Introduction to Robotics

Manipulation and Programming

Unit 2: Kinematics

INVERSE KINEMATICS FOR POSITION

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Objectives

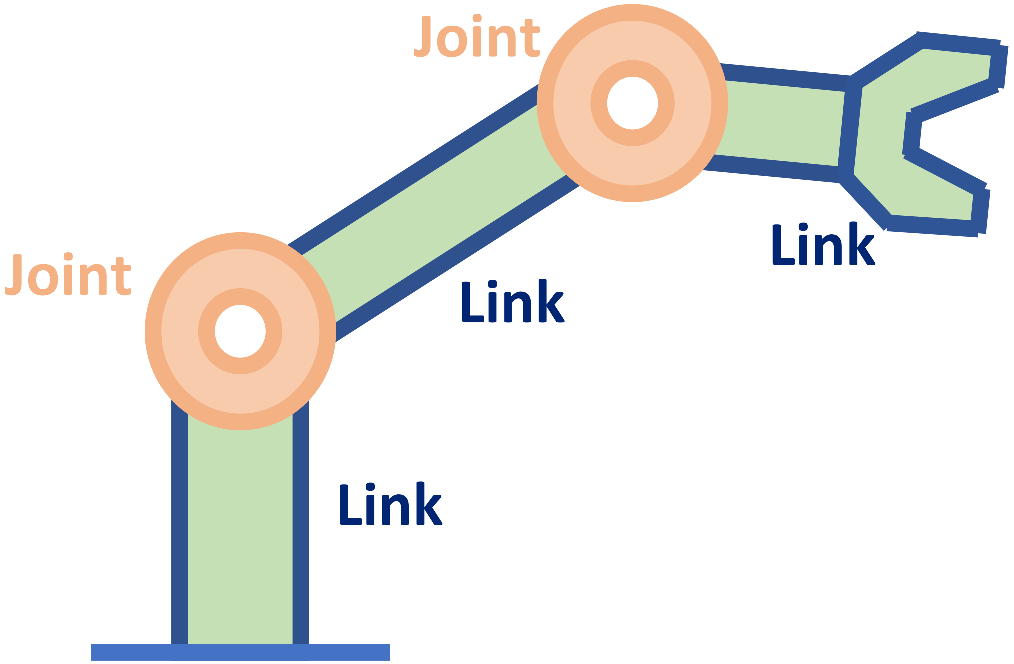
- Forward Kinematics Review
- Inverse Kinematics: How to find the kinematics parameters based on the end-effector's results.
- Graphical Methods

Forward Kinematics

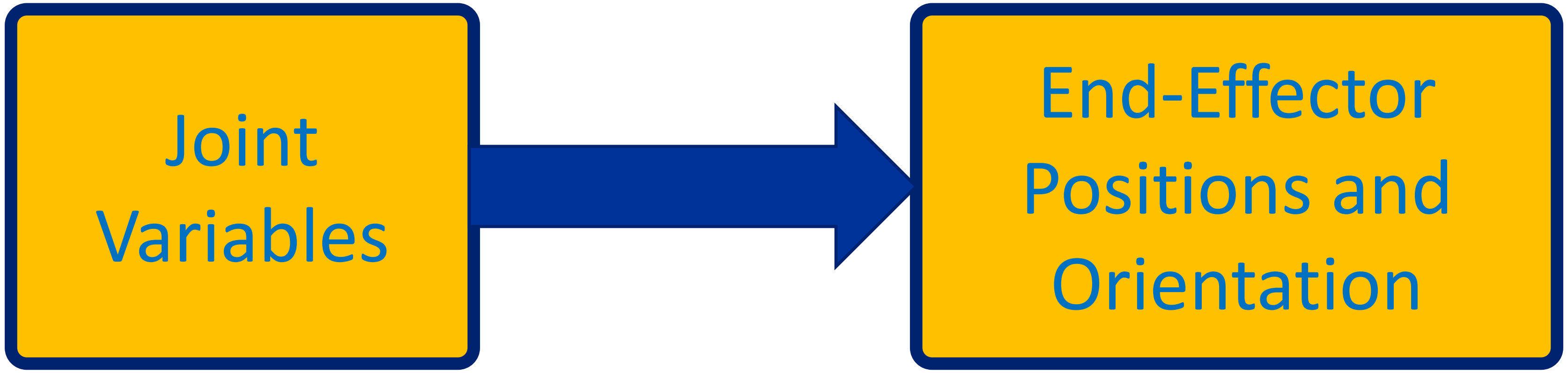
SECTION 1

Forward Kinematics

- Animator specifies joint variables: θ_1, θ_2, a_1, a_2, a_3, a_4
- Computer finds the positions of end-effector: [x, y, z]



