

## Lesson 3: Number System

### Number systems and conversions from one system to another.

The 4 number systems are:

#### Binary [ 2 ] .

- Uses 2 symbols: the digits 0 and 1.
- Some numbers in this system: 0, 000, 1010.

#### Decimal [ 10 ]

- Uses 10 symbols: the digits 0, 1, 2, ... ,9.
- Some numbers in this system: 111, 0, 1010.

#### Octal [ 8 ]

- Uses 8 symbols: the digits 0, 1, 2, 3, 4, 5, 6, 7.

#### Hexadecimal [ 16 ]

- Uses 16 symbols: 0, 1, 2, ..., 9, A, B, C, D, E, F.

#### Notes:

When written down, a number may be ambiguous regarding which system it belongs to. So, we will associate a subscript to clear such ambiguities. 1010 in the binary system will be denoted as 1010[2] to distinguish it from 1010 of the decimal system, which would be denoted as 1010[10]. Similarly, 1010 of the octal system, would be denoted as 1010[8], and 1010 of the Hex system would be denoted as 1010[16].

### Conversions from one system to another:

It is possible to convert between any of the number systems.

#### [ 2 ] $\rightarrow$ [ 10 ]

1010[2] = ?[10]

1010[2]

Based on multiplying with powers of 2.

$$1010 [2] = 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 0 * 2^0 = 2^3 + 2^1 = 8 + 2 = 10$$

i.e. 1010 [2] = 10 [10]

#### [ 10 ] $\rightarrow$ [ 2 ]

10 [10] = ? [2]

10 [10]

Based on dividing by 2. Record the quotients and the remainders. Stop dividing if quotient is 0, and then take the remainders bottom to top. This would be the binary number.

Number	Operation	Quotient	Remainder
10	10 / 2	5	0
5	5/2	2	1
2	2/2	1	0
1	$\frac{1}{2}$	0	1

Taking the remainders bottom-to-top on the above table forms the number 1010. This would be the binary number that is equal to 10 [10], i.e.  $10 [10] = 1010 [2]$ .

**[ 2 ] → [ 8 ]**

$$1010 [2] = ? [8]$$

Take chunks of 3 digits from right to left (pad with 0's at the left if there are not enough binary digits), and write the octal digit that corresponds to each chunk.

$$1010 [2] = 001\ 010 [2] = 1[8]\ 2[8] = 12[8].$$

**[ 8 ] → [ 2 ]**

$$12 [8] = ? [2]$$

Take each octal digit and map it to its binary representative with 3 binary digits.

$$12[8] \rightarrow 001 [2]\ 010[2] \rightarrow 001010[2] = 1010 [2]$$

**[ 2 ] → [ 16 ]**

$$11010 [2] = ? [16]$$

Take chunks of 4 digits from right to left (pad with 0's at the left if there are not enough binary digits), and write the hex digit that corresponds to each chunk.

$$11010[2] = 0001 [2]\ 1010 [2] = 1 [16]\ A [16] = 1A [16].$$

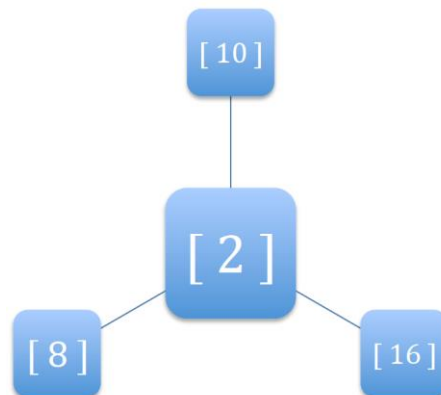
**[ 16 ] → [ 2 ]**

$$1A [16] = ? [2]$$

Take each hex digit and map it to its binary representative with 4 binary digits.

$$1A [16] = 0001 [2]\ 1010 [2] = 00011010 [2] = 11010 [2]$$

The diagram below shows all the conversion cases covered above.



This means that we can now handle any type of conversion, even conversions between the number systems that are not directly connected on the above diagram. For example, to convert  $[ 8 ] \rightarrow [ 10 ]$ , we can convert  $[ 8 ] \rightarrow [ 2 ]$  followed by  $[ 2 ] \rightarrow [ 10 ]$ . Or, to convert  $[ 8 ] \rightarrow [ 16 ]$ , we can convert  $[ 8 ] \rightarrow [ 2 ]$  followed by  $[ 2 ] \rightarrow [ 16 ]$ . Etc.