



CHAPTER 4B: BOOLEAN ALGEBRA

DR. ERIC CHOU

IEEE SENIOR MEMBER



Objectives

- Basic Boolean Algebra
- Logic Design
- Number Line Analysis
- Aggregated Logic Function (hasEven, allEven)
- Calendar Year
- Software Design Life Cycle
- Top-down Design Bottom-Up Implementation
- Pseudo Code
- PrintCalendar Project



Basic Boolean Algebra

LECTURE 1



Introduction

- •The most **obvious** way to **simplify** Boolean expressions is to manipulate them in the same way as normal **algebraic expressions** are manipulated.
- •With regards to logic relations in digital forms, a set of rules for symbolic manipulation is needed in order to solve for the unknowns.



Introduction

- •A set of rules formulated by the English mathematician George Boole describe certain propositions whose outcome would be either true or false.
- •With regard to digital logic, these rules are used to describe circuits whose state can be either, 1 (true) or 0 (false). In order to fully understand this, the relation between the AND gate, OR gate and NOT gate operations should be appreciated. A number of rules can be derived from these relations as Table 1 demonstrates.



Introduction

•A number of rules can be derived from these relations as Table 1 demonstrates.

- •Rule 1: X = 0 or X = 1
- •Rule 2: 0 * 0 = 0
- •Rule 3: 1 + 1 = 1
- •Rule 4: 0 + 0 = 0
- •Rule 5: 1 * 1 = 1
- •Rule 6: 1 * 0 = 0 * 1 = 0
- •Rule 7: 1 + 0 = 0 + 1 = 1

Table 1: Boolean Postulates



Laws of Boolean Algebra

- Table 2 shows the basic Boolean laws.
- •Note that every law has two expressions, (a) and (b). This is known as duality.
- •These are obtained by changing every AND(*) to OR(+), every OR(+) to AND(*) and all 1's to 0's and vice-versa.
- •It has become conventional to drop the * (AND symbol) i.e. A.B is written as AB.

Boolean Algebra Laws

T1: Commutative Law

(a)
$$A + B = B + A$$

(b)
$$A B = B A$$

T2: Associate Law

(a)
$$(A + B) + C = A + (B + C)$$

(b)
$$(A B) C = A (B C)$$

T3: Distributive Law

(a)
$$A (B + C) = A B + A C$$

(b)
$$A + (B C) = (A + B) (A + C)$$

T4: Identity Law

$$(a) A + A = A$$

(b)
$$AA = A$$

T5:

(a)
$$AB + A\overline{B} = A$$

(b)
$$(A+B)(A+\overline{B}) = A$$

T6: Redundance Law

(a)
$$A + AB = A$$

(b)
$$A (A + B) = A$$

T7:

(a)
$$0 + A = A$$

(b)
$$0 A = 0$$

T8:

(a)
$$1 + A = 1$$

(b)
$$1 A = A$$

T9:

(a)
$$\overline{A} + A = 1$$

(b)
$$\overline{A} A = 0$$

T10:

(a)
$$A + \overline{A} B = A + B$$

(b)
$$A(\overline{A} + B) = AB$$

T11 : De Morgan's Theorem

(a)
$$(\overline{A+B}) = \overline{A} \ \overline{B}$$

(b)
$$(\overline{A}\overline{B}) = \overline{A} + \overline{B}$$



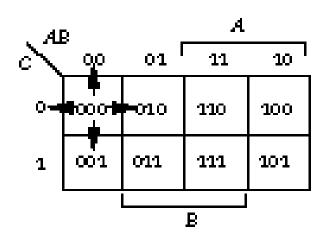
True Table for Logic Functions

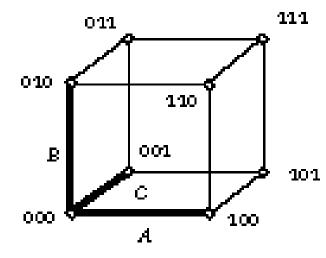
<expr A operator B> can have 16 possible outcomes which equals 2^4 (4 A/B combinations each has 2 possible outcome)

I																
Ν							(O	perato	ors)							
Ρ	Z									X	Ν		Ν		Ν	
U	Ε	A	A		В		×		Ν	Ν	0	A	0	В	A	0
T	R	Ν	>		>		0	0	0	0	Т	≤	T	≤	Ν	Ν
AB	0	D	В	Α	Α	В	R	R	R	R	'B'	В	'A'	Α	D	E
00	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
01	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
10	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
11	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Boolean Cube and Theorem of Symmetry

