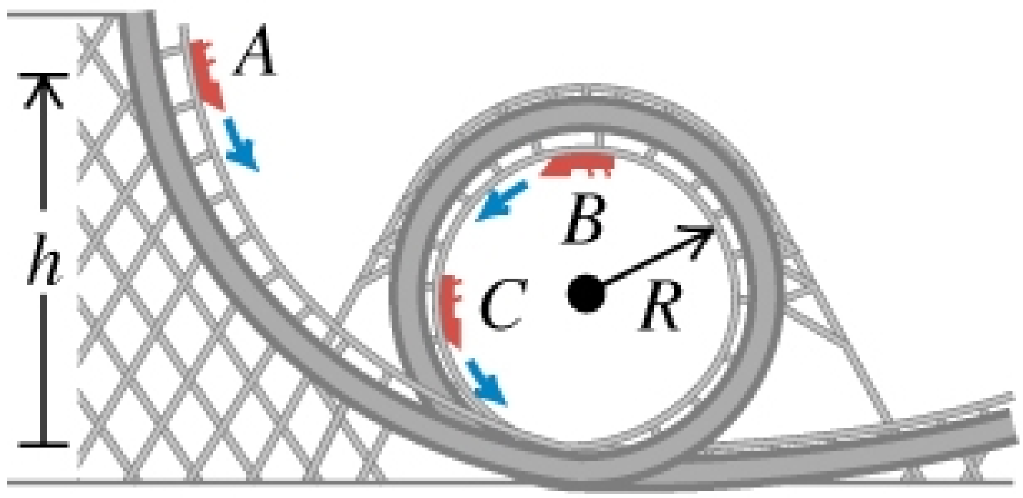
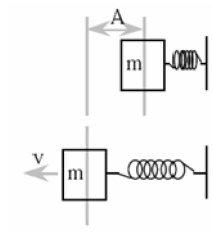
Chapter 4

Work, Power and Energy



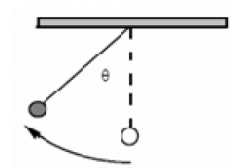
AP Physics Multiple Choice Practice – Work-Energy



1. A mass m attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the masses speed as it passes through its equilibrium position?
   (A) 0 (B) $A\sqrt{\frac{k}{m}}$ (C) $A\sqrt{\frac{m}{k}}$ (D) $\frac{1}{A}\sqrt{\frac{k}{m}}$

2. A force F at an angle θ above the horizontal is used to pull a heavy suitcase of weight mg a distance d along a level floor at constant velocity. The coefficient of friction between the floor and the suitcase is μ. The work done by the frictional force is:
   (A) –Fd cos θ (B) – μ Fd cos θ (C) -μmgd (D) -μ mgd cos θ

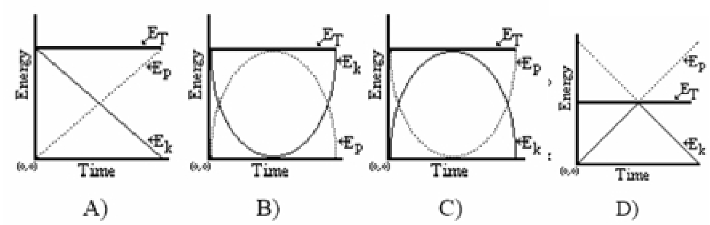
3. A 2 kg ball is attached to a 0.80 m string and whirled in a horizontal circle at a constant speed of 6 m/s. The work done on the ball during each revolution is:
   (A) 90 J (B) 72 J (D) 16 J (D) zero



4. A pendulum bob of mass m on a cord of length L is pulled sideways until the cord makes an angle θ with the vertical as shown in the figure to the right. The change in potential energy of the bob during the displacement is:
   (A) mgL (1–cos θ) (B) mgL (1–sin θ) (C) mgL sin θ
   (D) mgL cos θ

5. A softball player catches a ball of mass m, which is moving towards her with horizontal speed V. While bringing the ball to rest, her hand moved back a distance d. Assuming constant deceleration, the horizontal force exerted on the ball by the hand is
   (A) $mV^2/(2d)$ (B) mV^2/d (C) 2mV/d (D) mV/d

6. A pendulum is pulled to one side and released. It swings freely to the opposite side and stops. Which of the following might best represent graphs of kinetic energy (E_k), potential energy (E_p) and total mechanical energy (E_T)



Questions 7-8: A car of mass m slides across a patch of ice at a speed v with its brakes locked. It the hits dry pavement and skids to a stop in a distance d. The coefficient of kinetic friction between the tires and the dry road is μ.

7. If the car has a mass of 2m, it would have skidded a distance of
   (A) 0.5 d (B) d (C) 1.41 d (D) 2 d

8. If the car has a speed of 2v, it would have skidded a distance of
   (A) d (B) 1.41 d (C) 2 d (D) 4 d

9. A ball is thrown vertically upwards with a velocity v and an initial kinetic energy E_k. When half way to the top of its flight, it has a velocity and kinetic energy respectively of
   (A) $\frac{v}{2}, \frac{E_k}{2}$ (B) $\frac{v}{\sqrt{2}}, \frac{E_k}{2}$ (C) $\frac{v}{4}, \frac{E_k}{2}$ (D) $\frac{v}{2}, \frac{E_k}{\sqrt{2}}$

10. A football is kicked off the ground a distance of 50 yards downfield. Neglecting air resistance, which of the following statements would be INCORRECT when the football reaches the highest point?
(A) all of the balls original kinetic energy has been changed into potential energy
(B) the ball's horizontal velocity is the same as when it left the kickers foot
(C) the ball will have been in the air one-half of its total flight time
(D) the vertical component of the velocity is equal to zero

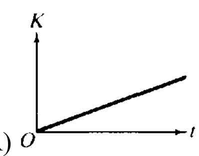
11. A mass m is attached to a spring with a spring constant k. If the mass is set into motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to equilibrium position?
(A) $v = \sqrt{\frac{kd}{m}}$ (B) $v = d\sqrt{\frac{k}{m}}$ (C) $v = \frac{kd}{mg}$ (D) $v^2 = \frac{mgd}{k}$

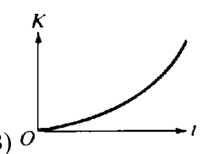
12. A fan blows the air and gives it kinetic energy. An hour after the fan has been turned off, what has happened to the kinetic energy of the air?
(A) it disappears (B) it turns into potential energy (C) it turns into thermal energy
(D) it turns into sound energy

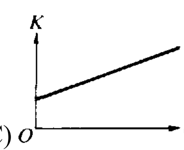
13. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) they strike the ground more than half a second apart.
(D) they strike the ground with the same kinetic energy.

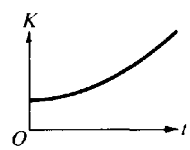
14. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(C) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
(D) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position

15. From the top of a high cliff, a ball is thrown horizontally with initial speed v_o. Which of the following graphs best represents the ball's kinetic energy K as a function of time t ?

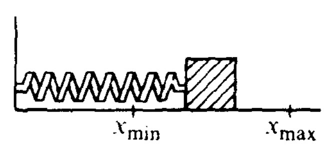
(A) 

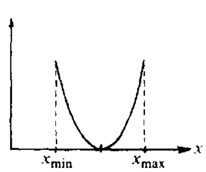
(B) 

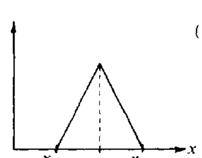
(C) 

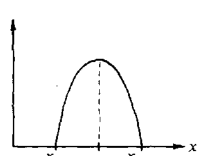
(D) 

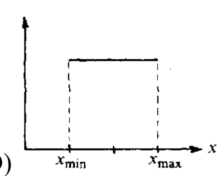
Questions 16-17: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



(A) 

(B) 

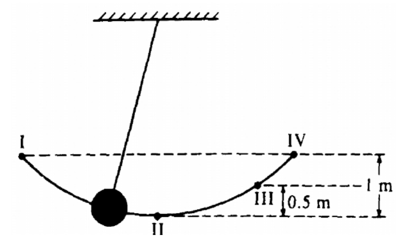
(C) 

(D) 

16. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C (D) D

17. Which graph can represent the kinetic energy of the block as a function of x ?
(A) A (B) B (C) C (D) D

Questions 18-19

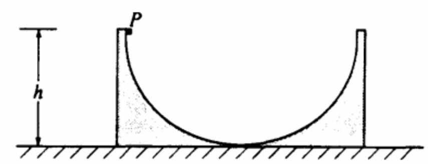


A ball swings freely back and forth in an arc from point I to point IV, as shown. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is I meter above II. Air resistance is negligible.

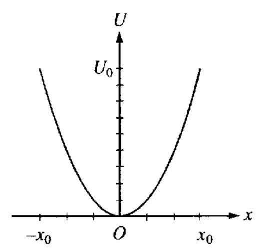
18. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?
(A) At point II (B) At some point between II and III
(C) At point III (D) At some point between III and IV

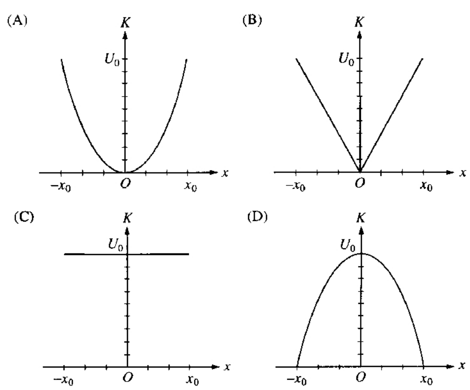
19. The speed of the ball at point II is most nearly
(A) 3.0 m/s (B) 4.5 m/s (C) 9.8 m/s (D) 14 m/s

20. The figure shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?
(A) It is equal to h/4 (B) It is equal to h/2
(C) It is equal to h
(D) It is between zero and h; the exact height depends on how much energy is lost to friction.



21. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?

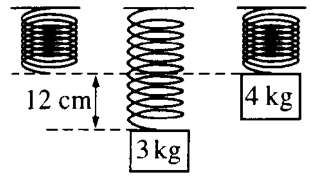




22. A child pushes horizontally on a box of mass m which moves with constant speed v across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

(A) μmgv (B) mgv (C) $\mu mg/v$ (D) $\mu mg/v$

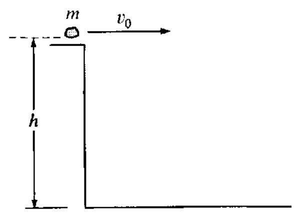
23. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?



(A) 18 cm (B) 24 cm
(C) 32 cm (D) 48 cm

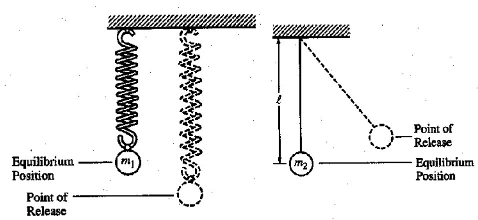
24. What is the kinetic energy of a satellite of mass m that orbits the Earth, of mass M, in a circular orbit of radius R?

(A) $\frac{1}{2}\frac{GMm}{R}$ (B) $\frac{1}{4}\frac{GMm}{R}$ (C) $\frac{1}{2}\frac{GMm}{R^2}$ (D) $\frac{GMm}{R^2}$



25. A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh (B) $\frac{1}{2} mv_0^2$ (C) $\frac{1}{2} mv_0^2 - mgh$ (D) $\frac{1}{2} mv_0^2 + mgh$



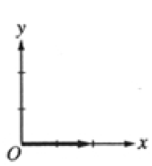
26. A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion Which of the following is true for both spheres?

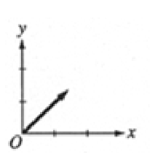
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position.
(B) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(C) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(D) The maximum total energy is attained only as the sphere passes through its equilibrium position.

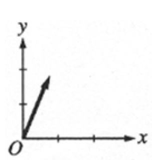
<u>Questions 27-28</u>

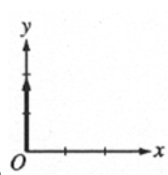
An object of mass m is initially at rest and free to move without friction in any direction in the xy-plane. A constant net force of magnitude F directed in the +x direction acts on the object for 1 s. Immediately thereafter a constant net force of the same magnitude F directed in the +y direction acts on the object for 1 s. After this, no forces act on the object.

27. Which of the following vectors could represent the velocity of the object at the end of 3 s, assuming the scales on the x and y axes are equal?

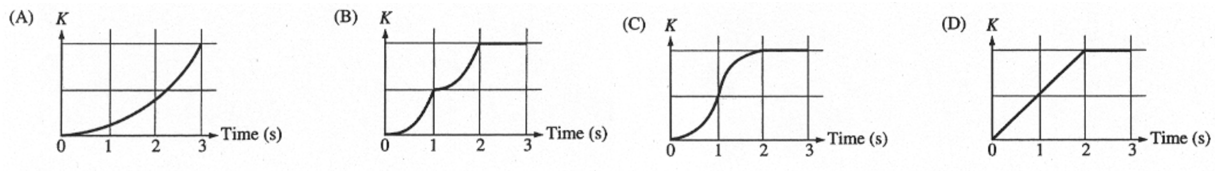
(A) 

(B) 

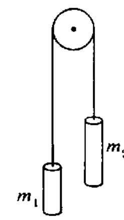
(C) 

(D) 

28. Which of the following graphs best represents the kinetic energy K of the object as a function of time?

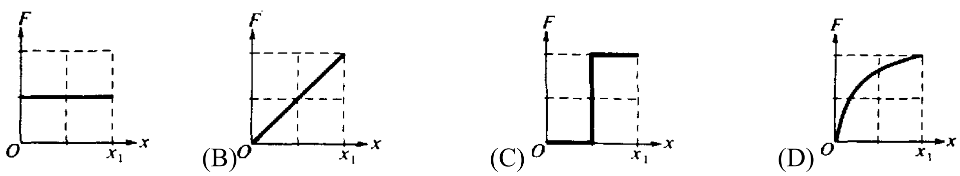


29. A system consists of two objects having masses m_1 and m_2 ($m_1 < m_2$). The objects are connected by a massless string, hung over a pulley as shown, and then released. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by

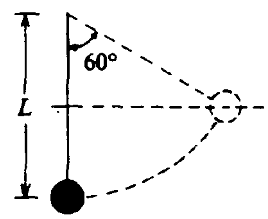


(A) $(m_2 - m_1)gh$ (B) m_2gh (C) $(m_1 + m_2)gh$ (D) $\frac{1}{2}(m_1 + m_2)gh$

30. The following graphs, all drawn to the same scale, represent the net force F as a function of displacement x for an object that moves along a straight line. Which graph represents the force that will cause the greatest change in the kinetic energy of the object from x = 0 to $x = x_1$?



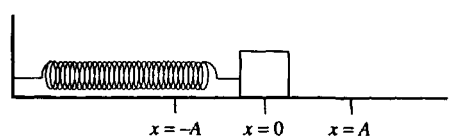
31. A pendulum consists of a ball of mass m suspended at the end of a massless cord of length L as shown. The pendulum is drawn aside through an angle of 60° with the vertical and released. At the low point of its swing, the speed of the pendulum ball is



(A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) ½gL (D) gL

32. A rock is lifted for a certain time by a force F that is greater in magnitude than the rock's weight W. The change in kinetic energy of the rock during this time is equal to the
(A) work done by the net force (F - W)
(B) work done by F alone
(C) work done by W alone
(D) difference in the potential energy of the rock before and after this time.

33. A block on a horizontal frictionless plane is attached to a spring, as shown. The block oscillates along the x-axis with amplitude A. Which of the following statements about energy is correct?

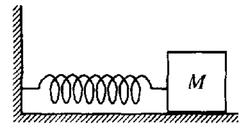


(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.

34. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

(A) h/4 (B) $\frac{h}{2\sqrt{2}}$ (C) h/2 (D) $\frac{h}{\sqrt{2}}$

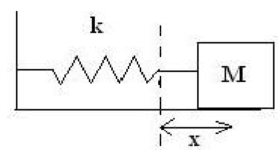
35. An ideal massless spring is fixed to the wall at one end, as shown. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is



(A) $\frac{Mgv_m}{2A}$ (B) $\frac{Mv_m^2}{2A}$ (C) $\frac{Mv_m^2}{A^2}$ (D) $\frac{Mv_m^2}{2A^2}$

36. A person pushes a block of mass M = 6.0 kg with a constant speed of 5.0 m/s straight up a flat surface inclined 30.0° above the horizontal. The coefficient of kinetic friction between the block and the surface is $\mu = 0.40$. What is the net force acting on the block?
(A) 0 N (B) 21 N (C) 30 N (D) 51 N

37. A block of mass M on a horizontal surface is connected to the end of a massless spring of spring constant k . The block is pulled a distance x from equilibrium and when released from rest, the block moves toward equilibrium. What coefficient of kinetic friction between the surface and the block would allow the block to return to equilibrium and stop?



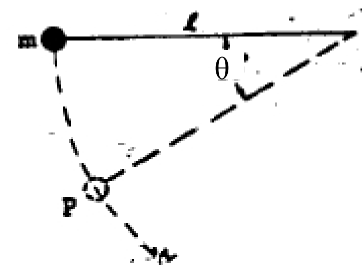
(A) $\frac{kx^2}{2Mg}$ (B) $\frac{kx}{Mg}$ (C) $\frac{kx}{2Mg}$ (D) $\frac{Mg}{2kx}$

38. An object is dropped from rest from a certain height. Air resistance is negligible. After falling a distance d, the object's kinetic energy is proportional to which of the following?
(A) $1 / d^2$ (B) $1 / d$ (C) $\sqrt{d}$ (D) d

39. An object is projected vertically upward from ground level. It rises to a maximum height H. If air resistance is negligible, which of the following must be true for the object when it is at a height H/2 ?
(A) Its speed is half of its initial speed.
(B) Its kinetic energy is half of its initial kinetic energy.
(C) Its potential energy is half of its initial potential energy.
(D) Its total mechanical energy is half of its initial value.

AP Physics Free Response Practice – Work Power Energy

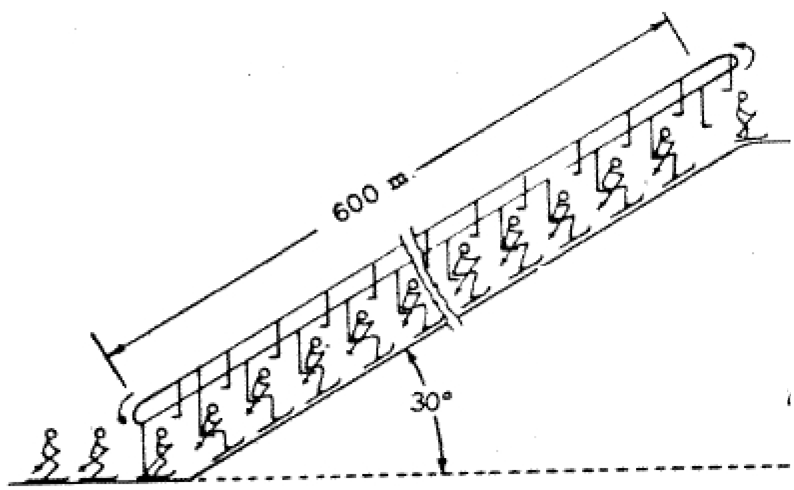
1974B1. A pendulum consisting of a small heavy ball of mass m at the end of a string of length L is released from a horizontal position. When the ball is at point P, the string forms an angle of θ with the horizontal as shown.

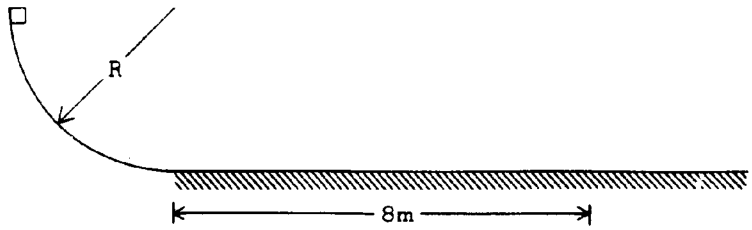


(a) In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

(b) Determine the speed of the ball at P.
(c) Determine the tension in the string when the ball is at P.

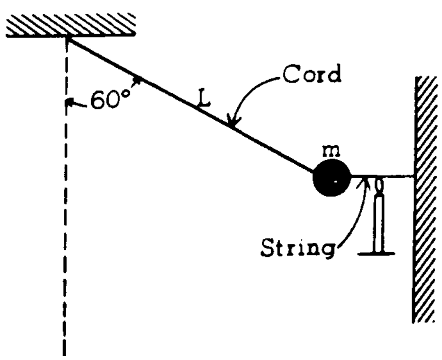
1974B7. A ski lift carries skiers along a 600 meter slope inclined at 30°. To lift a single rider, it is necessary to move 70 kg of mass to the top of the lift. Under maximum load conditions, six riders per minute arrive at the top. If 60 percent of the energy supplied by the motor goes to overcoming friction, what average power must the motor supply?





1975B1. A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume g = 10 meters per second^2.

a. Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.
b. How much time elapses while the block is sliding horizontally?
c. Calculate the radius of the incline in meters.



1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.
c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

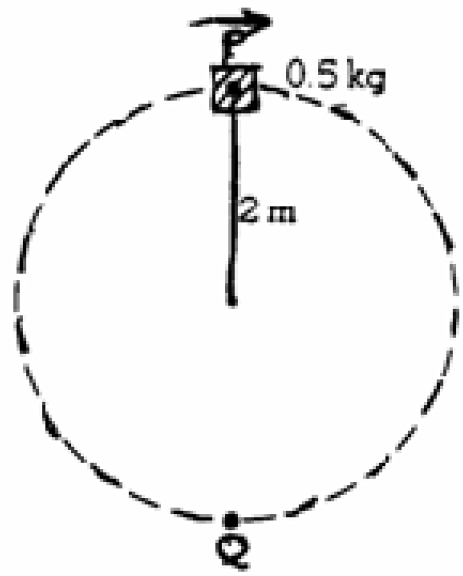
1977 B1. A block of mass 4 kilograms, which has an initial speed of 6 meters per second at time t = 0, slides on a horizontal surface.

a. Calculate the work W that must be done on the block to bring it to rest.

If a constant friction force of magnitude 8 newtons is exerted on the block by the surface, determine the following:

b. The speed v of the block as a function of the time t.

c. The distance x that the block slides as it comes to rest



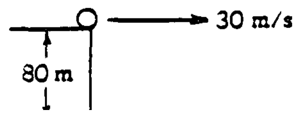
1978B1. A 0.5 kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume g = 10 meters per second squared.

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P.

(b) Calculate the speed of the object at point P.

(c) Calculate the increase in kinetic energy of the object as it moves from point P to point Q.

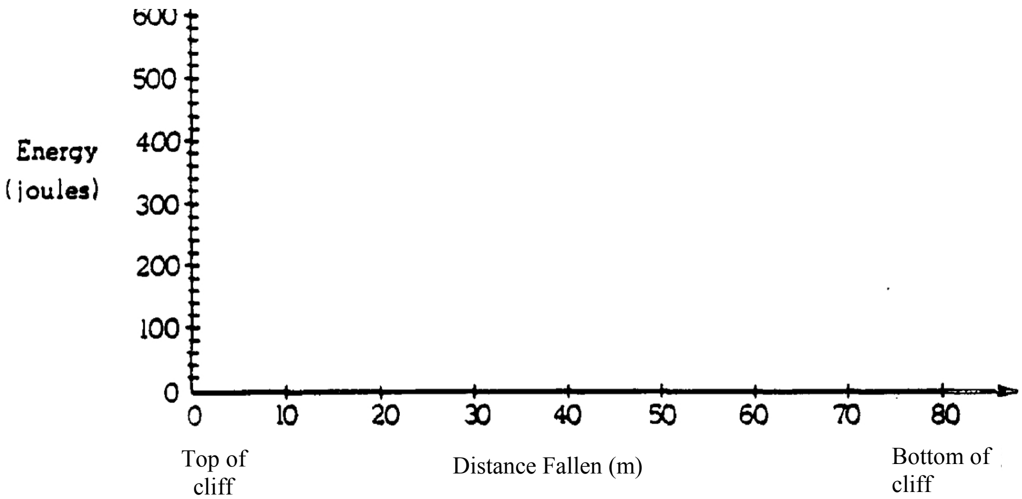
(d) Calculate the tension in the string as the object passes through point Q.



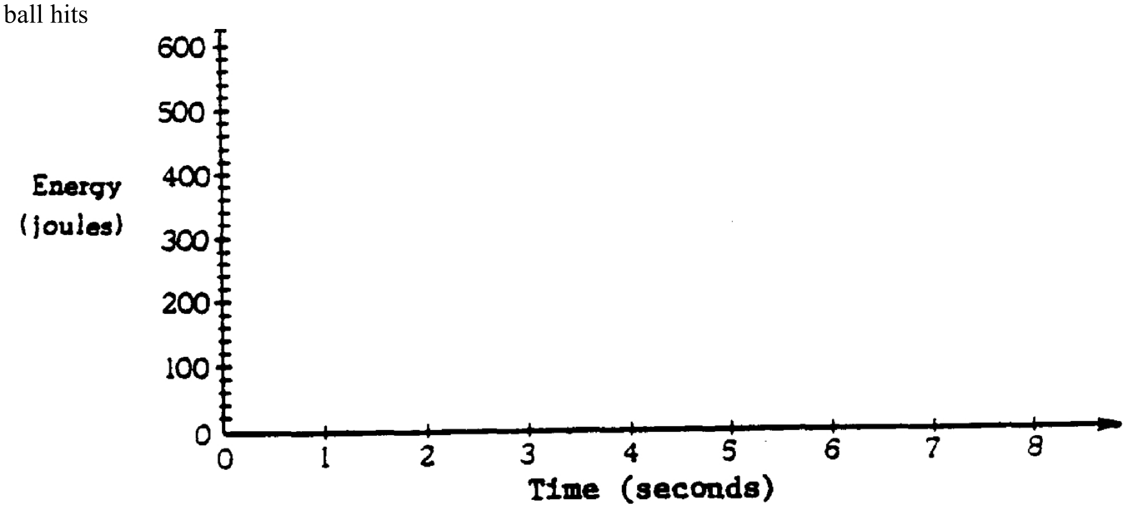
1979B1. From the top of a cliff 80 meters high, a ball of mass 0.4 kilogram is launched horizontally with a velocity of 30 meters per second at time t = 0 as shown above. The potential energy of the ball is zero at the bottom of the cliff. Use g = 10 meters per second squared.

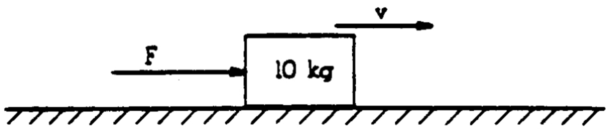
a. Calculate the potential, kinetic, and total energies of the ball at time t = 0.

b. On the axes below, sketch and label graphs of the potential, kinetic, and total energies of the ball as functions of the distance fallen from the top of the cliff



c. On the axes below sketch and label the kinetic and potential energies of the ball as functions of time until the ball hits



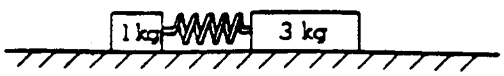


1981B1. A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time t = 0, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume g = 10 meters per second squared.

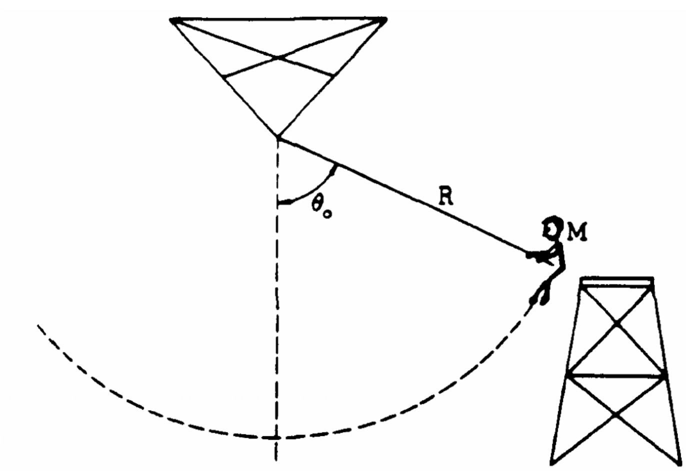
a. Calculate the force F necessary to keep the velocity constant.

The force is now changed to a larger constant value F'. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.

b. Calculate the force F'.
c. Calculate the acceleration of the block.

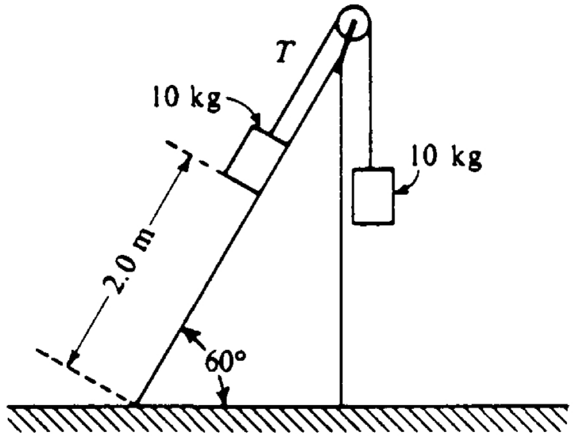


1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second. Determine the minimum work needed to compress the spring in this experiment.



1982B3. A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

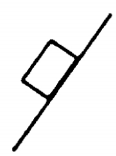
a. Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and cos θ_o
b. The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of cos θ_o.



1985B2. Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, $\sin 60° = 0.87$, and $\cos 60° = 0.50$.

a. What is the tension T in the string?

b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

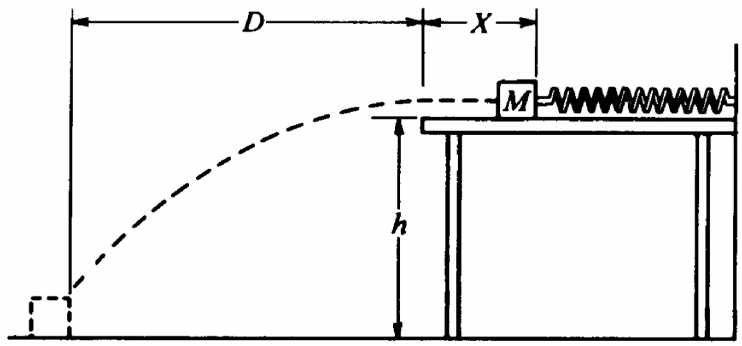


c. Determine the magnitude of the frictional force acting on the box on the plane.

The string is then cut and the left-hand box slides down the inclined plane.

d. Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

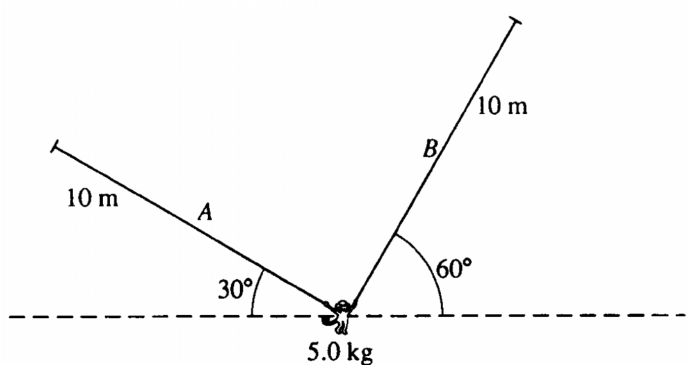
e. Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.



1986B2. One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance *h* above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible.

Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

a. The time elapsed from the instant the block leaves the table to the instant it strikes the floor
b. The horizontal component of the velocity of the block just before it hits the floor
c. The work done on the block by the spring
d. The spring constant



sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

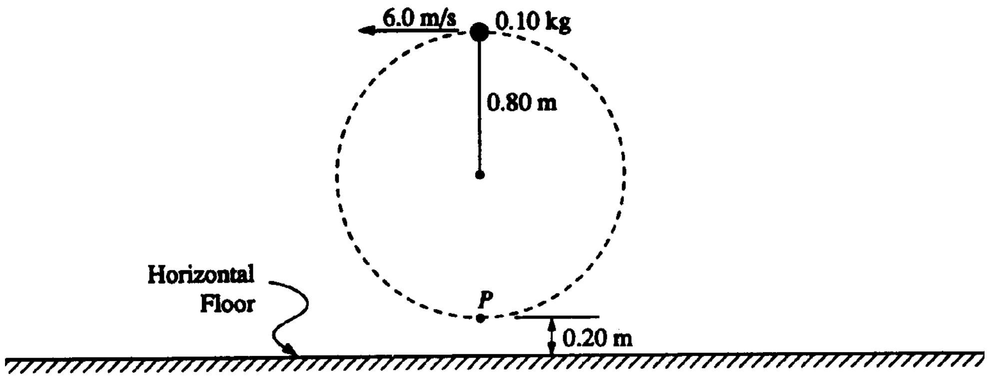
1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.

The monkey releases vine A and swings on vine B. Neglect air resistance.

c. Determine the speed of the monkey as it passes through the lowest point of its first swing.
d. Determine the tension in vine B as the monkey passes through the lowest point of its first swing.

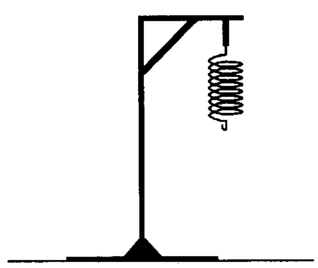


1992B1. A 0.10-kilogram solid rubber ball is attached to the end of an 0.80 meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

a. Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.
b. Determine the speed of the ball at point P, the lowest point of the circle.
c. Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.

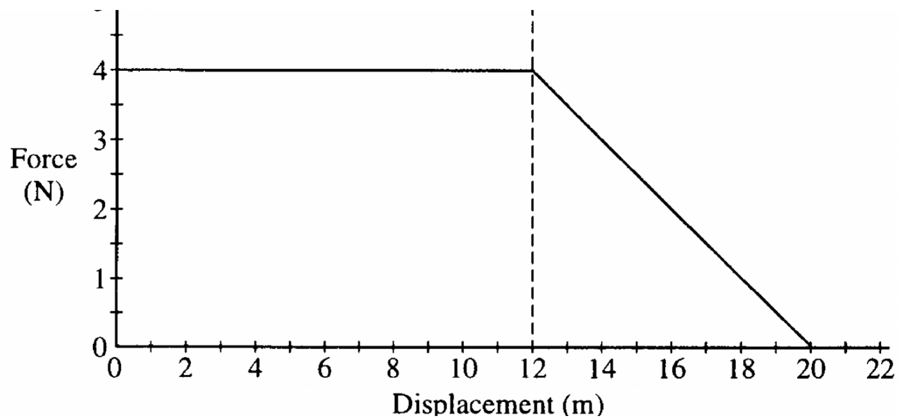
The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

d. Determine the horizontal distance that the ball travels before hitting the floor.



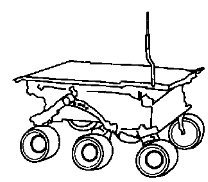
1996B2 (15 points) A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?
b. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

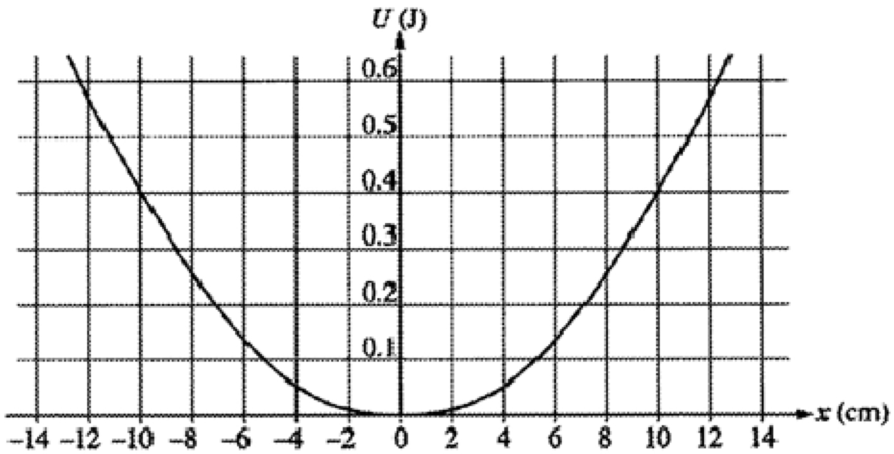
a. The acceleration of the particle when its displacement x is 6 m.
b. The time taken for the object to be displaced the first 12 m.
c. The amount of work done by the net force in displacing the object the first 12 m.
d. The speed of the object at displacement x = 12 m.
e. The final speed of the object at displacement x = 20 m.



1999B1. The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data	
Radius: 0.53 x Earth's radius	Mass of Sojourner vehicle:	11.5 kg
Mass: 0.11 x Earth's mass	Wheel diameter: 0.13 m	
	Stored energy available:	5.4×10^5 J
	Power required for driving under average conditions:	10 W
	Land speed:	6.7×10^{-3} m/s

a. Determine the acceleration due to gravity at the surface of Mars in terms of g, the acceleration due to gravity at the surface of Earth.
b. Calculate Sojourner's weight on the surface of Mars.
c. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
d. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
e. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
f. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.



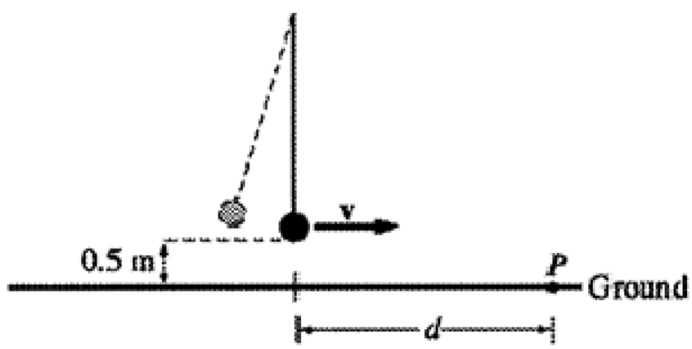
2002B2. A 3.0 kg object subject to a restoring force F is undergoing simple harmonic motion with a small amplitude. The potential energy U of the object as a function of distance x from its equilibrium position is shown above. This particular object has a total energy E: of 0.4 J.

(a) What is the object's potential energy when its displacement is +4 cm from its equilibrium position?

(b) What is the farthest the object moves along the x axis in the positive direction? Explain your reasoning.

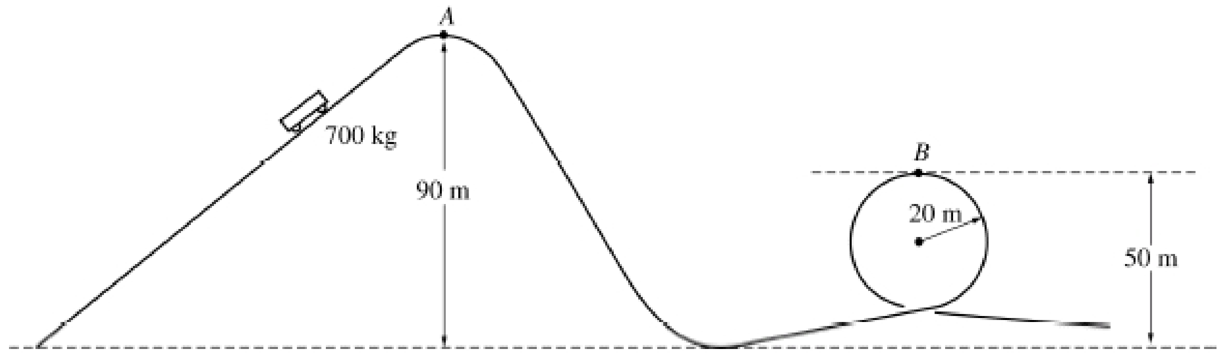
(c) Determine the object's kinetic energy when its displacement is – 7 cm.

(d) What is the object's speed at x = 0 ?



Note: Figure not drawn to scale.

(e) Suppose the object undergoes this motion because it is the bob of a simple pendulum as shown above. If the object breaks loose from the string at the instant the pendulum reaches its lowest point and hits the ground at point P shown, what is the horizontal distance d that it travels?



2004B1.
A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

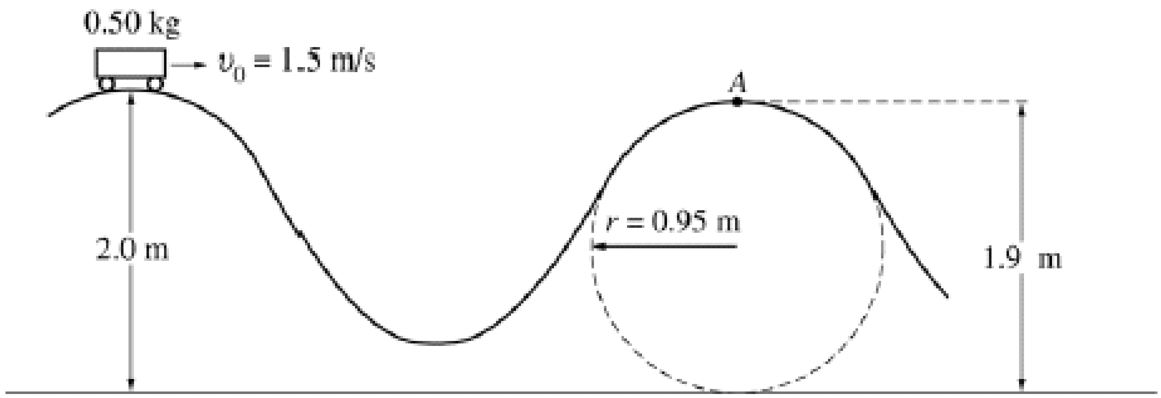
(a)
i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.
ii. Calculate the value v_{msx} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.

ii. Calculate the magnitude of all the forces identified in (c)

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.



B2004B1.
A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_o = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

(a) Calculate the speed of the car at point *A*.

(b) On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.

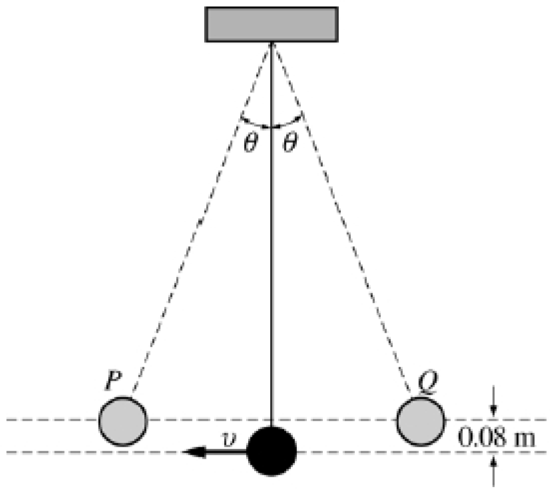
(c) Calculate the magnitude of the force of the track on the car at point *A*.

(d) In order to stop the car at point *A*, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

(e) Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

B2005B2

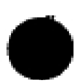
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point Q, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points P and Q as shown below.



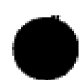
Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

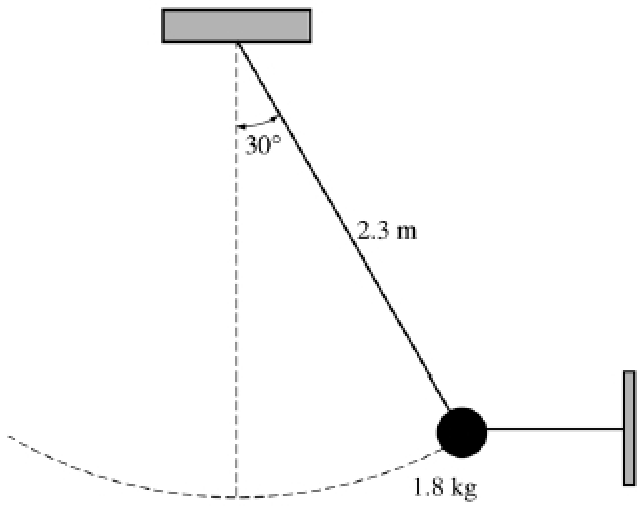


ii. When it is in motion at its lowest position

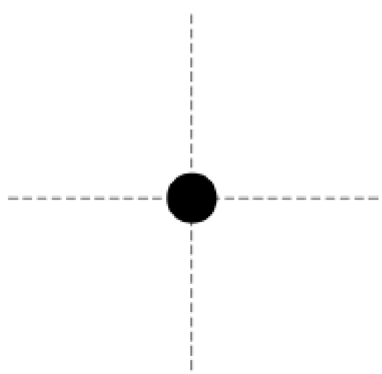


(b) Calculate the speed v of the bob at its lowest position.
(c) Calculate the tension in the string when the bob is passing through its lowest position.

2005B2.

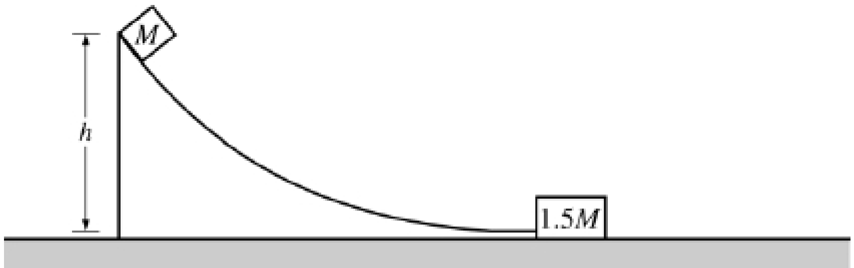


2. (10 points)
A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

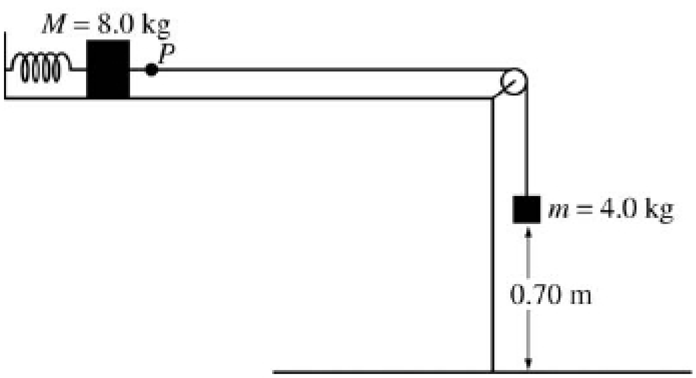
B2006B2



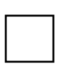
A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.

Express your answers to the following questions in terms of the given quantities and fundamental constants.
(a) Determine the height h of the ramp from which the small block was released.
(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

2006B1



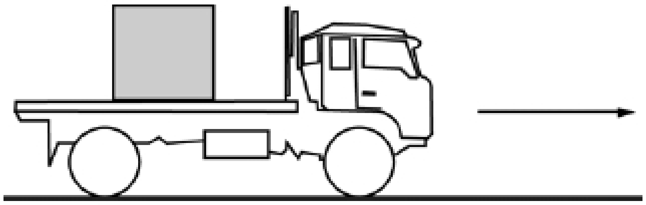
An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.
(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.
M = 8.0 kg $\qquad\qquad$ m = 4.0 kg



(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

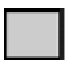
The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the maximum speed attained by the 8.0 kg block as it oscillates back and forth

B2008B2



A 4700 kg truck carrying a 900 kg crate is traveling at 25 m/s to the right along a straight, level highway, as shown above. The truck driver then applies the brakes, and as it slows down, the truck travels 55 m in the next 3.0 s. The crate does not slide on the back of the truck.

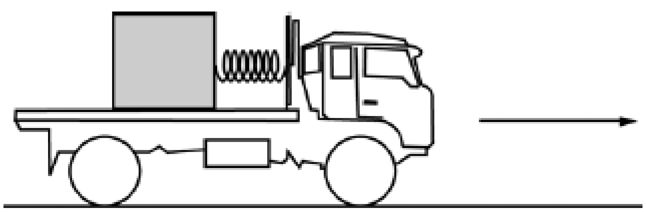
(a) Calculate the magnitude of the acceleration of the truck, assuming it is constant.
(b) On the diagram below, draw and label all the forces acting on the crate during braking.



(c)
i. Calculate the minimum coefficient of friction between the crate and truck that prevents the crate from sliding.

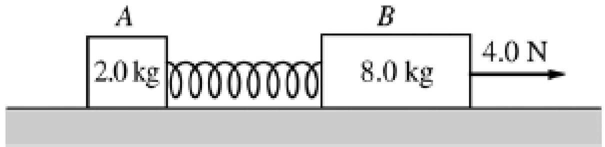
ii. Indicate whether this friction is static or kinetic.
____ Static ____Kinetic

Now assume the bed of the truck is frictionless, but there is a spring of spring constant 9200 N/m attaching the crate to the truck, as shown below. The truck is initially at rest.

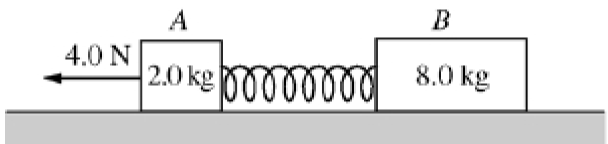


(d) If the truck and crate have the same acceleration, calculate the extension of the spring as the truck accelerates from rest to 25 m s in 10 s.
(e) At some later time, the truck is moving at a constant speed of 25 m/s and the crate is in equilibrium. Indicate whether the extension of the spring is greater than, less than, or the same as in part (d) when the truck was accelerating.
___ Greater ___ Less ___ The same
Explain your reasoning.

2008B2

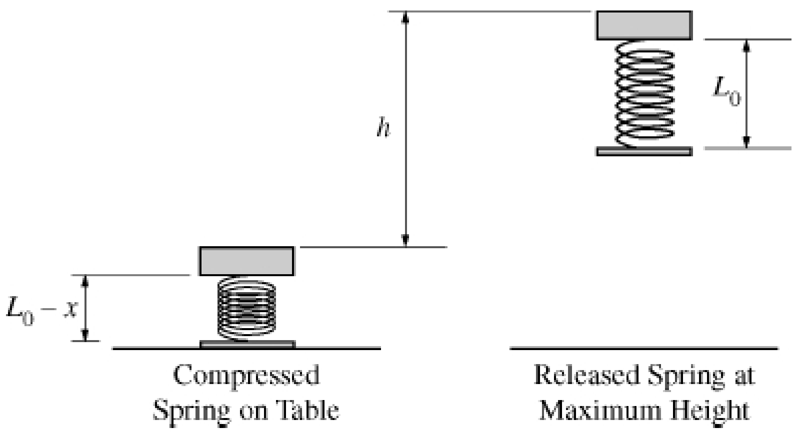


Block *A* of mass 2.0 kg and block *B* of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.
(a) Calculate the force that the spring exerts on the 2.0 kg block.
(b**)** Calculate the extension of the spring.
The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c**)** Is the magnitude of the acceleration greater than, less than, or the same as before?
____ Greater ____ Less ____ The same
Justify your answer.
(d) Is the amount the spring has stretched greater than, less than, or the same as before?
____ Greater ____ Less ____ The same
Justify your answer.
(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m s to the left. Block *A* then hits and sticks to a wall. Calculate the maximum compression of the spring.

