AP Computer Science B
Java Object-Oriented Programming [Ver. 2.0]

Unit 5: Algorithm Study

WEEK 13: CHAPTER 17 RECURSION (PART 1: RECURSIVE DESIGN)

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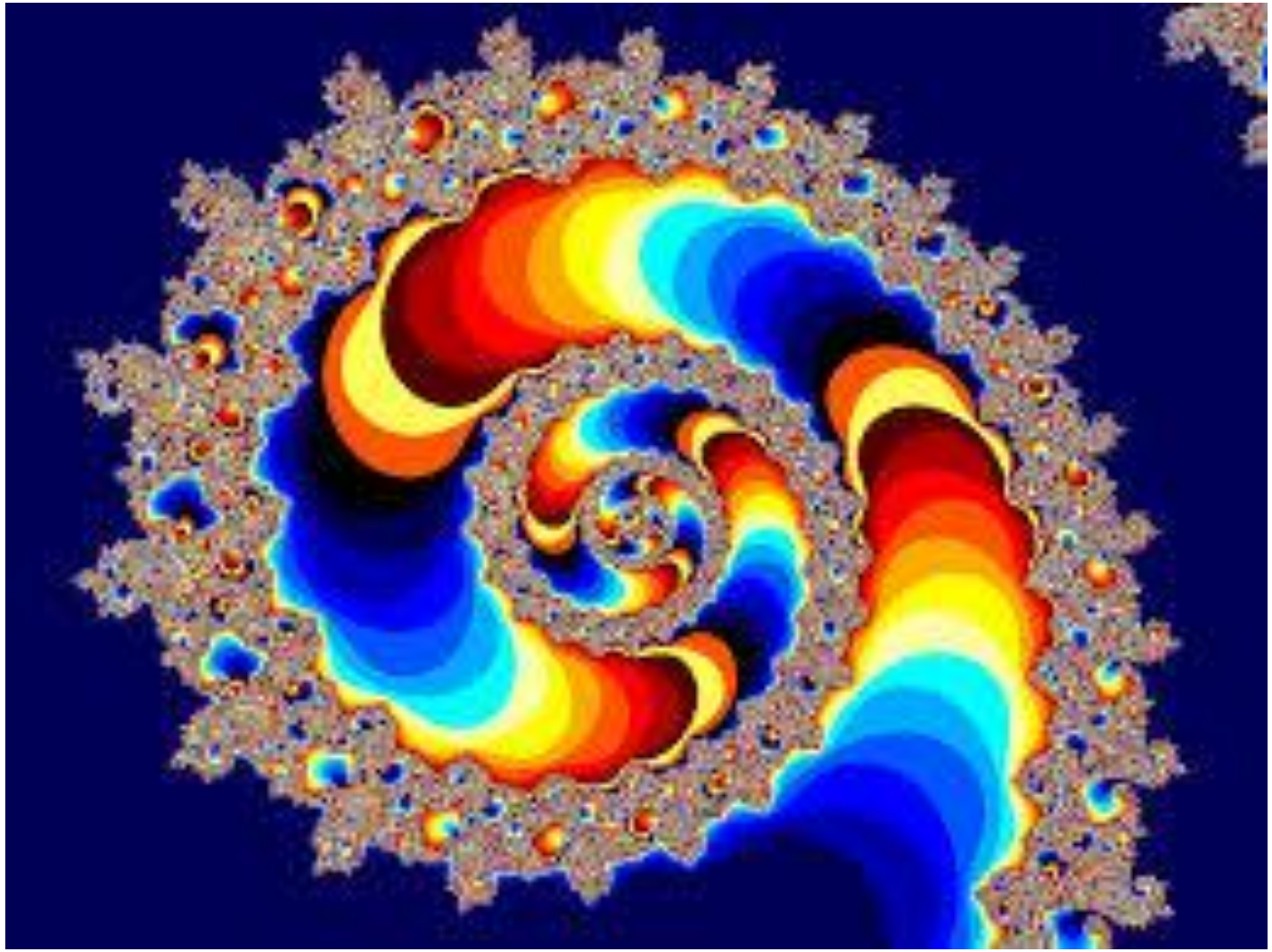
Objectives

- What is recursion?
- Computing Factorials and Tail Recursion
- Fibonacci Numbers

Real World Recursive Problems and Reading Recursion

LECTURE 1

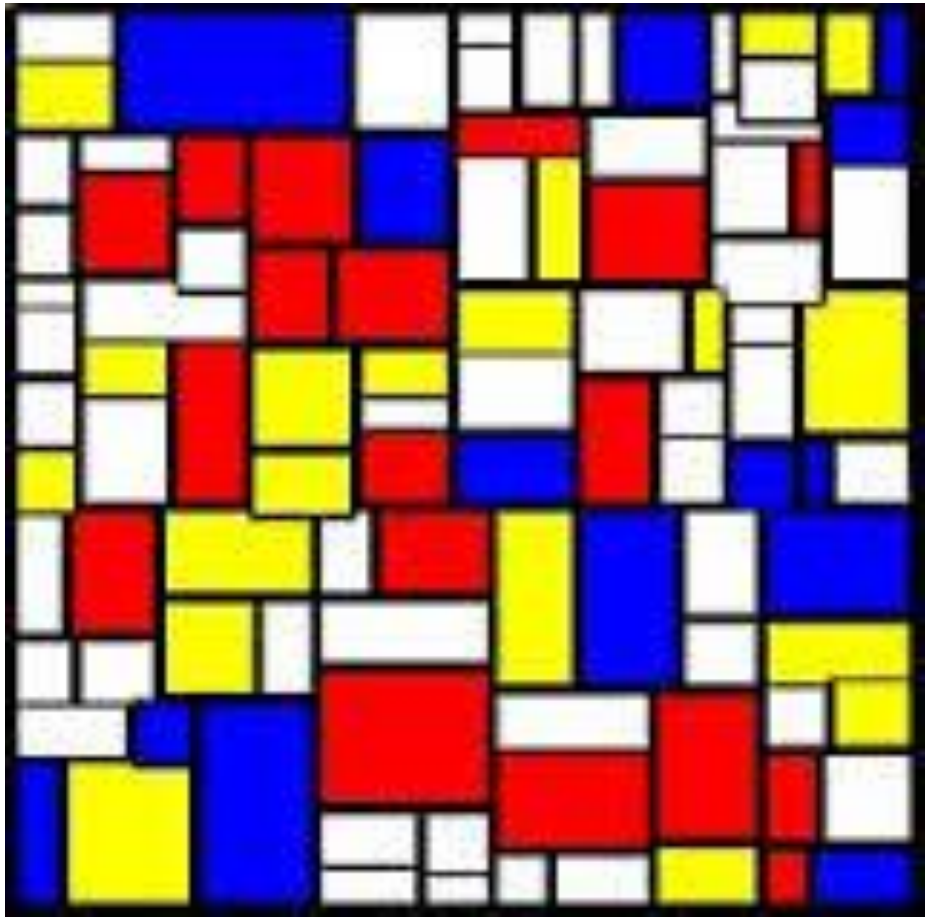
Mirror in Mirror

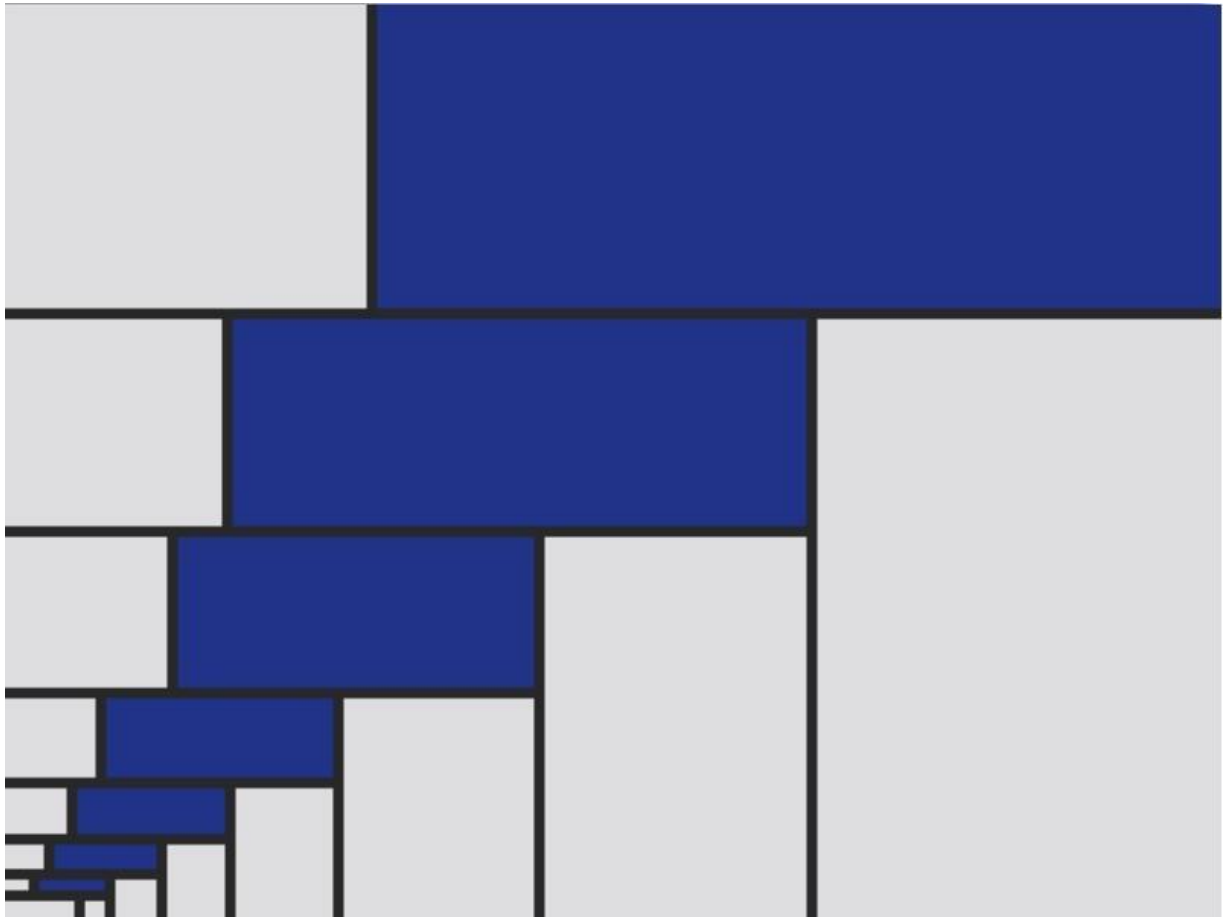


Fractals (Computer Art)