The dual of any Boolean property





If Boolean Expression

$$T(A, B, +, *, =, 1, 0)$$
 is valid.
 $T(A, B, *, +, = 0, 1)$ is also valid.

$$AB+A=A$$

$$(A+B)*A=A$$

$$A + (-A) = 1$$

 $A * (-A) = 0$



Example of De Morgan's Theorem

```
if ((x \% 5 != 0) \&\& (x \% 2 != 0)) // x is not multiple of 5 and x is not multiple of 2. equivalent to
```

```
if (!((x \% 5 == 0) | | (x \% 2 == 0))) // x is not (multiple of 5 or 2)
```

Download and work on BooleanQuiz.pdf



Logic Design

LECTURE 2



Student GPA Issues 1:

(smaller set should go first)

Smaller set go first:

```
if (math>=90) {mathGrade = 'A';}
else if (math>=80) {mathGrade = 'B';}
else if (math>=70) {mathGrade = 'C';}
else if (math>=60) {mathGrade = 'D';}
else
```

Smaller set go first:

```
if (math<60) {mathGrade = 'F';}
else if (math>=60){mathGrade = 'D';}
else if (math>=70){mathGrade = 'C';}
else if (math>=80){mathGrade = 'B';}
else if (math>=90){mathGrade = 'A';}
```



Student GPA Issue 2:

(Corner Cases)

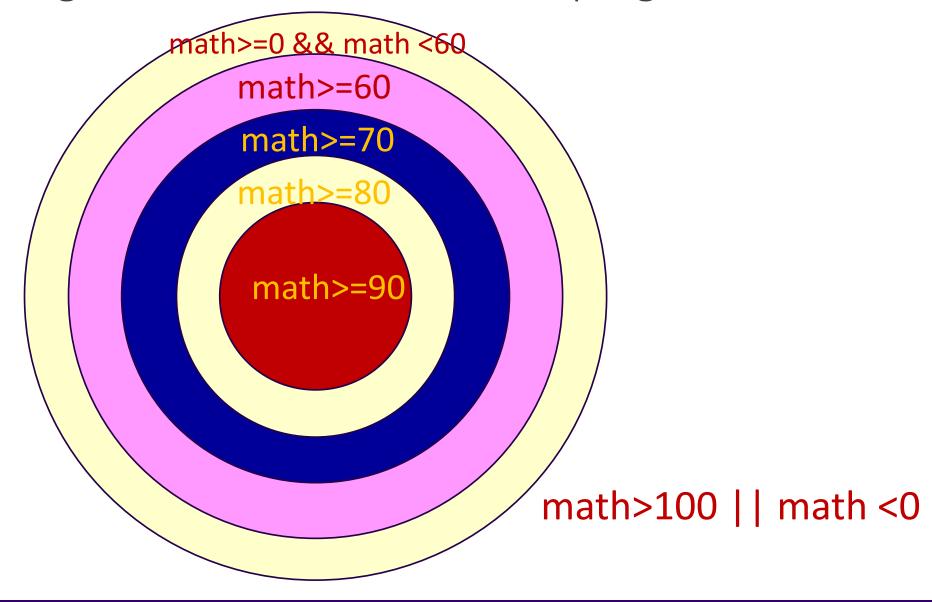
Smaller set go first:

```
if (math>=90) {mathGrade = 'A';}
else if (math>=80) {mathGrade = 'B';}
else if (math>=70) {mathGrade = 'C';}
else if (math>=60) {mathGrade = 'D';}
else {mathGrade = 'F';}
```

Corner Cases Handled:

```
if (math>=90 && math<=100)
{mathGrade = 'A';}
else if (math>=80 && math<=100)
{mathGrade = 'B';}
else if (math>=70 && math<=100)
{mathGrade = 'C';}
else if (math>=60 && math<=100)
{mathGrade = 'D';}
else if (math>= 0 && math<=100)
{mathGrade = 'F';}
else {mathGrade = 'N'; } // not graded yet
```

Venn Diagram for **StudentGPA** series programs





Set Theory and Logic Design (only positive numbers are shown)

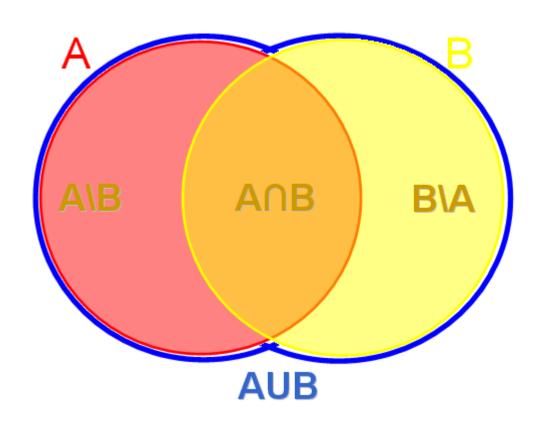
boolean m5 = (x % 5 == 0); // Set S5 = [0, 5, 10, 15, 20, ...]boolean m2 = (x % 2 == 0); // Set S2 = [0, 2, 4, 6, 8, 10, ...]

