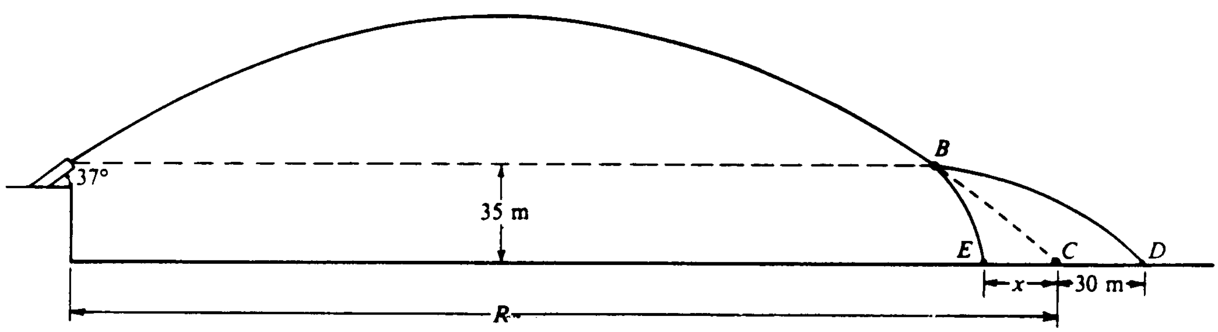
c. Determine the final speed v_f of the ramp and the final speed v' of the block after the block returns to the level surface. State whether the block is moving to the right or to the left.

1985M1. A projectile is launched from the top of a cliff above level ground. At launch the projectile is 35 meters above the base of the cliff and has a velocity of 50 meters per second at an angle 37° with the horizontal. Air resistance is negligible. Consider the following two cases and use $g = 10\ m/s^2$, $\sin 37° = 0.60$, and $\cos 37° = 0.80$.

Case I: The projectile follows the path shown by the curved line in the following diagram.

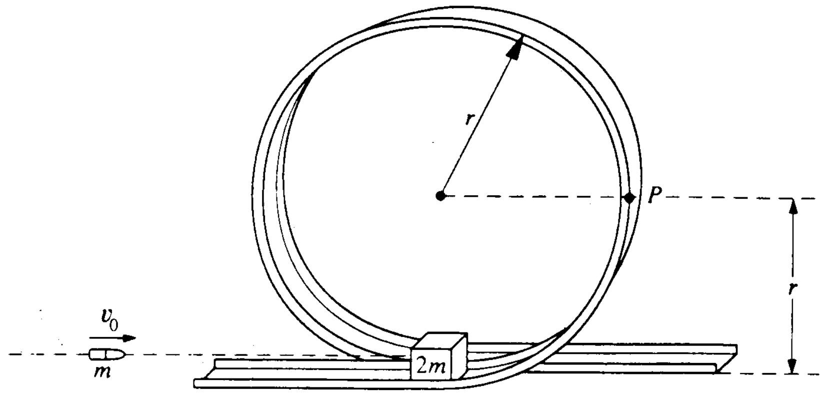


a. Calculate the total time from launch until the projectile hits the ground at point C.
b. Calculate the horizontal distance R that the projectile travels before it hits the ground.
c. Calculate the speed of the projectile at points A, B and C.



Case II: A small internal charge explodes at point B in the above diagram, causing the projectile to separate into two parts of masses 6 kilograms and 10 kilograms. The explosive force on each part is horizontal and in the plane of the trajectory. The 6-kilogram mass strikes the ground at point D, located 30 meters beyond point C, where the projectile would have landed had it not exploded The 10-kilogram mass strikes the ground at point E.

d. Calculate the distance x from C to *E.*



1991M1. A small block of mass 2m initially rests on a track at the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_o and remains embedded in the block as the block and bullet circle the loop. Determine each of the following in terms of m, v_0 r, and g.

a. The speed of the block and bullet immediately after impact

b. The kinetic energy of the block and bullet when they reach point P on the loop

c. The minimum initial speed v_{min} of the bullet if the block and bullet are to successfully execute a complete circuit of the loop

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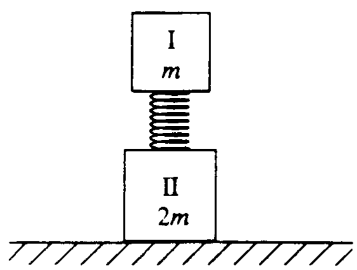
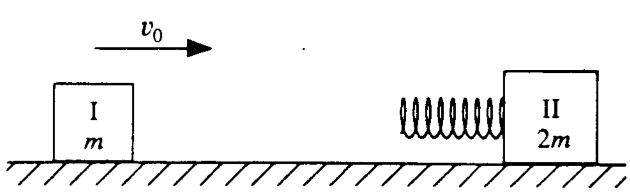


1991M3. The two blocks I and II shown above have masses m and 2m respectively. Block II has an ideal massless spring attached to one side. When block I is placed on the spring as shown. the spring is compressed a distance D at equilibrium. Express your answer to all parts of the question in terms of the given quantities and physical constants.

a. Determine the spring constant of the spring

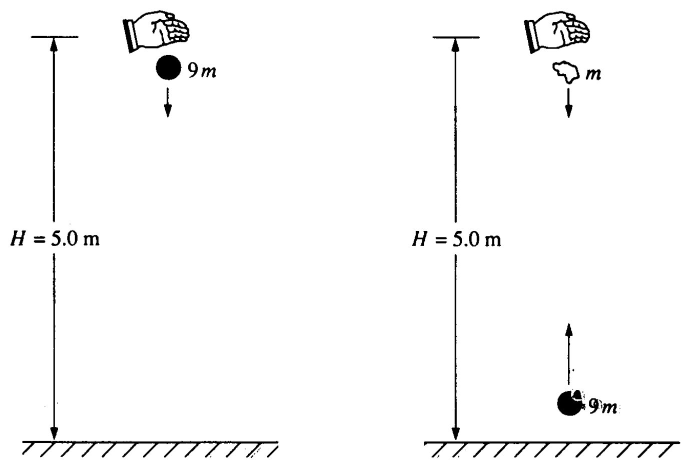


Later the two blocks are on a frictionless, horizontal surface. Block II is stationary and block I approaches with a speed v_o, as shown above.

b. The spring compression is a maximum when the blocks have the same velocity. Briefly explain why this is so.

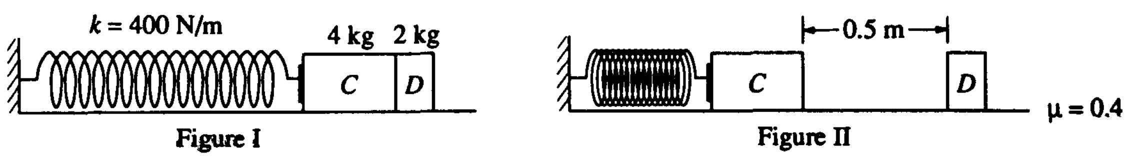
c. Determine the maximum compression of the spring during the collision.

d. Determine the velocity of block II after the collision when block I has again separated from the spring.



1992M1. A ball of mass 9m is dropped from rest from a height H = 5.0 meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, eventually collides with the ball, which is ascending. Assume that g = 10 m/s^2, that air resistance is negligible, and that the collision process takes negligible time.

a. Determine the speed of the ball immediately before it hits the ground.
b. Determine the time after the release of the clay blob at which the collision takes place.
c. Determine the height above the ground at which the collision takes place.
d. Determine the speeds of the ball and the clay blob immediately before the collision.
e. If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

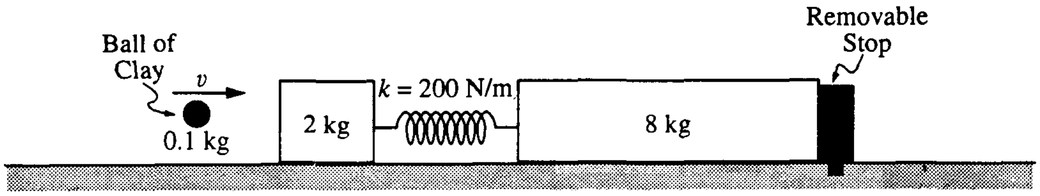


1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block *C* (mass m_c = 4.0 kilograms) and block *D* (mass m_D = 2.0 kilograms) rest on a horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block *C*. Block *C* is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block *C* is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right. Remember that the spring is not attached to block *C*. Determine each of the following.

b. The speed v_c of block *C* just before it collides with block *D*
c. The speed v_f blocks *C* and *D* just after they collide
d. The horizontal distance the blocks move before coming to rest



1994M1. A 2-kilogram block and an 8-kilogram block are both attached to an ideal spring (for which k = 200 N/m) and both are initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The 8-kilogram block is held still by a removable stop. As a result, the spring compresses a maximum distance of 0.4 meters.

a. Calculate the energy stored in the spring at maximum compression.

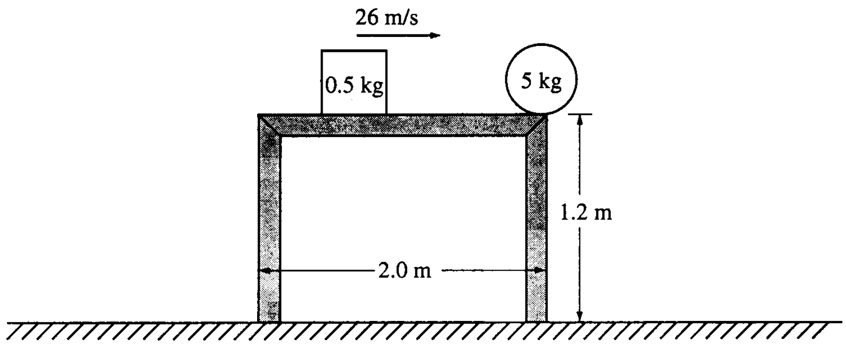
b. Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

c. Calculate the initial speed v of the clay.

In a second experiment, an identical ball of clay is thrown at another identical 2-kilogram block, but this time the stop is removed so that the 8-kilogram block is free to move.

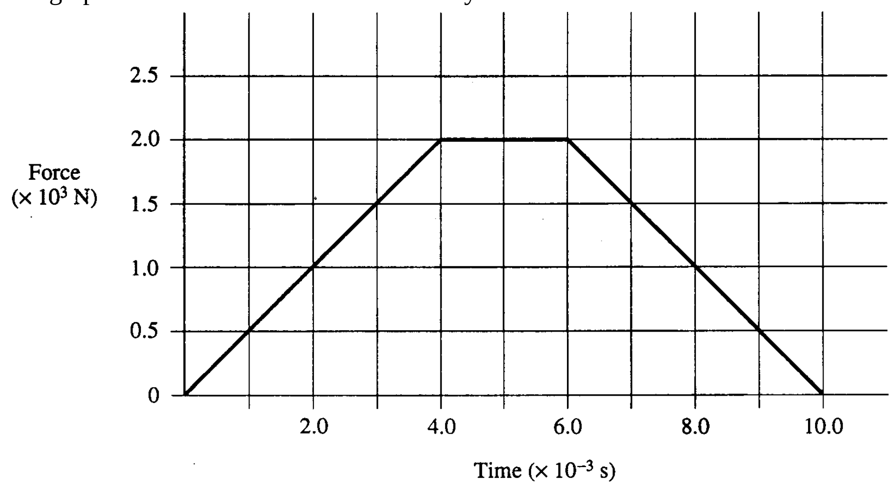
d. State whether the maximum compression of the spring will be greater than, equal to, or less than 0.4 meter. Explain briefly.

e. State the principle or principles that can be used to calculate the velocity of the 8-kilogram block at the instant that the spring regains its original length. Write the appropriate equation(s) and show the numerical substitutions, but do <u>not</u> solve for the velocity.



Note: Figure not drawn to scale.

1995M1. A 5-kilogram ball initially rests at the edge of a 2-meter-long, 1.2-meter-high frictionless table, as shown above. A hard plastic cube of mass 0.5 kilogram slides across the table at a speed of 26 meters per second and strikes the ball, causing the ball to leave the table in the direction in which the cube was moving. The figure below shows a graph of the force exerted on the ball by the cube as a function of time.



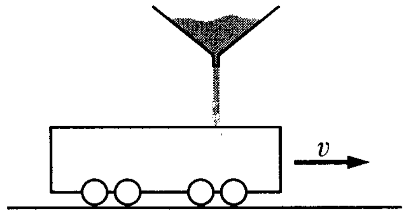
a. Determine the total impulse given to the ball.

b. Determine the horizontal velocity of the ball immediately after the collision.

c. Determine the following for the cube immediately after the collision.
   i. Its speed
   ii. Its direction of travel (right or left), if moving

d. Determine the kinetic energy dissipated in the collision.

e. Determine the distance between the two points of impact of the objects with the floor.



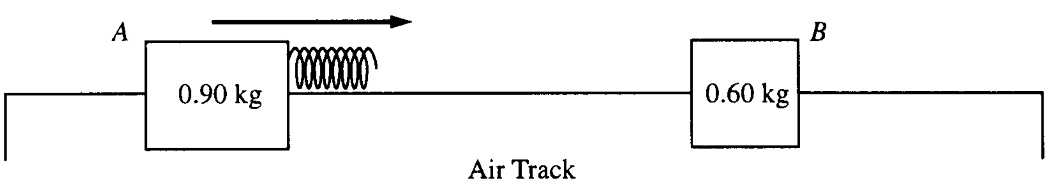
1997M2. An open-top railroad car (initially empty and of mass M_o) rolls with negligible friction along a straight horizontal track and passes under the spout of a sand conveyor. When the car is under the conveyor, sand is dispensed from the conveyor in a narrow stream at a steady rate $\Delta M/\Delta t = C$ and falls vertically from an average height h above the floor of the railroad car. The car has initial speed v_o and sand is filling it from time t = 0 to t = T. Express your answers to the following in terms of the given quantities and g.

a. Determine the mass M of the car plus the sand that it catches as a function of time t for $0 < t < T$.

b. Determine the speed v of the car as a function of time t for $0 < t < T$.

c. i. Determine the initial kinetic energy K_i of the empty car.
ii. Determine the final kinetic energy K_f of the car and its load.
iii. Is kinetic energy conserved? Explain why or why not.

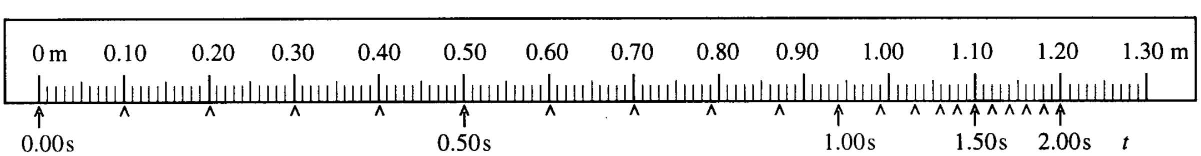
d. Determine expressions for the normal force exerted on the car by the tracks at the following times.
i. Before t = 0
ii. For $0 < t < T$
iii. After t = T



1998M1. Two gliders move freely on an air track with negligible friction, as shown above. Glider A has a mass of 0.90 kg and glider B has a mass of 0.60 kg. Initially, glider A moves toward glider B, which is at rest. A spring of negligible mass is attached to the right side of glider A. Strobe photography is used to record successive positions of glider A at 0.10 s intervals over a total time of 2.00 s, during which time it collides with glider B.

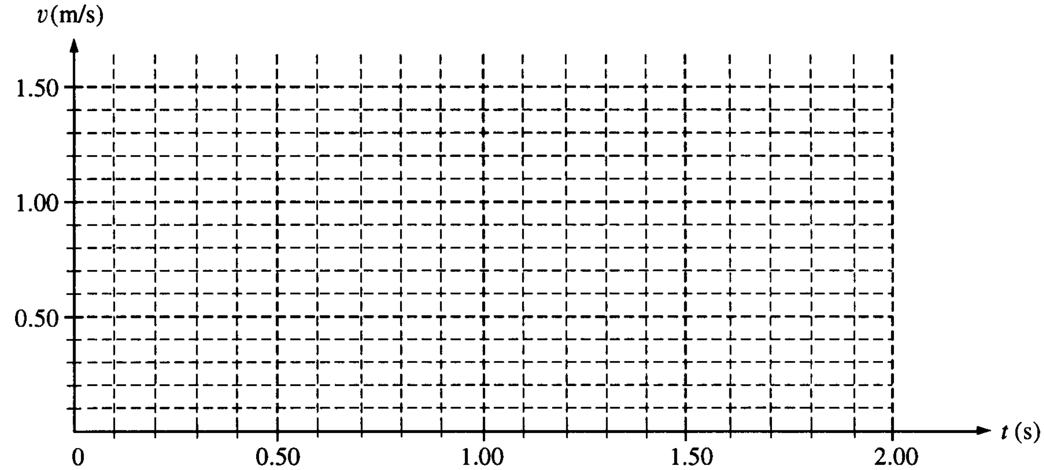
The following diagram represents the data for the motion of glider A. Positions of glider A at the end of each 0.10s interval are indicated by the symbol A against a metric ruler. The total elapsed time t after each 0.50 s is also indicated.

a. Determine the average speed of glider A for the following time intervals.

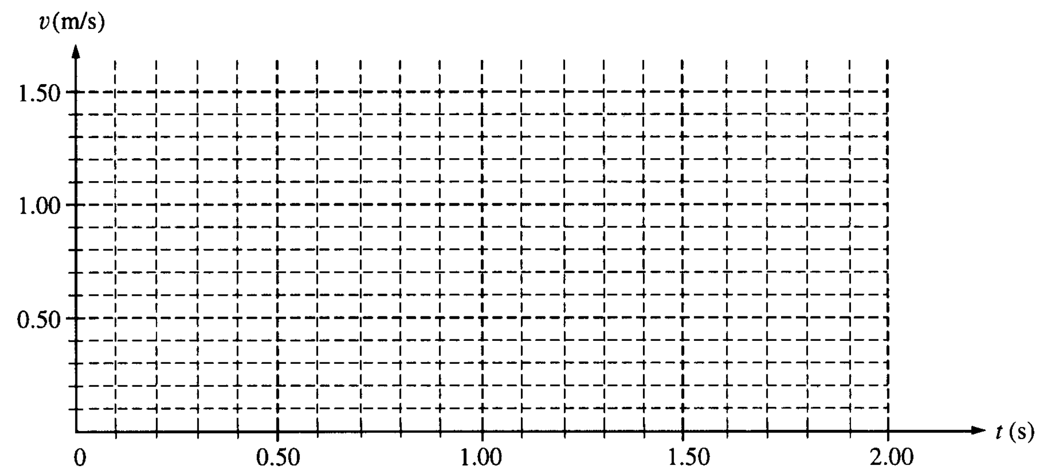


i. 0.10 s to 0.30 s ii. 0.90 s to 1.10 s iii. 1.70 s to 1.90 s

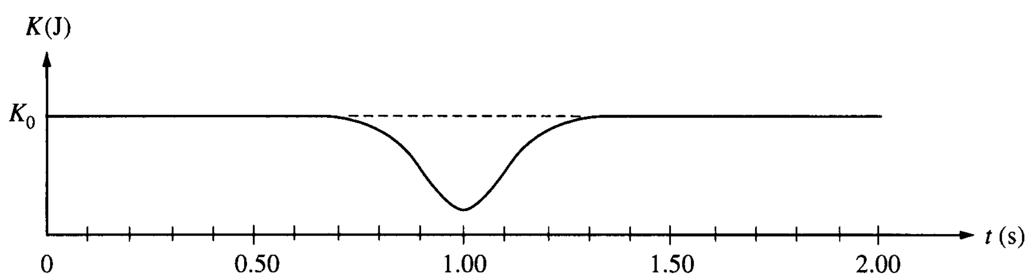
b. On the axes below, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.



c. i. Use the data to calculate the speed of glider B immediately after it separates from the spring.
ii. On the axes below, sketch a graph of the speed of glider B as a function of time t.

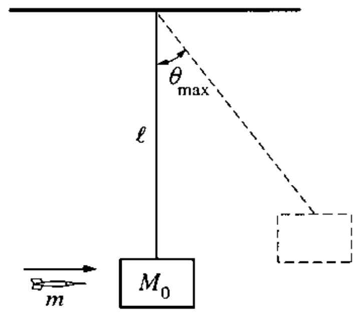


A graph of the total kinetic energy K for the two-glider system over the 2.00 s interval has the following shape. K_o is the total kinetic energy of the system at time t = 0.



d. i. Is the collision elastic? Justify your answer.

ii. Briefly explain why there is a minimum in the kinetic energy curve at t = 1.00 s.



1999M1 In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_0 which hangs from a cord of length *l* and negligible mass, as shown above. Assume the size of the block is negligible compared to *l*, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of m, M_0, *l*, θ_{max}, and g.

a. Determine the speed v_0 of the dart immediately before it strikes the block.

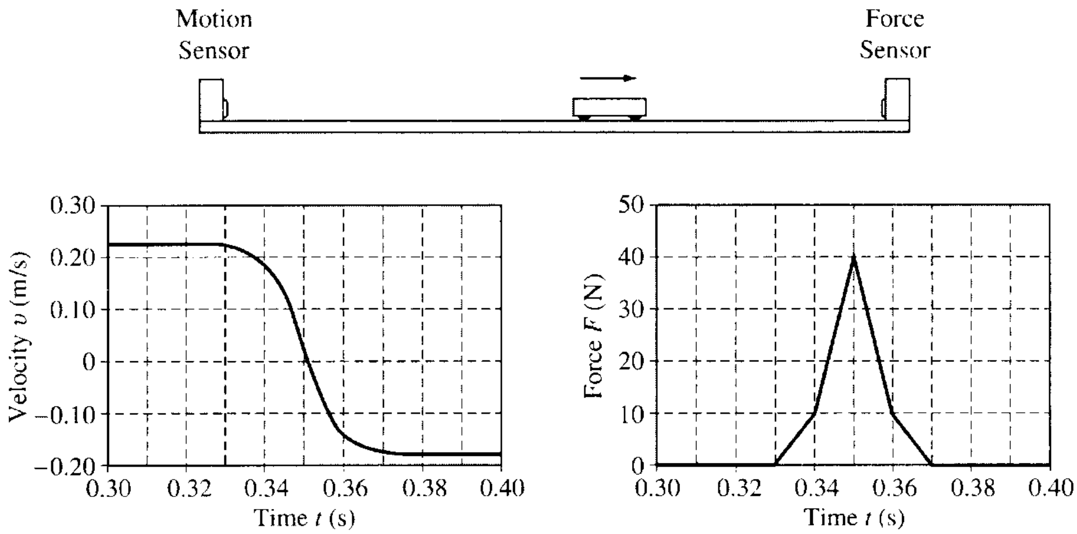
b. The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.

c. At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses
Protractor	5 m of string	Five more blocks of mass M_0
Spring		

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.

d. The dart is now shot into a block of wood that is fixed in place. The block exerts a force F on the dart that is proportional to the dart's velocity v and in the opposite direction, that is F = -bv, where b is a constant. Derive an expression for the distance L that the dart penetrates into the block, in terms of m, v_0, and b.



2001M1. A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the above graphs.

a. Determine the cart's average acceleration between t = 0.33 s and t = 0.37 s.
b. Determine the magnitude of the change in the cart's momentum during the collision.
c. Determine the mass of the cart.
d. Determine the energy lost in the collision between the force sensor and the cart

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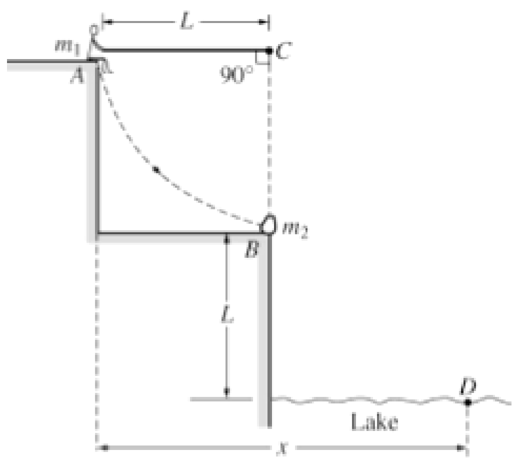
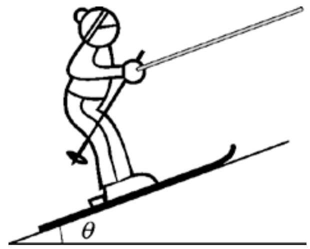


2004M1. A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown above. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

a. The speed of the person just before the collision with the object
b. The tension in the rope just before the collision with the object
c. The speed of the person and object just after the collision
d. The ratio of the kinetic energy of the person-object system before the collision to the kinetic energy after the collision
e. The total horizontal displacement *x* of the person from position *A* until the person and object land in the water at point *D.*



2010M3. A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{max} \sin(\pi t/T) \qquad (0 < t < T)$$
$$= 0 \qquad (t \leq T).$$

where $a_{\max}$ and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

a. Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

b. Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

c. Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

d. Derive an expression for the total impulse imparted to the skier during the acceleration.

	Solution	Answer
1.	The momentum of the heavier mass is m_2v, down; the momentum of the lighter is m_1v, up. As momentum is a vector, the net has a magnitude of $(m_2 - m_1)v$.	B
2.	2D collision. The y momentums are equal and opposite and will cancel out leaving only the x momentums which are also equal and will add together to give a total momentum equal to twice the x component momentum before hand. $p_{before} = p_{after}$ $2m_ov_o\cos60 = (2m_o)v_f$	B
3.	The wire is made of three identical pieces of length L, so each has a mass m. Let the left edge be located on the y axis. Then the center of mass of the assembly is $$(x_{cm}, y_{cm}) = \frac{m(0,\frac{1}{2}L) + m(\frac{1}{2}L,0) + m(L,\frac{1}{2}L)}{m+m+m} = \frac{1}{3}(\frac{3}{2}L, L) = (\frac{1}{2}L, \frac{1}{3}L)$$ Point B is above ¼ L, but below ½ L, so it's the best answer. (All the points are located at x = ½ L, which is obviously correct by symmetry.)	B
4.	The definition of the center of mass velocity is the total momentum divided by the total mass	E
5.	When equal masses collide elastically head-on, they exchange velocities	A
6.	In a perfectly inelastic (sticking) collision: $m_1v_1 + m_2v_2 = (m_1 + m_2)v'$	C
7.	First use the given kinetic energy of mass M1 to determine the projectile speed after. $K = ½M_1v_{1f}^2$ … $v_{1f} = \sqrt{(2K/M_1)}$. Now solve the explosion problem with $p_{before} = 0 = p_{after}$. Note that the mass of the gun is $M_2–M_1$ since M_2 was given as the total mass. $0 = M_1v_{1f} + (M_2–M_1)v_{2f}$ … now sub in from above for v_{1f} . $M_1(\sqrt{(2K/M_1)}) = –(M_2–M_1)v_{2f}$ and find v_{2f} … $v_{2f} = –M_1(\sqrt{(2K/M_1)}) / (M_2–M_1)$. Now sub this into $K_2 = ½(M_2–M_1)v_{2f}^2$ and simplify	D
8.	For a complete square loop, the center of mass is centered within the shape. With the upper left segment removed, the center of mass will shift slightly toward the lower right. That is, the new coordinates must be x > 1, y < 1	A
9.	Since there is no y momentum before, there cannot be any net y momentum after. The balls have equal masses so you need the y velocities of each ball to be equal after to cancel out the momenta. By inspection, looking at the given velocities and angles and without doing any math, the only one that could possibly make equal y velocities is choice D	D
10.	Since the only forces on the system are between parts of the system, the center of mass must remain stationary. As the smaller mass moves in one direction, the larger mass compensates by moving (a smaller amount) in the opposite direction	D
11.	Definition. $J_{net} = \Delta p$ $F_{net}t = \Delta p$	A
12.	Explosion with initial momentum. $p_{before} = p_{after}$ $mv_o = m_av_{af} + m_bv_{bf}$ $mv_o = (2/5\ m)(–v_o / 2) + (3/5\ m)(v_{bf})$ … solve for v_{bf}	E
13.	Let A be the origin. $x_{cm} = [(10\ kg)(0) + (5\ kg)(L)]/(10\ kg + 5\ kg) = L/3$	B
14.	The area of the Ft graph is the impulse which determines the momentum change. Since the net impulse is zero, there will be zero total momentum change.	C
15.	Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(m)(v) + (2m)(v / 2) = (3m)v_f$	C

16. As the system started at rest and the only forces on the system are between parts of the system, the center of mass must remain stationary — A

17. The total momentum vector before must match the total momentum vector after. Only choice E has a possibility of a resultant that matches the initial vector. — E

18. Since the angle and speed are the same, the x component velocity has been unchanged which means there could not have been any x direction momentum change. The y direction velocity was reversed so there must have been an upwards y impulse to change and reverse the velocity. — E

19. F = dp/dt — A

20. Simply add the energies ½ $(1.5)(2)^2$ + ½ $(4)(1)^2$ — B

21. Total momentum before must equal total momentum after. Before, there is an x momentum of (2)(1.5)=3 and a y momentum of (4)(1)=4 giving a total resultant momentum before using the Pythagorean theorem of 5. The total after must also be 5. — C

22. As the system started at rest and the only forces on the system are between parts of the system, the center of mass must remain stationary. While the ball is in motion toward the back of the boat, the boat will move slightly forward. Once the ball is stationary, so is the boat. — C

23. F = dp/dt = (dm/dt)Δv = (1 kg/s)(6 m/s) — B

24. As the system started at rest and the only forces on the system are between parts of the system, the center of mass of the system must remain stationary. When the person walks to the other side of the boat, it merely flips the system to its mirror image, there the center of the raft is now a distance d to the right of the center of mass of the system. In effect, the person and the center of the raft switch sides. — E

25. $K = p^2/2m$, for $K_1 > K_2$ then $p_1^2/2m_1 > p_2^2/2m_2$ but since $p_1 = p_2$, $m_2 > m_1$ and if $m_1 < m_2$ with the same momentum then $v_1 > v_2$. Alternately, $K = ½ mv^2 = ½ pv$ so if $K_1 > K_2$ then $pv_1 > pv_2$ — E

26. $\int F dt = m\Delta v$ — B

27. Inspecting the initial approach diagram, it is clear the system has a net monetum to the left (with no vertical momentum). After the collision, the system must still have a momentum to the left with no vertical component. Only one diagram illustrated this possibility. — D

28. As the system started at rest and the only forces on the system are between parts of the system, the center of mass of the system must remain stationary. E is the only arrangement that places the center of mass within the initial triangle. — E

1976M3

a. $p_i = p_f$
$mv_0 = mv_0/3 + MV$
$V = (2/3)mv_0/M$

b. $\Delta K = ½\ mv_f^2 – ½\ mv_i^2 = ½\ m(v_0/2)^2 – ½\ mv_0^2 = –(3/8)mv_0^2$

c. The work done on the bullet by a constant force $W = –FL = \Delta K$
This constant force can be calculated using the results above: $–FL_0 = –(3/8)mv_0^2$
$F = 3mv_0^2/8L_0$
To stop the bullet, $\Delta K = – ½\ mv_0^2$
$–FL = –(3mv_0^2/8L_0)L = – ½\ mv_0^2$
This gives $L = 4L_0/3$

d. When the block is free to move, the constant force acts over a greater distance, hence more kinetic energy will be lost. Or consider the block will carry off some kinetic energy when it is free to move.