Mondrimat Arts

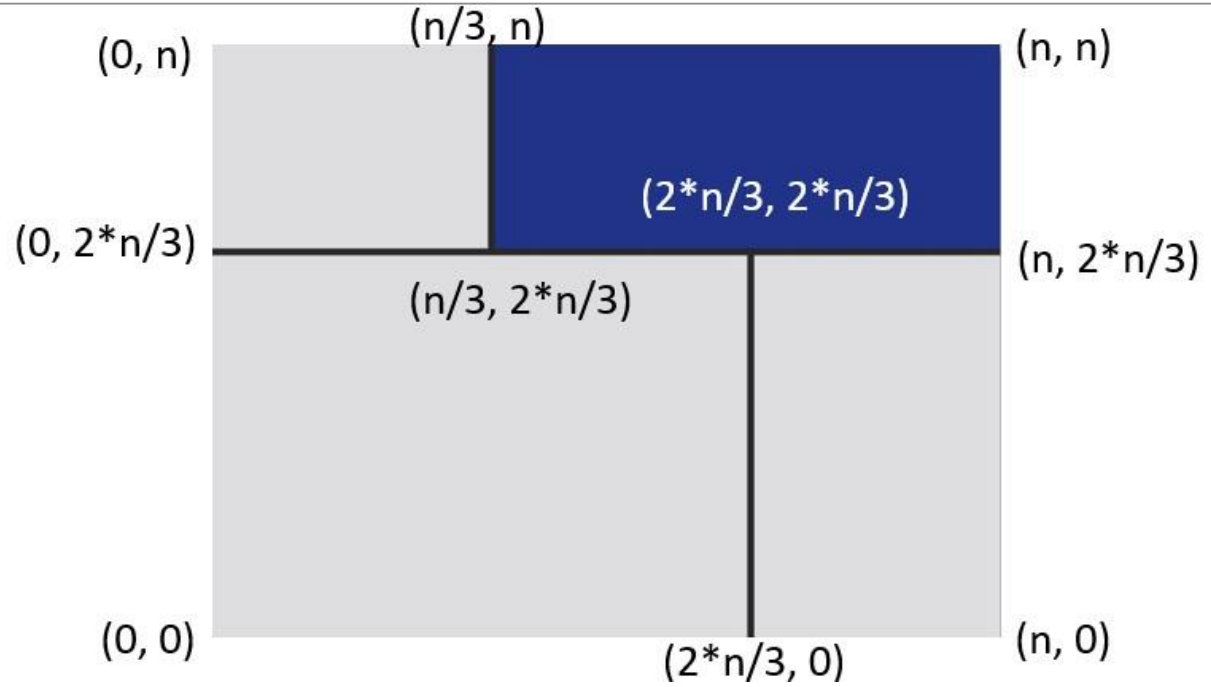
http://www.stephen.com/mondrimat/



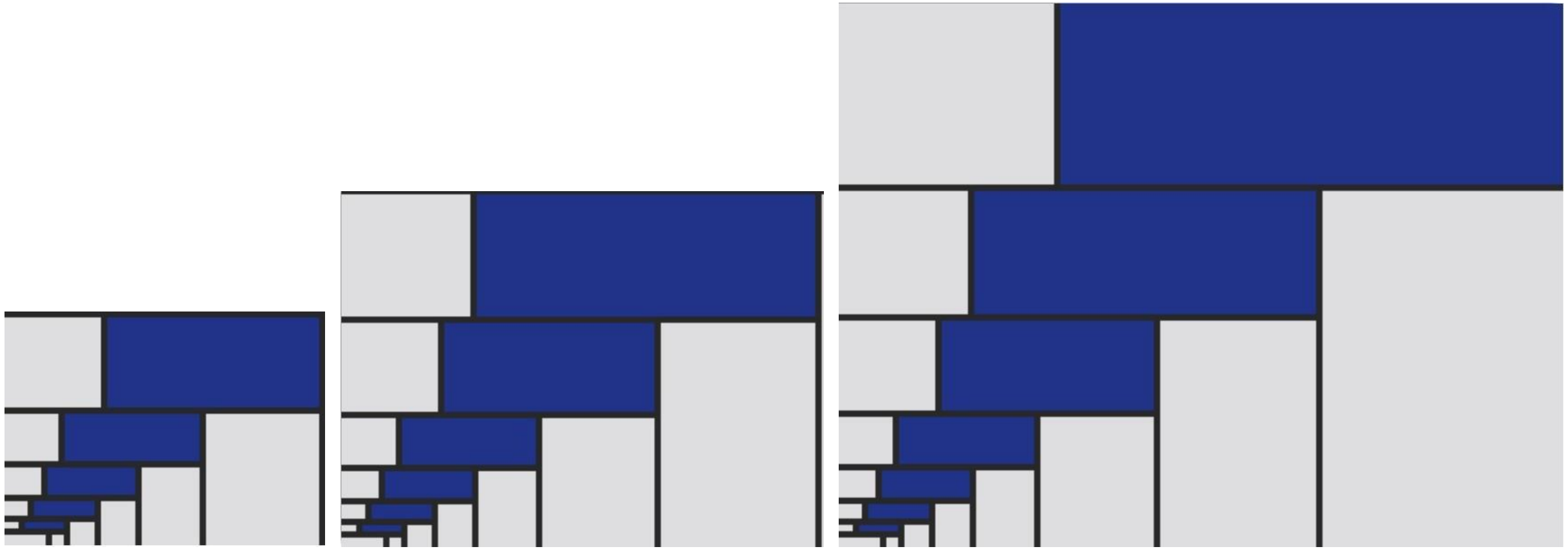


Reading the Recursive Pattern

draw(int n) { /* will draw the following pattern for one recursion call. */ }



Reading the Recursive Pattern



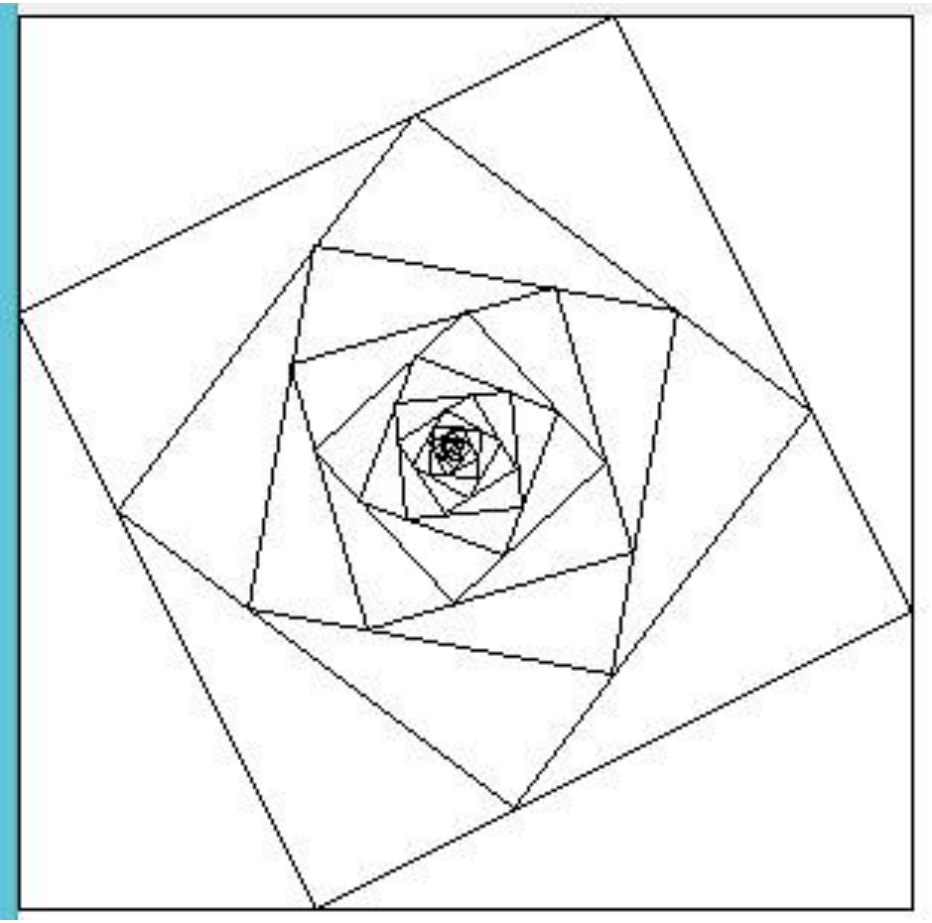
Three Principles in Recursion

(1) Divide and Conquer

(2) Recursive Function

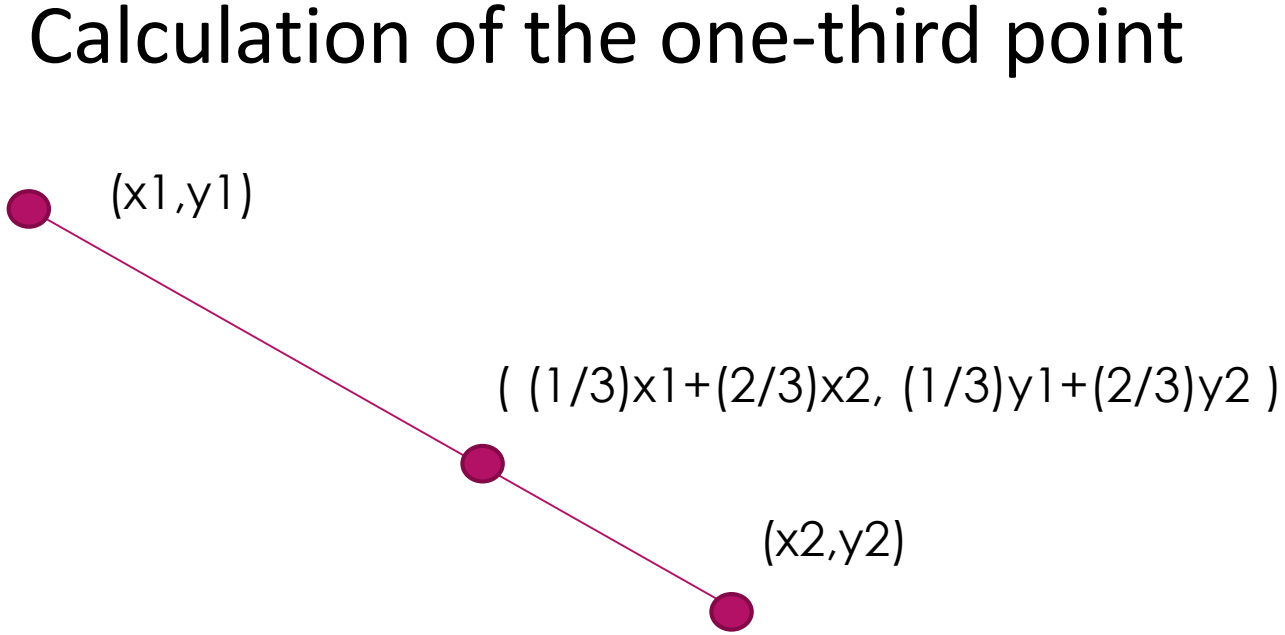
(3) Stop Condition (Otherwise, it will never stop.)

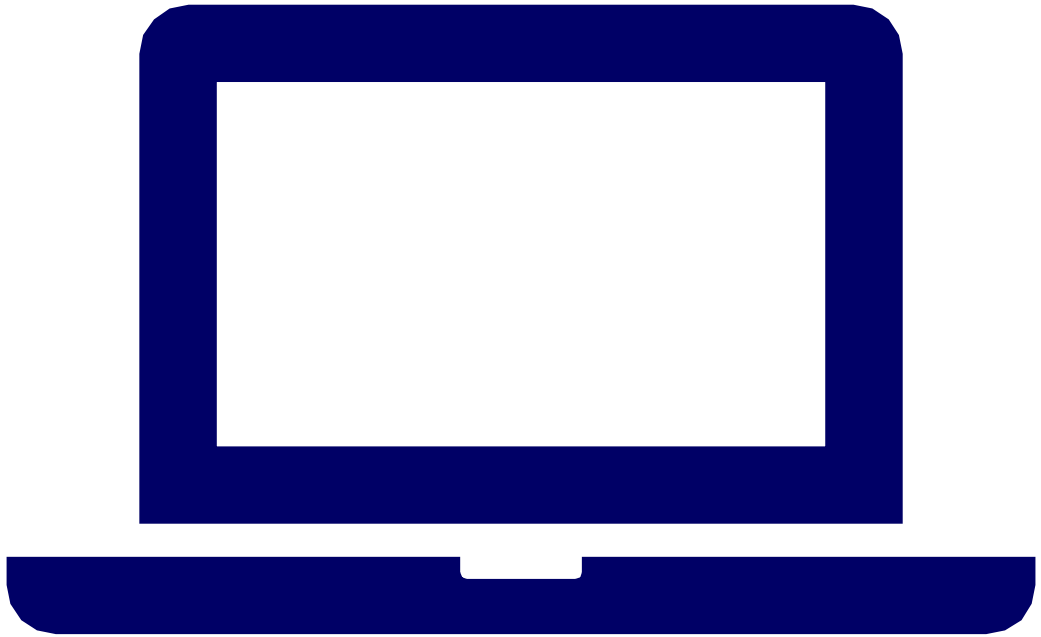
Demo Program: RecursiveRectangle.java



- This recursive rectangle project requires you to draw rectangles recursively on the one-third points of the line segments as shown on the left.
- After finishing a rectangle, you need to draw smaller rectangles based on the one-third points of the previous rectangle's line segments. (Recursion)
- Until the line segment's length is less than square root of ten, which also means the square of the line segment's length is less than 10. Then, you stop the recursion. (Stop Condition)
- Your results should look like the picture on the left side.

Demo Program: RecursiveRectangle.java





Demonstration Program

RECURSIVERECTANGLE.JAVA

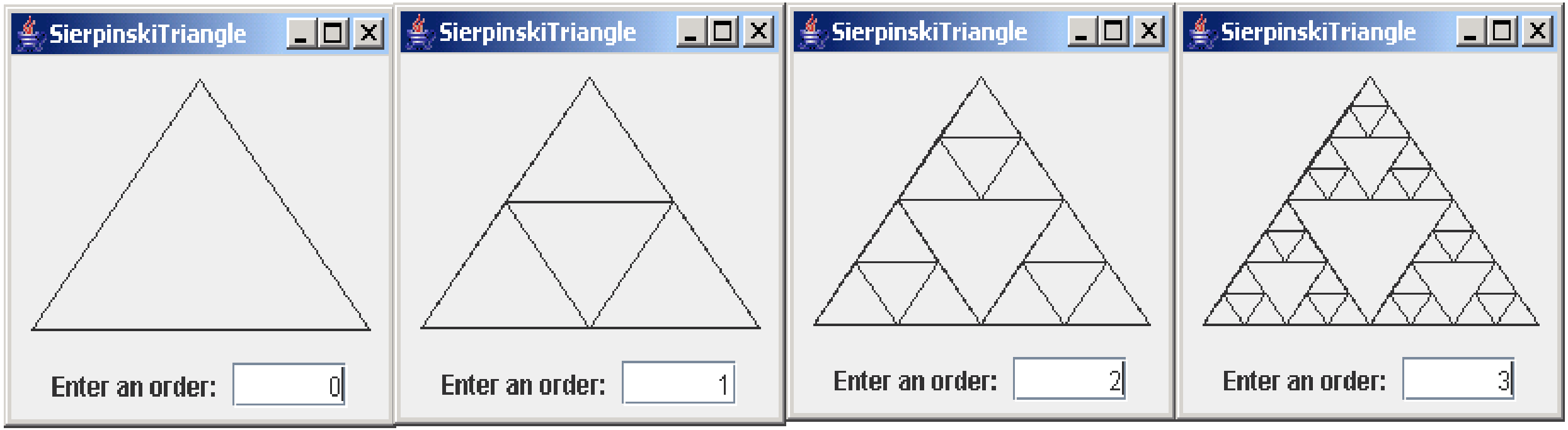
Fractals?

A fractal is a geometrical figure just like triangles, circles, and rectangles, but fractals can be divided into parts, each of which is a reduced-size copy of the whole. There are many interesting examples of fractals. This section introduces a simple fractal, called *Sierpinski triangle*, named after a famous Polish mathematician.