Boolean Expression	Set	Set Components
m5 && m2	<i>S</i> 5 ∩ <i>S</i> 2	[0, 10, 20, 30, 40,]
m5 m2	S5 U S2	[0, 2, 4, 5, 6, 8, 10, 12, 14, 15,]
m2 && !m5	S2 - S5	[2, 4, 6, 8, 12, 14, 16, 18,]
m5 && !m2	S5 - S2	[5, 15, 25, 35, 45,]
m2 ^ m5	<i>S</i> 5 ⊕ <i>S</i> 2	[2,4, 5, 6, 8, 12, 14, 15, 16,]



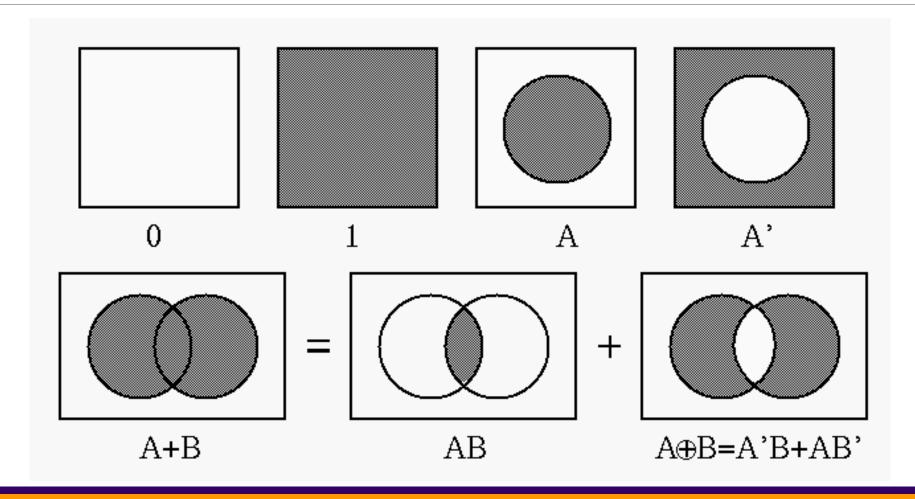


Venn Diagram Analysis





Venn Diagram Analysis



All 16 Boolean Functions can be expressed by (x, y, !, &&, ||)

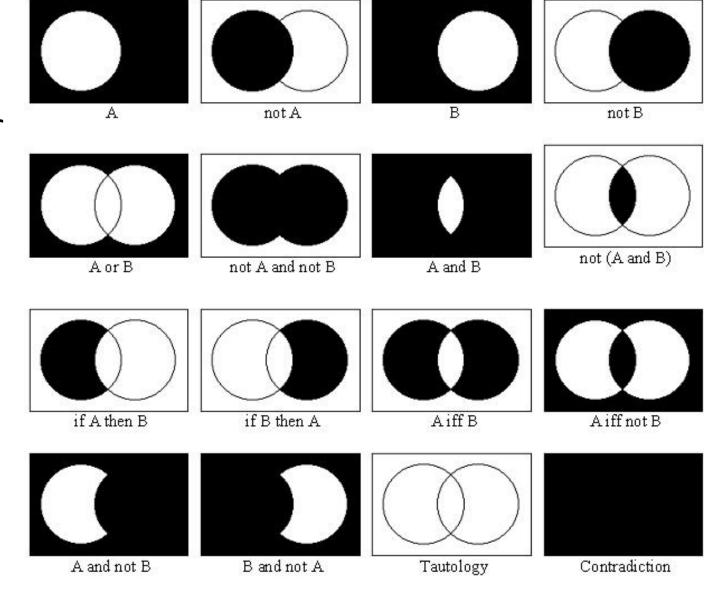
×	0	0	1	1
у	0	1.	0	1
0	0	0	0	0
х · у	0	0	0	1
x · y'	0	0	1	0
×	0	0	1	1
x'·y	0	1	0	0
у	0	1	0	1
x · y'+x' · y	0	1	1	0
x+y	0	1	1	1
(x+y)'	1	0	0	0
x · y+x' · y'	1	0	0	1
y'	1	0	1	0
x+y"	4	0	1	1
x'·y	1	1	0	0
x'+y	.1	1	0	1
(x · y)'	1	1	1	0
1	1	-1	1	1