1979M1

a. U = K; $mgh = ½\ mv^2$ gives $v = (2gh)^{1/2}$, since the collision is elastic this is the same speed after the collision. The direction is horizontally to the right due to the incline's 45º angle.

b. From P_1 to P_2 the ball maintains a horizontal speed of $(2gh)^{1/2}$ and travels a horizontal distance of $L/\sqrt{2}$
$d_x = v_xt$ gives $L/\sqrt{2} = (2gh)^{1/2}t$
vertically, $d_y = L/\sqrt{2} = ½\ gt^2$
Equating the two distances and solving for t gives $t = (8h/g)^{1/2}$

c. Substitution of t back into either distance expression gives $L = 4\sqrt{2}h$

d. U = K; $mgh + mgL/\sqrt{2} = ½\ mv_2^2$
$mgh + 4mgh = ½\ mv_2^2$
$v_2 = (10gh)^{1/2}$

1979M2

a. $M_1v_i = (M_1 + M_2)v_f$
substituting given values gives $M_2 = 10^5$ kg

b.
$$x = \int v dt$$
$$x = \int_0^\infty 2e^{-4t}dt = -\frac{1}{2}e^{-4t}\Big|_0^\infty = 0 - \left(-\frac{1}{2}\right) = 0.5\ m$$

c. $F = m(dv/dt) = m(d/dt)(1/3 + \beta t)^{-1}$
$= –m\beta(1/3 + \beta t)^{-2}$
$= –m\beta v^2$

1980M2

a. $p_i = p_f$
$mv_0 = mv_1 + 3mv_1$ (when the block reaches maximum height, they are traveling the same speed)
$v_1 = v_0/4$

b. Conservation of energy: $½\ mv_0^2 = ½\ (m + 3m)v_1^2 + mgh$
$h = 3v_0^2/8g$

c. This is an elastic collision: $mv_0 = mv' + 3mv_f$ and $v_0 = v_f – v'$
Solving the simultaneous equations gives $v_f = v_0/2$ and $v' = –v_0/2$ (the block moves to the left)

1985M1

a. $y = y_0 + v_{0y}t + ½ at^2$ where v_{0y} = (50 m/s) sin 37º = 30 m/s
   0 = (35 m) + (30 m/s)t – ½ (10 m/s^2)t^2
   t = 7 s

b. $R = v_x t$ = (50 m/s) cos 37º (7 s) = 280 m

c. $v_A = v_x$ = 40 m/s
   $v_B = v_0$ = 50 m/s
   $½ mv_C^2 = ½ mv_0^2 + mgh$ gives v_C = 56.6 m/s

d. The center of mass of the pieces will land at the same location as the intact projectile. Putting the center of mass at the same location gives x(10 kg) = (30 m)(6 kg) and x = 18 m

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1991M1

a. $p_i = p_f$
   $mv_0 = 3mv$
   $v = v_0/3$

b. Conservation of energy: $K_i = K_f + U_f$
   $½ (3m)(v_0/3)^2 = 3mgr + K_f$
   $K_f = mv_0^2/6 – 3mgr$

c. The minimum speed needed to execute a complete loop at the top of the loop is found form dynamics:
   $ΣF = ma_c$
   $F_N + mg = mv^2/r$, the minimum speed is found when $F_N = 0$
   $mg = mv_{top}^2/r$ giving $v_{top} = (gr)^{1/2}$
   When the bullet is moving at that minimum speed, the speed after the collision is $v_{min}/3$ as above
   Now use conservation of energy to find the speed needed by the bullet and block after the collision
   $K_i = K_f + U_f$
   $½ (3m)(v_{min}/3)^2 = 3mg(2r) + ½ (3m)(v_{top})^2$ giving $v_{min} = 3(5gr)^{1/2}$

1991M3

a. $F = kD = mg$
   $k = mg/D$

b. When $v_{relative} = 0$ the separation is neither increasing nor decreasing. This is the transition from approaching vs separating, where the spring is about to start expanding. Additionally, when they are moving the same speed, the kinetic energy is at a minimum (totally inelastic collision) therefore U_s is at a maximum.

c. $p_i = p_f$; $mv_0 = 3mv′$ and $K_i = K_f + U_s$; $½ mv_0^2 = ½ kx^2 + ½ (3m)v′^2$
   Using $v′ = v_0/3$ and $k = mg/D$ yields $x = v_0(2D/3g)^{1/2}$

d. This is an elastic collision: $mv_0 = mv_I + 2mv_{II}$ and $v_0 = v_{II} – v_I$
   Solving the simultaneous equations gives $v_{II} = (2/3)v_0$

1992M1

a. U = K; $mgH = ½ mv^2$ gives v = 10 m/s

b. $h_b = v_0t – ½ gt^2$ and $h_c = H – ½ gt^2$
   They will collide when $h_b = h_c$ which gives $v_0t = H$, or $t = H/v_0$ = (5 m)/(10 m/s) = 0.5 s

c. Substituting t into the height expressions gives h_c = 3.8 m

d. $v_b = v_0 – gt$ = 5 m/s
   $v_c = gt$ = 5 m/s

e. $m_bv_b – m_cv_c = (m_b + m_c)v′$ giving v′ = 4 m/s (up)

1993M1

a. $U = ½ kx^2 = 50$ J

b. $U + W_f = K_f$

 50 J – $\mu m_c gd = ½ m_c v_c^2$ giving $v_c = 4.58$ m/s

c. $m_c v_c = (m_c + m_D)v_f$ gives $v_f = 3.05$ m/s

d. When the blocks come to rest $K_i + W_f = 0$, or $½ (m_c + m_D)v_f^2 = \mu(m_c + m_D)gd$

 Solving gives d – 1.16 m

1994M1

a. $U = ½ kx^2 = 16$ J

b. $½ m_{tot}v^2 = U$; $v = (2U/m)^{1/2} = 3.9$ m/s

c. $m_c v_i = m_{tot} v$

 $v_i = m_{tot}v/m_c = 81.9$ m/s

d. Less. The kinetic energy of the center of mass of the system in this case is non-zero, so some of the initial energy remains as kinetic energy, so the spring compression is less.

e. This is an elastic collision: $Mv = Mv_1 + M_8v_2$ and $v = v_2 – v_1$

 3.9 m/s = $v_2 – v_1$ and (2.1 kg)(3.9 m/s) = (2.1 kg)v_1 + (8 kg)v_2

1995M1

a. Impulse is the area under the curve = 12 N-s

b. $J = \Delta p$; 12 N-s = $m_b\Delta v_b$ = (5 kg)(v_{fb} – 0)

 v_{fb} = 2.4 m/s

c. i. $J = \Delta p$; –12 N-s = $m_c\Delta v_c$ = (0.5 kg)(v_{fc} – 26 m/s)

 v_{fc} = 2 m/s

 ii. Since $v_{fc} > 0$ the cube is moving to the right

d. $\Delta K = K_f – K_i = ½ (m_c v_{fc}^2 + m_b v_{fb}^2 – m_c v_{ic}^2)$

 $|\Delta K|$ = 154 J

e. Both objects take the same time to reach the floor: $t = (2y/g)^{1/2} = 0.5$ s

 $x = vt$; $x_b = v_{fb}t$ = 1.2 m and $x_c = v_{fc}t$ = 1 m

 The separation is $\Delta x = x_b – x_c$ = 0.2 m

1997M2

a. The total mass is the sum of the car’s mass plus the mass of sand that has fallen up to that time

 $M = M_0 + (\Delta M/\Delta t)t = M_0 + Ct$

b. $p_i = p_f$

 $M_0v_0 = Mv' = (M_0 + Ct)v'$

 $v' = M_0v_0/(M_0 + Ct)$

c. i. $K_i = ½ M_0v_0^2$

 ii. $K_f = ½ Mv(T)^2 = ½ (M_0 + CT)(M_0v_0/(M_0 + CT))^2 = ½ M_0v_0^2(M_0/(M_0 + CT))$

 iii. KE is not conserved as it is dissipated in the inelastic collisions of the sand and car. Also, since momentum is constant and mass increases, $K = p^2/2m$ will decrease

d. i. $F_N = M_0g$

 ii. F_N = the weight of the cart plus the weight of the sand accumulated plus the impulsive force required to stop the vertical motion of the sand: $F_N = M_0g + M_sg + F_I$

 $M_s = Ct$ and $F_I = \Delta p/\Delta t = (\Delta M/\Delta t)v_y$ where $v_y = (2gh)^{1/2}$

 $F_N = (M_0 + Ct)g + C(2gh)^{1/2}$

 iii. $F_N = (M_0 + CT)g$

1998M1

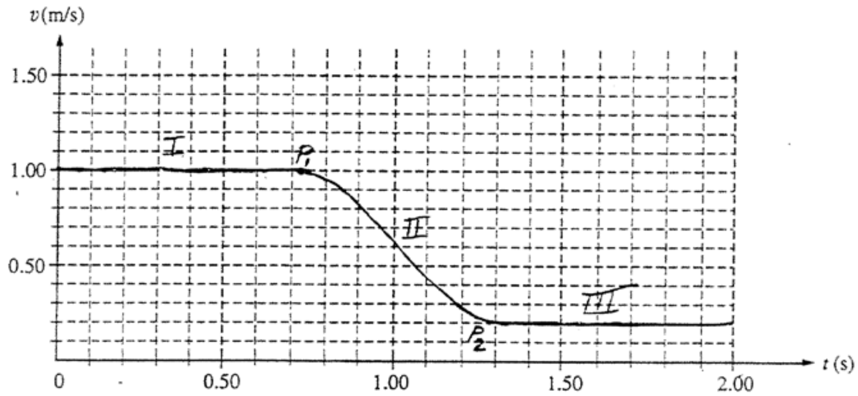
a. $v_{avg} = \Delta x/\Delta t$

i. v_{avg} = (0.3 m – 0.1 m)/(0.3 s – 0.1 s) = 1 m/s

ii. v_{avg} = (0.99 m – 0.87 m)/(1.1 s – 0.9 s) = 0.6 m/s

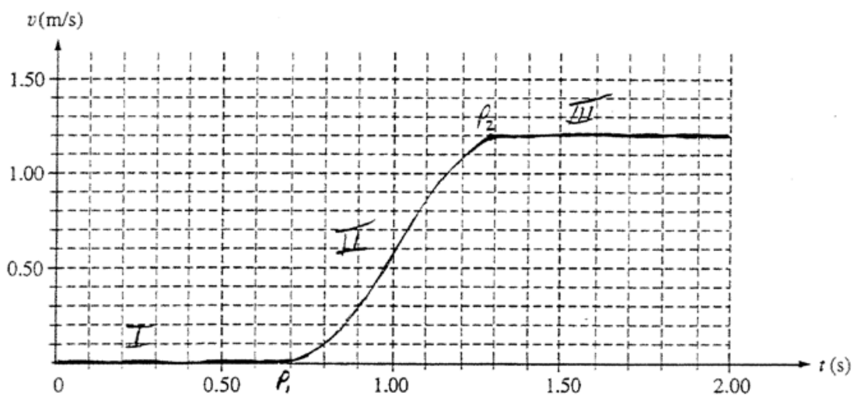
iii. v_{avg} = (1.18 m – 1.14 m)/(1.9 s – 1.7 s) = 0.2 m/s

b.



c. $m_A v_A = m_A v_{Af} + m_B v_{Bf}$ and $v_A = v_{Bf} - v_{Af}$ (elastic head on collision) gives v_B = 1.2 m/s

d.



e. i. Yes, the collision is elastic

The spring's force is conservative so the energy stored is the energy released also $K_f = K_i$

ii. At t = 1 s, the spring stores the lost KE

1999M1

a. conservation of momentum: $mv_0 = (m + M_0)v$ gives $v = mv_0/(m + M_0)$

conservation of energy: $\frac{1}{2} M_{total}v^2 = M_{total}gh$ where $h = \ell(1 - \cos\theta)$

Substituting for M_{total} and v gives $v_0 = (m + M_0)(2g\ell(1-\cos\theta))^{1/2}/m$

b. $\Sigma F = ma$

$T - M_{total}g = M_{total}v^2/\ell$

$T = (m + M_0)g + (m + M_0)(mv_0/(m + M_0))^2/\ell = (m + M_0)g(3 - 2\cos\theta)$

c. Points were awarded for the following:

A practical procedure that uses some or all of the apparatus listed and would work

Recognition of any assumptions that must be made

Indication of the proper mathematical computation using the variables measured

d. While traditional drag force methods will work here, here is an alternate solution:

$$\int F dt = \Delta p$$

$$\int_0^\infty -bv dt = -mv_0$$

v = dx/dt so we can replace v dt with dx also noting that as t reaches $\propto$, x = L

$$\int_0^L -b dx = -mv_0$$

$$-bL = -mv_0$$

$$L = mv_0/b$$

2001M1

a. $a_{avg} = \Delta v/\Delta t = (-0.18\text{ m/s} - 0.22\text{ m/s})/(0.37\text{ s} - 0.33\text{ s}) = -10\text{ m/s}^2$

b. Δp = area under the curve in the second graph = 0.6 N-s

c. $\Delta p = m\Delta v$
$m = \Delta p/\Delta v = (0.6\text{ N-s})/(0.4\text{ m/s}) = 1.5\text{ kg}$

d. $|\Delta E| = |E_f - E_i| = |\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2| = 0.012\text{ J}$

2004M1

a. $U = K$; $m_1gL = \frac{1}{2} m_1v_B^2$; $v_B = (2gL)^{1/2}$

b. $\Sigma F = ma$
$T - m_1g = m_1v_B^2/r$
$T = m_1g + m_1(2gL)/L = 3m_1g$

c. $m_1v_B = (m_1 + m_2)v_{after}$
$v_{after} = m_1v_B/(m_1 + m_2) = m_1(2gL)^{1/2}/(m_1 + m_2)$

d. $K_{before} = U_{before} = m_1gL$
$K_{after} = \frac{1}{2} (m_1 + m_2)v_{after}^2 = m_1^2gL/(m_1 + m_2)$
$K_b/K_a = (m_1 + m_2)/m_1$

e. To fall to the water: $y = \frac{1}{2} gt^2 = L$ so $t = (2L/g)^{1/2}$
From B to D, $x_{BD} = v_{after}t = (m_1(2gL)^{1/2}/(m_1 + m_2))\,(2L/g)^{1/2} = 2m_1L/(m_1 + m_2)$
From A to D: $x_{total} = x_{BD} + L = (3m_1 + m_2)L/(m_1 + m_2)$

2010M3

This is also from the work-energy chapter. Only part d is new here

a.

$$v = \int a\, dt = \int_0^t a_{max} sin \frac{\pi t}{T} dt = -\frac{a_{max}T}{\pi} cos \frac{\pi t}{T}\Big|_0^t = \frac{a_{max}T}{\pi}\left(1 - cos\frac{\pi t}{T}\right)$$

b. $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2)$
$v_f = v(T) = 2a_{max}T/\pi$
$v_i = v(0) = 0$
$W = 2ma_{max}^2T^2/\pi^2$

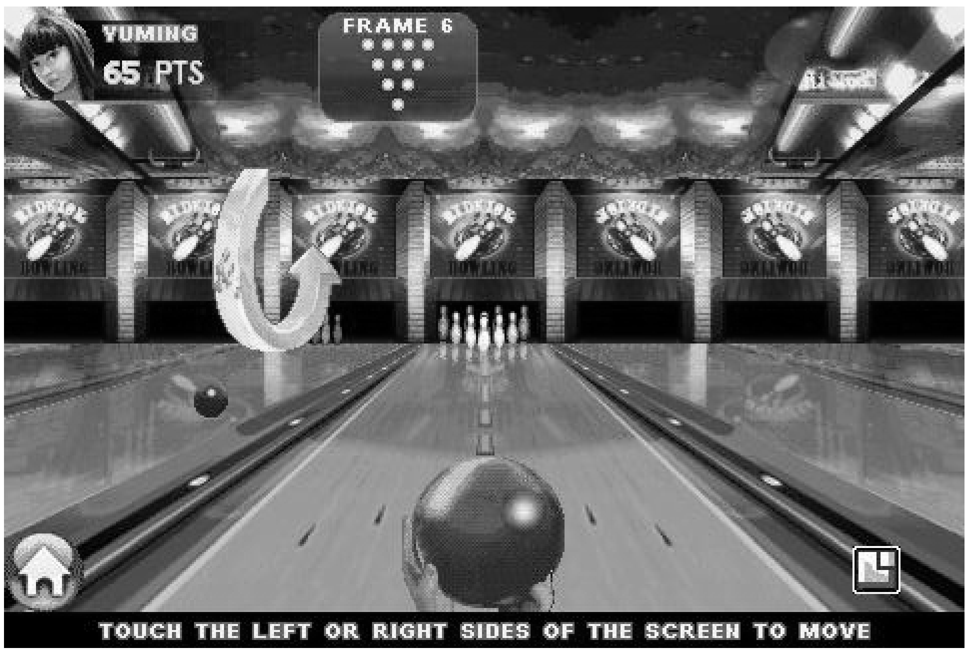
c. $\Sigma F = F_{rope} - mg \sin\theta = ma$ where $a = 0$ at terminal velocity
$F_{rope} = mg \sin\theta$

d.

$$J = \int F\, dt = \int_0^T ma_{max} sin \frac{\pi t}{T} dt = -\frac{ma_{max}T}{\pi} cos \frac{\pi t}{T}\Big|_0^T = \frac{2ma_{max}T}{\pi}$$

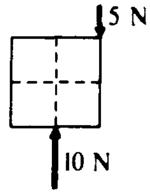
Chapter 5

Rotation

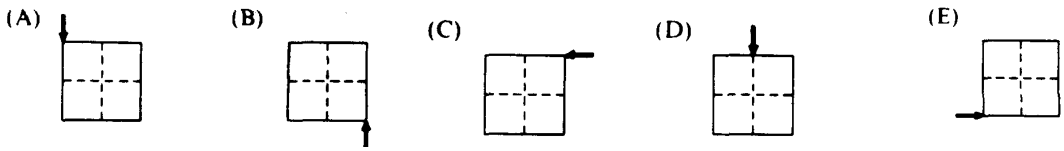


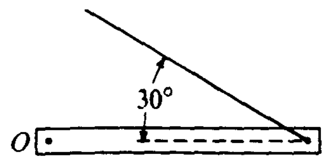
SECTION A – Torque and Statics

1. Torque is the rotational analogue of
(A) kinetic energy (B) linear momentum (C) acceleration (D) force (E) mass

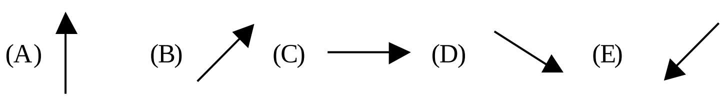


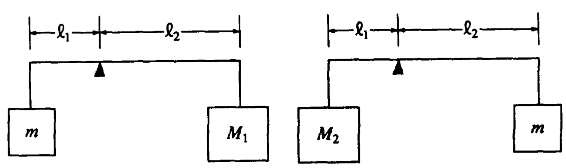
2. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown above. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?



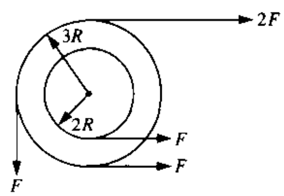


3. A uniform rigid bar of weight W is supported in a horizontal orientation as shown above by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?

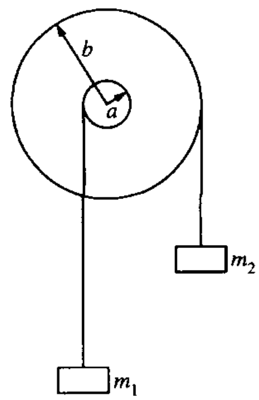




4. A rod of negligible mass is pivoted at a point that is off-center, so that length l_1 is different from length l_2. The figures above show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?
(A) $M_1 + M_2$ (B) $\frac{1}{2}(M_1 + M_2)$ (C) $M_1 M_2$ (D) $\frac{1}{2}M_1M_2$ (E) $\sqrt{M_1M_2}$



5. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown above. The magnitude of the net torque on the system about the axis is
(A) zero (B) FR (C) 2FR (D) 5FR (E) 14FR



6. For the wheel-and-axle system shown above, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$ (B) $am_1 = bm_2$ (C) $am_2 = bm_1$ (D) $a^2m_1 = b^2m^2$ (E) $b^2m_1 = a^2m_2$

SECTION B – Rotational Kinematics and Dynamics

Questions 1-2

A cylinder rotates with constant angular acceleration about a fixed axis. The cylinder's moment of inertia about the axis is 4 kg m^2. At time t = 0 the cylinder is at rest. At time t = 2 seconds its angular velocity is 1 radian per second.

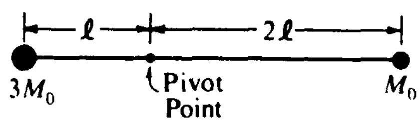
1. What is the angular acceleration of the cylinder between t = 0 and t = 2 seconds?
(A) 0.5 radian/s^2 (B) 1 radian/s^2 (C) 2 radian/s^2 (D) 4 radian/s^2 (E) 5 radian/s^2

2. What is the kinetic energy of the cylinder at time t = 2 seconds?
(A) 1 J (B) 2 J (C) 3 J (D) 4 J (E) cannot be determined without knowing the radius of the cylinder

3. A particle is moving in a circle of radius 2 meters according to the relation $\theta = 3t^2 + 2t$, where θ is measured in radians and t in seconds. The speed of the particle at t = 4 seconds is
(A) 13 m/s (B) 16 m/s (C) 26 m/s (D) 52 m/s (E) 338 m/s

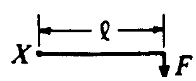
4. A uniform stick has length L. The moment of inertia about the center of the stick is I_o. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

(A) $I_0 + \frac{1}{4}ML^2$ (B) $I_0 + \frac{1}{2}ML^2$ (C) $I_0 + \frac{3}{4}ML^2$ (D) $I_0 + ML^2$ (E) $I_0 + \frac{5}{4}ML^2$

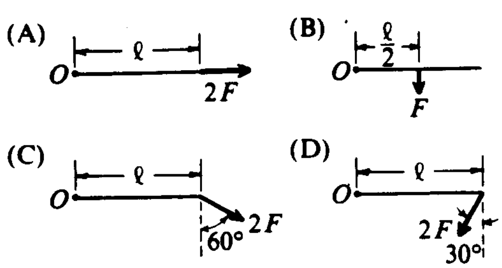


5. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude

(A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$ (E) $\frac{g}{l}$

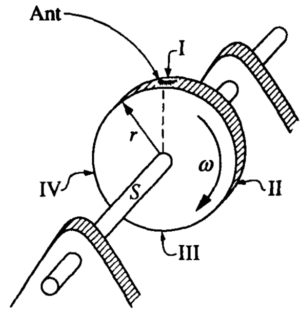


6. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram above? (All forces lie in the plane of the paper.)



(E) None of the above

Questions 7-8



An ant of mass m clings to the rim of a flywheel of radius r, as shown above. The flywheel rotates clockwise on a horizontal shaft S with constant angular velocity ω. As the wheel rotates, the ant revolves past the stationary points I, II, III, and IV. The ant can adhere to the wheel with a force much greater than its own weight.

7. It will be most difficult for the ant to adhere to the wheel as it revolves past which of the four points?
(A) I (B) II (C) III (D) IV
(E) It will be equally difficult for the ant to adhere to the wheel at all points.

8. What is the magnitude of the minimum adhesion force necessary for the ant to stay on the flywheel at point III?
(A) mg (B) $m\omega^2 r$ (C) $m\omega^2 r^2 + mg$ (D) $m\omega^2 r - mg$ (E) $m\omega^2 r + mg$

9. A turntable that is initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?
(A) $\sqrt{2\alpha}$ (B) $\sqrt{2\pi\alpha}$ (C) $\sqrt{4\pi\alpha}$ (D) 2α (E) $4\pi\alpha$

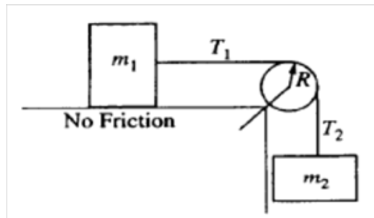
10. A 5-kilogram sphere is connected to a 10-kilogram sphere by a rigid rod of negligible mass, as shown above. The sphere-rod combination can be pivoted about an axis that is perpendicular to the plane of the page and that passes through one of the five lettered points. Through which point should the axis pass for the moment of inertia of the sphere-rod combination about this axis to be greatest?
(A) A (B) B (C) C (D) D (E) E

Questions 11-12

A wheel with rotational inertia *I* is mounted on a fixed, frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

11. What is the average net torque on the wheel during this time interval?
(A) $\frac{\omega_f}{T}$ (B) $\frac{\omega_f}{T^2}$ (C) $\frac{I\omega_f^2}{T}$ (D) $\frac{I\omega_f}{T^2}$ (E) $\frac{I\omega_f}{T}$

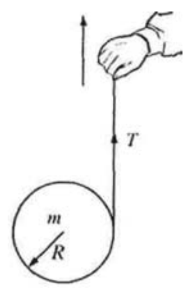
12. What is the average power input to the wheel during this time interval?
(A) $\frac{I\omega_f}{2T}$ (B) $\frac{I\omega_f^2}{2T}$ (C) $\frac{I\omega_f^2}{2T^2}$ (D) $\frac{I^2\omega_f}{2T^2}$ (E) $\frac{I^2\omega_f^2}{2T^2}$



13. Two blocks are joined by a light string that passes over the pulley shown above, which has radius R and moment of inertia I about its center. T_1 and T_2 are the tensions in the string on either side of the pulley and α is the angular acceleration of the pulley. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?
(A) $m_2gR = I\alpha$ (B) $(T_1 + T_2)R = I\alpha$ (C) $T_2R = I\alpha$ (D) $(T_2 - T_1)R = I\alpha$ (E) $(m_2 – m_1)gR = I\alpha$

14. a disk is free to rotate about an axis perpendicular to the disk through its center. If the disk starts from rest and accelerates uniformly at the rate of 3 radians/s^2 for 4 s, its angular displacement during this time is
(A) 6 radians (B) 12 radians (C) 18 radians (D) 24 radians (E) 48 radians

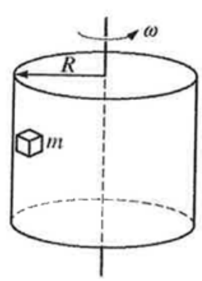
Questions 15-16



A solid cylinder of mass m and radius R has a string wound around it. A person holding the string pulls it vertically upward, as shown above, such that the cylinder is suspended in midair for a brief time interval Δt and its center of mass does not move. The tension in the string is T, and the rotational inertia of the cylinder about its axis is $\frac{1}{2}MR^2$

15. the net force on the cylinder during the time interval Δt is
(A) T (B) mg (C) $T – mgR$ (D) $mgR - T$ (E) zero

16. The linear acceleration of the person's hand during the time interval Δt is
(A) $\frac{T - mg}{m}$ (B) $2g$ (C) $\frac{g}{2}$ (D) $\frac{T}{m}$ (E) zero



17. A block of mass m is placed against the inner wall of a hollow cylinder of radius R that rotates about a vertical axis with a constant angular velocity ω, as shown above. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction μ between the mass and the wall must satisfy which of the following inequalities?