Forward Kinematics

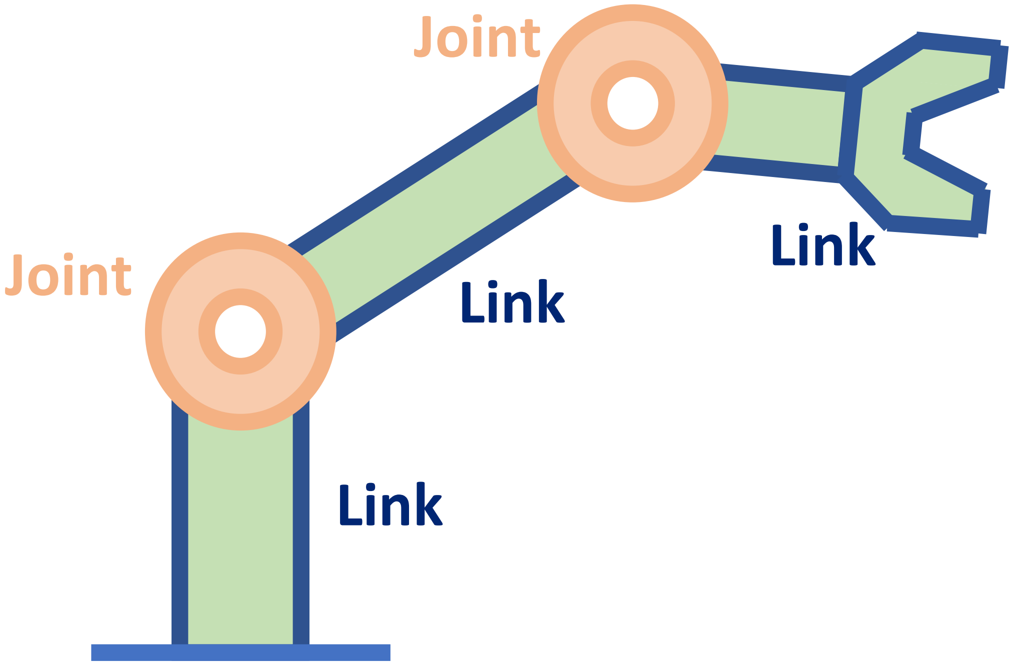


Inverse Kinematics

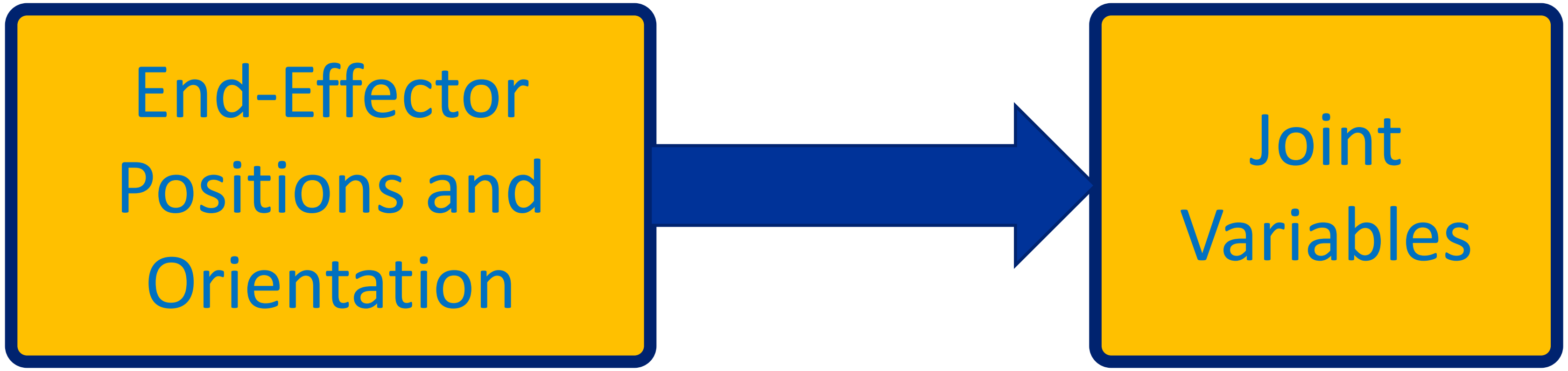
SECTION 2

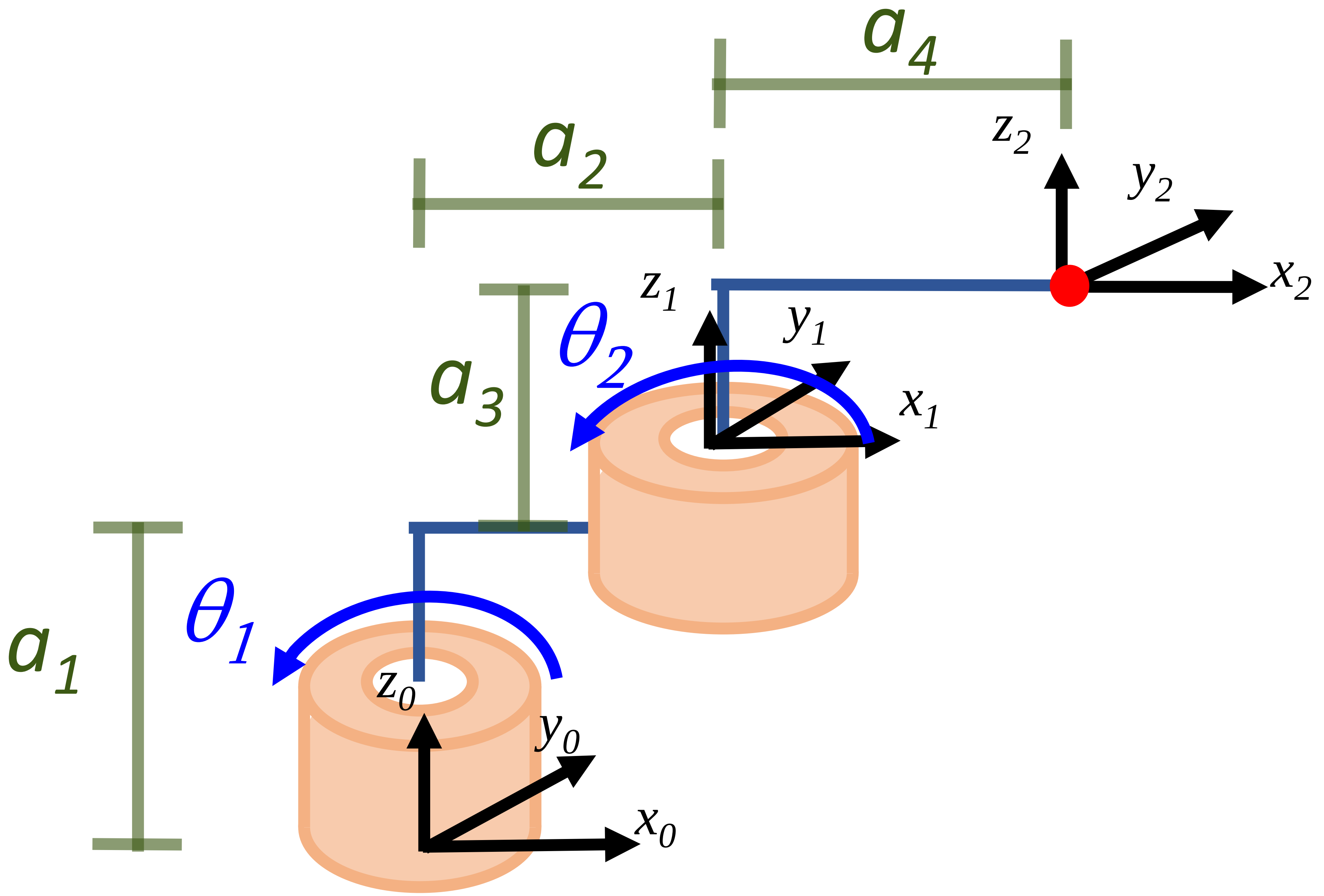
Inverse Kinematics

- Animator specifies the positions of end-effector: [x, y, z]
- Computer finds the joint variables: θ_1, θ_2. *Note:* a_1, a_2, a_3, a_4 *are fixed*