All 16 Boolean Functions can be expressed by (x, y, !, &&, ||)

Boolean Functions in Java						
boolean f0 = false;	boolean f8 = !(x+y);					
boolean f1 = x && y;	boolean $f9 = x & y + !x & !y;$					
boolean f2 = x && !y;	boolean f10 = !y;					
boolean f3 = x;	boolean $f11 = x + !y;$					
boolean f4 = !x && y;	boolean f12 = !x && y;					
boolean f5 = y;	boolean $f13 = !x + y;$					
boolean f6 = x && !y + !x && y;	boolean f14 = !(x && y)					
boolean $f7 = x + y$;	boolean f15 = true;					

Advanced Venn Diagram for all 16 Logic Functions for Two Inputs *f(A, B)*





Mutual Exclusive Sets

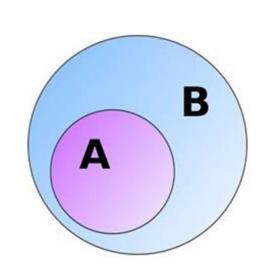
```
boolean male = (gender == 'M');
boolean female = (gender == 'F');
```

if (male) { // all male get here , and all female will not get here} if (female) { // all female get here, and all male will not get here}



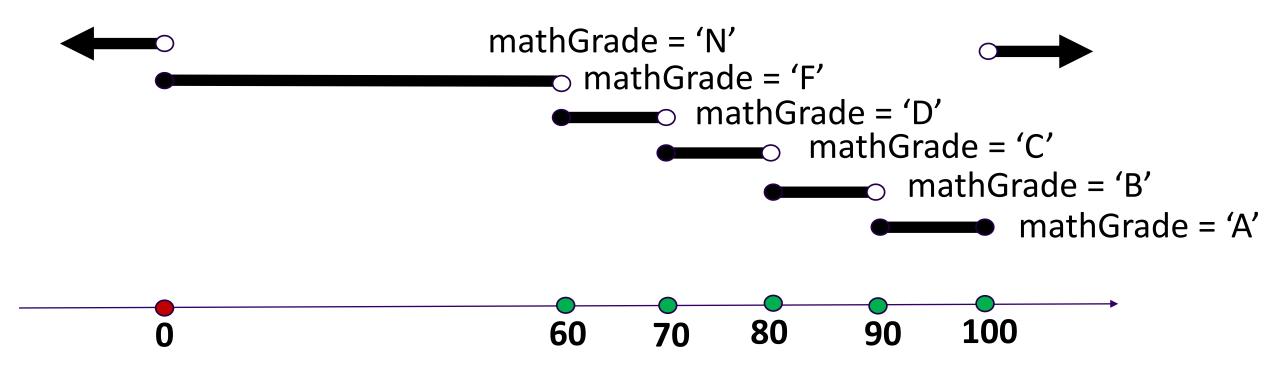
Set Contained by Another Set

```
Contained by (A == (x>2), B == (x>1))
if a is true then b must be true == !a || b
If x > 2 then x > 1 == !(x > 2) | | (x>1) (x>2) is contained by (x>1)
If you want to have A test first and B-A test:
     if (x > 2) { // all (x>2) get here }
     else if (x > 1) { // only x>1 && !(x>2) get here }
If you want to have A test and then B test:
     if (x>2) { // all x > 2 get here }
     if (x>1) { // all x > 1 get here }
Don't try this: if (x>1) { // all x>1 get here } else if (x>2) { // no x can get here}
```





Number Line Analysis (Letter Grade)



Sometimes the Logic Design Result Also Depends on the Data Type as Well



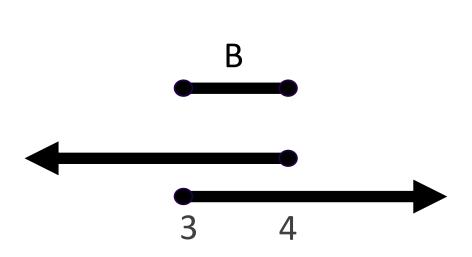
if
$$(x == 3 || x == 4) { // set A }$$

if $(x >= 3 && x <= 4) { // set B }$

For int data type, A is equivalent to B

For double data type, A is not equivalent to B

for example, x = 3.5 is in B but not in A





has Even, All Even

LECTURE 3



isEven – Full Logic

```
public static boolean isEven1(int n) {
    if (n%2==0) {
        return true;
    }
    else {
        return false;
    }
}
```



isEven – Exit by Return

```
public static boolean isEven2(int n){
    if (n%2==0) {
        return true;
    }
    // n%2!=0 after this line
    return false;
}
```



isEven – Exit by Return

```
public static boolean isEven3(int n) {
    return n%2==0;
}
```