2009B1



In an experiment, students are to calculate the spring constant k of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance x from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height h above the point of maximum compression. The students repeat the experiment several times, measuring h with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is m. The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped to it.

(a) Derive an expression for the height h in terms of m, x, k, and fundamental constants.

With the spring compressed a distance x = 0.020 m in each trial, the students obtained the following data for different values of m.

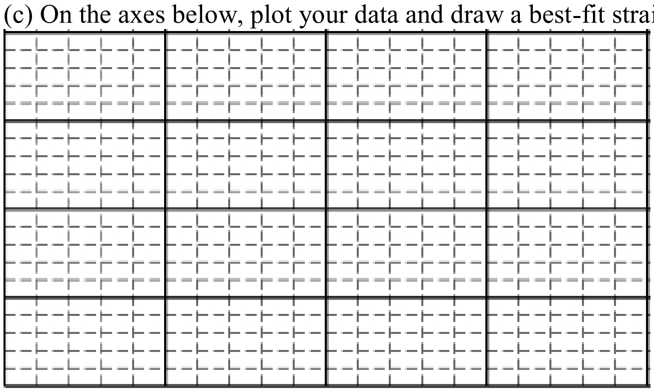
	m (kg)	h (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

(b)

i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant k ?

ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.

(c) On the axes below, plot your data and draw a best-fit straight line. Label the axes and indicate the scale.



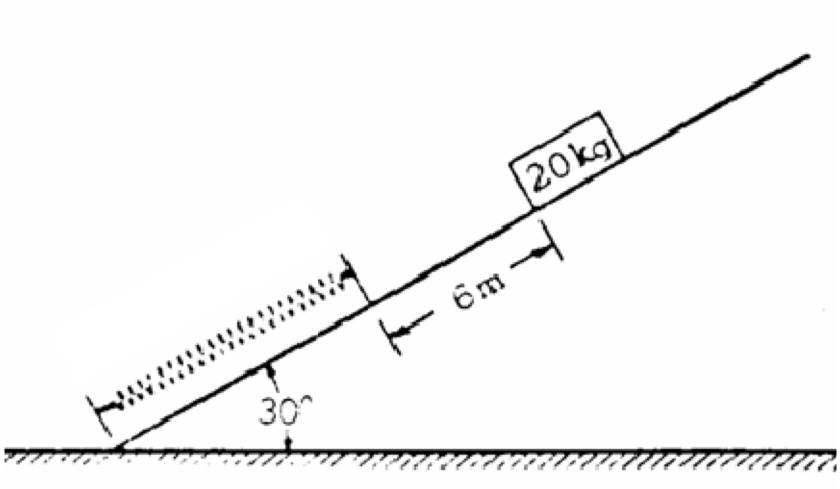
(d) Using your best-fit line, calculate the numerical value of the spring constant.

(e) Describe a procedure for measuring the height h in the experiment, given that the toy is only momentarily at that maximum height.

C1973M2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.

a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.

b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?

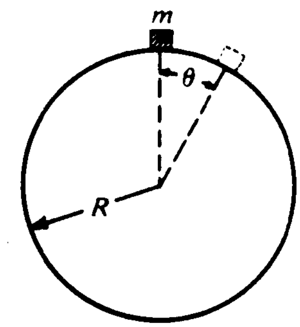


C1982M1. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30º with the horizontal and strikes a spring of unknown spring constant as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved.

a. Determine the speed of the block just before it hits the spring.

b. Determine the spring constant given that the distance the spring compresses along the incline is 3m when the block comes to rest.

c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

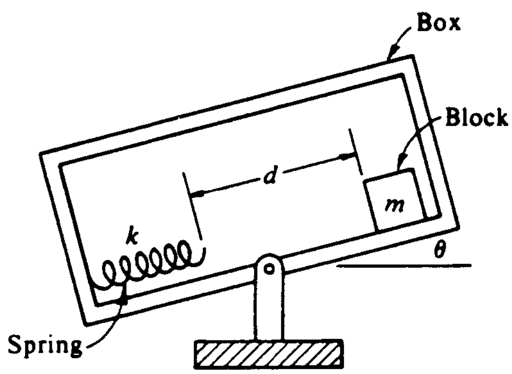


C1983M3. A particle of mass m slides down a fixed, frictionless sphere of radius R. starting from rest at the top.

a. In terms of m, g, R. and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass



C1985M2. An apparatus to determine coefficients of friction is shown above. At the angle θ shown with the horizontal, the block of mass m just starts to slide. The box then continues to slide a distance d at which point it hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities and fundamental constants, derive an expression for each of the following.

a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

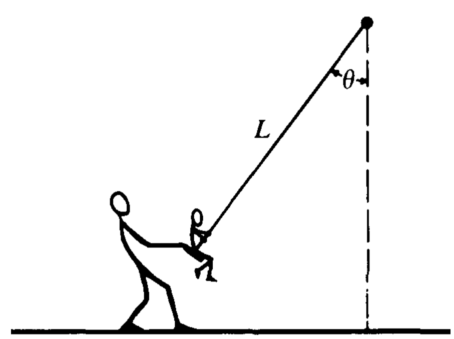
c. μ_k, the coefficient of kinetic friction.

C1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

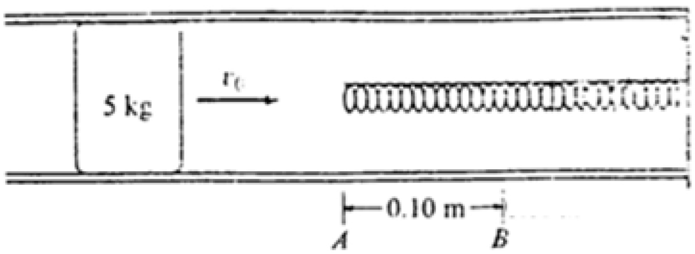
a) The tension in the rope

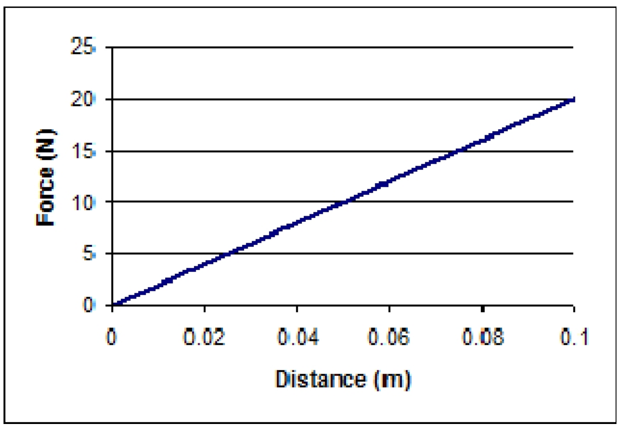
b) The horizontal force exerted by the adult.

c) The adult releases the swing from rest. In terms of W and θ determine the tension in the rope as the swing passes through its lowest point



C1988M2.



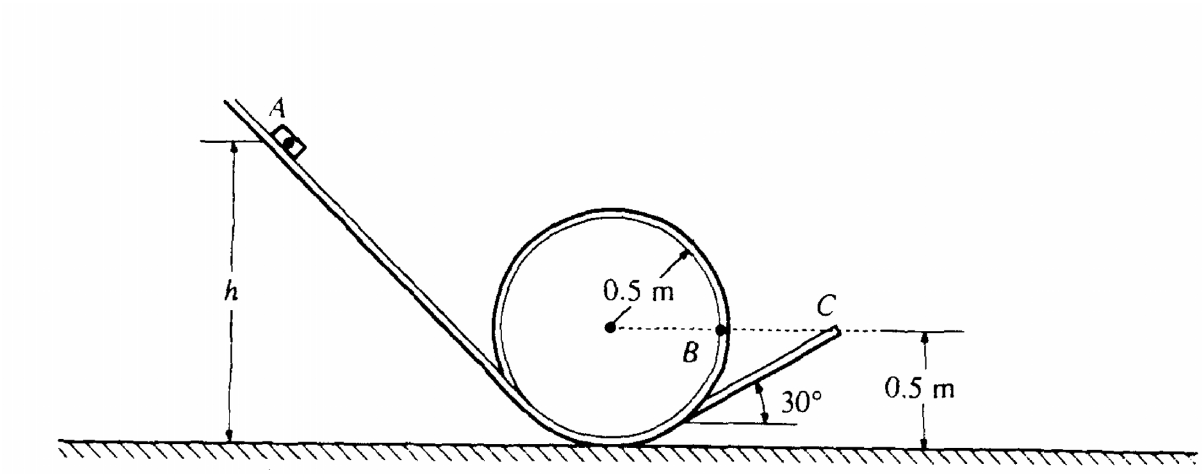


A 5-kilogram object initially slides with speed v_o in a hollow frictionless pipe. The end of the pipe contains a spring as shown. The object makes contact with the spring at point A and moves 0.1 meter before coming to rest at point B. The graph shows the magnitude of the force exerted on the object by the spring as a function of the objects distance from point A.

a. Calculate the spring constant for the spring.

b. Calculate the decrease in kinetic energy of the object as it moves from point A to point B.

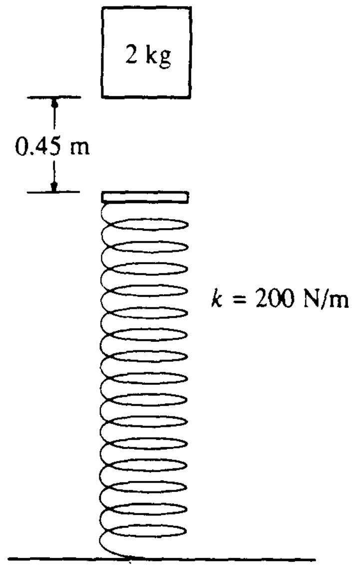
c. Calculate the initial speed v_o of the object



C1989M1. A 0.1 kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 m/s at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

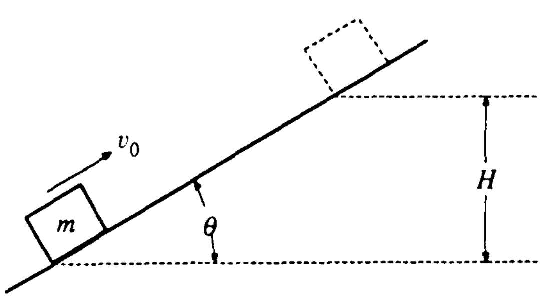
a. Determine the height h .
b. On the figure below, draw and label all the forces acting on the block when it is at point B. which is 0.5 meter above the ground.

c. Determine the magnitude of the velocity of the block when it is at point B.
d. Determine the magnitude of the force exerted by the track on the block when it is at point B.
e. Determine the maximum height above the ground attained by the block after it leaves the track.
f. Another track that has the same configuration, but is **NOT** frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 m/s, the height h must be 2 meters. Determine the work done by the frictional force.



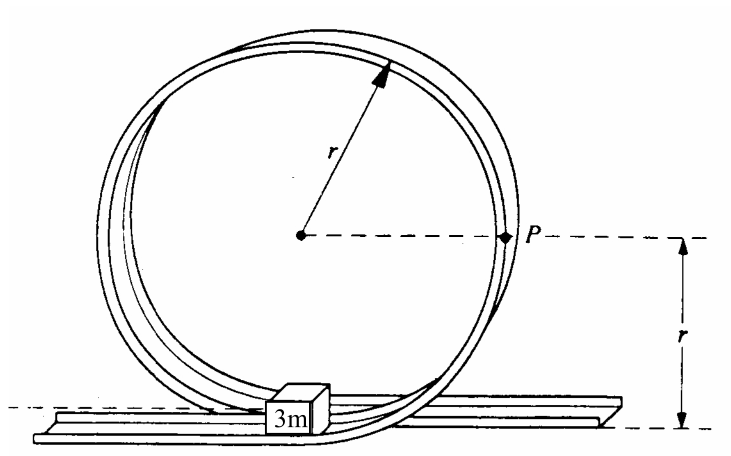
C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Is the speed of the block a maximum at the equilibrium position, explain.



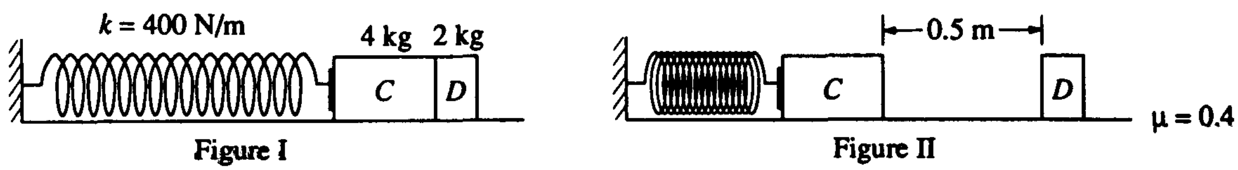
C1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, the box slides a distance $d = h_2 / \sin\theta$ along the length of the ramp as it reaches a new maximum height h_2. Determine the new maximum height h_2 in terms of the given quantities.



C1991M1. A small block of mass 3m moving at speed $v_0 / 3$ enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o , r, and g.

a. The kinetic energy of the block and bullet when they reach point P on the loop
b. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times
c The new required entry speed v_o' at the bottom of the loop such that the conditions in part b apply.

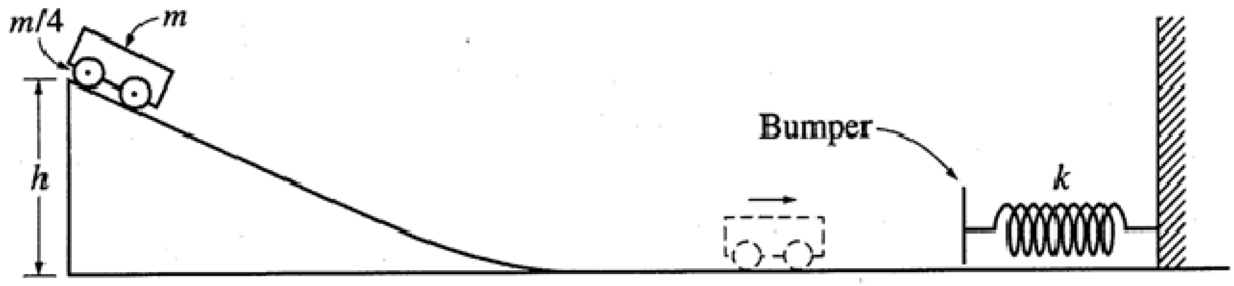


C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a rough horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D*. The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right at 3 m/s. Remember that the spring is not attached to block C. Determine each of the following.

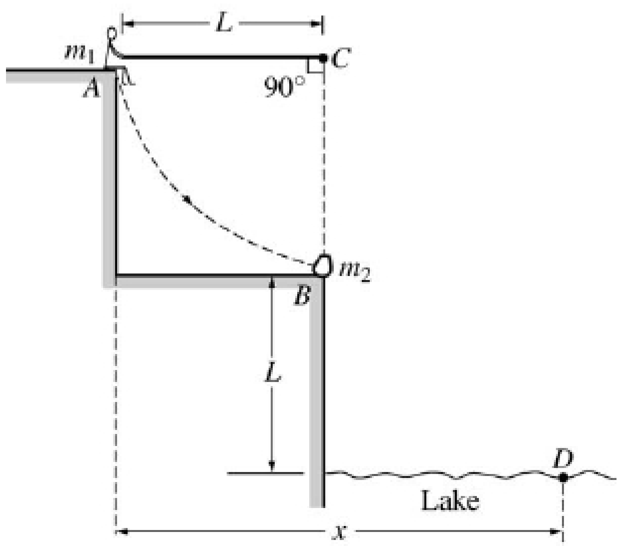
b. The speed v_c of block C just before it collides with block *D*
c. The horizontal distance the combined blocks move after leaving the spring before coming to rest



C2002M2. The cart shown above has a mass 2m. The cart is released from rest and slides from the top of an inclined frictionless plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the speed of the cart when it reaches the bottom of the incline.
b. After sliding down the incline and across the frictionless horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

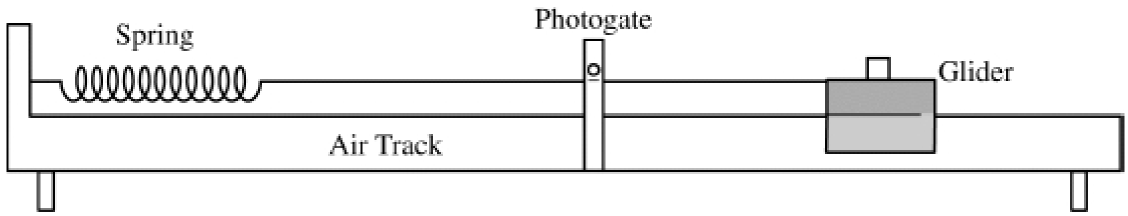
C2004M1



A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) After the person hits and grabs the rock, the speed of the combined masses is determined to be v'. In terms of v' and the given quantities, determine the total horizontal displacement x of the person from position A until the person and object land in the water at point D.

C2007M3.

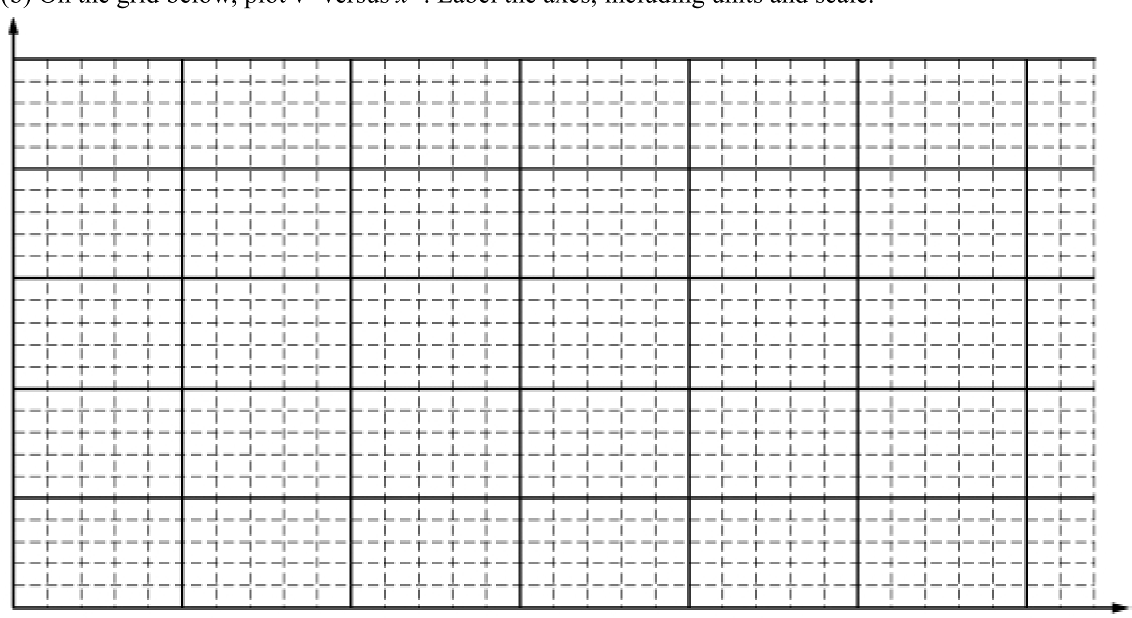


The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass *m*. The glider is pulled to stretch the spring an amount *x* from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time *t* that it takes the small block on top of the glider to pass through. Information about the distance *x* and the speed *v* of the glider as it passes through the photogate are given below.

Trial #	Extension of the Spring x (m)	Speed of Glider v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.

(b) On the grid below, plot v^2 versus x^2 . Label the axes, including units and scale.

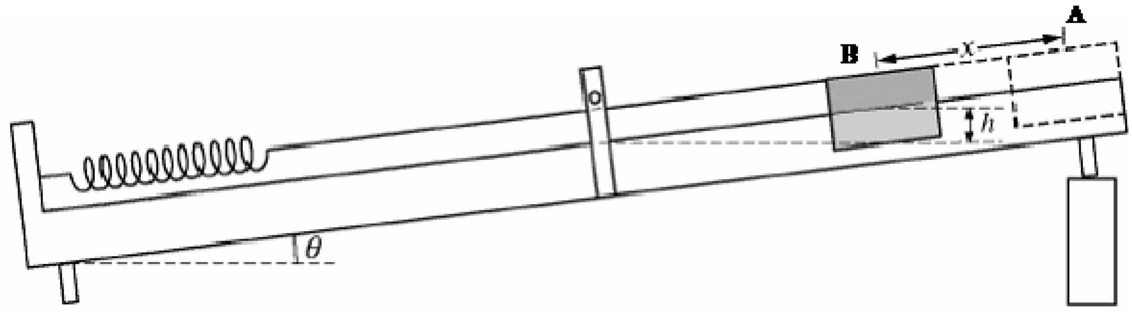


(c)

i. Draw a best-fit straight line through the data.

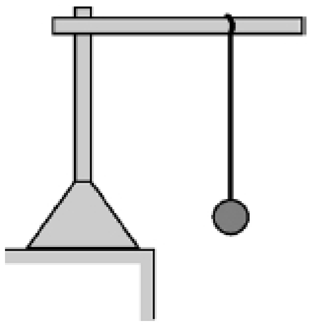
ii. Use the best-fit line to obtain the mass m of the glider.

(d) The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height h above the photogate. The experiment is repeated with a variety of values of x.



Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation starting from position A and ending at position B

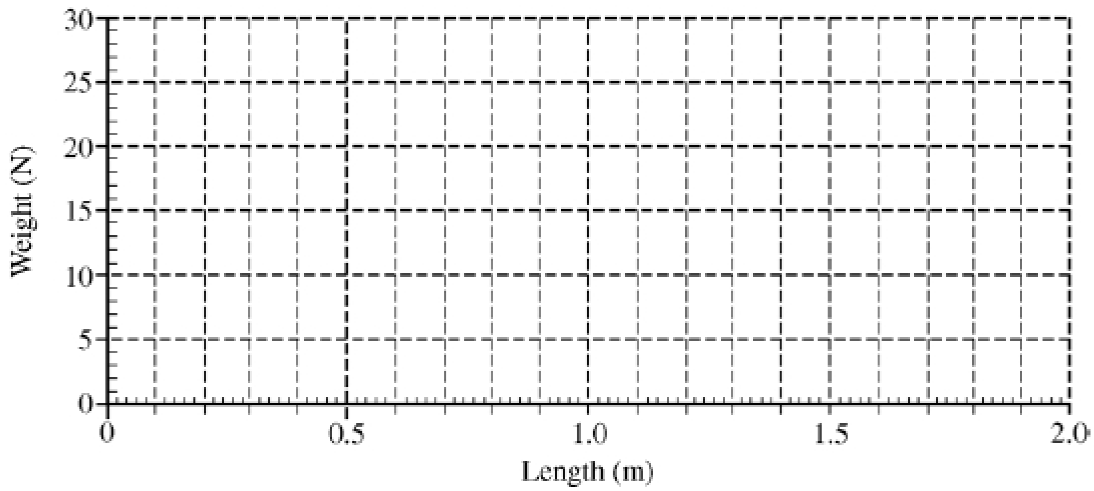
C2008M3



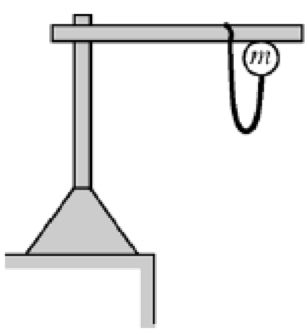
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



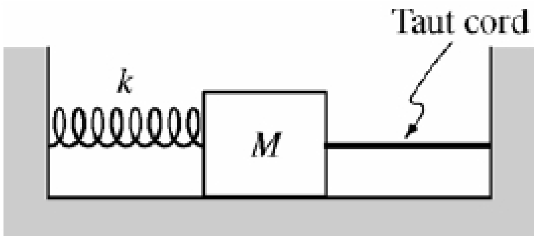
The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass m of the object.

(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Work-Energy

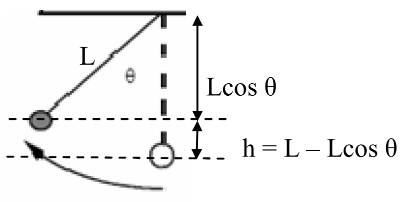
Solution — Answer

1. Conservation of Energy, $U_{sp} = K$, $\frac{1}{2} kA^2 = \frac{1}{2} mv^2$ solve for v — **B**

2. Constant velocity → $F_{net}=0$, $f_k = Fx = F\cos\theta$ $W_{fk} = -f_k d = -F\cos\theta\, d$ — **A**

3. In a circle moving at a constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle — **D**

4. — **A**



The potential energy at the first position will be the amount "lost" as the ball falls and this will be the change in potential. $U=mgh = mg(L-L\cos\theta)$

5. The work done by the stopping force equals the loss of kinetic energy. $-W=\Delta K$ — **A**
$-Fd = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$ $v_f = 0$ $F = mv^2/2d$

6. This is a conservative situation so the total energy should stay same the whole time. It should also start with max potential and min kinetic, which only occurs in choice C — **C**

7. Stopping distance is a work-energy relationship. Work done by friction to stop = loss of kinetic — **B**
$-f_k\, d = -\frac{1}{2} mv_i^2$ $\mu_k mg = \frac{1}{2} mv_i^2$
The mass cancels in the relationship above so changing mass doesn't change the distance

8. Same relationship as above … double the v gives 4x the distance — **D**

9. Half way up you have gained half of the height so you gained ½ of potential energy. Therefore you must have lost ½ of the initial kinetic energy so $E_2 = (E_k/2)$. — **B**
Subbing into this relationship $E_2 = (E_k/2)$
$\frac{1}{2} mv_2^2 = \frac{1}{2} m\, v^2 / 2$
$v_2^2 = v^2 / 2$ …. Sqrt both sides gives answer

10. At the top, the ball is still moving (v_x) so would still possess some kinetic energy — **A**

11. Same as question #1 with different variables used — **B**

12. Total energy is always conserved so as the air molecules slow and lose their kinetic energy, there is a heat flow which increases internal (or thermal) energy — **C**

13. Eliminating obviously wrong choices only leaves A as an option. The answer is A because since the first ball has a head start on the second ball it is moving at a faster rate of speed at all times. When both are moving in the air together for equal time periods the first faster rock will gain more distance than the slower one which will widen the gap between them. — **A**

14. For a mass on a spring, the max U occurs when the mass stops and has no K while the max K occurs when the mass is moving fast and has no U. Since energy is conserved it is transferred from one to the other so both maximums are equal — **C**

15. Since the ball is thrown with initial velocity it must start with some initial K. As the mass falls it gains velocity directly proportional to the time (V=Vi+at) but the K at any time is equal to 1/2 mv^2 which gives a parabolic relationship to how the K changes over time. D

16. Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates D

17. The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the K gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph. C

18. Point IV is the endpoint where the ball would stop and have all U and no K. Point II is the minimum height where the ball has all K and no U. Since point III is halfway to the max U point half the energy would be U and half would be K C

19. Apply energy conservation using points IV and II. $U_4 = K_2$ $mgh = ½ mv^2$ B

20. Since the track is rough there is friction and some mechanical energy will be lost as the block slides which means it cannot reach the same height on the other side. The extent of energy lost depends on the surface factors and cannot be determined without more information D

21. As the object oscillates its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D D

22. To push the box at a constant speed, the child would need to use a force equal to friction so $F=f_k=\mu mg$. The rate of work (W/t) is the power. Power is given by P=Fv → μmgv A

23. Two steps. I) use hookes law in the first situation with the 3 kg mass to find the spring constant (k). $F_{sp}=k\Delta x$, $mg=k\Delta x$, k = 30/.12 = 250. II) Now do energy conservation with the second scenario (note that the initial height of drop will be the same as the stretch Δx).
U top = U_{sp} bottom, $mgh = ½ k \Delta x^2$, $(4)(10)(\Delta x) = ½ (250) (\Delta x^2)$ C

24. In a circular orbit, the velocity of a satellite is given by $v = \sqrt{\frac{Gm_e}{r}}$ with m_e = M. Kinetic energy of the satellite is given by $K = ½ m v^2$. Plug in v from above to get answer A

25. Projectile. V_x doesn't matter $V_{iy} = 0$. Using $d = v_{iy}t + ½ at^2$ we get the answer D

26. A is true; both will be moving the fastest when they move through equilibrium. A

27. X and Y directions are independent and both start with the same velocity of zero in each direction. The same force is applied in each direction for the same amount of time so each should gain the same velocity in each respective direction. B

28. Kinetic energy is not a vector and the total resultant velocity should be used to determine the KE. For the 1st second the object gains speed at a uniform rate in the x direction and since KE is proportional to v^2 we should get a parabola. However, when the 2nd second starts the new gains in velocity occur only in the y direction and are at smaller values so the gains essentially start over their parabolic trend as shown in graph B B

29. As the system moves, m_2 loses energy over distance h and m_1 gains energy over the same distance h but some of this energy is converted to KE so there is a net loss of U. Simply subtract the $U_2 - U_1$ to find this loss — A

30. In a force vs. displacement graph, the area under the line gives the work done by the force and the work done will be the change in the K so the largest area is the most K change — D

31. Use energy conservation, U top = K bottom. As in problem #6 (in this document), the initial height is given by L – Lcos θ, with cos 60 = .5 so the initial height is ½ L. — A

32. Use application of the net work energy theorem which says … $W_{net} = \Delta K$. The net work is the work done by the net force which gives you the answer — A

33. There is no U_{sp} at position x=0 since there is no Δx here so this is the minimum U location — A

34. Using energy conservation in the first situation presented K=U gives the initial velocity as $v = \sqrt{2gh}$. The gun will fire at this velocity regardless of the angle. In the second scenario, the ball starts with the same initial energy but at the top will have both KE and PE so will be at a lower height. The velocity at the top will be equal to the v_x at the beginning — C

35. Use energy conservation $K=U_{sp}$ $\frac{1}{2} m v_m^2 = \frac{1}{2} k \Delta x^2$, with Δx=A, solve for k — D

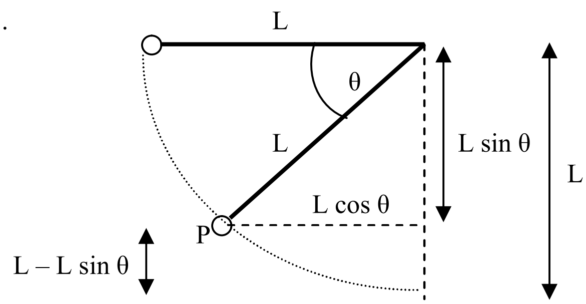
36. Based on net work version of work energy theorem. $W_{net} = \Delta K$, we see that since there is a constant speed, the ΔK would be zero, so the net work would be zero requiring the net force to also be zero. — A

37. As the block slides back to equilibrium, we want all of the initial spring energy to be dissipated by work of friction so there in no kinetic energy at equilibrium where all of the spring energy is now gone. So set work of friction = initial spring energy and solve for μ. The distance traveled while it comes to rest is the same as the initial spring stretch, d = x. — C
$\frac{1}{2} kx^2 = \mu mg(x)$

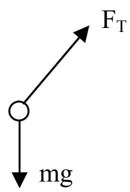
38. V at any given time is given by $v = v_i + at$, with $v_i = 0$ gives $v = at$, — D
V at any given distance is found by $v^2 = v_i^2 + 2ad$, with $v_i = 0$ gives $v^2 = 2ad$
This question asks for the relationship to distance.
The kinetic energy is given by $K = \frac{1}{2} m v^2$ and since $v^2 = 2ad$ we see a linear direct relationship of kinetic energy to distance (2*d → 2*K)
Another way of thinking about this is in relation to energy conservation. The total of $mgh+1/2mv^2$ must remain constant so for a given change in (h) the $\frac{1}{2} mv^2$ term would have to increase or decrease directly proportionally in order to maintain energy conservation.

39. Similar to the discussion above. Energy is conserved so the term $mgh + \frac{1}{2} mv^2$ must remain constant. As the object rises it loses K and gains U. Since the height is H/2 it has gained half of the total potential energy it will end up with which means it must have lost half of its kinetic energy, so its K is half of what it was when it was first shot. — B

1974B1.



(a) FBD



(b) Apply conservation of energy from top to point P

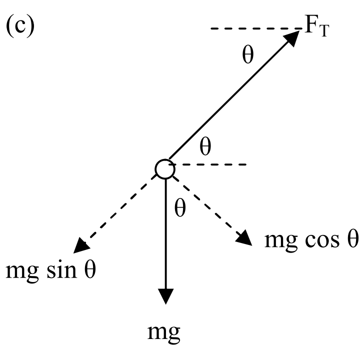
$U_{top} = U_p + K_p$

$mgh = mgh_p + ½ m v_p^2$

$gL = g(L – L\sin θ) + ½ v_p^2$

$$v = \sqrt{2gL\sin\theta}$$

(c)



$F_{NET(C)} = m v^2 / r$

$F_T – mg \sin θ = m v^2/ r$

$F_T – mg \sin θ = m (2gL\sin θ) / L$

$F_T = 2mg \sin θ + mg \sin θ$

$F_T = 3mg \sin θ$

1974B7.

6 riders per minute is equivalent to 6x(70kg)*9.8 = 4116 N of lifting force in 60 seconds

Work to lift riders = work to overcome gravity over the vertical displacement (600 sin 30)
Work lift = Fd = 4116N (300m) = $1.23x10^6$ J

P lift = W / t = $1.23x10^6$ J / 60 sec = 20580 W

But this is only 40% of the necessary power. ➔ 0.40 (total power) = 20580 W

Total power needed = 51450 W

1975B1.

(a) $F_{net} = ma$ $\quad - f_k = ma$ $\quad - 8 = 2a$ $\quad a = -4 \text{ m/s}^2$

(b) $v_f^2 = v_i^2 + 2ad$ $\quad (0)^2 = v_i^2 + 2(-4)(8)$ $\quad v_i = 8 \text{ m/s}$

$v_f = v_i + at$ $\quad t = 2 \text{ sec}$

(c) Apply energy conservation top to bottom

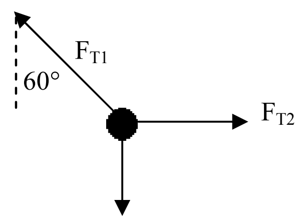
$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$(10)(R) = ½ (8)^2$ $\quad R = 3.2 \text{ m}$

1975 B7

(a)

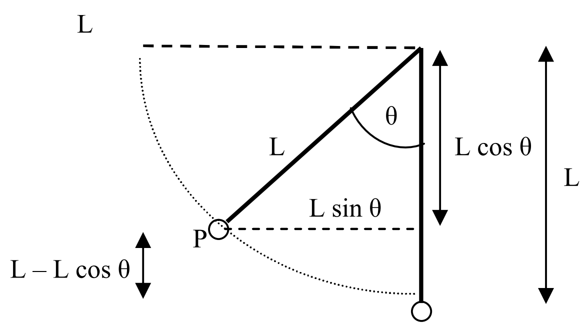


(b) $F_{NET(Y)} = 0$

$F_{T1} \cos\theta = mg$

$F_{T1} = mg / \cos(60) = 2mg$

(c) When string is cut it swing from top to bottom, similar to diagram for 1974B1 with θ moved as shown below



$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

1977B1.

(a) Apply work–energy theorem

$W_{NC} = \Delta ME$

$W_{fk} = \Delta K \quad (K_f - K_i)$

$W = -K_i$

$W = -\frac{1}{2} mv_i 2 \qquad -\frac{1}{2}(4)(6)^2 \qquad = -72$ J

(b) $F_{net} = ma$

$-f_k = m\,a \qquad v = v_i + at$

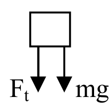
$a = -(8)/4 = -2$ m/s$^2 \qquad v = (6) + (-2)\,t$

(c) $W_{fk} = -f_k\,d$

$-72J = -(8)\,d \qquad d = 9$ m

1978B1,

(a)



(b) Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t + mg) = mv^2/r \qquad (20+0.5(10))=(0.5)v^2 / 2 \qquad v = 10$ m/s

(c) As the object moves from P to Q, it loses U and gains K. The gain in K is equal to the loss in U.

$\Delta U = mg\Delta h = (0.5)(10)(4) = 20$ J

(d) First determine the speed at the bottom using energy.

$K_{top} + K_{gain} = K_{bottom} \qquad \frac{1}{2} mv_{top}^2 + 20\text{ J} = \frac{1}{2} mv_{bot}^2 \qquad v_{bot} = 13.42$ m/s

At the bottom, Ft acts up (towards center) and mg acts down (away from center)

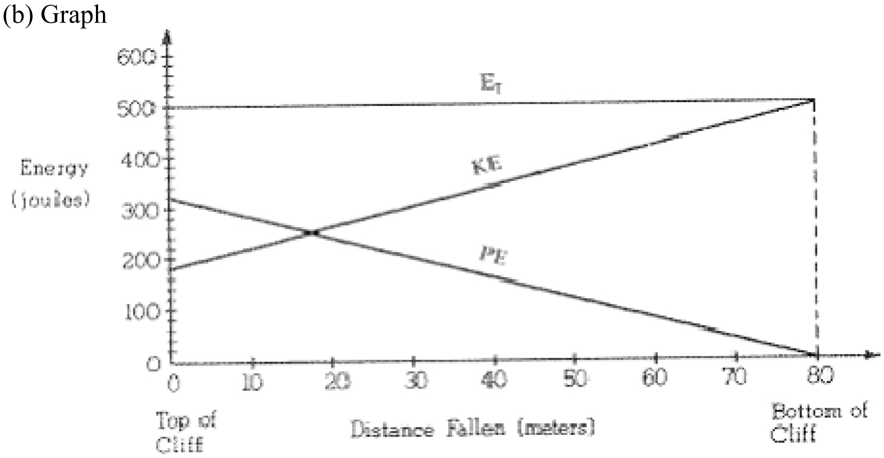
Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t - mg) = mv^2/r \qquad (F_t - 0.5(10)) = (0.5)(13.42)^2 / 2 \qquad F_t = 50$ N

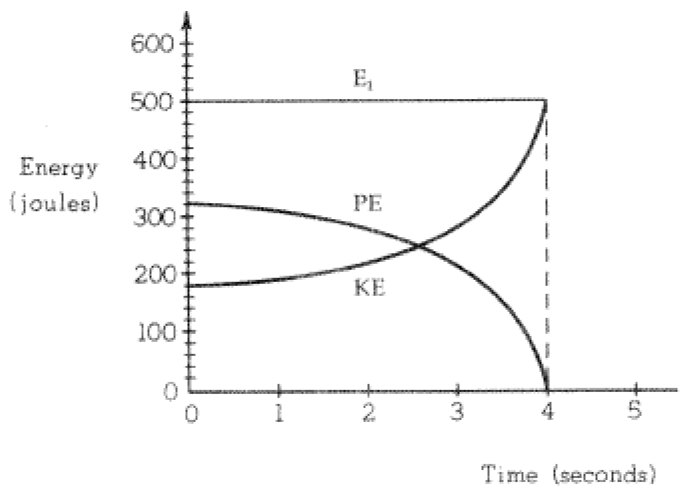
1979B1.

(a) U = mgh = 320 J
K = ½ m v^2 = 180 J
Total = U + K = 500 J

(b) Graph



(c) First determine the time at which the ball hits the ground, using $d_y = 0 + ½ g t^2$, to find it hits at 4 seconds.



1981B1.

(a) constant velocity means $F_{net} = 0$, $F - f_k = ma$ $F - \mu_k mg = 0$ $F - (0.2)(10)(10) = 0$
$F = 20$ N

(b) A change in K would require net work to be done. By the work–energy theorem:
$W_{net} = \Delta K$
$F_{net}\, d = 60$ J
$F_{net}\,(4\text{m}) = 60$ $F_{net} = 15$ N
$F' - f_k = 15$
$F' - 20 = 15$ $F = 35$ N

(c) $F_{net} = ma$
$(15) = (10)\, a$ $a = 1.5$ m/s^2

1981B2.

The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring
$W = ½\, m\, v^2 = ½\, (3)\, (10)^2 = 150$ J

1982B3.

Same geometry as in problem 1975B7.

(a) Apply energy conservation top to bottom
$U_{top} = K_{bot}$
$mgh = ½\, m\, v^2$
$mg\,(R - R\cos\theta) = ½\, m\, v^2$
$$v = \sqrt{2g(R - R\cos\theta)}$$

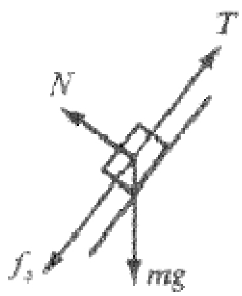
(b) Use $F_{NET(C)} = mv^2 / r$
$F_t - mg = m\,(2g(R\text{-}R\cos\theta)) / R$
$1.5\, mg - mg = 2mg(1 - 1\cos\theta)$
$.5 = 2(1 - \cos\theta)$

$2\cos\theta = 1.5 \rightarrow \cos\theta = ¾$

1985B2.

(a) The tension in the string can be found easily by isolating the 10 kg mass. Only two forces act on this mass, the Tension upwards and the weight down (mg) …. Since the systems is at rest, T = mg = 100 N

(b) FBD



(c) Apply $F_{net} = 0$ along the plane. $T - f_s - mg \sin\theta = 0$ $(100\text{ N}) - f_s - (10)(10)(\sin 60)$

$f_s = 13$ N

(d) Loss of mechanical energy = Work done by friction while sliding

First find kinetic friction force Perpendicular to plane $F_{net} = 0$ $F_n = mg\cos\theta$

$F_k = \mu_k F_n = \mu_k mg\cos\theta$

$W_{fk} = f_k d = \mu_k mg\cos\theta\,(d) = (0.15)(10)(10)(\cos(60)) = 15$J converted to thermal energy

(e) Using work-energy theorem … The U at the start – loss of energy from friction = K left over

$U - W_{fk} = K$

$mgh - W_{fk} = K$

$mg(d \sin 60) - 15 = K$

$(10)(10)(2) \sin 60 - 15 = K$ $K = 158$ J

1986B2.

(a) Use projectile methods to find the time. $d_y = v_{iy}t + ½ a t^2$ $h = 0 + g t_2 / 2$

$$t = \sqrt{\frac{2h}{g}}$$

(b) v_x at ground is the same as v_x top $V_x = d_x /t$ $$v_x = \frac{D}{\sqrt{\frac{2h}{g}}}$$

multiply top and bottom by reciprocal to rationalize

$$v_x = D\sqrt{\frac{g}{2h}}$$

(c) The work done by the spring to move the block is equal to the amount of K gained by it = K_f

$W = K_f = ½ m v^2 = (½ M (D^2 / (2h/g)) = MD^2 g / 4h$

(d) Apply energy conservation $U_{sp} = K$

$½ k\Delta x^2 = ½ mv^2$ (plug in V from part b) $$v_x = \frac{MD^2 g}{2hX^2}$$

If using $F=k\Delta x$ you have to plug use F_{avg} for the force

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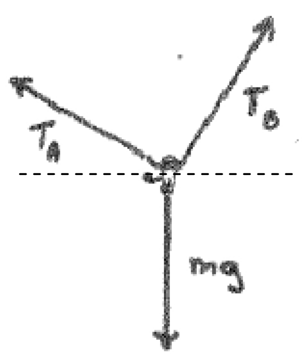
1991B1.

(a) FBD

30°  60°

(b) SIMULTANEOUS EQUATIONS

$F_{net(X)} = 0$ $F_{net(Y)} = 0$

$T_a \cos 30 = T_b \cos 60$ $T_a \sin 30 + T_b \sin 60 – mg = 0$

…. Solve above for T_b and plug into Fnet(y) eqn and solve

$T_a = 24$ N $T_b = 42$ N

(c) Using energy conservation with similar diagram as 1974B1 geometry

$U_{top} = U_p + K_p$

$mgh = ½ m v^2$

$g(L – L\sin \theta) = ½ v^2$

$(10)(10 – 10\sin 60) = ½ v^2$ $v = 5.2$ m/s

(d) $F_{net(C)}= mv^2/r$

$F_t – mg = mv^2 / r$ $F_t = m(g + v^2/r)$ $F_t = (5)(9.8+(5.2)^2/10) = 62$ N

1992B1.

(a) K + U ½ m v^2 + mgh ½ $(0.1)(6)^2 + (0.1)(9.8)(1.8) = 3.6$ J

(b) Apply energy conservation using ground as h=0

$E_{top} = E_p$

3.6 J = K + U

3.6 = ½ m v^2 + mgh

$3.6 = ½ (0.1)(v^2) + (0.1)(9.8)(.2)$ v = 8.2 m/s

(c) Apply net centripetal force with direction towards center as +

i) Top of circle = Ft points down and Fg points down

$F_{net(c)} = mv^2/r$

$Ft + mg = mv^2/r$ $F_t = mv^2/r – mg$ $(0.1)(6)^2/(0.8) – (.1)(9.8)$ $F_t = 3.5$ N

ii) Bottom of circle = Ft points up and Fg points down

$F_{net(c)} = mv^2/r$

$F_t – mg = mv^2/r$ $F_t = mv^2/r + mg$ $(0.1)(8.2)^2/(0.8) + (0.1)(9.8)$ $F_t = 9.5$ N

(d) Ball moves as a projectile.

First find time of fall in y direction

$d_y = v_{iy}t + ½ a t^2$

$(–0.2) = 0 + ½ (–9.8) t^2$

t = .2 sec

Then find range in x direction

$d_x = v_x t$

$d_x = (8.2)(0.2)$

$d_x = 1.6$ m

1996B2.

(a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k\ \Delta x$ and solve for k

(b) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan θ$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} – μ_s$ mg, Giving $mg = F_{spring} / μ$.

Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid dynamics unit.

1997B1.

(a) The force is constant, so simple $F_{net} = ma$ is sufficient. (4) = (0.2) a a = 20 m/s^2

(b) Use $d = v_i t + ½ a t^2$ 12 = (0) + ½ (20) t^2 t = 1.1 sec

(c) W = Fd   W = (4 N) (12 m) = 48 J

(d) Using work energy theorem   W = ΔK   (K_i = 0)   W = Kf - Ki

W = ½ m v_f^2

Alternatively, use $v_f^2 = v_i^2 + 2 a d$ 48J = ½ (0.2) (v_f^2) v_f = 22 m/s

(e) The area under the triangle will give the extra work for the last 8 m

½ (8)(4) = 16J   + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem   W = ½ m v_f^2 64 J = ½ (0.2) v_f^2 v_f = 25.3 m/s

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

1999B1.

(a) Plug into $g = GM_{planet} / r_{planet}^2$ lookup earth mass and radius

g_{mars} = 3.822 m/s^2 to get it in terms of g_{earth} divide by 9.8 g_{mars} = 0.39 g_{earth}

(b) Since on the surface, simply plug into F_g = mg = (11.5)(3.8) = 44 N

(c) On the incline, F_n = mg cos θ = (44) cos (20) = 41 N

(d) moving at constant velocity → F_{net} = 0

(e) W = P t   (5.4x10^5 J) = (10 W) t   t = 54000 sec

d = v t   (6.7x10^{-3})(54000 s)   d = 362 m

(f) P = Fv   (10) = F (6.7x10^{-3}) F_{push} =1492.54 N total pushing force used

* (.0001) use for drag

→ F_{drag} = 0.15 N

2002B2.

(a) From graph U = 0.05 J

(b) Since the total energy is 0.4 J, the farthest position would be when all of that energy was potential spring energy. From the graph, when all of the spring potential is 0.4 J, the displacement is 10 cm

(c) At –7 cm we read the potential energy off the graph as 0.18 J. Now we use energy conservation.

ME = U_{sp} + K   0.4J = 0.18 J + K   → K = 0.22 J

(d) At x=0 all of the energy is kinetic energy K= ½ m v^2 0.4 = ½ (3) v^2 v = 0.5 m/s

(e) The object moves as a horizontally launched projectile when it leaves.

First find time of fall in y direction

$d_y = v_{iy} t + ½ a t^2$

(– 0.5) = 0 + ½ (– 9.8) t^2

t = 0.3 sec

Then find range in x direction

$d_x = v_x t$

d_x = (0.5)(0.3)

d_x = 0.15 m

2004B1.

(a) i) fastest speed would be the lowest position which is the bottom of the first hill where you get all sick and puke your brains out.

ii) Applying energy conservation from the top of the hill where we assume the velocity is approximately zero we have

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$ $(9.8)(90) = ½ v^2$ $v = 42$ m/s

(b) Again applying energy conservation from the top to position B

$U_{top} = K_b + U_b$

$mgh = ½ m v_B{}^2 + mgh_B$

$(9.8)(90) = ½ v_B{}^2 + (9.8)(50)$ $v_B = 28$ m/s

(c) i) FBD

ii) $mg = (700)(9.8) = 6860$ N

$F_{net(C)} = mv^2 / r$

$F_n + mg = mv^2 / r$

$F_n = mv^2/r - mg = m (v^2/r - g) = (700) (28^2/20 - 9.8) = 20580$ N

(d) The friction will remove some of the energy so there will not be as much Kinetic energy at the top of the loop. In order to bring the KE back up to its original value to maintain the original speed, we would need less PE at that location. A lower height of the loop would reduce the PE and compensate to allow the same KE as before. To actually modify the track, you could flatten the inclines on either side of the loop to lower the height at B.

B2004B1.

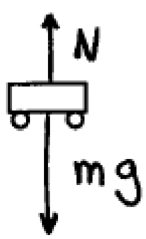
(a) set position A as the h=0 location so that the PE=0 there.

Applying energy conservation with have

$U_{top} + K_{top} = K_A$ $mgh + ½ m v^2 = ½ m v_A{}^2$

$(9.8)(0.1) + ½ (1.5)^2 = ½ v_A{}^2$ $v_A = 2.05$ m/s

(b) FBD



(c) $F_{net(C)} = mv^2 / r$

$mg - F_N = mv^2 / r$

$Fn = mg - mv^2/r = m (g - v^2/r) = (0.5)(9.8 - 2.05^2 / 0.95) = 2.7$ N

(c) To stop the cart at point A, all of the kinetic energy that would have existed here needs to be removed by the work of friction which does negative work to remove the energy.

$W_{fk} = - K_A$

$W_{fk} = - ½ m v_A{}^2 = - ½ (0.5)(2.05^2) = - 1.1$ J

(d) The car is rolling over a hill at point A and when F_n becomes zero the car just barely loses contact with the track. Based on the equation from part (c) the larger the quantity (mv^2 / r) the more likely the car is to lose contact with the track (since more centripetal force would be required to keep it there) … to increase this quantity either the velocity could be increased or the radius could be decreased. To increase the velocity of the car, make the initial hill higher to increase the initial energy. To decrease the radius, simply shorten the hill length.

B2005B2.

FBD i) ii)

(b) Apply energy conservation?

$U_{top} = K_{bottom}$

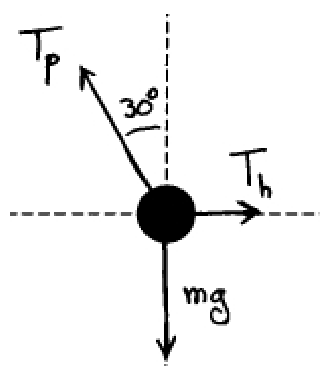
$mgh = \frac{1}{2} m v^2$ $(9.8)(.08) - \frac{1}{2} v^2$ $v = 1.3$ m/s

(c) $F_{net(c)} = mv^2/r$

$F_t - mg = mv^2/r$ $F_t = mv^2/r + mg$ $(0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $F_t = 0.93$ N

2005B2.