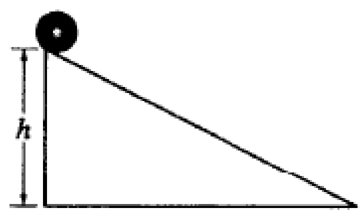
(A) $\mu \geq mg$ (B) $\mu \geq \frac{g}{\omega^2 R}$ (C) $\mu \geq \frac{\omega^2 R}{g}$ (D) $\mu \leq \frac{g}{\omega^2 R}$ (E) $\mu \leq \frac{\omega^2 R}{g}$

SECTION C – Rolling

1. A bowling ball of mass M and radius R. whose moment of inertia about its center is (2/5)MR^2, rolls without slipping along a level surface at speed *v*. The maximum vertical height to which it can roll if it ascends an incline is
(A) $\frac{v^2}{5g}$ (B) $\frac{2v^2}{5g}$ (C) $\frac{v^2}{2g}$ (D) $\frac{7v^2}{10g}$ (E) $\frac{v^2}{g}$

2. A wheel of mass M and radius R rolls on a level surface without slipping. If the angular velocity of the wheel is ω, what is its linear momentum ?
(A) $M\omega R$ (B) $M\omega^2 R$ (C) $M\omega R^2$ (D) $M\omega^2 R^2/2$ (E) Zero

Questions 3-4



A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown above.

3. If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?
(A) $\sqrt{2gh}$ (B) $\frac{2Mgh}{I}$ (C) $\frac{2Mghr^2}{I}$ (D) $\sqrt{\frac{2Mghr^2}{I}}$ (E) $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$

4. If the plane has friction so that the sphere rolls without slipping, what is the speed v_{cm} of the center of mass at the bottom of the incline?
(A) $\sqrt{2gh}$ (B) $\frac{2Mgh}{I}$ (C) $\frac{2Mghr^2}{I}$ (D) $\sqrt{\frac{2Mghr^2}{I}}$ (E) $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$

5. A wheel of 0.5 m radius rolls without slipping on a horizontal surface. The axle of the wheel advances at constant velocity, moving a distance of 20 m in 5 s. The angular speed of the wheel about its point of contact on the surface is
(A) 2 radians • s^{-1} (B) 4 radians • s^{-1} (C) 8 radians • s^{-1} (D) 16 radians • s^{-1} (E) 32 radians • s^{-1}

6. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
(A) vertically upward (B) horizontally forward (C) horizontally backward
(D) zero (E) upward and forward, at approximately 45° to the horizontal

SECTION D – Angular Momentum

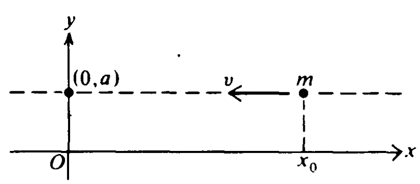
1. An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

	Angular Momentum	Kinetic Energy
(A)	Increases	Increases
(B)	Increases	Remains Constant
(C)	Remains Constant	Increases
(D)	Remains Constant	Remains Constant
(E)	Decreases	Remains Constant

2. A cylinder rotates with constant angular acceleration about a fixed axis. The cylinder's moment of inertia about the axis is 4 kg m^2. At time t = 0 the cylinder is at rest. At time t = 2 seconds its angular velocity is 1 radian per second. What is the angular momentum of the cylinder at time t = 2 seconds?
(A) 1 kg m^2/s (B) 2 kg m^2/s (C) 3 kg m^2/s (D) 4 kg m^2/s
(E) It cannot be determined without knowing the radius of the cylinder.

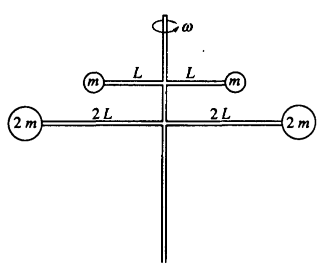
3. A figure skater is spinning on frictionless ice with her arms fully extended horizontally. She then drops her arms to her sides. Which of the following correctly describes her rotational kinetic energy and angular momentum as her arms fall?

	Rotational Kinetic Energy	Angular Momentum
(A)	Remains constant	Remains constant
(B)	Decreases	Increases
(C)	Decreases	Decreases
(D)	Increases	Decreases
(E)	Increases	Remains constant

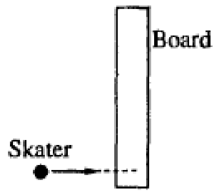


4. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is

(A) zero (B) mva (C) mvx_o (D) $mv\sqrt{x_0^2 + a^2}$ (E) $\frac{mva}{\sqrt{x_0^2 + a^2}}$

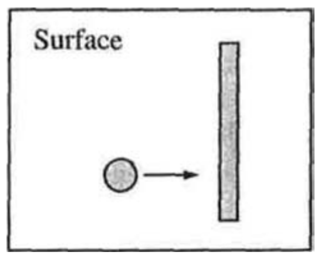


5. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?
(A) 2/1 (B) 1/1 (C) 1/2 (D) 1/4 (E) 1/8



Top View

6. A long board is free to slide on a sheet of frictionless ice. As shown in the top view above, a skater skates to the board and hops onto one end, causing the board to slide and rotate. In this situation, which of the following occurs?
   (A) Linear momentum is converted to angular momentum.
   (B) Kinetic energy is converted to angular momentum.
   (C) Rotational kinetic energy is conserved.
   (D) Translational kinetic energy is conserved.
   (E) Linear momentum and angular momentum are both conserved.



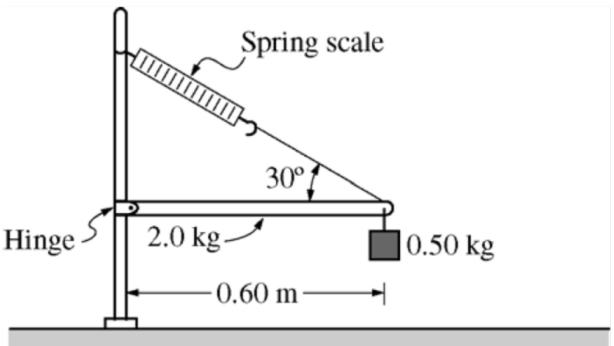
Top View

7. A disk sliding on a horizontal surface that has negligible friction collides with a rod that is free to move and rotate on the surface, as shown in the top view above. Which of the following quantities must be the same for the disk-rod system before and after the collision?
   I. Linear momentum
   II. Angular momentum
   III. Kinetic energy
   (A) I only
   (B) II only
   (C) I and II only
   (D) II and III only
   (E) I, II, and III

8. A figure skater goes into a spin with arms fully extended. Which of the following describes the changes in the rotational kinetic energy and angular momentum of the skater as the skater's arms are brought toward the body?

	Rotational Kinetic Energy	Angular Momentum
(A)	Remains the same	Increases
(B)	Remains the same	Remains the same
(C)	Increases	Remains the same
(D)	Decreases	Increases
(E)	Decreases	Remains the same

SECTION A – Torque and Statics



2008M2. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

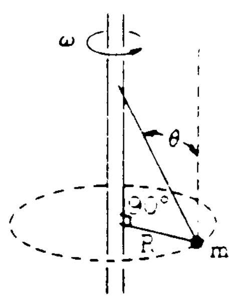
b. Calculate the reading on the spring scale.

The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length.

c. Calculate the rotational inertia of the rod-block system about the hinge.

d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

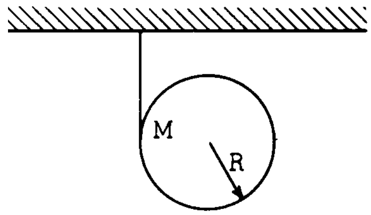
SECTION B – Rotational Kinematics and Dynamics



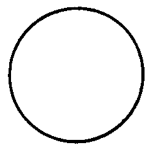
1973M3. A ball of mass m is attached by two strings to a vertical rod. as shown above. The entire system rotates at constant angular velocity ω about the axis of the rod.

a. Assuming ω is large enough to keep both strings taut, find the force each string exerts on the ball in terms of ω, m, g, R, and θ.

b. Find the minimum angular velocity, ω_{min} for which the lower string barely remains taut.



1976M2. A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $\frac{1}{2}MR^2$.



a. On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.
b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.
c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

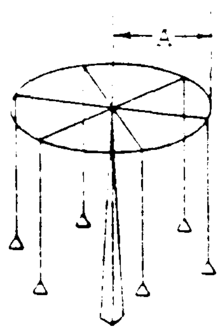


Figure I

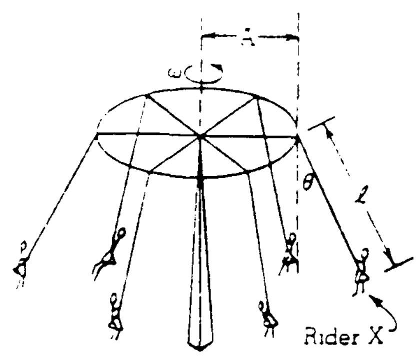
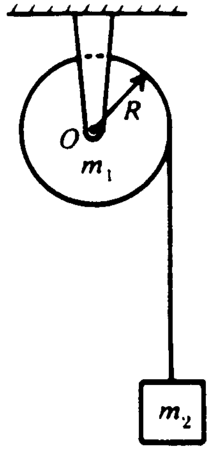


Figure II

1978M1. An amusement park ride consists of a ring of radius A from which hang ropes of length ℓ with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

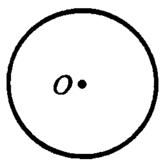
a. In the space below, draw and label all the forces acting on rider X (represented by the point below) under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

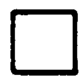
b. Derive an expression for ω in terms of A, ℓ, θ and the acceleration of gravity g.
c. Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, ℓ, θ, and the speed v of each rider.



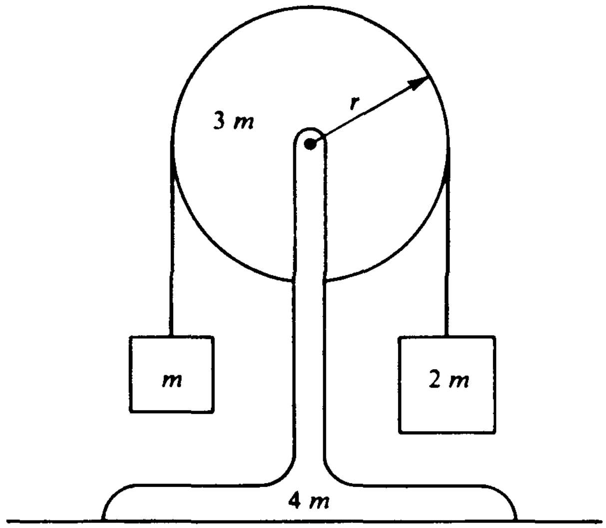
1983M2. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = ½m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.

a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.





b. In terms of m_1, m_2, R. and g, determine each of the following.
   i. The acceleration of the block
   ii. The tension in the cord



1985M3. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$.
Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

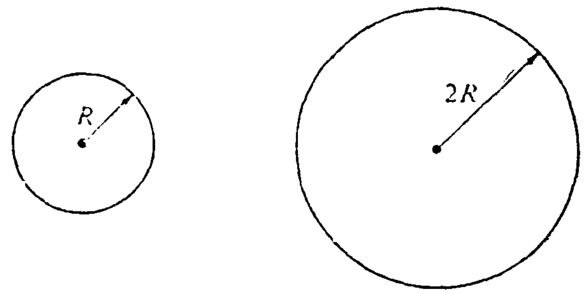
a. On the diagrams below, draw and label all the forces acting on each block.

b. Use the symbols identified in part a. to write each of the following.
   i. The equations of translational motion (Newton's second law) for each of the two blocks
   ii. The analogous equation for the rotational motion of the pulley

c. Solve the equations in part b. for the acceleration of the two blocks.

d. Determine the tension in the segment of the cord attached to the block of mass m.

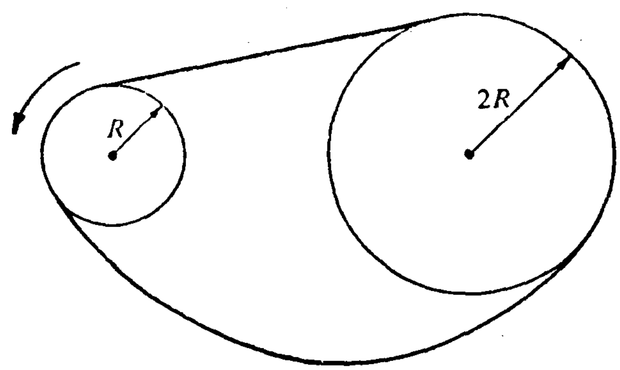
e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.



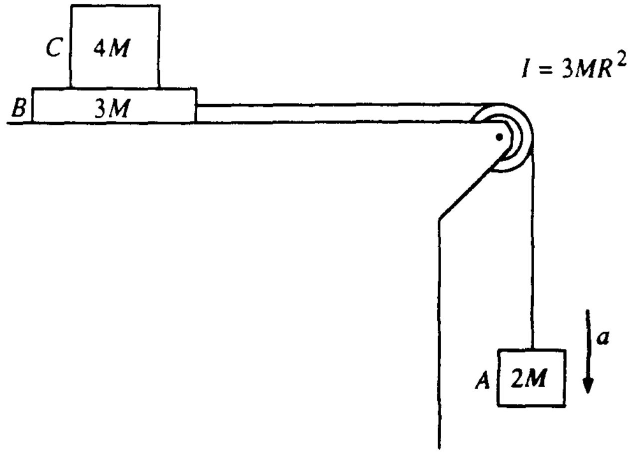
1988M3. The two uniform disks shown above have equal mass, and each can rotate on frictionless bearings about a fixed axis through its center. The smaller disk has a radius R and moment of inertia I about its axis. The larger disk has a radius 2R

a. Determine the moment of inertia of the larger disk about its axis in terms of I.

The two disks are then linked as shown below by a light chain that cannot slip. They are at rest when, at time t = 0, a student applies a torque to the smaller disk, and it rotates counterclockwise with constant angular acceleration α. Assume that the mass of the chain and the tension in the lower part of the chain, are negligible. In terms of I, R, α, and t, determine each of the following:



b. The angular acceleration of the larger disk
c. The tension in the upper part of the chain
d. The torque that the student applied to the smaller disk
e. The rotational kinetic energy of the smaller disk as a function of time



1989M2. Block A of mass 2M hangs from a cord that passes over a pulley and is connected to block B of mass 3M that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass 4M is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the

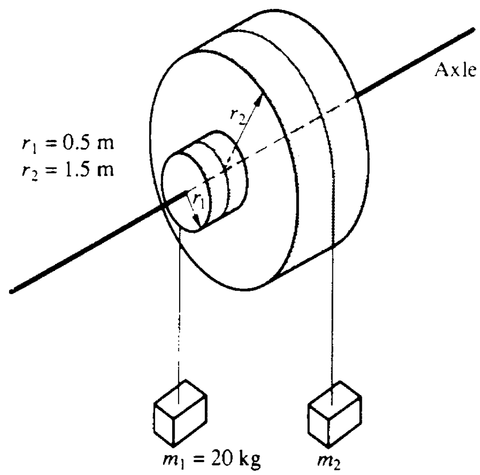
a. tension T_v in the vertical section of the cord

b. tension T_h in the horizontal section of the cord

If a = 2 meters per second squared, determine the

c. coefficient of kinetic friction between blocks B and C

d. acceleration of block C

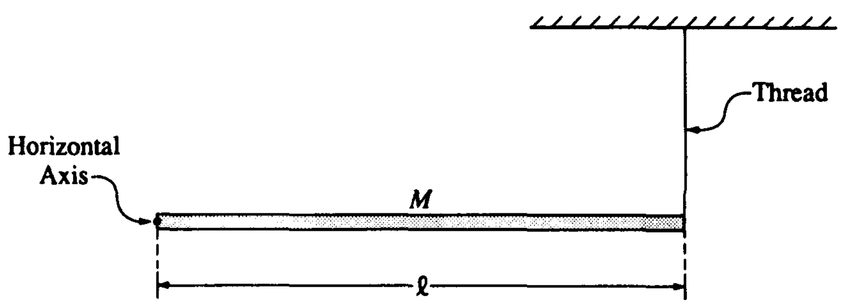


1991M2. Two masses. m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is I = 45 kg•m^2
Also r_1 = 0.5 meter, r_2 = 1.5 meters, and m_1 = 20 kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.
c. Determine the tension in the cable supporting m_1
d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.



1993M3. A long, uniform rod of mass *M* and length ℓ is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of *M*, ℓ, and *g*.

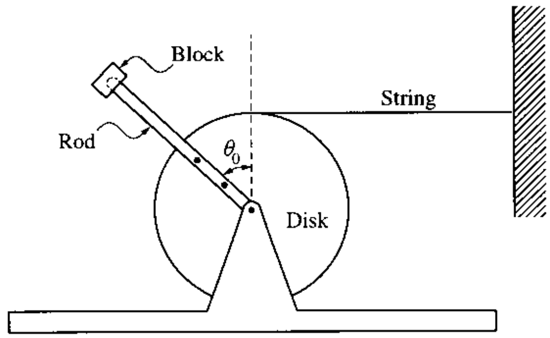
a. Determine the magnitude and direction of the force exerted on the rod by the axis.

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

b. The angular acceleration of the rod about the axis
c. The translational acceleration of the center of mass of the rod
d. The force exerted on the end of the rod by the axis

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod has swung.



1999M3 As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

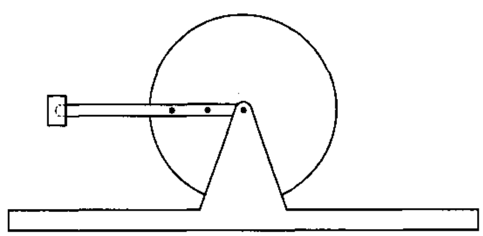
Disk: mass = 3m, radius = R, moment of inertia about center $I_D = 1.5mR^2$
Rod: mass = m, length = 2R, moment of inertia about one end $I_R = 4/3(mR^2)$
Block: mass = 2m

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

a. Determine the tension in the string.

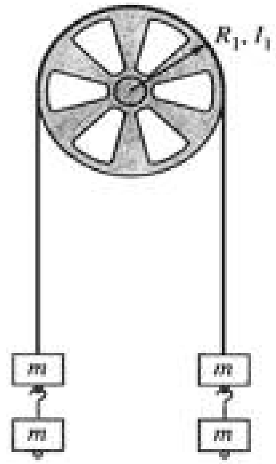
The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.
i. The magnitude of the angular acceleration of the disk
ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.



2000M3. A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley. Determine the answers to parts a. and b. in terms of m, R_1, I_1, and fundamental constants.

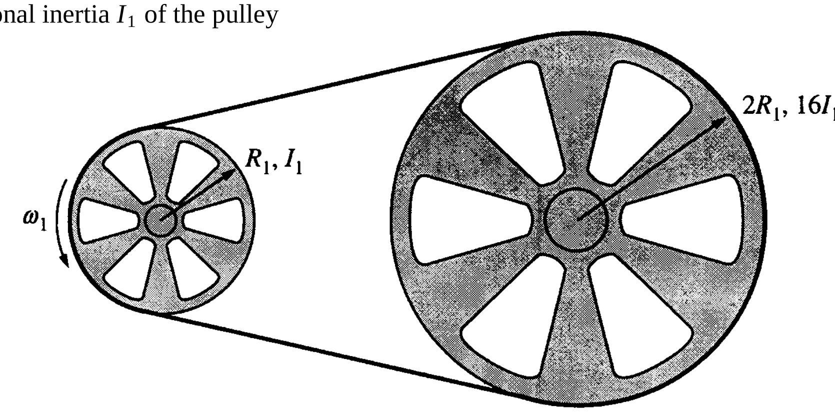
a. Determine the tension T in the cord.

b. One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration g/3 . Determine the following.

i. The tension T_3 in the section of cord supporting the three blocks on the left

ii. The tension T_1 in the section of cord supporting the single block on the right

iii. The rotational inertia I_1 of the pulley

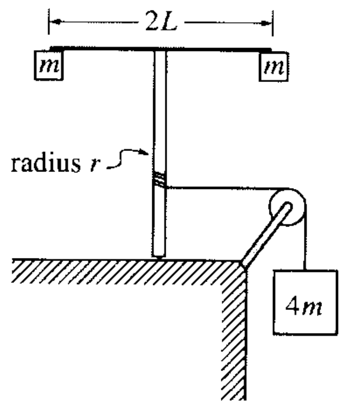


c. The blocks are now removed and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_I, and ω_1.

i. The angular speed ω_2 of the larger pulley

ii. The angular momentum L_2 of the larger pulley

iii. The total kinetic energy of the system



Experiment A

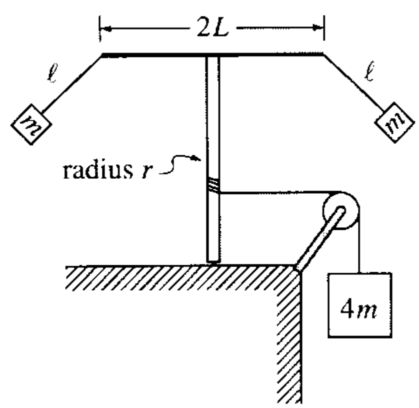
2001M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

b. Determine the downward acceleration of the large block.

c. When the large block has descended a distance *D*, how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD*? Check the appropriate space below and justify your answer.

Greater than 4mgD ______ Equal to 4mgD ___ Less than 4mgD _____



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? Check the appropriate space below and justify your answer.

Greater before ___ Equal to before ___ Less than before ___

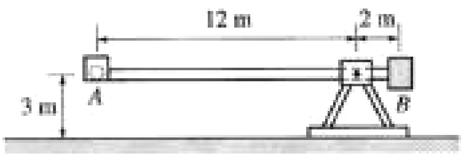


Figure 1

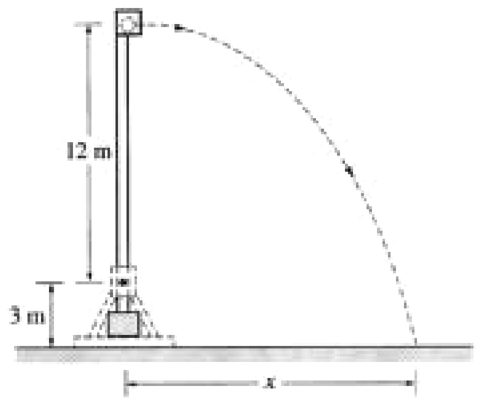


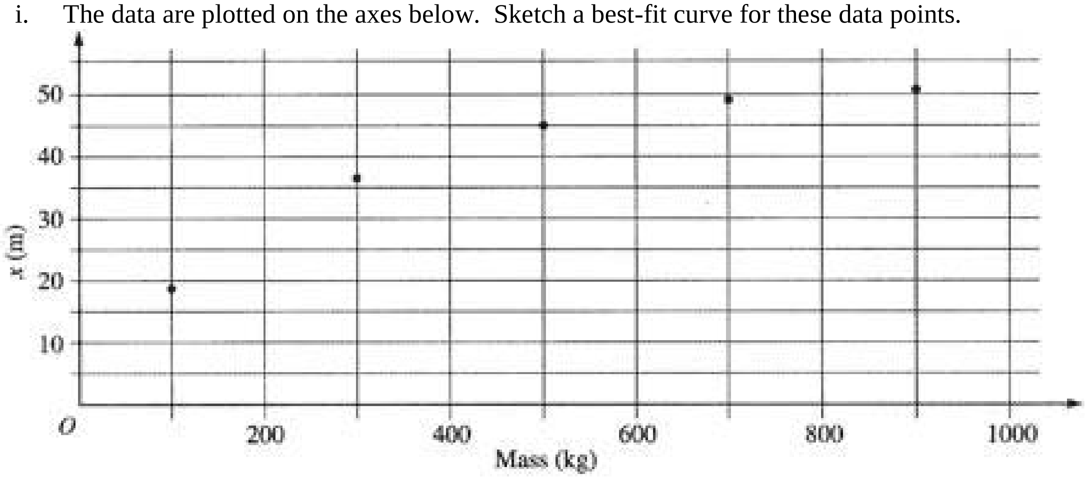
Figure 2

2003M3. Some physics students build a catapult, as shown above. The supporting platform is fixed firmly to the ground. The projectile, of mass 10 kg, is placed in cup *A* at one end of the rotating arm. A counterweight bucket *B* that is to be loaded with various masses greater than 10 kg is located at the other end of the arm. The arm is released from the horizontal position, shown in Figure 1, and begins rotating. There is a mechanism (not shown) that stops the arm in the vertical position, allowing the projectile to be launched with a horizontal velocity as shown in Figure 2.

a. The students load five different masses in the counterweight bucket, release the catapult, and measure the resulting distance x traveled by the 10 kg projectile, recording the following data.

Mass (kg)	100	300	500	700	900
x (m)	18	37	45	48	51

i. The data are plotted on the axes below. Sketch a best-fit curve for these data points.



ii. Using your best-fit curve, determine the distance *x* traveled by the projectile if 250 kg is placed in the counterweight bucket.

b. The students assume that the mass of the rotating arm, the cup, and the counterweight bucket can be neglected. With this assumption, they develop a theoretical model for *x* as a function of the counterweight mass using the relationship $x = v_x t$, where v, is the horizontal velocity of the projectile as it leaves the cup and *t* is the time after launch.

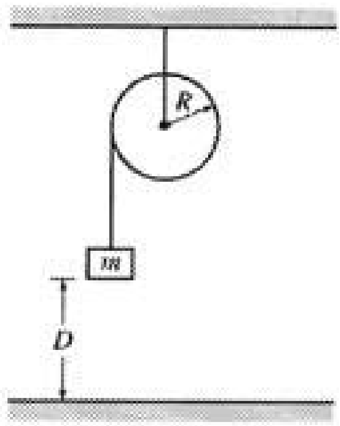
i. How many seconds after leaving the cup will the projectile strike the ground?

ii. Derive the equation that describes the gravitational potential energy of the system relative to the ground when in the position shown in Figure 1, assuming the mass in the counterweight bucket is *M*.

iii. Derive the equation for the velocity of the projectile as it leaves the cup, as shown in Figure 2.

c. i. Complete the theoretical model by writing the relationship for *x* as a function of the counterweight mass using the results from b. i and b. iii.

ii. Compare the experimental and theoretical values of *x* for a counterweight bucket mass of 300 kg. Offer a reason for any difference.



2004M2. A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

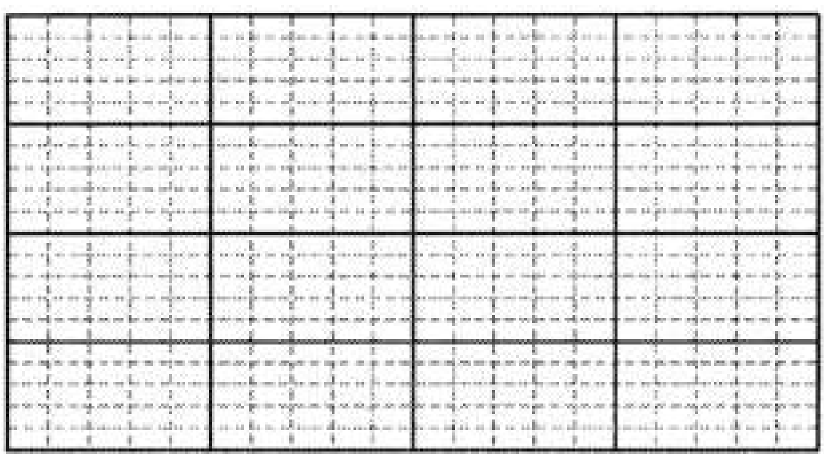
a. Calculate the linear acceleration a of the falling block in terms of the given quantities.

b. The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in b. i., label the axes, and draw the best-fit line to the data.

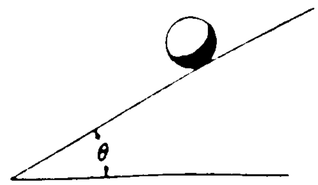


iii. Use your graph to calculate the magnitude of the acceleration.

c. Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

d. The value of acceleration found in b.iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

SECTION C – Rolling



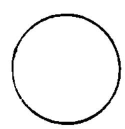
1974M2. The moment of inertia of a uniform solid sphere (mass M, radius R) about a diameter is $2MR^2/5$. The sphere is placed on an inclined plane (angle θ) as shown above and released from rest.

a. Determine the minimum coefficient of friction μ between the sphere and plane with which the sphere will roll down the incline without slipping

b. If μ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part a.? Explain your answer.

1977M2. A uniform cylinder of mass M, and radius R is initially at rest on a rough horizontal surface. The moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$. A string, which is wrapped around the cylinder, is pulled upwards with a force T whose magnitude is 0.6Mg and whose direction is maintained vertically upward at all times. In consequence, the cylinder both accelerates horizontally and slips. The coefficient of kinetic friction is 0.5.

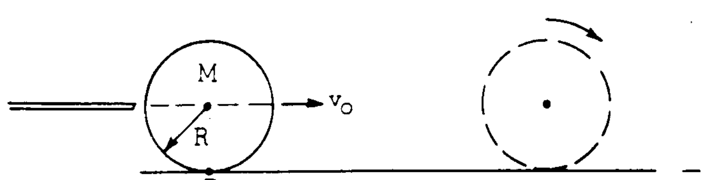
a. On the diagram below, draw vectors that represent each of the forces acting on the cylinder identify and clearly label each force.



b. Determine the linear acceleration a of the center of the cylinder.

c. Calculate the angular acceleration α of the cylinder.

d. Your results should show that a and αR are not equal. Explain.