Logic (Quantifier)

- • \forall x, x < 3 \rightarrow for all x, x < 3
- • $\exists x, x < 3 \rightarrow \text{Exists } x, x < 3$



Logic (Quantifier)

```
• \forall x, x % 2 == 0 → for all x, x is even

\neg \forall x, x % 2 == 0 → Not( for all x, x is even)

→ Exist x, x is odd

→ \exists x, x % 2 != 0
```



Logic (Quantifier)

- • $\exists x, x \% 2 == 0 \rightarrow Exists x, x is even$
- • \neg ($\exists x, x \% 2 == 0$) \rightarrow not (Exists x, x is even)
 - \rightarrow for all x, x is odd
 - $\rightarrow \forall x, x \% 2 != 0$

```
static int[] a = {1, 2, 3, 4, 5};
static int[] b = {1, 3, 5, 7, 9};
static int[] c = {2, 4, 6, 8, 10};
public static boolean allEven(int[] x){
 for (int i=0; i<x.length; i++){
     if (x[i]%2 !=0) return false;
 return true;
public static boolean allOdd(int[] x){
 for (int i=0; i<x.length; i++){
     if (x[i]\%2 ==0) return false;
 return true;
public static boolean hasEven(int[] x){
 for (int i=0; i<x.length; i++){
     if (x[i]%2 ==0) return true;
 return false;
public static boolean hasOdd(int[] x){
 for (int i=0; i<x.length; i++){
     if (x[i]%2 !=0) return true;
 return false;
```

```
public static void main(String[] args){
  System.out.print("\f");
  System.out.printf("a is allEven is %b\n", allEven(a));
  System.out.printf("a is allOdd is %b\n", allOdd(a));
  System.out.printf("a is hasEven is %b\n", hasEven(a));
  System.out.printf("a is hasOdd is %b\n", hasOdd(a));
  System.out.println();
  System.out.printf("b is allEven is %b\n", allEven(b));
  System.out.printf("b is allOdd is %b\n", allOdd(b));
  System.out.printf("b is hasEven is %b\n", hasEven(b));
  System.out.printf("b is hasOdd is %b\n", hasOdd(b));
  System.out.println();
  System.out.printf("c is allEven is %b\n", allEven(c));
  System.out.printf("c is allOdd is %b\n", allOdd(c));
  System.out.printf("c is hasEven is %b\n", hasEven(c));
  System.out.printf("c is hasOdd is %b\n", hasOdd(c));
  System.out.println();
```

```
a is allEven is false
a is allOdd is false
a is hasEven is true
a is hasOdd is true
b is allEven is false
b is allOdd is true
```

b is allodd is true
b is hasEven is false
b is hasOdd is true

c is allEven is true
c is allOdd is false
c is hasEven is true
c is hasOdd is false



Demonstration Program

ALLEVEN.JAVA



Calendar Year

LECTURE 4



Lab

CALENDAR YEAR

LEAPYEAR.JAVA



Background Information: (Leap Year)

- •Which Years are Leap Years?
- •In the Gregorian calendar 3 criteria must be taken into account to identify leap years:
- The year is evenly divisible by 4;
- If the year can be evenly divided by 100, it is NOT a leap year, unless;
- The year is also evenly divisible by 400. Then it is a leap year.
- •The year <u>2000</u> was somewhat special as it was the first instance when the third criterion was used in most parts of the world since the transition from the <u>Julian</u> to the <u>Gregorian Calendar</u>.



Year of the Rat

























Background Information:

(Chinese Zodiac)