(a) FBD



(b) Apply $F_{net(X)} = 0$ $F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $T_P \sin 30 = T_H$

$T_P = 20.37$ N $T_H = 10.18$ N

(c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = \frac{1}{2} m v^2$

$g(L - L \cos \theta) = \frac{1}{2} v^2$

$(10) (2.3 - 2.3 \cos 30) = \frac{1}{2} v^2$ $v_{bottom} = 2.5$ m/s

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B2006B2.

(a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = \frac{1}{2} m v^2$ $Mgh = \frac{1}{2} (M) (3.5v_o)^2$ $h = 6.125\, v_o^2 / g$

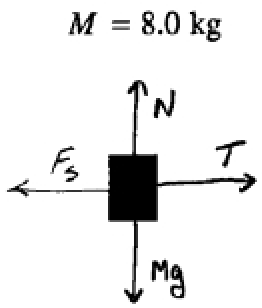
(b) $W_{NC} = \Delta K$ $(K_f - K_i)$ $K_f = 0$

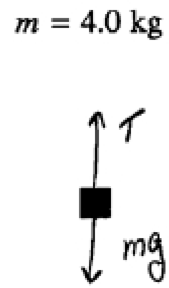
$- f_k d = 0 - \frac{1}{2} (1.5M)(2v_o)^2$

$\mu_k (1.5 M) g (d) = 3Mv_o^2$ $\mu_k = 2v_o^2 / gD$

2006B1.

(a) FBD





(b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

(c) Tension in string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k\,(0.05)$ $k = 780$ N/m

(d) 4 kg mass is in free fall. $D = v_i t + \frac{1}{2} g\, t^2$ $-0.7 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.38$ sec

(e) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $\frac{1}{2} k\, \Delta x^2 = \frac{1}{2} mv^2$ $\frac{1}{2}(780)(0.05) = \frac{1}{2}(8)\, v^2$ $v = 0.49$ m/s

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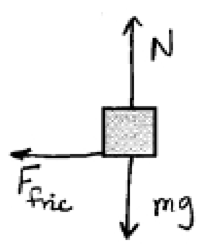
B2008B2.

(a) $d = v_i t + \frac{1}{2} a\, t^2$ $(55) = (25)(3) + \frac{1}{2} a\, (3)^2$ $a = -4.4$ m/s^2

(b) FBD



(c) using the diagram above and understanding that the static friction is actually responsible for decelerating the box to match the deceleration of the truck, we apply F_{net}
$F_{net} = ma$
$-f_s = -\mu_s F_n = ma$ $-\mu_s\, mg = ma$ $-\mu_s = a/g$ $-\mu_s = -4.4 / 9.8$ $\mu_s = 0.45$

Static friction applied to keep the box at rest relative to the truck bed.

(d) Use the given info to find the acceleration of the truck $a = \Delta v / t = 25/10 = 2.5$ m/s^2
To keep up with the trucks acceleration, the crate must be accelerated by the spring force, apply F_{net}
$F_{net} = ma$ $F_{sp} = ma$ $k\Delta x = ma$ $(9200)(\Delta x) = (900)(2.5)$ $\Delta x = 0.24$ m

(e) If the truck is moving at a constant speed the net force is zero. Since the only force acting directly on the crate is the spring force, the spring force must also become zero therefore the Δx would be zero and is **LESS** than before. Keep in mind the crate will stay on the frictionless truck bed because its inertia will keep it moving forward with the truck (remember you don't necessarily need forces to keep things moving)

2008B2.

(a) In a connected system, we must first find the acceleration of the system as a whole. The spring is internal when looking at the whole system and can be ignored.

$F_{net} = ma$ (4) = (10) a $a = 0.4\ m/s^2 \rightarrow$ the acceleration of the whole system and also of each individual block when looked at separate

Now we look at just the 2 kg block, which has only the spring force acting on its FBD horizontal direction.

$F_{net} = ma$ $F_{sp} = (2)(.4)$ $F_{sp} = 0.8$ N

(b) Use $F_{sp} = k\Delta x$ $0.8 = (80)\ \Delta x$ $\Delta x = 0.01$ m

(c) Since the same force is acting on the same total mass and $F_{net} = ma$, the acceleration is the same

(d) The spring stretch will be MORE. This can be shown mathematically by looking at either block. Since the 8 kg block has only the spring force on its FBD we will look at that one.

$F_{sp} = ma$ $k\Delta x = ma$ $(80)(\Delta x) = (8)(0.4)$ $\Delta x = 0.04$ m

(e) When the block A hits the wall it instantly stops, then block B will begin to compress the spring and transfer its kinetic energy into spring potential energy. Looking at block B energy conservation:

$K_b = U_{sp}$ $\frac{1}{2} m v_b^2 = \frac{1}{2} k \Delta x^2$ $(8)(0.5)^2 = (80)\Delta x^2$ $\Delta x = 0.16$ m

2009B1.

(a) Apply energy conservation. All of the spring potential becomes gravitational potential

$U_{sp} = U$

$\frac{1}{2} k \Delta x^2 = mgh$ $\quad$ $\frac{1}{2} k x^2 = mgh$ $\quad$ $h = kx^2 / 2mg$

(b) You need to make a graph that is of the form y = m x, with the slope having "k" as part of it and the y and x values changing with each other. Other constants can also be included in the slope as well to make the y and x variables simpler. h is dependent on the different masses used so we will make h our y value and use m as part of our x value. Rearrange the given equation so that is it of the form y = mx with h being y and mass related to x.

We get $\quad$ y = m x

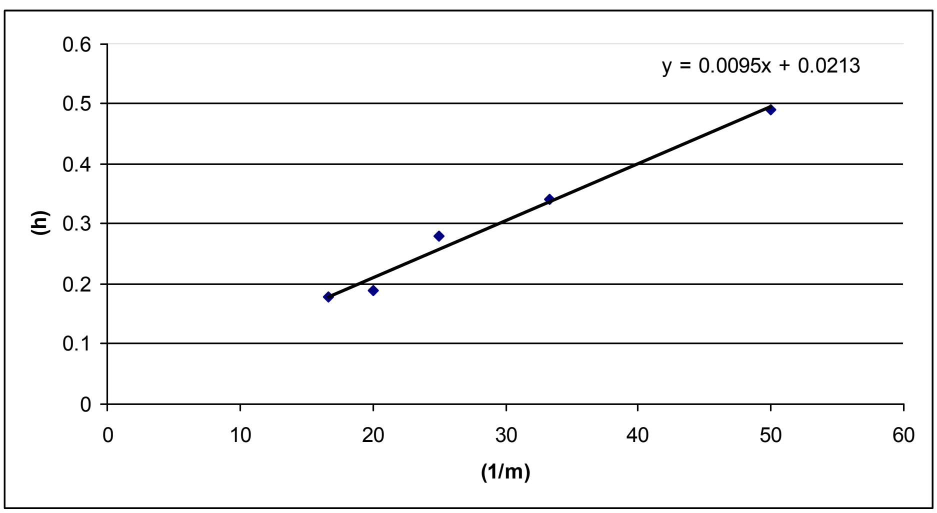
$$h = \left(\frac{kx^2}{2g}\right)\frac{1}{m}$$

so we use h as y and the value 1/m as x and graph it.

(note: we lumped all the things that do not change together as the constant slope term. Once we get a value for the slope, we can set it equal to this term and solve for k)

1/m	m (kg)	h (m)
50	0.020	0.49
33.33	0.030	0.34
25	0.040	0.28
20	0.050	0.19
16.67	0.060	0.18
X values		Y values

(c) Graph



(d) The slope of the best fit line is 0.01

We set this slope equal to the slope term in our equation, plug in the other known values and then solve it for k

$$0.01 = \left(\frac{kx^2}{2g}\right)$$

$$0.01 = \left(\frac{k(0.02)^2}{2(9.8)}\right)$$

Solving gives us $\quad k = 490$ N/m

(e) - Use a stopwatch, or better, a precise laser time measurement system (such as a photogate), to determine the time it takes the toy to leave the ground and raise to the max height (same as time it takes to fall back down as well). Since its in free fall, use the down trip with v_i=0 and apply $d = \frac{1}{2} g t^2$ to find the height.

- Or, videotape it up against a metric scale using a high speed camera and slow motion to find the max h.

C1973M2

(a) Apply work-energy theorem
$W_{nc} = \Delta ME$
$W_{fk} = \Delta K \quad (K_f - K_i) \qquad K_f = 0$
$-f_k d = -\frac{1}{2} mv_i^2 \qquad -f_k (0.12) = -\frac{1}{2} (0.030)(500)^2 \qquad f_k = 31250$ N

(b) Find find acceleration $\qquad -f_k = ma \qquad -(31250) = (0.03)a \qquad a = -1.04\text{x}10^6$ m/s^2

Then use kinematics $\qquad v_f = v_i + at \qquad 0 = 500 + (-1.04\text{x}10^6)t \qquad t = 4.8\text{x}10^{-4}$ sec

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C1982M1
(a) Apply energy conservation, set the top of the spring as h=0, therefore H at start = Lsin θ = 6 sin30 = 3 m

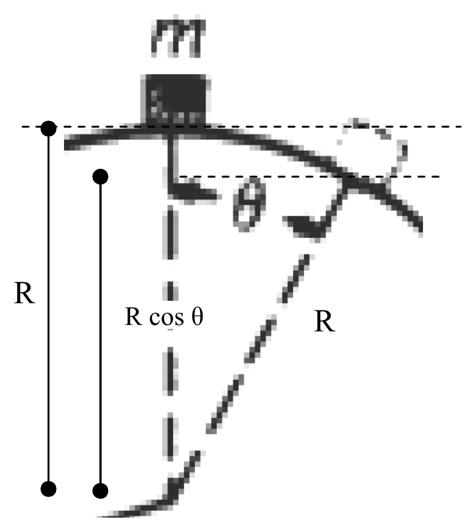
$U_{top} = K_{bot} \quad mgh = \frac{1}{2} mv^2 \quad (9.8)(3) = \frac{1}{2}(v^2) \qquad v = 7.67$ m/s

(b) Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.
$h = L\sin\theta + \Delta x \sin\theta$

$U_{top} = U_{sp}(\text{bot})$
$mgh = \frac{1}{2} k \Delta x^2$
$mg(L\sin\theta + \Delta x \sin\theta) = \frac{1}{2} k \Delta x^2$
$(20)(9.8)(6 \sin 30 + 3 \sin 30) = \frac{1}{2} k (3)^2 \qquad k = 196$ N/m

(c) The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

C1983M3.



$h = R - R\cos\theta = R(1 - \cos\theta)$

i) $K_2 = U_{top}$
$K_2 = mg(R(1 - \cos\theta))$

ii) From, $K = \frac{1}{2} m v^2 = mgR(1 - \cos\theta)$ … $v^2 = 2gR(1 - \cos\theta)$

Then $a_c = v^2 / R = \quad 2g(1 - \cos\theta)$

C1985M1

(a) We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} - f_k = 0$ $mg \sin\theta = \mu_s(F_n)$
$mg \sin\theta = \mu_s\, mg \cos\theta$ $\mu_s = \tan\theta$

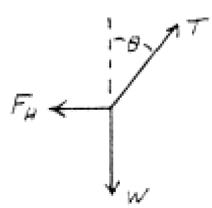
(b) Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so h= (d+x)sin θ
$U_f - U_i + U_{sp(F)} - U_{sp(i)}$
$0 - mgh + \frac{1}{2} k\Delta x^2 - 0$ $\frac{1}{2} kx^2 - mg(d+x)\sin\theta$

(c) To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k
$W_{NC} = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$ $-f_k(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$
$-\mu_k\, mg \cos\theta\,(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$\mu_k = [mg(d+x)\sin\theta - \frac{1}{2} kx^2] / [mg\,(d+x)\cos\theta]$

C1987M1

(a)



$F_{net(y)} = 0$
$T \cos\theta - W = 0$ $T = W / \cos\theta$

(b) Apply SIMULTANEOUS EQUATIONS
$F_{net(y)} = 0$ $F_{net(x)} = 0$
$T \cos\theta - W = 0$ $T \sin\theta - F_h = 0$
Sub T into X equation to get F_h $F_h = W \tan\theta$

(c) Using the same geometry diagram as solution 1975B7 solve for the velocity at the bottom using energy conservation

$U_{top} = K_{bot}$
$mgh = \frac{1}{2} mv^2$
$$v = \sqrt{2g(L - L\cos\theta)}$$
$$v = \sqrt{2gL(1 - \cos\theta)}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(T - W) = m(2gL(1-\cos\theta)) / L$

$T = W + 2mg - 2mg \cos\theta$
$T = W + 2W - 2W \cos\theta = W(3 - 2\cos\theta)$

C1988M2

(a) The graph is one of force vs Δx so the slope of this graph is the spring constant. Slope = 200 N/m

(b) Since there is no friction, energy is conserved and the decrease in kinetic energy will be equal to the gain in spring potential $|\Delta K| = U_{sp(f)} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2}(200)(0.1)^2 = 1$J.
Note: This is the same as the area under the line since the area would be the work done by the conservative spring force and the work done by a conservative force is equal to the amount of energy transferred.

(c) Using energy conservation. $K_i = U_{sp(f)}$ $\frac{1}{2} mv_o^2 = 1$ J $\frac{1}{2}(5)\, v_o^2 = 1$ $v_o = 0.63$ m/s

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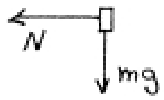
C1989M1

(a) Apply energy conservation from point A to point C setting point C as h=0 location
(note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

$U_A = K_C$ $mgh_a = ½ m v_c^2$ $(0.1)(9.8)(h_a) = ½ (0.1)(4)^2$ $h_a = 0.816m$
$h = h_a + 0.5 m = 1.32 m$

(b)



(c) Since the height at B and the height at C are the same, they would have to have the same velocities $v_b = 4$ m/s

(d) $F_{net(c)} = mv^2 / r$ $F_n = (0.1)(4)^2/(0.5) = 3.2$ N

(e) Using projectile methods … $V_{iy} = 4\sin 30 = 2$ m/s Then $v_{fy}^2 = v_{iy}^2 + 2 a d_y$
$(0) = (2)^2 + 2(-9.8)(d_y)$ $d_y = 0.2$
$h_{max} = d_y$ + initial height $= 0.7$ m

Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

(f) Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $U_{new} - U_{old} = mgh_{net} - mgh_{old}$ $(0.1)(9.8)(2-1.32) = 0.67$ J lost.

C1989M3

(a) Apply energy conservation from start to top of spring using h=0 as top of spring.
$U = K$ $mgh = ½ m v^2$ $(9.8)(0.45) = ½ v^2$ $v = 3$ m/s

(b) At equilibrium the forces are balances $F_{net} = 0$ $F_{sp} = mg =(2)(9.8) = 19.6$ N

(c) Using the force from part b, $F_{sp} = k \Delta x$ $19.6 = 200 \Delta x$ $\Delta x = 0.098$ m

(d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the start position is now increased by the amount of Δx found in part c $h_{new} = h+\Delta x = 0.45 + 0.098 = 0.548$ m)
$U_{top} = U_{sp} + K$
$mgh = ½ k \Delta x^2 + ½ mv^2$ $(2)(9.8)(0.548) = ½ (200)(0.098)^2 + ½ (2)(v^2)$ $v = 3.13$ m/s

(e) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

C1990M2

(a) Energy conservation, $K_{bot} = U_{top}$ $½ m v^2 = mgh$ $½ (v_o^2) = gh$ $h = v_o^2 / 2g$

(b) Work-Energy theorem. $W_{nc} = \Delta K + \Delta U$ $(U_i = 0, K_f = 0)$
$- f_k d = (mgh - 0) + (0 - ½ m v_o^2)$ $- (\mu_k mg\cos\theta) h_2 / \sin\theta = mgh_2 - ½ mv_o^2$

$\mu mg \cos\theta\, h_2 / \sin\theta + mgh_2 = ½ mv_o^2$ $h_2 (\mu g \cos\theta / \sin\theta + g) = ½ v_o^2$
$h_2 = v_o^2 / (2g(\mu \cot\theta+1))$

C1991M1

(a) Apply energy conservation.

$K_{bottom} = U_p + K_p$ $\qquad$ $\frac{1}{2} mv_{bot}^2 = mgh_p + K_p$

$\frac{1}{2}\, 3m\, (v_o/3)^2 = 3mg(r) + K_p$ $\qquad$ $K_p = mv_o^2/6 - 3mgr$

(b) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set $F_n=0$ at the top of the loop so that only mg is acting down on the block. Then apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\qquad$ $3mg = 3m\, v^2 /r$ $\qquad$ $v = \sqrt{rg}$

(c) Energy conservation, top of loop to bottom of loop

$U_{top} + K_{top} = K_{bot}$

$mgh + \frac{1}{2} m\, v_{top}^2 = \frac{1}{2} m\, v_{bot}^2$ $\qquad$ $g(2r) + \frac{1}{2} (\sqrt{rg})^2 = \frac{1}{2} (vo')^2$ $\qquad$ $v_o{}' = \sqrt{5gr}$

C1993M1

- since there is friction on the surface the whole time, this is not an energy conservation problem, use work-energy.

(a) $U_{sp} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2} (400)(0.5)^2 = 50$ J

(b) Using work-energy

$W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)} - U_{sp(i)}) + (K_f - K_i)$

$- f_k d = (\ 0 - 50J\ ) + (\ \frac{1}{2} m\, v_f^2 - 0)$

$- \mu\, mg\, d = \frac{1}{2} mv_f^2 - 50$

$- (0.4)(4)(9.8)(0.5) = \frac{1}{2} (4)(v_c^2) - 50$ $\qquad$ $v_c = 4.59$ m/s

(c) $W_{nc} = (K_f - K_i)$

$-f_k d = (0 - \frac{1}{2} m\, v_i^2\ )$ $\qquad$ $- \mu\, mg\, d = - \frac{1}{2} m\, v_i^2$ $\qquad$ $(0.4)(6)(9.8)\, d = \frac{1}{2} (6)(3)^2$ $\qquad$ d = 1.15 m

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C2002M2

(a) Energy conservation, potential top = kinetic bottom $\qquad$ $v = \sqrt{2gh}$

(b) Energy conservation, potential top = spring potential $\qquad$ $U=U_{sp}$ $\qquad$ $(2m)gh = \frac{1}{2} k\, x_m^2$

$$x_m = 2\sqrt{\frac{mgh}{k}}$$

C2004M1

(a) Energy conservation with position B set as h=0. $U_a = K_b$ $\quad$ $v_b = \sqrt{2gL}$

(b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = mv_b^2 / r$ $\qquad$ $F_t - mg = m(2gL) / L$ $\qquad$ $F_t = 3mg$

(c) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + \frac{1}{2} g\, t^2$ $\qquad$ $L = 0 + gt^2 / 2$

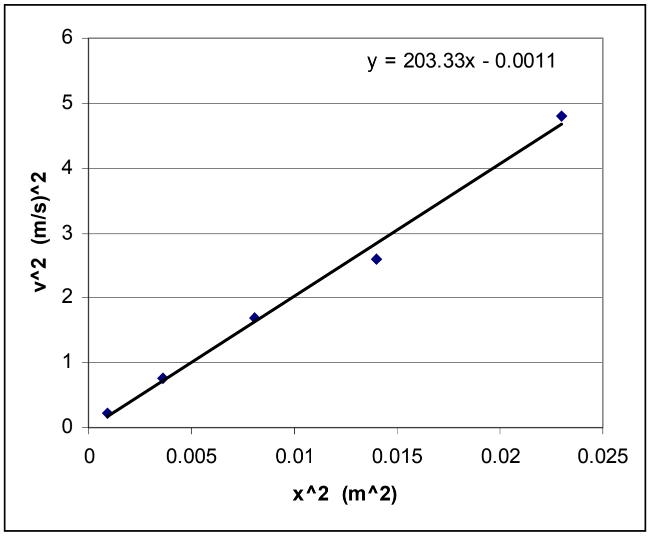
$t = \sqrt{\frac{2L}{g}}$ $\quad$ Then use $v_x = d_x / t$ $\quad$ $d_x = v'\sqrt{\frac{2L}{g}}$ $\quad$ total distance $\quad$ $x = v'\sqrt{\frac{2L}{g}} + L$

total distance includes the initial horizontal displacement L so it is added to the range

C2007M3

(a) Spring potential energy is converted into kinetic energy $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

(b) (c) i)



ii) using the equation above and rearrange to the form $y = m\,x$ with v^2 as y and x^2 as x

$y = m \quad x$

$v^2 = (k/m)\, x^2$

Slope = 200 = k /m

200 = (40) / m

m = 0.2 kg

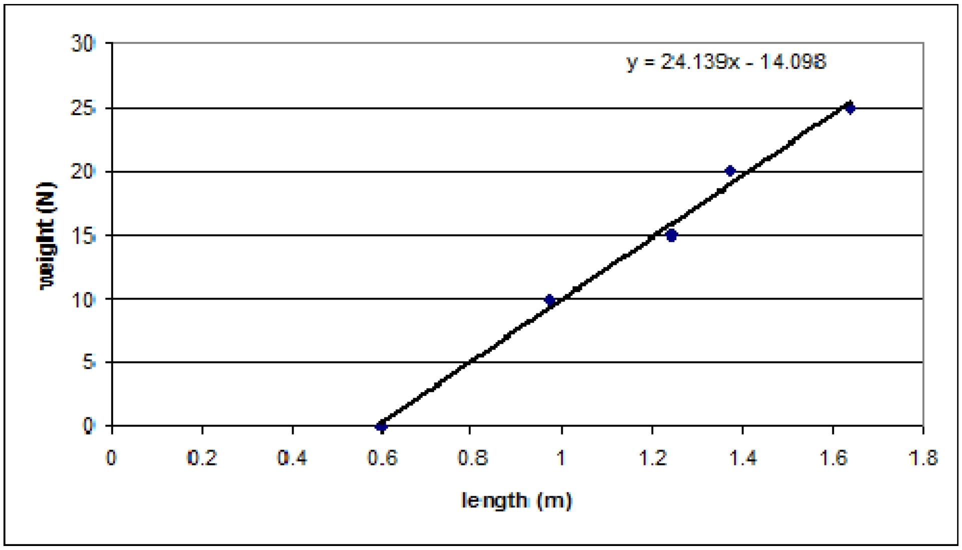
(d) Now you start with spring potential and gravitational potential and convert to kinetic. Note that at position A the height of the glider is given by h + the y component of the stretch distance x. $h_{initial} = h + x \sin\theta$

$U + U_{sp} = K$

$mgh + \frac{1}{2} k\, x^2 = \frac{1}{2} m\, v^2 \qquad mg(h + x \sin\theta) + \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

C2008M3

(a)



(b) The slope of the line is $F / \Delta x$ which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

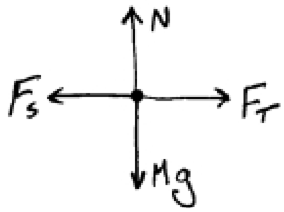
$mgh = \frac{1}{2} k \Delta x^2$ $\quad m(9.8)(1.5) = \frac{1}{2}(24)(0.9)^2$ $\quad m = 0.66$ kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $\quad F_{sp} = (0.66)(9.8)$ $\quad F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $\quad 6.5 = (24)\, \Delta x$ $\quad \Delta x = 0.27$ m

Supplemental

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} k\Delta x_2^2 + \frac{1}{2} mv^2$

$k (\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$

$$v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$$

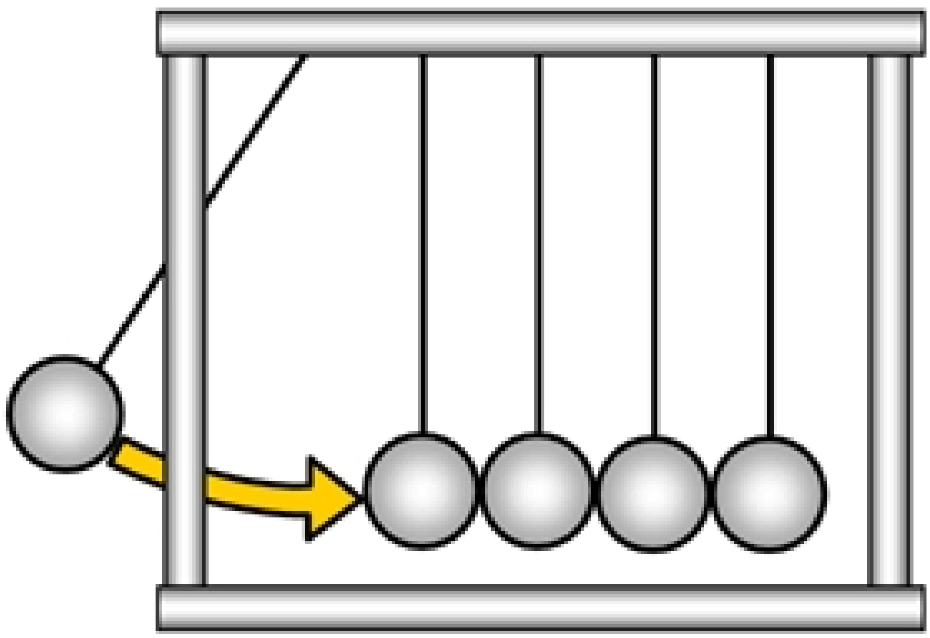
(d) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get

$F_{net} = ma$ $F_{sp} - f_k = ma$

From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$

Chapter 5

Momentum and Impulse



AP Physics Multiple Choice Practice – Momentum and Impulse

1. A car of mass m, traveling at speed v, stops in time t when maximum braking force is applied. Assuming the braking force is independent of mass, what time would be required to stop a car of mass $2m$ traveling at speed v?
(A) ½ t (B) t (C) $\sqrt{2}\,t$ (D) $2t$

2. A block of mass M is initially at rest on a frictionless floor. The block, attached to a massless spring with spring constant k, is initially at its equilibrium position. An arrow with mass m and velocity v is shot into the block. The arrow sticks in the block. What is the maximum compression of the spring?

$$(A)\ v\sqrt{\frac{m}{k}} \qquad (B)\ v\sqrt{\frac{m+M}{k}} \qquad (C)\ \frac{(m+M)v}{\sqrt{mk}} \qquad (D)\ \frac{mv}{\sqrt{(m+M)k}}$$

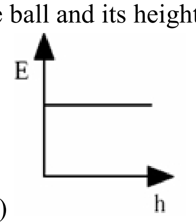
3. Two objects, P and Q, have the same momentum. Q can have more kinetic energy than P if it has:
(A) More mass than P (B) The same mass as P (C) More speed than P (D) The same speed at P

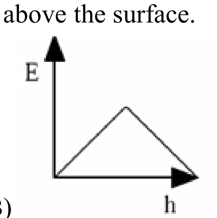
4. A spring is compressed between two objects with unequal masses, m and M, and held together. The objects are initially at rest on a horizontal frictionless surface. When released, which of the following is true?
(A) The total final kinetic energy is zero.
(B) The two objects have equal kinetic energy.
(C) The speed of one object is equal to the speed of the other.
(D) The total final momentum of the two objects is zero.

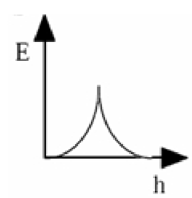
5. Two football players with mass 75 kg and 100 kg run directly toward each other with speeds of 6 m/s and 8 m/s respectively. If they grab each other as they collide, the combined speed of the two players just after the collision would be:
(A) 2 m/s (B) 3.4 m/s (C) 4.6 m/s (D) 7.1 m/s

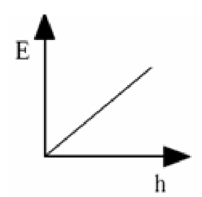
6. A 5000 kg freight car moving at 4 km/hr collides and couples with an 8000 kg freight car which is initially at rest. The approximate common final speed of these two cars is
(A) 1 km/h (B) 1.3 km/h (C) 1.5 km/h (D) 2.5 km/h

7. A rubber ball is held motionless a height h_o above a hard floor and released. Assuming that the collision with the floor is elastic, which one of the following graphs best shows the relationship between the total energy E of the ball and its height h above the surface.

(A)


(B)


(C)


(D)


8. Two carts are held together. Cart 1 is more massive than Cart 2. As they are forced apart by a compressed spring between them, which of the following will have the same magnitude for both carts.
(A) change of velocity (B) force (C) speed (D) velocity

9. A ball with a mass of 0.50 kg and a speed of 6 m/s collides perpendicularly with a wall and bounces off with a speed of 4 m/s in the opposite direction. What is the magnitude of the impulse acting on the ball?
(A) 1 Ns (B) 5 Ns (C) 2 m/s (D) 10 m/s

10. A cart with mass 2m has a velocity v before it strikes another cart of mass 3m at rest. The two carts couple and move off together with a velocity of
(A) $v/5$ (B) $2v/5$ (C) $2v/3$ (D) $(2/5)^{1/2}\,v$

11. **Multiple Correct:** Consider two laboratory carts of different masses but identical kinetic energie . Which of the following statements must be correct? Select two answers.
    (A) The one with the greatest mass has the greatest momentum
    (B) The same impulse was required to accelerate each cart from rest
    (C) Both can do the same amount of work as they come to a stop
    (D) The same amount of force was required to accelerate each cart from rest

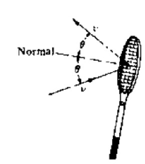
12. A mass m has speed v. It then collides with a stationary object of mass 2m. If both objects stick together in a perfectly inelastic collision, what is the final speed of the newly formed object?
    (A) v / 3 (B) v / 2 (C) 2v / 3 (D) 3v / 2

13. A 50 kg skater at rest on a frictionless rink throws a 2 kg ball, giving the ball a velocity of 10 m/s. Which statement describes the skater's subsequent motion?
    (A) 0.4 m/s in the same direction as the ball's motion.
    (B) 0.4 m/s in the opposite direction of the ball's motion.
    (C) 2 m/s in the same direction as the ball's motion.
    (D) 2 m/s in the opposite direction of the ball's motion.

14. A student initially at rest on a frictionless frozen pond throws a 1 kg hammer in one direction. After the throw, the hammer moves off in one direction while the student moves off in the other direction. Which of the following correctly describes the above situation?
    (A) The hammer will have the momentum with the greater magnitude
    (B) The student will have the momentum with the greater magnitude
    (C) The hammer will have the greater kinetic energy
    (D) The student will have the greater kinetic energy

15. Two toy cars with different masses originally at rest are pushed apart by a spring between them. Which TWO of the following statements would be true?
    (A) both toy cars will acquire equal but opposite momenta
    (B) both toy cars will acquire equal kinetic energies
    (C) the more massive toy car will acquire the least speed
    (D) the smaller toy car will experience an acceleration of the greatest magnitude

16. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially as shown. The magnitude of the momentum change of the ball is

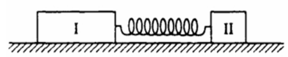


    (A) 0 (B) 2mv (C) 2mv sinθ (D) 2mv cosθ

17. Two bodies of masses 5 and 7 kilograms are initially at rest on a horizontal frictionless surface. A light spring is compressed between the bodies, which are held together by a thin thread. After the spring is released by burning through the thread, the 5 kilogram body has a speed of 0.2 m/s. The speed of the 7 kilogram body is (in m/s)

    (A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{1}{5}$ (D) $\frac{1}{\sqrt{35}}$

18. **Multiple Correct:** A satellite of mass M moves in a circular orbit of radius R at a constant speed v around the Earth which has mass M_E. Which of the following statements must be true? Select two answers:
    (A) The net force on the satellite is equal to $Mv^2/2$ and is directed toward the center of the orbit.
    (B) The net work done on the satellite by gravity in one revolution is zero.
    (C) The angular momentum of the satellite is a constant.
    (D) The net force on the satellite is equal to GMM_E/R



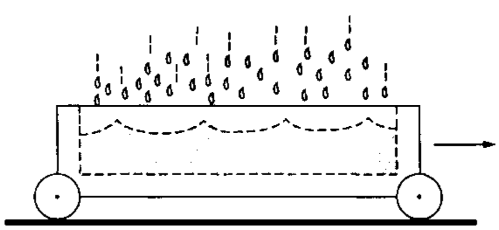
19. Two pucks are firmly attached by a stretched spring and are initially held at rest on a frictionless surface, as shown above. The pucks are then released simultaneously. If puck I has three times the mass of puck II, which of the following quantities is the same for both pucks as the spring pulls the two pucks toward each other?
(A) Speed (B) Magnitude of acceleration (C) Kinetic energy (D) Magnitude of momentum

20. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?
(A) The collision is elastic.
(B) The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.
(C) The speed of the objects that are stuck together will be less than the initial speed of the less massive object.
(D) The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

21. Two objects having the same mass travel toward each other on a flat surface each with a speed of 1.0 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 2.0 meters per second relative to the surface. Which of the following assessments of this report is most accurate?
(A) Momentum was not conserved therefore the report is false.
(B) If potential energy was released to the objects during the collision the report could be true.
(C) If the objects had different masses the report could be true.
(D) If the surface was inclined the report could be true.

22. A solid metal ball and a hollow plastic ball of the same external radius are released from rest in a large vacuum chamber. When each has fallen 1 meter, they both have the same
(A) inertia (B) speed (C) momentum (D) change in potential energy

23. A railroad car of mass *m* is moving at speed *v* when it collides with a second railroad car of mass *M* which is at rest. The two cars lock together instantaneously and move along the track. What is the kinetic energy of the cars immediately after the collision?
(A) $\frac{1}{2} mv^2$ (B) $\frac{1}{2}(M+m)(mv/M)^2$ (C) $\frac{1}{2}(M+m)(Mv/m)^2$ (D) $\frac{1}{2}(M+m)(mv/(m+M))^2$

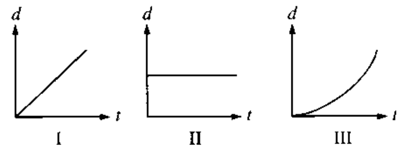


24. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of mechanical energy
(B) decrease because of conservation of momentum
(C) decrease because of conservation of mechanical energy
(D) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

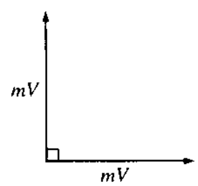
Questions 25-26

Three objects can only move along a straight, level path. The graphs below show the position *d* of each of the objects plotted as a function of time *t*.

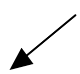
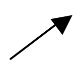
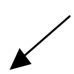


25. The magnitude of the momentum of the object is increasing in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only

26. The sum of the forces on the object is zero in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only

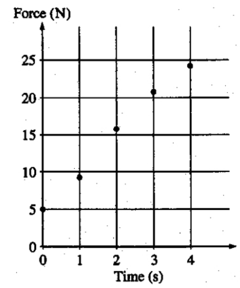


27. A stationary object explodes, breaking into three pieces of masses *m*, *m*, and *3m*. The two pieces of mass *m* move off at right angles to each other with the same magnitude of momentum mV, as shown in the diagram above. What are the magnitude and direction of the velocity of the piece having mass *3m* ?

Magnitude	Direction
(A) $\frac{V}{\sqrt{2}}$	
(B) $\frac{V}{\sqrt{2}}$	
(C) $\frac{\sqrt{2}V}{3}$	
(D) $\frac{\sqrt{2}V}{3}$	

28. An empty sled of mass M moves without friction across a frozen pond at speed v_o. Two objects are dropped vertically into the sled one at a time: first an object of mass m and then an object of mass 2m. Afterward the sled moves with speed v_f. What would be the final speed of the sled if the objects were dropped into it in reverse order?
(A) $v_f / 3$
(B) $v_f / 2$
(C) v_f
(D) $2v_f$

29. A student obtains data on the magnitude of force applied to an object as a function of time and displays the data on the graph shown. The increase in the momentum of the object between t=0 s and t=4 s is most nearly

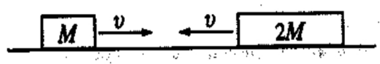


(A) 40 N·s
(B) 50 N·s
(C) 60 N·s
(D) 80 N·s

30. How does an air mattress protect a stunt person landing on the ground after a stunt?
(A) It reduces the kinetic energy loss of the stunt person.
(B) It reduces the momentum change of the stunt person.
(C) It shortens the stopping time of the stunt person and increases the force applied during the landing.
(D) It lengthens the stopping time of the stunt person and reduces the force applied during the landing.

31. Two objects, A and B, initially at rest, are "exploded" apart by the release of a coiled spring that was compressed between them. As they move apart, the velocity of object A is 5 m/s and the velocity of object B is – 2 m/s. The ratio of the mass of object A to the mass object B, m_a/m_b is

(A) 4/25 (B) 2/5 (C) 5/2 (D) 25/4

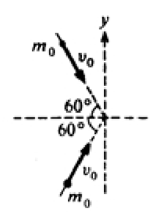


32. The two blocks of masses M and 2M shown above initially travel at the same speed v but in opposite directions. They collide and stick together. How much mechanical energy is lost to other forms of energy during the collision?

(A) 1/2 M v^2
(B) 3/4 M v^2
(C) 4/3 M v^2
(D) 3/2 M v^2

33. Two particles of equal mass m_o, moving with equal speeds v_o along paths inclined at 60° to the x-axis as shown, collide and stick together. Their velocity after the collision has magnitude

(A) $\frac{v_0}{4}$ (B) $\frac{v_0}{2}$ (C) $\frac{\sqrt{3}v_0}{2}$ (D) v_o



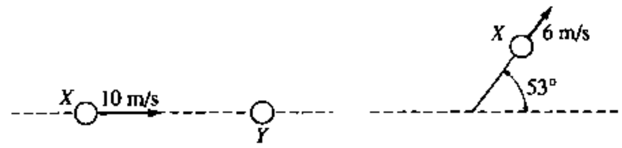
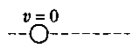
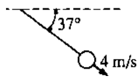
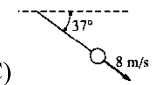


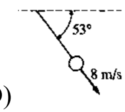
Figure I   Figure II

34. Two balls are on a frictionless horizontal tabletop. Ball X initially moves at 10 meters per second, as shown in Figure I above. It then collides elastically with identical ball Y which is initially at rest. After the collision, ball X moves at 6 meters per second along a path at 53° to its original direction, as shown in Figure II above. Which of the following diagrams best represents the motion of ball Y after the collision?

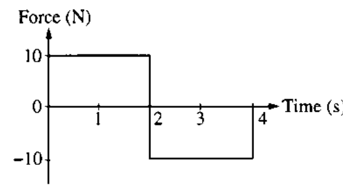
(A) 

(B) 

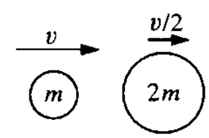
(C) 

(D) 

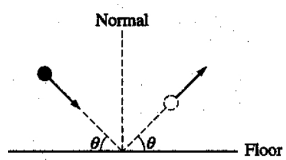
35. The graph shows the force on an object of mass M as a function of time. For the time interval 0 to 4 s, the total change in the momentum of the object is
(A) 40 kg m/s (B) 20 kg m/s
(C) 0 kg m/s (D) – 20 kg m/s



36. As shown in the top view, a disc of mass m is moving horizontally to the right with speed v on a table with negligible friction when it collides with a second disc of mass $2m$. The second disc is moving horizontally to the right with speed $v/2$ at the moment before impact. The two discs stick together upon impact. The kinetic energy of the composite body immediately after the collision is
(A) $(1/6)mv^2$ (B) $(1/2)mv^2$ (C) $2/3mv^2$ (D) $(9/8)mv^2$



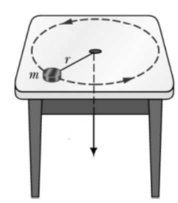
Top View



37. A 2 kg ball collides with the floor at an angle θ and rebounds at the same angle and speed as shown above. Which of the following vectors represents the impulse exerted on the ball by the floor?

(A) ↘ (B) → (C) ↗ (D) ↑

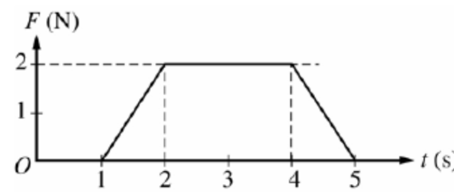
38. An object m, on the end of a string, moves in a circle (initially of radius r) on a horizontal frictionless table as shown. As the string is pulled very slowly through a small hole in the table, which of the following is correct for an observer measuring from the hole in the table?
(A) The angular momentum of m remains constant.
(B) The angular momentum of m decreases.
(C) The kinetic energy of m remains constant
(D) The kinetic energy of m decreases



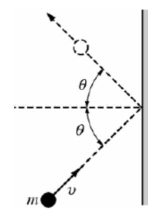
39. A boy of mass m and a girl of mass $2m$ are initially at rest at the center of a frozen pond. They push each other so that she slides to the left at speed v across the frictionless ice surface and he slides to the right. What is the total work done by the children?
(A) mv (B) mv^2 (C) $2mv^2$ (D) $3mv^2$

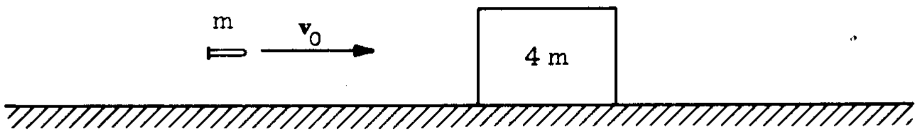
40. An object of mass M travels along a horizontal air track at a constant speed v and collides elastically with an object of identical mass that is initially at rest on the track. Which of the following statements is true for the two objects after the impact?
(A) The total momentum is Mv and the total kinetic energy is ½ Mv^2
(B) The total momentum is Mv and the total kinetic energy is less than ½ Mv^2
(C) The total momentum is less than Mv and the total kinetic energy is ½ Mv^2
(D) The momentum of each object is ½ Mv

41. A 2 kg object initially moving with a constant velocity is subjected to a force of magnitude F in the direction of motion. A graph of F as a function of time t is shown. What is the increase, if any, in the velocity of the object during the time the force is applied?
(A) 0 m/s (B) 3.0 m/s (C) 4.0 m/s (D) 6.0 m/s



42. A ball of mass m with speed v strikes a wall at an angle θ with the normal, as shown. It then rebounds with the same speed and at the same angle. The impulse delivered by the ball to the wall is
(A) mv sin θ (B) mv cos θ (C) 2mv sin θ (D) 2mv cos θ

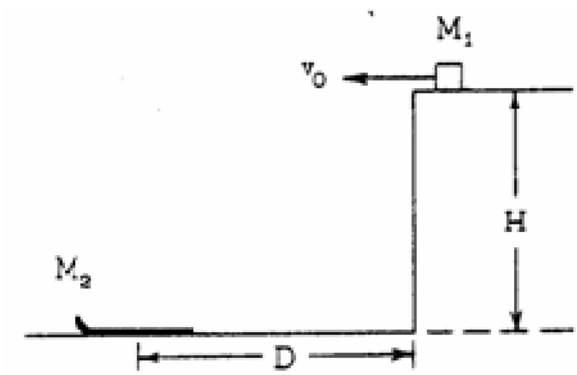




1976B2.

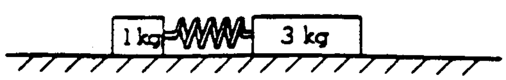
A bullet of mass m and velocity v_o is fired toward a block of mass 4m. The block is initially at rest on a frictionless horizontal surface. The bullet penetrates the block and emerges with a velocity of $\frac{v_o}{3}$

(a) Determine the final speed of the block.
(b) Determine the loss in kinetic energy of the bullet.
(c) Determine the gain in the kinetic energy of the block.



1978B2. A block of mass M_1 travels horizontally with a constant speed v_o on a plateau of height H until it comes to a cliff. A toboggan of mass M_2 is positioned on level ground below the cliff as shown above. The center of the toboggan is a distance D from the base of the cliff.

(a) Determine D in terms of v_o, H, and g so that the block lands in the center of the toboggan.
(b) The block sticks to the toboggan which is free to slide without friction. Determine the resulting velocity of the block and toboggan.



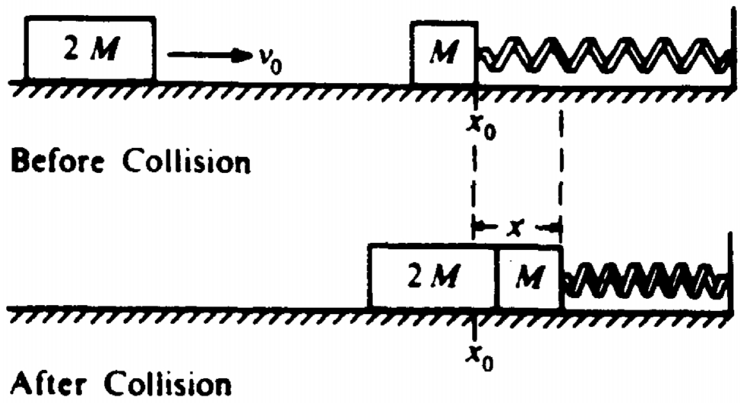
1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table.

In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second.

a. Determine the minimum work needed to compress the spring in this experiment.

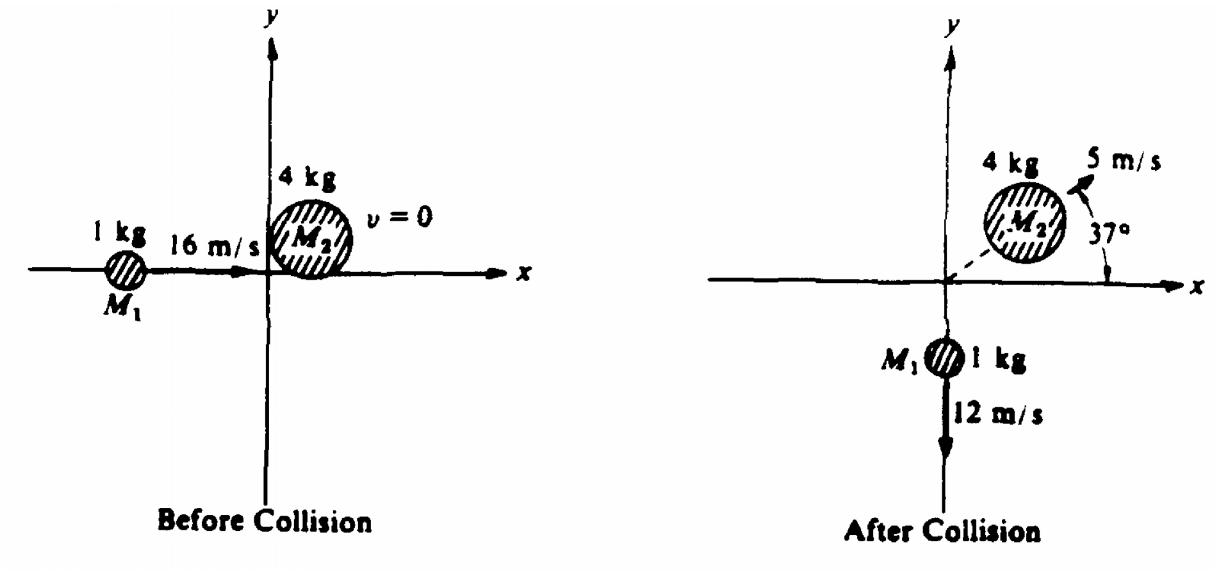
In a different experiment, the spring is compressed again exactly as above, but this time both masses are released simultaneously and each mass moves off separately at unknown speeds.

b. Determine the final velocity of each mass relative to the cable after the masses are released.



1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact
b. x, the maximum distance the spring is compressed



View From Above

1984B2. Two objects of masses M_1 = 1 kilogram and M_2 = 4 kilograms are free to slide on a horizontal frictionless surface. The objects collide and the magnitudes and directions of the velocities of the two objects before and after the collision are shown on the diagram above. (sin 37° = 0.6, cos 37° = 0.8, tan 37° = 0.75)

a. Calculate the x and y components (p_x and p_y, respectively) of the momenta of the two objects before and after the collision, and write your results in the proper places in the following table.

	M_1 = 1 kg		M_2 = 4 kg	
	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$
Before Collision				
After Collision				

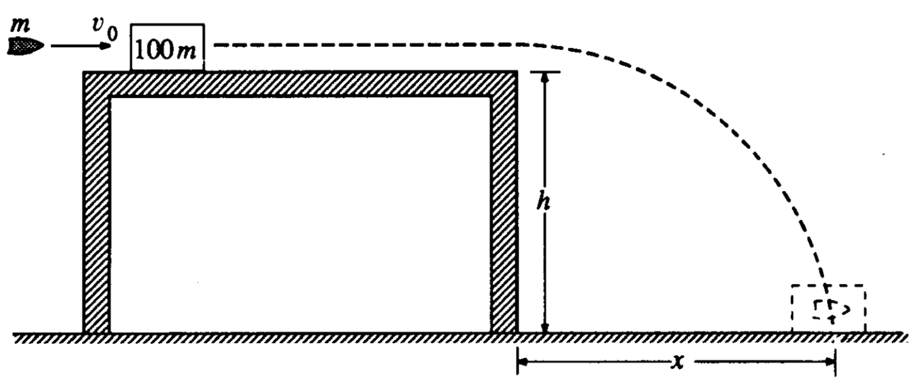
b. Show, using the data that you listed in the table, that linear momentum is conserved in this collision.

c. Calculate the kinetic energy of the two-object system before and after the collision.

d. Is kinetic energy conserved in the collision?

1985B1. A 2-kilogram block initially hangs at rest at the end of two 1-meter strings of negligible mass as shown on the left diagram above. A 0.003-kilogram bullet, moving horizontally with a speed of 1000 meters per second, strikes the block and becomes embedded in it. After the collision, the bullet/ block combination swings upward, but does not rotate.

a. Calculate the speed v of the bullet/ block combination just after the collision.
b. Calculate the ratio of the initial kinetic energy of the bullet to the kinetic energy of the bullet/ block combination immediately after the collision.
c. Calculate the maximum vertical height above the initial rest position reached by the bullet/block combination.



1990B1. A bullet of mass m is moving horizontally with speed v_o when it hits a block of mass 100m that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact, the bullet and the block slide off the table and hit the floor a distance x from the edge of the table. Derive expressions for the following quantities in terms of m, h, v_o, and appropriate constants:

a. the speed of the block as it leaves the table
b. the change in kinetic energy of the bullet-block system during impact
c. the distance x

Suppose that the bullet passes through the block instead of remaining in it.