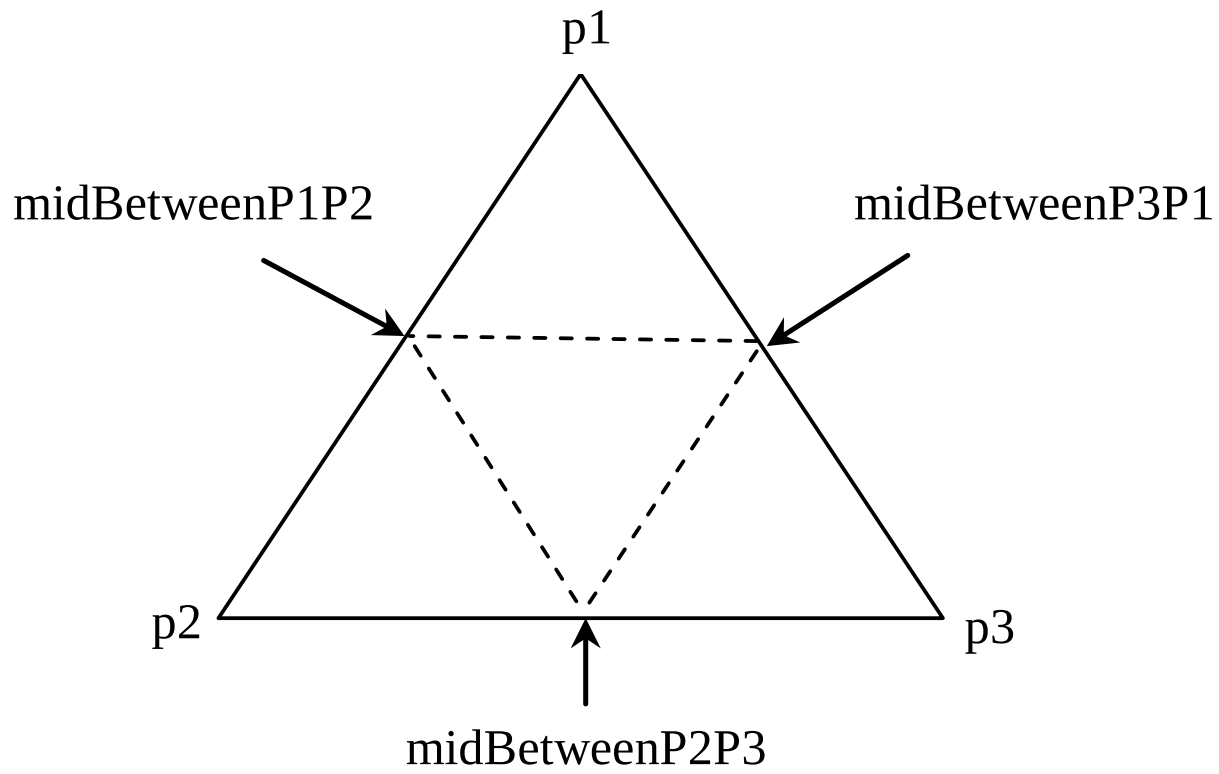
Sierpinski Triangle

- It begins with an equilateral triangle, which is considered to be the Sierpinski fractal of order (or level) 0, as shown in Figure (a).
- Connect the midpoints of the sides of the triangle of order 0 to create a Sierpinski triangle of order 1, as shown in Figure (b).
- Leave the center triangle intact. Connect the midpoints of the sides of the three other triangles to create a Sierpinski of order 2, as shown in Figure (c).
- You can repeat the same process recursively to create a Sierpinski triangle of order 3, 4, ..., and so on, as shown in Figure (d).

Recursion

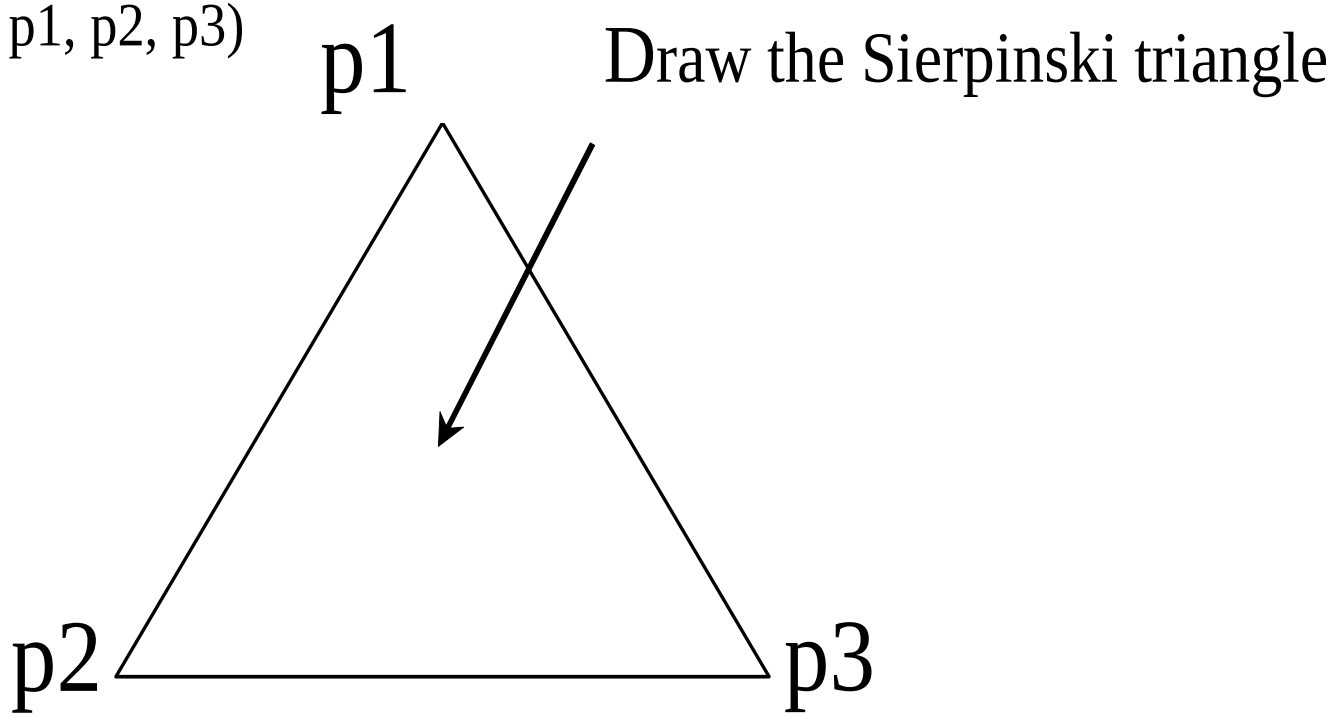

SierpinskiTriangle
Enter an order: 0
SierpinskiTriangle
Enter an order: 1
SierpinskiTriangle
Enter an order: 2
SierpinskiTriangle
Enter an order: 3

Sierpinski Triangle Solution



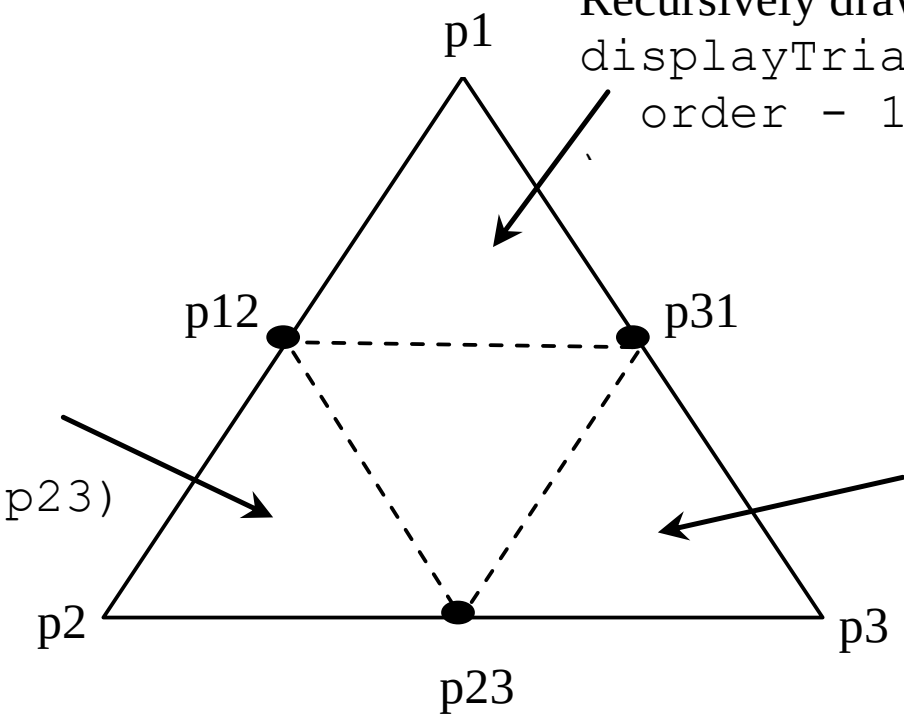
Sierpinski Triangle Solution

displayTriangles(g, order, p1, p2, p3)



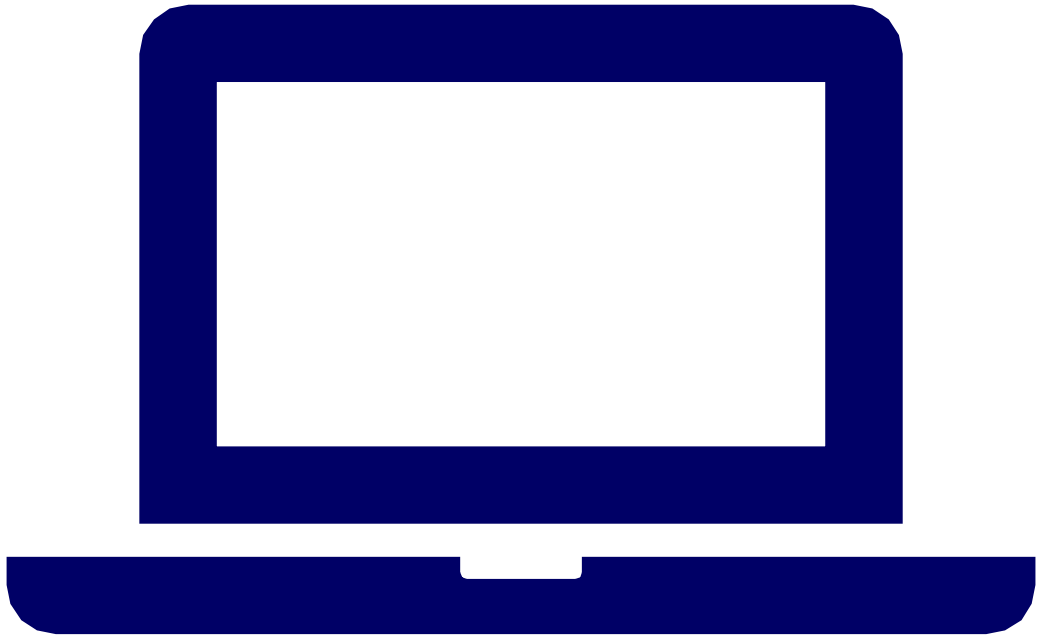
Sierpinski Triangle Solution

Recursively draw the small Sierpinski triangle
`displayTriangles(g, order - 1, p1, p12, p31)`



Recursively draw the small Sierpinski triangle
`displayTriangles(g, order - 1, p12, p2, p23)`

Recursively draw the small Sierpinski triangle
`displayTriangles(g, order - 1, p31, p23, p3)`



Demonstration Program

SIERPINSKITRIANGLES.JAVA

Basic Recursion

LECTURE 1

Recursion

- Base Condition (Termination Condition)
- Recursive Structure
  - Divide and Conquer: Split a big problem to smaller ones
  - Incremental Design: explore the relationship between small ones and the large one
  - Recursive Formula: describe the recursive relationship into Java code

Countdown Recursion

- Call a recursive function with a counter.
- The counter decreases for each sub-case.
- When counter reaches 0, the rocket blasts off.