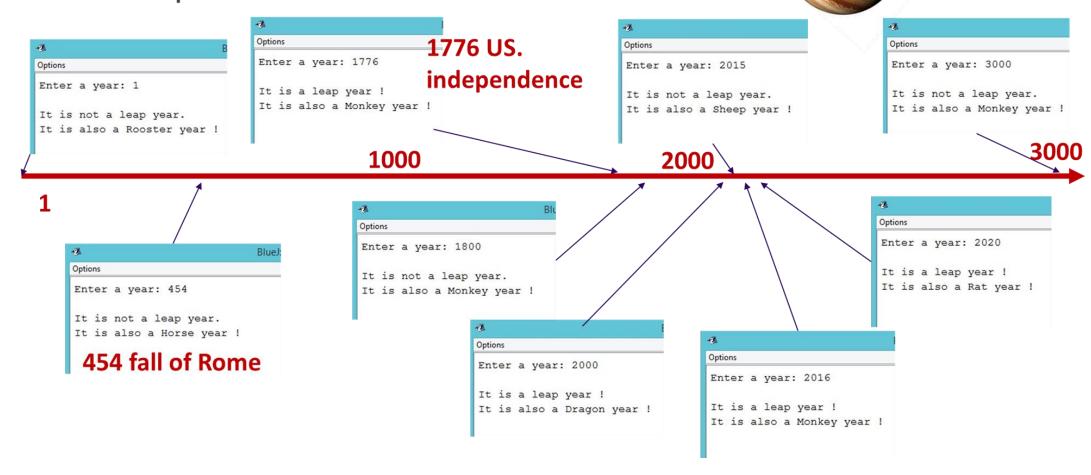
- •Jupiter goes around the sun every 11.87 years. (approximately 12 years.) Chinese call it *Planet of Years*. They use it to calculate for the Zodiac. Every 12 years is one rotation.
- •The year 1948 is year of Rat. The year 1950 is year of Tiger. The year 2015 is year of Sheep (sometimes also called goat/ram. Chinese believe they are in one category.)
- •If the year 2015 is 67 (2015-1948) years away from 1948 and 67 % 12 is 7, 7 years away from rat year is year of Sheep.
- •So, you may use y = (x-1948) % 12 to know that Year x is y years away from Rat year. Then, look up from the table.

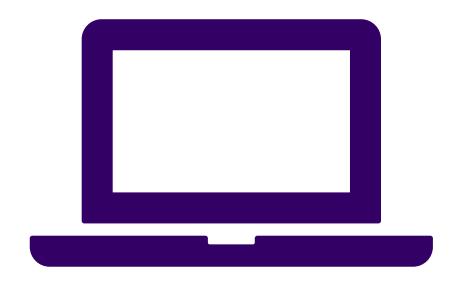


Lab: Calendar Year (LeapYear.java)

- •Write a program to ask the calendar year between 1948 and now, to determine 2 things:
 - (1) Is it a leap year?
 - (2) What Chinese Zodiac Year it is?
- •Then, Print out a Calendar Year Report with these information.

Expected Result:





Demonstration Program

CHINESEZODIAC. JAVA



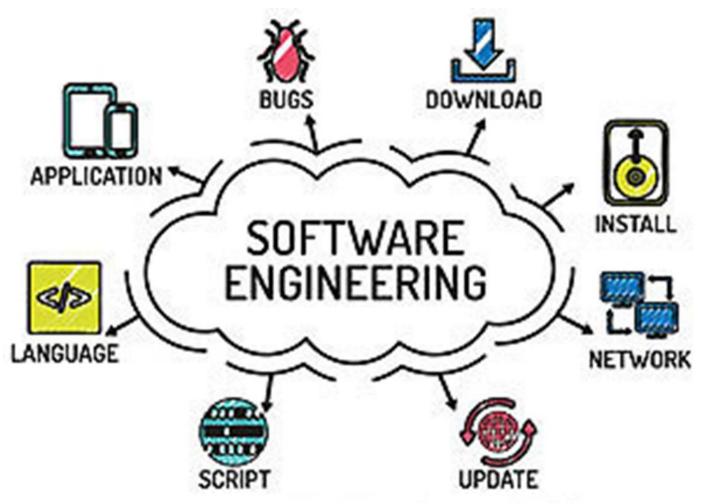
Software Design Life Cycle

LECTURE 5



Software Engineering

- •Software engineering is the branch of computer science that deals with the design, development, testing, and maintenance of software applications.
- •Software engineers apply engineering principles and knowledge of programming languages to build software solutions for end users.