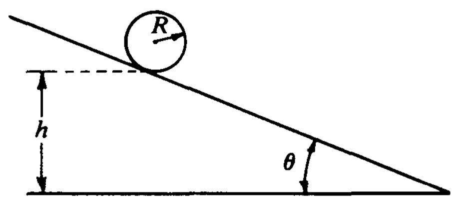
1980M3. A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c = 2MR^2/5$
The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_o as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.

a. Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

b. Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

c. Determine the time at which the ball begins to roll without slipping.

d. When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

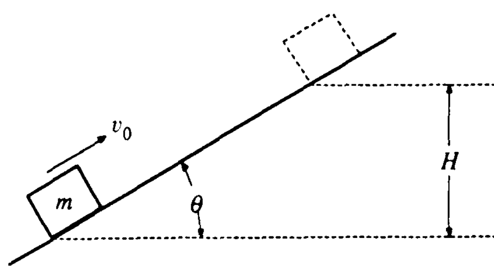


1986M2. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height *h* above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R. *h,* g, and θ.

a. Determine the following for the sphere when it is at the bottom of the plane:
   i. Its translational kinetic energy
   ii. Its rotational kinetic energy
b. Determine the following for the sphere when it is on the plane.
   i. Its linear acceleration
   ii. The magnitude of the frictional force acting on it

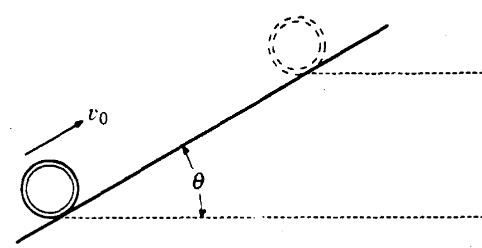
The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?
d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.



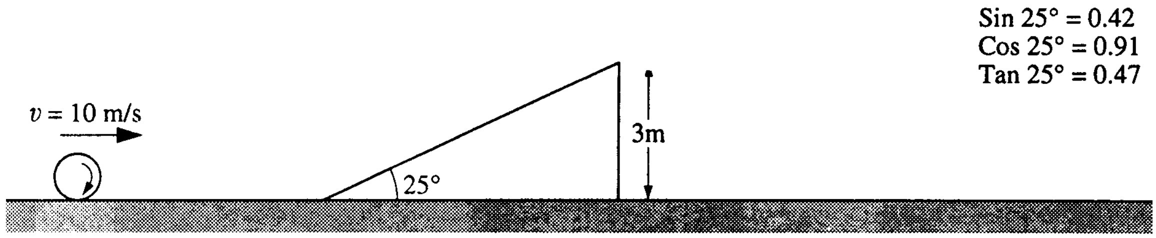
1990M2. A block of mass m slides up the incline shown above with an initial speed v_O in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_O in the position shown.

c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities.
d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.



Note: Diagram not drawn to scale.

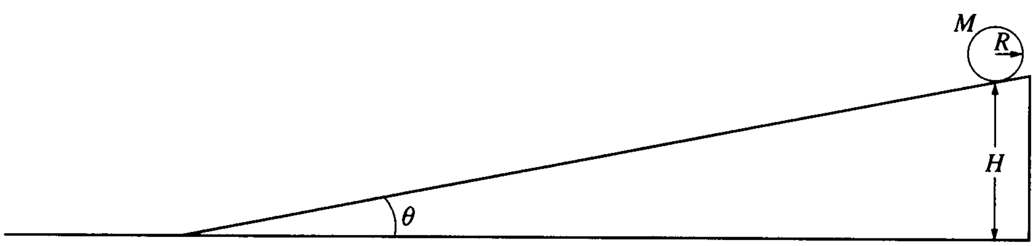
1994M2. A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass m of 25 kilograms, and a radius r of 0.2 meter. The moment of inertia of the sphere about its center of mass is $I = 2mr^2/5$. The sphere approaches a 25° incline of height 3 meters as shown above and rolls up the incline without slipping.

a. Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b. i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.
ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

c. Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

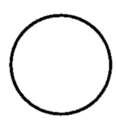
d. Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in b. Explain briefly.



1997M3. A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

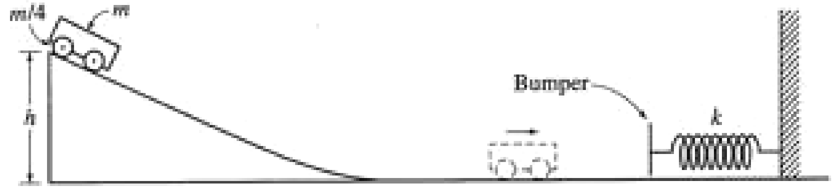
b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is $(2/3)g \sin\theta$.

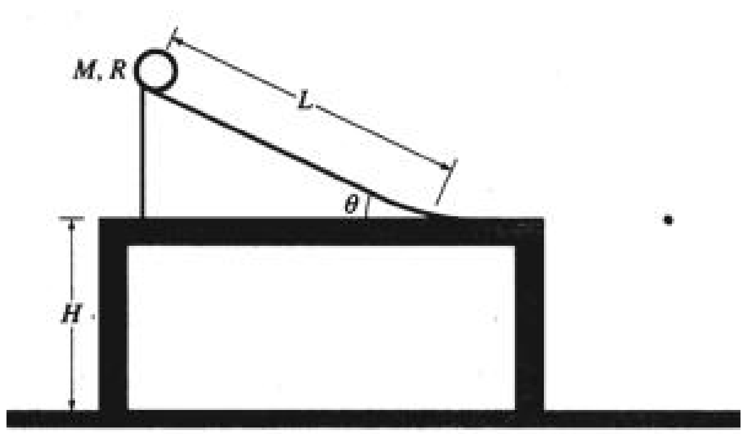
d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction μ is now made less than the value determined in part d., so that the cylinder both rotates and slips.
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part a. Justify your answer.
ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.



2002M2. The cart shown above is made of a block of mass m and four solid rubber tires each of mass m/4 and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia ½ ML^2, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the total rotational inertia of all four tires.
b. Determine the speed of the cart when it reaches the bottom of the incline.
c. After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.
d. Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part c.. Give a reasonable explanation for this decrease.

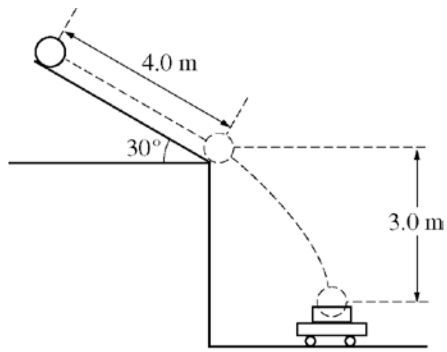


2006M3. A thin hoop of mass *M*, radius *R*, and rotational inertia MR^2 is released from rest from the top of the ramp of length *L* above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height *H* above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.
b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.
c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.
d. Suppose that the hoop is now replaced by a disk having the same mass *M* and radius *R*. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

Less than____ The same as____ Greater than____

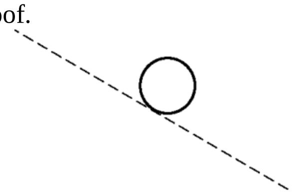
Briefly justify your response.



Note: Figure not drawn to scale.

2010M2. A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass *M* and radius *R* about its center of mass is $2MR^2/5$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.

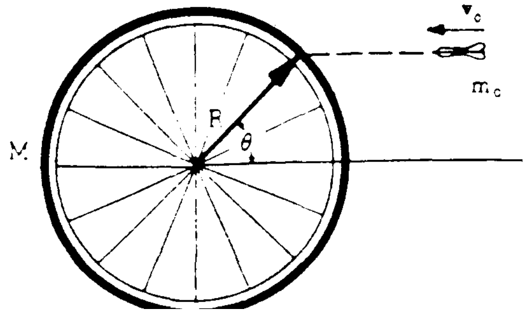


b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part a. to assist in your solution, use the space below. Do NOT add anything to the figure in part a.

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

SECTION D – Angular Momentum



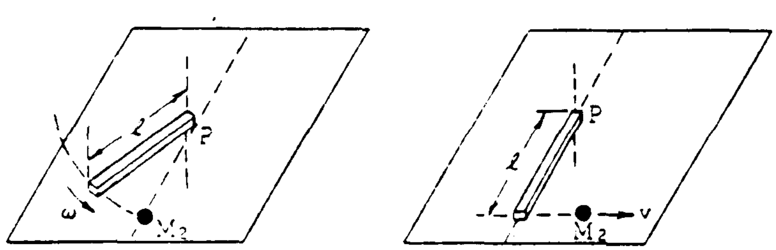
View From Above

1975M2. A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_o is thrown with velocity v_o as shown above and sticks in the tire.

a. If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

b. In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$



1978M2. A system consists of a mass M_2 and a uniform rod of mass M_1 and length *l*. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, *l*, and ω.

b. Determine the linear momentum of this system just before the collision in terms of M_1, *l*, and ω.

c. Determine the linear momentum of this system just after the collision in terms of M_1 *l*, and ω.

d. What is responsible for the change in the linear momentum of this system during the collision?

e. Why is the angular momentum of this system about point P conserved during the collision?

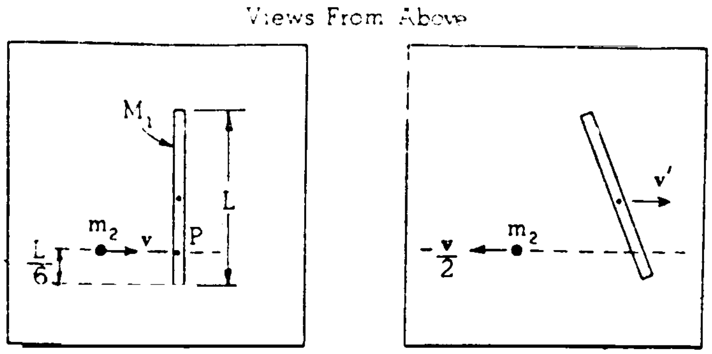


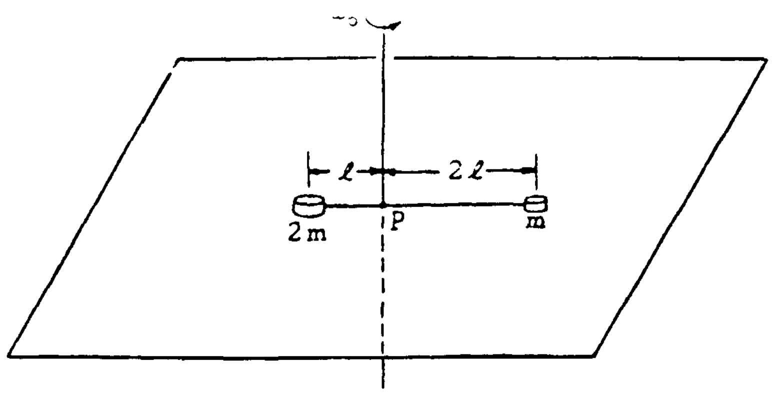
Figure I: Before   Figure II: After

1981M3. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $M_1L^2/12$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity
–½**v** as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity **v**' of the center of mass of this rod after the collision.

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

c. Determine the change in kinetic energy of the system resulting from the collision.

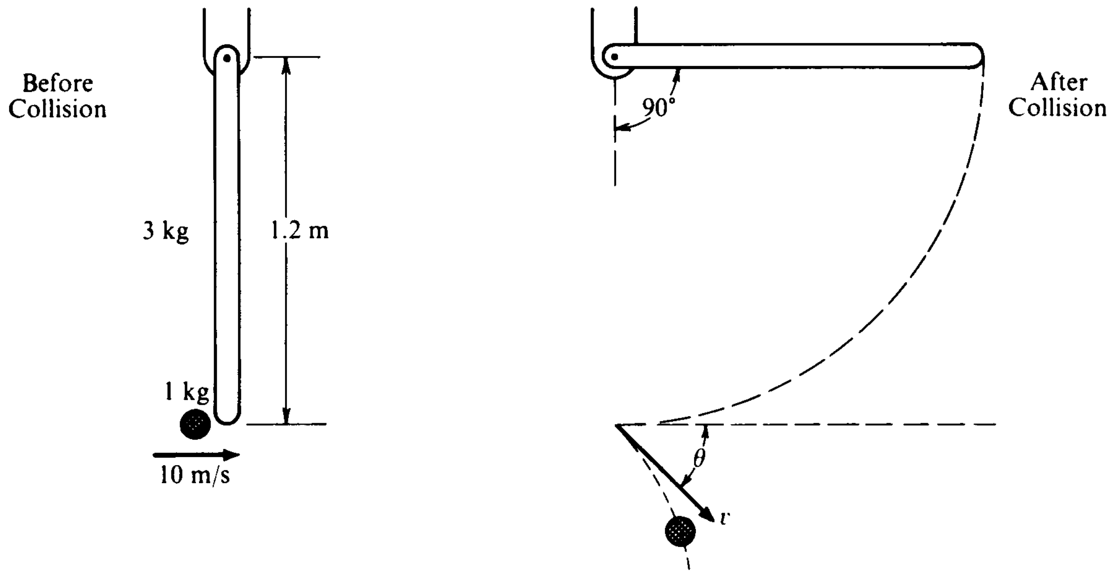


1982M3. A system consists of two small disks, of masses m and 2m, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time t = 0, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction. Develop expressions for the following quantities in terms of μ m, l, g, and ω_o

a. The initial angular momentum of the system about the axis through P

b. The frictional torque acting on the system about the axis through P

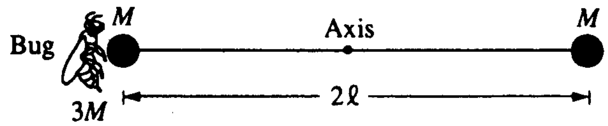
c. The time T at which the system will come to rest.



Note: You may use $g = 10\ \mathrm{m/s^2}$.

1987M3. A l.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.

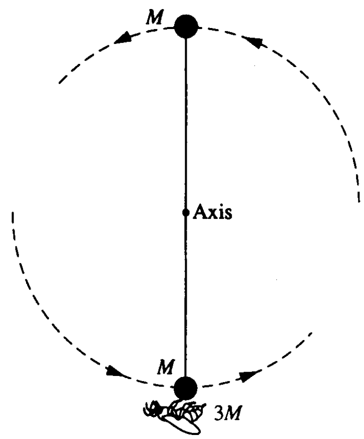
a. Determine the angular velocity of the bar immediately after the collision.
b. Determine the speed v of the l-kilogram object immediately after the collision.
c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.
d. Determine the angle θ.



1992M2. Two identical spheres, each of mass M and negligible radius, are fastened to opposite ends of a rod of negligible mass and length 2*l*. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass 3M, lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of M, *l*, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

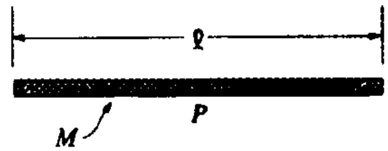


The rod-spheres-bug system swings about the axis. At the instant that the rod is vertical, as shown above, determine each of the following.

c. The angular speed of the bug

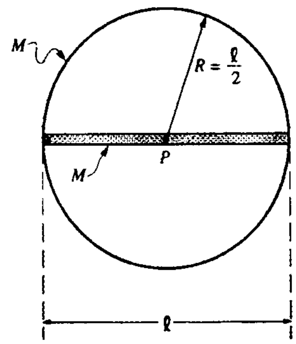
d. The angular momentum of the system

e. The magnitude and direction of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere



1996M3. Consider a thin uniform rod of mass M and length l, as shown above.

a. Show that the rotational inertia of the rod about an axis through its center and perpendicular to its length is $Ml^2/12$.



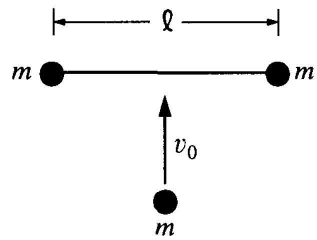
The rod is now glued to a thin hoop of mass M and radius R/2 to form a rigid assembly, as shown above. The centers of the rod and the hoop coincide at point P. The assembly is mounted on a horizontal axle through point P and perpendicular to the page.

b. What is the rotational inertia of the rod-hoop assembly about the axle?

Several turns of string are wrapped tightly around the circumference of the hoop. The system is at rest when a cat, also of mass M, grabs the free end of the string and hangs vertically from it without swinging as it unwinds, causing the rod-hoop assembly to rotate. Neglect friction and the mass of the string.

c. Determine the tension T in the string.
d. Determine the angular acceleration a of the rod-hoop assembly.
e. Determine the linear acceleration of the cat.
f. After descending a distance $H = 5l/3$, the cat lets go of the string. At that instant, what is the angular momentum of the cat about point P?

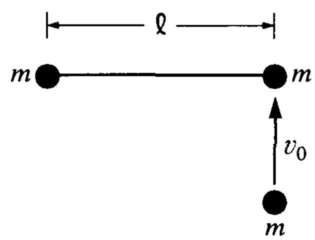
1998M2. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length l and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0 . Express your answers in terms of m, v_0 l. and fundamental constants.



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision.

ii. Determine the change in kinetic energy as a result of the collision.



b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly.

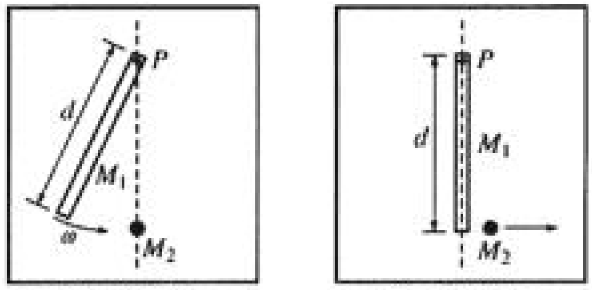
i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.)

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision.

iii. Determine the speed of the center of mass immediately after the collision.

iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

v. Determine the change in kinetic energy as a result of the collision.



Before Collision After Collision

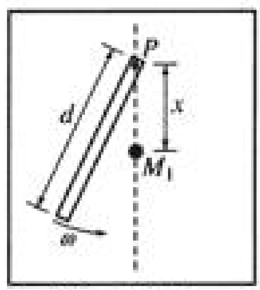
TOP VIEWS

2005M3. A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

a. Derive an expression for the angular momentum of the rod about point P before the collision.

b. Derive an expression for the speed v of the ball after the collision.

c. Assuming that this collision is elastic, calculate the numerical value of the ratio M_1 / M_2

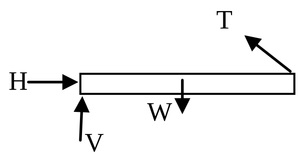


Before Collision

d. A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

SECTION A – Torque and Statics

	Solution	Answer
1.	Definition of Torque	D
2.	To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed	A
3.	FBD — Since the rope is at an angle it has x and y components of force. Therefore, H would have to exist to counteract T_x. Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar	B



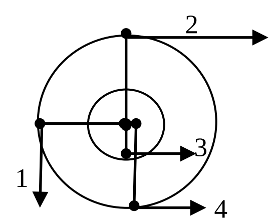
4. Applying rotational equilibrium to each diagram gives — **E**

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$
$L_1 = M_1(L_2)/m$
(sub this L_1) into the Diagram 2 eqn, and solve.

DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$
$M_2(L_1) = m(L_2)$

5. Find the torques of each using proper signs and add up. — **C**

+ (1) – (2) + (3) + (4)
+F(3R) – (2F)(3R) + F(2R) +F(3R) = 2FR



6. Simply apply rotational equilibrium — **B**

$(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
$m_1a = m_2b$

SECTION B – Rotational Kinematics and Dynamics

1.	$\alpha = \Delta\omega/\Delta t$	A
2.	$K_{rot} = ½ I\omega^2$	B
3.	$\omega = d\theta/dt = 6t + 2$; $v = \omega r$	D
4.	$I_{tot} = \Sigma I = I_0 + I_M = I_0 + M(½L)^2$	A
5.	$\Sigma\tau = I\alpha$ where $\Sigma\tau = (3M_0)(\ell) - (M_0)(2\ell) = M_0\ell$ and $I = (3M_0)(\ell)^2 + (M_0)(2\ell)^2 = 7M_0\ell^2$	A
6.	$\tau_X = F\ell$; $\tau_O = F_OL_O \sin\theta$, solve for the correct combination of F_O and L_O	C

7. Just as the tension in a rope is greatest at the bottom of as vertical circle, the force needed to maintain circular motion in any vertical circle is greatest at the bottom as the applied force must balance the weight of the object and additionally provide the necessary centripetal force C

8. $\Sigma F_{bottom} = F_{adhesion} - mg = F_{centripetal} = m\omega^2 r$ E

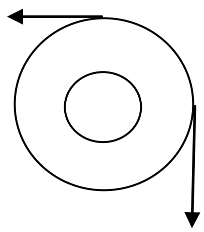
9. For one complete revolution $\theta = 2\pi$; $\omega^2 = \omega_0{}^2 + 2\alpha\theta$ C

10. The moment of inertia is least about the object's center of mass. The greater the distance of the axis from the center of mass, the greater the moment of inertia ($I = I_{cm} + MD^2$). The center of mass is at point B. E

11. $\tau = \Delta L/\Delta t = (I\omega_f - 0)/T$ E

12. $P_{avg} = \tau\omega_{avg} = (I\omega_f/T)(½\omega_f)$ or $P_{avg} = \Delta K/T$ B

13. D



$\Sigma\tau = T_2R - T_1R = I\alpha$

14. $\theta = \omega_0 t + ½\,\alpha t^2$ D

15. If the cylinder is "suspended in mid air" (i.e. the linear acceleration is zero) then $\Sigma F = 0$ E

16. $\Sigma\tau = TR = I\alpha = ½\,MR^2\alpha$ which gives $\alpha = 2T/MR$ and since $\Sigma F = 0$ then $T = Mg$ so $\alpha = 2g/R$ the acceleration of the person's hand is equal to the linear acceleration of the string around the rim of the cylinder $a = \alpha R = 2g$ B

17. In order that the mass not slide down $f = \mu F_N \geq mg$ and $F_N = m\omega^2 R$ B
solving for μ gives $\mu \geq g/\omega^2 R$

SECTION C – Rolling

1. $K_{tot} = K_{rot} + K_{trans} = ½\,I\omega^2 + ½\,Mv^2 = ½\,(2/5)MR^2\omega^2 + ½\,Mv^2 = (1/5)Mv^2 + ½\,Mv^2 = (7/10)Mv^2 = MgH$, solving gives $H = 7v^2/10g$ D

2. Rolling without slipping: $v = \omega R$. Linear momentum $= Mv = M\omega R$ A

3. $Mgh = K_{tot} = K_{rot} + K_{trans}$, however without friction, there is no torque to cause the sphere to rotate so $K_{rot} = 0$ and $Mgh = ½\,Mv^2$ A

4. $Mgh = K_{tot} = K_{rot} + K_{trans} = ½\,I\omega^2 + ½\,Mv^2$; substituting v/r for ω gives E
$Mgh = ½(I/r^2 + M)v^2$ and solving for v gives $v^2 = 2Mgh/(I/r^2 + M)$, multiplying by r^2/r^2 gives desired answer

5. $v = d/t$ and $\omega = v/r$ C

6. The first movement of the point of contact of a rolling object is vertically upward as there is no side to side (sliding) motion for the point in contact A

SECTION D – Angular Momentum

1. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater — C

2. $L = I\omega$ — D

3. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater — E

4. $L = mvr_\perp$ where $r_\perp$ is the perpendicular line joining the origin and the line along which the particle is moving — B

5. $L = I\omega$ and since ω is uniform the ratio $L_{upper}/L_{lower} = I_{upper}/I_{lower} = 2mL^2/2(2m)(2L)^2 = 1/8$ — E

6. Since it is a perfectly inelastic (sticking) collision, KE is not conserved. As there are no external forces or torques, both linear and angular momentum are conserved — E

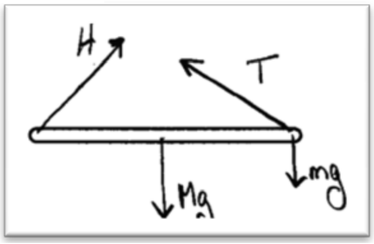
7. As there are no external forces or torques, both linear and angular momentum are conserved. As the type of collision is not specified, we cannot say kinetic energy *must* be the same. — C

8. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater — C

SECTION A – Torque and Statics

2008M2

a.



b. $\Sigma\tau = 0$
About the hinge: $TL \sin 30° - mgL - Mg(L/2) = 0$ gives $T = 29$ N

c. $I_{total} = I_{rod} + I_{block}$ where $I_{rod,end} = I_{cm} + MD^2 = ML^2/12 + M(L/2)^2 = ML^2/3$
$I_{total} = ML^2/3 + mL^2 = 0.42$ kg-m^2

d. $\Sigma\tau = I\alpha$
$mgL + MgL/2 = I\alpha$ gives $\alpha = 21$ rad/s^2

SECTION B – Rotational Kinematics and Dynamics

1973M3

a. Define a coordinate system with the x-axis directed to the vertical rod and the y-axis directed upwards and perpendicular to the first. Let T_1 be the tension in the horizontal string. Let T_2 be the tension in the string tilted upwards.
Applying Newton's Second Law: $\Sigma F_x = T_1 + T_2 \sin\theta = m\omega^2 R$; $\Sigma F_y = T_2 \cos\theta - mg = 0$
Solving yields: $T_2 = mg/\cos\theta$ and $T_1 = m(\omega^2 R - g \tan\theta)$

b. Let $T_1 = 0$ and solving for ω gives $\omega = (g \tan\theta/R)^{1/2}$

1976M2

a.

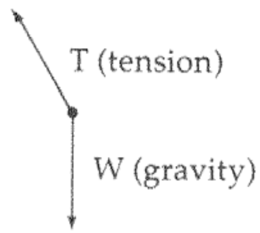
b. $\Sigma\tau = I\alpha$ (about center of mass) (one could also choose about the point at which the tape comes off the cylinder)
$TR = ½\ MR^2 \times (a/R)$
$T = ½\ Ma$

$\Sigma F = ma$
$Mg - T = Ma$
$Mg = 3Ma/2$
$a = 2g/3$

c. As there are no horizontal forces, the cylinder moves straight down.