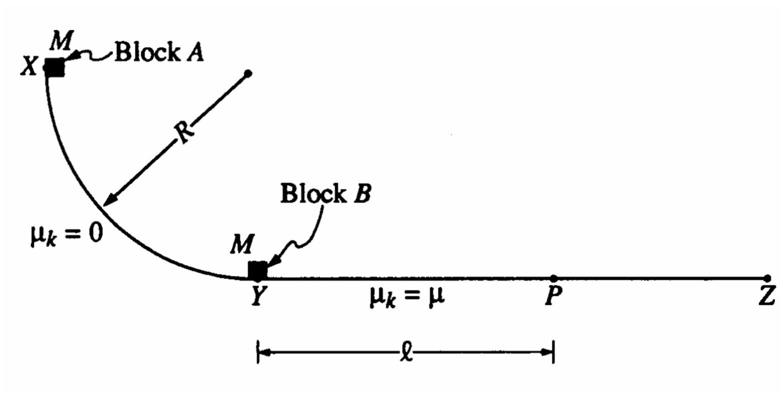
d. State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.
e. State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

1992B2. A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

a. Determine the speed of the child-sled system after the child jumps onto the sled.
b. Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.

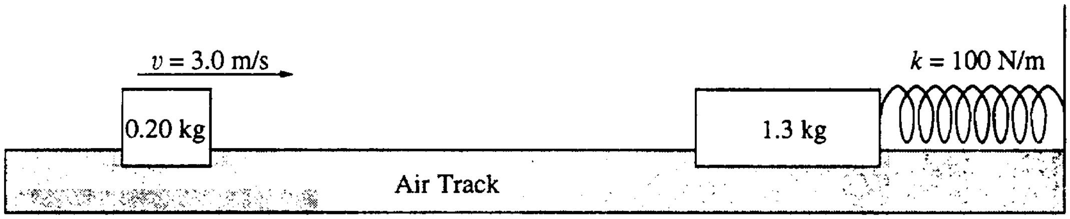
c. Determine the speed of the sled after the child jumps off it.
d. Determine the kinetic energy of the child-sled system when the child is at rest on the ice.
e. Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?



Side View

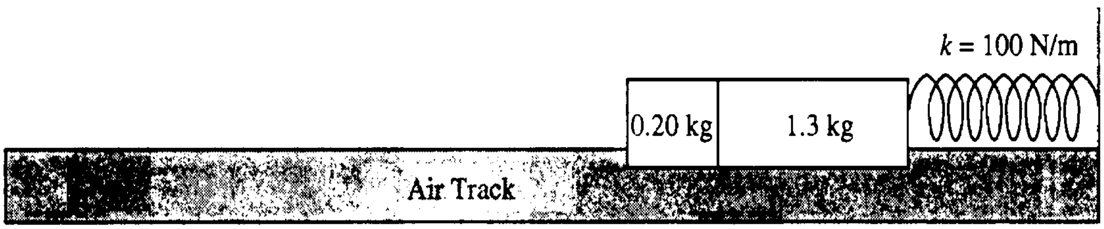
1994B2. A track consists of a frictionless arc XY, which is a quarter-circle of radius R, and a rough horizontal section YZ. Block A of mass M is released from rest at point X, slides down the curved section of the track, and collides instantaneously and inelastically with identical block B at point Y. The two blocks move together to the right, sliding past point P, which is a distance *L* from point Y. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ Express your answers in terms of M, *L*, μ, R, and g.

a. Determine the speed of block A just before it hits block B.
b. Determine the speed of the combined blocks immediately after the collision.
c. Assuming that no energy is transferred to the track or to the air surrounding the blocks. Determine the amount of internal energy transferred in the collision
d. Determine the additional thermal energy that is generated as the blocks move from Y to P



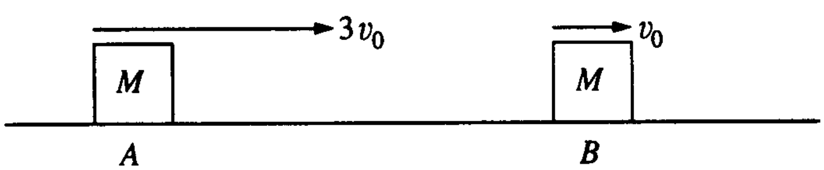
1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

a. Determine the following for the 0.20-kilogram mass immediately before the impact.
   i. Its linear momentum ii. Its kinetic energy
b. Determine the following for the combined masses immediately after the impact.
   i. The linear momentum ii. The kinetic energy



After the collision, the two masses compress the spring as shown.

c. Determine the maximum compression distance of the spring.

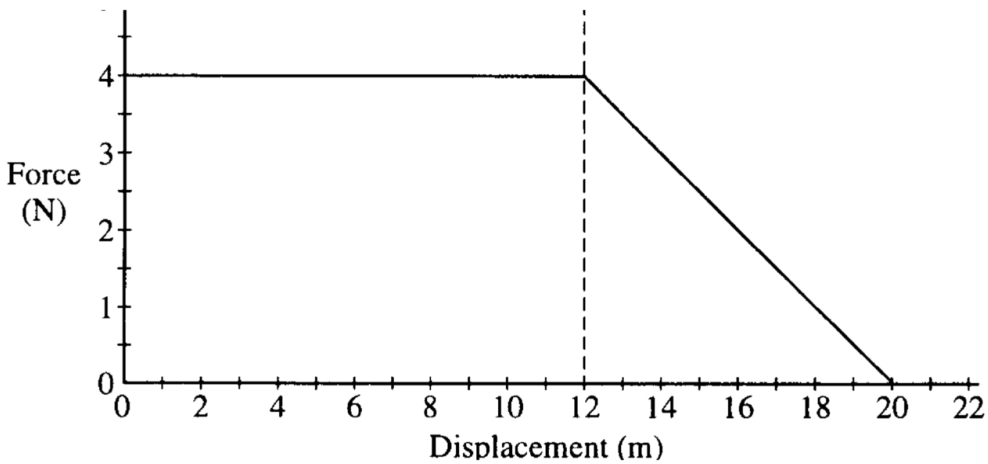


1996B1. Two identical objects A and B of mass M move on a one-dimensional, horizontal air track. Object B initially moves to the right with speed v_o. Object A initially moves to the right with speed $3v_o$, so that it collides with object B. Friction is negligible. Express your answers to the following in terms of M and v_o.

a. Determine the total momentum of the system of the two objects.
b. A student predicts that the collision will be totally inelastic (the objects stick together on collision). Assuming this is true, determine the following for the two objects immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)

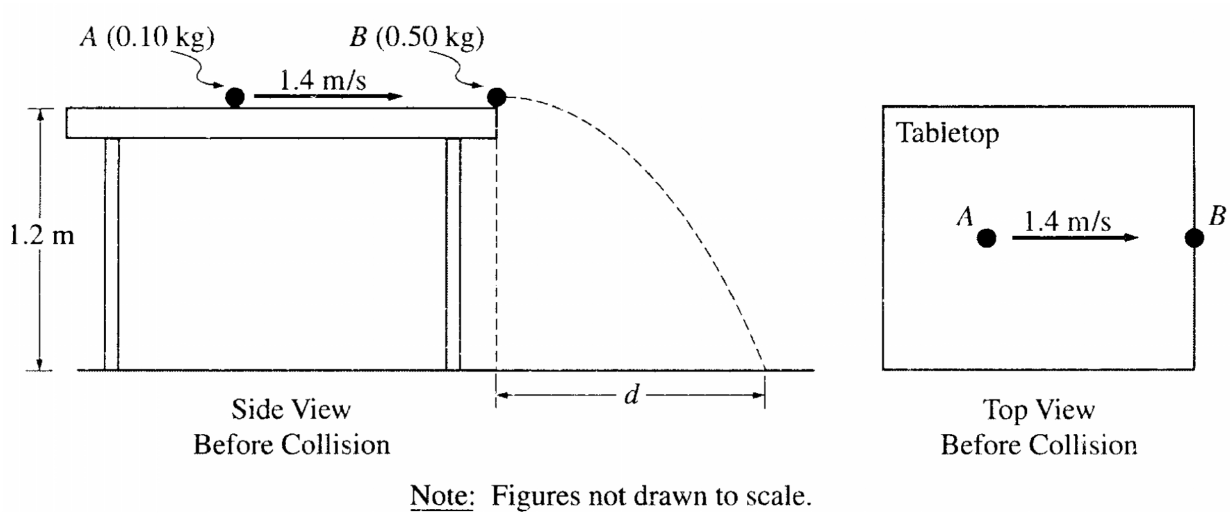
When the experiment is performed, the student is surprised to observe that the objects separate after the collision and that object B subsequently moves to the right with a speed $2.5v_o$.

c. Determine the following for object A immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)
d. Determine the kinetic energy dissipated in the actual experiment.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

a. The acceleration of the particle when its displacement x is 6 m.
b. The time taken for the object to be displaced the first 12 m.
c. The amount of work done by the net force in displacing the object the first 12 m.
d. The speed of the object at displacement x = 12 m.
e. The final speed of the object at displacement x = 20 m.
f. The change in the momentum of the object as it is displaced from x = 12 m to x = 20 m



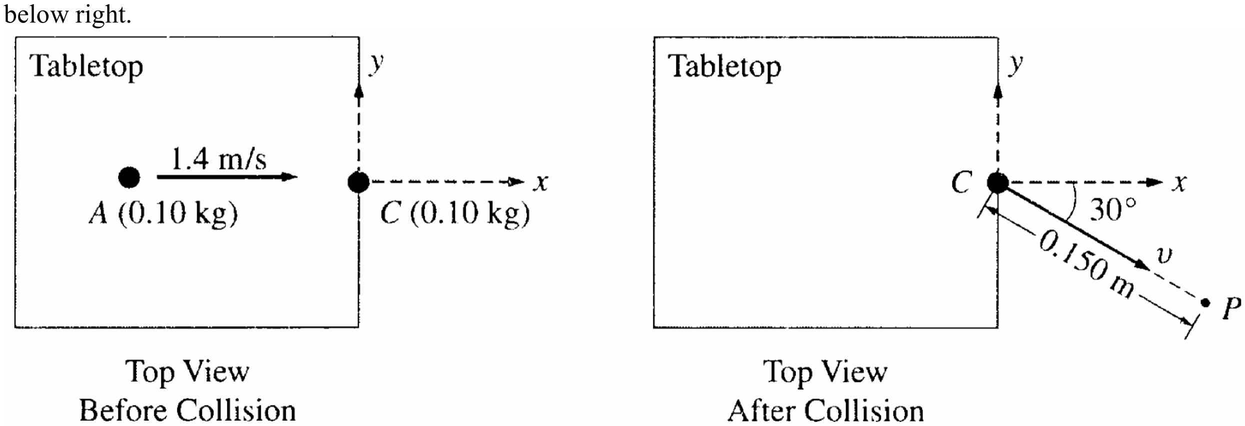
2001B2. An incident ball A of mass 0.10 kg is sliding at 1.4 m/s on the horizontal tabletop of negligible friction as shown above. It makes a head-on collision with a target ball B of mass 0.50 kg at rest at the edge of the table. As a result of the collision, the incident ball rebounds, sliding backwards at 0.70 m/s immediately after the collision.

a. Calculate the speed of the 0.50 kg target ball immediately after the collision.

The tabletop is 1.20 m above a level, horizontal floor. The target ball is projected horizontally and initially strikes the floor at a horizontal displacement *d* from the point of collision.

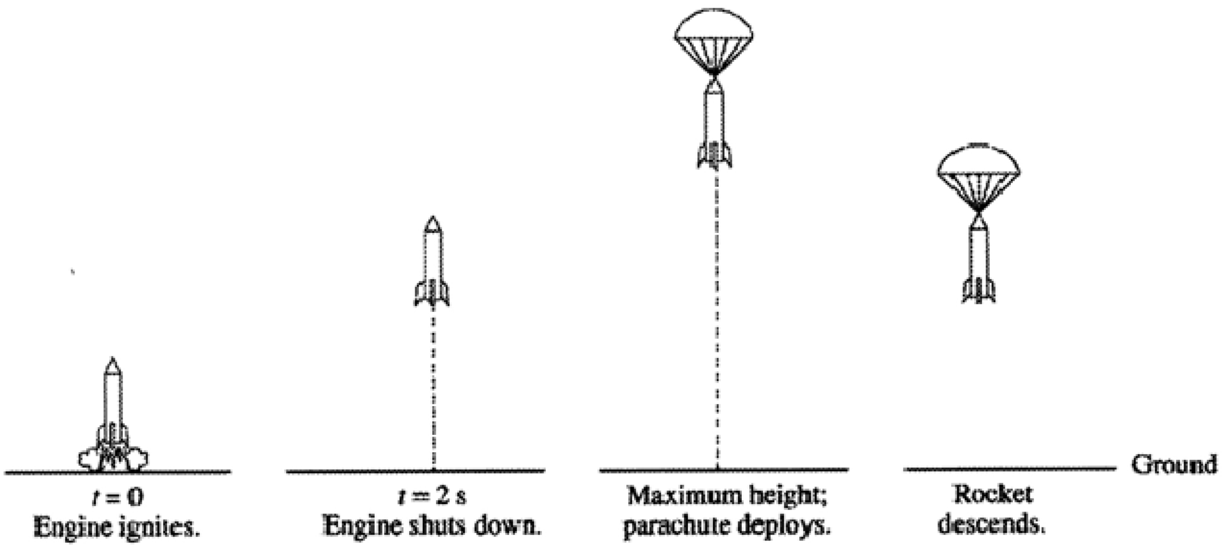
b. Calculate the horizontal displacement

In another experiment on the same table, the target ball B is replaced by target ball C of mass 0.10 kg. The incident ball A again slides at 1.4 m/s, as shown below left, but this time makes a glancing collision with the target ball C that is at rest at the edge of the table. The target ball C strikes the floor at point P, which is at a horizontal displacement of 0.15 m from the point of the collision, and at a horizontal angle of 30° from the +x-axis, as shown below right.



c. Calculate the speed v of the target ball C immediately after the collision.

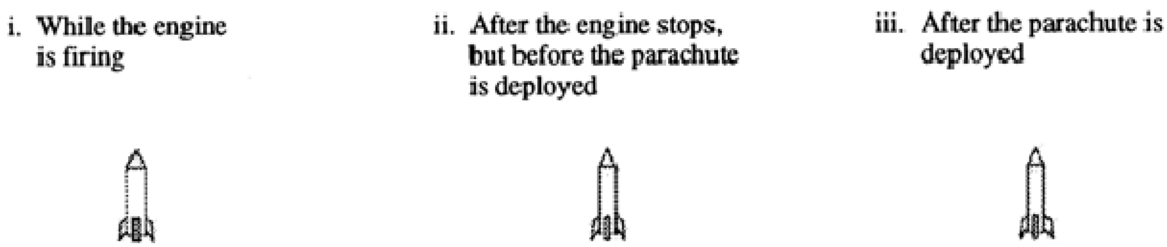
d. Calculate the y-component of incident ball A's momentum immediately after the collision.



Note: Figures not drawn to scale.

2002B1. A model rocket of mass 0.250 kg is launched vertically with an engine that is ignited at time t = 0, as shown above. The engine provides an impulse of 20.0 N•s by firing for 2.0 s. Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

(a) On the figures below, draw and label a free-body diagram for the rocket during each of the following intervals.

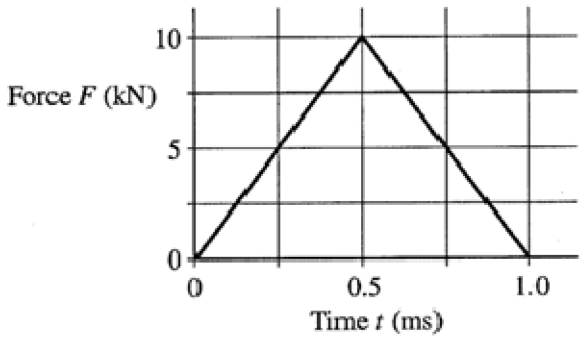


(b) Determine the magnitude of the average acceleration of the rocket during the 2 s firing of the engine.

(c) What maximum height will the rocket reach?

(d) At what time after t = 0 will the maximum height be reached?

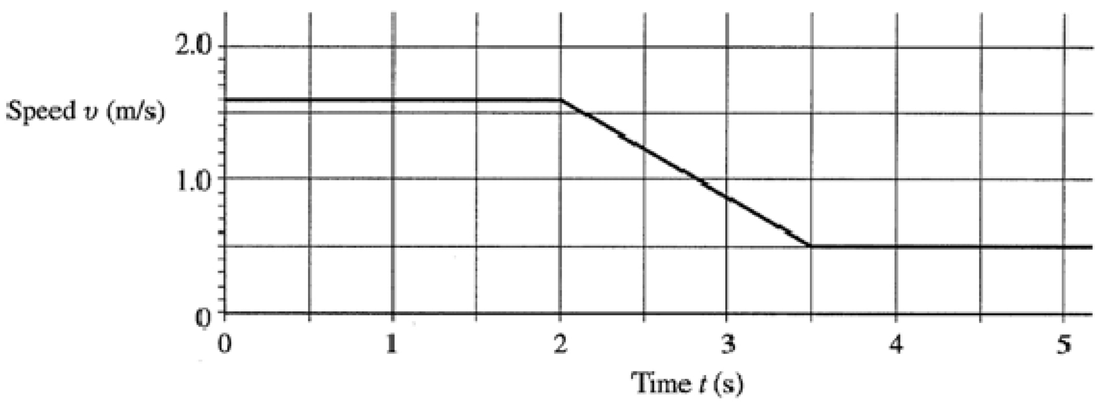
2002B1B. A 2.0 kg frictionless cart is moving at a constant speed of 3.0 m/s to the right on a horizontal surface, as shown above, when it collides with a second cart of undetermined mass m that is initially at rest. The force F of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 m/s to the right. Assume that friction is negligible before, during, and after the collision.



(a) Calculate the magnitude and direction of the velocity of the 2.0 kg cart after the collision.

(b) Calculate the mass m of the second cart.

After the collision, the second cart eventually experiences a ramp, which it traverses with no frictional losses. The graph below shows the speed v of the second cart as a function of time t for the next 5.0 s, where $t = 0$ is now the instant at which the carts separate.

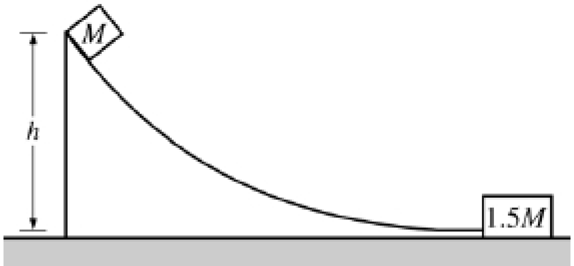


(c) Calculate the acceleration of the cart at $t = 3.0$ s.

(d) Calculate the distance traveled by the second cart during the 5.0 s interval after the collision (0 s < t < 5.0 s).

(e) State whether the ramp goes up or down **and** calculate the maximum elevation (above or below the initial height) reached by the second cart on the ramp during the 5.0 s interval after the collision (0 s < t < 5.0 s).

2006B2B



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2\ v_o$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

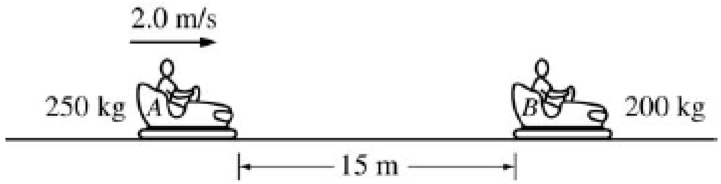
(a) Determine the height h of the ramp from which the small block was released.
(b) Determine the speed of the small block after the collision.
(c) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.
(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

2008B1B

A 70 kg woman and her 35 kg son are standing at rest on an ice rink, as shown above. They push against each other for a time of 0.60 s, causing them to glide apart. The speed of the woman immediately after they separate is 0.55 m/s. Assume that during the push, friction is negligible compared with the forces the people exert on each other.

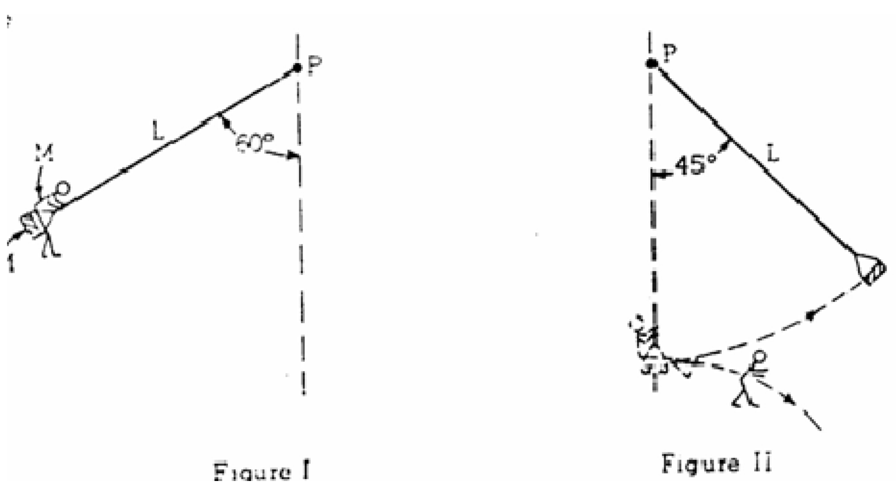
(a) Calculate the initial speed of the son after the push.
(b) Calculate the magnitude of the average force exerted on the son by the mother during the push.
(c) How do the magnitude and direction of the average force exerted on the mother by the son during the push compare with those of the average force exerted on the son by the mother? Justify your answer.
(d) After the initial push, the friction that the ice exerts cannot be considered negligible, and the mother comes to rest after moving a distance of 7.0 m across the ice. If their coefficients of friction are the same, how far does the son move after the push?

2008B1



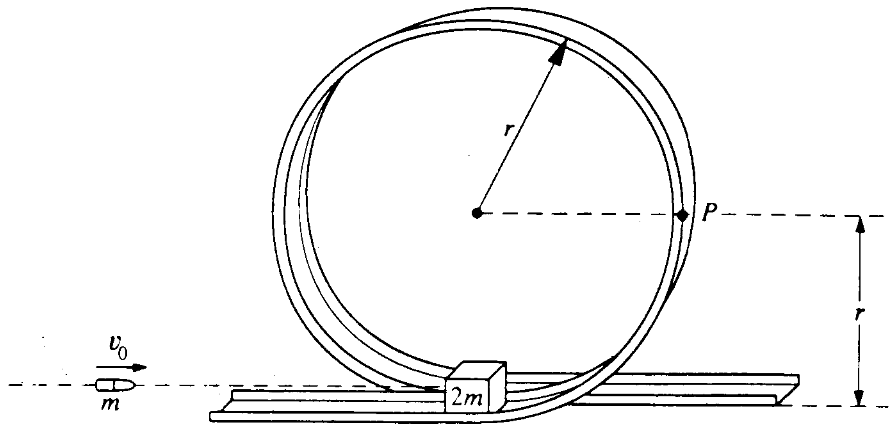
Several students are riding in bumper cars at an amusement park. The combined mass of car *A* and its occupants is 250 kg. The combined mass of car *B* and its occupants is 200 kg. Car *A* is 15 m away from car *B* and moving to the right at 2.0 m/s, as shown, when the driver decides to bump into car *B*, which is at rest.

(a) Car *A* accelerates at 1.5 m/s^2 to a speed of 5.0 m/s and then continues at constant velocity until it strikes car *B*. Calculate the total time for car *A* to travel the 15 m.
(b) After the collision, car *B* moves to the right at a speed of 4.8 m/s .
i. Calculate the speed of car *A* after the collision.
ii. Indicate the direction of motion of car *A* after the collision.
____ To the left ____ To the right ____ None; car *A* is at rest.
(c) Is this an elastic collision?
____ Yes ____ No
Justify your answer.



C1981M2. A swing seat of mass M is connected to a fixed point P by a massless cord of length L. A child also of mass M sits on the seat and begins to swing with zero velocity at a position at which the cord makes a 60° angle with the vertical is shown in Figure I. The swing continues down until the cord is exactly vertical at which time the child jumps off in a horizontal direction. The swing continues in the same direction until its cord makes a 45° angle with the vertical as shown in Figure II: at that point it begins to swing in the reverse direction.

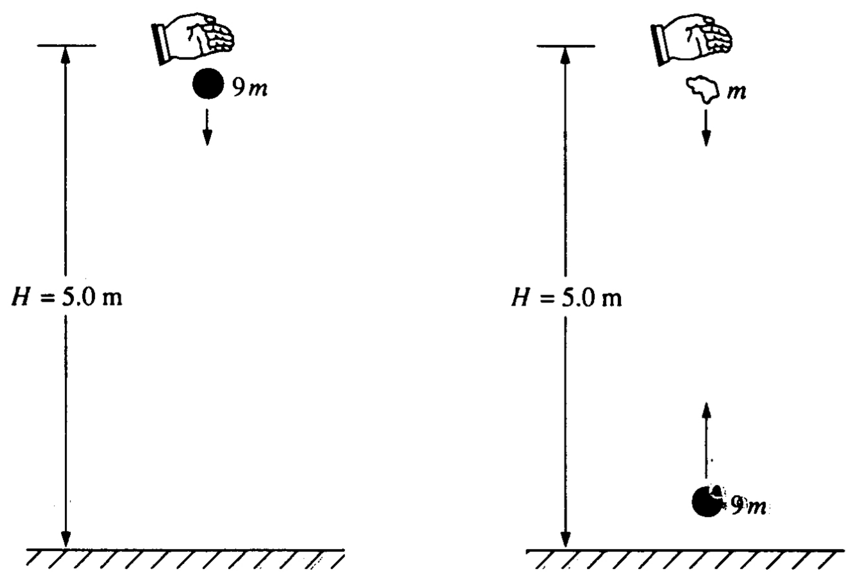
a) Determine the speed of the child and seat just at the lowest position prior to the child's dismount from the seat
b) Determine the speed if the seat immediately after the child dismounts
c) Determine the speed of the child immediately after he dismounts from the swing?



C1991M1. A small block of mass 2m initially rests on a track at the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_o and remains embedded in the block as the block and bullet circle the loop.

Determine each of the following in terms of m, v_o, r, and g.

a. The speed of the block and bullet immediately after impact
b. The kinetic energy of the block and bullet when they reach point P on the loop
c. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times
d The new required entry speed v_o' of the block and bullet at the bottom of the loop such that the conditions in part c apply.
e. The new initial speed of the bullet to produce the speed from part d above.



C1992M1. A ball of mass 9m is dropped from rest from a height H = 5.0 meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, collides with the ball 0.5 seconds later, which is ascending. Assume that g = 10 m/s^2, that air resistance is negligible, and that the collision process takes negligible time.

a. Determine the speed of the ball immediately before it hits the ground.

b. Determine the rebound speed of the ball immediately after it collides with the ground, justify your answer.

c. Determine the height above the ground at which the clay-ball collision takes place.

d. Determine the speeds of the ball and the clay blob immediately before the collision.

e. If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

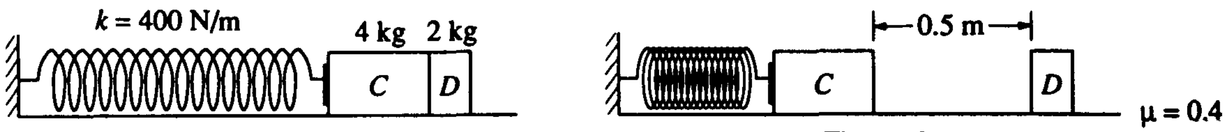


Figure I
Figure II

C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

c. The speed v_f of blocks C and *D* just after they collide

d. The horizontal distance the blocks move before coming to rest

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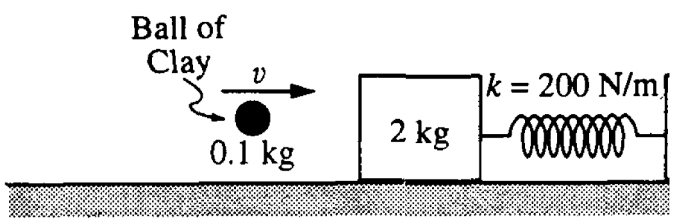


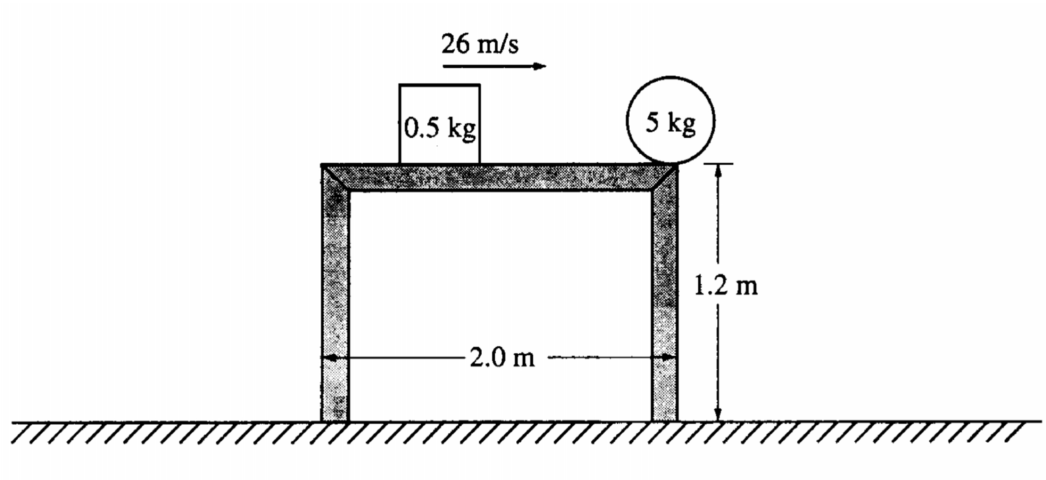
C1994M1. A 2-kilogram block is attached to an ideal spring (for which k = 200 N/m) and initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The spring is attached to a wall as shown. As a result, the spring compresses a maximum distance of 0.4 meters.

a. Calculate the energy stored in the spring at maximum compression.

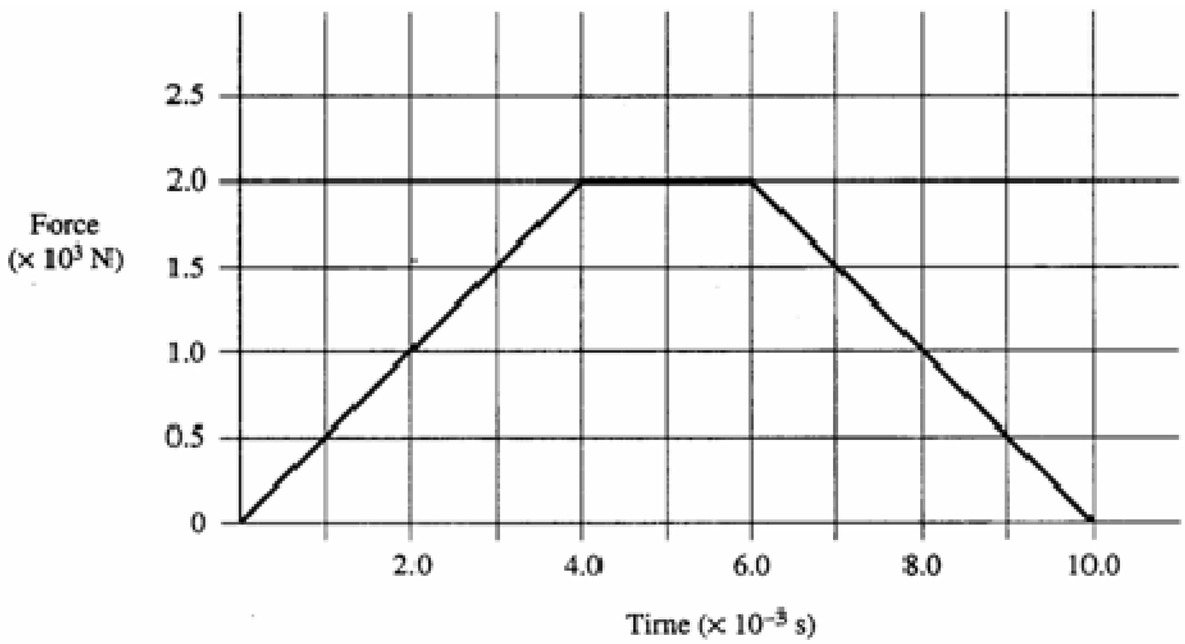
b. Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

c. Calculate the initial speed v of the clay.

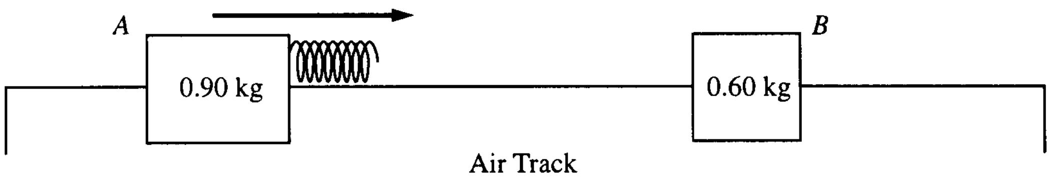


Note: Figure not drawn to scale.

C1995M1. A 5-kilogram ball initially rests at the edge of a 2-meter-long, 1.2-meter-high frictionless table, as shown above. A hard plastic cube of mass 0.5 kilogram slides across the table at a speed of 26 meters per second and strikes the ball, causing the ball to leave the table in the direction in which the cube was moving. The figure below shows a graph of the force exerted **on the ball** by the cube as a function of time.

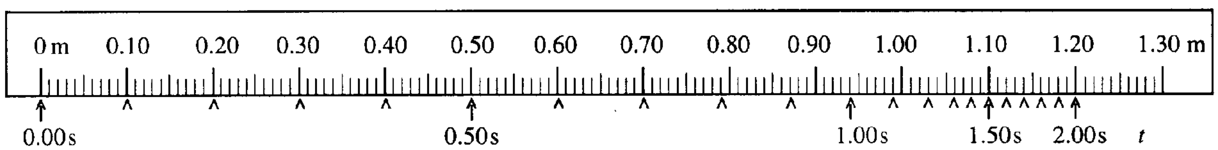


a. Determine the total impulse given to the ball.
b. Determine the horizontal velocity of the ball immediately after the collision.
c. Determine the following for the cube immediately after the collision.
   i. Its speed
   ii. Its direction of travel (right or left), if moving
d. Determine the kinetic energy dissipated in the collision.
e. Determine the distance between the two points of impact of the objects with the floor.



C1998M1. Two gliders move freely on an air track with negligible friction, as shown above. Glider A has a mass of 0.90 kg and glider B has a mass of 0.60 kg. Initially, glider A moves toward glider B, which is at rest. A spring of negligible mass is attached to the right side of glider A. Strobe photography is used to record successive positions of glider A at 0.10 s intervals over a total time of 2.00 s, during which time it collides with glider B.

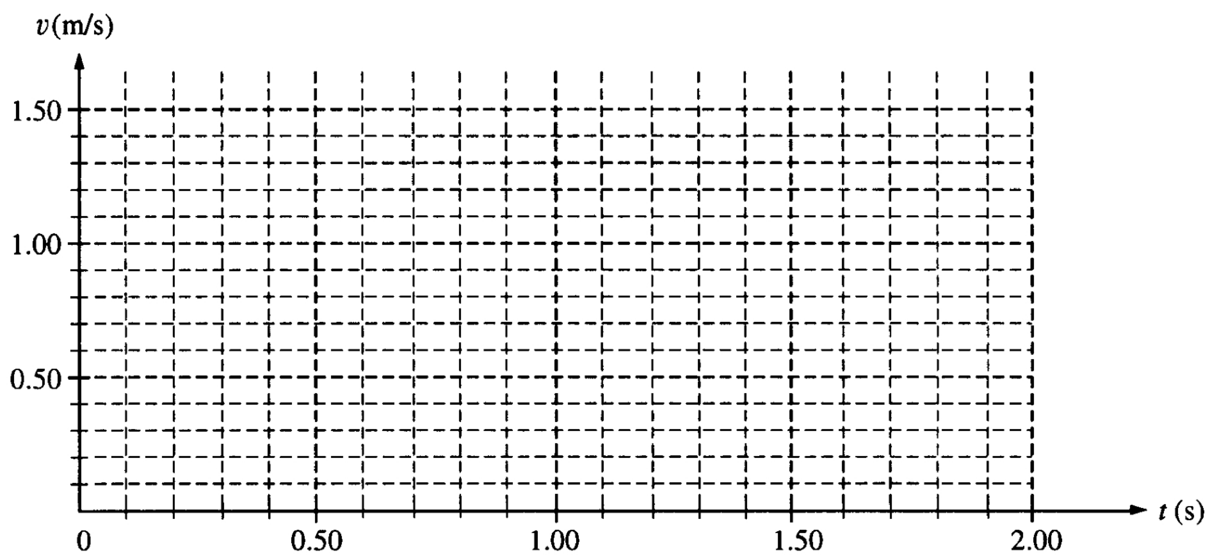
The following diagram represents the data for the motion of glider A. Positions of glider A at the end of each 0.10s interval are indicated by the symbol ʌ against a metric ruler. The total elapsed time t after each 0.50 s is also indicated.



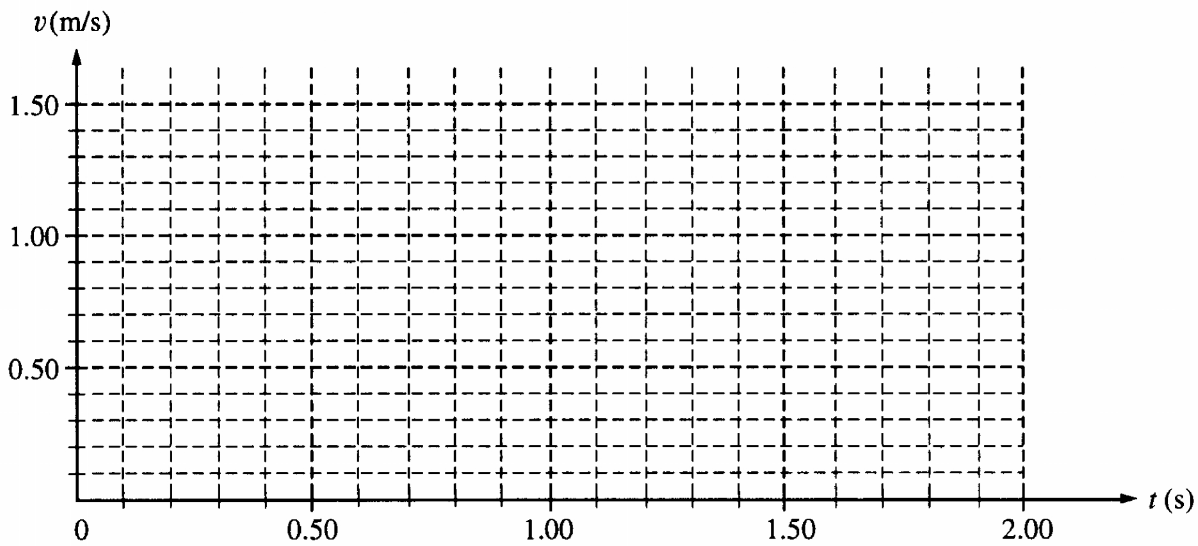
a. Determine the average speed of glider A for the following time intervals.

i. 0.0 s to 0.30 s ii. 0.90 s to 1.10 s iii. 1.70 s to 1.90 s

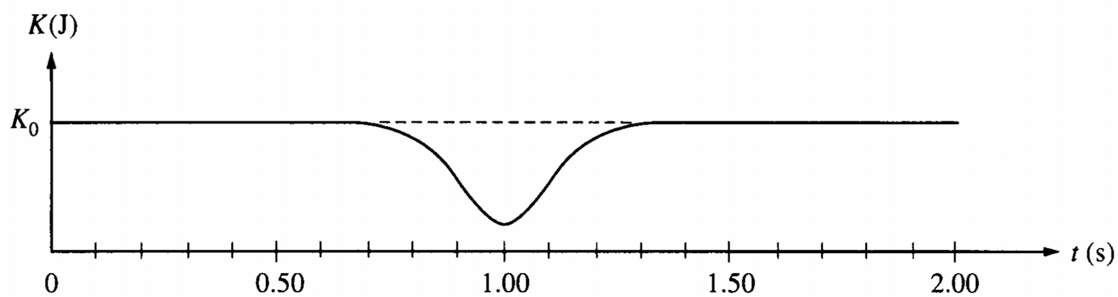
b. On the axes below, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.



c. i. Use the data to calculate the speed of glider B immediately after it separates from the spring.
ii. On the axes below, sketch a graph of the speed of glider B as a function of time t.



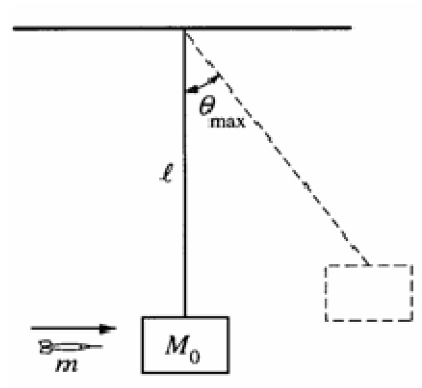
A graph of the total kinetic energy K for the two-glider system over the 2.00 s interval has the following shape. K_o is the total kinetic energy of the system at time t = 0.



d. i. Is the collision elastic? Justify your answer.

ii. Briefly explain why there is a minimum in the kinetic energy curve at t = 1.00 s.

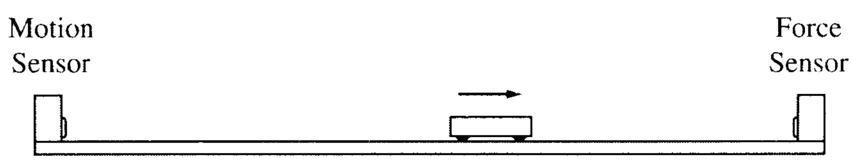
C1999M1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_o which hangs from a cord of length l and negligible mass, as shown. Assume the size of the block is negligible compared to l, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of
m, M_0, l, θ_{max}, and g.

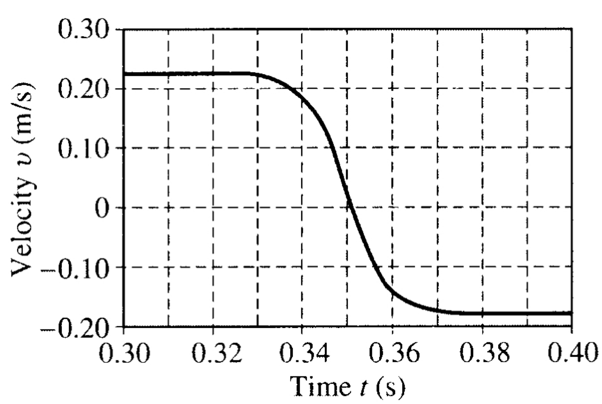


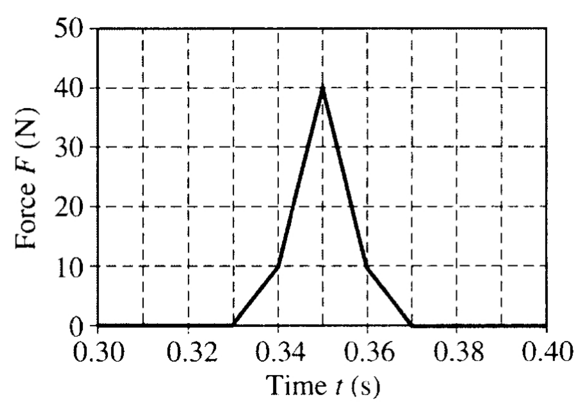
a. Determine the speed v_o of the dart immediately before it strikes the block.
b. The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.
c. At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses	Protractor
5 m of string	Five more blocks of mass M_0	Spring	

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.







2001M1. A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the following graphs.
a. Determine the cart's average acceleration between t = 0.33 s and t = 0.37 s.
b. Determine the magnitude of the change in the cart's momentum during the collision.
c. Determine the mass of the cart.
d. Determine the energy lost in the collision between the force sensor and the cart

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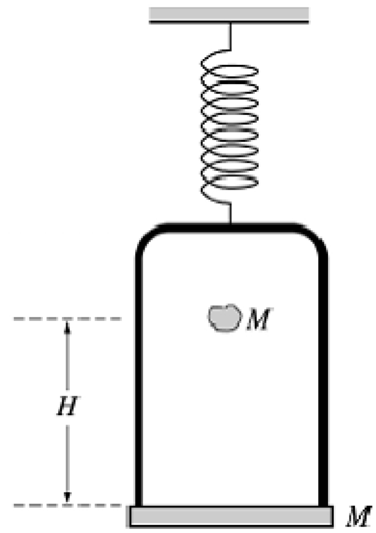
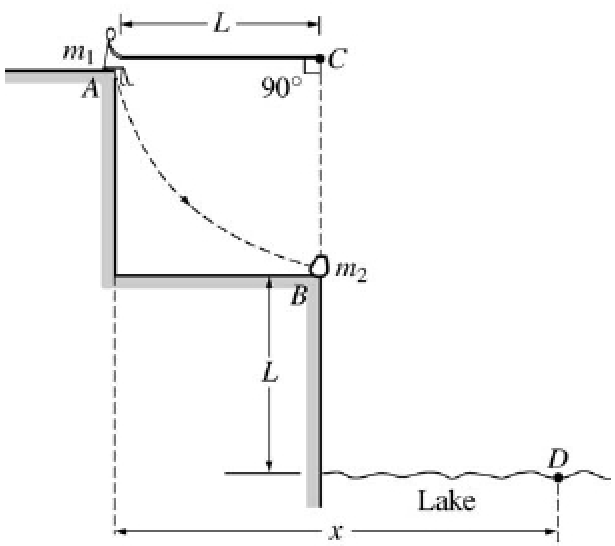
C2003M2.

An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.



(a) Determine the speed of the clay at the instant it hits the pan.
(b) Determine the speed of the pan just after the clay strikes it.
(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

C2004M1.



A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) The speed of the person and object just after the collision
(d) The total horizontal displacement x of the person from position A until the person and object land in the water at point D.

ANSWERS - AP Physics Multiple Choice Practice – Momentum and Impulse

	Solution	Answer
1.	Based on $Ft = m\Delta v$, doubling the mass would require twice the time for same momentum change	D
2.	Two step problem. I) find velocity after collision with arrow. $m_a v_{ai} = (m_a+m_b) v_f$ $v_f = mv / (m+M)$ II) now use energy conservation. $K_i = U_{sp(f)}$ $\frac{1}{2}(m+M)v_f^2 = \frac{1}{2} k \Delta x^2$, sub in v_f from I	D
3.	Since the momentum is the same, that means the quantity $m_1v_1 = m_2v_2$. This means that the mass and velocity change proportionally to each other so if you double m_1 you would have to double m_2 or v_2 on the other side as well to maintain the same momentum. Now we consider the energy formula $KE= \frac{1}{2} mv^2$ since the v is squared, it is the more important term to increase in order to make more energy. So if you double the mass of 1, then double the velocity of 2, you have the same momentum but the velocity of 2 when squared will make a greater energy, hence we want more velocity in object 2 to have more energy.	C
4.	Due to momentum conservation, the total before is zero therefore the total after must also be zero	D
5.	Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(75)(6) + (100)(-8) = (175) v_f$	A
6.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(5000)(4) = (13000)v_f$	C
7.	Energy is conserved during fall and since the collision is elastic, energy is also conserved during the collision and always has the same total value throughout.	A
8.	To conserve momentum, the change in momentum of each mass must be the same so each must receive the same impulse. Since the spring is in contact with each mass for the same expansion time, the applied force must be the same to produce the same impulse.	B
9.	Use $J=\Delta p$ $J = mv_f - mv_i$ $J = (0.5)(-4) - (0.5)(6)$	B
10.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(2m)(v) = (5m) v_f$	B
11.	First of all, if the kinetic energies are the same, then when brought to rest, the non conservative work done on each would have to be the same based on work-energy principle. Also, since both have the same kinetic energies we have $\frac{1}{2} m_1v_1^2 = \frac{1}{2} m_2v_2^2$ … since the velocity is squared an increase in mass would need a proportionally smaller decrease in velocity to keep the terms the same and thus make the quantity mv be higher for the larger mass. This can be seen through example: If mass m_1 was double mass m_2 its velocity would be $v / \sqrt{2}$ times in comparison to mass m_2's velocity. So you get double the mass but less than half of the velocity which makes a larger mv term.	A,C
12.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (3m) v_f$	A
13.	Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (50)(v_{1f}) + (2)(10)$	B
14.	Explosion, momentum before is zero and after must also be zero. To have equal momentum the heavier student must have a much smaller velocity and since that smaller velocity is squared it has the effect of making the heavier object have less energy than the smaller one	C

15. Based on momentum conservation both carts have the same magnitude of momentum "mv" but based on $K = ½ m v^2$ the one with the larger mass would have a directly proportional smaller velocity that then gets squared. So by squaring the smaller velocity term it has the effect of making the bigger mass have less energy. This can be shown with an example of one object of mass m and speed v compared to a second object of mass 2m and speed v/2. The larger mass ends up with less energy even through the momenta are the same. B

16. A 2d collision must be looked at in both x-y directions always. Since the angle is the same and the v is the same, v_y is the same both before and after therefore there is no momentum change in the y direction. All of the momentum change comes from the x direction. D
$v_{ix} = v \cos\theta$ and $v_{fx} = -v \cos\theta$. $\Delta p = mv_{fx} - mv_{ix}$ … $-mv\cos\theta - mv\cos\theta$

17. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (7)(v_{1f}) + (5)(0.2)$ B

18. In a circle at constant speed, work is zero since the force is parallel to the incremental distance moved during revolution. Angular momentum is given by mvr and since none of those quantities are changing it is constant. However the net force is NOT = MR, its Mv^2/R B,C

19. Since the momentum before is zero, the momentum after must also be zero. Each mass must have equal and opposite momentum to maintain zero total momentum. D

20. In a perfect inelastic collision with one of the objects at rest, the speed after will always be less no matter what the masses. The 'increase' of mass in 'mv' is offset by a decrease in velocity C

21. Since the total momentum before and after is zero, momentum conservation is not violated, however the objects gain energy in the collision which is not possible unless there was some energy input which could come in the form of inputting stored potential energy in some way. B

22. The plastic ball is clearly lighter so anything involving mass is out, this leaves speed which makes sense based on free-fall B

23. Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (m+M)\,v_f$ D

24. As the cart moves forward it gains mass due to the rain but in the x direction the rain does not provide any impulse to speed up the car so its speed must decrease to conserve momentum B

25. Momentum increases if velocity increases. In a d-t graph, III shows increasing slope (velocity) B

26. The net force is zero if velocity (slope) does not change, this is graphs I and II C

27. Since the initial object was stationary and the total momentum was zero it must also have zero total momentum after. To cancel the momentum shown of the other two pieces, the 3m piece would need an x component of momentum $p_x = mV$ and a y component of momentum $p_y = mV$ giving it a total momentum of $\sqrt{2}\,mV$ using Pythagorean theorem. Then set this total momentum equal to the mass * velocity of the 3rd particle. D
$\sqrt{2}\,mV = (3m)\,V_{m3}$ and solve for V_{m3}

28. Its does not matter what order to masses are dropped in. Adding mass reduces momentum proportionally. All that matters is the total mass that was added. This can be provided by finding the velocity after the first drop, then continuing to find the velocity after the second drop. Then repeating the problem in reverse to find the final velocity which will come out the same C

29. Increase in momentum is momentum change which is the area under the curve C

30. Basic principle of impulse. Increased time lessens the force of impact. D

31. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = m_1(5) + m_2(-2)$ B

32. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $Mv + (-2Mv) = (3M)\,v_f$ gives $v_f = v/3$. C
Then to find the energy loss subtract the total energy before – the total energy after
$[\,\frac{1}{2}Mv^2 + \frac{1}{2}(2M)v^2\,] - \frac{1}{2}(3M)(v/3)^2 = 3/6\,Mv^2 + 6/6\,Mv^2 - 1/6\,Mv^2$

33. 2D collision. The y momentums are equal and opposite and will cancel out leaving only the x momentums which are also equal and will add together to give a total momentum equal to twice the x component momentum before hand. $p_{before} = p_{after}$ $2m_ov_o\cos 60 = (2m_o)\,v_f$ B

34. Since there is no y momentum before, there cannot be any net y momentum after. The balls have equal masses so you need the y velocities of each ball to be equal after to cancel out the momenta. By inspection, looking at the given velocities and angles and without doing any math, the only one that could possibly make equal y velocities is choice D C

35. The area of the Ft graph is the impulse which determines the momentum change. Since the net impulse is zero, there will be zero total momentum change. C

36. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(m)(v) + (2m)(v/2) = (3m)v_f$ C

37. Since the angle and speed are the same, the x component velocity has been unchanged which means there could not have been any x direction momentum change. The y direction velocity was reversed so there must have been an upwards y impulse to change and reverse the velocity. D

38. Just as linear momentum must be conserved, angular momentum must similarly be conserved. Angular momentum is given by L = mvr, so to conserve angular momentum, these terms must all change proportionally. In this example, as the radius decreases the velocity increases to conserve momentum. A

39. Each child does work by pushing to produce the resulting energy. This kinetic energy is input through the stored energy in their muscles. To transfer this energy to each child, work is done. The amount of work done to transfer the energy must be equal to the amount of kinetic energy gained. Before hand, there was zero energy so if we find the total kinetic energy of the two students, that will give us the total work done. First, we need to find the speed of the boy using momentum conservation, explosion: D
$p_{before} = 0 = p_{after}$ $0 = m_bv_b + m_gv_g$ $0 = (m)(v_b) = (2m)(v_g)$ so $v_b = 2v$
Now we find the total energy $K_{tot} = K_b + K_g = \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2 = 2mv^2 + mv^2 = 3mv^2$

40. Since it is an elastic collision, the energy after must equal the energy before, and in all collisions momentum before equals momentum after. So if we simply find both the energy before and the momentum before, these have the same values after as well. $p = Mv$, $K = \frac{1}{2}Mv^2$ A

41. The area under the F-t graph will give the impulse which is equal to the momentum change. With the momentum change we can find the velocity change. B
J = area = 6 Then $J = \Delta p = m\Delta v$ $6 = (2)\Delta v$ $\Delta v = 3$ m/s

42. This is the same as question 16 except oriented vertically instead of horizontally. D

1976B2.

a) Apply momentum conservation. $p_{before} = p_{after}$ $mv_o = (m)(v_o/3) + (4m)(v_{f2})$ $v_{f2} = v_o / 6$

b) $KE_f – KE_i = ½\ mv_o^2 – ½\ m\ (v_o / 3)^2 = 4/9\ mv_o^2$

c) $KE = ½\ (4m)(v_o / 6)^2 = 1/18\ mv_o^2$

1978B1.

a) Projectile methods. Find t in y direction. $d_y = v_{iy}t + ½\ a\ t^2$ $t = \sqrt{\dfrac{2H}{g}}$

D is found with $v_x = d_x / t$ $D = v_o t$ $v_o\sqrt{\dfrac{2H}{g}}$

b) Apply momentum conservation in the x direction. $p_{before(x)} = p_{after(x)}$ $M_1v_o = (M_1+M_2)v_f$ $v_f = M_1v_0 / (M_1+M_2)$

1981B2.

a) The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring $W = ½\ m\ v^2 = ½\ (3)\ (10)^2 = 150$ J

b) Apply momentum conservation to the explosion
$p_{before} = 0 = p_{after}$ $0 = m_1v_{1f} + m_2v_{2f}$ $0 = (1)v_{1f} + (3)v_{2f}$ $v_{1f} = 3\ v_{2f}$

Apply energy conservation … all of the spring energy is converted into the kinetic energy of the masses
$150\ J = K_1 + K_2$ $150 = ½\ m\ v_{1f}^2 + ½\ m\ v_{2f}^2$ sub in above for v_{2f}
$150 = ½\ (1)(3v_{2f})^2 + ½\ (3)(v_{2f})^2$
$v_{2f} = 5$ m/s $v_{1f} = 15$ m/s

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $½\ (3M)(2/3\ v_o)^2 = ½\ k\ \Delta x^2$ $\sqrt{\dfrac{4Mv_o^{\ 2}}{3k}}$

1984B2.

a) Before the collision there is only an x direction momentum of mass M_1 … $p_x = m_1v_{1x} = 16$, all the rest are 0
After the collision, M_1 has y direction momentum $= m_1v_{1fy} = 12$ and M_2 has x and y direction momentums.
Using trig to find the x and y velocities of mass M_2 … $v_x = 5 \cos 37 = 3$, and $v_y = 5 \sin 37 = 3.75$.
Then plug into mv to get each x and y momentum after.