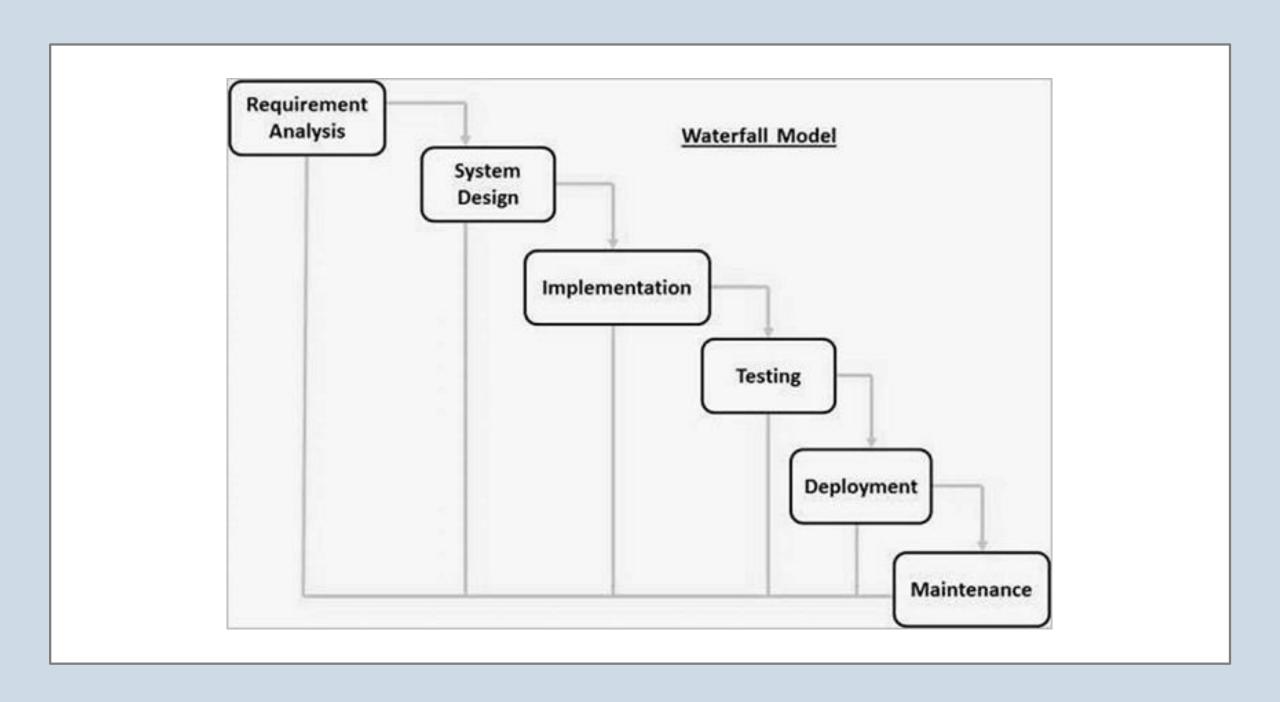


ComputerHope.com



Software Design Life Cycle

- •The Software Development Life Cycle (SDLC) refers to a methodology with clearly defined processes for creating high-quality software. in detail, the SDLC methodology focuses on the following phases of software development:
 - Requirement analysis
 - Planning
 - Software design such as architectural design
 - Software development
 - Testing
 - Deployment



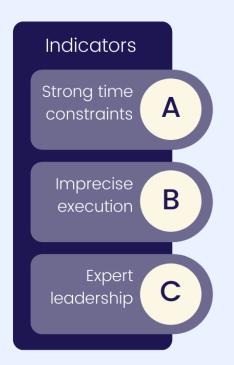


Top-down versus Bottom Up

- •Top down approach starts with the big picture, then breaks down from there into smaller segments.
- •A bottom-up approach is the piecing together of systems to give rise to more complex systems, thus making the original systems subsystems of the emergent system.

Top Down Approach



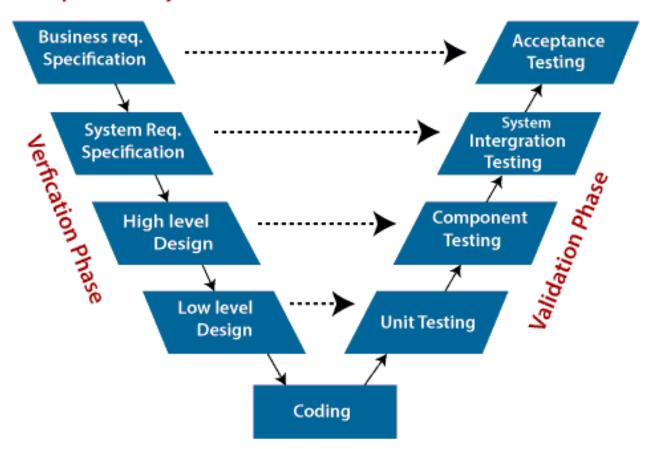




V- Model

Developer's life Cycle

Tester's Life Cycle



Top-Down Design and Bottom-Up Implementation



Pseudo Code

LECTURE 6



What is pseudo code?

Simple definition:

Use **English** or **English-like** language to describe a program for a computation.

Formal Definition:

<u>Pseudocode</u> is an **artificial** and **informal** language that helps programmers develop algorithms. **Pseudocode** is a "text-based" detail (algorithmic) design tool.

The rules of Pseudocode are reasonably straightforward. All statements showing "dependency" are to be indented. These include while, do, for, if, switch.