1978M1

a.



b. $\Sigma F = ma$; $T \cos\theta = mg$ and $T \sin\theta = m\omega^2 r = m\omega^2(A + \ell \sin\theta)$

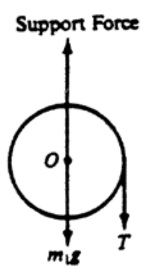
$$\omega = \sqrt{\frac{g\ tan\theta}{A + l\ sin\theta}}$$

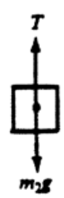
c. $W = \Delta E = \Delta K + \Delta U = ½\ mv^2 + mg\ell(1 - \cos\theta)$ for each rider

$W = 6(½\ mv^2 + mg\ell(1 - \cos\theta))$

1983M2

a.





b. i./ii. On the disk: $\Sigma\tau = I\alpha = TR = ½\ m_1R^2\alpha$

For the block $a = \alpha R$ so $\alpha = a/R$ and $\Sigma F = m_2g - T = m_2a$

Solving yields

$$a = \frac{2m_2g}{2m_2 + m_1}\ and\ \ T = \frac{m_1m_2g}{2m_2 + m_1}$$

1985M3

a.

b. i. $\Sigma F = ma$; $T_1 - mg = ma$ and $2mg - T_2 = 2ma$
ii. $\Sigma\tau = I\alpha$; $(T_2 - T_1)r = I\alpha$

c. $\alpha = a/r$
Combining equations from b.i. gives $T_2 - T_1 = mg - 3ma$
Substituting for $(T_2 - T_1)$ into torque equation gives $a = 2g/9$

d. $T_1 = m(g + a) = 11mg/9$

e. $F_N = 7mg + T_1 + T_2$ (the table counters all the downward forces on the *apparatus*)
$T_2 = 2m(g - a) = 14mg/9$
$F_N = 88mg/9$

1988M3

a. I is proportional to mR^2; masses are equal and R becomes 2R
$I_{2R} = 4I$

b. The disks are coupled by the chain along their rims, which means the linear motion of the rims have the same displacement, velocity and acceleration.
$v_R = v_{2R}$; $R\omega_R = 2R\omega_{2R}$; $R\alpha t = 2R\alpha_{2R}t$ gives $\alpha_{2R} = \alpha/2$

c. $\tau_{2R} = T(2R) = I_{2R}\alpha_{2R} = (4I)(\alpha/2) = 2I\alpha$ giving $T = I\alpha/R$

d. $\Sigma\tau = \tau_{student} - TR = I\alpha$
$\tau_{student} = I\alpha + TR = I\alpha + (I\alpha/R)R = 2I\alpha$

e. $K = ½ I\omega^2 = ½ I(\alpha t)^2$

1989M2

a. $\Sigma F = ma$; $2Mg - T_v = 2Ma$ so $T_v = 2M(g - a)$

b. $\Sigma\tau = T_vR - T_hR = I\alpha = 3MR^2(a/R)$
$T_h = (T_vR - 3MRa)/R = 2M(g - a) - 3Ma = 2Mg - 5Ma$

c. $F_f = \mu F_N = \mu(4Mg)$
$T_h - F_f = 3Ma$
$2Mg - 5Ma - 4\mu Mg = 3Ma$
$4\mu Mg = 2Mg - 8Ma$
$\mu = (2g - 8a)/4g$
plugging in given values gives $\mu = 0.1$

d. $F_f = 4\mu Mg = ma_C = 4Ma_C$
$a_C = 1\ m/s^2$

1991M2

a. $\Sigma\tau = 0$; $m_2gr_2 = m_1gr_1$; $m_2 = m_1r_1/r_2 = 6.67$ kg

b./c. $\tau = I\alpha$; $Tr_1 = (45\ kg\text{-}m^2)\alpha$
$\Sigma F = ma$; $(20\ kg)g - T = (20\ kg)a$
Combining with $a = \alpha r$ gives $\alpha = 2\ rad/s^2$ and $T = 180$ N

d. $mgh = ½ mv^2 + ½ I\omega^2 = ½ mv^2 + ½ I(v^2/r^2)$ giving $v = 1.4$ m/s

1993M3

a. $\Sigma\tau = F_a\ell - Mg\ell/2 = 0$ giving $F_a = Mg/2$

b. $\Sigma\tau = Mg\ell/2 = I\alpha = (M\ell^2/3)\alpha$; $\alpha = 3g/2\ell$

c. $a = \alpha r$ where $r = \ell/2$

   $a = (3g/2\ell)(\ell/2) = 3g/4$

d. $\Sigma F = Ma$; $Mg - F_a = Ma = M(3g/4)$

   $F_a = Mg/4$

e. $\Delta U = \Delta K_{rot}$

   $mgh = mg(\ell/2)\sin\theta = ½ I\omega^2 = ½ (M\ell^2/3)\omega^2$

   solving gives $\omega = (3g\sin\theta/\ell)^{1/2}$

1999M3

a. $\Sigma\tau = 0$ so $\tau_{cw} = \tau_{ccw}$ and $\tau_{cw} = TR$ (from the string) so we just need to find τ_{ccw} as the sum of the torques from the various parts of the system

   $\Sigma\tau_{ccw} = \tau_{rod} + \tau_{block} = mgR\sin\theta_0 + 2mg(2R)\sin\theta_0 = 5mgR\sin\theta_0 = TR$ so $T = 5mg\sin\theta_0$

b. i. $I = I_{disk} + I_{rod} + I_{block} = 3mR^2/2 + 4mR^2/3 + 2m(2R)^2 = 65mR^2/6$

   $\alpha = \tau/I = (5mgR\sin\theta_0)/(65mR^2/6) = 6g\sin\theta_0/13R$

   ii. $a = \alpha r$ where $r = 2R$ so $a = 12g\sin\theta_0/13$

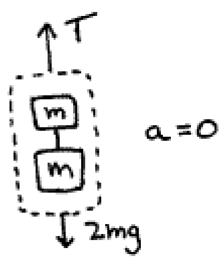
c. ΔU (from each component) $= K = ½ I\omega^2$

   $mgR\cos\theta_0 + 2mg(2R)\cos\theta_0 = ½ (65mR^2/6)\omega^2$

   $\omega = (12g\cos\theta_0/13R)^{1/2}$ and $v = \omega r = \omega(2R) = 4(3gR\cos\theta_0/13)^{1/2}$

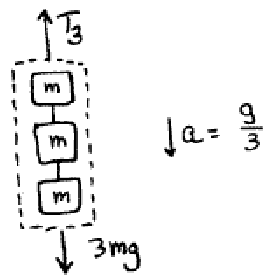
2000M3

a.



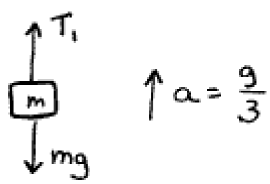
$\Sigma F = ma = 0$ so $T = 2mg$

b. i.



$\Sigma F = ma$
$3mg - T_3 = 3m(g/3)$
$T_3 = 2mg$

ii.



$\Sigma F = ma$
$T_1 - mg = m(g/3)$
$T_1 = 4mg/3$

iii. $\Sigma\tau = (T_3 - T_1)R_1 = I\alpha$ and $\alpha = a/R_1 = g/3R_1$
$(2mg - 4mg/3)R_1 = I_1(g/3R_1)$
$I_1 = 2mR_1^2$

c. i. Tangential speeds are equal; $\omega_1 R_1 = \omega_2 R_2 = \omega_2(2R_1)$ therefore $\omega_2 = \omega_1/2$
ii. $L = I\omega = (16I_1)(\omega_1/2) = 8I_1\omega_1$
iii. $K = ½\, I_1\omega_1^2 + ½\, I_2\omega_2^2 = (5/2)I_1\omega_1^2$

<u>2001M3</u>

a. $I = \Sigma mr^2 = mL^2 + mL^2 = 2mL^2$

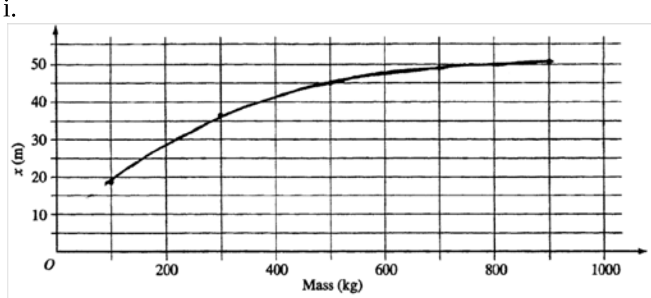
b. $\Sigma F = ma$; $4mg - T = 4ma$
$\Sigma\tau = I\alpha$; $Tr = I\alpha$; $T = I\alpha/r = 4mg - 4ma$ and $\alpha = a/r$, solving gives $a = 2gr^2/(L^2 + 2r^2)$

c. Equal, total energy is conserved

d. Less, the small blocks rise and gain potential energy. The total energy available is still 4mgD, therefore the kinetic energy must be less than in part c.

<u>2003M3</u>

a. i.



ii. x = 33 m

b. i. $y = ½ gt^2$; $t = (2y/g)^{1/2} = 1.75$ s
ii. $U_{initial} = U_{bucket} + U_{projectile} = M(9.8\ m/s^2)(3\ m) + (10\ kg)(9.8\ m/s^2)(3\ m) = 29.4M + 294$
iii. $U_{initial} = U_{final} + K$ where $U_{final} = Mg(1\ m) + (10\ kg)g(15\ m) = 9.8M + 1470$
$K_{projectile} = ½\ 10v_x^2$ and $K_{bucket} = ½\ Mv_b^2$ where $v_b = v_x/6$
putting it all together gives $29.4M + 294 = 9.8M + 1470 + 5v_x^2 + (M/72)v_x^2$

$$v_x = \sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

c. i. $x = v_x t$

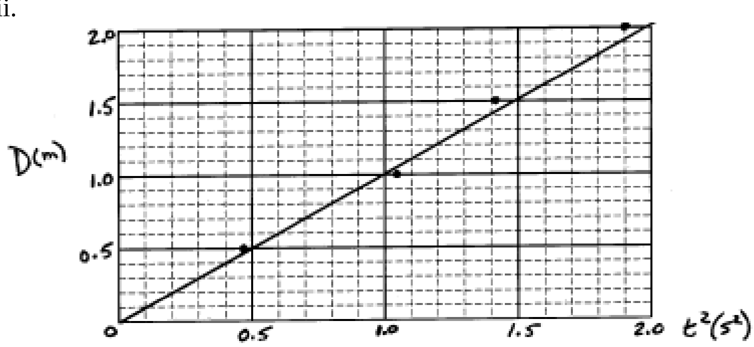
$$x = 1.75\sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

d. x(300 kg) = 39.7 m (greater than the experimental value)
possible reasons include friction at the pivot, air resistance, neglected masses not negligible

<u>2004M2</u>

a. $x = v_0 t + ½ at^2$
$x = D$ and $v_0 = 0$ so $D = ½ at^2$ and $a = 2D/t^2$

b. i. graph D vs. t^2 (as an example)
ii.



iii. a = 2(slope) = 2.04 m/s^2

c. $\Sigma\tau = TR = I\alpha$ and $\alpha = a/R$ so $I = TR^2/a$
$\Sigma F = mg - T = ma$ so $T = m(g - a)$
$I = m(g - a)R^2/a = mR^2((g/a) - 1)$

d. The string was wrapped around the pulley several times, causing the effective radius at which the torque acted to be larger than the radius of the pulley used in the calculation.

The string slipped on the pulley, allowing the block to accelerate faster than it would have otherwise, resulting in a smaller experimental moment of inertia.

Friction is not a correct answer, since the presence of friction would make the experimental value of the moment of inertia too large

SECTION C – Rolling

NOTE: Rolling problems may be solved considering rotation about the center of mass or the point of contact. The solutions below will only show one of the two methods, but for most, if not all cases, the other method is applicable. When considering rotation about the point of contact, remember to use the parallel axis theorem for the moment of inertia of the rolling object.

1974M2

a. Torque provided by friction; at minimum μ, $F_f = \mu F_N = \mu Mg \cos\theta$
   $\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma = \mu Mg \cos\theta$ giving $a = (5/2)\mu g \cos\theta$
   $\Sigma F = Ma$; $Mg \sin\theta - \mu Mg \cos\theta = Ma = (5/2)\mu Mg \cos\theta$ giving $\mu = (2/7) \tan\theta$

b. Energy at the bottom is the same in both cases, however with $\mu = 0$, there is no torque and no energy in rotation, which leaves more (all) energy in translation and velocity is higher

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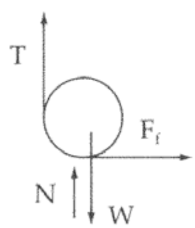
1977M2

a.



b. $\Sigma F_y = 0$; $T + N = W$; $N = W - T = Mg - (3/5)Mg = (2/5)Mg$
   $\Sigma F_x = ma$; $F_f = ma$; $\mu N = ma$; $½ (2/5)Mg = Ma$; $a = g/5$

c. $\Sigma\tau = I\alpha$; $(T - F_f)R = ½ MR^2\alpha$
   $(3/5)Mg - (1/5)Mg = ½ MR\alpha$
   $(2/5)g = ½ R\alpha$
   $\alpha = 4g/5R$

d. The cylinder is slipping on the surface and does not meet the condition for pure rolling

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1980M3

a. $\Sigma F = ma$; $F_f = \mu F_N$;
   $-\mu Mg = Ma$
   $a = -\mu g$
   $v = v_0 + at$
   $v = v_0 - \mu gt$

b. $\tau = I\alpha$ where the torque is provided by friction $F_f = \mu Mg$
   $\mu MgR = (2MR^2/5)\alpha$
   $\alpha = (5\mu g/2R)$
   $\omega = \omega_0 + \alpha t = (5\mu g/2R)t$

c. Slipping stops when the tangential velocity si equal to the velocity of the center of mass, or the condition for pure rolling has been met: $v(t) = \omega(t)R$
   $v_0 - \mu gt = R(5g/2R)t$, which gives $T = (2/7)(v_0/\mu g)$

d. Since the line of action of the frictional force passes through P, the net torque about point P is zero. Thus, the time rate of change of the angular momentum is zero and the angular momentum is constant.

1986M2

a. $U = K$

$Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$

i. $K_{trans} = ½ Mv^2 = (5/7)Mgh$

ii. $K_{rot} = ½ I\omega^2 = (2/7)Mgh$ (or $Mgh – K_{trans}$)

b. i. $\tau = F_fR = I\alpha = I(a/R)$

$F_fR = (2/5)MR^2(a/R)$

$F_f = (2/5)Ma$

$\Sigma F = ma$

$Mg \sin\theta – F_f = Ma$

$Mg \sin\theta – (2/5)Ma = Ma$

$g \sin\theta = (7/5) a$

$a = (5/7) g \sin\theta$

ii. $F_f = (2/5)Ma = (2/5)M(5/7)g \sin\theta = (2/7)Mg \sin\theta$

c. $K_{tot} = Mgh$

d. Greater, the moment of inertia of the hollow sphere is greater and will be moving slower at the bottom of the incline. Since the translational speed is less, the translational KE is taking a smaller share of the same total energy as the solid sphere.

1990M2

a. $K = U$

$½ mv_0^2 = mgH$; $H = v_0^2/2g$

b. $K + W_f = U$ where $W_f = –F_fd$ and $F_f = \mu mg \cos\theta$ and $d = h/\sin\theta$

$½ mv_0^2 – (\mu mg \cos\theta)(h/\sin\theta) = mgh$

$½ mv_0^2 = mgh(\mu \cot\theta + 1)$

$h = v_0^2/(2g(\mu \cot\theta + 1)) = H/(\mu \cot\theta + 1)$

c. $K_{trans} + K_{rot} = U$ where $K_{rot} = ½ I\,\omega^2 = ½ (mR^2)(v/R)^2 = ½ mv_0^2$

$½ mv_0^2 + ½ mv_0^2 = mgh'$

$h' = v_0^2/g = 2H$

d. Rotational energy will not change therefore $½ mv_0^2 = mgh''$ and $h'' = v_0^2/2g = H$

1994M2

a. $K_{tot} = K_{trans} + K_{rot} = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$K_{tot} = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10 = 1750$ J

b. i. $K_{total,bottom} = K_{top} + U_{top} = 7Mv_{top}^2/10 + Mgh$; $v_{top} = 7.56$ m/s

ii. It is directed parallel to the incline: 25º

c. $y = y_0 + v_{oy}t + ½ a_yt^2$

0 m = 3 m + (7.56 m/s)(sin 25º)t + ½ (–9.8 m/s^2)t^2 which gives t = 1.16 s (positive root)

$x = v_xt$ = (7.56 m/s)(cos 25º)(1.16 s) = 7.93 m

d. The speed would be less than in b.

The gain in potential energy is entirely at the expense of the translational kinetic energy as there is no torque to slow the rotation.

1997M3

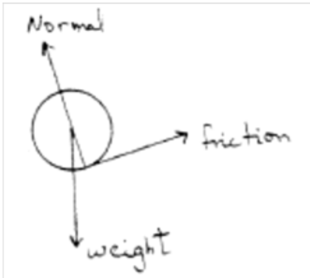
a. $U = K$

$MgH = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$MgH = ½ Mv^2 + ½ (½)MR^2(v/R)^2 = ½ Mv^2 + ¼ Mv^2 = 3Mv^2/4$

$v = (4gH/3)^{1/2}$

b.



c. For a change of pace, we can use kinematics:
$v_f^2 = v_i^2 + 2ad$
$4gH/3 = 0 + 2a(H/\sin\theta)$
$a = (2/3)g \sin\theta$

d. $\Sigma F = Ma$
$Mg \sin\theta - F_f = Ma = M(2/3)g \sin\theta$
$Mg \sin\theta - \mu Mg \cos\theta = (2/3)Mg \sin\theta$
$\mu \cos\theta = (1/3) \sin\theta$
$\mu = (1/3) \tan\theta$

e. i. The translational speed is greater, less energy is transferred to the rotational motion so more goes into the translational motion. Additionally, with a smaller frictional force, the translational acceleration is greater.
ii. Total kinetic energy is less. Energy is dissipated as heat due to friction.

2002M2

a. For each tire: $I = ½ ML^2 = ½ (m/4)r^2$
$I_{total} = 4 \times I = ½ mr^2$

b. U = K; total mass = 2m
$2mgh = ½ (2m)v^2 + ½ I\omega^2$ and $\omega = v/R$
$2mgh = mv^2 + ½ (½)mr^2(v/r)^2 = ½ mv^2 + (¼)mv^2 = 5mv^2/4$
$v = (8gh/5)^{1/2}$

c. $U_g = U_s$
$2mgh = ½ kx_m^2$; $x_m = 2(mgh/k)^{1/2}$

d. In an inelastic collision, energy is lost. With less energy after the collision, the spring is compressed less.

2006M3

a. $\Sigma\tau = I\alpha$
$F_fR = I\alpha = MR^2(a/R)$; $F_f = Ma$
$\Sigma F = ma$
$Mg \sin\theta - F_f = Ma$
$Mg \sin\theta - Ma = Ma$
$a = ½ g \sin\theta$

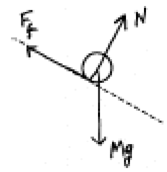
b. $v_f^2 = 2aL = gL \sin\theta$
$v_f = (gL \sin\theta)^{1/2}$

c. $H = ½ gt^2$; $t = (2H/g)^{1/2}$
$d = v_xt = (gL \sin\theta)^{1/2}(2H/g)^{1/2} = (2LH \sin\theta)^{1/2}$

d. Greater. A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

2010M2

a.



b. Torque provided by friction; $F_f = \mu F_N = \mu Mg \cos\theta$

$\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma$; $Ma = (5/2)F_f$

$\Sigma F = Ma$

$Mg \sin\theta - F_f = (5/2)F_f$

$F_f = (2/7)Mg \sin\theta = 8.4$ N

c. $Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$; $v = 5.3$ m/s

d. The horizontal speed of the wagon is due to the horizontal component of the ball in the collision:

$M_i v_{ix} = M_f v_{fx}$; where $M_f = M_{ball} + M_{wagon} = 18$ kg

(6 kg)(5.3 m/s)(cos 30º) = (18 kg)v_f

$v_f = 1.5$ m/s

SECTION D – Angular Momentum

1975M2

a. $L_i = L_f$

$m_0 v_0 R \sin\theta = I\omega$

$\omega = m_0 v_0 R \sin\theta / I$; $I = (M + m_0)R^2$

$\omega = m_0 v_0 \sin\theta/(M + m_0)R$

b. $K_i = ½ m_0 v_0^2$

$K_f = ½ I\omega^2 = ½ (M + m_0)R^2(m_0 v_0 \sin\theta/(M + m_0)R)^2$

$K_f/K_i = m_0 \sin^2\theta/(M + m_0)$

1978M2

a. $L_i = L_f$

$I\omega = mvr$

$(1/3)M_1 \ell^2 \omega = M_2 v \ell$

$v = M_1 \ell \omega / 3M_2$

b. $p_{system} = p_{cm\ of\ rod} = M_1 v_{cm} = M_1 \omega(\ell/2)$

c. $P_f = M_2 v_f = M_1 \omega \ell / 3M_2$

d. There is a net external force on the system from the axis at point P.

e. Since the net external force acts at point P (the pivot), the net torque about point P is zero, hence angular momentum is conserved.

1981M3

a. $m_2 v = m_2(-v/2) + M_1 v'$

$v' = 3m_2 v/2M_1$

b. $L_i = L_f$

$m_2 v(L/3) = m_2(-v/2)(L/3) + (1/12)M_1 L^2 \omega$

$\omega = 6m_2 v/M_1 L$

c. $\Delta K = K_f - K_i = \frac{1}{2} m_2(-v/2)^2 + \frac{1}{2} M_1 v'^2 + \frac{1}{2} I\omega^2 - \frac{1}{2} m_2 v^2$
$= -3m_2v^2/8 + 21m_2{}^2v^2/8M_1$

<u>1982M3</u>

a. $L = I\omega$ where $I = \Sigma mr^2 = (2m)\ell^2 + m(2\ell)^2 = 6m\ell^2$
$L = 6m\ell^2\omega$

b. $F_f = \mu mg$
$\Sigma\tau = -(\mu(2m)g\ell + \mu mg(2\ell)) = -4\mu mg\ell$

c. $\alpha = \tau/I = -4\mu mg\ell/6m\ell^2 = -2\mu g/3\ell$
$\omega = \omega_0 + \alpha t$; setting $\omega = 0$ and solving for T gives $T = 3\omega_0\ell/2\mu g$

<u>1987M3</u>

a. $K = U$
$\frac{1}{2} I\omega^2 = mgh_{cm}$
$\frac{1}{2} (m\ell^2/3)\omega^2 = mg(\ell/2)$ which gives $\omega = 5$ rad/s

b. $K_i = K_f$
$\frac{1}{2} m_0 v_0{}^2 = \frac{1}{2} m_0 v^2 + \frac{1}{2} I\omega^2$
$v = 8$ m/s

c. $L = mvr = (1\text{ kg})(10\text{ m/s})(1.2\text{ m}) = 12\text{ kg-m}^2/\text{s}$

d. $L_i = L_f$
$12\text{ kg-m}^2/\text{s} = m_0(v_\perp)\ell + I\omega = m_0(v\cos\theta)\ell + I\,\omega$
$\theta = 60^\circ$

<u>1992M2</u>

a. $\Sigma\tau = (3M + M)g\ell - Mg\ell = 3Mg\ell$

b. $I = \Sigma mr^2 = 4M\ell^2 + M\ell^2 = 5m\ell^2$
$\alpha = \tau/I = 3Mg\ell/5m\ell^2 = 3g/5\ell$

c. $\Delta U_{bug} + \Delta U_{left\ sphere} + \Delta U_{right\ sphere} = \Delta K_{rot}$
since $\Delta U_{left\ sphere} = -\Delta U_{right\ sphere}$, we only need to consider ΔU_{bug}
$3Mg\ell = \frac{1}{2} I\omega^2 = \frac{1}{2} (5M\ell^2)\omega^2$
$\omega = (6g/5\ell)^{1/2}$

d. $L = I\omega = 5M\ell^2(6g/5\ell)^{1/2} = (30M^2g\ell^3)^{1/2}$

e. Let T be the force we are looking for
$\Sigma F = ma_c$
$T - 3Mg = M\omega^2\ell$
$T = 3Mg + 3M(6g/5\ell)\ell = 33Mg/5$

<u>1996M3</u>

a.

$$I = \int r^2 dm$$

$$dm = \frac{M}{l} dr$$

$$I = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{M}{l} r^2 dr$$

$$I = \frac{M}{l} \frac{r^3}{3}\Bigg|_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

b. $I = \Sigma I = M\ell^2/12 + M(\ell/2)^2 = M\ell^2/3$

c./d./e.

$\Sigma F = ma$

for cat: $Mg - T = Ma$

$\Sigma\tau = I\alpha$ where $\alpha = a/r = a/(\ell/2)$

for hoop: $T\ell/2 = (M\ell^2/3)(a/(\ell/2))$ which gives $a = 3T/4M$

substituting gives $Mg - T = 3T/4$

$T = 4Mg/7$

$\alpha = T\ell/2I = 6g/7\ell$

$a = \alpha(\ell/2) = 3g/7$

f. $L = Mv(\ell/2)$ where v is found from $v^2 = v_0^2 + 2aH = 2(3g/7)(5\ell/3) = 10g\ell/7$

$L = ½ M\ell(10g\ell/7)^{1/2}$

<u>1998M2</u>

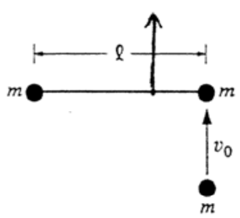
a. i. $mv_0 = (3m)v_f$; $v_f = v_0/3$

$K_f = ½ (3m)(v_0/3)^2 = mv_0^2/6$

ii. $\Delta K = K_f - K_i = mv_0^2/6 - ½ mv_0^2 = -mv_0^2/3$

b. i. $r_{cm} = \Sigma m_i r_i/\Sigma m = m(0) + 2m(\ell)/(m + 2m) = (2/3)\ell$

ii.



iii. $p_i = p_f$

$mv_0 = (3m)v_f$; $v_f = v_0/3$

iv. $L_i = L_f$

$mv_0R \sin\theta = mv_0(\ell/3) = I\omega$ where $I = \Sigma mr^2 = m(2\ell/3)^2 + 2m(\ell/3)^2 = (2/3)m\ell^2$

solving yields $\omega = v_0/2\ell$

v. $K_i = ½ mv_0^2$

$K_f = ½ mv_f^2 + ½ I\omega^2 = ½ (3m)(v_0/3)^2 + ½ (2/3)m\ell^2(v_0/2\ell)^2 = ¼ mv_0^2$

$\Delta K = -¼ mv_0^2$

<u>2005M3</u>

a. $L = I\omega = (1/3)M_1d^2\omega$

b. $L_f = L_i$

$M_2vd = (1/3)M_1d^2\omega$

$v = M_1d\omega/3M_2$

c. $K_f = K_i$

$½ M_2v^2 = ½ I\omega^2$

$M_2v^2 = I\omega^2$

$M_2(M_1d\omega/3M_2) = (1/3)M_1d^2\omega^2$ $\quad$ $M_2(1/9)(M_1/M_2)^2d^2\omega^2 = (1/3)M_1d^2\omega^2$ $\quad$ $(1/9)(M_1^2/M_2) = M_1/3$

$M_1/M_2 = 3$

d. $L_f = L_i$

$M_1vx = (1/3)M_1d^2\omega$

$v = d^2\omega/3x$

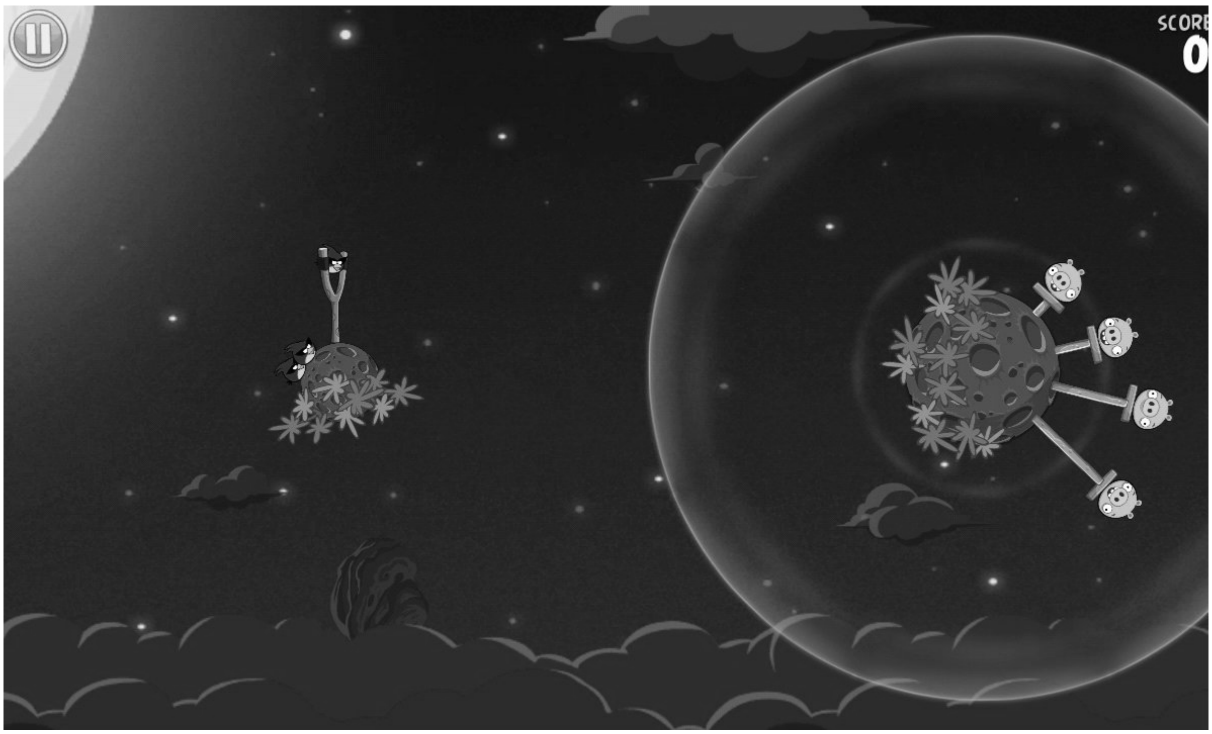
$½ M_1v^2 = ½ I\omega^2$ $\quad$ $M_1v^2 = I\omega^2 = (1/3)M_1d^2\omega^2$

$v^2 = d^2\omega^2/3$ $\quad$ substituting from above $\quad$ $(d^2\omega/3x)^2 = d^2\omega^2/3$

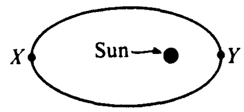
solving for x gives $x = d/\sqrt{3}$

Chapter 6

Gravitation



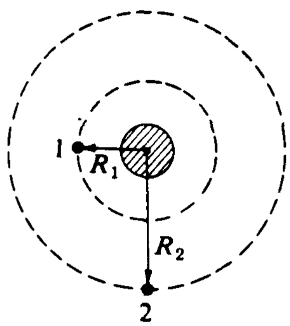
1. The mass of Planet X is one-tenth that of the Earth, and its diameter is one-half that of the Earth. The acceleration due to gravity at the surface of Planet X is most nearly
(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 7 m/s^2 (E) 10 m/s^2



2. A satellite travels around the Sun in an elliptical orbit as shown above. As the satellite travels from point X to point Y. which of the following is true about its speed and angular momentum?