Demo Program:

Rocket.java

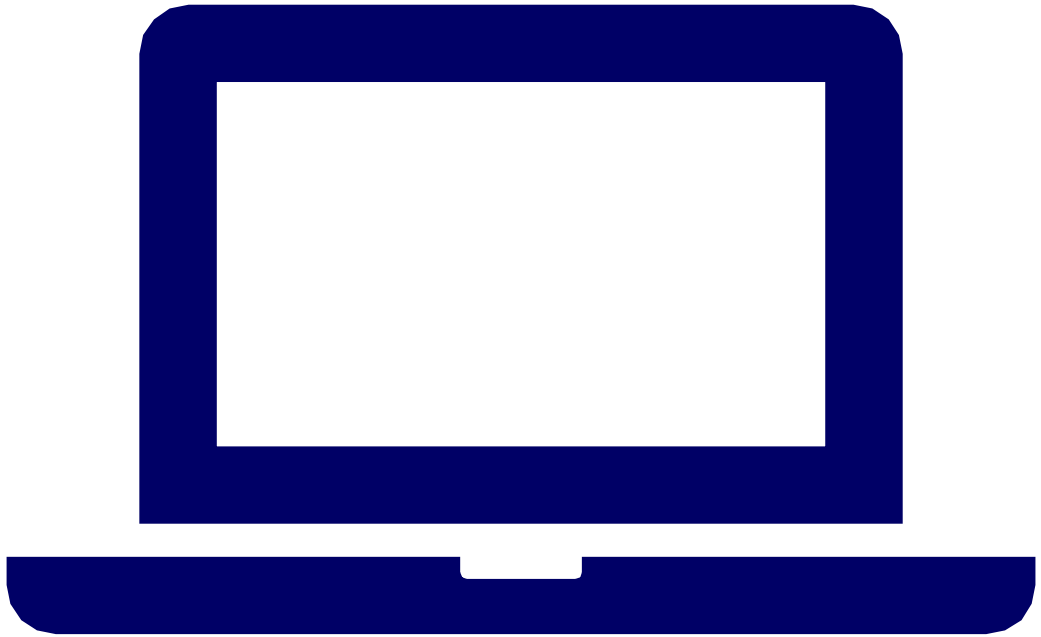
```
public class Rocket
{
    public static void countDown(int n){
        if (n==0) { System.out.println("Blast off!!!"); return; }
        System.out.println(n);
        countDown(n-1);
    }

    public static void main(String[] args){
        countDown(10);
    }
}
```

Base Condition

Incremental Design

Recursive Call



Demonstration Program

ROCKET.JAVA

Countdown Recursion with Return Value

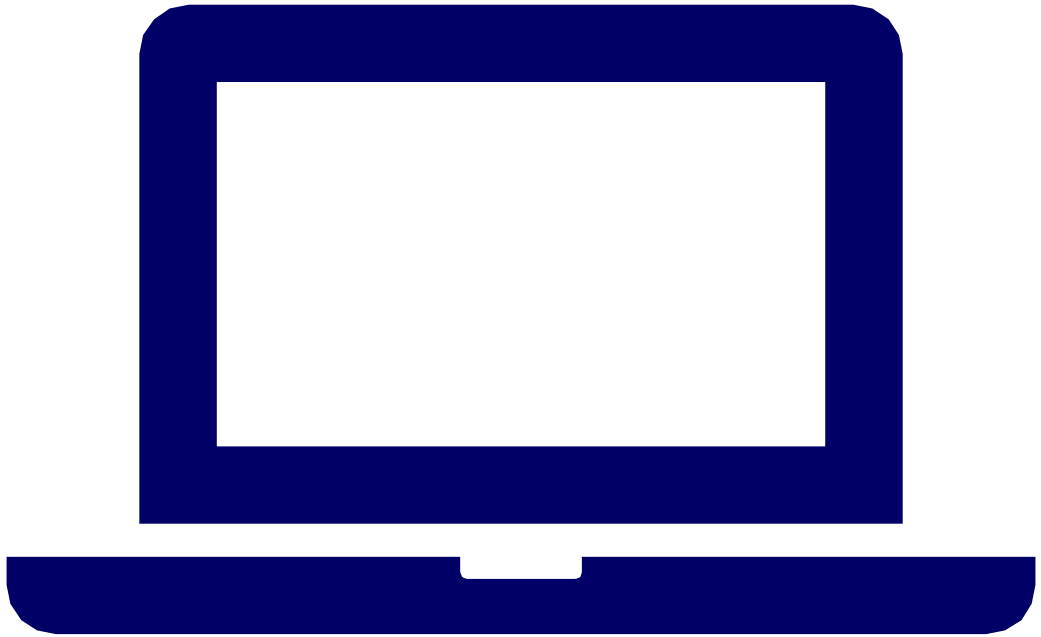
- Call a recursive function with a counter.
- The counter decreases for each sub-case.
- When counter reaches 0, the base case return a value.
- Each bigger case receive a return value and returning another value.

Demo Program:

RecursiveSum.java

```
public class RecursiveSum
{
    public static int sum(int n){
      if (n==0) return 0;
      return sum(n-1)+n;
    }

    public static void main(String[] args){
      System.out.print("\f");
      System.out.println(sum(10));
    }
}
```



Demonstration Program

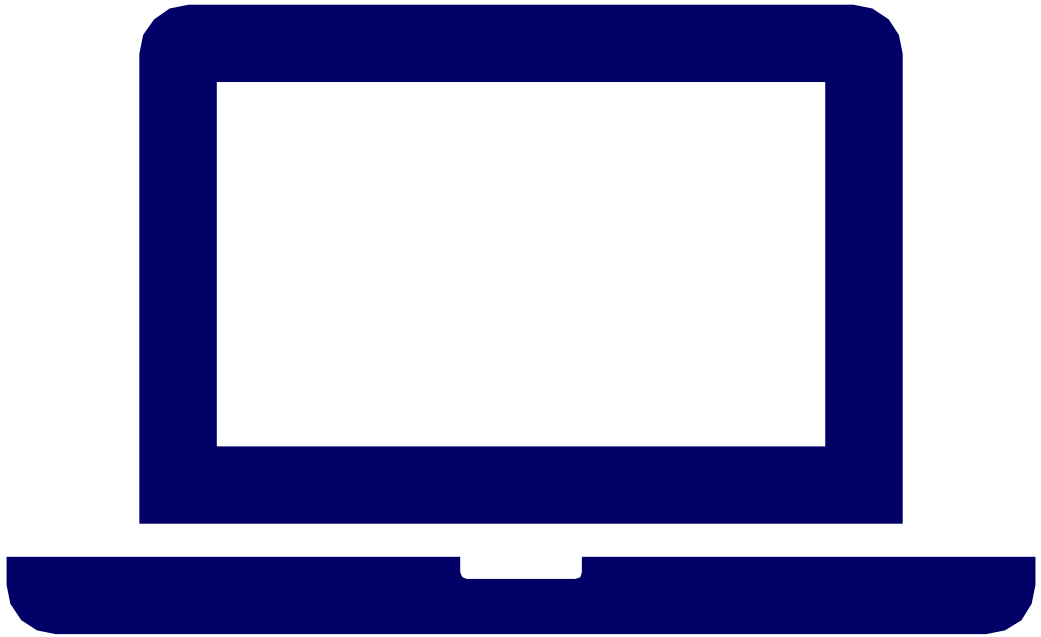
RECURSIVESUM.JAVA

Demo Program:

RecursiveProduct.java

```
public class RecursiveProduct
{
    public static int product(int n){
        if (n==0) return 1;
        return product(n-1)*n;
    }

    public static void main(String[] args){
        System.out.print("\f");
        System.out.println(product(5));
    }
}
```

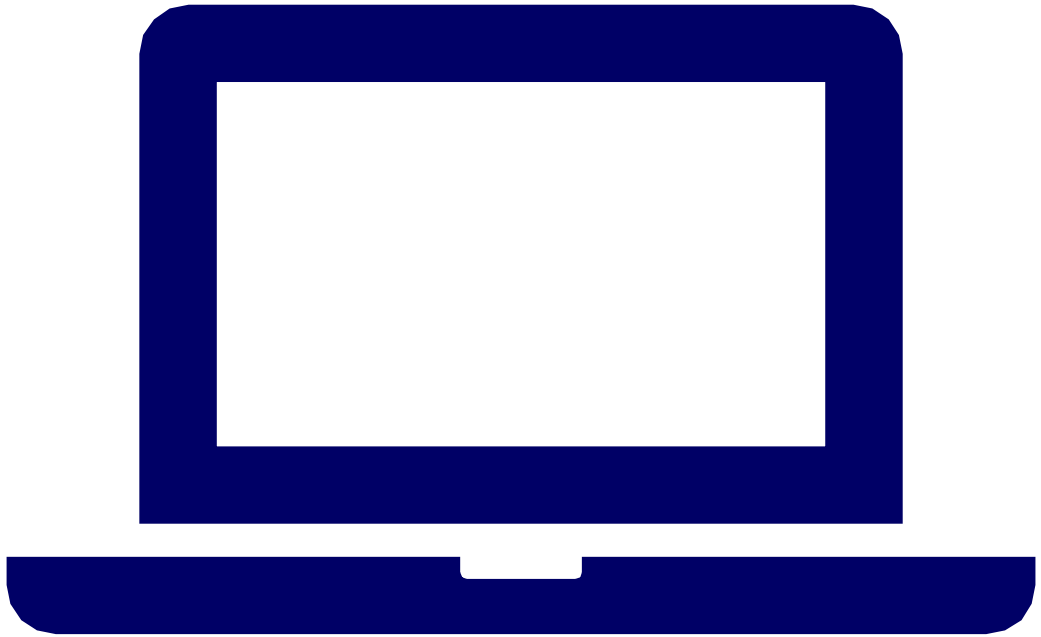


Demonstration Program

RECURSIVEPRODUCT.JAVA

Recursion with Accumulator

- Call a recursive function with an accumulator as a parameter.
- In the base case, the accumulator is returned.
- Each recursive call, the accumulator is updated (accumulates).



Demonstration Program

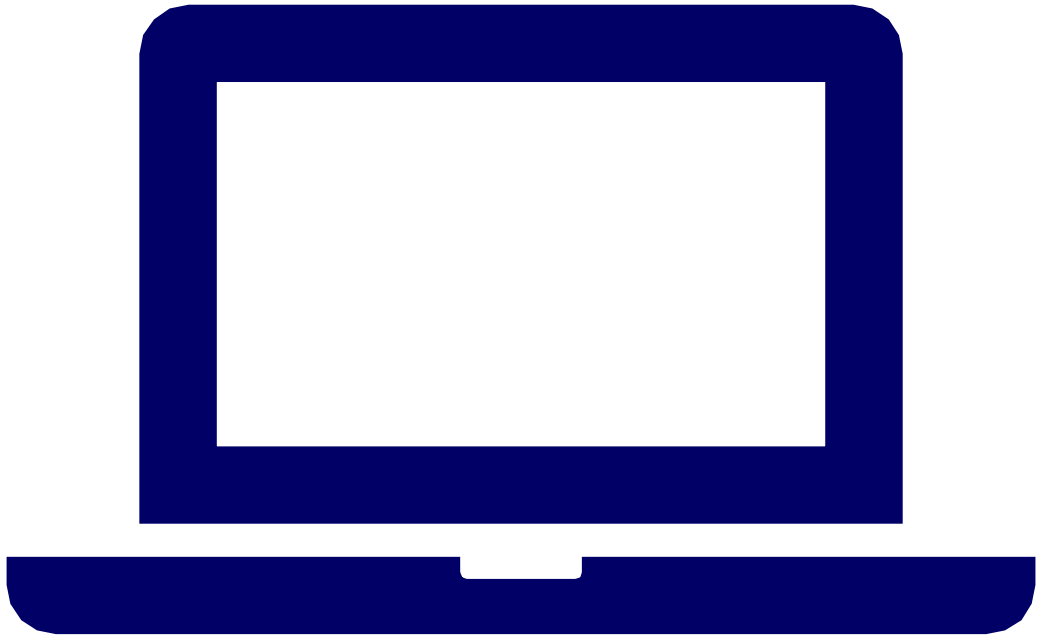
ACCUMULATEDSUM.JAVA

Demo Program:

AccumulatedSum.java

```
public class AccumulatedSum
{
    public static int sum(int n, int s){
        if (n==0) return s;
        return sum(n-1, s+n);
    }

    public static void main(String[] args){
        System.out.print("\f");
        System.out.println(sum(10, 0));
    }
}
```



Demonstration Program

ACCUMULATEDPRODUCT.JAVA

Demo Program:

AccumulatedProduct.java

```
public class AccumultedProduct
{
    public static int product(int n, int s){
        if (n==0) return s;
        return product(n-1, s*n);
     }

     public static void main(String[] args){
        System.out.print("\f");
        System.out.println(product(5, 1));
     }
}
```

Why with accumulator?

- Tail recursion.
- More efficient than normal recursive function.
- As a recursive helper function.