Inverse Kinematics





a_4
a_2
z_2
y_2
x_2
z_1
y_1
θ_2
a_3
x_1
a_1
θ_1
z_0
y_0
x_0

Inverse Kinematics

$$H_2^0 = H_1^0 \, H_2^1$$

$$H_2^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_2\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_2\sin(\theta_1) \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_4\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_4\sin(\theta_2) \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p = H_2^0 \, p'$$

Inverse Kinematics (New Notation)

$$H_2^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_2\, C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_2\, S\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_4\, C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_4\, S\theta_2 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & a_4 C(\theta_1+\theta_2) + a_2\, C\theta_1 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & a_4 S(\theta_1+\theta_2) + a_2\, S\theta_1 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p = H_2^0\, p'$$

Given Original Point of Frame 2

$$p = H_2^0 \, p'$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & a_4 C(\theta_1+\theta_2) + a_2\, C\theta_1 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & a_4 S(\theta_1+\theta_2) + a_2\, S\theta_1 \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$x = \; a_4 C(\theta_1+\theta_2) + a_2\, C\theta_1$$
$$y = \; a_4 S(\theta_1+\theta_2) + a_2\, S\theta_1$$

Non-linear Equation

Given x, y, to find θ_1 and θ_2

$$x = a_4 C(\theta_1 + \theta_2) + a_2\, C\theta_1$$
$$y = a_4 S(\theta_1 + \theta_2) + a_2\, S\theta_1$$

- It is a non-linear system of equation. Only numerical solution can be easily found. Symbolic reduction is almost impossible.