Pseudo Code Standard

http://www.engr.sjsu.edu/bjfurman/courses/ME30/ME30pdf/Notes_on_Algorithms.pdf

It does not need a standard because it is free style.

But if you need some reference, this one can be a good reference source.



Method for Developing an Algorithm

- 1. **Define the problem**: State the problem you are trying to solve in clear and concise terms.
- 2. List the inputs (information needed to solve the problem) and the outputs (what the algorithm will produce as a result)
- 3. **Describe the steps** needed to convert or manipulate the inputs to produce the outputs. Start at a high level first, and keep refining the steps until they are effectively computable operations.
- 4. **Test the algorithm**: choose data sets and verify that your algorithm works!



Pseudocode (or Program Design Language)

- Consists of natural language-like statements that precisely describe the steps of an algorithm or program
- Statements describe actions
- Focuses on the logic of the algorithm or program
- Avoids language-specific elements
- Written at a level so that the desired programming code can be generated almost automatically from each statement.
- Steps are numbered. Subordinate numbers and/or indentation are used for dependent statements in selection and repetition structures.



- Computation/Assignment
 - Compute var1 as the sum of x and y
 - Assign expression to var2
 - Increment counter1

- Input/Output
 - Input: Get var1, var2, ...
 - Output: Display var1, var2, ...



• <u>Selection</u>

Single-Selection IF

- 1. IF condition THEN (IF condition is true, then do subordinate statement 1, etc. If condition is false, then skip statements)
 - 1.1 statement 1
 - 1.2 etc.



Double-Selection IF

- 2. IF condition THEN (IF condition is true, then do subordinate statement 1, then skip statements and execute statements under ELSE)
 - 2.1 statement 1
 - 2.2 etc.
 - 3. **ELSE** (else if condition is not true, then do subordinate statement 2, etc.)
 - 3.1 statement 2
 - 3.2 statement 3
 - 4. **SWITCH** expression **TO**
 - 4.1 case 1: action1
 - 4.2 case 2: action2
 - 4.3 etc.
 - 4.4 default: actionx

etc. If condition is false,



- Repetition
 - 5. WHILE condition (while condition is true, then do subordinate statements)
 - 5.1 statement 1
 - 5.2 etc.
 - DO WHILE structure (like WHILE, but tests condition at the end of the loop. Thus, statements in the structure will always be executed at least once.)
 - 6. **DO**
 - 6.1 statement 1
 - 6.2 etc.
 - 7. WHILE condition



FOR structure (a specialized version of WHILE for repeating execution of statements a specific number of times)

- 8. FOR bounds on repetition
 - 8.1 statement 1
 - 8.2 etc.





To find maximum from a group of data:

set the maximum to one of the element.

each time compare the maximum to one of the rest of elements in the group.

If the element picked is larger than our maximum

Then, set the element to be the new maximum

look at the next element to repeat the comparison

After all of the elements have been compared, then the maximum is found.



Pseudocode Example 2:

(in your own language. This is only an example.)

```
FIND MAX (ARRAY X):
                                 ; program start
 SET MAXIMUM to the first element of X.
                                                ; set initial condition
 WHILE i < X's length THEN
                                                ; iterate through X array
   BEGIN
      IF X[i] > MAXIMUM THEN
                                                ; update maximum if
          ASSIGN X[i] to MAXIMUM
                                                ; a greater element found
                                               ; find next element
      INCREMENT i
   END
 RETURN MAXIMUM
                                               ; return maximum
```





Personal Experience

- I personally just use C/C++ or Java-like language as pseudo language to describe a program if it is not coded yet.
- Java is a good candidate for pseudo language as well.



Homework (Honor code)

Write a pseudo code to perform a task of

How to prepare breakfast with:

English Muffins

Bacon

A Fried Egg

Orange Juice

Fruit Cup



Print Calendar

LECTURE 7

AUGUST 2023

SUN	MON	TUE	WED	THU	FRI	SAT
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31	1	2 Hememade



Top Level Analysis

•Input: Month, Year

Output: Monthly Calendar for the specific month.



Unit Project: printCalendar.java

Write a program to print out a monthly calendar from any month after Jan. 1800. (Wednesday).

Using the top down design and bottom up implementation guidelines from the previous two lectures.



Expected Result: (Jan. 2016)

Sample Answer Program: printCalendar.java

```
70
                                   BlueJ: Terminal Window - Unit2
Options
Enter full year (e.g., 2016): 2016
Enter month in number between 1 and 12: 1
          January 2016
 Sun Mon Tue Wed Thu Fri Sat
  31
```