	Speed	Angular Momentum
(A)	Remains constant	Remains constant
(B)	Increases	Increases
(C)	Decreases	Decreases
(D)	Increases	Remains constant
(E)	Decreases	Remains constant

3. A newly discovered planet, "Cosmo," has a mass that is 4 times the mass of the Earth. The radius of the Earth is R_e. The gravitational field strength at the surface of Cosmo is equal to that at the surface of the Earth if the radius of Cosmo is equal to
(A) ½R_e (B) R_e (C) $2R_e$ (D) $\sqrt{R_e}$ (E) R_e^2



4. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations a_2 and a_1 of the two satellites are related by which of the following?
(A) $a_2 = 4a_1$ (B) $a_2 = 2a_1$ (C) $a_2 = a_1$ (D) $a_2 = a_1/2$ (E) $a_2 = a_1/4$

5. The radius of the Earth is approximately 6,000 kilometers. The acceleration of an astronaut in a perfectly circular orbit 300 kilometers above the Earth would be most nearly
(A) 0 m/s^2 (B) 0.05 m/s^2 (C) 5 m/s^2 (D) 9 m/s^2 (E) 11 m/s^2

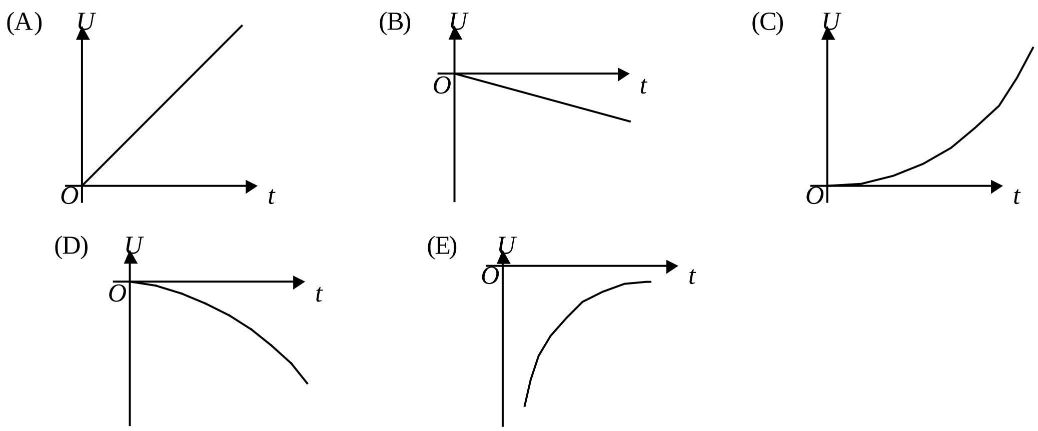
6. A satellite moves in a stable circular orbit with speed v_o at a distance R from the center of a planet. For this satellite to move in a stable circular orbit a distance $2R$ from the center of the planet, the speed of the satellite must be

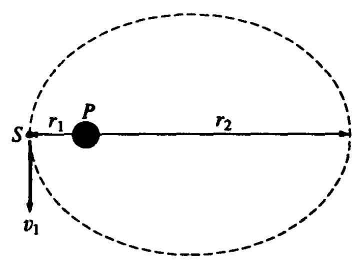
(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) v_o (D) $\sqrt{2v_0}$ (E) $2v_o$

7. A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface. The radius of the new planet in terms of the radius R of Earth is

(A) ½R (B) $\frac{\sqrt{2}}{2}$ R (C) $\sqrt{2}$ R (D) 2R (E) 4R

8. A small mass is released from rest at a very great distance from a larger stationary mass. Which of the following graphs best represents the gravitational potential energy U of the system of the two masses as a function of time t?





9. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?

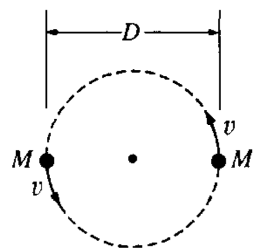
(A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $(r_2 - r_2)v_1$ (D) $\frac{r_1 + r_2}{2}v_1$ (E) $\frac{r_2 - r_1}{r_1 + r_2}v_1$

Questions 10-11 refer to a ball that is tossed straight up from the surface of a small, spherical asteroid with no atmosphere. The ball rises to a height equal to the asteroid's radius and then falls straight down toward the surface of the asteroid.

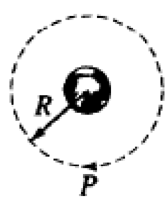
10. What forces, if any, act on the ball while it is on the way up?
(A) Only a decreasing gravitational force that acts downward
(B) Only an increasing gravitational force that acts downward
(C) Only a constant gravitational force that acts downward
(D) Both a constant gravitational force that acts downward and a decreasing force that acts upward
(E) No forces act on the ball.

11. The acceleration of the ball at the top of its path is
    (A) at its maximum value for the ball's flight
    (B) equal to the acceleration at the surface of the asteroid
    (C) equal to one-half the acceleration at the surface of the asteroid
    (D) equal to one-fourth the acceleration at the surface of the asteroid
    (E) zero

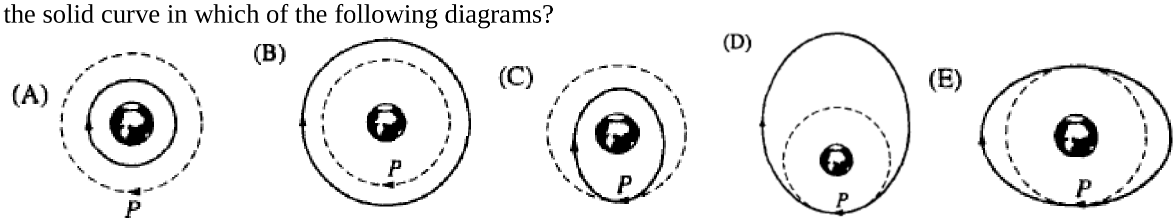
12. A satellite of mass M moves in a circular orbit of radius R with constant speed v. True statements about this satellite include which of the following?
    I. Its angular speed is v/R.
    II. Its tangential acceleration is zero.
    III. The magnitude of its centripetal acceleration is constant.
    (A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, and III

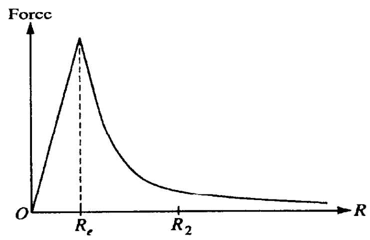


13. Two identical stars, a fixed distance D apart, revolve in a circle about their mutual center of mass, as shown above. Each star has mass M and speed *v*. G is the universal gravitational constant. Which of the following is a correct relationship among these quantities?
    (A) v^2 = GM/D (B) v^2 = GM/2D (C) v^2 = GM/D^2 (D) v^2 = MGD (E) v^2 = 2GM^2/D



14. A spacecraft orbits Earth in a circular orbit of radius *R*, as shown above. When the spacecraft is at position *P* shown, a short burst of the ship's engines results in a small increase in its speed. The new orbit is best shown by the solid curve in which of the following diagrams?



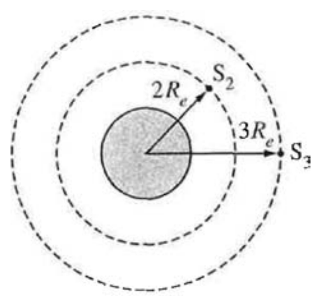


15. The graph above shows the force of gravity on a small mass as a function of its distance R from the center of the Earth of radius R_e, if the Earth is assumed to have a uniform density. The work done by the force of gravity when the small mass approaches Earth from far away and is placed into a circular orbit of radius R_2 *is* best represented by the area under the curve between
(A) $R = 0$ and $R = R_e$, (B) $R = 0$ and $R = R_2$ (C) $R = R_e$, and $R = R_2$
(D) $R = R_e$ and $R = \infty$ (E) $R = R_2$ and $R = \infty$

16. The escape speed for a rocket at Earth's surface is v_e . What would be the rocket's escape speed from the surface of a planet with twice Earth's mass and the same radius as Earth?

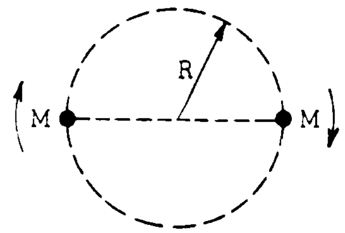
(A) $2\, v_e$ (B) $\sqrt{2}v_e$ (C) v_e (D) $\frac{v_e}{\sqrt{2}}$ (E) $v_e/2$

17. A student is asked to determine the mass of Jupiter. Knowing which of the following about Jupiter and one of its moons will allow the determination to be made?
I. The time it takes for Jupiter to orbit the Sun
II. The time it takes for the moon to orbit Jupiter
III. The average distance between the moon and Jupiter
(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III



18. The figure above represents satellites S_2 and S_3 of equal mass orbiting Earth in circles of radii $2R_e$ and $3R_e$, respectively, where R_e is the radius of Earth. How do the kinetic energy and the angular momentum of S_3 compare with those of S_2 ?

	Kinetic Energy	Angular Momentum
(A)	Less for S_3	Greater for S_3
(B)	Greater for S_3	Greater for S_3
(C)	The same for both	The same for both
(D)	Less for S_3	Less for S_3
(E)	Greater for S_3	Less for S_3



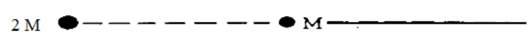
1977M3. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

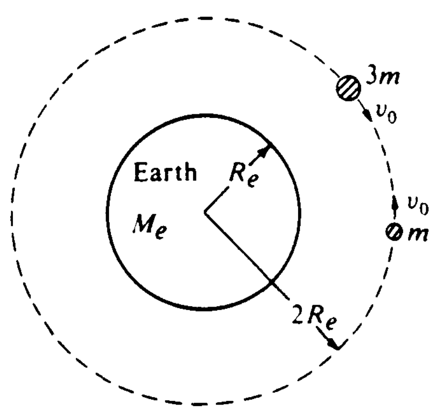
b. Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.



d. Find the ratio of the speeds, $v_{2M}/v_{M.}$

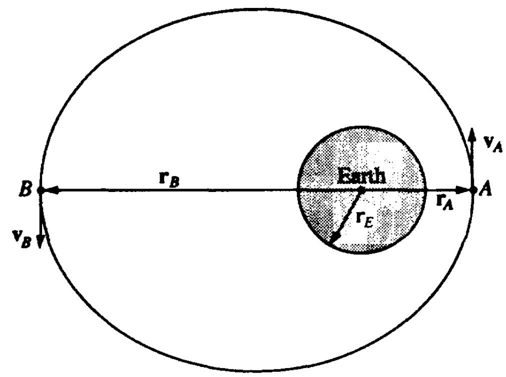


1984M2. Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



1992M3. A spacecraft of mass 1,000 kilograms is in an elliptical orbit about the Earth, as shown above. At point A the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .

a. The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

b. The magnitude of the angular momentum of the spacecraft about the center of the Earth.

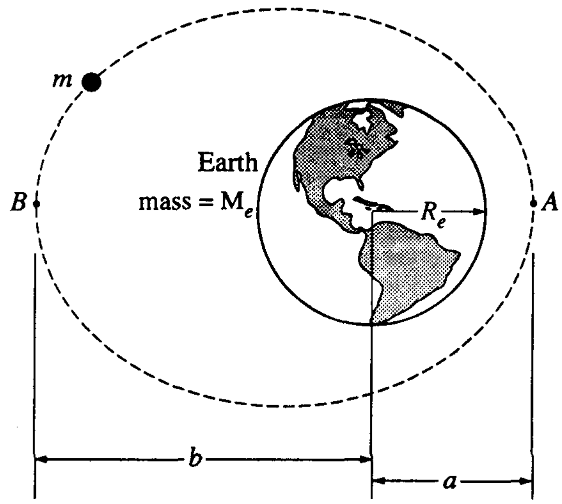
Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.

c. Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth. Determine each of the following.

d. The speed of the spacecraft if it is in a <u>circular</u> orbit around the Earth

e. The minimum speed of the spacecraft at point A if it is to escape completely from the Earth

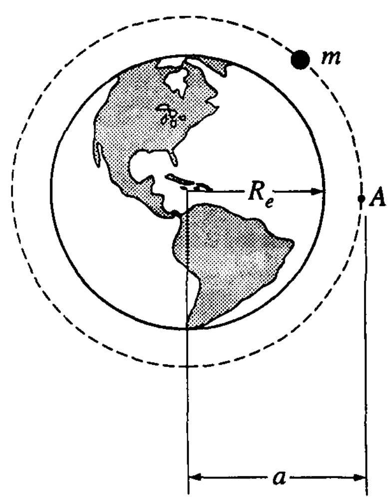


1994M3 A satellite of mass *m* is in an elliptical orbit around the Earth, which has mass M_e and radius R_e. The orbit varies from a closest approach of distance *a* at point A to maximum distance of b from the center of the Earth at point B. At point A, the speed of the satellite is v_o. Assume that the gravitational potential energy $U_g = 0$ when masses are an infinite distance apart. Express your answers in terms of *a*, b, *m*, M_e, R_e, v_o, and G.

a. Write the appropriate definite integral, including limits, that can be evaluated to show that the potential energy of the satellite when it is a distance r from the center of the Earth is given by

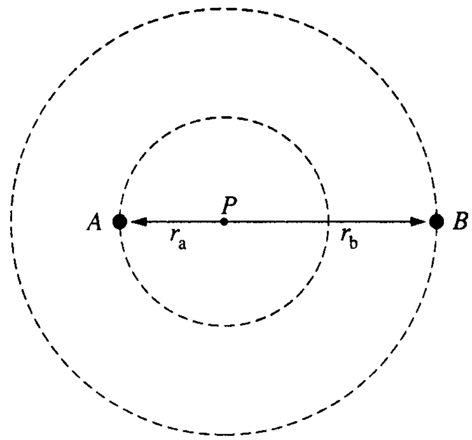
$$U_g = -\frac{GM_e m}{r}$$

b. Determine the total energy of the satellite when it is at A.
c. What is the magnitude of the angular momentum of the satellite about the center of the Earth when it is at A?
d. Determine the velocity of the satellite as it passes point B in its orbit.



As the satellite passes point A, a rocket engine on the satellite is fired so that its orbit is changed to a circular orbit of radius a about the center of the Earth.

e. Determine the speed of the satellite for this circular orbit.
f. Determine the work done by the rocket engine to effect this change.



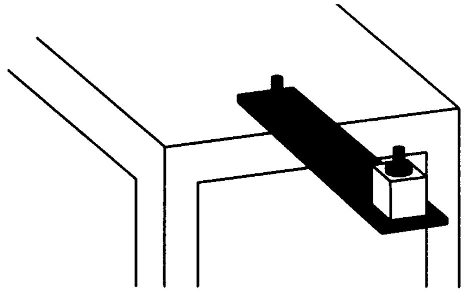
1995M3 Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.

Determine expressions for the following three quantities in terms of r_a, r_b, T, and fundamental constants.

a. The centripetal acceleration of star A
b. The mass M_b of star B
c. The mass M_a of star A

Determine expressions for the following two quantities in terms of M_a, M_b, r_a, r_b, T, and fundamental constants.

d. The moment of inertia of the two-star system about its center of mass.
e. The angular momentum of the system about the center of mass.

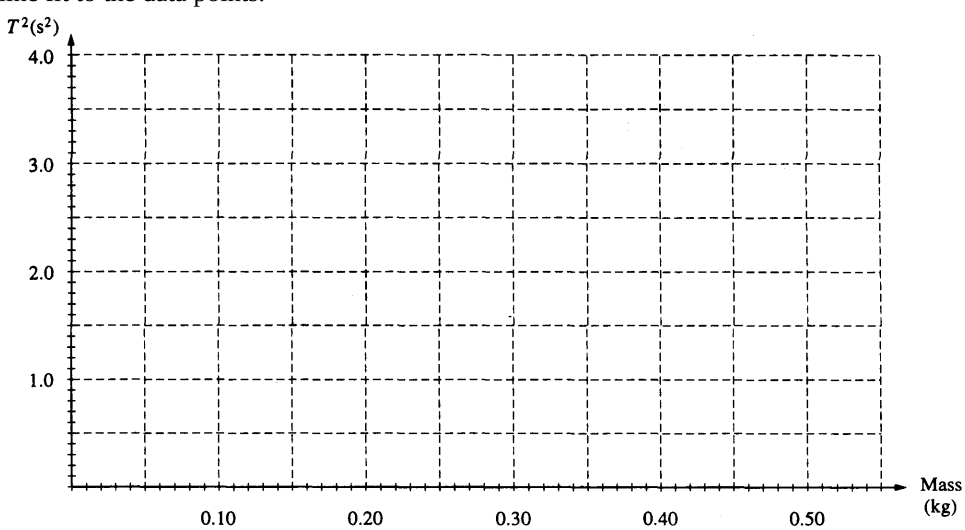


1996M1. A thin, flexible metal plate attached at one end to a platform, as shown above, can be used to measure mass. When the free end of the plate is pulled down and released, it vibrates in simple harmonic motion with a period that depends on the mass attached to the plate. To calibrate the force constant, objects of known mass are attached to the plate and the plate is vibrated, obtaining the data shown below.

a. Fill in the blanks in the data table.

Mass (kg)	Average Time for Ten Vibrations (s)	Period T (s)	T^2 (s^2)
0.10	8.86		
0.20	10.6		
0.30	13.5		
0.40	14.7		
0.50	17.7		

b. On the graph below, plot T^2 versus mass. Draw on the graph the line that is your estimate of the best straight-line fit to the data points.



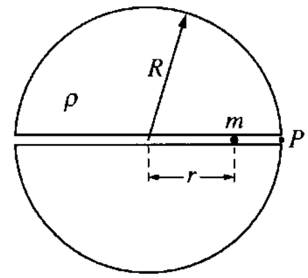
c. An object whose mass is not known is vibrated on the plate, and the average time for ten vibrations is measured to be 16.1 s. From your graph, determine the mass of the object. Write your answer with a reasonable number of significant digits.

d. Explain how one could determine the force constant of the metal plate.

e. Can this device be used to measure mass aboard the space shuttle Columbia as it orbits the Earth? Explain briefly.

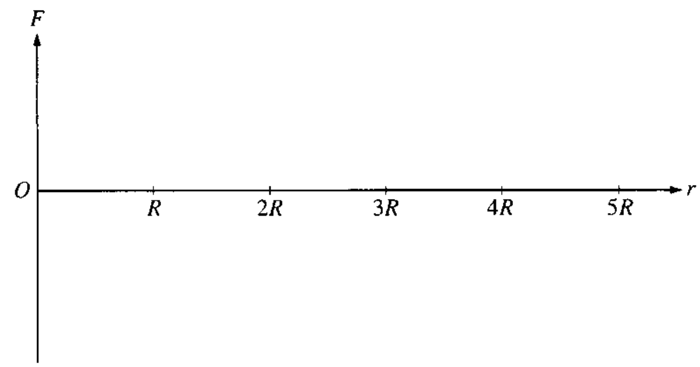
f. If Columbia is orbiting at 0.3×10^6 m above the Earth's surface, what is the acceleration of Columbia due to the Earth's gravity? (Radius of Earth = 6.4×10^6 m, mass of Earth = 6.0×10^{24} kg)

g. Since the answer to part (f) is not zero, briefly explain why objects aboard the orbiting Columbia seem weightless.



1999 M2 A spherical, nonrotating planet has a radius R and a uniform density ρ throughout its volume. Suppose a narrow tunnel were drilled through the planet along one of its diameters, as shown in the figure above, in which a small ball of mass m could move freely under the influence of gravity. Let r be the distance of the ball from the center of the planet.

a. Show that the magnitude of the force on the ball at a distance $r < R$ from the center of the planet is given by $F = -Cr$. where $C = 4\pi G\rho m/3$
b. On the axes below, sketch the force F on the ball as a function of distance r from the center of the planet.



The ball is dropped into the tunnel from rest at point P at the planet's surface.

c. Determine the work done by gravity as the ball moves from the surface to the center of the planet.
d. Determine the speed of the ball when it reaches the center of the planet.
e. Fully describe the subsequent motion of the ball from the time it reaches the center of the planet.
f. Write an equation that could be used to calculate the time it takes the ball to move from point P to the center of the planet. It is not necessary to solve this equation.

2001M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by

$$v = \sqrt{\frac{GM_J}{R}}$$

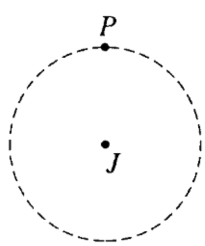
ii. The period of the orbit is given by

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

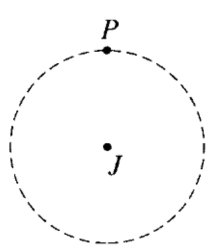
b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.



2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

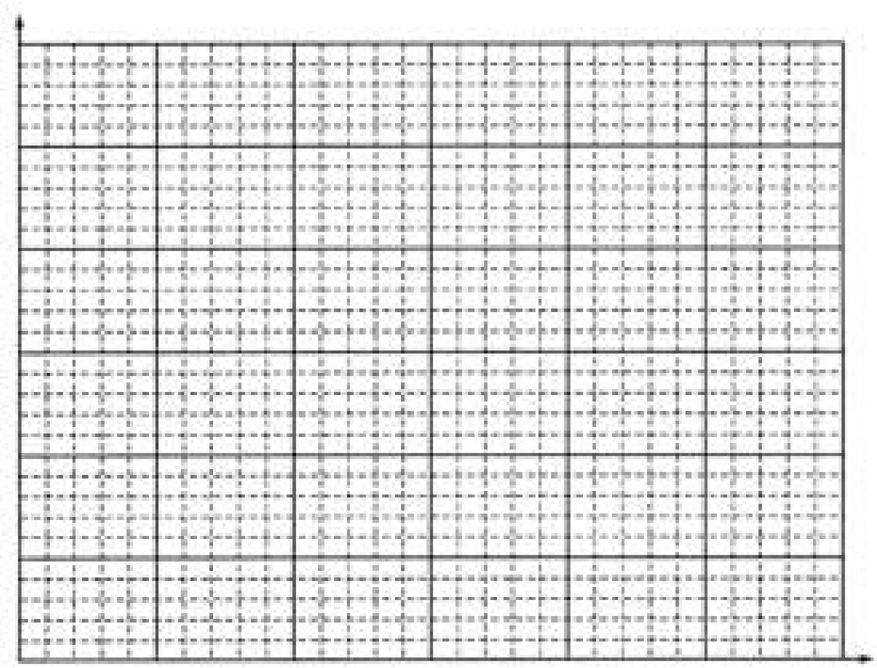
a. Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

2007M2. In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a. Calculate the radius of the GS orbit.
b. Calculate the mass of Mars.
c. Calculate the total mechanical energy of the GS in this orbit.
d. If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_________Greater than _________ Less than
Justify your answer.

e. In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

	Solution	Answer
1.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10	B
2.	Kepler's second law (Law of areas) is based on conservation of angular momentum, which remains constant. In order for angular momentum to remain constant, as the satellite approaches the sun, its speed increases.	D
3.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 4 = g × 4 and if the net effect is $g = g_{Earth}$ then r must be twice that of Earth.	C
4.	$a = g = \frac{GM}{r^2}$, if $R_2 = 2R_1$ then $a_2 = ¼\ a_1$	E
5.	$g = \frac{GM}{r^2}$. 300 km above the surface of the Earth is only a 5% increase in the distance (1.05 times the distance). This will produce only a small effect on g ($\div 1.05^2$)	D
6.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. If r is doubled, v decreases by $\sqrt{2}$	B
7.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and if the net effect is $g = g_{Earth}$ then r must be $\sqrt{2}$ times that of Earth	C
8.	$U = -\frac{Gm_1m_2}{r}$ and as the masses get closer, the potential energy becomes more negative. The slope of the graph is proportional to the force and as the masses get closer the force (and the slope) increases	D
9.	From conservation of angular momentum $v_1r_1 = v_2r_2$	A
10.	As the ball moves away, the force of gravity decreases due to the increasing distance.	A
11.	$g = \frac{GM}{r^2}$ At the top of its path, it has doubled its original distance from the center of the asteroid.	D
12.	Angular speed (in radians per second) is v/R. Since the satellite is not changing speed, there is no tangential acceleration and v^2/r is constant.	E
13.	The radius of each orbit is ½ D, while the distance between them is D. This gives $\frac{GMM}{D^2} = \frac{Mv^2}{D/2}$	B

14. An burst of the ships engine produces an increase in the satellite's energy. Now the satellite is moving at too large a speed for a circular orbit. The point at which the burst occurs must remain part of the ship's orbit, eliminating choices A and B. The Earth is no longer at the focus of the ellipse in choice E. D

15. The work is the area under the graph between the two points which the particle is traveling between E

16. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is $\frac{1}{2}mv^2 = \left|-\frac{GMm}{r}\right|$ which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$ B

17. Kepler's third law E

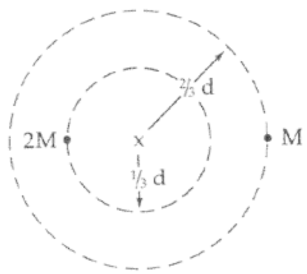
18. $K = \frac{GMm}{2r}$ (greater for closer satellites) A
L = mvr and since v_{orbit} is proportional to $r^{-1/2}$, L is proportional to $r^{1/2}$

1977M3

a. $F_g = F_c$ gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b. E = PE + KE = $-\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$

c.



d. $F_{g2} = F_{g1} = F_c$

$\frac{(2M){v_2}^2}{{}^1/_3d} = \frac{M{v_1}^2}{{}^2/_3d}$ gives v_2/v_1 = ½

1984M2

a. $F_g = F_c$ gives $\frac{GM_em}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives $(3m)v_0 - mv_0 = (4m)v'$ giving $v' = ½\ v_0$

c. E = PE + KE = $-\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_em}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_em}{4R_e}$

1992M3

a. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2$ = -8.1×10^9 J

b. L = mvr = 8.5×10^{13} kg-m²/s

c. Angular momentum is conserved so $mv_ar_a = mv_br_b$ giving $v_b = 2.4 \times 10^3$ m/s

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM/R}$ = 5.8×10^3 m/s

e. Escape occurs when E = PE + KE = 0 giving $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0\ and\ v_{esc} = \sqrt{2GM/R}$ = 8.2×10^3 m/s

1994M3

a.

$$U_a - U_b = -\int_b^a \vec{F} \cdot \overrightarrow{dr}$$

$$U(r) - U(\infty) = -\int_\infty^r -\frac{GM_e m}{r^2} dr$$

$$U(r) - 0 = -\frac{GM_e m}{r}\bigg|_\infty^r$$

b. $E = PE + KE = -\frac{GM_e m}{a} + \frac{1}{2}m{v_0}^2$

c. $L = mvr = mv_0 a$

d. Conservation of angular momentum gives $mv_0a = mv_bb$, or $v_b = v_0a/b$

e. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_e/a}$

f. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2}m\left(\frac{GM_e}{a} - {v_0}^2\right)$

1995M3

a. $v = \frac{2\pi r}{T}$ $and\ a = \frac{v^2}{r} = \frac{4\pi^2 r_a}{T^2}$

b. The centripetal force on star A is due to the gravitational force exerted by star B. $M_a a_a = \frac{GM_a M_b}{(r_a+r_b)^2}$ and substituting part (a) gives $M_b = \frac{4\pi^2 r_a (r_a+r_b)^2}{GT^2}$

c. The same calculations can be performed with the roles of star A and star B switched. $M_a = \frac{4\pi^2 r_b (r_a+r_b)^2}{GT^2}$

d. $I = \Sigma mr^2 = M_a r_a^2 + M_b r_b^2$

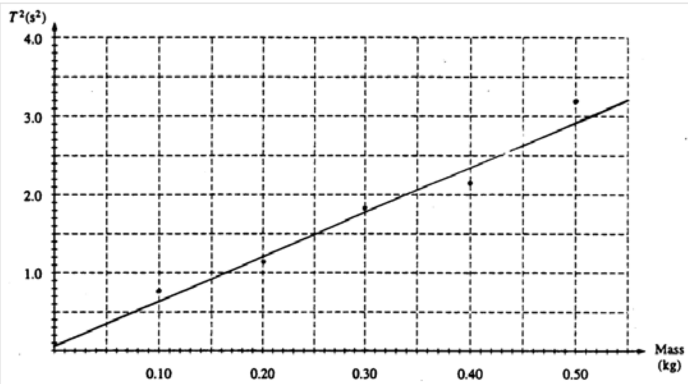
e. $L_{total} = M_a v_a r_a + M_b v_b r_b = M_a \frac{2\pi r_a}{T} r_a + M_b \frac{2\pi r_b}{T} r_b = \frac{2\pi}{T}(M_a {r_a}^2 + M_b {r_b}^2)$

1996M1

a.

Mass (kg)	Average Time for Ten Vibrations (s)	Period T (s)	T^2 (s^2)
0.10	8.86	.886	.785
0.20	10.6	1.06	1.12
0.30	13.5	1.35	1.82
0.40	14.7	1.47	2.16
0.50	17.7	1.77	3.13

b.



c. 0.45 kg

d. This is simple harmonic motion. Using the equation for the period of a mass on a spring and solving for the spring constant we get $k = 4\pi^2 m/T^2$ where m/T^2 is the inverse of the slope of the line of best fit.

e. Yes, it can be used in space as the period of oscillation is independent of gravity.

f. $F = GMm/r^2 = ma$ which gives $a = GM/r^2 = 8.9\ m/s^2$

g. All objects aboard the shuttle, and the shuttle itself, are all accelerating toward the Earth at the same rate (they are in free fall). The normal force is zero and there is no sensation of weight.