	$M_1 = 1$ kg		$M_2 = 4$ kg	
	p_x (kg m /s)	p_y (kg m /s)	p_x (kg m /s)	p_y (kg m /s)
Before	16	0	0	0
After	0	-12	16	12

b) SUM = 16 -12 16 12
When adding x's before they = x's after 16=16, when adding y's before they equal y's after |-12|=12

c) Kinetic Energy Before
$K = ½ m_1v_{1ix}^2$
$K = ½ (1)(16)^2 = 128$ J

Kinetic Energy After
$K = ½ m_1v_{1fy}^2 + ½ m_2v_2^2$
$K = ½ (1)(12)^2 + ½ (4)(5)^2 = 122$ J

d) From above, K is not conserved.

1985B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_{1i} = (m+M)v_f$ $v_f = 1.5$ m/s

b) KE_i / KE_f $½ m v_{1i}^2 / ½ (m+M)vf^2 = 667$

c) Apply conservation of energy of combined masses $K = U$ $½ (m+M)v^2 = (m+M)gh$ $h = 0.11$ m

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1990B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_o = (101m)v_f$ $v_f = v_o / 101$

b) $\Delta K = K_f - K_i = ½ (101m)v_f^2 - ½ mv_o^2 = ½ (101m)(v_o/101)^2 - ½ mv_o^2 = -(50/101) mv_o^2$

c) Using projectile methods. Find t in y direction. $d_y = v_{iy}t + ½ a t^2$ $t = \sqrt{\dfrac{2h}{g}}$

D is found with $v_x = d_x / t$ $D = v_xt$ $\dfrac{v_0}{101}\sqrt{\dfrac{2h}{g}}$

d) The velocity of the block would be different but the change in the x velocity has no impact on the time in the y direction due to independence of motion. v_{iy} is still zero so t is unchanged.

e) In the initial problem, all of the bullets momentum was transferred to the block. In the new scenario, there is less momentum transferred to the block so the block will be going slower. Based on $D = v_xt$ with the same time as before but smaller velocity the distance x will be smaller.

1992 B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$
$m_1 v_{1i} = (m_1+m_2)v_f$ $(30)(4) = (80)v_f$ $v_f = 1.5$ m/s

b) $K= ½ (m_1+m_2)\ v_f^2 = ½ (80)(1.5)^2 = 90$ J

c) Apply momentum conservation explosion.
$p_{before} = p_{after}$ $(m_1+m_2)v = m_1v_{1f} + m_2v_{2f}$ $(80)(1.5) = 0 + (50)v_{2f}$ $v_{2f} = 2.4$ m/s

d) $K = ½\ m_2v_{2f}^2 = ½ (50)(2.4)^2 = 144$ J

e) By inspection the energy in d is greater. The energy increased due to an energy input from the work of the child's muscles in pushing on the sled.

1994 B2.

a) Apply energy conservation top to bottom. $U = K$ $mgh = ½\ mv^2$ $(gR) = ½\ v^2$ $v = \sqrt{2gR}$

b) Apply momentum conservation

$p_{before} = p_{after}$ $m_a v_{ai} = (m_a+m_b)v_f$ $M(\sqrt{2gR}) = 2Mv_f$ $v_f = \dfrac{\sqrt{2gR}}{2}$

c) The loss of the kinetic energy is equal to the amount of internal energy transferred

$\Delta K = K_f - K_i = ½\ 2M \left(\dfrac{\sqrt{2gR}}{2}\right)^2 - ½\ M \left(\sqrt{2gR}\right)^2$ $= -MgR / 2$ lost ➔ $MgR / 2$ internal energy gain.

d) Find the remaining kinetic energy loss using work-energy theorem which will be equal the internal energy gain.
$W_{nc} = \Delta K$ $-f_kd = -\mu F_nd = -\mu(2m)gL = -2\ \mu MgL$, kinetic loss = internal E gain ➔ $2\mu MgL$

1995 B1.

a) i) $p = mv$ $= (0.2)(3) = 0.6$ kg m/s
ii) $K = ½\ mv^2$ $= ½\ (0.2)(3)^2 = 0.9$ J

b) i.) Apply momentum conservation $p_{before} = p_{after} = 0.6$ kg m/s
ii) First find the velocity after using the momentum above
$0.6 = (1.3+0.2)\ v_f$ $v_f = 0.4$ m/s $K = ½\ (m_1+m_2)\ v_f^2 = ½\ (1.3+0.2)(0.4)^2 = 0.12$ J

c) Apply energy conservation $K = U_{sp}$ $0.12\text{ J} = ½\ k\Delta x^2 = ½\ (100)\ \Delta x^2$ $\Delta x = 0.05$ m

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1996 B1.

a) $p_{tot} = M(3v_o) + (M)(v_o) = 4mv_o$

b) i) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$

$4Mv_o = (m_1+m_2)v_f$ $\quad$ $4Mv_o = (2M)\ v_f$ $\quad$ $v_f = 2v_o$

ii) Since they are both moving right they would have to be moving right after

c) i) Apply momentum conservation $p_{before} = p_{after}$

$4Mv_o = m_1v_{1f} + m_2v_{2i}$ $\quad$ $4Mv_o = Mv_{af} + M(2.5v_o)$ $\quad$ $v_{af} = 1.5\ v_o$

ii) As before, the would have to be moving right.

d) $\Delta K = K_f - K_i = (½\ m_av_{af}^2 + ½\ m_bv_{bf}^2) - (\ ½\ m_av_{ai}^2 + ½\ m_bv_{bi}^2) = 4.25\ Mv_o^2 - 5\ Mv_o^2 = -0.75\ Mv_o^2$

1997 B1.

a) The force is constant, so simple $F_{net} = ma$ is sufficient. $(4) = (0.2)\ a$ $\quad a = 20\ m/s^2$

b) Use $d = v_it + ½\ a\ t^2$ $\quad 12 = (0) + ½\ (20)\ t^2$ $\quad t = 1.1$ sec

c) $W = Fd$ $\quad W = (4\ N)\ (12\ m) = 48\ J$

d) Using work energy theorem $\quad W = \Delta K$ $\quad (K_i = 0)$ $\quad W = Kf - Ki$

$W = ½\ m\ v_f^2$

Alternatively, use $v_f^{\ 2} = v_i^2 + 2\ a\ d$ $\quad 48J = ½\ (0.2)\ (v_f^2)$ $\quad v_f = 21.9$ m/s

e) The area under the triangle will give the extra work for the last 8 m

½ (8)(4) = 16J + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem $\quad W = ½\ m\ v_f^2$ $\quad 64\ J = ½\ (0.2)\ v_f^2$ $\quad v_f = 25.3$ m/s

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

f) The momentum change can simply be found with $\Delta p = m\Delta v = m(v_f - v_i) = 0.2\ (25.3 - 21.9) = 0.68$ kg m/s

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2001B2.

a) Apply momentum conservation $p_{before} = p_{after}$

$m_av_{ai} = m_av_{af} + m_bv_{bf}$ $\quad (0.1)(1.4) = (0.1)(-0.7) + (0.5)v_{bf}$ $\quad v_{bf} = 0.42$ m/s

b) Using projectile methods. Find t in y direction. $\quad d_y = v_{iy}t + ½\ a\ t^2$ $\quad -1.2m = 0 + ½\ (-9.8)\ t^2$ $\quad t = 0.49$

D is found with $v_x = d_x / t$ $\quad D = v_xt$ $\quad (0.42)(0.49)$ $\quad D = 0.2$ m

c) The time of fall is the same as before since it's the same vertical distance. t = 0.49 s

The velocity of ball C leaving the table can be found using projectile methods. $v_x = d / t = 0.15/0.49 = 0.31$ m/s

d) Looking that the y direction. $p_{y(before)} = p_{y(after)}$

$0 = p_{ay} - p_{cy}$ $\quad 0 = p_{ay} - m_cv_{cy}$ $\quad 0 = p_{ay} - (0.1)(0.31)\sin 30$ $\quad p_{ay} = 0.015$ kg m/s

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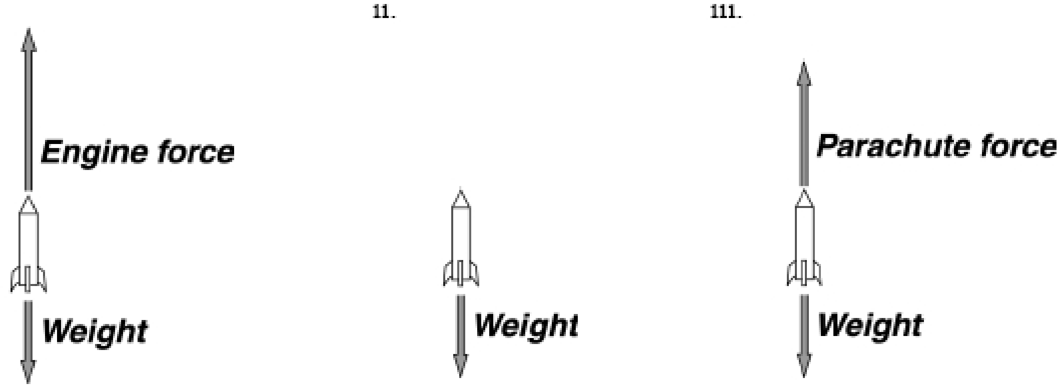
2002B1.

a)

i. ii. iii.



b) $J_{engine} = F_{eng}\, t$ (20) = F_{eng} (2) F_{eng} = 10 N

$F_{net} = ma$ $(F_{eng} – mg) = ma$ $(10 – 0.25(9.8)) = (0.25)a$ $a = 30\ m/s^2$

c) Find distance traveled in part (i) $d_1 = v_it + ½\, a\, t^2 = 0 + ½\, (30)(2)^2 = 60$ m
Find distance in part (ii) free fall.
first find velocity at end of part (i) = v_i for part ii
$v_{1f} = v_{1i} + a_1t_1 = (0) + (30)(2) = 60$ m/s
then find distance traveled in part ii
$v_{2f}^2 = v_{2i}^2 + 2gd_2 = (60)^2 + 2(–9.8)(d_2)$ $d_2 = 184$ m

$d_{total} = 244$ m

d) Find time in part ii. $v_{2f} = v_{2i} + gt$ $0 = 60 + –9.8\, t$ $t = 6.1$ s
then add it to the part I time (2 s) total time ➔ 8.1 sec

2002B1B

a) The graph of force vs time uses area to represent the Impulse and the impulse equals change in momentum.
Area = 2 x ½ bh = (0.5 ms)(10kN). Milli and kilo cancel each other out. Area = 5 Ns = J

VERY IMPORTANT – Based on the problem, the force given and therefore impulse is actually negative because the graph is for the 2 kg cart and clearly the force would act opposite the motion of the cart.

$J = \Delta p = mv_f – mv_i$ $(–5) = (2)(v_f) – 2(3)$ $v_f = 0.5$ m/s (for the 2 kg cart)

b) Apply momentum conservation
$m_av_{ai} = m_av_{af} + m_bv_{bf}$
$p_{before} = p_{after}$
$(2)(3) = (2)(0.5) + (m_b)(1.6)$ $m_b = 3.125$ kg

c) slope = acceleration = $\Delta y / \Delta x = (0.5 – 1.6) / (3.5 – 3) = – 0.73\ m/s^2$

d) distance = area under line, using four shapes.
0-2 rectangle, 2-3.5 triangle top + rectangle bottom, 3.5-5, rectangle ➔ 5.5 m

e) Since the acceleration is negative the cart is slowing so it must be going up the ramp. Use energy conservation to find the max height. $K_{bot} = U_{top}$ $½\, mv^2 = mgh$ $½\, (1.6)^2 = (9.8)\, h$ $h = 0.13$ m

2006B2B.

a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$ $Mgh = ½ (M) (3.5v_o)^2$ $h = 6.125 v_o^2 / g$

b) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(M)(3.5v_o) = (M)v_{af} + (1.5M)(2v_o)$ $v_{af} = ½ v_o$

c) $W_{NC} = \Delta K$ $(K_f\text{-}K_i)$ $K_f = 0$

$- f_k d = 0 - ½ (1.5M)(2v_o)^2$

$\mu_k (1.5 M) g (d) = 3Mv_o^2$ $\mu_k = 2v_o^2 / gD$

d) Compare the kinetic energies before and after

Before After

$K = ½ M (3.5v_o)^2$ $½ M(½ v_o)^2 + ½ (1.5M)(2v_o)^2$ there are not equal so its inelastic

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2008B1B.

a) Apply momentum conservation to the explosion $p_{before} = 0 = p_{after}$

$0 = m_a v_{af} + m_b v_{bf}$ $0 = (70)(- 0.55) + (35)(v_{bf})$ $v_{bf} = 1.1$ m/s

b) $J_{son} = \Delta p_{son}$ $F_{on\text{-}son} t = m(v_f - v_i)$ $F (0.6) = (35)(0 - 1.1) =$ $F = - 64$ N

c) Based on newtons third law action/reaction, the force on the son must be the same but in the opposite direction as the force on the mother.

d) On the son $W_{fk} = \Delta K$ $- f_k d = ½ mv_f^2 - ½ mv_i^2$ $- \mu mg\, d = ½ m (0 - v_i^2)$ $d = v_i^2 / (2\mu)$

This would be the same formula for the mother's motion with a different initial velocity. Since the mass cancels out we see the distance traveled is proportional to the velocity squared. The boy moves at twice the speed of the mother, so based on this relationship should travel 4 x the distance. The mother traveled 7 m so the son would have a sliding distance of 28 m.

(Alternatively, you could plug in the numbers for the mother to solve for μ and then plug in again using the same value of μ and the sons velocity to find the distance. μ is the same for both people.)

2008B1.

a) First determine the time to travel while the car accelerates. $v_{1f} = v_{1i} + a_1 t_1$ $(5) = (2) + (1.5)\, t_1$ $t_1 = 2$ sec

Also determine the distance traveled while accelerating $d_1 = v_{1i} t_1 + ½ a_1 t_1^2$ $d_1 = (2)(2) + ½ (1.5)(2)^2 = 7$ m

This leaves 8 m left for the constant speed portion of the trip.

The velocity at the end of the 7m is the average constant velocity for the second part of the trip

$v_2 = d_2 / t_2$ $5 = 8 / t_2$ $t_2 = 1.6$ sec ➔ total time $= t_1 + t_2 =$ 3.6 seconds

b) i) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(250)(5) = (250)v_{af} + (200)(4.8)$ $v_{af} = 1.2$ m/s

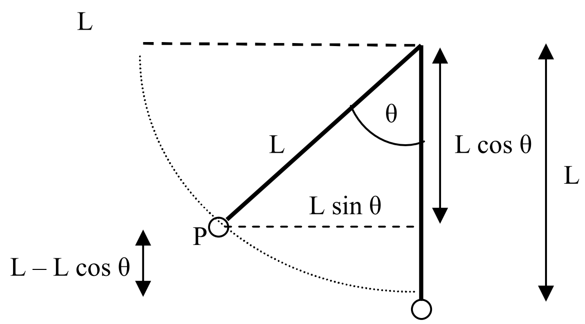
ii) Since the velocity is + the car is moving right

c) Check kinetic energy before vs after

$K_i = ½ (250)(5)^2 = 3125$ J $K_f = ½ (250) (1.2)^2 + ½ (200)(4.8)^2 = 2484$ J

Since the energies are not the same, it is inelastic

C1981M2.



a) $U_{top} = K_{bot}$
$mgh = ½ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$
$$v = \sqrt{2g(L - \frac{L}{2})}$$
$$v = \sqrt{gL}$$

b) Use the max rise height on the opposite site to find the seats speed
$K_{bot} = U_{top}$ $\quad ½ mv^2 = mgh$

$$v = \sqrt{2g(L - L\cos 45)}$$
$$v = \sqrt{2g(L - \frac{\sqrt{2}L}{2})}$$
$$v = \sqrt{2gL\ (1 - \frac{\sqrt{2}}{2})} \qquad v = \sqrt{gL\ (2 - \sqrt{2})}$$

c) Apply momentum conservation $\quad p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $\quad (2m)\sqrt{gL} = m\ v_{af} + m(\sqrt{gL\ (2 - \sqrt{2}}) \quad = \sqrt{gL}\left(2 - \sqrt{(2 - \sqrt{2})}\right)$

C1991M1.

(a) Apply momentum conservation perfect inelastic $\quad p_{before} = p_{after}$
$mv_o = (m+2m)\, v_f$ $\quad v_f = v_o / 3$

(b) Apply energy conservation.
$K_{bottom} = U_p + K_p$ $\quad ½ mv_{bot}^2 = mgh_p + K_p$
$½\, 3m\, (v_o/3)^2 = 3mg(r) + K_p$ $\quad K_p = mv_o^2/6 - 3mgr$

(c) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set F_n=0 at the top of the loop so that only mg is acting down on the block. The apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\quad 3mg = 3m\, v^2 /r$ $\quad v = \sqrt{rg}$

(d) Energy conservation top of loop to bottom of loop
$U_{top} + K_{top} = K_{bot}$
$mgh + ½\, m\, v_{top}^2 = ½\, m\, v_{bot}^2$ $\quad g(2r) + ½\, (\sqrt{rg})^2 = ½\, (vo')^2$ $\quad v_o' = \sqrt{5gr}$

(e) Apply momentum conservation, perfect inelastic with v_f as the speed found above and v_i unknown

$p_{before} = p_{after}$ $\quad mv_b' = (m+2m)\, v_f$ $\quad v_b' = 3v_f =$ $\quad v_o' = 3\sqrt{5gr}$

C1992M1.

a) $U_{top} = K_{bot}$ $mgh = ½ mv^2$ $(10)(5) = ½ v^2$ $v = 10$ m/s

b) Since the ball hits the ground elastically, it would rebound with a speed equal to that it hit with 10 m/s

c) Free fall of clay $d = v_i t + ½ gt^2 = 0 + ½ (–10)(0.5)^2$
$d = –1.25$ m displaced down, so height from ground would be 3.75 m

d) Clay free fall (down) $v_f = v_i + gt = 0 +(–10)(0.5) = –5$ m/s speed = 5 m/s
Ball free fall (up) $v_f = v_i + gt = 10 + (–10)(0.5) = 5$ m/s speed = 5 m/s

e) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai} + m_b v_{bi} = (m_a+m_b)\, v_f$ $(9m)(5) + (m)(–5) = (10m)\, v_f$ $v_f = 4$ m/s, up (since +)

C1993M1. - since there is friction on the surface the whole time, energy conservation cannot be used

a) $U_{sp} = ½ k\, \Delta x^2 = ½ (400)(0.5)^2 = 50$ J

b) Using work-energy $W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)}–U_{sp(i)}) + (K_f – K_i)$
$– f_k d = (0 – 50J) + (½ m v_f^2 – 0)$
$– \mu\, mg\, d = ½ mv_f^2 – 50$
$– (0.4)(4)(9.8)(0.5) = ½ (4)(v_c^2) – 50$ $v_c = 4.59$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ci} = (m_c+m_d)\, v_f$ $(4)(4.59) = (4+2)\, v_f$ $v_f = 3.06$ m/s

d) $W_{nc} = (K_f – K_i)$
$– f_k d = (0 – ½ m v_i^2)$ $– \mu\, mg\, d = – ½ m v_i^2$ $(0.4)(6)(9.8)\, d = ½ (6)(3.06)^2$ $d = 1.19$ m

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C1994M1.

a) $U_{sp} = ½ k\, \Delta x^2 = ½ (200)(0.4)^2 = 16$ J

b) Apply energy conservation $K_{before\ compression} = U_{sp\text{-}after\ compression}$
$½ (m_a+m_b)\, v^2 = U_{sp}$ $½ (0.1+2)\, v^2 = 16$ $v = 3.9$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai} = (m_a+m_b)\, v_f$ $(0.1)\, v_{ai} = (0.1+2)\, (3.9)$ $v_{ai} = 81.9$ m/s

---

C1995M1.

a) In the F vs t curve the impulse is the area under the curve. Area of triangle + rectangle + triangle = 12 Ns

b) $J_{on\text{-}ball} = \Delta p_{ball}$ $J = m(v_{bf} - v_{bi})$ $12 = 5(v_{bf} - 0)$ $v_{bf} = 2.4$ m/s

c) i) Due to action reaction, the force on the cube is the same as that on the ball but in the opposite direction so the impulse applied to it is –12 Ns. $J_{on\text{-}cube} = \Delta p_{cube}$ $J = m(v_{cf} - v_{ci})$ $-12 = 0.5(v_{cf} - 26)$ $v_{cf} = 2$ m/s

ii) since +, moving right

d) $\frac{1}{2} mv_{cf}^2 + \frac{1}{2} mv_{bf}^2 - \frac{1}{2} mv_{ci}^2 =$ 154 J

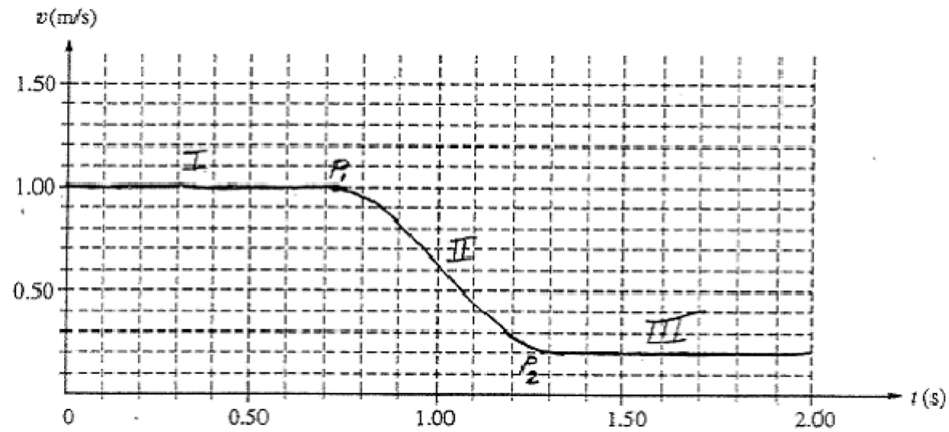
e) Using projectiles … both take same time to fall since $v_{iy} = 0$ for both and distance of fall same for both
$d_y = v_{iy}t + \frac{1}{2} g\, t_2$ $-1.2 = 0 + \frac{1}{2}(-9.8)\, t^2$ $t = 0.5$ sec

Each d_x is found using $d_x = v_x t$ for each respetive speed of cube and ball.
Gives d_x(cube) = 1m d_x(ball) = 1.2 m so they are spaced by 0.2 m when they hit.

C1998M1.

a) use v = d / t for each interval i) 1 m/s ii) 0.6 m/s iii) 0.2 m/s

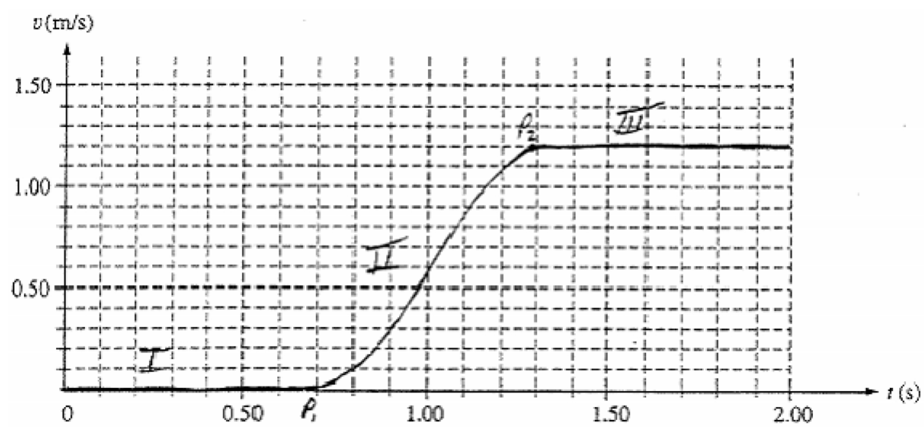
b) Based on the pattern of the Λ shapes of the ruler we can see the glider moves at a constant speed up until 0.70 s where the spacings start to change and it decelerates up until around the 1.3 second time where the speed becomes constant again. So the first constant speed is the initial velocity of the glider (1 m/s) and the second constant speed is the final velocity of the glider after the collision (0.2 m/s)



c)) i) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(0.9)(1) = (0.9)(0.2) + (0.6)(v_{bf})$ $v_{bf} = 1.2$ m/s

ii) Glider B is at rest up until 0.7 seconds where the collision accelerates to a final constant speed of 1.2 m/s



d) i) The collision is elastic because the kinetic energy before and after is the same

ii) The kinetic energy becomes a minimum because the energy is momentarily transferred to the spring

C1991M1. - The geometry of this problem is similar to C1981M2 in this document.

a) First determine the speed of the combined dart and block using energy conservation.

$K_{bot} = U_{top}$

$\frac{1}{2} mv^2 = mgh$ Then apply momentum conservation bullet to block collision

$$v = \sqrt{2g(L - L\cos\theta)}$$
$$v = \sqrt{2gL(1-\cos\theta)}$$

perfect inelastic …$p_{before} = p_{after}$

$mv_o = (m+M_o)\,v$ $$v_0 = \frac{(m+M_o)}{m}\sqrt{2gL(1-\cos\theta)}$$

b) Apply $F_{net(c)} = mv^2 / r$, at the lowest point (tension acts upwards weight acts down)

$F_t - mg = mv^2/r$ $F_t = m(g+v^2/r)$ substitute v from above

$F_t = (m+M_o)(g + 2gL(1-\cos\theta)/L) = (m+M_o)(g + 2g - 2g\cos\theta) = (m+M_o)\,g\,(3 - 2\cos\theta)$

c) One way would be to hang the spring vertically, attach the five known masses, measure the spring stretch, and use these results to find the spring constant based on $F=k\Delta x$. Then attach the block to the spring and measure the spring stretch again. Fire the dart vertically at the block and measure the maximum distance traveled. Similar to the problem above, use energy conservation to find the initial speed of the block+dart then use momentum conservation in the collision to find the darts initial speed.

C2001M1.

a) Pick velocity from the graph and use $a = (v_f - v_i)/t$ $a = -10\ \text{m/s}^2$

b) The area of the force time graph gives the impulse which equals the momentum change. You can break the graph into three triangles and 1 rectangle and find the area = 0.6 Ns = 0.6 kg m/s of momentum change

c) Using the value above. $\Delta p = m(v_f - v_i)$ $-0.6 = m(-0.22 - 0.18)$ $m = 1.5$ kg.
The force sensor applies a – momentum since it would push in the negative direction as the cart collides with it.

d) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2}(1.5)(0.18^2 - 0.22^2) = -0.012$ J

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ $mgh = \frac{1}{2} mv^2$ $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\,v_f$ $M(\sqrt{2gH}) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Even though the position shown has an unknown initial stretch and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

Apply energy conservation $K + U = U_{sp\,(gained)}$ $\frac{1}{2} mv^2 + mgh = \frac{1}{2} k\,\Delta x^2$

Plug in mass (2m), h = H/2 and Δx = H/2 ➔ $\frac{1}{2}(2m)v^2 + (2m)g(H/2) = \frac{1}{2} k(H/2)^2$

plug in v_f from part b $m(2gH/4) + mgH = kH^2/8$ ….

Both sides * (1/H) ➔ $mg/2 + mg = kH/8$ ➔ $3/2\, mg = kH/8$ $k = 12mg/H$

C2004M1.

a) Energy conservation with position B set as h=0. $U_a = K_b$ $\quad v_b = \sqrt{2gL}$

b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = m_1 v_b^2 / r$ $\qquad F_t - m_1 g = m_1(2gL) / L$ $\qquad F_t = 3m_1 g$

c) Apply momentum conservation perfect inelastic $\qquad p_{before} = p_{after}$

$m_1 v_{1i} = (m_1+m_2)\, v_f$ $\qquad v_f = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}$

d) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy} t + ½\, g\, t^2$ $\qquad L = 0 + gt^2 / 2$

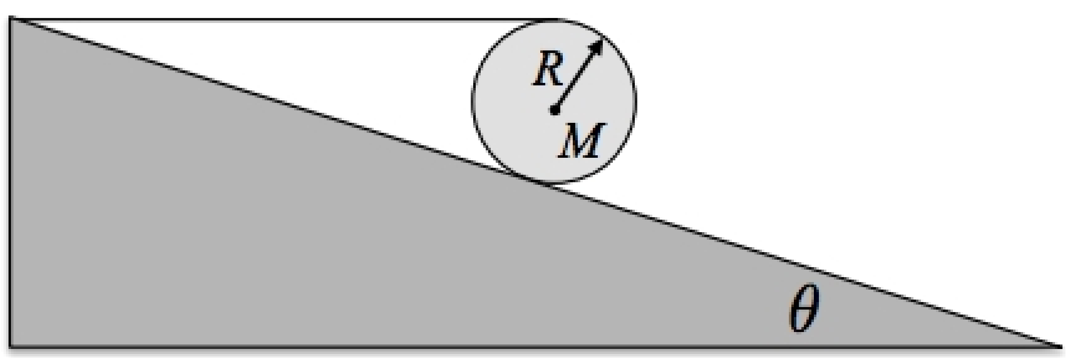
$t = \sqrt{\dfrac{2L}{g}}$ Then use $v_x = d_x / t$ $\qquad d_x = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}\sqrt{\dfrac{2L}{g}} = \dfrac{m_1}{(m_1+m_2)} 2L$

The dx found is measured from the edge of the second lower cliff so the total horizontal distance would have to include the initial x displacement (L) starting from the first cliff.

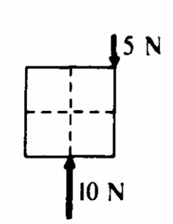
➔ $\dfrac{m_1}{(m_1+m_2)} 2L + L = L\left[\dfrac{2m_1}{(m_1+m_2)} + 1\right]$

Chapter 6

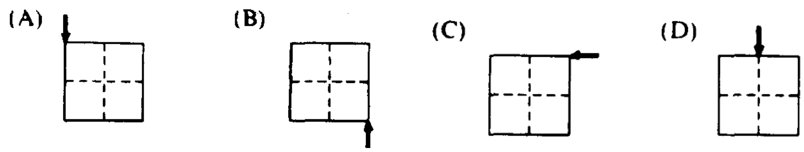
Rotation

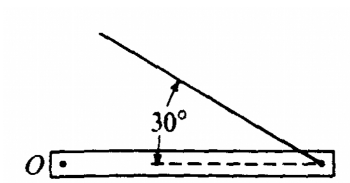


SECTION A – Torque and Statics

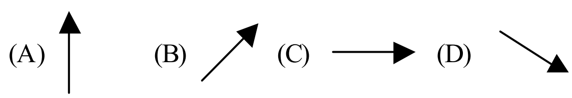


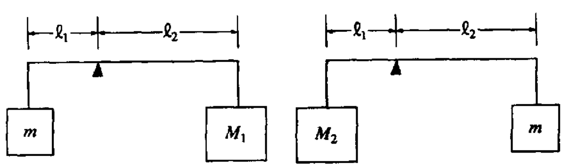
1. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown above. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?





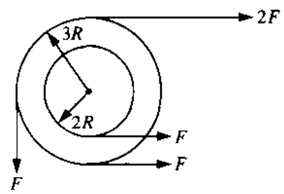
2. A uniform rigid bar of weight W is supported in a horizontal orientation as shown above by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?



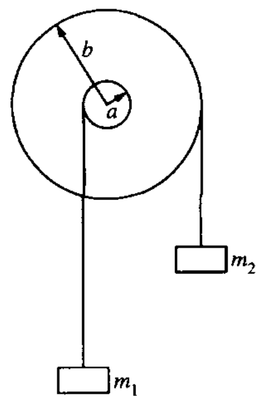


3. A rod of negligible mass is pivoted at a point that is off-center, so that length l_1 is different from length l_2. The figures above show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

(A) $M_1 + M_2$ (B) $\frac{1}{2}(M_1 + M_2)$ (C) $M_1 M_2$ (D) $\sqrt{M_1 M_2}$



4. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown above. The magnitude of the net torque on the system about the axis is
(A) FR (B) 2FR (C) 5FR (D) 14FR

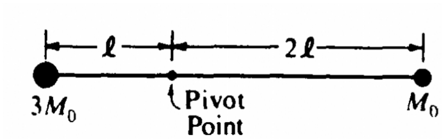


5. For the wheel-and-axle system shown above, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$ (B) $am_1 = bm_2$ (C) $am_2 = bm_1$ (D) $a^2m_1 = b^2m^2$

SECTION B – Rotational Kinematics and Dynamics

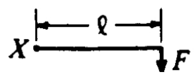
1. A uniform stick has length L. The moment of inertia about the center of the stick is I_o. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

   (A) $I_0+\frac{1}{4}ML^2$ (B) $I_0+\frac{1}{2}ML^2$ (C) $I_0+\frac{3}{4}ML^2$ (D) I_0+ML^2

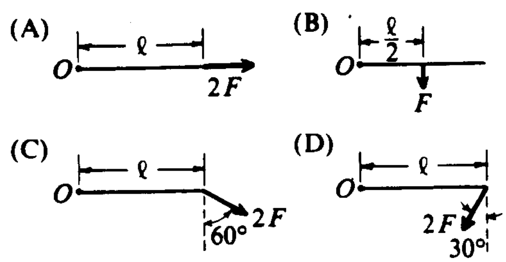


2. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude

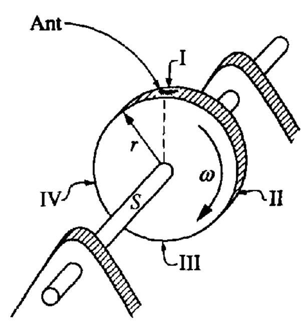
   (A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$



3. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram above? (All forces lie in the plane of the paper.)



Questions 4-5



An ant of mass m clings to the rim of a flywheel of radius r, as shown above. The flywheel rotates clockwise on a horizontal shaft S with constant angular velocity ω. As the wheel rotates, the ant revolves past the stationary points I, II, III, and IV. The ant can adhere to the wheel with a force much greater than its own weight.

4. It will be most difficult for the ant to adhere to the wheel as it revolves past which of the four points?
   (A) I (B) II (C) III (D) IV

5. What is the magnitude of the minimum adhesion force necessary for the ant to stay on the flywheel at point III?
   (A) mg (B) $m\omega^2r$ (C) $m\omega^2r$ - mg (D) $m\omega^2r$ + mg

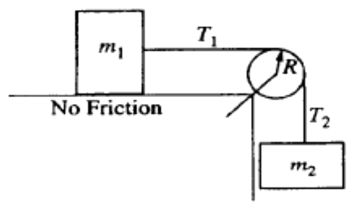
6. A turntable that is initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?
(A) $\sqrt{2\alpha}$ (B) $\sqrt{2\pi\alpha}$ (C) $\sqrt{4\pi\alpha}$ (D) $4\pi\alpha$

Questions 7-8

A wheel with rotational inertia I is mounted on a fixed, frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

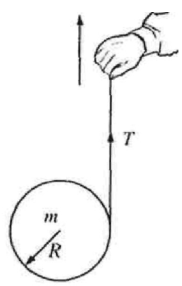
11. What is the average net torque on the wheel during this time interval?
(A) $\frac{\omega_f}{T}$ (B) $\frac{I\omega_f^2}{T}$ (C) $\frac{I\omega_f}{T^2}$ (D) $\frac{I\omega_f}{T}$

12. What is the average power input to the wheel during this time interval?
(A) $\frac{I\omega_f}{2T}$ (B) $\frac{I\omega_f^2}{2T}$ (C) $\frac{I\omega_f^2}{2T^2}$ (D) $\frac{I^2\omega_f}{2T^2}$



9. Two blocks are joined by a light string that passes over the pulley shown above, which has radius R and moment of inertia I about its center. T_1 and T_2 are the tensions in the string on either side of the pulley and α is the angular acceleration of the pulley. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?
(A) $m_2gR = I\alpha$ (B) $T_2R = I\alpha$ (C) $(T_2 - T_1)R = I\alpha$ (D) $(m_2 - m_1)gR = I\alpha$

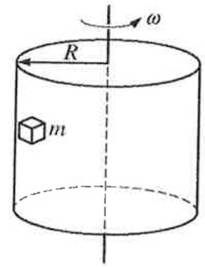
Questions 10-11



A solid cylinder of mass m and radius R has a string wound around it. A person holding the string pulls it vertically upward, as shown above, such that the cylinder is suspended in midair for a brief time interval Δt and its center of mass does not move. The tension in the string is T, and the rotational inertia of the cylinder about its axis is $\frac{1}{2}MR^2$

10. the net force on the cylinder during the time interval Δt is
(A) mg (B) $T - mgR$ (C) $mgR - T$ (D) zero

11. The linear acceleration of the person's hand during the time interval Δt is
(A) $\frac{T - mg}{m}$ (B) $2g$ (C) $\frac{g}{2}$ (D) $\frac{T}{m}$



12. A block of mass *m* is placed against the inner wall of a hollow cylinder of radius *R* that rotates about a vertical axis with a constant angular velocity *ω,* as shown above. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction *μ* between the mass and the wall must satisfy which of the following inequalities?

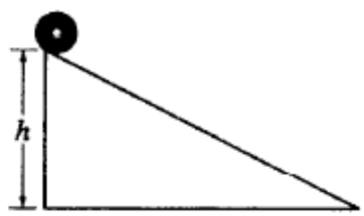
    (A) $\mu \square \dfrac{g}{\square^2 R}$ (B) $\mu \square \dfrac{\square^2 R}{g}$ (C) $\mu \square \dfrac{g}{\square^2 R}$ (D) $\mu \square \dfrac{\square^2 R}{g}$

SECTION C – Rolling

1. A bowling ball of mass M and radius R. whose moment of inertia about its center is (2/5)MR^2, rolls without slipping along a level surface at speed *v*. The maximum vertical height to which it can roll if it ascends an incline is

    (A) $\dfrac{v^2}{5g}$ (B) $\dfrac{2v^2}{5g}$ (C) $\dfrac{v^2}{2g}$ (D) $\dfrac{7v^2}{10g}$

Questions 2-3



A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown above.

2. If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?

    (A) $\sqrt{2gh}$ (B) $\dfrac{2Mghr^2}{I}$ (C) $\sqrt{\dfrac{2Mghr^2}{I}}$ (D) $\sqrt{\dfrac{2Mghr^2}{I+Mr^2}}$

3. If the plane has friction so that the sphere rolls without slipping, what is the speed v_{cm} of the center of mass at the bottom of the incline?

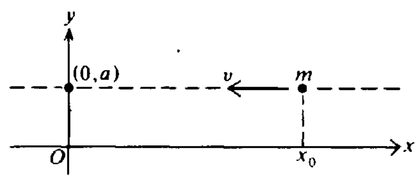
    (A) $\sqrt{2gh}$ (B) $\dfrac{2Mghr^2}{I}$ (C) $\sqrt{\dfrac{2Mghr^2}{I}}$ (D) $\sqrt{\dfrac{2Mghr^2}{I+Mr^2}}$

4. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
    (A) vertically upward (B) horizontally forward (C) horizontally backward
    (D) upward and forward, at approximately 45° to the horizontal

SECTION D – Angular Momentum

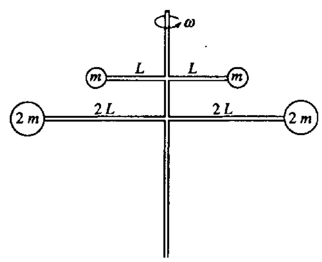
1. An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

	Angular Momentum	Kinetic Energy
(A)	Increases	Increases
(B)	Increases	Remains Constant
(C)	Remains Constant	Increases
(D)	Remains Constant	Remains Constant

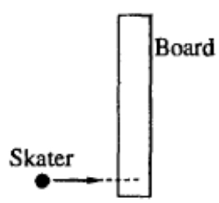


2. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is

(A) zero (B) mva (C) mvx_o (D) $mv\sqrt{x_0^2 + a^2}$

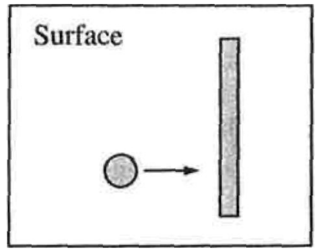


3. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?
(A) 2/1 (B) 1/2 (C) 1/4 (D) 1/8



Top View

4. A long board is free to slide on a sheet of frictionless ice. As shown in the top view above, a skater skates to the board and hops onto one end, causing the board to slide and rotate. In this situation, which of the following occurs?
(A) Linear momentum is converted to angular momentum.
(B) Rotational kinetic energy is conserved.
(C) Translational kinetic energy is conserved.
(D) Linear momentum and angular momentum are both conserved.

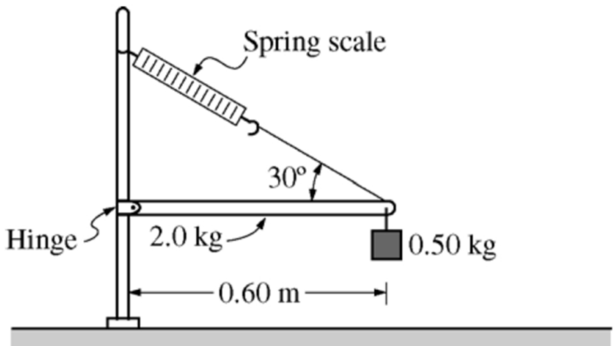


Top View

5. **Multiple Correct.** A disk sliding on a horizontal surface that has negligible friction collides with a rod that is free to move and rotate on the surface, as shown in the top view above. Which of the following quantities must be the same for the disk-rod system before and after the collision? Select two answers.
   I. Linear momentum
   II. Angular momentum
   III. Kinetic energy
   (A) Linear Momentum
   (B) Angular Momentum
   (C) Kinetic Energy
   (D) Mechanical Energy

<u>WARNING: These are AP Physics C Free Response Practice – Use with caution!</u>

<u>SECTION A – Torque and Statics</u>



2008M2. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

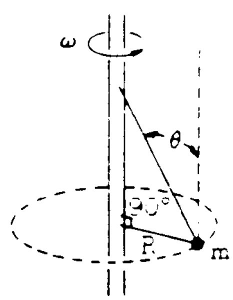
b. Calculate the reading on the spring scale.

The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length.

c. Calculate the rotational inertia of the rod-block system about the hinge.

d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

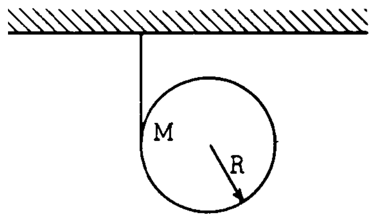
<u>SECTION B – Rotational Kinematics and Dynamics</u>



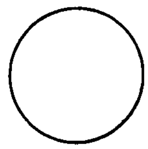
1973M3. A ball of mass m is attached by two strings to a vertical rod. as shown above. The entire system rotates at constant angular velocity ω about the axis of the rod.

a. Assuming ω is large enough to keep both strings taut, find the force each string exerts on the ball in terms of ω, m, g, R, and θ.

b. Find the minimum angular velocity, ω_{min} for which the lower string barely remains taut.



1976M2. A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $\frac{1}{2}MR^2$.



a. On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.
b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.
c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

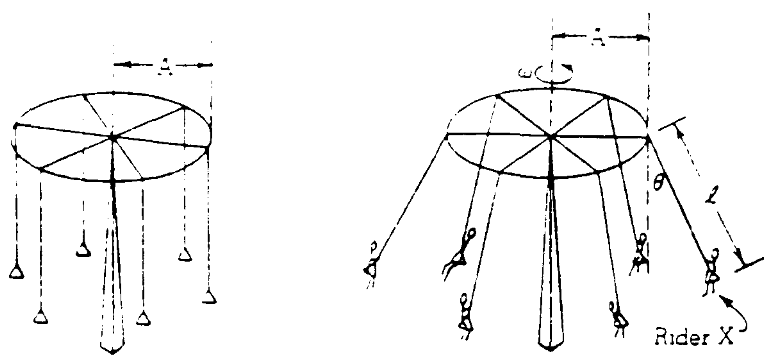


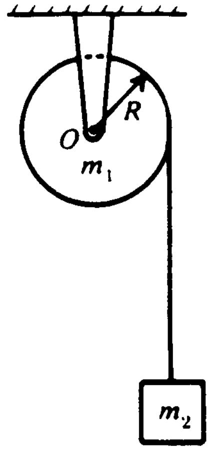
Figure I Figure II

1978M1. An amusement park ride consists of a ring of radius A from which hang ropes of length ℓ with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

a. In the space below, draw and label all the forces acting on rider X (represented by the point below) under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

•

b. Derive an expression for ω in terms of A, ℓ, θ and the acceleration of gravity g.
c. Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, ℓ, θ, and the speed v of each rider.



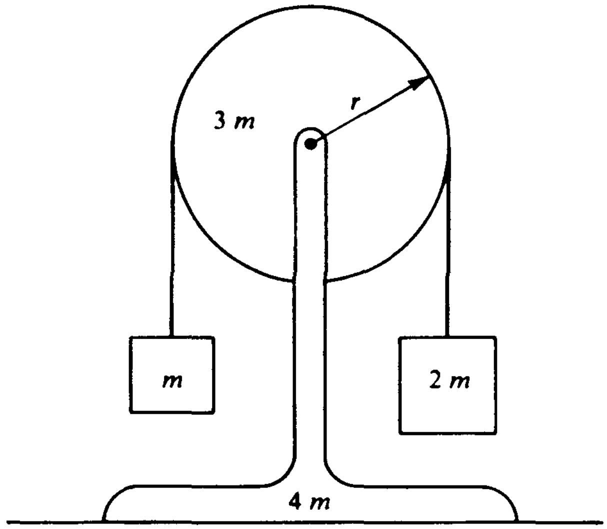
1983M2. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = \frac{1}{2}m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.

a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.

b. In terms of m_1, m_2, R. and g, determine each of the following.

i. The acceleration of the block

ii. The tension in the cord



1985M3. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$.
Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

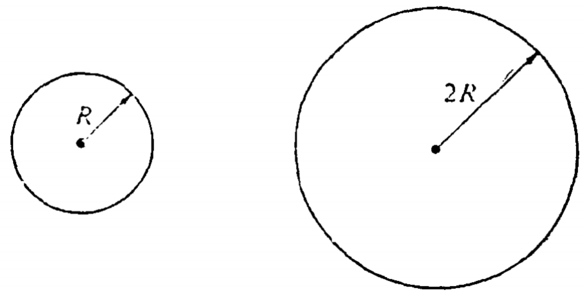
a. On the diagrams below, draw and label all the forces acting on each block.

b. Use the symbols identified in part a. to write each of the following.
   i. The equations of translational motion (Newton's second law) for each of the two blocks
   ii. The analogous equation for the rotational motion of the pulley

c. Solve the equations in part b. for the acceleration of the two blocks.

d. Determine the tension in the segment of the cord attached to the block of mass m.