Tail Recursion

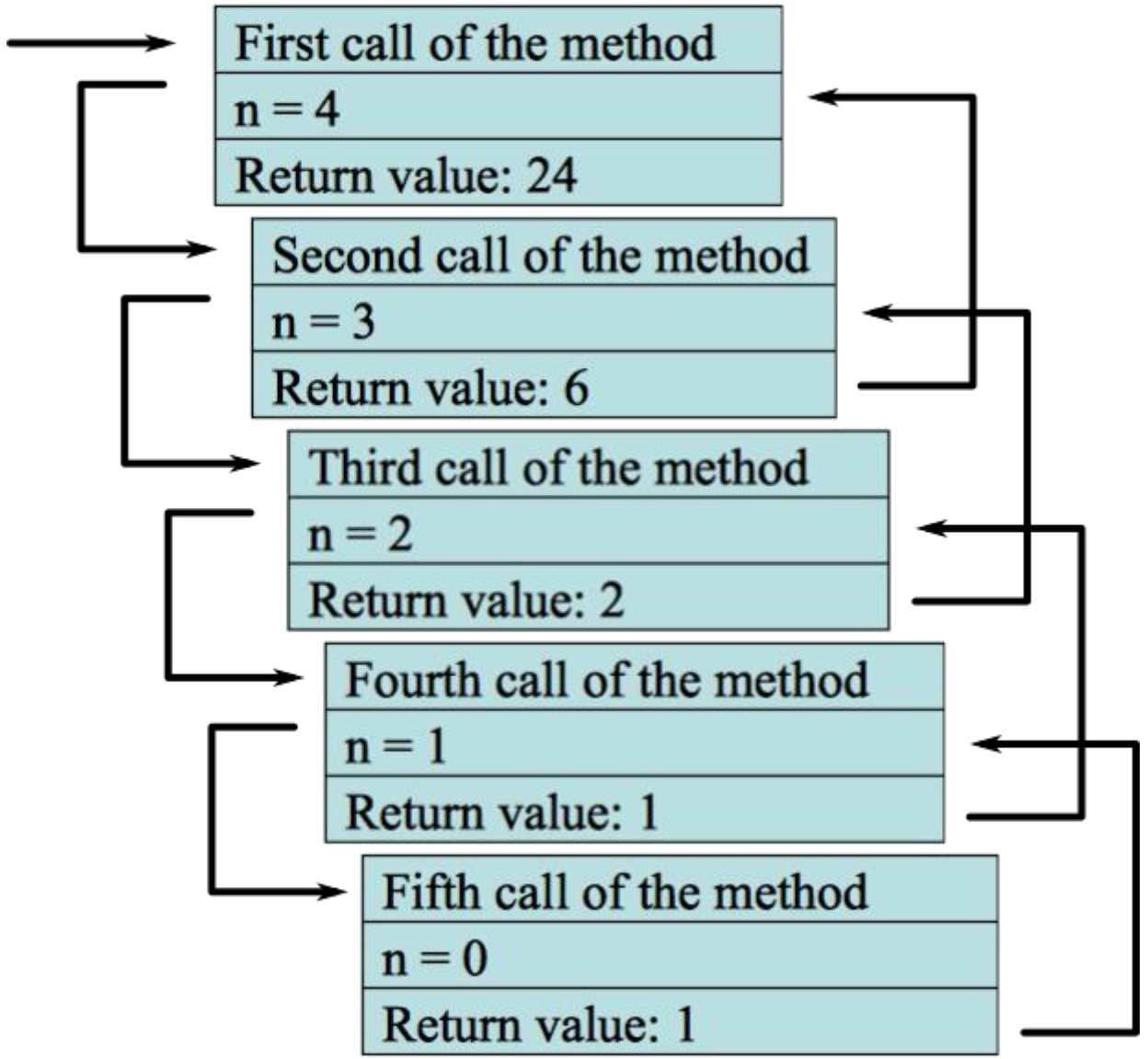
LECTURE 1

Tail Recursion

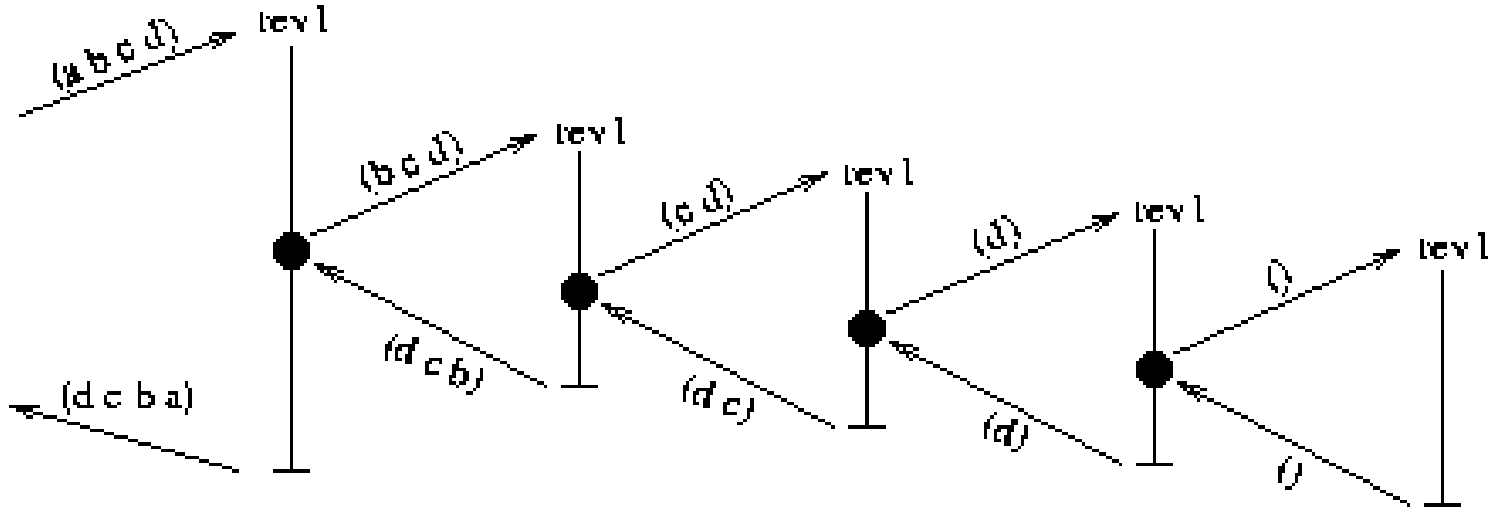
- A **tail recursion** is a **recursive function** where the **function** calls itself at the end ("**tail**") of the **function** in which no computation is done after the return of **recursive** call.
- Many compilers optimize to change a **recursive** call to a **tail recursive** or an iterative call. ...
- Hence the compiler can do away with a stack.

Tail Recursion

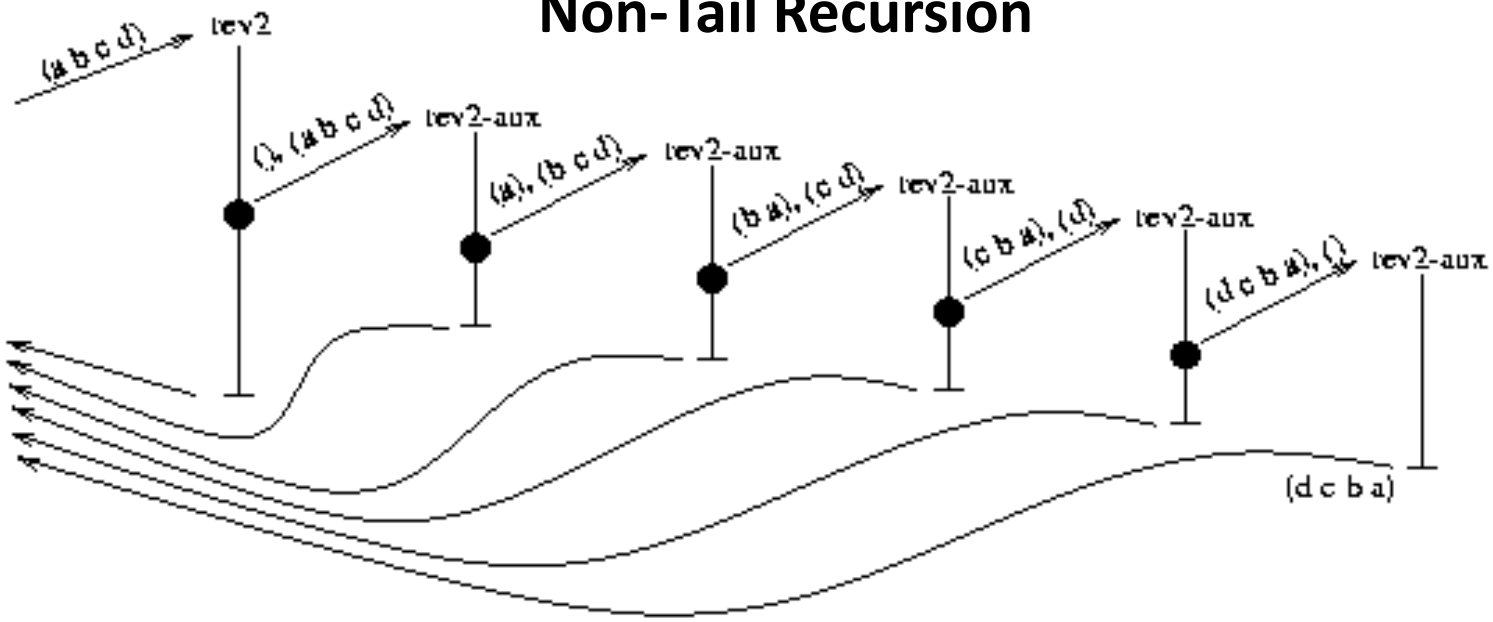
- A tail-recursive function is just a function whose very the last action is a call to itself. Tail-Call Optimisation (TCO) lets us convert regular recursive calls into tail calls to make recursions practical for large inputs, which was earlier leading to stack overflow error in normal recursion scenario.
- Java does not directly support TCO at the compiler level, but with the introduction of lambda expressions and functional interfaces in JAVA 8, we can implement this concept in a few lines of code.


First call of the method
n = 4
Return value: 24
Second call of the method
n = 3
Return value: 6
Third call of the method
n = 2
Return value: 2
Fourth call of the method
n = 1
Return value: 1
Fifth call of the method
n = 0
Return value: 1

Non-Tail Recursion



Non-Tail Recursion



Computing Factorials and Tail Recursion

LECTURE 1

Computing Factorial

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

```
n! = n * (n-1)!
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) =n*factorial(n-1);
```

```
factorial(3)
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) =
  n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) =n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * ( 2 * (1 * factorial(0)))
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * ( 2 * (1 * factorial(0)))
             = 3 * ( 2 * ( 1 * 1))
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * ( 2 * (1 * factorial(0)))
         = 3 * ( 2 * ( 1 * 1))
         = 3 * ( 2 * 1)
```

Computing Factorial

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * ( 2 * (1 * factorial(0)))
             = 3 * ( 2 * ( 1 * 1))
             = 3 * ( 2 * 1)
                 = 3 * 2
```

Computing Factorial

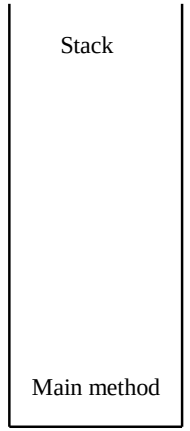
```
factorial(4) = 4 * factorial(3)
             = 4 * 3 * factorial(2)
             = 4 * 3 * (2 * factorial(1))
             = 4 * 3 * ( 2 * (1 * factorial(0)))
             = 4 * 3 * ( 2 * ( 1 * 1))
             = 4 * 3 * ( 2 * 1)
             = 4 * 3 * 2
             = 4 * 6
             = 24
```