1999M2

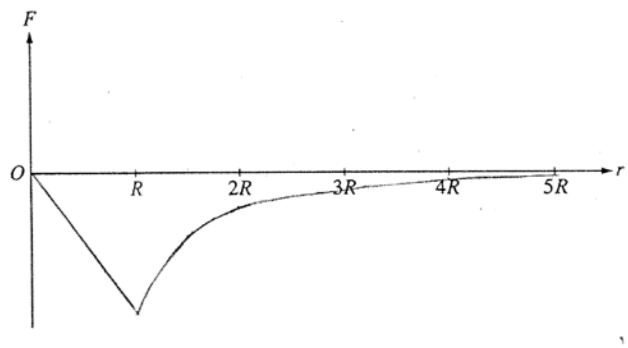
a.

$$F = -\frac{GM_{under}m}{r^2}$$

$$M_{under} = \rho V_{under} = \rho \frac{4}{3}\pi r^3$$

$$F = -\frac{G\left(\rho \frac{4}{3}\pi r^3\right)m}{r^2} = -\frac{4\pi G\rho m}{3} r$$

b.



c.

$$W = \int F dr = \int -Cr\, dr$$

$$W = \int_R^0 -Cr\, dr = \frac{CR^2}{2}$$

d. W = ΔK
$CR^2/2 = ½\ mv^2$
$v = (CR^2/m)^{1/2}$

e. The ball will continue through the center of the planet and travel to the surface, where it will stop and return through the center and continue oscillating in this manner.

f. F = ma = –Cr
$m(d^2r/dt^2) = –Cr$
$d^2r/dt^2 + (C/m)r = 0$ (simple harmonic motion)

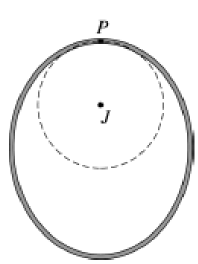
2001M2

a. i. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_J/R}$

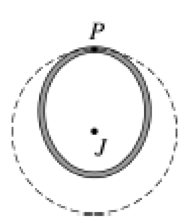
ii. v = d/T = 2πR/T giving $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM_J/R}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$

b. Plugging numerical values into a.ii. above gives R = 1.59 × 10^8 m

c. i.



ii.



2005M2

a. $F = \frac{GM_S m}{R^2}$

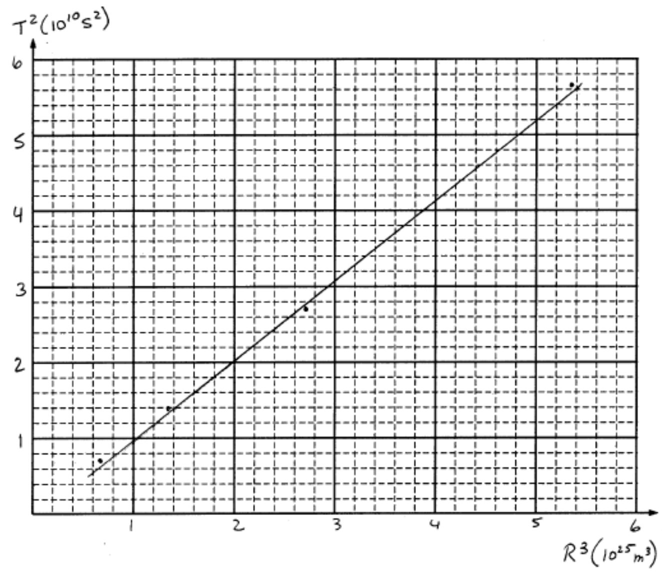
b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

e.



f. From part c. we have an expression for the slope of the line.
Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg

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2007M2

a. $v = 2\pi R/T$ gives $R = 3.83 \times 10^6$ m

b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ M = \frac{v^2 R}{G}$ = 6.64×10^{23} kg

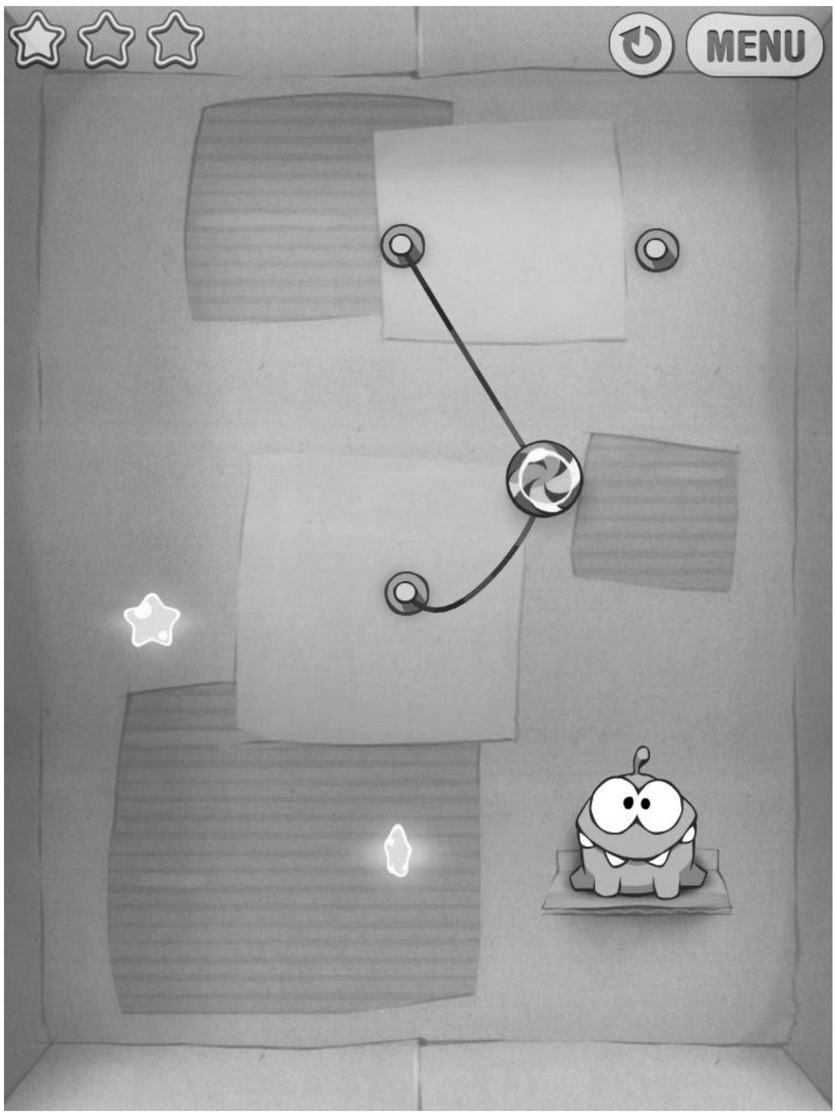
c. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2$ = -5.38×10^9 J

d. From Kepler's third law r^3/T^2 = constant so if r decreases, then T must also.

e. Conservation of angular momentum gives $mv_1r_1 = mv_2r_2$ so $v_2 = r_1v_1/r_2$, but the distances *above the surface* are given so the radius of Mars must be added to the given distances before plugging them in for each r. This gives $v_2 = 3.34 \times 10^3$ m/s.

Chapter 7

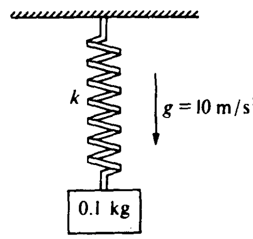
Oscillations



1. A simple pendulum of length *l*. whose bob has mass m, oscillates with a period T. If the bob is replaced by one of mass 4m, the period of oscillation is

   (A) $\frac{1}{4}$ T (B) $\frac{1}{2}$ T (C) T (D) 2T (E)4T

2. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
   (A) The kinetic and potential energies are equal at all times.
   (B) The kinetic and potential energies are both constant.
   (C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
   (D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
   (E) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position.



Questions 3-4

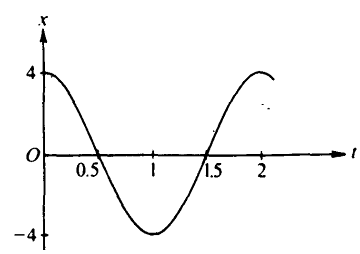
A 0. l -kilogram block is attached to an initially unstretched spring of force constant k = 40 newtons per meter as shown above. The block is released from rest at time t = 0.

3. What is the amplitude of the resulting simple harmonic motion of the block?

   (A) $\frac{1}{40}$m (B) $\frac{1}{20}$m (C) $\frac{1}{4}$m (D) $\frac{1}{2}$m (E) 1 m

4. At what time after release will the block first return to its initial position?

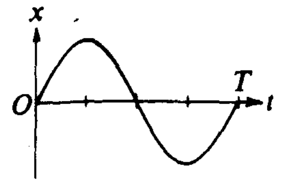
   (A) $\frac{\pi}{40}$s (B) $\frac{\pi}{20}$s (C) $\frac{\pi}{10}$s (D) $\frac{\pi}{5}$s (E) $\frac{\pi}{4}$s



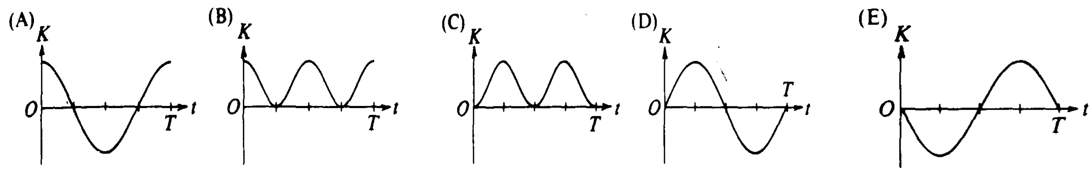
5. A particle moves in simple harmonic motion represented by the graph above. Which of the following represents the velocity of the particle as a function of time?
   (A) v(t) = 4 cos πt (B) v(t)= π cosπt
   (C) v(t) = $-\pi^2$ cos πt (D) v(t) = –4 sin πt (E) v(t) = –4π sin πt

6. A ball is dropped from a height of 10 meters onto a hard surface so that the collision at the surface may be assumed elastic. Under such conditions the motion of the ball is
   (A) simple harmonic with a period of about 1.4 s
   (B) simple harmonic with a period of about 2.8 s
   (C) simple harmonic with an amplitude of 5 m
   (D) periodic with a period of about 2.8 s but not simple harmonic
   (E) motion with constant momentum

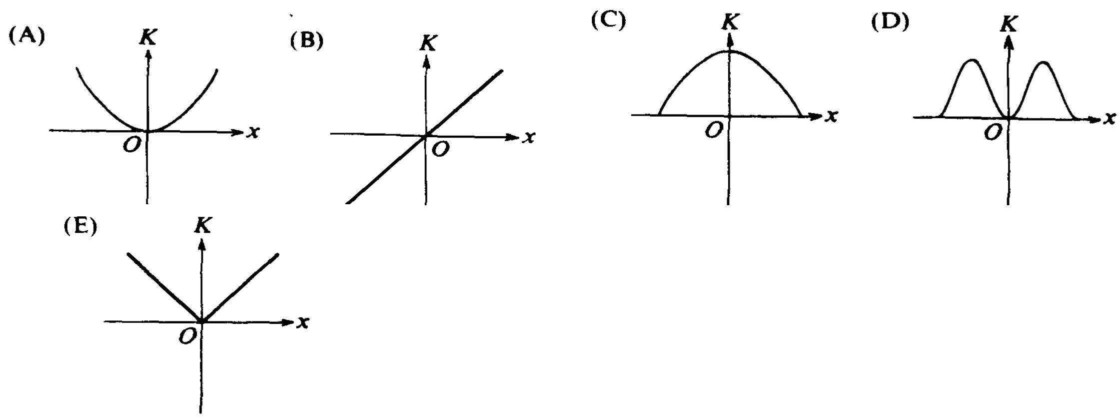
Questions 7-8 refer to the graph below of the displacement x versus time t for a particle in simple harmonic motion.



7. Which of the following graphs shows the kinetic energy K of the particle as a function of time t for one cycle of motion?



8. Which of the following graphs shows the kinetic energy K of the particle as a function of its displacement x ?



Questions 9-10

A particle moves in a circle in such a way that the x and y-coordinates of its motion are given in meters as functions of time t in seconds by:

$$x = 5\cos(3t) \qquad y = 5\sin(3t)$$

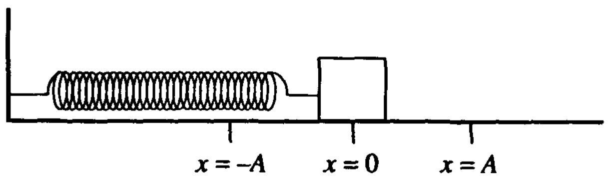
9. What is the period of revolution of the particle?
(A) 1/3 s (B) 3 s (C) $2\pi/3$ s (D) $3\pi/2$ s (E) 6π s

10. Which of the following is true of the speed of the particle?
(A) It is always equal to 5 m/s.
(B) It is always equal to 15 m/s.
(C) It oscillates between 0 and 5 m/s.
(D) It oscillates between 0 and 15 m/s.
(E) It oscillates between 5 and 15 m/s.

11. When a mass *m* is hung on a certain ideal spring, the spring stretches a distance *d*. If the mass is then set oscillating on the spring, the period of oscillation is proportional to

(A) $\sqrt{\frac{d}{g}}$ (B) $\sqrt{\frac{g}{d}}$ (C) $\sqrt{\frac{d}{mg}}$ (D) $\sqrt{\frac{m^2 g}{d}}$ (E) $\sqrt{\frac{m}{g}}$

12. Two objects of equal mass hang from independent springs of unequal spring constant and oscillate up and down. The spring of greater spring constant must have the
   (A) smaller amplitude of oscillation
   (B) larger amplitude of oscillation
   (C) shorter period of oscillation
   (D) longer period of oscillation
   (E) lower frequency of oscillation



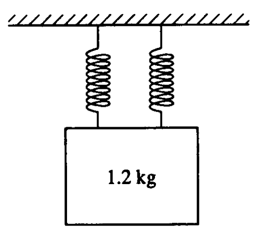
<u>Questions 13-14</u>

A block on a horizontal frictionless plane is attached to a spring, as shown above. The block oscillates along the x-axis with simple harmonic motion of amplitude A.

13. Which of the following statements about the block is correct?
   (A) At x = 0, its velocity is zero.
   (B) At x = 0, its acceleration is at a maximum.
   (C) At x = A, its displacement is at a maximum.
   (D) At x = A, its velocity is at a maximum.
   (E) At x = A, its acceleration is zero.

14. Which of the following statements about energy is correct?
   (A) The potential energy of the spring is at a minimum at x = 0.
   (B) The potential energy of the spring is at a minimum at x = A.
   (C) The kinetic energy of the block is at a minimum at x =0.
   (D) The kinetic energy of the block is at a maximum at x = A.
   (E) The kinetic energy of the block is always equal to the potential energy of the spring.

<u>Questions 15-16</u>



Two identical massless springs are hung from a horizontal support. A block of mass 1.2 kilograms is suspended from the pair of springs, as shown above. When the block is in equilibrium, each spring is stretched an additional 0.15 meter.

15. The force constant of each spring is most nearly
   (A) 40 N/m (B) 48 N/m (C) 60 N/m (D) 80 N/m (E) 96 N/m

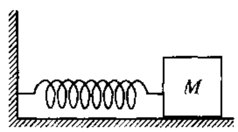
16. When the block is set into oscillation with amplitude A, it passes through its equilibrium point with a speed v. In which of the following cases will the block, when oscillating with amplitude A, also have speed v when it passes through its equilibrium point?
   I. The block is hung from only one of the two springs.
   II. The block is hung from the same two springs, but the springs are connected in series rather than in parallel.
   III. A 0.5 kilogram mass is attached to the block.
   (A) None (B) III only (C) I and II only (D) II and III only (E) I, II, and III

17. A simple pendulum consists of a l.0-kilogram brass bob on a string about 1.0 meter long. It has a period of 2.0 seconds. The pendulum would have a period of 1.0 second if the
(A) string were replaced by one about 0.25 meter long
(B) string were replaced by one about 2.0 meters long
(C) bob were replaced by a 0.25-kg brass sphere
(D) bob were replaced by a 4.0-kg brass sphere
(E) amplitude of the motion were increased

18. The equation of motion of a simple harmonic oscillator is $d^2x/dt^2 = -9x$, where x is displacement and t is time. The period of oscillation is
(A) 6π (B) $9/2\pi$ (C) $3/2\pi$ (D) $2\pi/3$ (E) $2\pi/9$

19. A pendulum with a period of 1 s on Earth, where the acceleration due to gravity is g, is taken to another planet, where its period is 2 s. The acceleration due to gravity on the other planet is most nearly
(A) g/4 (B) g/2 (C) g (D) 2g (E) 4g

20. A particle moves in the xy-plane with coordinates given by
$x = A \cos\omega t$ and $y = A \sin \omega t$,
where A = 1.5 meters and ω = 2.0 radians per second. What is the magnitude of the particle's acceleration?
(A) Zero (B) 1.3 m/s^2 (C) 3.0 m/s^2 (D) 4.5 m/s^2 (E) 6.0 m/s^2



21. An ideal massless spring is fixed to the wall at one end, as shown above. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is
(A) $\frac{Mg}{A}$ (B) $\frac{Mgv_m}{2A}$ (C) $\frac{Mv_m^2}{2A}$ (D) $\frac{Mv_m^2}{A^2}$ (E) $\frac{Mv_m^2}{2A^2}$

Questions 22-23
A simple pendulum has a period of 2 s for small amplitude oscillations.

22. The length of the pendulum is most nearly
(A) 1/6 m (B) 1/4 m (C) 1/2 m (D) 1 m (E) 2 m

23. Which of the following equations could represent the angle θ that the pendulum makes with the vertical as a function of time t ?
(A) $\theta = \theta_{max} \sin \frac{\pi}{2} t$ (B) $\theta = \theta_{max} \sin \pi t$ (C) $\theta = \theta_{max}, \sin 2\pi t$ (D) $\theta = \theta_{max} \sin 4\pi t$ (E) $\theta = \theta_{max} \sin 8\pi t$

24. A mass *M* suspended by a spring with force constant *k* has a period *T* when set into oscillation on Earth. Its period on Mars, whose mass is about 1/9 and radius 1/2 that of Earth, is most nearly
(A) 1/3 *T* (B) 2/3 *T* (C) *T* (D) 3/2 *T* (E) 3 *T*

25. A 1.0 kg mass is attached to the end of a vertical ideal spring with a force constant of 400 N/m. The mass is set in simple harmonic motion with an amplitude of 10 cm. The speed of the 1.0 kg mass at the equilibrium position is
(A) 2 m/s (B) 4 m/s (C) 20 m/s (D) 40 m/s (E) 200 m/s

Questions 26-27

A 2 kg mass connected to a spring oscillates on a horizontal, frictionless surface with simple harmonic motion of amplitude 0.4 m. The spring constant is 50 N/m.

26. The period of this motion is
(A) 0.04π s (B) 0.08π s (C) 0.4π s (D) 0.8π s (E) 1.26π s

27. The maximum velocity occurs where the
(A) potential energy is a maximum
(B) kinetic energy is a minimum
(C) displacement from equilibrium is equal to the amplitude of 0.4 meter
(D) displacement from equilibrium is half the amplitude
(E) displacement from equilibrium is equal to zero

Questions 28-30

The following pairs of equations show how the *x*- and *y*-coordinates of a particle vary with time *t*. In the equations, *A*, *B*, and ω are nonzero constants. Choose the pair of equations that best answers each of the following questions. A choice may be used once, more than once, or not at all.

(A) $x = A\cos\omega t$, $y = A\sin\omega t$

(B) $x = A\cos\omega t$, $y = 2A\sin\omega t$

(C) $x = At$, $y = Bt$

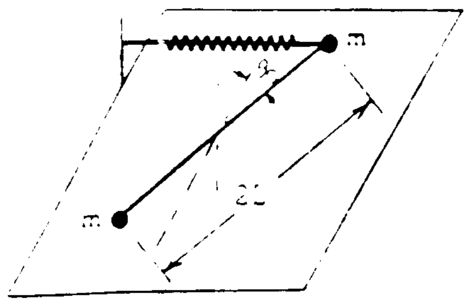
(D) $x = At^2$, $y = Bt^2$

(E) $x = At$, $y = Bt^2$

28. Which pair of equations can describe the path of a particle moving with zero acceleration?

29. Which pair of equations can describe the path of a particle moving with an acceleration that is perpendicular to the velocity of the particle at $t = 0$ and remains constant in magnitude and direction?

30. Which pair of equations can describe the path of a particle that moves with a constant speed and with a nonzero acceleration that is constant in magnitude?



Note: the diagram shows a total length of 2L and an angle of θ from the vertical

1978M3. A stick of length 2L and negligible mass has a point mass m affixed to each end. The stick is arranged so that it pivots in a horizontal plane about a frictionless vertical axis through its center. A spring of force constant k is connected to one of the masses as shown above. The system is in equilibrium when the spring and stick are perpendicular. The stick is displaced through a small angle θ_o as shown and then released from rest at t = 0 .

a. Determine the restoring torque when the stick is displaced from equilibrium through the small angle θ_o.
b. Determine the magnitude of the angular acceleration of the stick just after it has been released.
c. Write the differential equation whose solution gives the behavior of the system after it has been released.
d. Write the expression for the angular displacement θ of the stick as a function of time t after it has been released from rest.

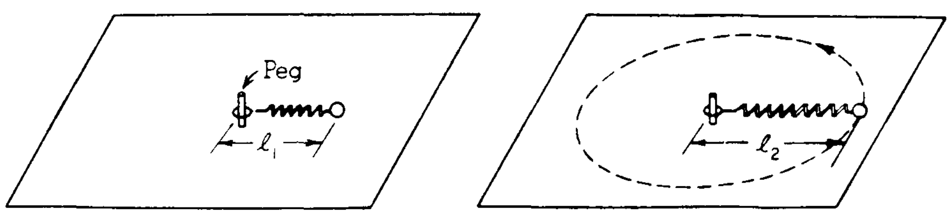
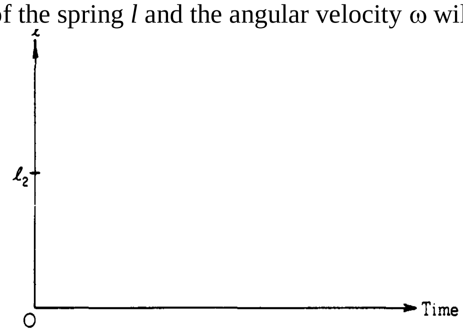
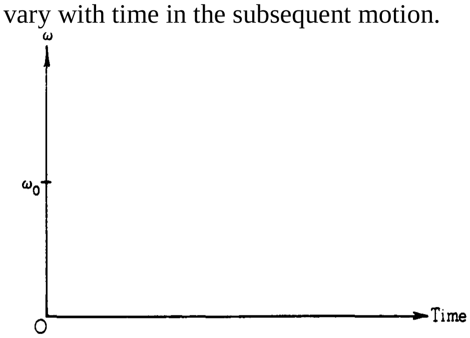


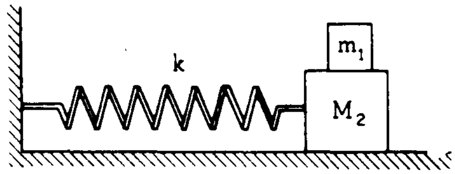
Figure 1 Figure 2

1979M3. A mass m constrained to move on a frictionless horizontal surface is attached to a frictionlpeg by a massless spring having force constant k. The unstretched length of the spring is l_1, as shown in Figure 1. When the mass moves in a circle about the peg with constant angular velocity ω_o. the length of the spring is l_2 as shown in Figure 2. Express your answers to parts a, b, and c in terms of m, k, ω_o and l_1.

a. Determine the length l_2
b. Assume the total energy of the system in Figure 1 is zero. Determine the total energy of the rotating system in Figure 2.
c. Determine the magnitude of the angular momentum of the system.
d. While the mass is rotating about the peg with angular velocity ω_o, it is struck by a hammer that provides a small impulse directed inward. On the axes below, sketch graphs to indicate qualitatively the manner in which the length of the spring l and the angular velocity ω will vary with time in the subsequent motion.







1980M1. A small mass m_1 rests on but is not attached to a large mass M_2 that slides on its base without friction. The maximum frictional force between m_1 and M_2 is f. A spring of spring constant k is attached to the large mass M_2 and to the wall as shown above.

a. Determine the maximum horizontal acceleration that M_2 may have without causing m_1 to slip.

b. Determine the maximum amplitude A for simple harmonic motion of the two *masses if* they are to move together, i.e., m_1 must not slip on M_2.

c. The two-mass combination is pulled to the right the maximum amplitude A found in part b. and released. Describe the frictional force on the small mass m_1 during the first half cycle of oscillation.

d. The two-mass combination is now pulled to the right a distance of A' greater than A and released.
   i. Determine the acceleration of m_1 at the instant the masses are released.
   ii. Determine the acceleration of M_2 at the instant the masses are released.

1986M3. A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the *cube of the displacement;* i.e., $F = -kx^3$

This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

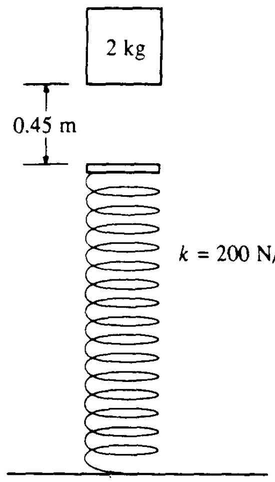
a. The potential energy in the spring at the instant the mass is released

b. The maximum speed of the mass

c. The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

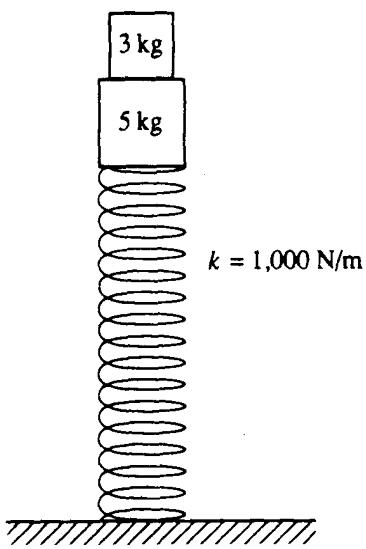
The amplitude of the oscillation is now increased:

d. State whether the period of the oscillation increases, decreases, or remains the same. Justify your answer.



1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the period of the simple harmonic motion that ensues.
c. Determine the distance that the spring is compressed at the instant the speed of the block is maximum.
d. Determine the maximum compression of the spring.
e. Determine the amplitude of the simple harmonic motion.



1990M3. A 5-kilogram block is fastened to a vertical spring that has a spring constant of 1,000 newtons per meter. A 3-kilogram block rests on top of the 5-kilogram block, as shown above.

a. When the blocks are at rest, how much is the spring compressed from its original length?

The blocks are now pushed down and released so that they oscillate.

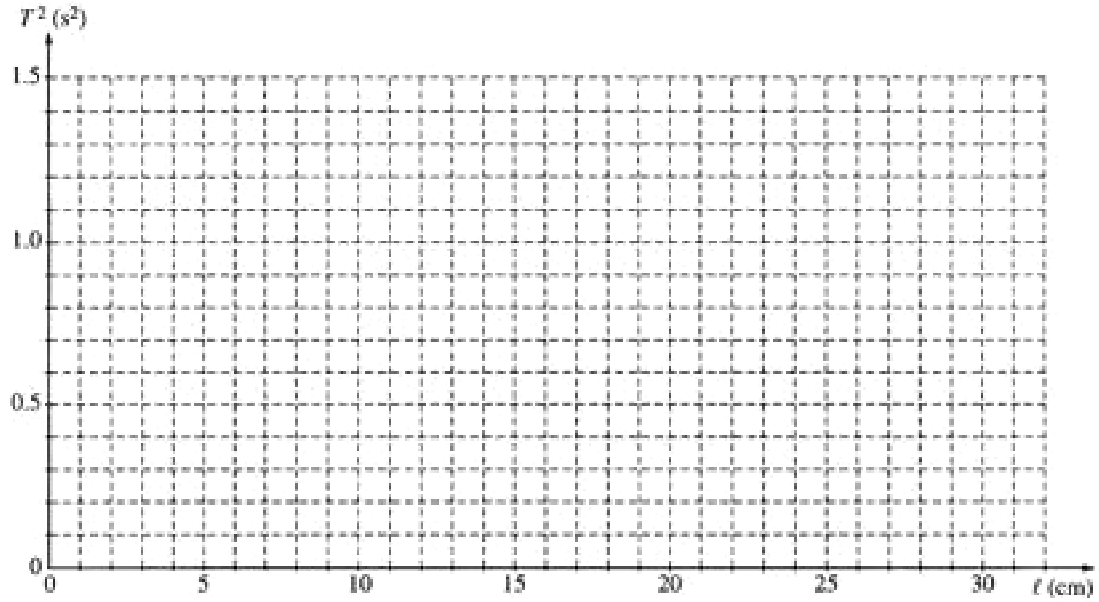
b. Determine the frequency of this oscillation.
c. Determine the magnitude of the maximum acceleration that the blocks can attain and still remain in contact at all times.
d. How far can the spring be compressed beyond the compression in part a. without causing the blocks to exceed the acceleration value in part c?
e. Determine the maximum speed of the blocks if the spring is compressed the distance found in part d.

2000M1. You are conducting an experiment to measure the acceleration due to gravity g_u at an unknown location. In the measurement apparatus, a simple pendulum swings past a photogate located at the pendulum's lowest point, which records the time t_{10} for the pendulum to undergo 10 full oscillations. The pendulum consists of a sphere of mass m at the end of a string and has a length l. There are four versions of this apparatus, each with a different length. All four are at the unknown location, and the data shown below are sent to you during the experiment.