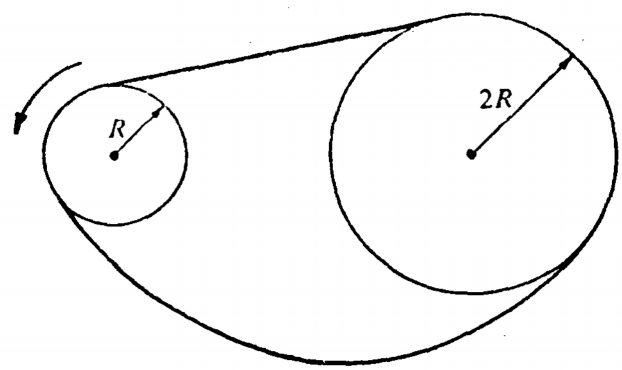
e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.



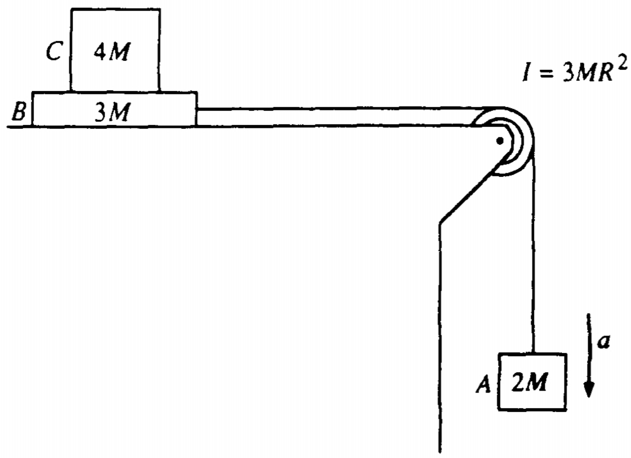
1988M3. The two uniform disks shown above have equal mass, and each can rotate on frictionless bearings about a fixed axis through its center. The smaller disk has a radius R and moment of inertia I about its axis. The larger disk has a radius 2R

a. Determine the moment of inertia of the larger disk about its axis in terms of I.

The two disks are then linked as shown below by a light chain that cannot slip. They are at rest when, at time t = 0, a student applies a torque to the smaller disk, and it rotates counterclockwise with constant angular acceleration α. Assume that the mass of the chain and the tension in the lower part of the chain, are negligible. In terms of I, R, α, and t, determine each of the following:



b. The angular acceleration of the larger disk
c. The tension in the upper part of the chain
d. The torque that the student applied to the smaller disk
e. The rotational kinetic energy of the smaller disk as a function of time



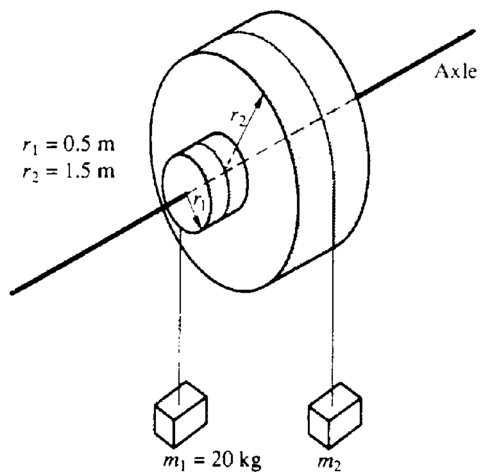
1989M2. Block A of mass 2M hangs from a cord that passes over a pulley and is connected to block B of mass 3M that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass 4M is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the

a. tension T_v in the vertical section of the cord
b. tension T_h in the horizontal section of the cord

If a = 2 meters per second squared, determine the

c. coefficient of kinetic friction between blocks B and C
d. acceleration of block C

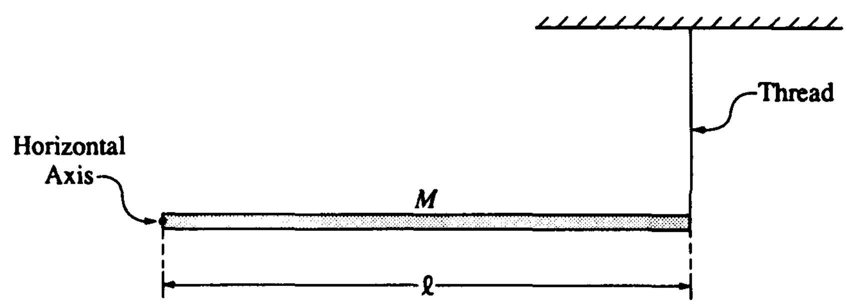


1991M2. Two masses. m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is I = 45 kg•m^2 Also r_1 = 0.5 meter, r_2 = 1.5 meters, and m_1 = 20 kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.
c. Determine the tension in the cable supporting m_1
d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.



1993M3. A long, uniform rod of mass M and length ℓ is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of M, ℓ and g.

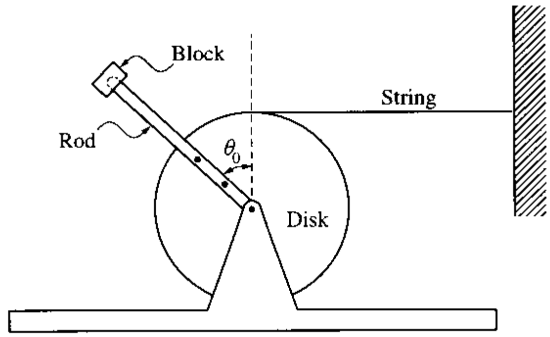
a. Determine the magnitude and direction of the force exerted on the rod by the axis.

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

b. The angular acceleration of the rod about the axis
c. The translational acceleration of the center of mass of the rod
d. The force exerted on the end of the rod by the axis

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod has swung.



1999M3 As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass = 3m, radius = R, moment of inertia about center $I_D = 1.5mR^2$

Rod: mass = m, length = 2R, moment of inertia about one end $I_R = 4/3(mR^2)$

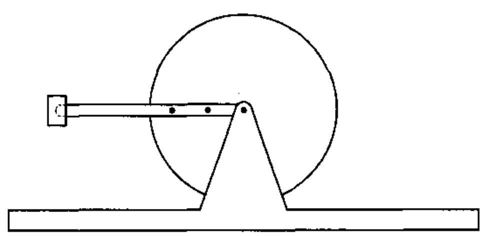
Block: mass = 2m

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

a. Determine the tension in the string.

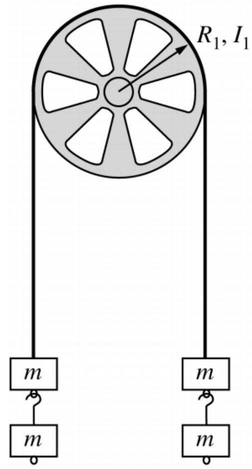
The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.
i. The magnitude of the angular acceleration of the disk
ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

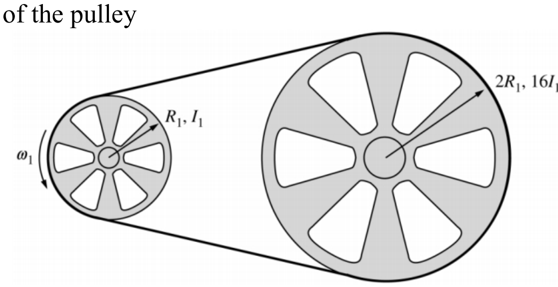
c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.



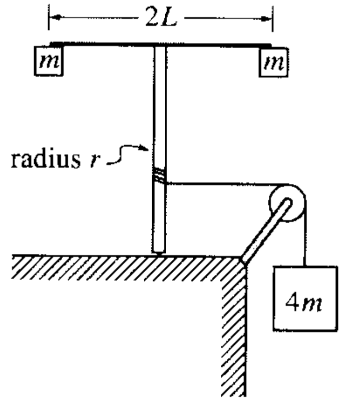
2000M3. A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley. Determine the answers to parts a. and b. in terms of m, R_1, I_1, and fundamental constants.

a. Determine the tension T in the cord.

b. One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration g/3 . Determine the following.
   i. The tension T_3 in the section of cord supporting the three blocks on the left
   ii. The tension T_1 in the section of cord supporting the single block on the right
   iii. The rotational inertia I_1 of the pulley



c. The blocks are now removed and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_I, and ω_1.
   i. The angular speed ω_2 of the larger pulley
   ii. The angular momentum L_2 of the larger pulley
   iii. The total kinetic energy of the system



Experiment A

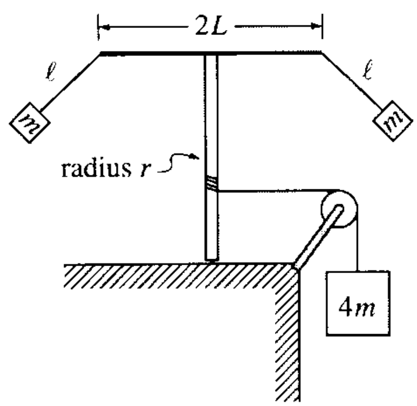
2001M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.

b. Determine the downward acceleration of the large block.

c. When the large block has descended a distance *D,* how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD*? Check the appropriate space below and justify your answer.

Greater than 4mgD ______ Equal to 4mgD ____ Less than 4mgD _____



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? Check the appropriate space below and justify your answer.

Greater before ___ Equal to before ___ Less than before ___

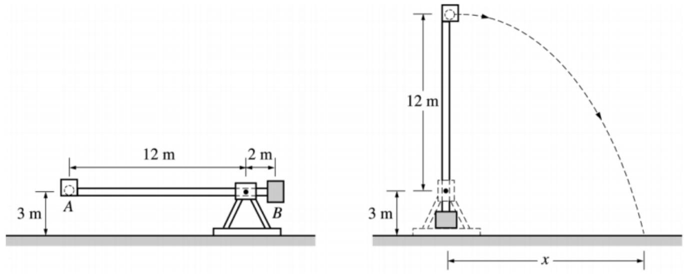


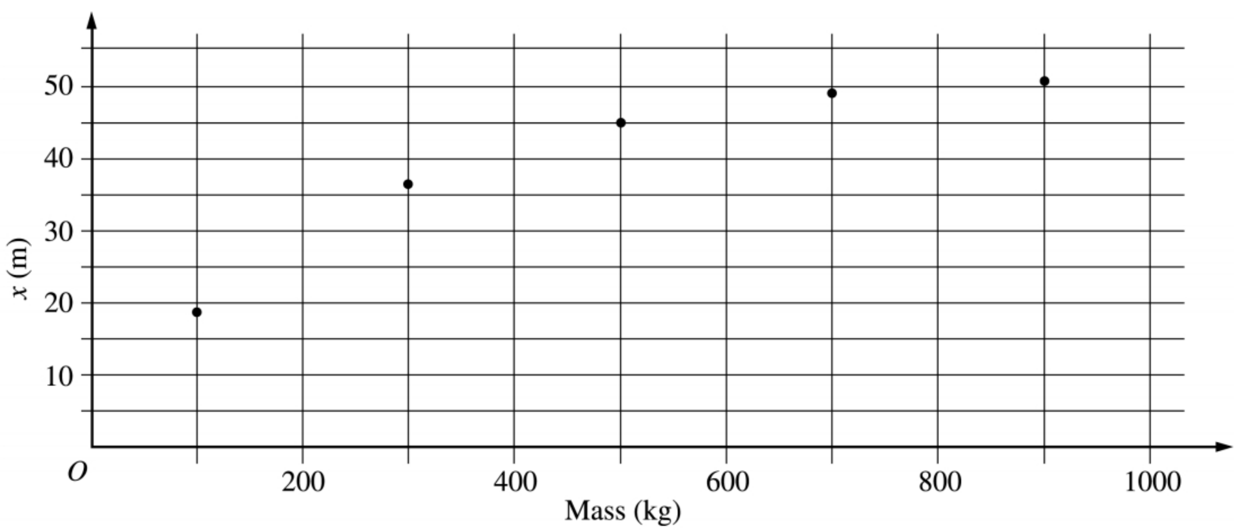
Figure 1 Figure 2

2003M3. Some physics students build a catapult, as shown above. The supporting platform is fixed firmly to the ground. The projectile, of mass 10 kg, is placed in cup *A* at one end of the rotating arm. A counterweight bucket *B* that is to be loaded with various masses greater than 10 kg is located at the other end of the arm. The arm is released from the horizontal position, shown in Figure 1, and begins rotating. There is a mechanism (not shown) that stops the arm in the vertical position, allowing the projectile to be launched with a horizontal velocity as shown in Figure 2.

a. The students load five different masses in the counterweight bucket, release the catapult, and measure the resulting distance x traveled by the 10 kg projectile, recording the following data.

Mass (kg)	100	300	500	700	900
x (m)	18	37	45	48	51

i. The data are plotted on the axes below. Sketch a best-fit curve for these data points.



ii. Using your best-fit curve, determine the distance x traveled by the projectile if 250 kg is placed in the counterweight bucket.

b. The students assume that the mass of the rotating arm, the cup, and the counterweight bucket can be neglected. With this assumption, they develop a theoretical model for x as a function of the counterweight mass using the relationship $x = v_x t$, where v, is the horizontal velocity of the projectile as it leaves the cup and t is the time after launch.

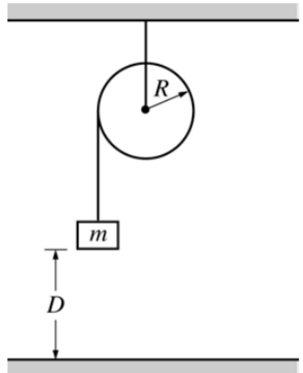
i. How many seconds after leaving the cup will the projectile strike the ground?

ii. Derive the equation that describes the gravitational potential energy of the system relative to the ground when in the position shown in Figure 1, assuming the mass in the counterweight bucket is M.

iii. Derive the equation for the velocity of the projectile as it leaves the cup, as shown in Figure 2.

c. i. Complete the theoretical model by writing the relationship for x as a function of the counterweight mass using the results from b. i and b. iii.

ii. Compare the experimental and theoretical values of x for a counterweight bucket mass of 300 kg. Offer a reason for any difference.



2004M2. A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

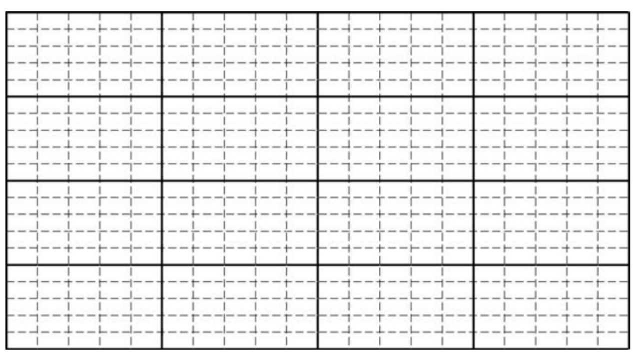
a. Calculate the linear acceleration a of the falling block in terms of the given quantities.

b. The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in b. i., label the axes, and draw the best-fit line to the data.

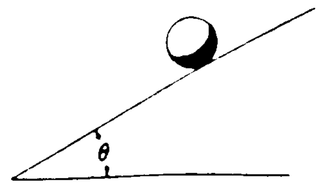


iii. Use your graph to calculate the magnitude of the acceleration.

c. Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

d. The value of acceleration found in b.iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

SECTION C – Rolling



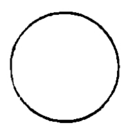
1974M2. The moment of inertia of a uniform solid sphere (mass M, radius R) about a diameter is $2MR^2/5$. The sphere is placed on an inclined plane (angle θ) as shown above and released from rest.

a. Determine the minimum coefficient of friction μ between the sphere and plane with which the sphere will roll down the incline without slipping

b. If μ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part a.? Explain your answer.

1977M2. A uniform cylinder of mass M, and radius R is initially at rest on a rough horizontal surface. The moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$. A string, which is wrapped around the cylinder, is pulled upwards with a force T whose magnitude is 0.6Mg and whose direction is maintained vertically upward at all times. In consequence, the cylinder both accelerates horizontally and slips. The coefficient of kinetic friction is 0.5.

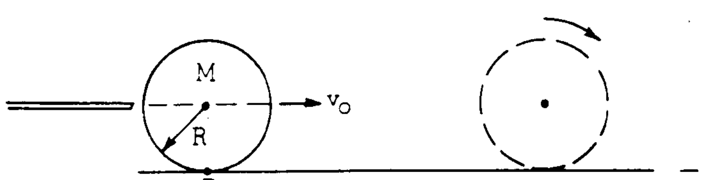
a. On the diagram below, draw vectors that represent each of the forces acting on the cylinder identify and clearly label each force.



b. Determine the linear acceleration a of the center of the cylinder.

c. Calculate the angular acceleration α of the cylinder.

d. Your results should show that a and αR are not equal. Explain.



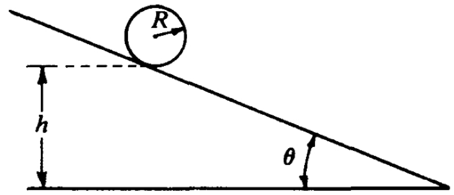
1980M3. A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c = 2\ MR^2/5$
The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_o as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.

a. Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

b. Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

c. Determine the time at which the ball begins to roll without slipping.

d. When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

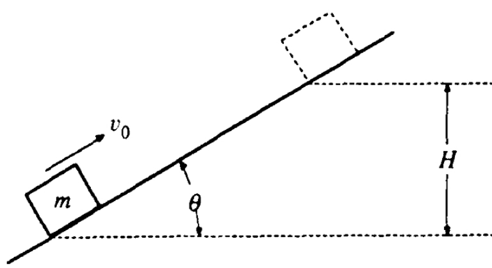


1986M2. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height *h* above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R. *h,* g, and θ.

a. Determine the following for the sphere when it is at the bottom of the plane:
   i. Its translational kinetic energy
   ii. Its rotational kinetic energy
b. Determine the following for the sphere when it is on the plane.
   i. Its linear acceleration
   ii. The magnitude of the frictional force acting on it

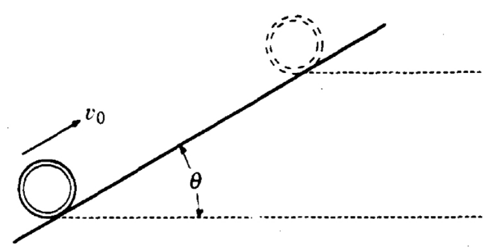
The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?
d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.



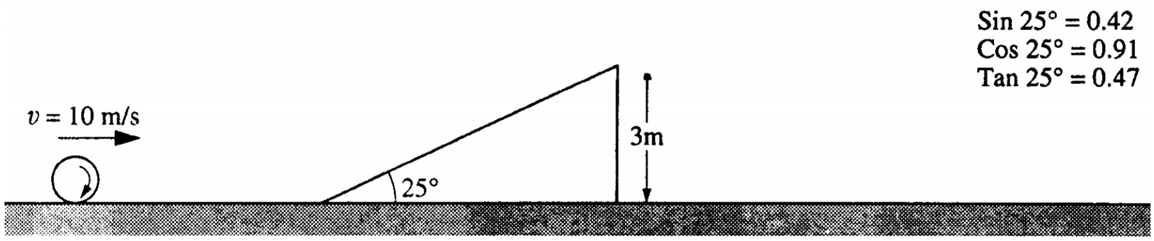
1990M2. A block of mass m slides up the incline shown above with an initial speed v_O in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_O in the position shown.

c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities.
d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.



Note: Diagram not drawn to scale.

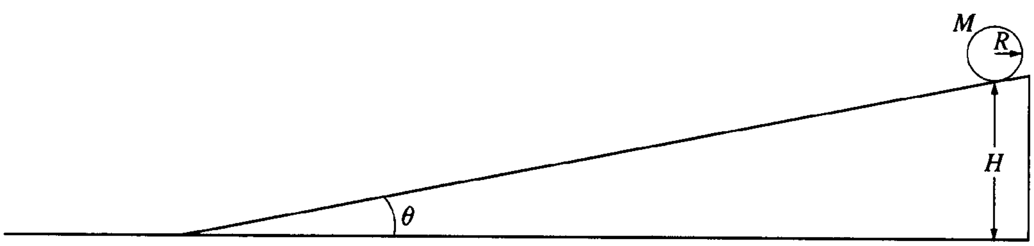
1994M2. A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass m of 25 kilograms, and a radius r of 0.2 meter. The moment of inertia of the sphere about its center of mass is $I = 2mr^2/5$. The sphere approaches a 25° incline of height 3 meters as shown above and rolls up the incline without slipping.

a. Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b. i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.
ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

c. Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

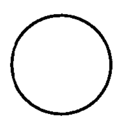
d. Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in b. Explain briefly.



1997M3. A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



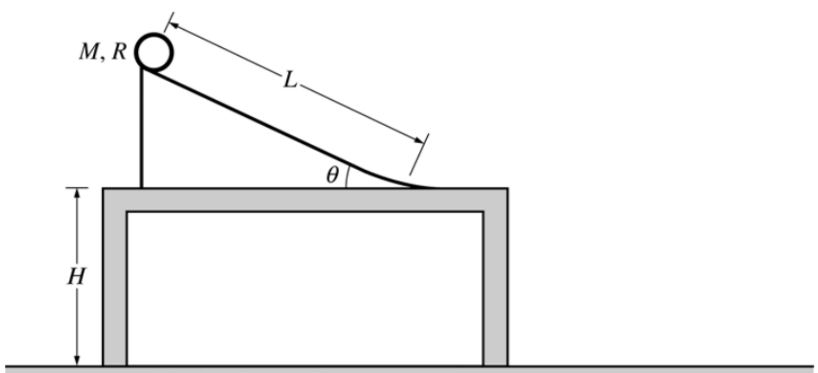
c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is $(2/3)g \sin\theta$.

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction μ is now made less than the value determined in part d., so that the cylinder both rotates and slips.
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part a. Justify your answer.
ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

2002M2. The cart shown above is made of a block of mass m and four solid rubber tires each of mass m/4 and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia ½ ML^2, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the total rotational inertia of all four tires.
b. Determine the speed of the cart when it reaches the bottom of the incline.
c. After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.
d. Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part c.. Give a reasonable explanation for this decrease.

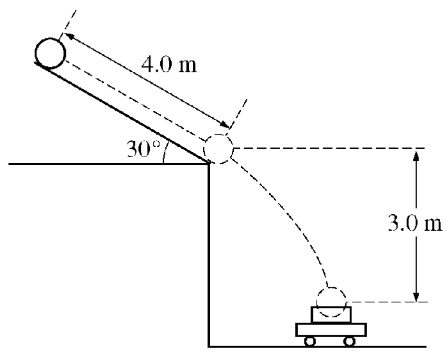


2006M3. A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.
b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.
c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.
d. Suppose that the hoop is now replaced by a disk having the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

Less than____ The same as____ Greater than____

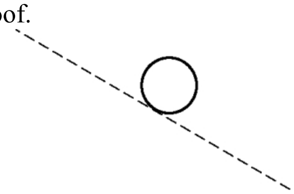
Briefly justify your response.



Note: Figure not drawn to scale.

2010M2. A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $2MR^2/5$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.

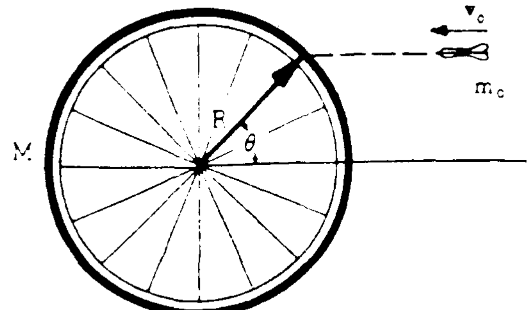


b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part a. to assist in your solution, use the space below. Do NOT add anything to the figure in part a.

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

SECTION D – Angular Momentum



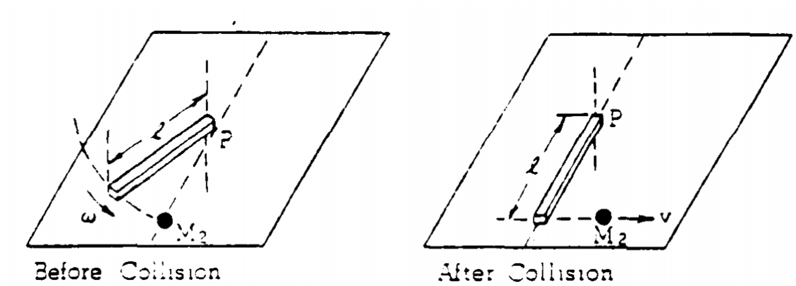
View From Above

1975M2. A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_o is thrown with velocity v_o as shown above and sticks in the tire.

a. If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

b. In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$



1978M2. A system consists of a mass M_2 and a uniform rod of mass M_1 and length l. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, l, and ω.

b. Determine the linear momentum of this system just before the collision in terms of M_1, l, and ω.

c. Determine the linear momentum of this system just after the collision in terms of M_1 l, and ω.

d. What is responsible for the change in the linear momentum of this system during the collision?

e. Why is the angular momentum of this system about point P conserved during the collision?

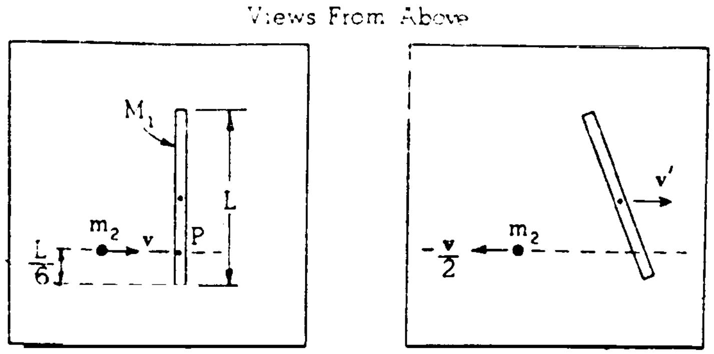


Figure I: Before

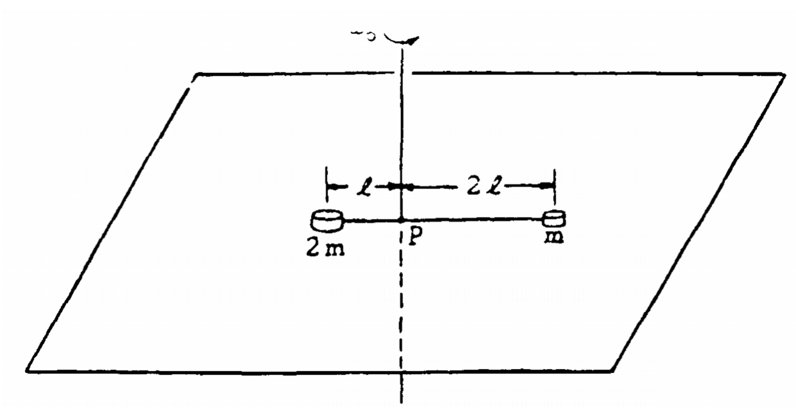
Figure II: After

1981M3. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $M_1L^2/12$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity $-\frac{1}{2}\mathbf{v}$ as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity **v**' of the center of mass of this rod after the collision.

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

c. Determine the change in kinetic energy of the system resulting from the collision.

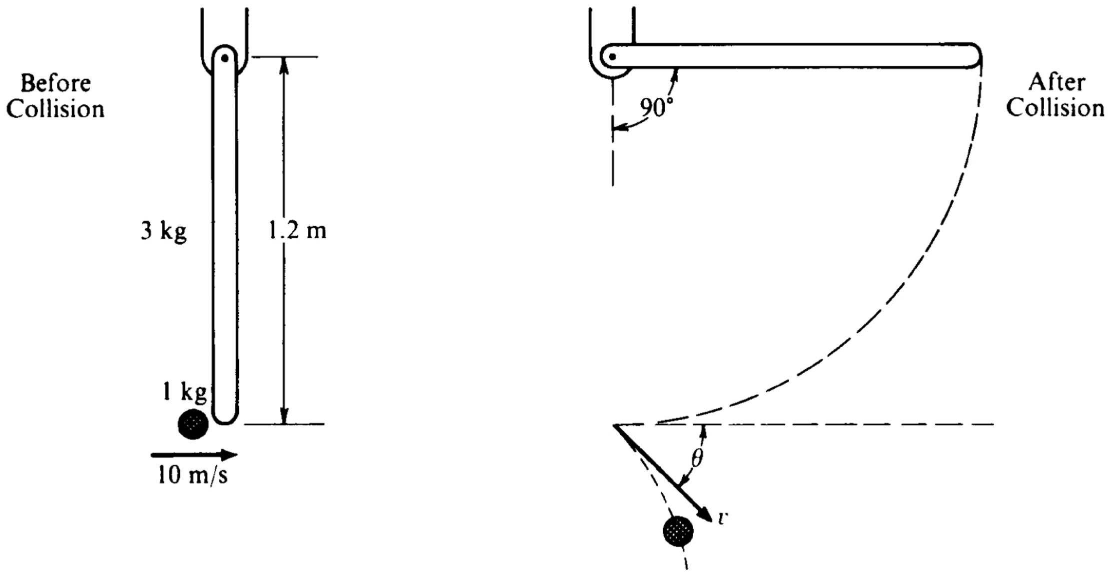


1982M3. A system consists of two small disks, of masses m and 2m, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time t = 0, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction. Develop expressions for the following quantities in terms of μ m, l, g, and ω_o

a. The initial angular momentum of the system about the axis through P

b. The frictional torque acting on the system about the axis through P

c. The time T at which the system will come to rest.



Note: You may use $g = 10\ \text{m/s}^2$.

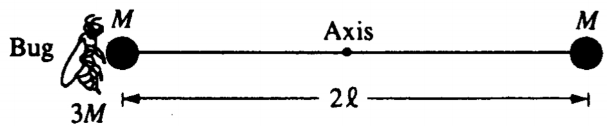
1987M3. A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.

a. Determine the angular velocity of the bar immediately after the collision.

b. Determine the speed v of the 1-kilogram object immediately after the collision.

c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.

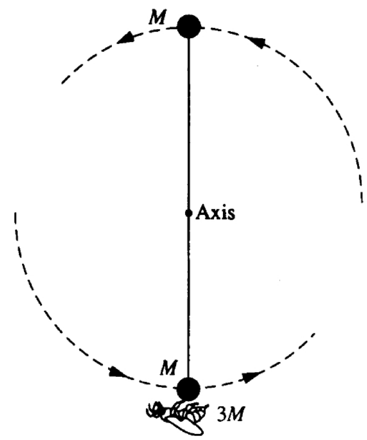
d. Determine the angle θ.



1992M2. Two identical spheres, each of mass M and negligible radius, are fastened to opposite ends of a rod of negligible mass and length 2*l*. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass 3M, lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of M, *l*, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

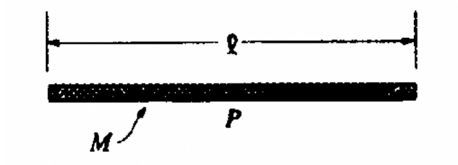


The rod-spheres-bug system swings about the axis. At the instant that the rod is vertical, as shown above, determine each of the following.

c. The angular speed of the bug

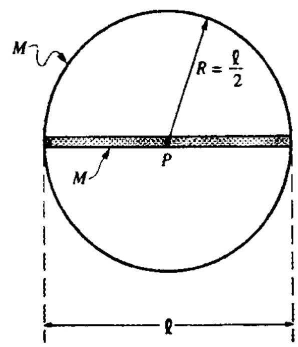
d. The angular momentum of the system

e. The <u>magnitude</u> and <u>direction</u> of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere



1996M3. Consider a thin uniform rod of mass M and length l, as shown above.

a. Show that the rotational inertia of the rod about an axis through its center and perpendicular to its length is $Ml^2/12$.



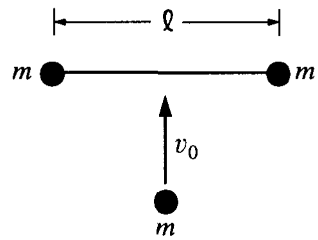
The rod is now glued to a thin hoop of mass M and radius R/2 to form a rigid assembly, as shown above. The centers of the rod and the hoop coincide at point P. The assembly is mounted on a horizontal axle through point P and perpendicular to the page.

b. What is the rotational inertia of the rod-hoop assembly about the axle?

Several turns of string are wrapped tightly around the circumference of the hoop. The system is at rest when a cat, also of mass M, grabs the free end of the string and hangs vertically from it without swinging as it unwinds, causing the rod-hoop assembly to rotate. Neglect friction and the mass of the string.

c. Determine the tension T in the string.
d. Determine the angular acceleration a of the rod-hoop assembly.
e. Determine the linear acceleration of the cat.
f. After descending a distance H = $5l/3$, the cat lets go of the string. At that instant, what is the angular momentum of the cat about point P?

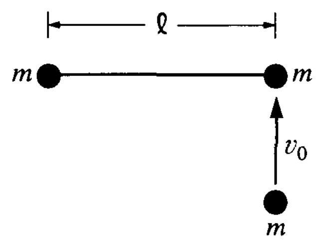
1998M2. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length l and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0 . Express your answers in terms of m, v_0 l. and fundamental constants.



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision.

ii. Determine the change in kinetic energy as a result of the collision.



b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly.

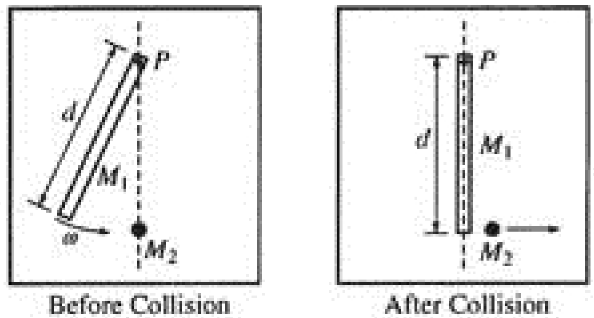
i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.)

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision.

iii. Determine the speed of the center of mass immediately after the collision.

iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

v. Determine the change in kinetic energy as a result of the collision.



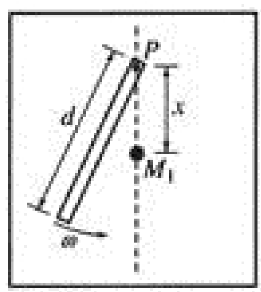
TOP VIEWS

2005M3. A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

a. Derive an expression for the angular momentum of the rod about point P before the collision.

b. Derive an expression for the speed v of the ball after the collision.

c. Assuming that this collision is elastic, calculate the numerical value of the ratio M_1 / M_2



Before Collision

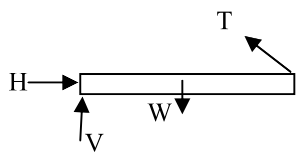
d. A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

SECTION A – Torque and Statics

Solution | Answer

1. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed — A

2. FBD



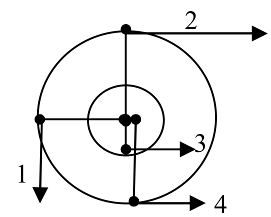
Since the rope is at an angle it has x and y components of force.
Therefore, H would have to exist to counteract T_x.
Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar — B

3. Applying rotational equilibrium to each diagram gives — D

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$
$L_1 = M_1(L_2)/m$
(sub this L_1) into the Diagram 2 eqn, and solve.

DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$
$M_2(L_1) = m(L_2)$

4. Find the torques of each using proper signs and add up. — B
$+(1) - (2) + (3) + (4)$
$+F(3R) - (2F)(3R) + F(2R) + F(3R) = 2FR$



5. Simply apply rotational equilibrium — B
$(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
$m_1a = m_2b$

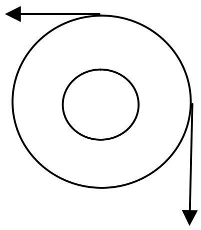
SECTION B – Rotational Kinematics and Dynamics

1. $I_{tot} = \Sigma I = I_0 + I_M = I_0 + M(\tfrac{1}{2}L)^2$ — A
2. $\Sigma\tau = I\alpha$ where $\Sigma\tau = (3M_0)(l) - (M_0)(2l) = M_0l$ and $I = (3M_0)(l)^2 + (M_0)(2l)^2 = 7M_0l^2$ — A
3. $\tau_X = Fl$; $\tau_O = F_OL_O \sin\theta$, solve for the correct combination of F_O and L_O — C
4. Just as the tension in a rope is greatest at the bottom of as vertical circle, the force needed to maintain circular motion in any vertical circle is greatest at the bottom as the applied force must balance the weight of the object and additionally provide the necessary centripetal force — C
5. $\Sigma F_{bottom} = F_{adhesion} - mg = F_{centripetal} = m\omega^2 r$ — D
6. For one complete revolution $\theta = 2\pi$; $\omega^2 = \omega_0^2 + 2\alpha\theta$ — C

7. $\tau = \Delta L/\Delta t = (I\omega_f - 0)/T$ D

8. $P_{avg} = \tau\omega_{avg} = (I\omega_f/T)(\frac{1}{2}\omega_f)$ or $P_{avg} = \Delta K/T$ B

9. C



$\Sigma\tau = T_2R - T_1R = I\alpha$

10. If the cylinder is "suspended in mid air" (i.e. the linear acceleration is zero) then $\Sigma F = 0$ D

11. $\Sigma\tau = TR = I\alpha = \frac{1}{2} MR^2\alpha$ which gives $\alpha = 2T/MR$ and since $\Sigma F = 0$ then $T = Mg$ so $\alpha = 2g/R$ the acceleration of the person's hand is equal to the linear acceleration of the string around the rim of the cylinder $a = \alpha R = 2g$ B

12. In order that the mass not slide down $f = \mu F_N \geq mg$ and $F_N = m\omega^2R$ solving for μ gives $\mu \geq g/\omega^2R$ A

SECTION C – Rolling

1. $K_{tot} = K_{rot} + K_{trans} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2 = \frac{1}{2} (2/5)MR^2\omega^2 + \frac{1}{2} Mv^2 = (1/5)Mv^2 + \frac{1}{2} Mv^2 = (7/10)Mv^2 = MgH$, solving gives $H = 7v^2/10g$ D

2. $Mgh = K_{tot} = K_{rot} + K_{trans}$, however without friction, there is no torque to cause the sphere to rotate so $K_{rot} = 0$ and $Mgh = \frac{1}{2} Mv^2$ A

3. $Mgh = K_{tot} = K_{rot} + K_{trans} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$; substituting v/r for ω gives $Mgh = \frac{1}{2}(I/r^2 + M)v^2$ and solving for v gives $v^2 = 2Mgh/(I/r^2 + M)$, multiplying by r^2/r^2 gives desired answer D

4. The first movement of the point of contact of a rolling object is vertically upward as there is no side to side (sliding) motion for the point in contact A

SECTION D – Angular Momentum

1. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater — C

2. $L = mvr_\perp$ where $r_\perp$ is the perpendicular line joining the origin and the line along which the particle is moving — B

3. $L = I\omega$ and since ω is uniform the ratio $L_{upper}/L_{lower} = I_{upper}/I_{lower} = 2mL^2/2(2m)(2L)^2 = 1/8$ — D

4. Since it is a perfectly inelastic (sticking) collision, KE is not conserved. As there are no external forces or torques, both linear and angular momentum are conserved — D

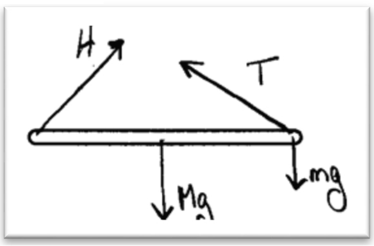
5. As there are no external forces or torques, both linear and angular momentum are conserved. As the type of collision is not specified, we cannot say kinetic or mechanical energy *must* be the same. — A/B

WARNING: These are AP Physics C Free Response Practice – Rotation – ANSWERS
Use with caution!

SECTION A – Torque and Statics

2008M2

a.



b. $\Sigma\tau = 0$
About the hinge: $TL \sin 30^\circ - mgL - Mg(L/2) = 0$ gives $T = 29$ N

c. $I_{total} = I_{rod} + I_{block}$ where $I_{rod,end} = I_{cm} + MD^2 = ML^2/12 + M(L/2)^2 = ML^2/3$
$I_{total} = ML^2/3 + mL^2 = 0.42$ kg-m^2

d. $\Sigma\tau = I\alpha$
$mgL + MgL/2 = I\alpha$ gives $\alpha = 21$ rad/s^2

SECTION B – Rotational Kinematics and Dynamics

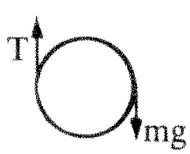
1973M3

a. Define a coordinate system with the x-axis directed to the vertical rod and the y-axis directed upwards and perpendicular to the first. Let T_1 be the tension in the horizontal string. Let T_2 be the tension in the string tilted upwards.
Applying Newton's Second Law: $\Sigma F_x = T_1 + T_2 \sin\theta = m\omega^2 R$; $\Sigma F_y = T_2 \cos\theta - mg = 0$
Solving yields: $T_2 = mg/\cos\theta$ and $T_1 = m(\omega^2 R - g \tan\theta)$

b. Let $T_1 = 0$ and solving for ω gives $\omega = (g \tan\theta/R)^{1/2}$

1976M2

a.



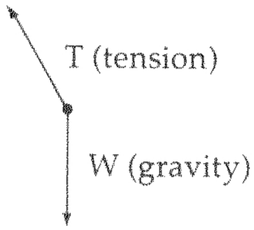
b. $\Sigma\tau = I\alpha$ (about center of mass) (one could also choose about the point at which the tape comes off the cylinder)
$TR = ½ MR^2\times(a/R)$
$T = ½ Ma$

$\Sigma F = ma$
$Mg - T = Ma$
$Mg = 3Ma/2$
$a = 2g/3$

c. As there are no horizontal forces, the cylinder moves straight down.

<u>1978M1</u>

a.



b. $\Sigma F = ma$; $T \cos \theta = mg$ and $T \sin \theta = m\omega^2 r = m\omega^2(A + \ell \sin \theta)$

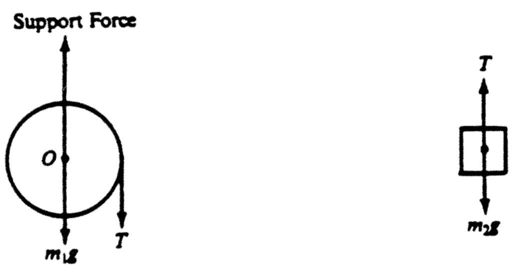
$$\omega = \sqrt{\frac{g \tan\theta}{A + l \sin\theta}}$$

c. $W = \Delta E = \Delta K + \Delta U = ½ mv^2 + mg\ell(1 – \cos \theta)$ for each rider

$W = 6(½ mv^2 + mg\ell(1 – \cos \theta))$

<u>1983M2</u>

a.



b. i./ii. On the disk: $\Sigma\tau = I\alpha = TR = ½ m_1R^2\alpha$

For the block $a = \alpha R$ so $\alpha = a/R$ and $\Sigma F = m_2g – T = m_2a$

Solving yields

$$a = \frac{2m_2 g}{2m_2 + m_1} \text{ and } T = \frac{m_1 m_2 g}{2m_2 + m_1}$$

1985M3

a.

b. i. $\Sigma F = ma$; $T_1 - mg = ma$ and $2mg - T_2 = 2ma$
ii. $\Sigma\tau = I\alpha$; $(T_2 - T_1)r = I\alpha$

c. $\alpha = a/r$
Combining equations from b.i. gives $T_2 - T_1 = mg - 3ma$
Substituting for $(T_2 - T_1)$ into torque equation gives $a = 2g/9$

d. $T_1 = m(g + a) = 11mg/9$

e. $F_N = 7mg + T_1 + T_2$ (the table counters all the downward forces on the *apparatus*)
$T_2 = 2m(g - a) = 14mg/9$
$F_N = 88mg/9$

1988M3

a. I is proportional to mR^2; masses are equal and R becomes 2R
$I_{2R} = 4I$

b. The disks are coupled by the chain along their rims, which means the linear motion of the rims have the same displacement, velocity and acceleration.
$v_R = v_{2R}$; $R\omega_R = 2R\omega_{2R}$; $R\alpha t = 2R\alpha_{2R}t$ gives $\alpha_{2R} = \alpha/2$

c. $\tau_{2R} = T(2R) = I_{2R}\alpha_{2R} = (4I)(\alpha/2) = 2I\alpha$ giving $T = I\alpha/R$

d. $\Sigma\tau = \tau_{student} - TR = I\alpha$
$\tau_{student} = I\alpha + TR = I\alpha + (I\alpha/R)R = 2I\alpha$

e. $K = ½ I\omega^2 = ½ I(\alpha t)^2$

1989M2

a. $\Sigma F = ma$; $2Mg - T_v = 2Ma$ so $T_v = 2M(g - a)$

b. $\Sigma\tau = T_vR - T_hR = I\alpha = 3MR^2(a/R)$
$T_h = (T_vR - 3MRa)/R = 2M(g - a) - 3Ma = 2Mg - 5Ma$

c. $F_f = \mu F_N = \mu(4Mg)$
$T_h - F_f = 3Ma$
$2Mg - 5Ma - 4\mu Mg = 3Ma$
$4\mu Mg = 2Mg - 8Ma$
$\mu = (2g - 8a)/4g$
plugging in given values gives $\mu = 0.1$

d. $F_f = 4\mu Mg = ma_C = 4Ma_C$
$a_C = 1\ m/s^2$

1991M2

a. $\Sigma\tau = 0$; $m_2gr_2 = m_1gr_1$; $m_2 = m_1r_1/r_2 = 6.67$ kg

b./c. $\tau = I\alpha$; $Tr_1 = (45\ kg\text{-}m^2)\alpha$
$\Sigma F = ma$; $(20\ kg)g - T = (20\ kg)a$
Combining with $a = \alpha r$ gives $\alpha = 2\ rad/s^2$ and $T = 180$ N

d. $mgh = ½ mv^2 + ½ I\omega^2 = ½ mv^2 + ½ I(v^2/r^2)$ giving $v = 1.4$ m/s

1993M3

a. $\Sigma\tau = F_a\ell - Mg\ell/2 = 0$ giving $F_a = Mg/2$

b. $\Sigma\tau = Mg\ell/2 = I\alpha = (M\ell^2/3)\alpha$; $\alpha = 3g/2\ell$

c. $a = \alpha r$ where $r = \ell/2$

$a = (3g/2\ell)(\ell/2) = 3g/4$

d. $\Sigma F = Ma$; $Mg - F_a = Ma = M(3g/4)$

$F_a = Mg/4$

e. $\Delta U = \Delta K_{rot}$

$mgh = mg(\ell/2)\sin\theta = \frac{1}{2} I\omega^2 = \frac{1}{2}(M\ell^2/3)\omega^2$

solving gives $\omega = (3g\sin\theta/\ell)^{1/2}$

1999M3

a. $\Sigma\tau = 0$ so $\tau_{cw} = \tau_{ccw}$ and $\tau_{cw} = TR$ (from the string) so we just need to find τ_{ccw} as the sum of the torques from the various parts of the system

$\Sigma\tau_{ccw} = \tau_{rod} + \tau_{block} = mgR\sin\theta_0 + 2mg(2R)\sin\theta_0 = 5mgR\sin\theta_0 = TR$ so $T = 5mg\sin\theta_0$

b. i. $I = I_{disk} + I_{rod} + I_{block} = 3mR^2/2 + 4mR^2/3 + 2m(2R)^2 = 65mR^2/6$

$\alpha = \tau/I = (5mgR\sin\theta_0)/(65mR^2/6) = 6g\sin\theta_0/13R$

ii. $a = \alpha r$ where $r = 2R$ so $a = 12g\sin\theta_0/13$

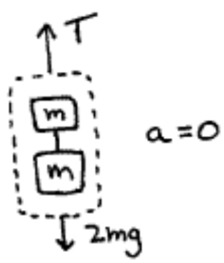
c. ΔU (from each component) $= K = \frac{1}{2} I\omega^2$

$mgR\cos\theta_0 + 2mg(2R)\cos\theta_0 = \frac{1}{2}(65mR^2/6)\omega^2$

$\omega = (12g\cos\theta_0/13R)^{1/2}$ and $v = \omega r = \omega(2R) = 4(3gR\cos\theta_0/13)^{1/2}$

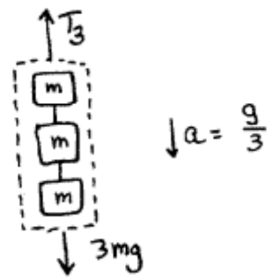
2000M3

a.



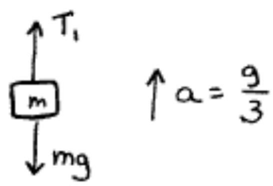
$\Sigma F = ma = 0$ so $T = 2mg$

b. i.



$\Sigma F = ma$
$3mg - T_3 = 3m(g/3)$
$T_3 = 2mg$

ii.



$\Sigma F = ma$
$T_1 - mg = m(g/3)$
$T_1 = 4mg/3$

iii. $\Sigma\tau = (T_3 - T_1)R_1 = I\alpha$ and $\alpha = a/R_1 = g/3R_1$
$(2mg - 4mg/3)R_1 = I_1(g/3R_1)$
$I_1 = 2mR_1^2$

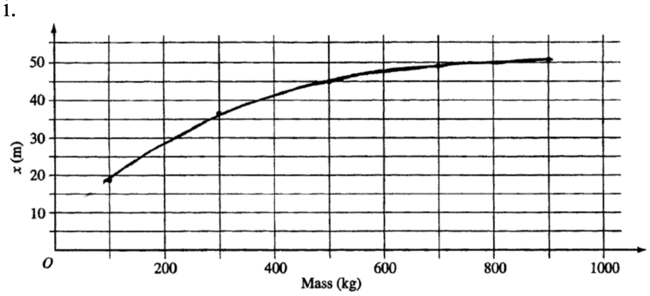
c. i. Tangential speeds are equal; $\omega_1 R_1 = \omega_2 R_2 = \omega_2(2R_1)$ therefore $\omega_2 = \omega_1/2$
ii. $L = I\omega = (16I_1)(\omega_1/2) = 8I_1\omega_1$
iii. $K = ½ I_1\omega_1^2 + ½ I_2\omega_2^2 = (5/2)I_1\omega_1^2$

2001M3

a. $I = \Sigma mr^2 = mL^2 + mL^2 = 2mL^2$
b. $\Sigma F = ma$; $4mg - T = 4ma$
$\Sigma\tau = I\alpha$; $Tr = I\alpha$; $T = I\alpha/r = 4mg - 4ma$ and $\alpha = a/r$, solving gives $a = 2gr^2/(L^2 + 2r^2)$
c. Equal, total energy is conserved
d. Less, the small blocks rise and gain potential energy. The total energy available is still 4mgD, therefore the kinetic energy must be less than in part c.