```
factorial(0) = 1;
factorial(n) = n*factorial(n-1);
```

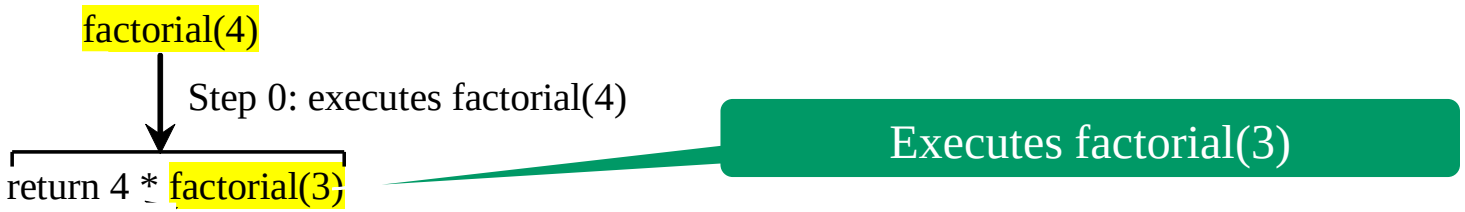
Trace Recursive factorial

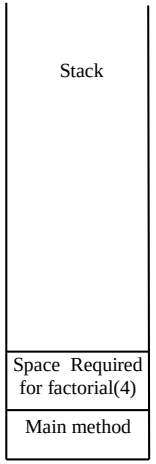
Executes factorial(4)

factorial(4)

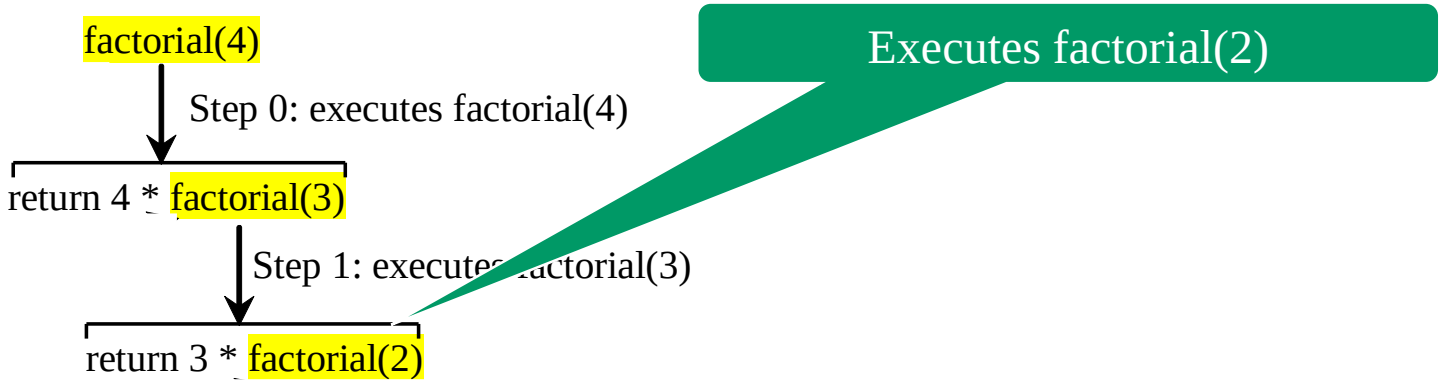


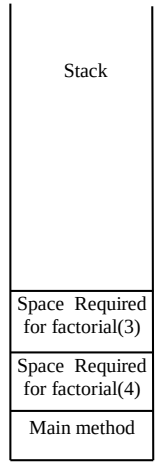
Trace Recursive factorial



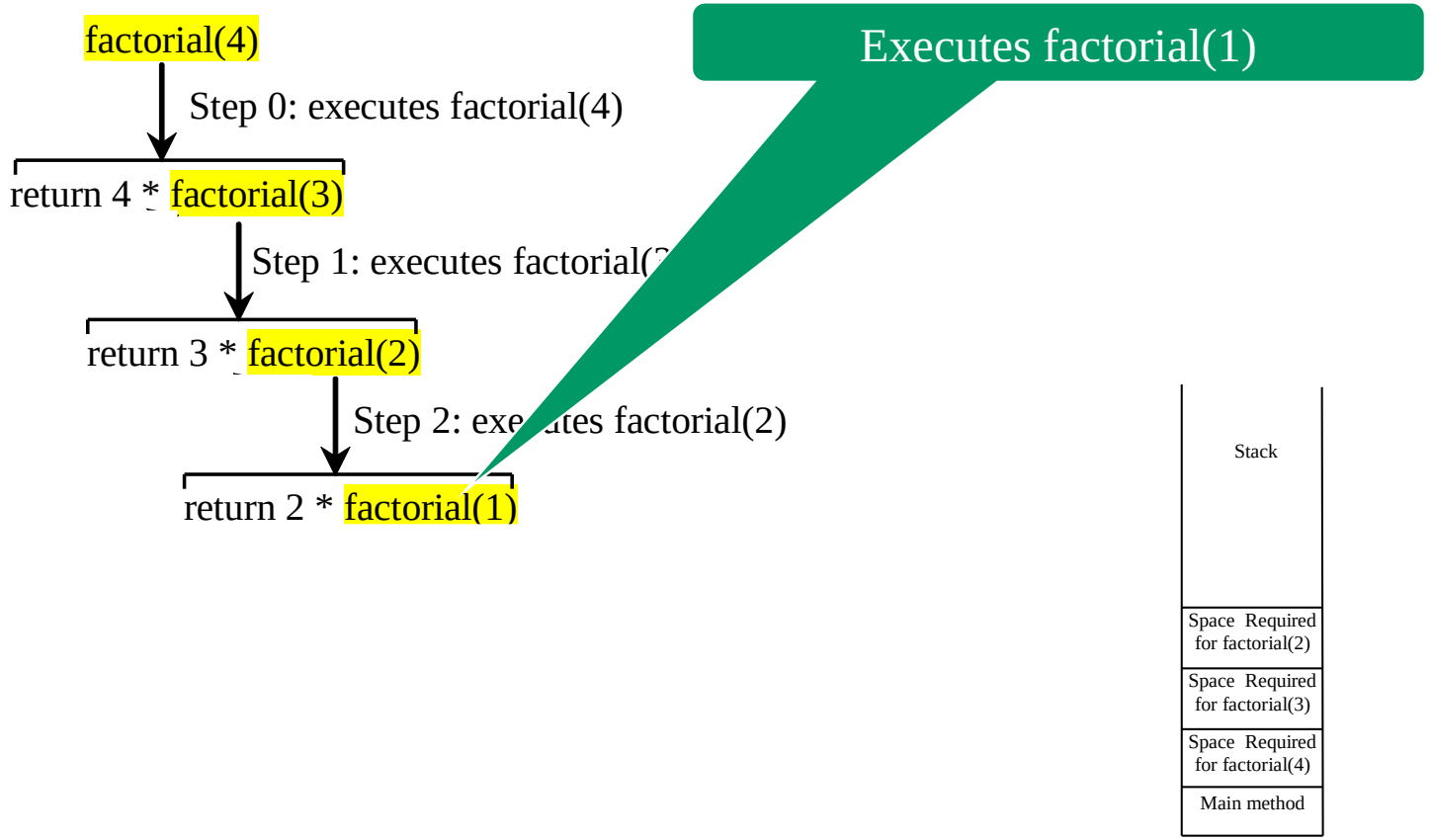


Trace Recursive factorial

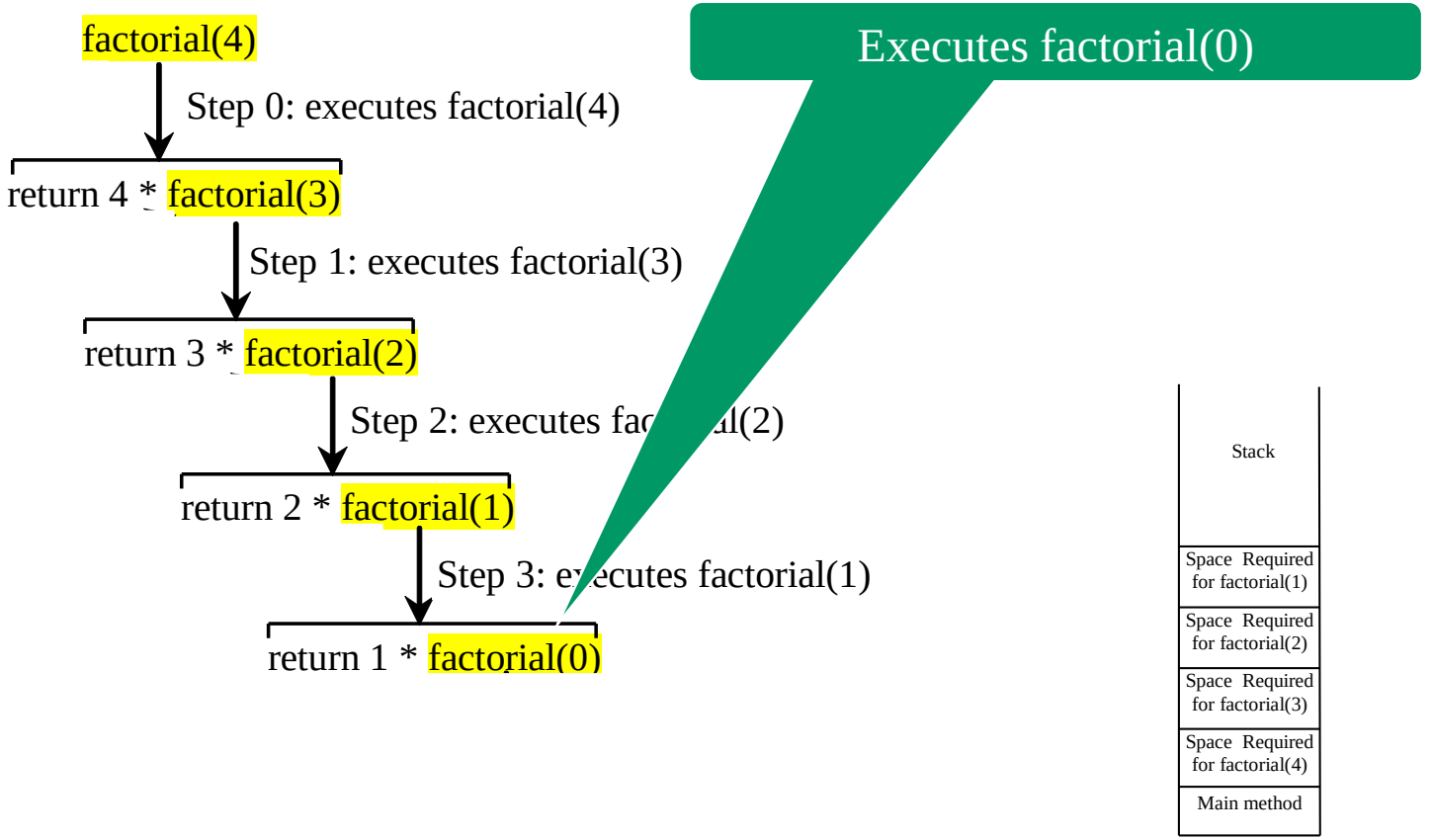




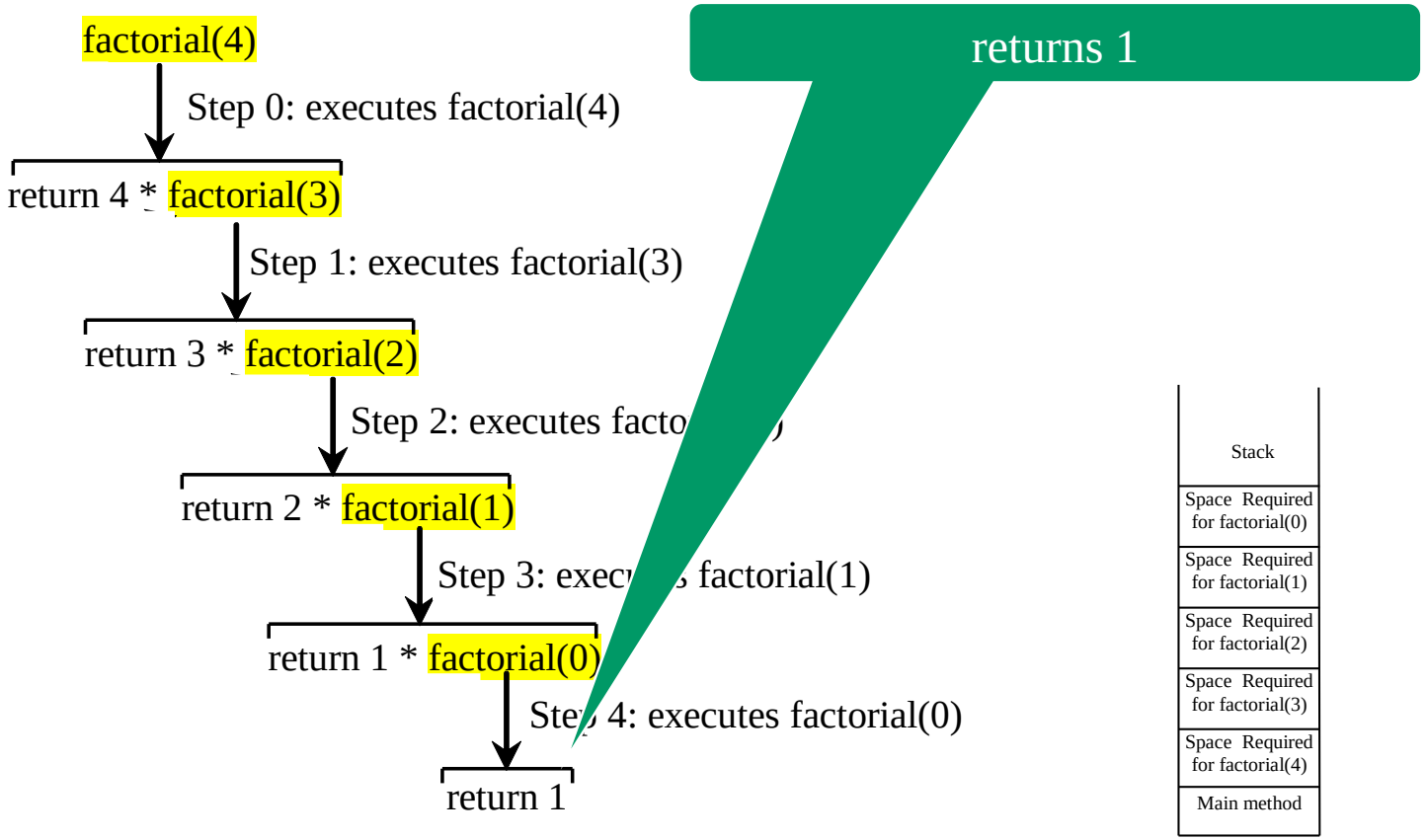
Trace Recursive factorial



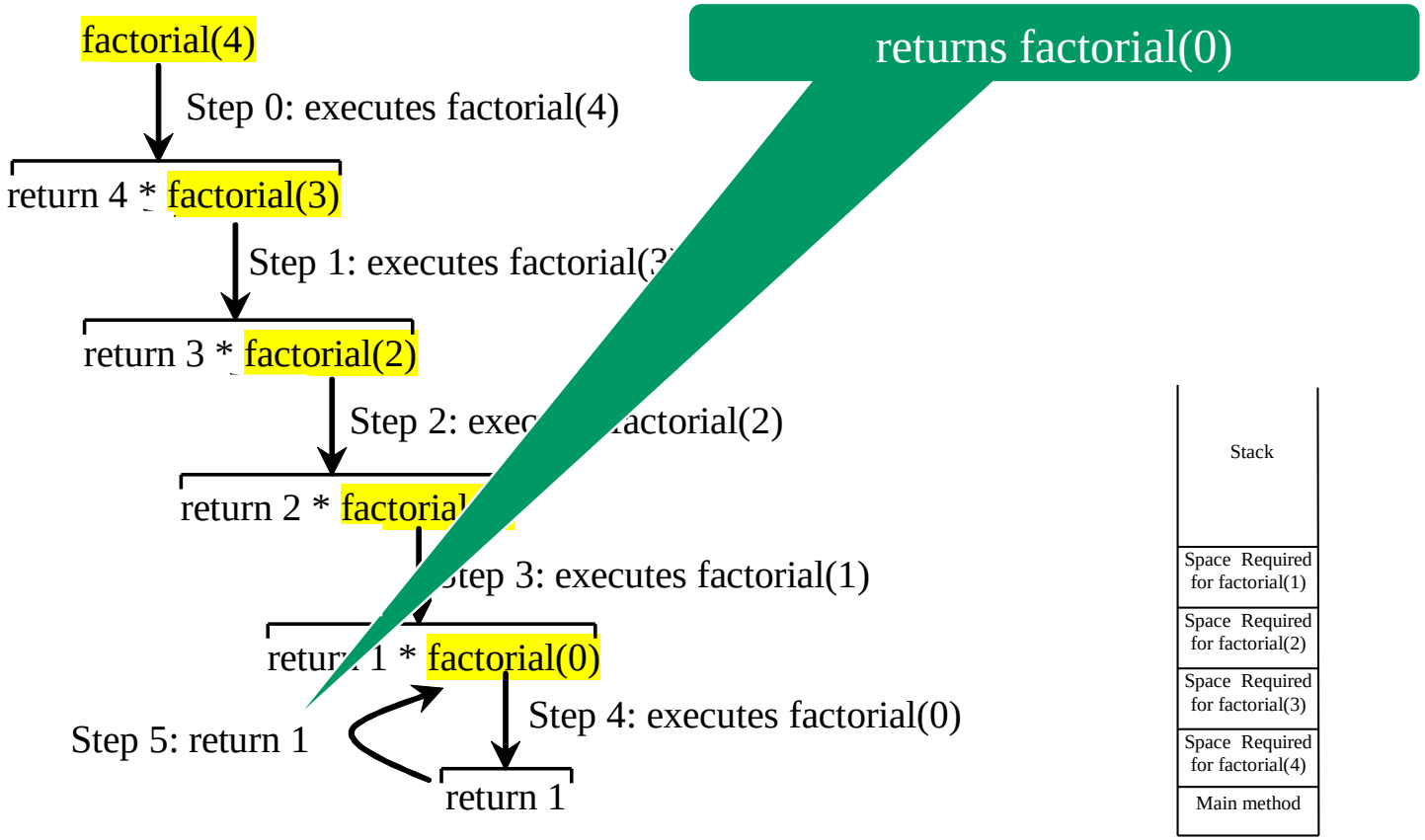
Trace Recursive factorial



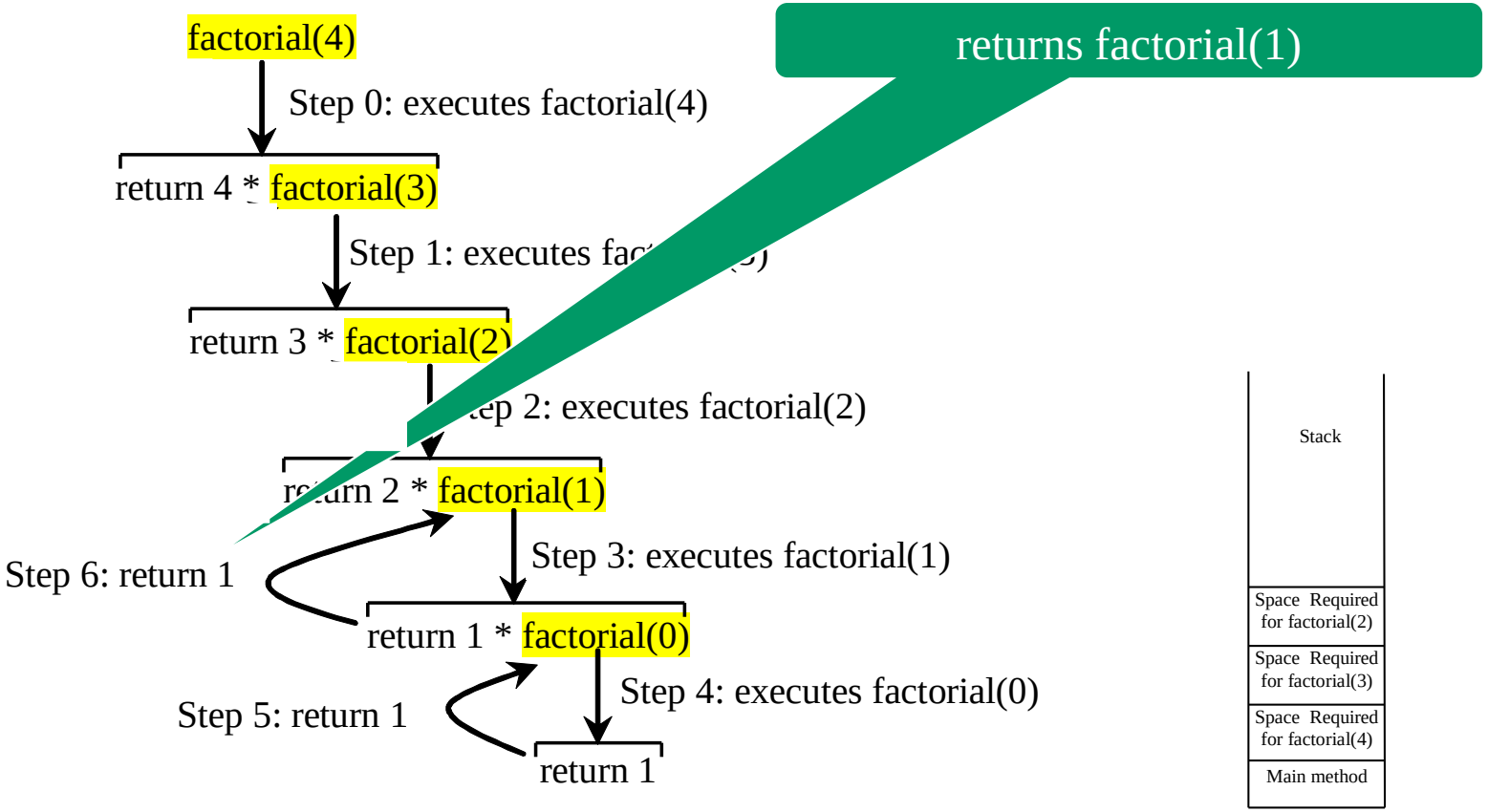
Trace Recursive factorial



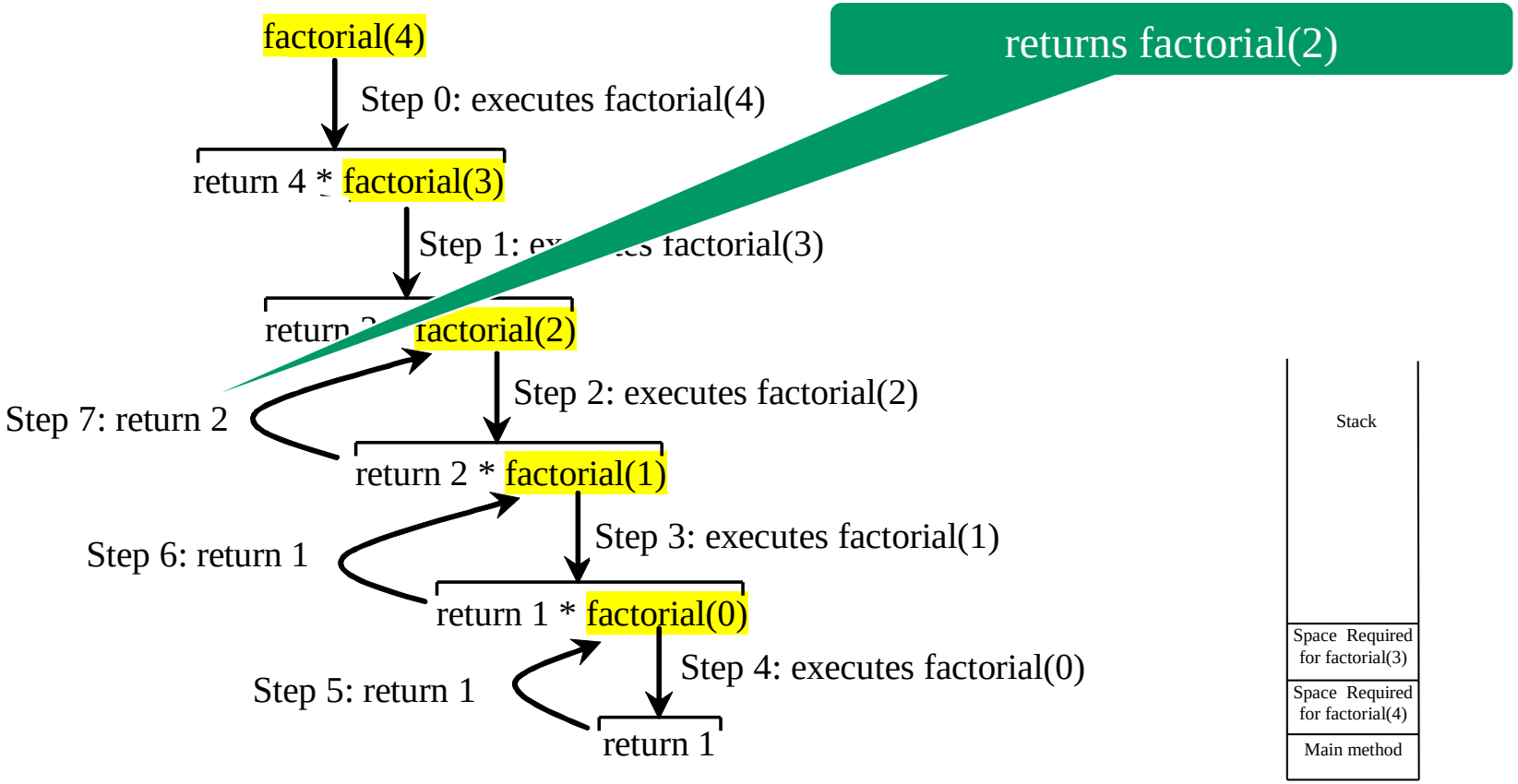
Trace Recursive factorial



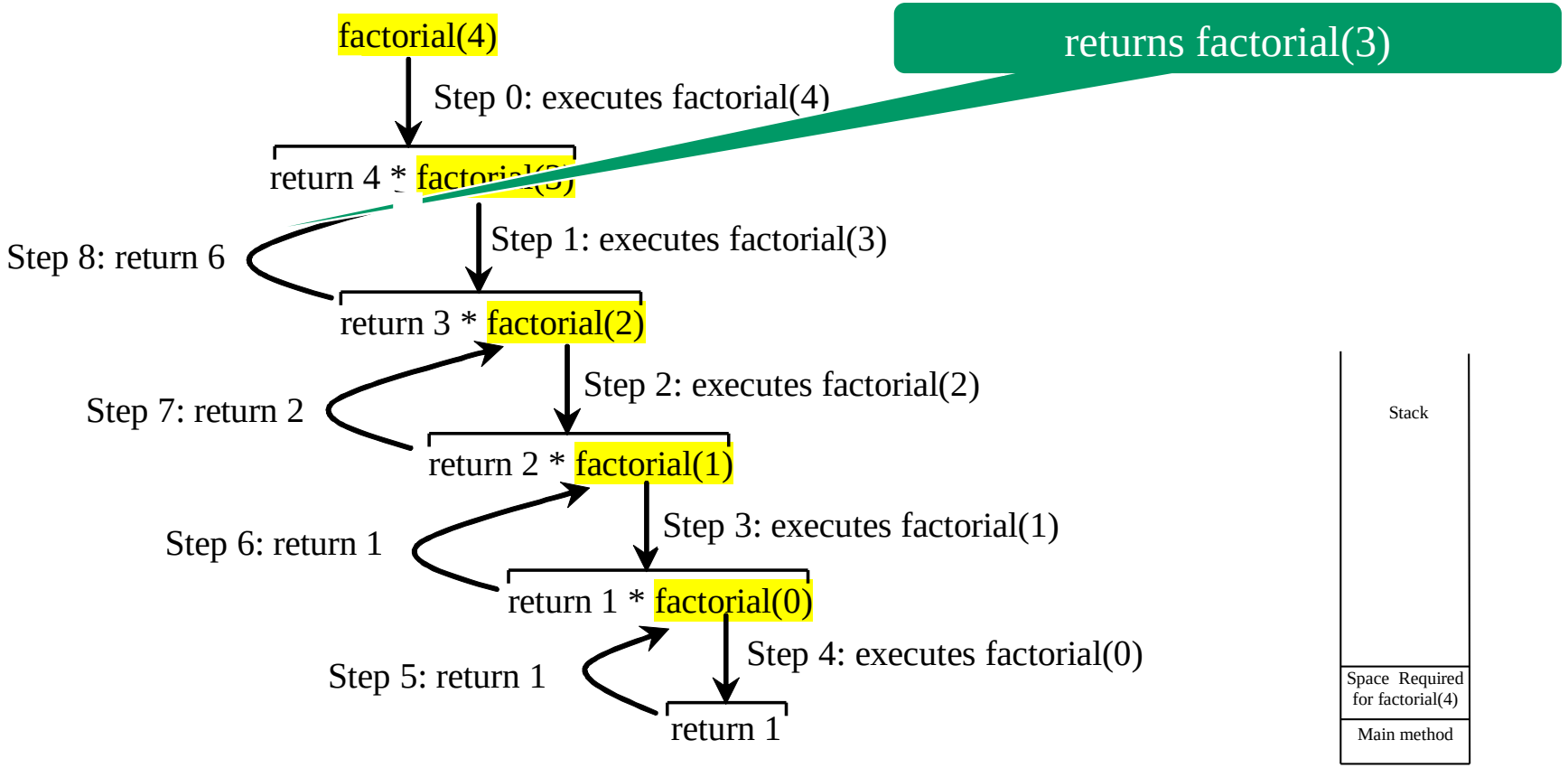
Trace Recursive factorial



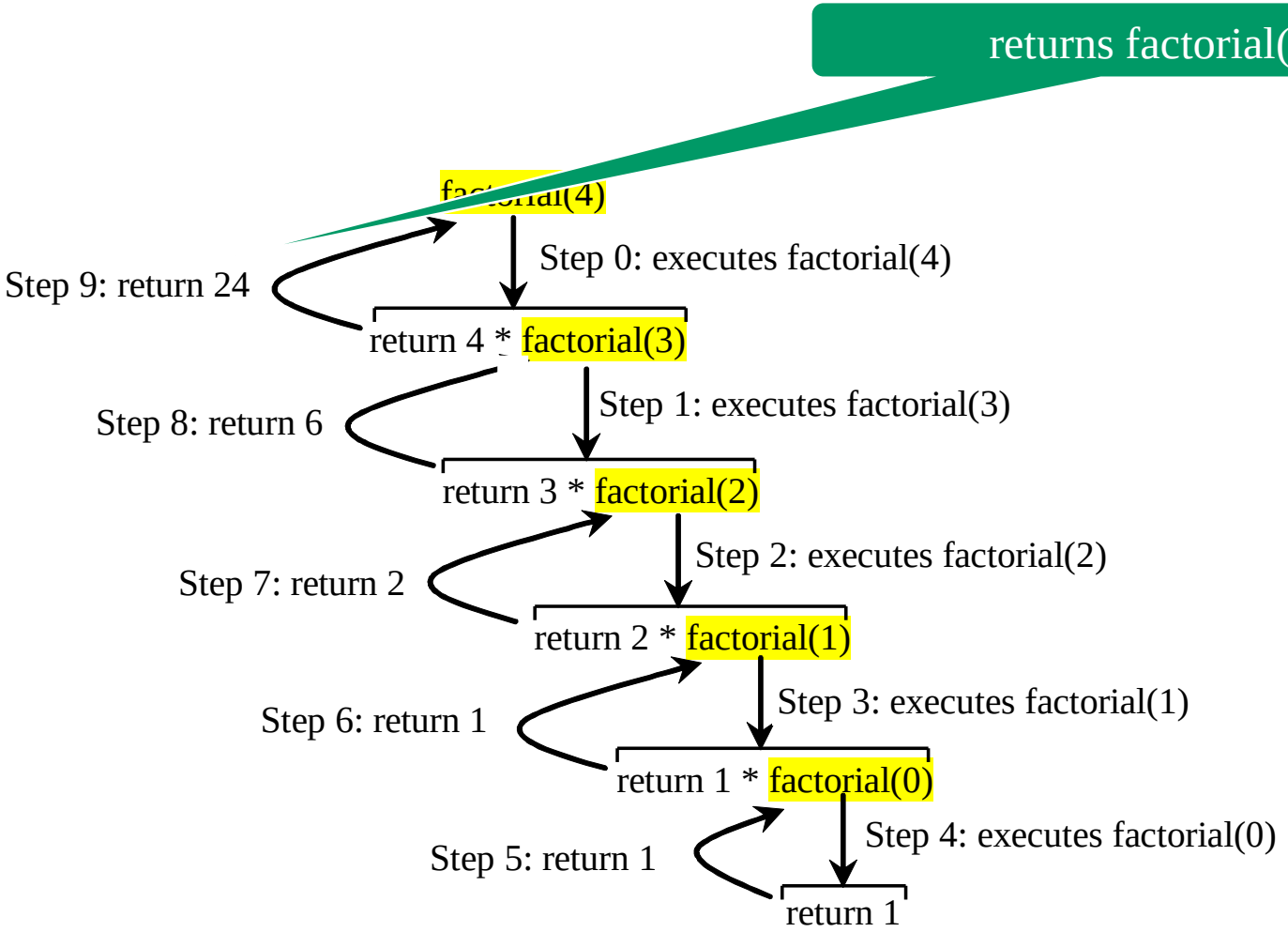
Trace Recursive factorial