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62		
18	8.89		
32	12.08		

a. For each pendulum, calculate the period T and the square of the period. Use a reasonable number of significant figures. Enter these results in the table above.

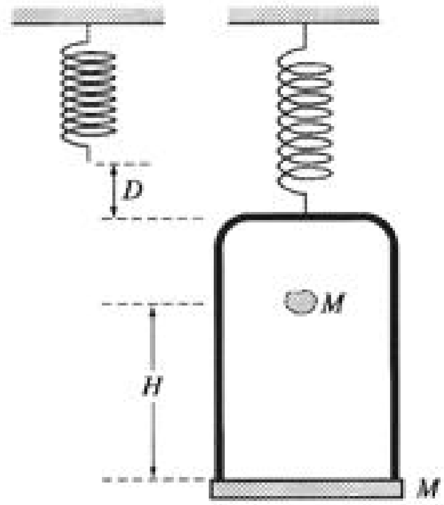
b. On the axes below, plot the square of the period versus the length of the pendulum. Draw a best-fit straight line for this data.



c. Assuming that each pendulum undergoes small amplitude oscillations, from your fit determine the experimental value g_{exp} of the acceleration due to gravity at this unknown location. Justify your answer.

d. If the measurement apparatus allows a determination of g_u that is accurate to within 4%, is your experimental value in agreement with the value 9.80 m/s^2? Justify your answer.

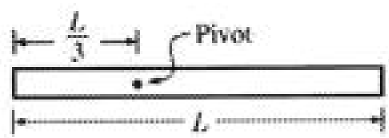
e. Someone informs you that the experimental apparatus is in fact near Earth's surface, but that the experiment has been conducted inside an elevator with a constant acceleration a. Assuming that your experimental value g is exact, determine the magnitude and direction of the elevator's acceleration.



2003M2. An ideal spring is hung from the ceiling and a pan of mass M is suspended from the end of the spring, stretching it a distance D as shown above. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the speed of the clay at the instant it hits the pan.
b. Determine the speed of the pan just after the clay strikes it.
c. Determine the period of the simple harmonic motion that ensues.
d. Determine the distance the spring is stretched (from its initial unstretched length) at the moment the speed of the pan is a maximum. Justify your answer.
e. The clay is now removed from the pan and the pan is returned to equilibrium at the end of the spring. A rubber ball, also of mass M, is dropped from the same height H onto the pan, and after the collision is caught in midair before hitting anything else.

   Indicate below whether the period of the resulting simple harmonic motion of the pan is greater than, less than, or the same as it was in part c.
   _____Greater than _____Less than _____The same as
   Justify your answer.



2004M3. A uniform rod of mass M and length L is attached to a pivot of negligible friction as shown above. The pivot is located at a distance $L/3$ from the left end of the rod. Express all answers in terms of the given quantities and fundamental constants.

a. Calculate the rotational inertia of the rod about the pivot.
   The rod is then released from rest from the horizontal position shown above.
b. Calculate the linear speed of the bottom end of the rod when the rod passes through the vertical.

The rod is brought to rest in the vertical position shown above and hangs freely. It is then displaced slightly from this position by a small angle θ. Write the differential equation that governs the motion of the rod as it swings.

c. Calculate the period of oscillation as it swings, assuming that the angle of oscillation is small.

2009M1. A 3.0 kg object is moving along the x-axis in a region where its potential energy as a function of x is given as $U(x) = 4.0x^2$, where U is in joules and x is in meters. When the object passes the point $x = -0.50$ m, its velocity is +2.0 m/s. All forces acting on the object are conservative.

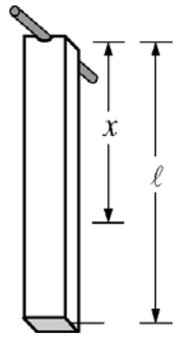
a. Calculate the total mechanical energy of the object.

b. Calculate the x-coordinate of any points at which the object has zero kinetic energy.

c. Calculate the magnitude of the momentum of the object at $x = 0.60$ m.

d. Calculate the magnitude of the acceleration of the object as it passes $x = 0.60$ m.

e. On the axes below, sketch graphs of the object's position x versus time t and kinetic energy K versus time t. Assume that $x = 0$ at time $t = 0$. The two graphs should cover the same time interval and use the same scale on the horizontal axes.

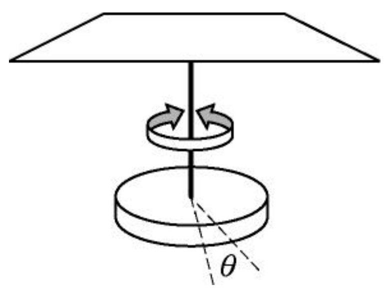


2009M2. You are given a long, thin, rectangular bar of known mass M and length ℓ with a pivot attached to one end. The bar has a nonuniform mass density, and the center of mass is located a known distance x from the end with the pivot. You are to determine the rotational inertia I_b of the bar about the pivot by suspending the bar from the pivot, as shown above, and allowing it to swing. Express all algebraic answers in terms of I_b, the given quantities, and fundamental constants.

a.
   i. By applying the appropriate equation of motion to the bar, write the differential equation for the angle θ the bar makes with the vertical.
   ii. By applying the small-angle approximation to your differential equation, calculate the period of the bar's motion.

b. Describe the experimental procedure you would use to make the additional measurements needed to determine I_b . Include how you would use your measurements to obtain I_b and how you would minimize experimental error.

c. Now suppose that you were not given the location of the center of mass of the bar. Describe an experimental procedure that you could use to determine it, including the equipment that you would need.



2011M3. The torsion pendulum shown above consists of a disk of rotational inertia I suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle θ, the twisted rod exerts a restoring torque τ that is proportional to the angular displacement: $\tau = -\beta\theta$, where β is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.

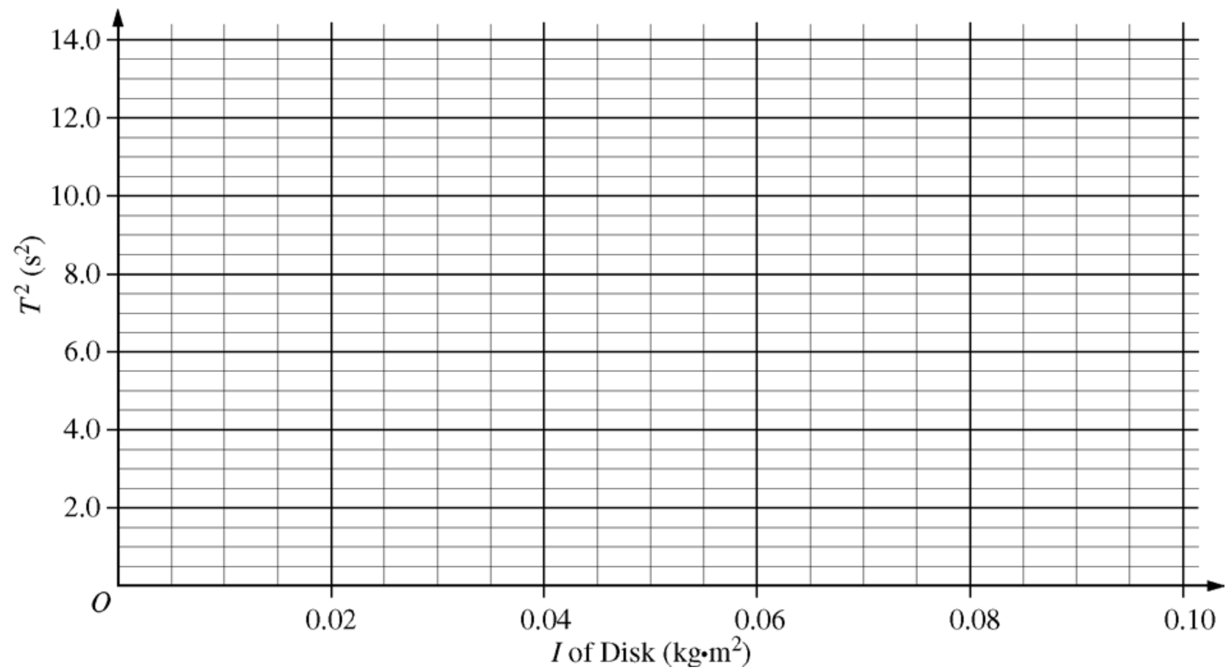
a. In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement θ of the torsion pendulum as a function of time t.

b. Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

To determine the torsion constant β of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

Rotational Inertia I of Disk $(kg{\bullet}m^2)$	Average Time for Ten Oscillations (s)	Period T (s)	T^2 (s^2)
0.025	22.4	2.24	5.0
0.036	26.8	2.68	7.2
0.049	29.5	2.95	8.7
0.064	33.3	3.33	11.1
0.081	35.9	3.59	12.9

c. On the graph below, plot the data points. Draw a straight line that best represents the data.



d. Determine the equation for your line.

e. Calculate the torsion constant β of the rod from your line.

f. What is the physical significance of the intercept of your line with the vertical axis?

	Solution	Answer
1.	Mass does not affect the period of a pendulum	C
2.	Energy is conserved here and switches between kinetic and potential which have maximums at different locations	D
3.	At the current location all of the energy is gravitational potential. As the spring stretches to its max location all of that gravitational potential will become spring potential when it reaches its lowest position. When the box oscillates back up it will return to its original location converting all of its energy back to gravitational potential and will oscillate back and forth between these two positions. As such the maximum stretch bottom location represents twice the amplitude so simply halving that max Δx will give the amplitude. Finding the max stretch: → The initial height of the box h and the stretch Δx have the same value ($h=\Delta x$) $U = U_{sp}$ $mg(\Delta x_1) = ½ k\Delta x_1^2$ $mg = ½ k \Delta x_1$ $\Delta x_1 = .05$ m. This is 2A, so the amplitude is 0.025 m or 1/40 m. Alternatively, we could simply find the equilibrium position measured from the initial top position based on the forces at equilibrium, and this equilibrium stretch measured from the top will be the amplitude directly. To do this: $F_{net} = 0$ $F_{sp} = mg$ $k\Delta x_2 = mg$ $\Delta x_2 = 0.025$ m, which is the amplitude	A
4.	Plug into period for mass-spring system $T = 2\pi \sqrt{(m/k)}$	C
5.	From the graph, the amplitude of the oscillation is 4 (meters, one presumes) and the period is 2 (seconds?). That means $\omega = 2\pi/T = \pi$, and so the position function is given by $x = 4 \cos \pi t$ The velocity is given by the derivative of this function; $v = d(4 \cos \pi t)=dt = -4\pi \sin \pi t$	E
6.	Based on free fall, the time to fall down would be 1.4 seconds. Since the collision with the ground is elastic, all of the energy will be returned to the ball and it will rise back up to its initial height completing 1 cycle in a total time of 2.8 seconds. It will continue doing this oscillating up and down. However, this is not simple harmonic because to be simple harmonic the force should vary directly proportional to the displacement but that is not the case in this situation	D
7.	Energy will never be negative. The max kinetic occurs at zero displacement and the kinetic energy become zero when at the maximum displacement	B
8.	Same reasoning as above, it must be C	C
9.	in the form $x = A$ sin/cos ωt, for this function, $\omega = 3$ and $T = 2\pi/\omega = 2\pi/3$	C
10.	$v_x = dx/dt = -15 \sin(3t)$ and $v_y = dy/dt = 15 \cos(3t)$. Speed is the magnitude of $(v_x^2 + v_y^2)^{1/2}$ = 15 m/s (constant)	B
11.	First use the initial stretch to find the spring constant. $F_{sp} = mg = k\Delta x$ $k = mg / d$ Then plug that into $T = 2\pi \sqrt{(m/k)}$ $T = 2\pi\sqrt{\frac{m}{(\frac{mg}{d})}}$	A
12.	Based on $T = 2\pi \sqrt{(m/k)}$ the larger spring constant makes a smaller period	C
13.	Basic fact about SHM. Amplitude is max displacement	C
14.	Basic fact about SHM. Spring potential energy is a min at x=0 with no spring stretch	A

15.	With two springs acting upward $mg = 2F_s$, or $F_s = (12\text{ N})/2 = 6\text{ N}$ on each spring. Also, $F_s = kx$, so $k = (6\text{ N})/(0.15\text{ m})$	A
16.	The 'effective' spring constant for the two-spring configuration is $k' = 2k$ which makes the speed at equilibrium $v = A\times\sqrt{(k'/m)}$ From one spring only, A will be twice as large, and k will be halved, this is not the same speed In series, $k' = k/2$ and A will be four times as large, not the same speed Changing m only will change the speed	A
17.	Based on $T = 2\pi\sqrt{(L/g)}$, ¼ the length equates to ½ the period	A
18.	Equations of the form $d^2x/dt^2 + Cx = 0$ are simple harmonic with $\omega = C^{1/2}$ For this equation $\omega = 3$ and $T = 2\pi/\omega$	D
19.	Based on $T = 2\pi\sqrt{(L/g)}$, ¼ g would double the period	A
20.	In circular motion, $a = \omega^2 r = (2\text{ rad/s})^2(1.5\text{ m})$	E
21.	Using energy conservation. $U_{sp} = K$ $\frac{1}{2}kA^2 = \frac{1}{2}mv_m^2$ solve for k	D
22.	Plug into $T = 2\pi\sqrt{(L/g)}$	D
23.	With a period of 2 s, $\omega = 2\pi/T = \pi$ so the equation must have the form $\sin \pi t$	B
24.	Based on $g = GM_p / R^2$, g of mars is 4/9 that of earth. Then based on $T = 2\pi\sqrt{(L/g)}$, with g changing to "4/9 g" gives a period changing by $\sqrt{9/4}$ or 3/2 T	D
25.	Using energy conservation. $U_{sp} = K$ $\frac{1}{2}kA^2 = \frac{1}{2}mv_m^2$ solve for v	A
26.	$T = 2\pi\sqrt{(m/k)}$	C
27.	Maximum velocity occurs at equilibrium (x = 0), where potential energy ($\frac{1}{2}kx^2$) is zero.	E
28.	For zero acceleration, velocity must be constant, which means position is a linear function of time.	C
29.	An example of an acceleration constant in magnitude and direction is the acceleration due to gravity. For this acceleration to be perpendicular to the velocity means the particle is moving horizontally at t = 0. (E) satisfies these conditions since v_x = constant and $v_y(0) = 0$	E
30.	Constant speed while accelerating is circular motion. (A) represents equations of motion for an object moving in a circle	A

1978M3

a. $F = -kx$ where for small angles $x = L\theta$ ($x = L\sin\theta$ is also acceptable)
$F = -kL\theta$
$\tau = FL\cos\theta \approx FL = -kL^2\theta$

b. $\tau = I\alpha$ where $I = 2mL^2$
$\alpha = \tau/I = -k\theta/2m$

c. $\alpha = d^2\theta/dt^2$
$d^2\theta/dt^2 = -k\theta/2m$

d. The equation above is the form of simple harmonic motion with an angular frequency $\omega = (k/2m)^{1/2}$
$\theta = \theta_0 \cos (k/2m)^{1/2}t$

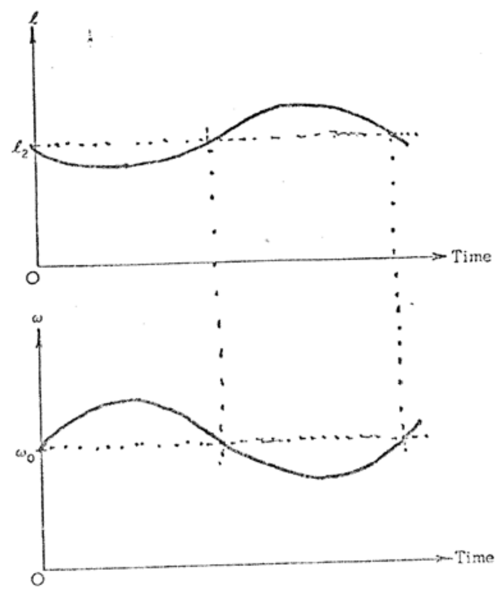
1979M3

a. $\Sigma F = ma$
$kx = m\omega^2 r$
$k(\ell_2 - \ell_1) = m\omega_0^2\ell_2$
$\ell_2 = k\ell_1/(k - m\omega_0^2)$

b. $E = U + K = ½ k(\ell_2 - \ell_1)^2 + ½ mv^2$
$= ½ k(\ell_2 - \ell_1)^2 + ½ m(\ell_2\omega_0)^2 = ½ mk\ell_1^2\omega_0^2(k + m\omega_0^2)/(k - m\omega_0^2)^2$

c. $L = I\omega = m\ell_2^2\omega_0 = m\omega_0(k\ell_1/(k - m\omega_0^2))$

d.



1980M1

a. For the blocks not to slip $a_1 = a_2$
   $a_1 = f/m_1 = a_2$

b. $-kx = ma$
   applied to the "no slip" condition $f = kA = (m_1 + M_2)a = (m_1 + M_2)(f/m_1)$
   $A = (m_1 + M_2)f/m_1k$

c. The frictional force on mass m_1 is proportional to the sinusoidal acceleration of the masses as they oscillate.

d. i. $f = m_1a_1$; $a_1 = f/m_1$
   ii. $F_s - f = M_2a_2$; $a_2 = (F_s - f)/M_2 = (kA' - f)/M_2$
   note: A′ may also be interpreted as the additional distance beyond A from the wording of the question

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1986M3

a through c is also in the work-energy chapter

a. $\Delta U = -\int F dx$
   $$U = -\int_0^A -kx^3 dx = \frac{kA^4}{4}$$

b. $\frac{1}{2} Mv_{max}^2 = \frac{1}{4} kA^4$
   $v_{max} = A^2(k/2M)^{1/2}$

c. $E_{total} = K + U$; When $K = U$ then $E_{tot} = U + U = 2U$
   $U = \frac{1}{2} E_{tot}$
   $\frac{1}{4} (kx^4) = \frac{1}{2} (kA^4/4)$
   $x = A(2^{-1/4})$

d. The period decreases. For a spring that obeys Hooke's law, F is proportional to x, and the period is independent of amplitude. For this problem, F is proportional to x^3 so the force, and hence the acceleration increase at a greater rate.

1989M3

a. Conservation of energy: $mgh = \frac{1}{2} mv^2$ gives $v = (2gh)^{1/2} = 3$ m/s
b. $T = 2\pi/\omega = 2\pi(m/k)^{1/2} = 0.63$ s
c. v is a maximum when $\Sigma F = 0$ or $ky = mg$, which gives $y = mg/k = 0.1$ m
d. Conservation of energy: $U_g = U_s$; $mg(y + 0.45 \text{ m}) = \frac{1}{2} ky^2$ gives $y = 0.41$ m
e. The amplitude is the distance from equilibrium (answer to c.) to the maximum compression (answer to d.)
   $A = (0.41 \text{ m} - 0.1 \text{ m}) = 0.31$ m

1990M3

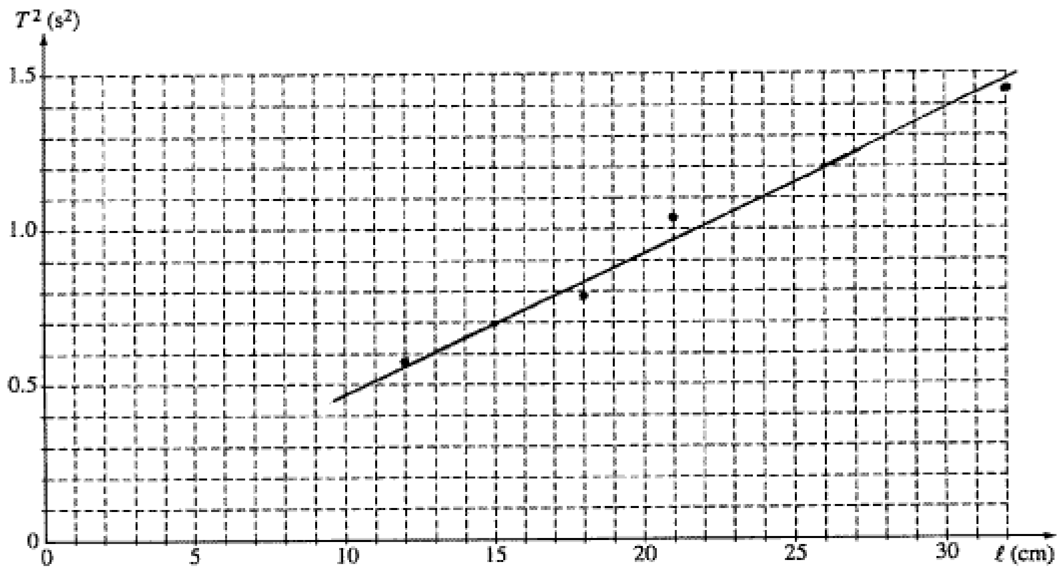
a. $F = kx$
   $x = F/k = mg/k = 0.078$ m
b. $f = (1/2\pi)(k/m)^{1/2} = 1.8$ Hz
c. If the spring pulls the 5 kg block downward such that $a > g$ then the 5 kg block will move faster than the 3 kg block can fall and they will lose contact, therefore $a_{max} = g = 9.8 \text{ m/s}^2$
d. Maximum acceleration occurs at the extremes of motion, when $x = A$, that is, $a_{max} = kA/m$
   $A = ma_{max}/k = 0.078$ m
e. $v_{max} = A(k/m)^{1/2} = 0.87$ m/s

2000M1

a.

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62	0.762	0.581
18	8.89	0.889	0.790
21	10.09	1.009	1.018
32	12.08	1.208	1.459

b.



c. The slope is $\Delta(T^2)/\Delta\ell$
$T = 2\pi(\ell/g)^{1/2}$ which gives $T^2/\ell = 4\pi^2/g$ so $g = 4\pi^2$/slope
From the above graph $g_{exp} = 8.77\ m/s^2$

d. No, it is not in agreement. The percent difference is $(9.8 - 8.77)/9.8 = .117$ or 11.7%

e. $\Sigma F = ma$; $F_N - mg = ma$ where $F_N = mg_{exp}$ which gives $a = g_{exp} - g = -1.03\ m/s^2$ (down)

2003M2

a. $U = K$; $MgH = ½\ Mv_c^2$; $v_c = (2gH)^{1/2}$

b. conservation of momentum
$Mv_c = 2Mv_p$; $v_p = ½\ (2gH)^{1/2}$

c. $T = 2\pi(m/k)^{1/2}$ where k is found from $Mg = kD$; $k = Mg/D$
$T = 2\pi(2D/g)^{1/2}$

d. v is maximized at the equilibrium point, which is where $\Sigma F = 0$, or $kx = 2Mg$
$(Mg/D)x = 2Mg$
$x = 2D$

e. Less. There is less mass oscillating.

2004M3

a.

$$I = \int r^2 dm$$

$$dm = \frac{M}{L} dr$$

$$I = \int_{-\frac{L}{3}}^{\frac{2L}{3}} \frac{M}{L} r^2 dr$$

$$I = \frac{M}{L} \frac{r^3}{3} \Big|_{-L/3}^{2L/3} = \frac{ML^2}{9}$$

b. Conservation of energy
$U = K$
$Mgh_{cm} = ½ I\omega^2$
$Mg(L/6) = ½ (ML^2/9)(v/r)^2$ where $r = 2L/3$
solving gives $v = 2(gL/3)^{1/2}$

c. $T = 2\pi(I/mgd)^{1/2}$ where $d = L/6$
$T = 2\pi((ML^2/9)/(MgL/6))^{1/2} = 2\pi(2L/3g)^{1/2}$

2009M1

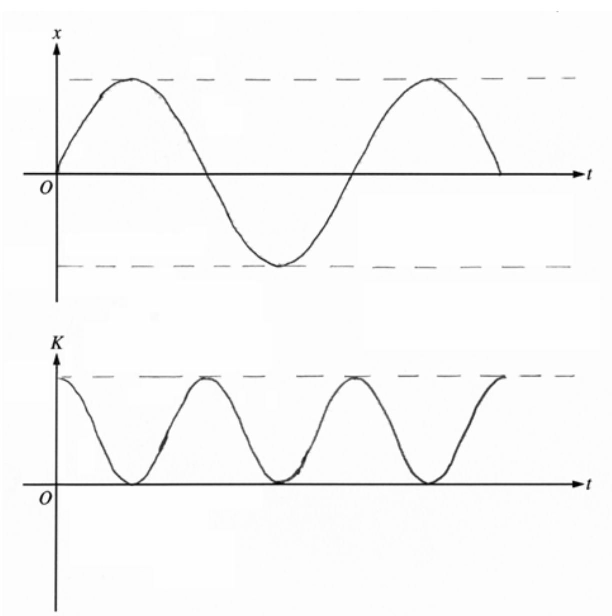
a. $E = U(x) + K(x) = 4x^2 + ½ m(v(x))^2$
$= (4.0\ J/m^2)(–0.5\ m)^2 + ½\ (3\ kg)(2\ m/s)^2 = 7\ J$

b. $E = U$ when $K = 0$
$E = U(x)$
$7\ J = (4\ J/m^2)\ x^2$ which gives $x = \pm 1.3\ m$

c. $K = E – U = 7\ J – (4\ J/m^2)(0.6\ m)^2 = 5.56\ J = ½\ mv^2$
$v = 1.92\ m/s$
$p = mv = 5.8$ kg-m/s (or $p = (2mK)^{1/2}$)

d. $F = –dU/dx = –(d/dx)(4x^2) = –8x$
$a = F/m = –8x/m$
$|a| = (8\ kg/s^2)(0.6\ m)/(3\ kg) = 1.6\ m/s^2$

e.



2009M2

a. i. $\tau = I\alpha$

$-Mgx \sin \theta = I_b \alpha$

$-Mgx \sin \theta = I_b(d^2\theta/dt^2)$

ii. $(d^2\theta/dt^2) + (Mgx/I_b)\theta = 0$

In this form, the angular frequency of the simple harmonic motion is $\omega^2 = Mgx/I_b$

$T = 2\pi/\omega = 2\pi(I_b/Mgx)^{1/2}$

b. Example:

Displace the bar by a small angle and release it to oscillate. To reduce errors, time 10 complete oscillations with a stopwatch. Calculate the average value of the time for 10 oscillations and then divide by 10 to determine the period *T*. Calculate I_b from $T = 2\pi(I_b/Mgx)^{1/2}$, using known values of *M* and *x*.

c. Example:

Place the bar on top of a fulcrum, e.g., the top of a prism. Adjust the position of the bar until it is balanced horizontally. The point at which this occurs is the center of mass.

2011M3

a. $\Sigma\tau = I\alpha$

$I\alpha = -\beta\theta$

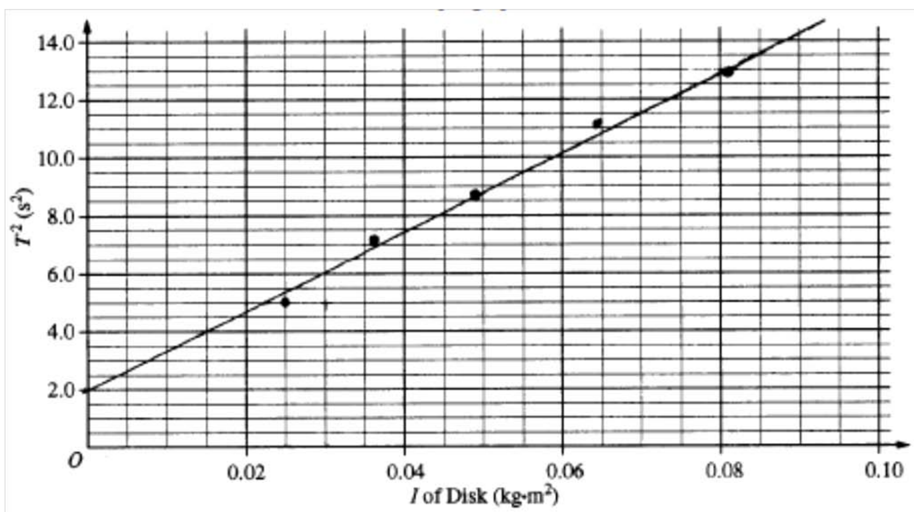
$I(d^2\theta/dt^2) = -\beta\theta$

b. where $m(d^2x/dt^2) = -kx$ has angular frequency $\omega = (k/m)^{1/2}$

we can relate that to $I(d^2\theta/dt^2) = -\beta\theta$ where its angular frequency should be $(\beta/I)^{1/2}$

The period $T = 2\pi/\omega = 2\pi(I/\beta)^{1/2}$

c.



d. The general equation for a line is $y(x) = mx + b$, we can use $T^2 = mI + b$

The slope is found from $m = \Delta(T^2)/\Delta I$ and using points on the line gives $m = 135$ s^2/kg-m^2

The y intercept is $b = 2$ s^2

This gives the equation for the line as $T^2 = (135$ s^2/kg-m$^2)I + 2.0$ s

e. Using $T = 2\pi(I/\beta)^{1/2}$

$T^2 = (4\pi^2/\beta)I$ gives us the slope $m = 4\pi^2/\beta$

$\beta = 4\pi^2/m = 0.292$ kg-m^2/s^2

f. They- intercept represents the square of the period of oscillation of the flexible rod.