2003M3

a. i.



ii. x = 33 m

b. i. $y = \frac{1}{2} gt^2$; $t = (2y/g)^{1/2} = 1.75$ s

ii. $U_{initial} = U_{bucket} + U_{projectile} = M(9.8\ m/s^2)(3\ m) + (10\ kg)(9.8\ m/s^2)(3\ m) = 29.4M + 294$

iii. $U_{initial} = U_{final} + K$ where $U_{final} = Mg(1\ m) + (10\ kg)g(15\ m) = 9.8M + 1470$

$K_{projectile} = \frac{1}{2}\, 10v_x^2$ and $K_{bucket} = \frac{1}{2}\, Mv_b^2$ where $v_b = v_x/6$

putting it all together gives $29.4M + 294 = 9.8M + 1470 + 5v_x^2 + (M/72)v_x^2$

$$v_x = \sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

c. i. $x = v_x t$

$$x = 1.75\sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

d. x(300 kg) = 39.7 m (greater than the experimental value)

possible reasons include friction at the pivot, air resistance, neglected masses not negligible

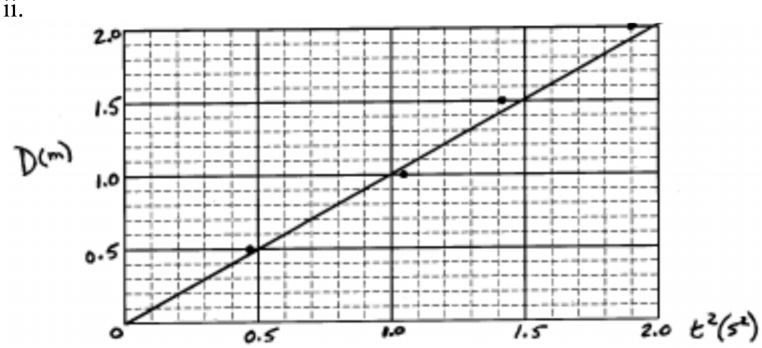
<u>2004M2</u>

a. $x = v_0 t + \frac{1}{2} at^2$

$x = D$ and $v_0 = 0$ so $D = \frac{1}{2} at^2$ and $a = 2D/t^2$

b. i. graph D vs. t^2 (as an example)

ii.



iii. $a = 2(\text{slope}) = 2.04\ m/s^2$

c. $\Sigma\tau = TR = I\alpha$ and $\alpha = a/R$ so $I = TR^2/a$

$\Sigma F = mg - T = ma$ so $T = m(g - a)$

$I = m(g - a)R^2/a = mR^2((g/a) - 1)$

d. The string was wrapped around the pulley several times, causing the effective radius at which the torque acted to be larger than the radius of the pulley used in the calculation.

The string slipped on the pulley, allowing the block to accelerate faster than it would have otherwise, resulting in a smaller experimental moment of inertia.

Friction is not a correct answer, since the presence of friction would make the experimental value of the moment of inertia too large

SECTION C – Rolling

NOTE: Rolling problems may be solved considering rotation about the center of mass or the point of contact. The solutions below will only show one of the two methods, but for most, if not all cases, the other method is applicable. When considering rotation about the point of contact, remember to use the parallel axis theorem for the moment of inertia of the rolling object.

1974M2

a. Torque provided by friction; at minimum μ, $F_f = \mu F_N = \mu Mg \cos\theta$
$\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma = \mu Mg\cos\theta$ giving $a = (5/2)\mu g\cos\theta$
$\Sigma F = Ma$; $Mg\sin\theta - \mu Mg\cos\theta = Ma = (5/2)\mu Mg\cos\theta$ giving $\mu = (2/7)\tan\theta$

b. Energy at the bottom is the same in both cases, however with $\mu = 0$, there is no torque and no energy in rotation, which leaves more (all) energy in translation and velocity is higher

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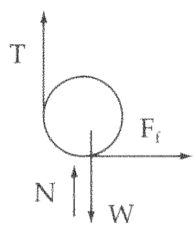
1977M2

a.



b. $\Sigma F_y = 0$; $T + N = W$; $N = W - T = Mg - (3/5)Mg = (2/5)Mg$
$\Sigma F_x = ma$; $F_f = ma$; $\mu N = ma$; $½\,(2/5)Mg = Ma$; $a = g/5$

c. $\Sigma\tau = I\alpha$; $(T - F_f)R = ½\,MR^2\alpha$
$(3/5)Mg - (1/5)Mg = ½\,MR\alpha$
$(2/5)g = ½\,R\alpha$
$\alpha = 4g/5R$

d. The cylinder is slipping on the surface and does not meet the condition for pure rolling

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1980M3

a. $\Sigma F = ma$; $F_f = \mu F_N$;
$-\mu Mg = Ma$
$a = -\mu g$
$v = v_0 + at$
$v = v_0 - \mu gt$

b. $\tau = I\alpha$ where the torque is provided by friction $F_f = \mu Mg$
$\mu MgR = (2MR^2/5)\alpha$
$\alpha = (5\mu g/2R)$
$\omega = \omega_0 + \alpha t = (5\mu g/2R)t$

c. Slipping stops when the tangential velocity si equal to the velocity of the center of mass, or the condition for pure rolling has been met: $v(t) = \omega(t)R$
$v_0 - \mu gt = R(5g/2R)t$, which gives $T = (2/7)(v_0/\mu g)$

d. Since the line of action of the frictional force passes through P, the net torque about point P is zero. Thus, the time rate of change of the angular momentum is zero and the angular momentum is constant.

1986M2

a. U = K

$Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$

i. $K_{trans} = ½ Mv^2 = (5/7)Mgh$

ii. $K_{rot} = ½ I\omega^2 = (2/7)Mgh$ (or $Mgh – K_{trans}$)

b. i. $\tau = F_fR = I\alpha = I(a/R)$

$F_fR = (2/5)MR^2(a/R)$

$F_f = (2/5)Ma$

$\Sigma F = ma$

$Mg \sin\theta – F_f = Ma$

$Mg \sin\theta – (2/5)Ma = Ma$

$g \sin\theta = (7/5) a$

$a = (5/7) g \sin\theta$

ii. $F_f = (2/5)Ma = (2/5)M(5/7)g \sin\theta = (2/7)Mg \sin\theta$

c. $K_{tot} = Mgh$

d. Greater, the moment of inertia of the hollow sphere is greater and will be moving slower at the bottom of the incline. Since the translational speed is less, the translational KE is taking a smaller share of the same total energy as the solid sphere.

1990M2

a. K = U

$½ mv_0^2 = mgH$; $H = v_0^2/2g$

b. $K + W_f = U$ where $W_f = –F_fd$ and $F_f = \mu mg \cos\theta$ and $d = h/\sin\theta$

$½ mv_0^2 – (\mu mg \cos\theta)(h/\sin\theta) = mgh$

$½ mv_0^2 = mgh(\mu \cot\theta + 1)$

$h = v_0^2/(2g(\mu \cot\theta + 1)) = H/(\mu \cot\theta + 1)$

c. $K_{trans} + K_{rot} = U$ where $K_{rot} = ½ I \omega^2 = ½ (mR^2)(v/R)^2 = ½ mv_0^2$

$½ mv_0^2 + ½ mv_0^2 = mgh'$

$h' = v_0^2/g = 2H$

d. Rotational energy will not change therefore $½ mv_0^2 = mgh''$ and $h'' = v_0^2/2g = H$

1994M2

a. $K_{tot} = K_{trans} + K_{rot} = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$K_{tot} = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10 = 1750$ J

b. i. $K_{total,bottom} = K_{top} + U_{top} = 7Mv_{top}^2/10 + Mgh$; $v_{top} = 7.56$ m/s

ii. It is directed parallel to the incline: 25º

c. $y = y_0 + v_{oy}t + ½ a_yt^2$

0 m = 3 m + (7.56 m/s)(sin 25º)t + ½ (–9.8 m/s^2)t^2 which gives t = 1.16 s (positive root)

$x = v_xt$ = (7.56 m/s)(cos 25º)(1.16 s) = 7.93 m

d. The speed would be less than in b.

The gain in potential energy is entirely at the expense of the translational kinetic energy as there is no torque to slow the rotation.

1997M3

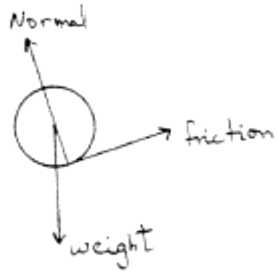
a. U = K

$MgH = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$MgH = ½ Mv^2 + ½ (½)MR^2(v/R)^2 = ½ Mv^2 + ¼ Mv^2 = 3Mv^2/4$

$v = (4gH/3)^{1/2}$

b.



c. For a change of pace, we can use kinematics:
$v_f^2 = v_i^2 + 2ad$
$4gH/3 = 0 + 2a(H/\sin\theta)$
$a = (2/3)g \sin\theta$

d. $\Sigma F = Ma$
$Mg \sin\theta - F_f = Ma = M(2/3)g \sin\theta$
$Mg \sin\theta - \mu Mg \cos\theta = (2/3)Mg \sin\theta$
$\mu \cos\theta = (1/3) \sin\theta$
$\mu = (1/3) \tan\theta$

e. i. The translational speed is greater, less energy is transferred to the rotational motion so more goes into the translational motion. Additionally, with a smaller frictional force, the translational acceleration is greater.
ii. Total kinetic energy is less. Energy is dissipated as heat due to friction.

2002M2

a. For each tire: $I = \frac{1}{2} ML^2 = \frac{1}{2} (m/4)r^2$
$I_{total} = 4 \times I = \frac{1}{2} mr^2$

b. $U = K$; total mass = 2m
$2mgh = \frac{1}{2} (2m)v^2 + \frac{1}{2} I\omega^2$ and $\omega = v/R$
$2mgh = mv^2 + \frac{1}{2} (\frac{1}{2})mr^2(v/r)^2 = \frac{1}{2} mv^2 + (\frac{1}{4})mv^2 = 5mv^2/4$
$v = (8gh/5)^{1/2}$

c. $U_g = U_s$
$2mgh = \frac{1}{2} kx_m^2$; $x_m = 2(mgh/k)^{1/2}$

d. In an inelastic collision, energy is lost. With less energy after the collision, the spring is compressed less.

2006M3

a. $\Sigma\tau = I\alpha$
$F_fR = I\alpha = MR^2(a/R)$; $F_f = Ma$
$\Sigma F = ma$
$Mg \sin\theta - F_f = Ma$
$Mg \sin\theta - Ma = Ma$
$a = \frac{1}{2} g \sin\theta$

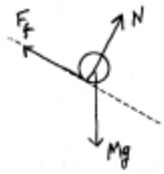
b. $v_f^2 = 2aL = gL \sin\theta$
$v_f = (gL \sin\theta)^{1/2}$

c. $H = \frac{1}{2} gt^2$; $t = (2H/g)^{1/2}$
$d = v_xt = (gL \sin\theta)^{1/2}(2H/g)^{1/2} = (2LH \sin\theta)^{1/2}$

d. Greater. A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

2010M2

a.



b. Torque provided by friction; $F_f = \mu F_N = \mu Mg \cos\theta$

$\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma$; $Ma = (5/2)F_f$

$\Sigma F = Ma$

$Mg \sin\theta - F_f = (5/2)F_f$

$F_f = (2/7)Mg \sin\theta = 8.4$ N

c. $Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$; $v = 5.3$ m/s

d. The horizontal speed of the wagon is due to the horizontal component of the ball in the collision:

$M_i v_{ix} = M_f v_{fx}$; where $M_f = M_{ball} + M_{wagon} = 18$ kg

$(6\text{ kg})(5.3\text{ m/s})(\cos 30°) = (18\text{ kg})v_f$

$v_f = 1.5$ m/s

SECTION D – Angular Momentum

1975M2

a. $L_i = L_f$

$m_0 v_0 R \sin\theta = I\omega$

$\omega = m_0 v_0 R \sin\theta/I$; $I = (M + m_0)R^2$

$\omega = m_0 v_0 \sin\theta/(M + m_0)R$

b. $K_i = ½ m_0 v_0^2$

$K_f = ½ I\omega^2 = ½ (M + m_0)R^2(m_0 v_0 \sin\theta/(M + m_0)R)^2$

$K_f/K_i = m_0 \sin^2\theta/(M + m_0)$

1978M2

a. $L_i = L_f$

$I\omega = mvr$

$(1/3)M_1\ell^2\omega = M_2 v\ell$

$v = M_1\ell\omega/3M_2$

b. $p_{system} = p_{cm\ of\ rod} = M_1 v_{cm} = M_1\omega(\ell/2)$

c. $P_f = M_2 v_f = M_1\omega\ell/3M_2$

d. There is a net external force on the system from the axis at point P.

e. Since the net external force acts at point P (the pivot), the net torque about point P is zero, hence angular momentum is conserved.

1981M3

a. $m_2 v = m_2(-v/2) + M_1 v'$

$v' = 3m_2 v/2M_1$

b. $L_i = L_f$

$m_2 v(L/3) = m_2(-v/2)(L/3) + (1/12)M_1 L^2\omega$

$\omega = 6m_2 v/M_1 L$

c. $\Delta K = K_f - K_i = ½\, m_2(-v/2)^2 + ½\, M_1 v'^2 + ½\, I\omega^2 - ½\, m_2 v^2$
$= -3m_2v^2/8 + 21m_2^2v^2/8M_1$

<u>1982M3</u>

a. $L = I\omega$ where $I = \Sigma mr^2 = (2m)\ell^2 + m(2\ell)^2 = 6m\ell^2$
$L = 6m\ell^2\omega$

b. $F_f = \mu mg$
$\Sigma\tau = -(\mu(2m)g\ell + \mu mg(2\ell)) = -4\mu mg\ell$

c. $\alpha = \tau/I = -4\mu mg\ell/6m\ell^2 = -2\mu g/3\ell$
$\omega = \omega_0 + \alpha t$; setting $\omega = 0$ and solving for T gives $T = 3\omega_0\ell/2\mu g$

<u>1987M3</u>

a. $K = U$
$½\, I\omega^2 = mgh_{cm}$
$½\,(m\ell^2/3)\omega^2 = mg(\ell/2)$ which gives $\omega = 5$ rad/s

b. $K_i = K_f$
$½\, m_0v_0^2 = ½\, m_0v^2 + ½\, I\omega^2$
$v = 8$ m/s

c. $L = mvr = (1\text{ kg})(10\text{ m/s})(1.2\text{ m}) = 12\text{ kg-m}^2/\text{s}$

d. $L_i = L_f$
$12\text{ kg-m}^2/\text{s} = m_0(v_\perp)\ell + I\omega = m_0(v\cos\theta)\ell + I\,\omega$
$\theta = 60°$

<u>1992M2</u>

a. $\Sigma\tau = (3M + M)g\ell - Mg\ell = 3Mg\ell$

b. $I = \Sigma mr^2 = 4M\ell^2 + M\ell^2 = 5m\ell^2$
$\alpha = \tau/I = 3Mg\ell/5m\ell^2 = 3g/5\ell$

c. $\Delta U_{bug} + \Delta U_{left\ sphere} + \Delta U_{right\ sphere} = \Delta K_{rot}$
since $\Delta U_{left\ sphere} = -\Delta U_{right\ sphere}$, we only need to consider ΔU_{bug}
$3Mg\ell = ½\, I\omega^2 = ½\,(5M\ell^2)\omega^2$
$\omega = (6g/5\ell)^{1/2}$

d. $L = I\omega = 5M\ell^2(6g/5\ell)^{1/2} = (30M^2g\ell^3)^{1/2}$

e. Let T be the force we are looking for
$\Sigma F = ma_c$
$T - 3Mg = M\omega^2\ell$
$T = 3Mg + 3M(6g/5\ell)\ell = 33Mg/5$

<u>1996M3</u>

a.

$$I = \quad r^2 dm$$

$$dm = \frac{M}{l}dr$$

$$I = {}_{-\frac{l}{2}}^{\frac{l}{2}}\frac{M}{l}r^2 dr$$

$$I = \frac{M}{l}\frac{r^3}{3}{}_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

b. $I = \Sigma I = M\ell^2/12 + M(\ell/2)^2 = M\ell^2/3$

c./d./e.

$\Sigma F = ma$

for cat: $Mg - T = Ma$

$\Sigma\tau = I\alpha$ where $\alpha = a/r = a/(\ell/2)$

for hoop: $T\ell/2 = (M\ell^2/3)(a/(\ell/2))$ which gives $a = 3T/4M$

substituting gives $Mg - T = 3T/4$

$T = 4Mg/7$

$\alpha = T\ell/2I = 6g/7\ell$

$a = \alpha(\ell/2) = 3g/7$

f. $L = Mv(\ell/2)$ where v is found from $v^2 = v_0^2 + 2aH = 2(3g/7)(5\ell/3) = 10g\ell/7$

$L = ½ M\ell(10g\ell/7)^{1/2}$

1998M2

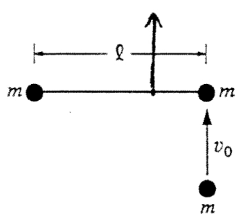
a. i. $mv_0 = (3m)v_f$; $v_f = v_0/3$

$K_f = ½ (3m)(v_0/3)^2 = mv_0^2/6$

ii. $\Delta K = K_f - K_i = mv_0^2/6 - ½ mv_0^2 = -mv_0^2/3$

b. i. $r_{cm} = \Sigma m_i r_i/\Sigma m = m(0) + 2m(\ell)/(m + 2m) = (2/3)\ell$

ii.



iii. $p_i = p_f$

$mv_0 = (3m)v_f$; $v_f = v_0/3$

iv. $L_i = L_f$

$mv_0 R \sin\theta = mv_0(\ell/3) = I\omega$ where $I = \Sigma mr^2 = m(2\ell/3)^2 + 2m(\ell/3)^2 = (2/3)m\ell^2$

solving yields $\omega = v_0/2\ell$

v. $K_i = ½ mv_0^2$

$K_f = ½ mv_f^2 + ½ I\omega^2 = ½ (3m)(v_0/3)^2 + ½ (2/3)m\ell^2(v_0/2\ell)^2 = ¼ mv_0^2$

$\Delta K = -¼ mv_0^2$

2005M3

a. $L = I\omega = (1/3)M_1d^2\omega$

b. $L_f = L_i$

$M_2vd = (1/3)M_1d^2\omega$

$v = M_1d\omega/3M_2$

c. $K_f = K_i$

$½ M_2v^2 = ½ I\omega^2$

$M_2v^2 = I\omega^2$

$M_2(M_1d\omega/3M_2) = (1/3)M_1d^2\omega^2$ $\quad$ $M_2(1/9)(M_1/M_2)^2d^2\omega^2 = (1/3)M_1d^2\omega^2$ $\quad$ $(1/9)(M_1^2/M_2) = M_1/3$

$M_1/M_2 = 3$

d. $L_f = L_i$

$M_1vx = (1/3)M_1d^2\omega$

$v = d^2\omega/3x$

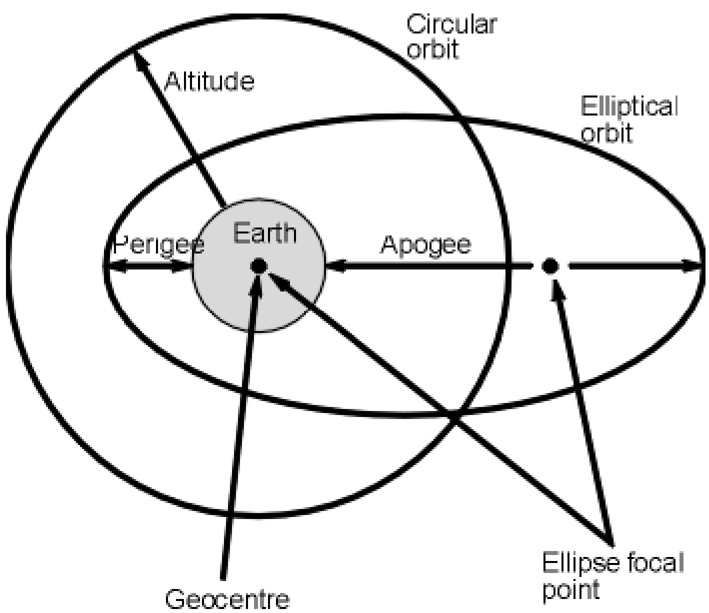
$½ M_1v^2 = ½ I\omega^2$ $\quad$ $M_1v^2 = I\omega^2 = (1/3)M_1d^2\omega^2$

$v^2 = d^2\omega^2/3$ $\quad$ substituting from above $\quad$ $(d^2\omega/3x)^2 = d^2\omega^2/3$

solving for x gives $x = d/\sqrt{3}$

Chapter 7

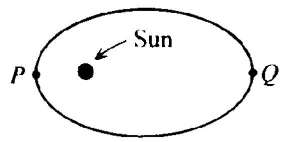
Gravitation



AP Physics Multiple Choice Practice – Gravitation

1. Each of five satellites makes a circular orbit about an object that is much more massive than any of the satellites. The mass and orbital radius of each satellite are given below. Which satellite has the greatest speed?

	Mass	Radius
(A)	½m	R
(B)	m	½R
(C)	m	R
(D)	m	2R

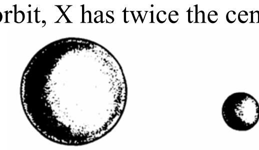


2. An asteroid moves in an elliptic orbit with the Sun at one focus as shown above. Which of the following quantities increases as the asteroid moves from point P in its orbit to point Q?
(A) Speed (B) Angular momentum (C) Kinetic energy (D) Potential energy

3. A person weighing 800 newtons on Earth travels to another planet with twice the mass and twice the radius of Earth. The person's weight on this other planet is most nearly
(A) 400 N (B) $\frac{800}{\sqrt{2}}$ N (C) $800\sqrt{2}$ (D) 1,600 N

4. Mars has a mass 1/10 that of Earth and a diameter 1/2 that of Earth. The acceleration of a falling body near the surface of Mars is most nearly
(A) 0.25 m/s^2 (B) 0.5 m/s^2 (C) 2 m/s^2 (D) 4 m/s^2

5. **Multiple correct:** If Spacecraft X has twice the mass of Spacecraft Y, then true statements about X and Y include which of the following? Select two answers.
(A) On Earth, X experiences twice the gravitational force that Y experiences.
(B) On the Moon, X has twice the weight of Y.
(C) The weight of the X on Earth will always be equal to the weight of Y on the Moon.
(D) When both are in the same circular orbit, X has twice the centripetal acceleration of Y



6. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?
(A) Acceleration (B) Kinetic energy (C) Displacement from the center of mass (D) Gravitational force

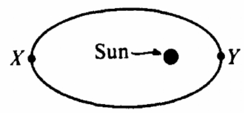
7. An object has a weight *W* when it is on the surface of a planet of radius *R*. What will be the gravitational force on the object after it has been moved to a distance of *4R* from the center of the planet?
(A) 16W (B) 4W (C)1/4W (D) 1/16 W

8. A new planet is discovered that has twice the Earth's mass and twice the Earth's radius. On the surface of this new planet, a person who weighs 500 N on Earth would experience a gravitational force of
(A) 125 N (B) 250 N (C) 500 N (D) 1000 N

9. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are the same. (B) Both are longer.
(C) The period of the mass on the spring is shorter, that of the pendulum is the same.
(D) The period of the pendulum is shorter; that of the mass on the spring is the same.

10. A satellite of mass m and speed v moves in a stable, circular orbit around a planet of mass M. What is the radius of the satellite's orbit?

(A) $\frac{Gv}{mM}$ (B) $\frac{GM}{v^2}$ (C) $\frac{GmM}{v}$ (D) $\frac{GmM}{v^2}$

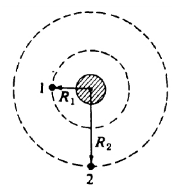


11. A satellite travels around the Sun in an elliptical orbit as shown above. As the satellite travels from point X to point Y. which of the following is true about its speed and angular momentum?

	Speed	Angular Momentum
(A)	Increases	Increases
(B)	Decreases	Decreases
(C)	Increases	Remains constant
(D)	Decreases	Remains constant

12. A newly discovered planet has a mass that is 4 times the mass of the Earth. The radius of the Earth is R_e. The gravitational field strength at the surface of the new planet is equal to that at the surface of the Earth if the radius of the new planet is equal to

(A) ½R_e (B) 2R_e (C) $\overline{R_e}$ (D) R_e^2



13. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations a_2 and a_1 of the two satellites are related by which of the following?

(A) $a_2 = 4a_1$ (B) $a_2 = 2a_1$ (C) $a_2 = a_1/2$ (D) $a_2 = a_1/4$

14. A satellite moves in a stable circular orbit with speed v_o at a distance R from the center of a planet. For this satellite to move in a stable circular orbit a distance $2R$ from the center of the planet, the speed of the satellite must be

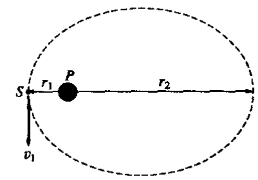
(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) $\sqrt{2v_0}$ (D) $2v_o$

15. If F_1 is the magnitude of the force exerted by the Earth on a satellite in orbit about the Earth and F_2 is the magnitude of the force exerted by the satellite on the Earth, then which of the following is true?

(A) F_1 is much greater than F_2. (B) F_1 is slightly greater than F_2.
(C) F_1 is equal to F_2. (D) F_2 is slightly greater than F_1

16. A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface. The radius of the new planet in terms of the radius R of Earth is

(A) ½R (B) $\frac{\sqrt{2}}{2}$ R (C) $\sqrt{2}$ R (D) 2R



17. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?

(A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $\frac{r_1+r_2}{2}v_1$ (D) $\frac{r_2-r_1}{r_1+r_2}v_1$

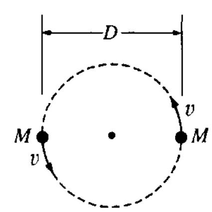
Questions 18-19

A ball is tossed straight up from the surface of a small, spherical asteroid with no atmosphere. The ball rises to a height equal to the asteroid's radius and then falls straight down toward the surface of the asteroid.

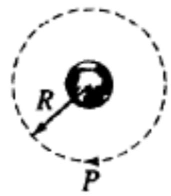
18. What forces, if any, act on the ball while it is on the way up?
(A) Only a decreasing gravitational force that acts downward
(B) Only a constant gravitational force that acts downward
(C) Both a constant gravitational force that acts downward and a decreasing force that acts upward
(C) No forces act on the ball.

19. The acceleration of the ball at the top of its path is
(A) equal to the acceleration at the surface of the asteroid
(B) equal to one-half the acceleration at the surface of the asteroid
(C) equal to one-fourth the acceleration at the surface of the asteroid
(D) zero

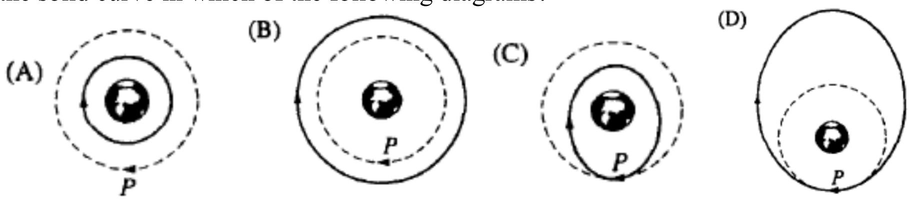
20. **Multiple Correct:** A satellite of mass M moves in a circular orbit of radius R with constant speed v. True statements about this satellite include which of the following? Select two answers.
(A) Its angular speed is v/R.
(B) The gravitational force does work on the satellite.
(C) The magnitude and direction of its centripetal acceleration is constant.
(D) Its tangential acceleration is zero.



21. Two identical stars, a fixed distance D apart, revolve in a circle about their mutual center of mass, as shown above. Each star has mass M and speed *v*. G is the universal gravitational constant. Which of the following is a correct relationship among these quantities?
(A) v^2 = GM/D (B) v^2 = GM/2D (C) $v^2 = GM/D^2$ (D) $v^2 = 2GM^2/D$



22. A spacecraft orbits Earth in a circular orbit of radius R, as shown above. When the spacecraft is at position P shown, a short burst of the ship's engines results in a small increase in its speed. The new orbit is best shown by the solid curve in which of the following diagrams?



23. The escape speed for a rocket at Earth's surface is v_e . What would be the rocket's escape speed from the surface of a planet with twice Earth's mass and the same radius as Earth?

    (A) $2v_e$ (B) $\sqrt{2}v_e$ (C) v_e (D) $\frac{v_e}{\sqrt{2}}$

24. A hypothetical planet orbits a star with mass one-half the mass of our sun. The planet's orbital radius is the same as the Earth's. Approximately how many Earth years does it take for the planet to complete one orbit?
    (A) ½ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) 2

25. Two artificial satellites, 1 and 2, are put into circular orbit at the same altitude above Earth's surface. The mass of satellite 2 is twice the mass of satellite 1. If the period of satellite 1 is T, what is the period of satellite 2?
    (A) $T/2$ (B) T (C) $2T$ (D) $4T$

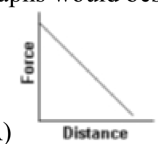
26. A planet has a radius one-half that of Earth and a mass one-fifth the Earth's mass. The gravitational acceleration at the surface of the planet is most nearly
    (A) 4.0 m/s^2 (B) 8.0 m/s^2 (C) 12.5 m/s^2 (D) 25 m/s^2

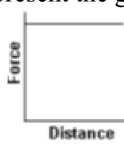
27. In the following statements, the word "weight" refers to the force a scale registers. If the Earth were to stop rotating, but not change shape,
    (A) the weight of an object at the equator would increase.
    (B) the weight of an object at the equator would decrease.
    (C) the weight of an object at the north pole would increase.
    (D) the weight of an object at the north pole would decrease.

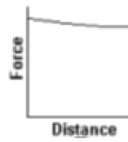
28. What happens to the force of gravitational attraction between two small objects if the mass of each object is doubled and the distance between their centers is doubled?
    (A) It is doubled (B) It is quadrupled (C) It is halved (D) It remains the same

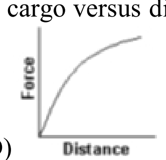
29. One object at the surface of the Moon experiences the same gravitational force as a second object at the surface of the Earth. Which of the following would be a reasonable conclusion?
    (A) both objects would fall at the same acceleration
    (B) the object on the Moon has the greater mass
    (C) the object on the Earth has the greater mass
    (D) both objects have identical masses

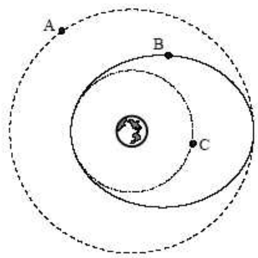
30. Consider an object that has a mass, m, and a weight, W, at the surface of the moon. If we assume the moon has a nearly uniform density, which of the following would be closest to the object's mass and weight at a distance halfway between Moon's center and its surface?
(A) ½ m & ½ W (B) ¼ m & ¼ W (C) 1 m & ½ W (D) 1 m & ¼ W

31. As a rocket blasts away from the earth with a cargo for the international space station, which of the following graphs would best represent the gravitational force on the cargo versus distance from the surface of the Earth?
(A) 

(B) 

(C) 

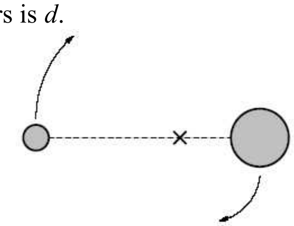
(D) 



32. Three equal mass satellites A, B, and C are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B, and L_C. Which of the following statements is correct?
(A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

<u>Questions 33-34</u>

Two stars orbit their common center of mass as shown in the diagram below. The masses of the two stars are $3M$ and M. The distance between the stars is d.



33. What is the value of the gravitational potential energy of the two star system?
(A) $-\frac{GM^2}{d}$ (B) $\frac{3GM^2}{d}$ (C) $-\frac{GM^2}{d^2}$ (D) $-\frac{3GM^2}{d}$

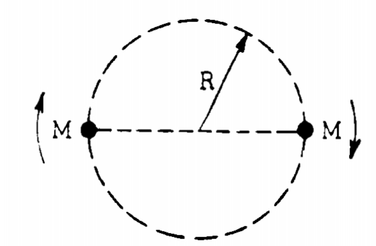
34. Determine the period of orbit for the star of mass $3M$.
(A) $\pi\sqrt{\frac{d^3}{GM}}$ (B) $\pi\sqrt{\frac{d^3}{3GM}}$ (C) $2\pi\sqrt{\frac{d^3}{GM}}$ (D) $\frac{\pi}{4}\sqrt{\frac{d^3}{GM}}$

35. Two iron spheres separated by some distance have a minute gravitational attraction, F. If the spheres are moved to one half their original separation and allowed to rust so that the mass of each sphere increases 41%, what would be the resulting gravitational force?
(A) $2F$ (B) $4F$ (C) $6F$ (D) $8F$

36. A ball thrown upward near the surface of the Earth with a velocity of 50 m/s will come to rest about 5 seconds later. If the ball were thrown up with the same velocity on Planet X, after 5 seconds it would be still moving upwards at nearly 31 m/s. The magnitude of the gravitational field near the surface of Planet X is what fraction of the gravitational field near the surface of the Earth?
(A) 0.16 (B) 0.39 (C) 0.53 (D) 0.63

37. Two artificial satellites I and II have circular orbits of radii R and $2R$, respectively, about the same planet. The orbital velocity of satellite I is v. What is the orbital velocity of satellite II?
(A) $\frac{v}{2}$ (B) $\frac{v}{\sqrt{2}}$ (C) $\sqrt{2}v$ (D) $2v$

AP Physics Free Response Practice – Gravitation



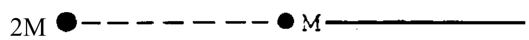
*1977M3. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

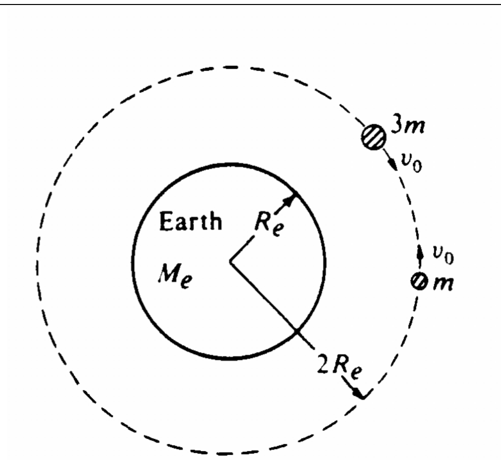
b. Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.



d. Find the ratio of the speeds, $v_{2M}/v_{M.}$

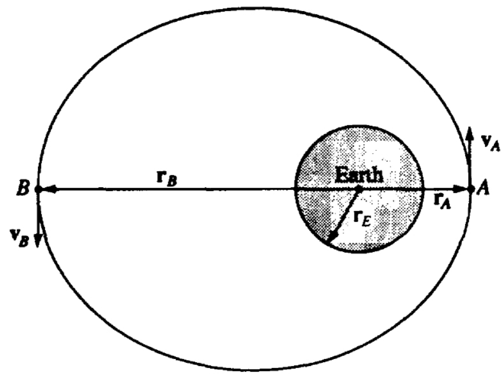


1984M2. Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



*1992M3. A spacecraft of mass 1,000 kilograms is in an elliptical orbit about the Earth, as shown above. At point A the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .

a. The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

b. The magnitude of the angular momentum of the spacecraft about the center of the Earth.

Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.

c. Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth. Determine each of the following.

d. The speed of the spacecraft if it is in a <u>circular</u> orbit around the Earth

e. The minimum speed of the spacecraft at point A if it is to escape completely from the Earth

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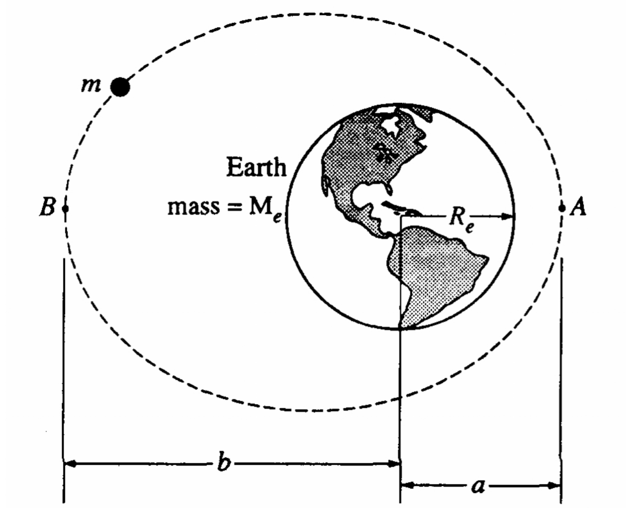
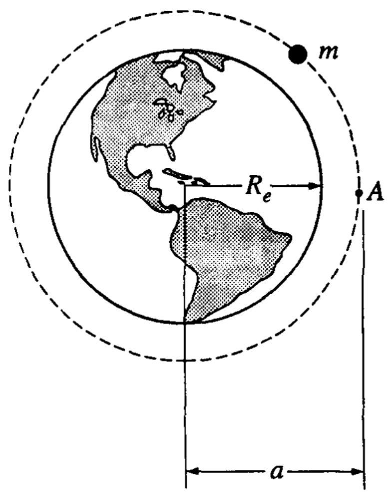


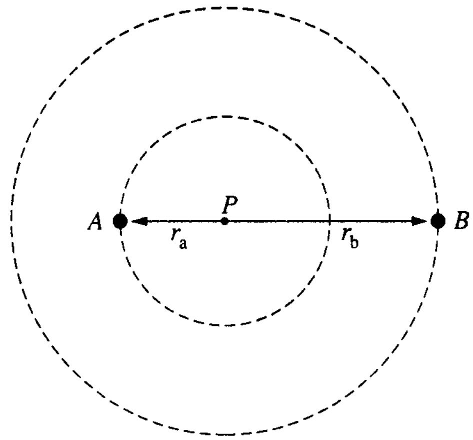
*1994M3 (modified) A satellite of mass m is in an elliptical orbit around the Earth, which has mass M_e and radius R_e. The orbit varies from a closest approach of distance a at point A to maximum distance of b from the center of the Earth at point B. At point A, the speed of the satellite is v_o. Assume that the gravitational potential energy $U_g = 0$ when masses are an infinite distance apart. Express your answers in terms of a, b, m, M_e, R_e, v_o, and G.

a. Determine the total energy of the satellite when it is at A.
b. What is the magnitude of the angular momentum of the satellite about the center of the Earth when it is at A?
c. Determine the velocity of the satellite as it passes point B in its orbit.



As the satellite passes point A, a rocket engine on the satellite is fired so that its orbit is changed to a circular orbit of radius a about the center of the Earth.

d. Determine the speed of the satellite for this circular orbit.
e. Determine the work done by the rocket engine to effect this change.



*1995M3 (modified) Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.

Determine expressions for the following three quantities in terms of r_a, r_b, T, and fundamental constants.

a. The centripetal acceleration of star A
b. The mass M_b of star B
c. The mass M_a of star A
d. Determine an expressions for the angular momentum of the system about the center of mass in terms of M_a, M_b, r_a, r_b, T, and fundamental constants.

2007M2. In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a. Calculate the radius of the GS orbit.
b. Calculate the mass of Mars.
c. Calculate the total mechanical energy of the GS in this orbit.
d. If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_________Greater than _________ Less than

Justify your answer.

e. In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

2001M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by

$$v = \sqrt{\frac{GM_J}{R}}$$

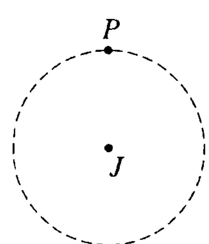
ii. The period of the orbit is given by

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

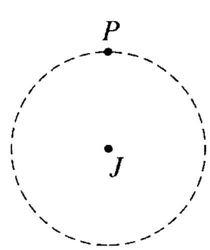
b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.



2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

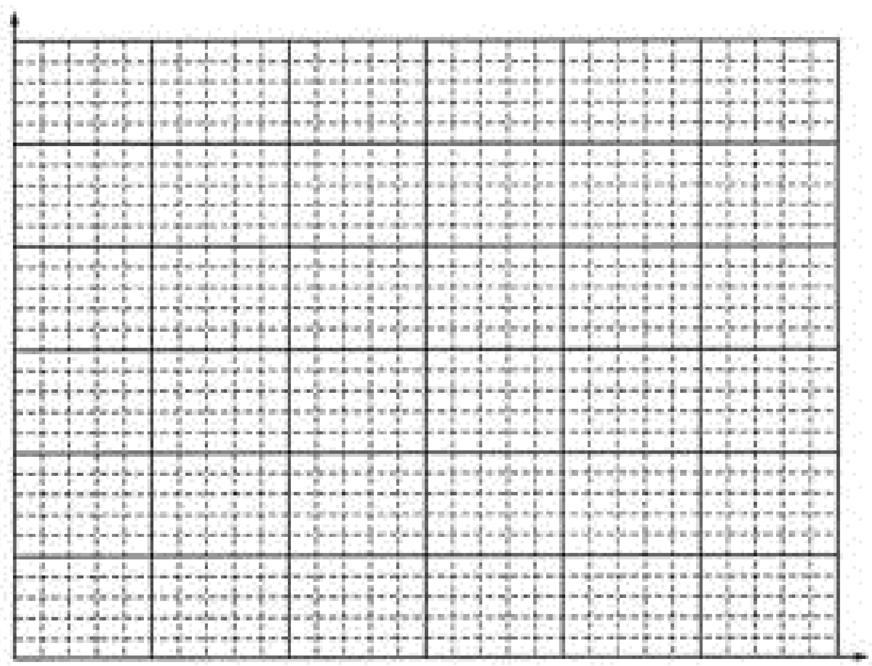
a. Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

ANSWERS - AP Physics Multiple Choice Practice – Gravitation

	Solution	Answer
1.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed. The smallest radius of orbit will be the fastest satellite.	B
2.	As a satellite moves farther away, it slows down, also decreasing its angular momentum and kinetic energy. The total energy remains the same in the absence of resistive or thrust forces. The potential energy becomes less negative, which is an increase.	D
3.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	A
4.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10	D
5.	The gravitational force on an object *is* the weight, and is proportional to the mass. In the same circular orbit, it is only the mass of the body being orbited and the radius of the orbit that contributes to the orbital speed and acceleration.	A,B
6.	Newton's third law	D
7.	Force is inversely proportional to distance between the centers squared. R × 4 = F ÷ 16	D
8.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	B
9.	A planet of the same size and twice the mass of Earth will have twice the acceleration due to gravity. The period of a mass on a spring has no dependence on g, while the period of a pendulum is inversely proportional to g.	D
10.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$	B
11.	Kepler's second law (Law of areas) is based on conservation of angular momentum, which remains constant. In order for angular momentum to remain constant, as the satellite approaches the sun, its speed increases.	C
12.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 4 = g × 4 and if the net effect is $g = g_{Earth}$ then r must be twice that of Earth.	B
13.	$a = g = \frac{GM}{r^2}$, if $R_2 = 2R_1$ then $a_2 = \frac{1}{4} a_1$	D

14. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. If r is doubled, v decreases by $\sqrt{2}$ B

15. Newton's third law C

16. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and if the net effect is $g = g_{Earth}$ then r must be $\sqrt{2}$ times that of Earth C

17. From conservation of angular momentum $v_1r_1 = v_2r_2$ A

18. As the ball moves away, the force of gravity decreases due to the increasing distance. A

19. $g = \frac{GM}{r^2}$ At the top of its path, it has doubled its original distance from the center of the asteroid. C

20. Angular speed (in radians per second) is v/R. Since the satellite is not changing speed, there is no tangential acceleration and v^2/r is constant. A,D

21. The radius of each orbit is ½ D, while the distance between them is D. This gives B

$$\frac{GMM}{D^2} = \frac{Mv^2}{D/2}$$

22. An burst of the ships engine produces an increase in the satellite's energy. Now the satellite is moving at too large a speed for a circular orbit. The point at which the burst occurs must remain part of the ship's orbit, eliminating choices A and B. The Earth is no longer at the focus of the ellipse in choice E. D

23. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is B

$\frac{1}{2}mv^2 = \left|-\frac{GMm}{r}\right|$ which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$

24. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 7 = g × 7 and r × 2 = g ÷ 4, so the net effect is g × 7/4 C

25. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed or period. B

26. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 5 = g ÷ 5 and r ÷ 2 = g × 4, so the net effect is g × 4/5 B

27. Part of the gravitational force acting on an object at the equator is providing the necessary centripetal force to move the object in a circle. If the rotation of the earth were to stop, this part of the gravitational force is no longer required and the "full" value of this force will hold the object to the Earth. A

28. $F = \frac{GMm}{r^2}$. F is proportional to each mass and inversely proportional to the distance between their centers squared. If each mass is doubled, F is quadrupled. If r is doubled F is quartered. D

29. Since the acceleration due to gravity is less on the surface of the moon, to have the same gravitational force as a second object on the Earth requires the object on the Moon to have a larger mass. B

30. The mass of an object will not change based on its location. As one digs into a sphere of uniform density, the acceleration due to gravity (and the weight of the object) varies directly with distance from the center of the sphere. C

31. $F = \frac{GMm}{r^2}$ so F is proportional to $1/r^2$. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration due to gravity is only slightly smaller in orbit compared to the surface of the Earth. C

32. The angular momentum of each satellite is conserved independently so we can compare the orbits at any location. Looking at the common point between orbit A and B shows that satellite A is moving faster at that point than satellite B, showing $L_A > L_B$. A similar analysis at the common point between B and C shows $L_B > L_C$ A

33. $U = -\frac{GMm}{r}$ D

34. Since they are orbiting their center of mass, the larger mass has a radius of orbit of $\frac{1}{4}d$. The speed can be found from $\frac{G(3M)M}{d^2} = \frac{(3M)v^2}{d/4}$ which gives $v = \sqrt{\frac{GM}{4d}} = \frac{2\pi(d/4)}{T}$ A

35. $F = \frac{GMm}{r^2}$; If r ÷ 2, F × 4. If each mass is multiplied by 1.41, F is doubled (1.41×1.41) D

36. $g = \Delta v/t = (31\text{ m/s} - 50\text{ m/s})/(5\text{ s}) = -3.8\text{ m/s}^2$ B

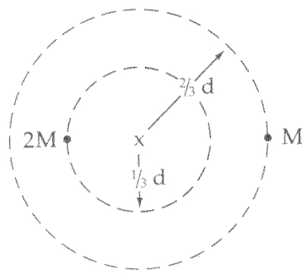
37. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. B

1977M3

a. $F_g = F_c$ gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b. E = PE + KE = $-\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$

c.



d. $F_{g2} = F_{g1} = F_c$

$\frac{(2M)v_2^2}{{}^1/_3 d} = \frac{Mv_1^2}{{}^2/_3 d}$ gives $v_2/v_1 = ½$

1984M2

a. $F_g = F_c$ gives $\frac{GM_e m}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives $(3m)v_0 – mv_0 = (4m)v'$ giving $v' = ½ v_0$

c. E = PE + KE = $-\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_e m}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_e m}{4R_e}$

1992M3

a. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2 = –8.1 \times 10^9$ J

b. L = mvr = 8.5×10^{13} kg-m²/s

c. Angular momentum is conserved so $mv_a r_a = mv_b r_b$ giving $v_b = 2.4 \times 10^3$ m/s

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM/R}$ = 5.8×10^3 m/s

e. Escape occurs when E = PE + KE = 0 giving $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0\ and\ v_{esc} = \sqrt{2GM/R}$ = 8.2×10^3 m/s

1994M3

a. E = PE + KE = $-\frac{GM_e m}{a} + \frac{1}{2}mv_0^2$

b. L = mvr = $mv_0 a$

c. Conservation of angular momentum gives $mv_0 a = mv_b b$, or $v_b = v_0 a/b$

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_e/a}$

e. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2}m\left(\frac{GM_e}{a} - v_0^2\right)$

1995M3

a. $v = \frac{2\pi r}{T}$ and $a = \frac{v^2}{r} = \frac{4\pi^2 r_a}{T^2}$

b. The centripetal force on star A is due to the gravitational force exerted by star B.
$M_a a_a = \frac{GM_a M_b}{(r_a + r_b)^2}$ and substituting part (a) gives $M_b = \frac{4\pi^2 r_a (r_a + r_b)^2}{GT^2}$

c. The same calculations can be performed with the roles of star A and star B switched.
$M_a = \frac{4\pi^2 r_b (r_a + r_b)^2}{GT^2}$

d. $L_{total} = M_a v_a r_a + M_b v_b r_b = M_a \frac{2\pi r_a}{T} r_a + M_b \frac{2\pi r_b}{T} r_b = \frac{2\pi}{T}(M_a {r_a}^2 + M_b {r_b}^2)$

2007M2

a. $v = 2\pi R/T$ gives $R = 3.83 \times 10^6$ m

b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $M = \frac{v^2 R}{G} = 6.64 \times 10^{23}$ kg

c. $E = PE + KE = -\frac{GMm}{R} + \frac{1}{2}mv^2 = -5.38 \times 10^9$ J

d. From Kepler's third law r^3/T^2 = constant so if r decreases, then T must also.

e. Conservation of angular momentum gives $mv_1 r_1 = mv_2 r_2$ so $v_2 = r_1 v_1 / r_2$, but the distances *above the surface* are given so the radius of Mars must be added to the given distances before plugging them in for each r. This gives $v_2 = 3.34 \times 10^3$ m/s.

2001M2

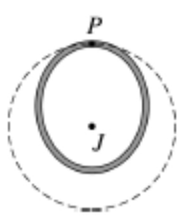
a. i. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{GM_J/R}$

ii. $v = d/T = 2\pi R/T$ giving $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM_J/R}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$

b. Plugging numerical values into a.ii. above gives $R = 1.59 \times 10^8$ m

c. i.

ii.