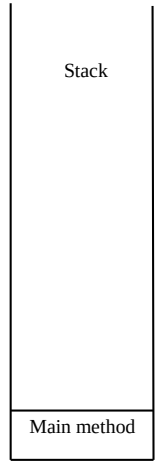


Trace Recursive factorial

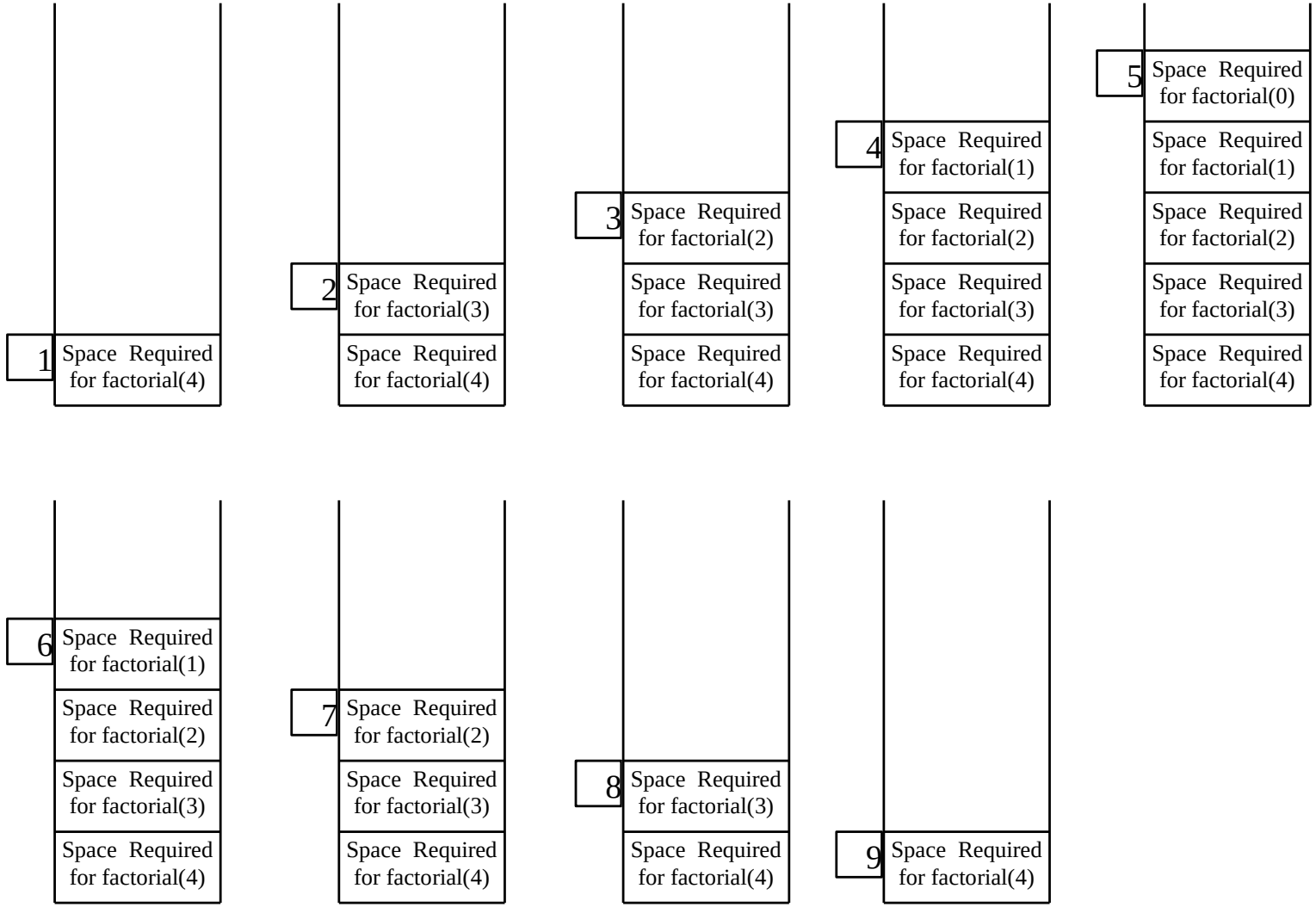


Trace Recursive factorial





factorial(4) Stack Trace



Tail Recursion

- A recursive method is said to be *tail recursive* if there are no pending operations to be performed on return from a recursive call.

Demo Program:

Non-Tail-Recursive: ComputeFactorial.java

Tail-Recursive: ComputeFactorialTailRecursion.java

Fibonacci Numbers

LECTURE 1

Recursive Method

A method calling itself.

- A recursive method is defined as a method which calls itself.
- It is realized by call-stacks. **Call-stack** keep track of which call frame is currently active. Call-stack‘s push and pop operations works as the branching out new method call or returning from a branch-out method call.