2005M2

a. $F = \frac{GM_S m}{R^2}$

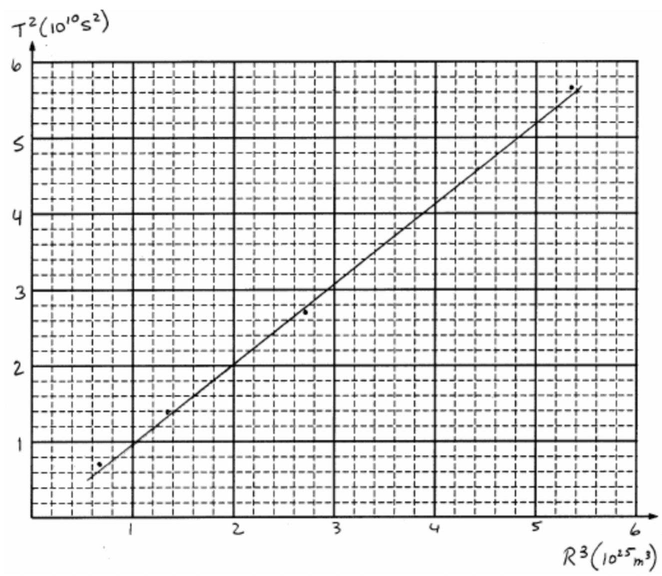
b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

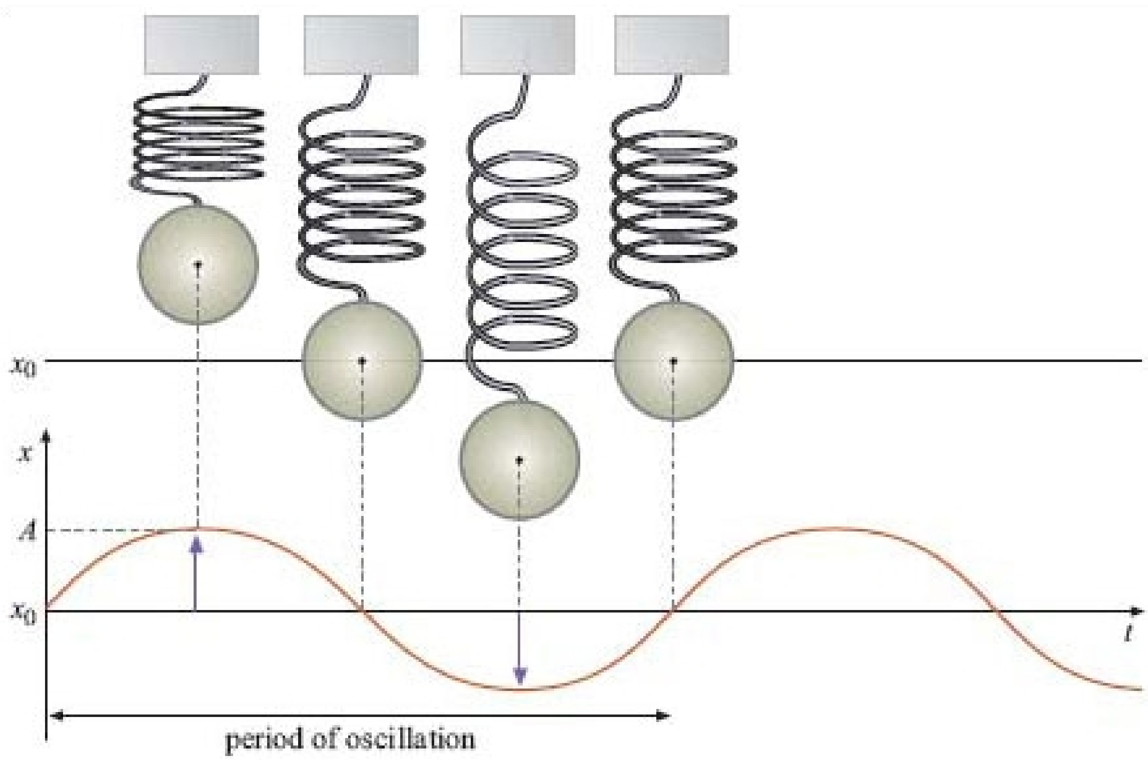
e.



f. From part c. we have an expression for the slope of the line.
Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg

Chapter 8

Oscillations



AP Physics Multiple Choice Practice – Oscillations

1. A mass m, attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the mass's speed as it passes through its equilibrium position?

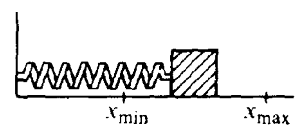
$$(A)\ A\sqrt{\frac{k}{m}} \quad (B)\ A\sqrt{\frac{m}{k}} \quad (C)\ \frac{1}{A}\sqrt{\frac{k}{m}} \quad (D)\ \frac{1}{A}\sqrt{\frac{m}{k}}$$

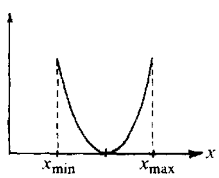
2. A mass m is attached to a spring with a spring constant k. If the mass is set into simple harmonic motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to the equilibrium position?

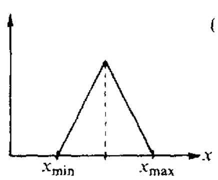
$$(A)\ v=\sqrt{\frac{md}{k}} \quad (B)\ v=\sqrt{\frac{kd}{m}} \quad (C)\ v=\sqrt{\frac{kd}{mg}} \quad (D)\ v=d\sqrt{\frac{k}{m}}$$

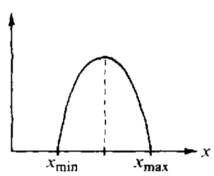
3. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
   (A) The kinetic and potential energies are equal to each other at all times.
   (B) The kinetic and potential energies are both constant.
   (C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
   (D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.

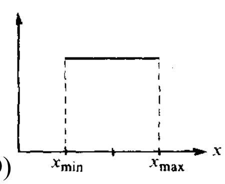
Questions 4-5: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



(A) 

(B) 

(C) 

(D) 

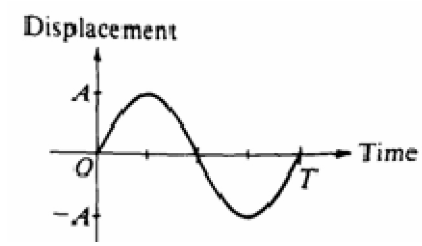
4. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
   (A) A (B) B (C) C (D) D

5. Which graph can represent the kinetic energy of the block as a function of x ?
   (A) A (B) B (C) C (D) D

6. An object swings on the end of a cord as a simple pendulum with period T. Another object oscillates up and down on the end of a vertical spring also with period T. If the masses of both objects are doubled, what are the new values for the Periods?

Pendulum	Mass on Spring
(A) $\frac{T}{\sqrt{2}}$	$T\sqrt{2}$
(B) T	$T\sqrt{2}$
(C) $T\sqrt{2}$	T
(D) $T\sqrt{2}$	$\frac{T}{\sqrt{2}}$

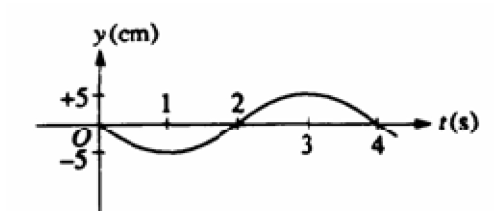
7. An object is attached to a spring and oscillates with amplitude A and period T, as represented on the graph. The nature of the velocity v and acceleration a of the object at time T/4 is best represented by which of the following?
(A) v > 0, a > 0 (B) v > 0, a < 0
(C) v > 0, a = 0 (D) v = 0, a < 0



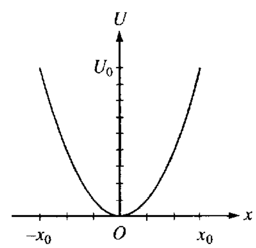
8. When an object oscillating in simple harmonic motion is at its maximum displacement from the equilibrium position. Which of the following is true of the values of its speed and the magnitude of the restoring force?

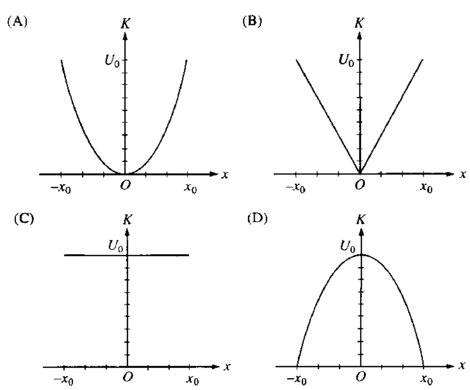
Speed	Restoring Force
(A) Zero	Maximum
(B) Zero	Zero
(C) Maximum	½ maximum
(D) Maximum	Zero

9. A particle oscillates up and down in simple harmonic motion. Its height y as a function of time t is shown in the diagram. At what time t does the particle achieve its maximum positive acceleration?
(A) 1 s (B) 2 s (C) 3 s (D) 4 s

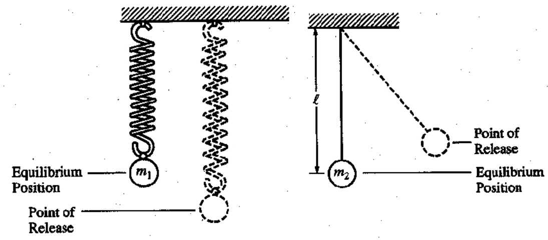


10. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?





Questions 11-12

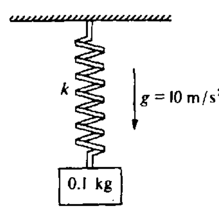


A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion

11. Which of the following is true for both spheres?
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position
(B) The maximum kinetic energy is attained as the sphere reaches its point of release.
(C) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(D) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(E) The maximum total energy is attained only as the sphere passes through its equilibrium position.

12. If both spheres have the same period of oscillation, which of the following is an expression for the spring constant
(A) $L / m_1 g$
(B) $g / m_2 L$
(C) $m_2 g / L$
(D) $m_1 g / L$

13. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are shorter.
(B) Both are the same.
(C) The period of the mass on the spring is shorter; that of the pendulum is the same.
(D) The period of the pendulum is shorter; that of the mass on the spring is the same



Questions 14-15
A 0.1 -kilogram block is attached to an initially unstretched spring of force constant k = 40 newtons per meter as shown above. The block is released from rest at time t = 0.

14. What is the amplitude, in meters, of the resulting simple harmonic motion of the block?

(A) $\frac{1}{40}m$ (B) $\frac{1}{20}m$ (C) $\frac{1}{4}m$ (D) $\frac{1}{2}m$

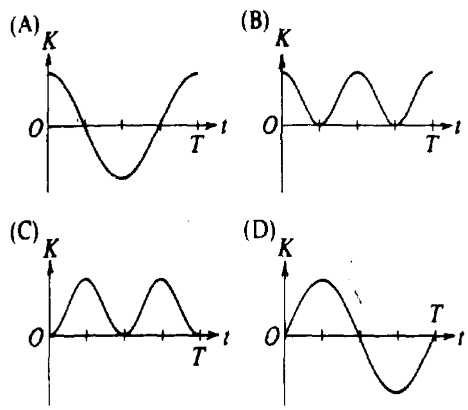
15. What will the resulting period of oscillation be?

(A) $\frac{\pi}{40}s$ (B) $\frac{\pi}{20}s$ (C) $\frac{\pi}{10}s$ (D) $\frac{\pi}{4}s$

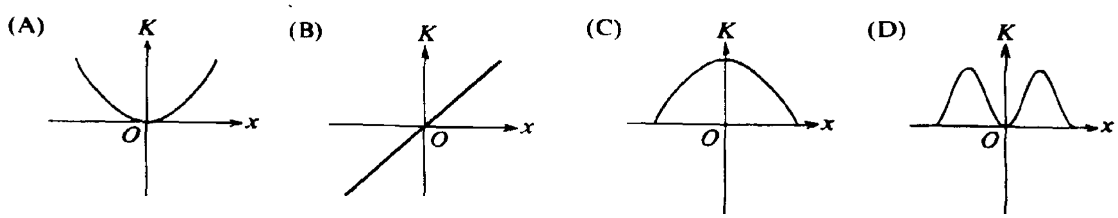
16. A ball is dropped from a height of 10 meters onto a hard surface so that the collision at the surface may be assumed elastic. Under such conditions the motion of the ball is
(A) simple harmonic with a period of about 1.4 s
(B) simple harmonic with a period of about 2.8 s
(C) simple harmonic with an amplitude of 5 m
(D) periodic with a period of about 2.8 s but not simple harmonic

Questions 17-18 refer to the graph below of the displacement x versus time t for a particle in simple harmonic motion.

17. Which of the following graphs shows the kinetic energy K of the particle as a function of time t for one cycle of motion?



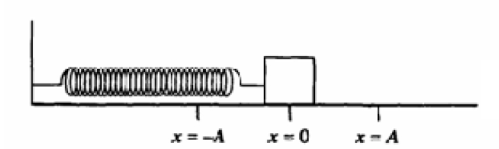
18. Which of the following graphs shows the kinetic energy K of the particle as a function of its displacement x ?



19. A mass m is attached to a vertical spring stretching it distance d. Then, the mass is set oscillating on a spring with an amplitude of A, the period of oscillation is proportional to

(A) $\sqrt{\frac{d}{g}}$ (B) $\sqrt{\frac{g}{d}}$ (C) $\sqrt{\frac{d}{mg}}$ (D) $\sqrt{\frac{m^2 g}{d}}$

20. Two objects of equal mass hang from independent springs of unequal spring constant and oscillate up and down. The spring of greater spring constant must have the
(A) smaller amplitude of oscillation
(B) larger amplitude of oscillation
(C) shorter period of oscillation
(D) longer period of oscillation



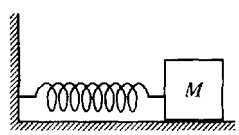
Questions 21-22. A block on a horizontal frictionless plane is attached to a spring, as shown above. The block oscillates along the x-axis with simple harmonic motion of amplitude A.

21. Which of the following statements about the block is correct?
    (A) At x = 0, its acceleration is at a maximum. (B) At x = A, its displacement is at a maximum.
    (C) At x = A, its velocity is at a maximum. (D) At x = A, its acceleration is zero.

22. Which of the following statements about energy is correct?
    (A) The potential energy of the spring is at a minimum at x = 0.
    (B) The potential energy of the spring is at a minimum at x = A.
    (C) The kinetic energy of the block is at a minimum at x =0.
    (D) The kinetic energy of the block is at a maximum at x = A.

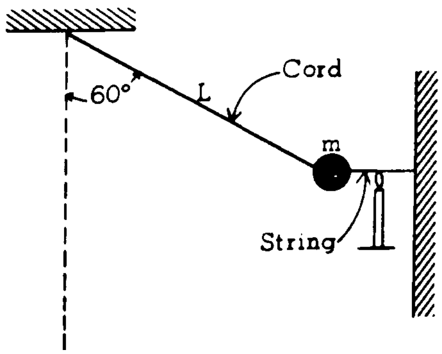
23. A simple pendulum consists of a l.0 kilogram brass bob on a string about 1.0 meter long. It has a period of 2.0 seconds. The pendulum would have a period of 1.0 second if the
    (A) string were replaced by one about 0.25 meter long
    (B) string were replaced by one about 2.0 meters long
    (C) bob were replaced by a 0.25 kg brass sphere
    (D) bob were replaced by a 4.0 kg brass sphere

24. A pendulum with a period of 1 s on Earth, where the acceleration due to gravity is g, is taken to another planet, where its period is 2 s. The acceleration due to gravity on the other planet is most nearly
    (A) g/4 (B) g/2 (C) 2g (D) 4g



25. An ideal massless spring is fixed to the wall at one end, as shown above. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is
    (A) $\frac{Mg}{A}$ (B) $\frac{Mgv_m}{2A}$ (C) $\frac{Mv_m^2}{2A}$ (D) $\frac{Mv_m^2}{A^2}$

AP Physics Free Response Practice – Oscillations



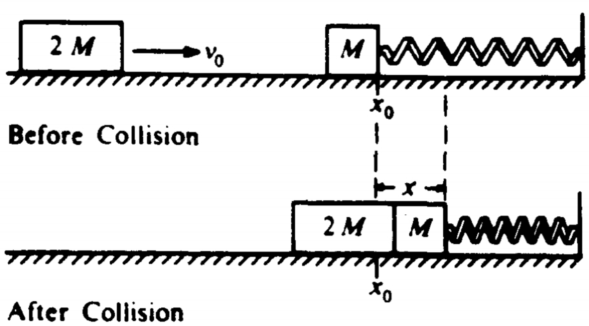
1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.

c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

d. The motion of the pendulum after the string is burned is periodic. Is it also simple harmonic? Why, or why not?

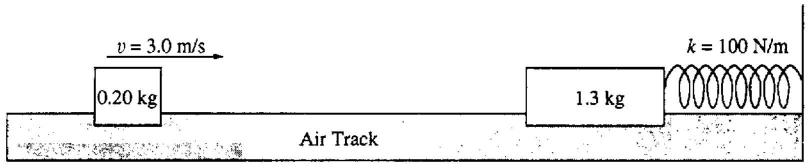


1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact

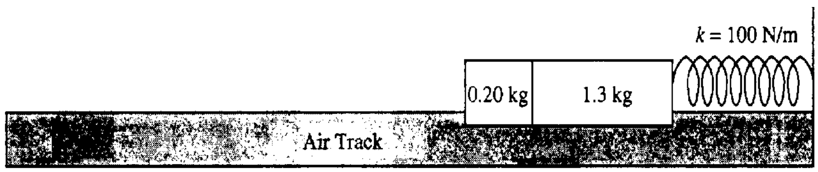
b. x, the maximum distance the spring is compressed

c. T, the period of the subsequent simple harmonic motion



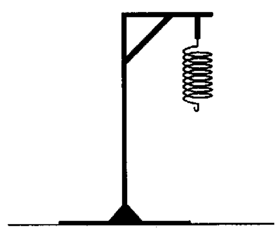
1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

a. Determine the following for the 0.20-kilogram mass immediately before the impact.
   i. Its linear momentum ii. Its kinetic energy
b. Determine the following for the combined masses immediately after the impact.
   i. The linear momentum ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

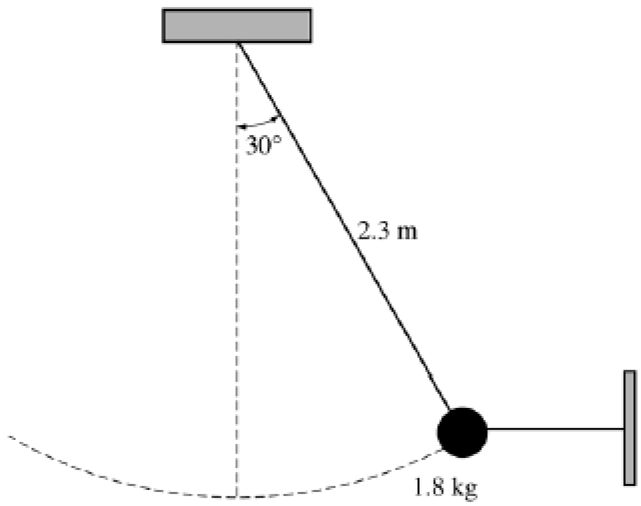
c. Determine the amplitude of the harmonic motion.
d. Determine the period of the harmonic motion.



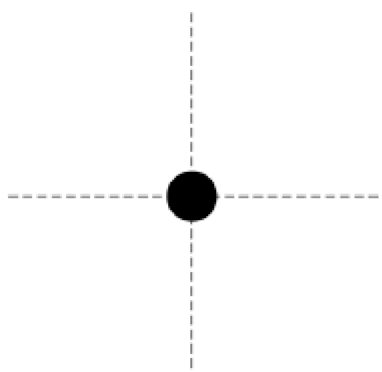
1996B2. A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?
b. Assume that the spring constant is determined to be 500 N/m. A 2.0-kg mass is attached to the lower end of the spring and released from rest. Determine the frequency of oscillation of the mass.
c. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.

2005B2



A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.

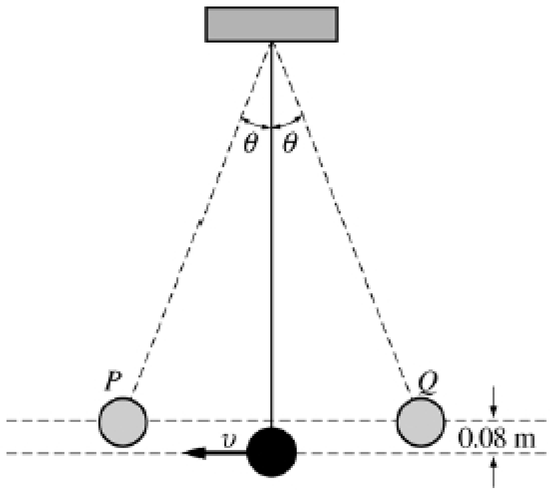


(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

(d) How long will it take the bob to reach the lowest position for the first time?

2005B2B

A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

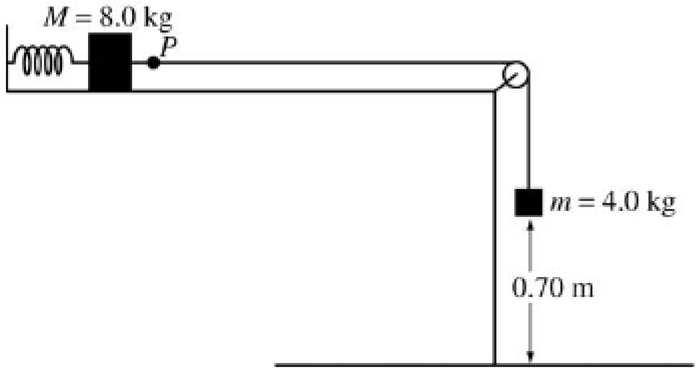
ii. When it is in motion at its lowest position

(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

(d) Describe one modification that could be made to double the period of oscillation.

2006B1



An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.

(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.

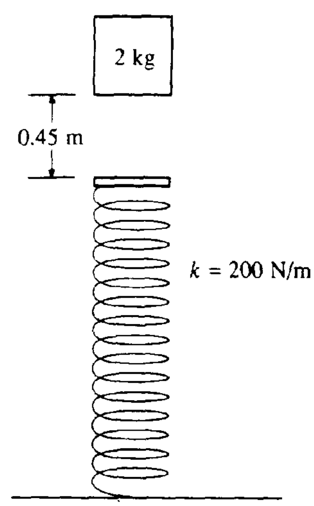
M = 8.0 kg m = 4.0 kg

(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

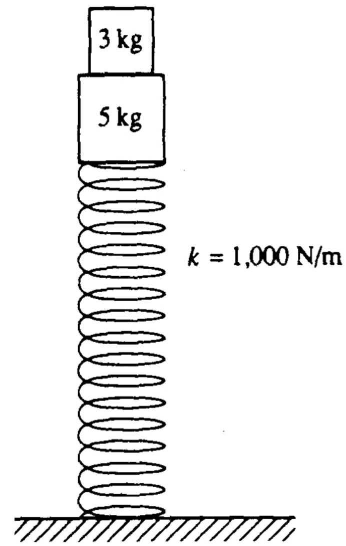
The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the frequency of oscillation of the 8.0 kg block.
(f) Calculate the maximum speed attained by the 8.0 kg block.

C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Determine the resulting amplitude of the oscillation that ensues
f. Is the speed of the block a maximum at the equilibrium position, explain.
g. Determine the period of the simple harmonic motion that ensues



1990M3. A 5-kilogram block is fastened to an ideal vertical spring that has an unknown spring constant. A 3-kilogram block rests on top of the 5-kilogram block, as shown above.



a. When the blocks are at rest, the spring is compressed to its equilibrium position a distance of $\Delta x_1 = 20$ cm, from its original length. Determine the spring constant of the spring

The 3 kg block is then raised 50 cm above the 5 kg block and dropped onto it.

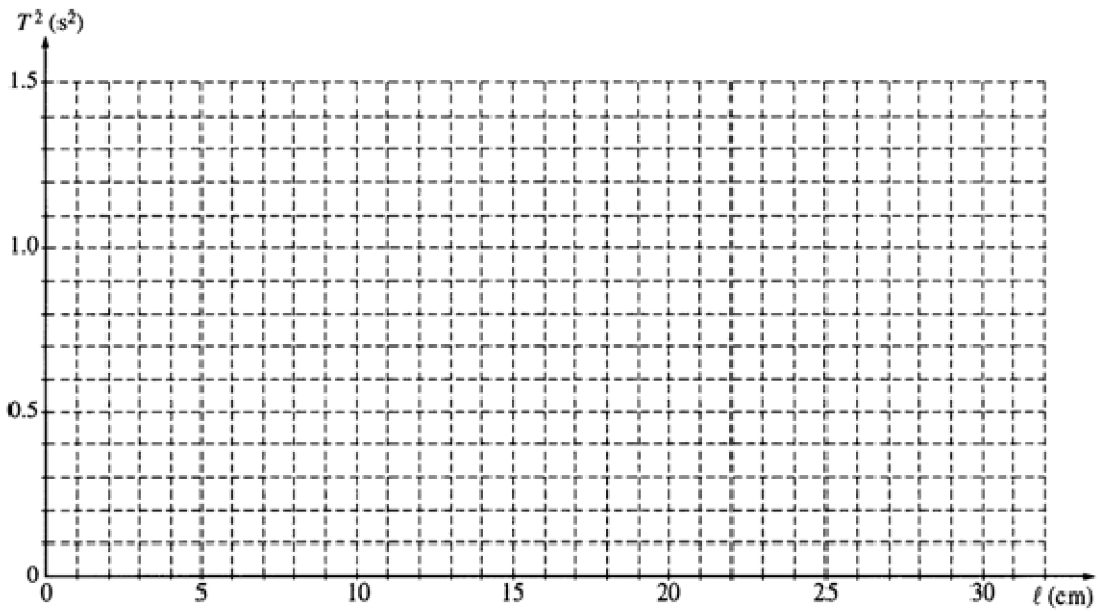
b. Determine the speed of the combined blocks after the collision
c. Setup, plug in known values, but do not solve an equation to determine the amplitude Δx_2 of the resulting oscillation
d. Determine the resulting frequency of this oscillation.
e. Where will the block attain its maximum speed, explain.
f. Is this motion simple harmonic.

(2000 M1) You are conducting an experiment to measure the acceleration due to gravity g_u at an unknown location. In the measurement apparatus, a simple pendulum swings past a photogate located at the pendulum's lowest point, which records the time t_{10} for the pendulum to undergo 10 full oscillations. The pendulum consists of a sphere of mass m at the end of a string and has a length l. There are four versions of this apparatus, each with a different length. All four are at the unknown location, and the data shown below are sent to you during the experiment.

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62		
18	8.89		
21	10.09		
32	12.08		

a. For each pendulum, calculate the period T and the square of the period. Use a reasonable number of significant figures. Enter these results in the table above.

b. On the axes below, plot the square of the period versus the length of the pendulum. Draw a best-fit straight line for this data.

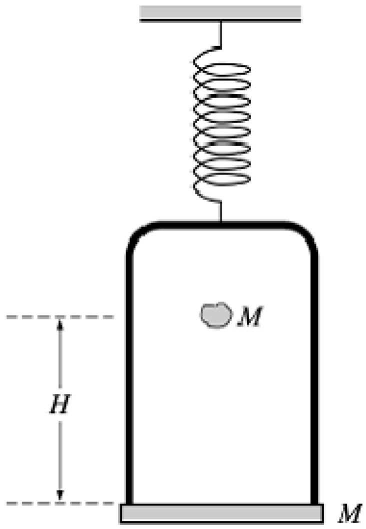


c. Assuming that each pendulum undergoes small amplitude oscillations, from your fit, determine the experimental value g_{exp} of the acceleration due to gravity at this unknown location. Justify your answer.

d. If the measurement apparatus allows a determination of g_u that is accurate to within 4%, is your experimental value in agreement with the value 9.80 m/s^2 ? Justify your answer.

e. Someone informs you that the experimental apparatus is in fact near Earth's surface, but that the experiment has been conducted inside an elevator with a constant acceleration a. If the elevator is moving down, determine the direction of the elevator's acceleration, justify your answer.

C2003M2.



An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

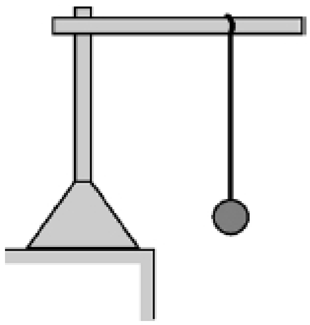
(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the clay and pan just after the clay strikes it.

(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

(d) Determine the resulting period of oscillation

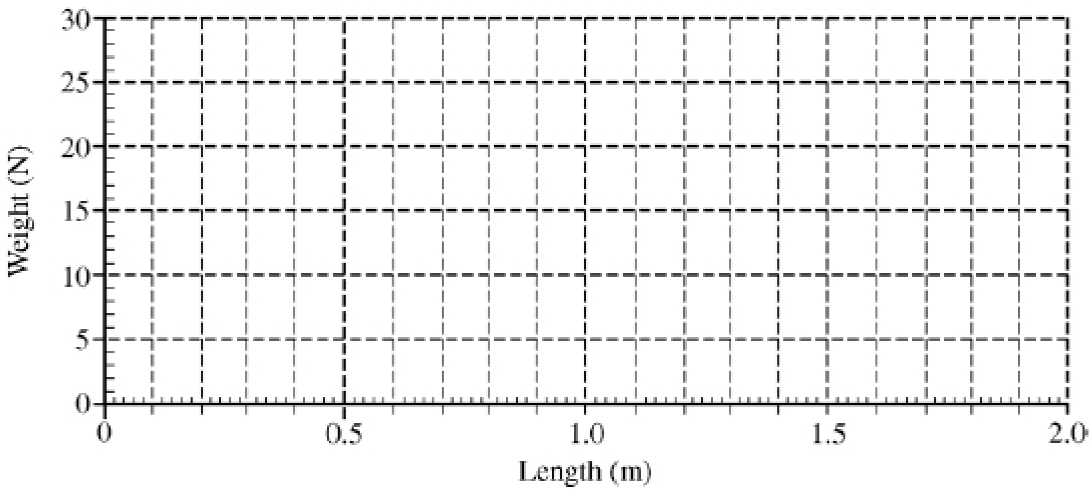
C2008M3



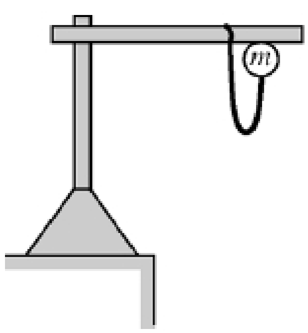
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass m of the object.

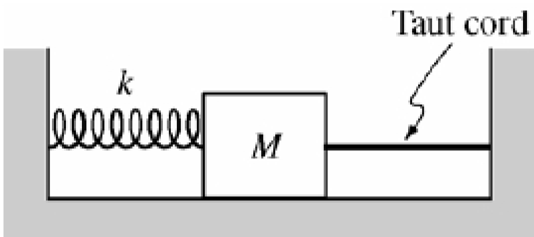
(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

iii. Calculate the speed of the object at the equilibrium position

iv. Is the speed in part iii above the maximum speed, explain your answer.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Calculate the time after the cord breaks until the block first reaches its position furthest to the left.
(e) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Oscillations

	Solution	Answer
1.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} mv^2$	A
2.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k d^2 = \frac{1}{2} mv^2$	D
3.	Energy is conserved here and switches between kinetic and potential which have maximums at different locations	D
4.	Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates	D
5.	The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the KE gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph.	C
6.	Pendulum is unaffected by mass. Mass-spring system has mass causing the T to change proportional to $\sqrt{m}$ so since the mass is doubled the period is changed by $\sqrt{2}$	B
7.	At T/4 the mass reaches maximum + displacement where the restoring force is at a maximum and pulling in the opposite direction and hence creating a negative acceleration. At maximum displacement the mass stops momentarily and has zero velocity	D
8.	See #7 above	A
9.	+ Acceleration occurs when the mass is at negative displacements since the force will be acting in the opposite direction of the displacement to restore equilibrium. Based on $F=k\Delta x$ the most force, and therefore the most acceleration occurs where the most displacement is	A
10.	As the object oscillates, its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D	D
11.	For the spring, equilibrium is shown where the maximum transfer of kinetic energy has occurred and likewise for the pendulum the bottom equilibrium position has the maximum transfer of potential energy into spring energy.	A
12.	Set period formulas equal to each other and rearrange for k	D
13.	In a mass-spring system, both mass and spring constant (force constant) affect the period.	D
14.	At the current location all of the energy is gravitational potential. As the spring stretches to its max location all of that gravitational potential will become spring potential when it reaches its lowest position. When the box oscillates back up it will return to its original location converting all of its energy back to gravitational potential and will oscillate back and forth between these two positions. As such the maximum stretch bottom location represents twice the amplitude so simply halving that max Δx will give the amplitude. Finding the max stretch: → The initial height of the box h and the stretch Δx have the same value ($h=\Delta x$) $U = U_{sp}$ $mg(\Delta x_1) = \frac{1}{2} k\Delta x_1^2$ $mg = \frac{1}{2} k \Delta x_1$ $\Delta x_1 = .05$ m. This is 2A, so the amplitude is 0.025 m or 1/40 m. Alternatively, we could simply find the equilibrium position measured from the initial top position based on the forces at equilibrium, and this equilibrium stretch measured from the top will be the amplitude directly. To do this:	A

$F_{net} = 0$ $F_{sp} = mg$ $k\Delta x_2 = mg$ $\Delta x_2 = 0.025$ m, which is the amplitude

15. Plug into period for mass-spring system $T = 2\pi \sqrt{(m/k)}$ C

16. Based on free fall, the time to fall down would be 1.4 seconds. Since the collision with the ground is elastic, all of the energy will be returned to the ball and it will rise back up to its initial height completing 1 cycle in a total time of 2.8 seconds. It will continue doing this oscillating up and down. However, this is not simple harmonic because to be simple harmonic the force should vary directly proportional to the displacement but that is not the case in this situation D

17. Energy will never be negative. The max kinetic occurs at zero displacement and the kinetic energy become zero when at the maximum displacement B

18. Same reasoning as above, it must be C C

19. First use the initial stretch to find the spring constant. $F_{sp} = mg = k\Delta x$ $k = mg / d$ D

    Then plug that into $T = 2\pi \sqrt{(m/k)}$ $T = 2\pi\sqrt{\dfrac{m}{(\frac{mg}{d})}}$

20. Based on $T = 2\pi \sqrt{(m/k)}$ the larger spring constant makes a smaller period C

21. Basic fact about SHM. Amplitude is max displacement B

22. Basic fact about SHM. Spring potential energy is a min at x=0 with no spring stretch D

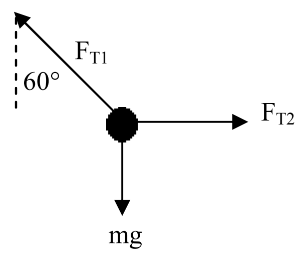
23. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ the length equates to ½ the period A

24. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ g would double the period A

25. Using energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$ solve for k D

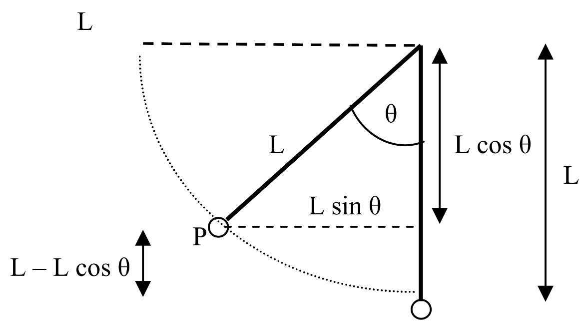
1975B7.

(a)



(b) $F_{NET(Y)} = 0$
$F_{T1} \cos \theta = mg$
$F_{T1} = mg / \cos(60) = 2mg$

(c) When the string is cut it swings from top to bottom, similar to the diagram for 1974B1 from work-energy problems with θ on the opposite side as shown below



$U_{top} = K_{bot}$
$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

(d) This motion is not simple harmonic because the restoring force, $(F_{gx}) = mg \sin \theta$, is not directly proportional to the displacement due to the sin function. For small angles of θ the motion is approximately SHM, though not exactly, but in this example the larger value of θ creates and even larger disparity.

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $\frac{1}{2}(3M)(2/3\ v_o)^2 = \frac{1}{2}k\,\Delta x^2$ $\sqrt{\frac{4Mv_o^{\,2}}{3k}}$

c) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3m}{k}}$

1995 B1.

a) i) $p = mv$ $= (0.2)(3) = 0.6$ kg m/s
ii) $K = \frac{1}{2}mv^2$ $= \frac{1}{2}(0.2)(3)^2 = 0.9$ J

b) i.) Apply momentum conservation $p_{before} = p_{after} = 0.6$ kg m/s
ii) First find the velocity after, using the momentum above
$0.6 = (1.3+0.2)\ v_f$ $v_f = 0.4$ m/s, then find K, $K = \frac{1}{2}(m_1+m_2)\ v_f^2 = \frac{1}{2}(1.3+0.2)(0.4)^2 = 0.12$ J

c) Apply energy conservation $K = U_{sp}$ $0.12\text{ J} = \frac{1}{2}k\Delta x^2 = \frac{1}{2}(100)\ \Delta x^2$ $\Delta x = 0.05$ m

d) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.5}{100}} = 0.77s$

1996B2.

a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k\ \Delta x$ and solve for k

b) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{500}} = 0.4s$

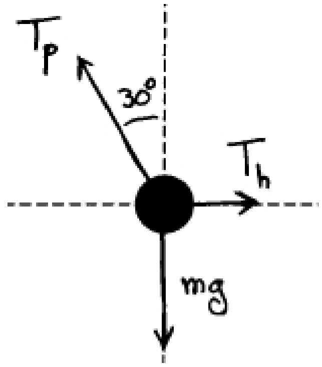
… then the frequency is given by $f = 1/T = 2.5$ Hz

c) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan\theta$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} - \mu_s\ mg$, Giving $mg = F_{spring} / \mu$.
Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid mechanics unit.

2005B2.

a) FBD



b) Apply $F_{net(X)} = 0$ $\quad$ $F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $\quad$ $T_P \sin 30 = T_H$

$T_P = 20.37$ N $\quad$ $T_H = 10.18$ N

c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½\ m\ v^2$

$g(L - L\cos\theta) = ½\ v^2$

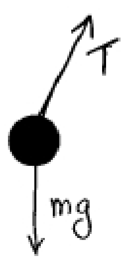
$(10)(2.3 - 2.3\cos 30) = ½\ v^2$ $\quad$ $v_{bottom} = 2.5$ m/s

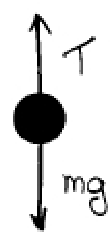
d) The bob will reach the lowest position in ¼ of the period.

$$T = \frac{1}{4}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\pi}{2}\sqrt{\frac{2.3}{9.8}} = 0.76\ s$$

B2005B2.

FBD $\quad$ i) $\quad$ ii)





b) Apply energy conservation?

$U_{top} = K_{bottom}$

$mgh = ½\ m\ v^2$ $\quad$ $(9.8)(.08) - ½\ v^2$ $\quad$ $v = 1.3$ m/s

c) $F_{net(c)} = mv^2/r$

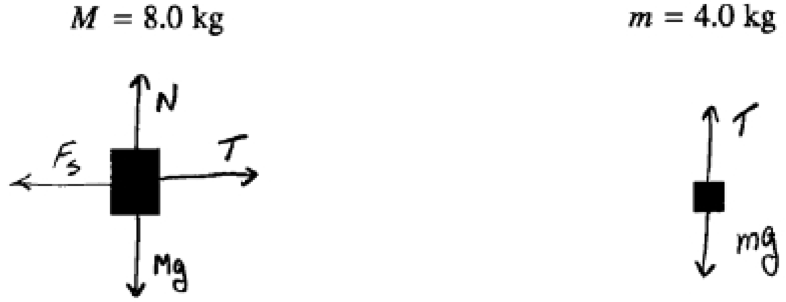
$F_t - mg = mv^2/r$ $\quad$ $F_t = mv^2/r + mg$ $\quad$ $(0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $\quad$ $F_t = 0.93$ N

d) "g" and "L" are the two factors that determine the pendulum period based on $\left(T = 2\pi\sqrt{\frac{L}{g}}\right)$

To double the value of T, L should be increased by 4x or g should be decreased by ¼. The easiest modification would be simply to increase the length by 4 x

2006B1.

a) FBD



b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

c) Tension in the string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k(0.05)$ $k = 780$ N/m

d) 4 kg mass is in free fall. $D = v_i t + \frac{1}{2} g t^2$ $-0.7 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.38$ sec

e) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{780}} = 0.63s$

… then the frequency is given by $f = 1/T = 1.6$ Hz

f) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $\frac{1}{2} k\,\Delta x^2 = \frac{1}{2} mv^2$ $\frac{1}{2}(780)(0.05)^2 = \frac{1}{2}(8)v^2$ $v = 0.49$ m/s

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C1989M3.

a) Apply energy conservation from top to end of spring using h=0 as end of spring.
$U = K$ $mgh = \frac{1}{2} m v^2$ $(9.8)(0.45) = \frac{1}{2} v^2$ $v = 3$ m/s

b) At equilibrium the forces are balanced $F_{net} = 0$ $F_{sp} = mg = (2)(9.8) = 19.6$ N

c) Using the force from part b, $F_{sp} = k\,\Delta x$ $19.6 = 200\,\Delta x$ $\Delta x = 0.098$ m

d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the top position is now increased by the amount of Δx found in part c $h_{new} = h + \Delta x = 0.45 + 0.098 = 0.548$ m
$U_{top} = U_{sp} + K_{\text{(at equil)}}$
$mgh_{new} = \frac{1}{2} k\,\Delta x^2 + \frac{1}{2} mv^2$ $(2)(9.8)(0.548) = \frac{1}{2}(200)(0.098)^2 + \frac{1}{2}(2)(v^2)$ $v = 3.13$ m/s

e) Use the turn horizontal trick. Set equilibrium position as zero spring energy then solve it as a horizontal problem where $K_{equil} = U_{sp(\text{at max amp.})}$ $\frac{1}{2} mv^2 = \frac{1}{2} k\Delta x^2$ $\frac{1}{2}(2)(3.13)^2 = \frac{1}{2}(200)(A^2)$ $A = 0.313$ m

f) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

g) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 0.63s$

C1990M3.

a) Equilibrium so $F_{net} = 0$, $F_{sp} = mg$ $k\Delta x = mg$ $k (0.20) = (8)(9.8)$ $k = 392$ N/m

b) First determine the speed of the 3 kg block prior to impact using energy conservation

$U = K$ $mgh = ½ m v^2$ $(9.8)(0.50) = ½ v^2$ $v = 3.13$ m/s

Then solve perfect inelastic collision. $p_{before} = p_{after}$ $m_1v_{1i} = (m_1+m_2) v_f$ $(3)(3.13)=(8)v_f$ $v_f = 1.17$ m/s

c) Since we do not know the speed at equilibrium nor do we know the amplitude Δx_2 the turn horizontal trick would not work initially. If you first solve for the speed at equilibrium as was done in 1989M3 first, you could then use the turn horizontal trick. However, since this question is simply looking for an equation to be solved, we will use energy conservation from the top position to the lowest position where the max amplitude is reached. For these two positions, the total distance traveled is equal to the distance traveled to equilibrium + the distance traveled to the max compression $(\Delta x_1 + \Delta x_2) = (0.20 + \Delta x_2)$ which will serve as both the initial height as well as the total compression distance. We separate it this way because the distance traveled to the maximum compression from equilibrium is the resulting amplitude Δx_2 that the question is asking for.

Apply energy conservation

$U_{top} + K_{top} = U_{sp(max\text{-}comp)}$

$mgh + ½ m v^2 = ½ k \Delta x_2^2$ $(8)(9.8)(0.20 + \Delta x_2) + ½ (8)(1.17)^2 = ½ (392)(0.20 + \Delta x_2)^2$

The solution of this quadratic would lead to the answer for Δx_2 which is the amplitude.

d) First find period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{392}} = 0.90s$ Then find frequency $f = 1/T = 1.11$ Hz

e) The maximum speed will occur at equilibrium because the net force is zero here and the blocks stop accelerating in the direction of motion momentarily. Past this point, an upwards net force begins to exist which will slow the blocks down as they approach maximum compressions and begin to oscillate.

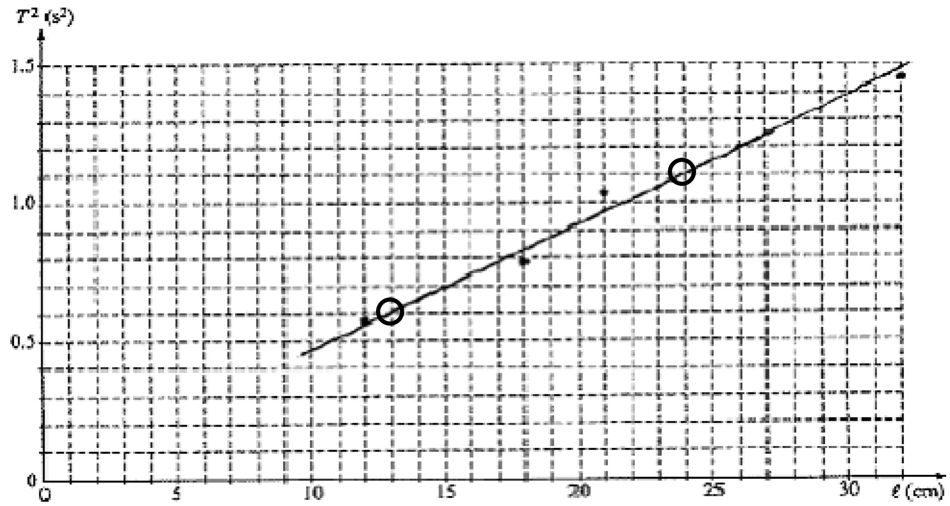
f) This motion is simple harmonic because the force acting on the masses is given by $F=k\Delta x$ and is therefore directly proportional to the displacement meeting the definition of simple harmonic motion

C2000M1.

a)

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62	0.762	0.581
18	8.89	0.889	0.790
21	10.09	1.009	1.018
32	12.08	1.208	1.459

b)



c) We want a linear equation of the form y = mx.

Based on $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\qquad T^2 = 2^2\pi^2\dfrac{L}{g}$ $\qquad T^2 = \dfrac{4\pi^2}{g}\,L$

y = m x

This fits our graph with y being T^2 and x being L. Finding the slope of the line will give us a value that we can equate to the slope term above and solve it for g. Since the points don't fall on the line we pick random points as shown circled on the graph and find the slope to be = 4.55. Set this = to $4\pi^2$ / g and solve for g = 8.69 m/s^2

d) A +/- 4% deviation of the answer (8.69) puts its possible range in between 8.944 – 8.34 so this result does not agree with the given value 9.8

e) Since the value of g is less than it would normally be (you feel lighter) the elevator moving down would also need to be **accelerating down** to create a lighter feeling and smaller F_n. Using down as the positive direction we have the following relationship, $F_{net} = ma$ $\qquad mg - F_n = ma$ $\qquad F_n = mg - ma$
For F_n the be smaller than usual, a would have to be + which we defined as down.

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ $mgh = ½ mv^2$ $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\, v_f$ $M\,(\sqrt{2gH}\,) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Again we cannot use the turn horizontal trick because we do not know information at the equilibrium position. While the tray was initially at its equilibrium position, its collision with the clay changed where this location would be.

Even though the initial current rest position immediately after the collision has an unknown initial stretch to begin with due to the weight of the tray and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

Apply energy conservation $K + U = U_{sp\,(gained)}$ $½ mv^2 + mgh = ½ k\, \Delta x^2$

Plug in mass (2m), h = H/2 and Δx = H/2 ➔ $½ (2m)v^2 + (2m)g(H/2) = ½ k(H/2)^2$

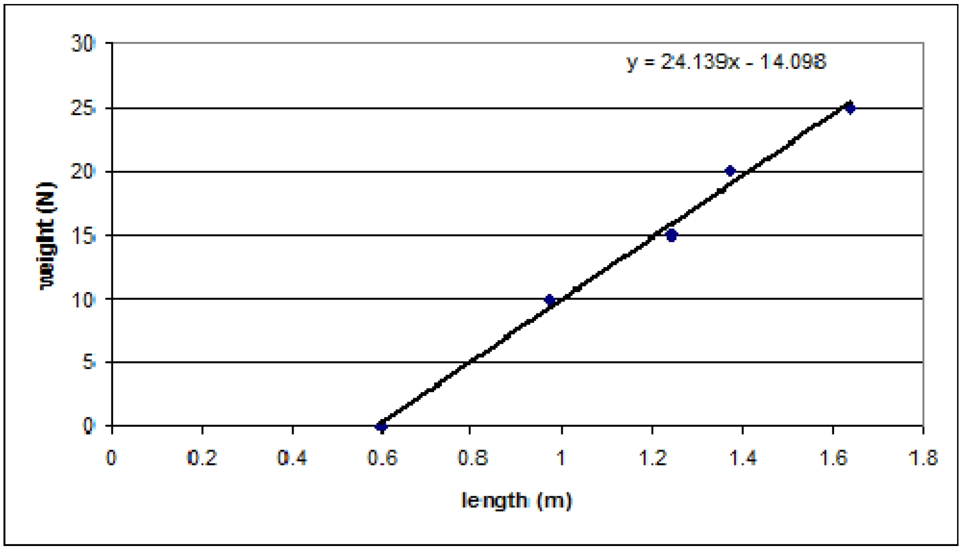
plug in v_f from part b $m(2gH/4) + mgH = kH^2/8$ ….

Both sides * (1/H) ➔ mg/2 + mg = kH/8 ➔ 3/2 mg = kH/8 k = 12mg / H

d) Based on $T = 2\pi\sqrt{\dfrac{2M}{\dfrac{12Mg}{H}}}$ = $2\pi\sqrt{\dfrac{H}{6g}}$

C2008M3

(a)



(b) The slope of the line is F / Δx which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

$mgh = \frac{1}{2} k \Delta x^2$ $\quad m(9.8)(1.5) = \frac{1}{2}(24)(0.9)^2$ $\quad m = 0.66$ kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $\quad F_{sp} = (0.66)(9.8)$ $\quad F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $\quad 6.5 = (24)\Delta x$ $\quad \Delta x = 0.27$ m

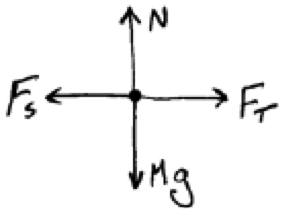
iii) Measured from the starting position of the mass, the equilibrium position would be located at the location marked by the unstretched cord length + the stretch found above. 0.6+0.27 = 0.87m. Set this as the h=0 location and equate the U_{top} to the U_{sp} + K here.

$mgh = \frac{1}{2} k \Delta x^2 + 1/2\, mv^2$ $\quad (0.66)(9.8)(0.87) = \frac{1}{2}(24)(0.27)^2 + \frac{1}{2}(0.66) v^2$ $\quad v = 3.8$ m/s

iv) This is the maximum speed because this is the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the mass down until it reaches its maximum compression and stops momentarily.

Supplemental.

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k\, \Delta x^2 = \frac{1}{2} k \Delta x_2^2 + \frac{1}{2} mv^2$
$k\,(\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$ $$v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$$

(d) To reach the position from the far left will take ½ of a period of oscillation.

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad t = \frac{1}{2}2\pi\sqrt{\frac{M}{k}} \qquad = \pi\sqrt{\frac{M}{k}}$$

(e) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get
$F_{net} = ma$ $F_{sp} - f_k = ma$
From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$