- Stop condition will stop a method to continue to branch out and return certain value back to the calling frame.

Fibonacci Numbers

Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89…

indices: 0 1 2 3 4 5 6 7 8 9 10 11

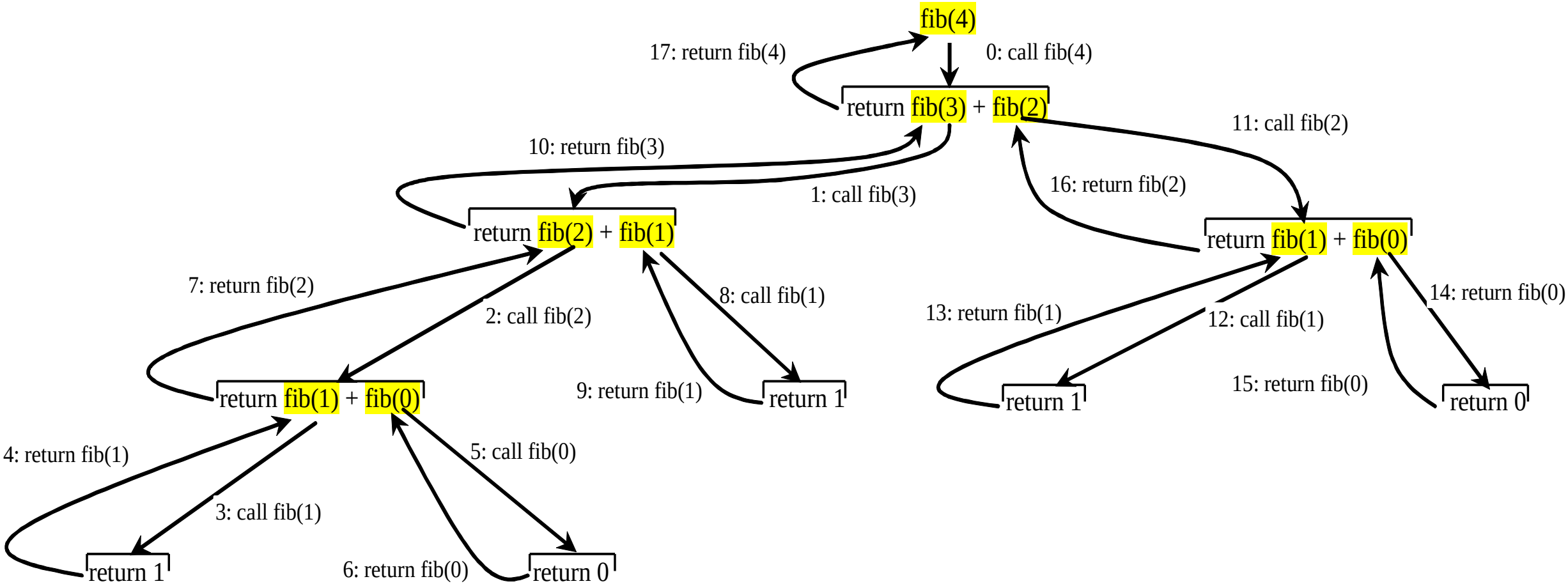
fib(0) = 0;

fib(1) = 1;

fib(index) = fib(index -1) + fib(index -2); index >=2

fib(3) = fib(2) + fib(1) = (fib(1) + fib(0)) + fib(1) = (1 + 0) +fib(1) = 1 + fib(1) = 1 + 1 = 2

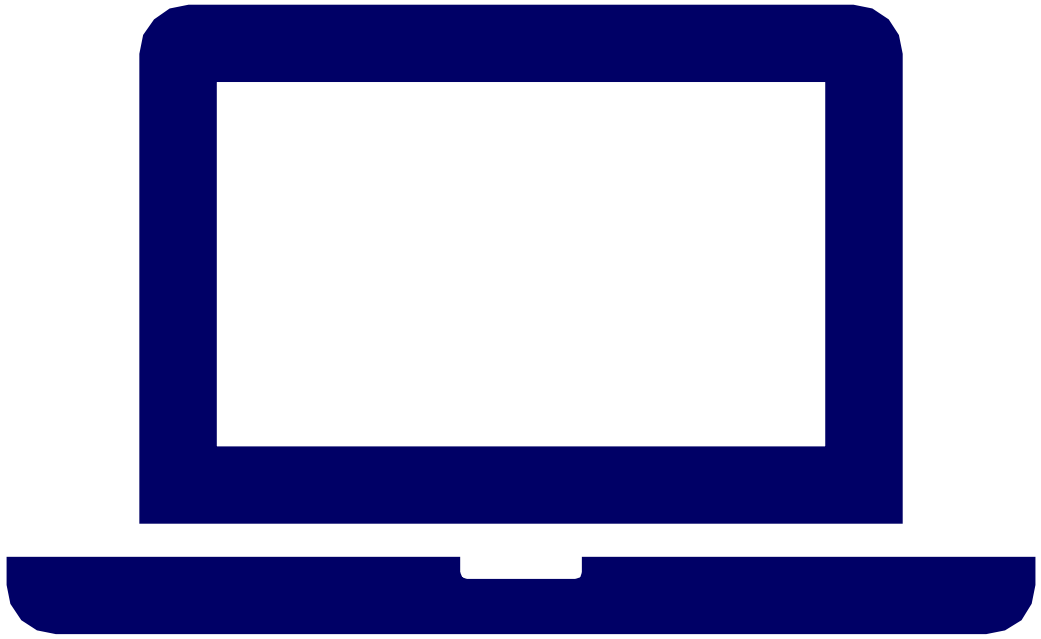
Fibonnaci Numbers, cont.



Fibonacci Method is a 2nd Order Linear Recurrence Equation

http://mathworld.wolfram.com/LinearRecurrenceEquation.html

- For recursive method which branch out two method call, it is called 2nd order recursive method.
- 2nd order recursive method may need 2 stop conditions.
- The time complexity grows in order of ***O(2^n)***



Demonstration Program

COMPUTEFIBONACCI.JAVA