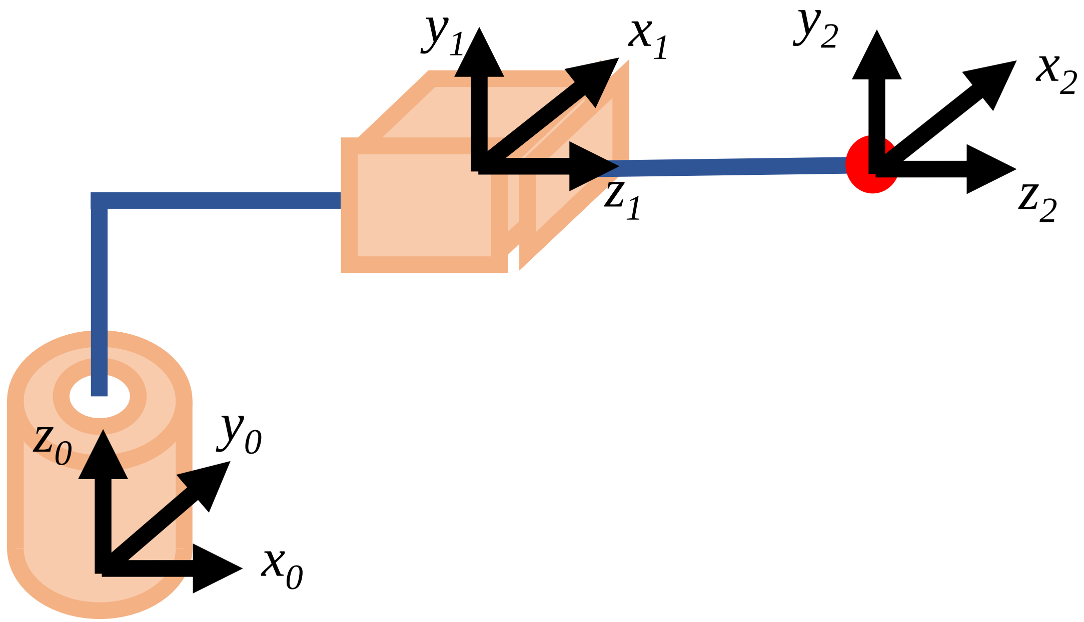
Graphical Method

SECTION 3

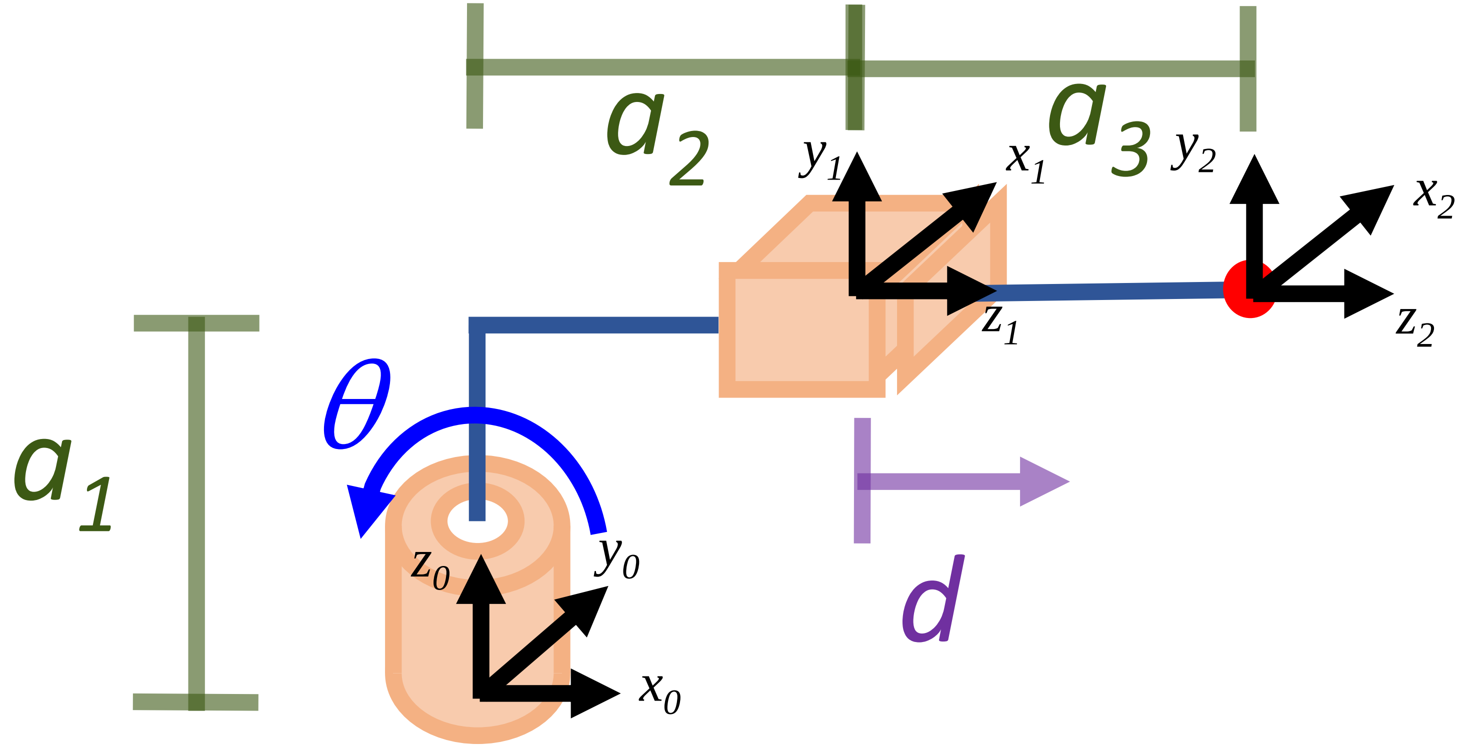
Graphical Method

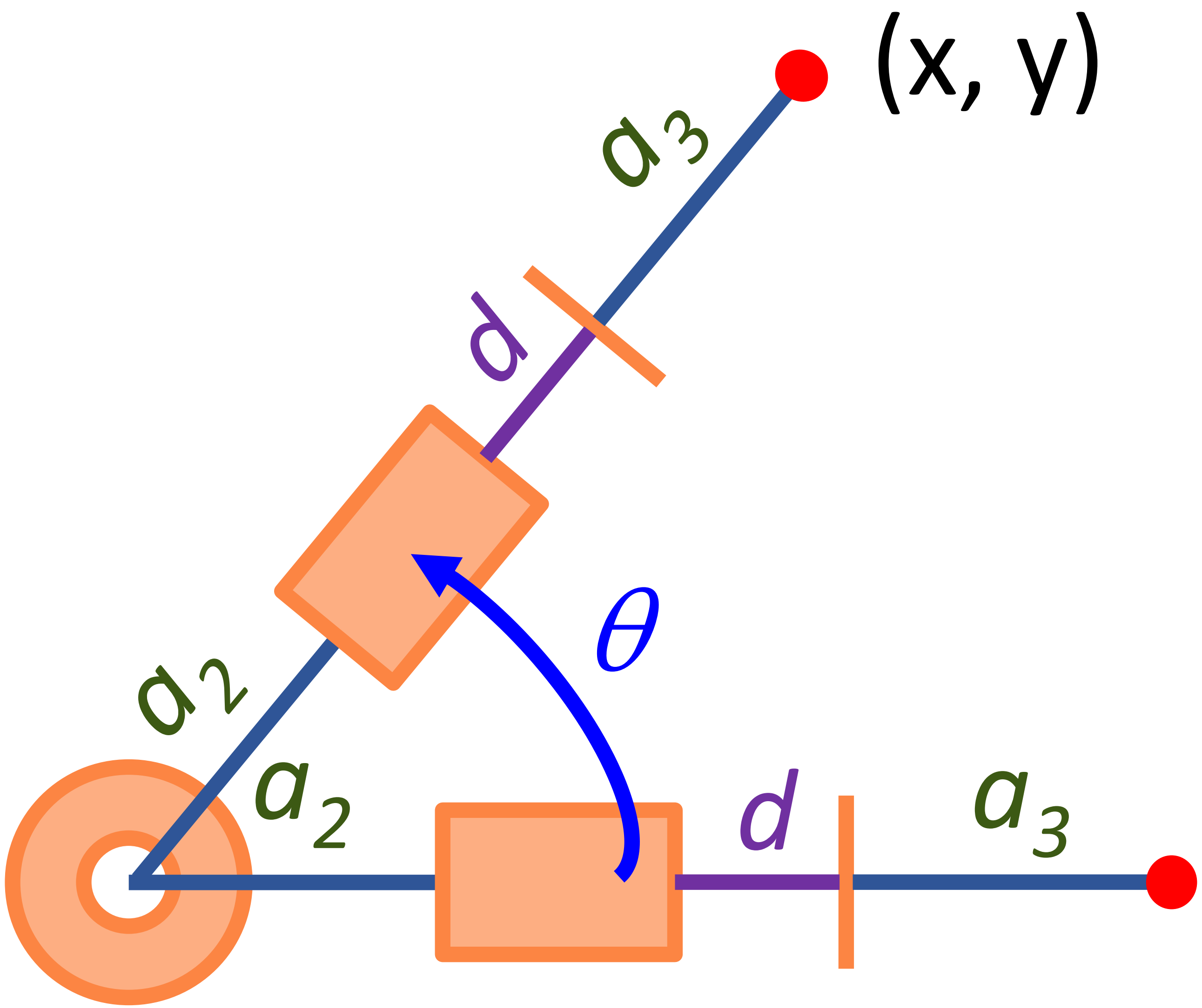
- Inverse kinematics is the problem in which we know a position we want the end-effector to go to, and we need to find the values of the joint variables that move the end-effector to that position.
- In this section, we learn the 'graphical approach' to inverse kinematics, see some examples, and use the inverse kinematics equations to manipulate a robot arm of the similar model.

Example 1: Cylindrical Manipulator (2 DOF)



Example 1: Cylindrical Manipulator (2 DOF)





Given x, y

Solve (d, θ)

(1) $r = a_2 + a_3$

(2) $x = (r + d)\,cos(\theta)$

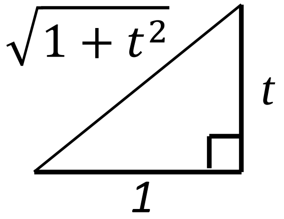
(3) $y = (r + d)\,sin(\theta)$

(4) $\frac{y}{x} = tan(\theta) = t$

$\theta = \tan^{-1}(t)$

$sin(\theta) = \frac{t}{\sqrt{1+t^2}} = \frac{y}{r+d}$

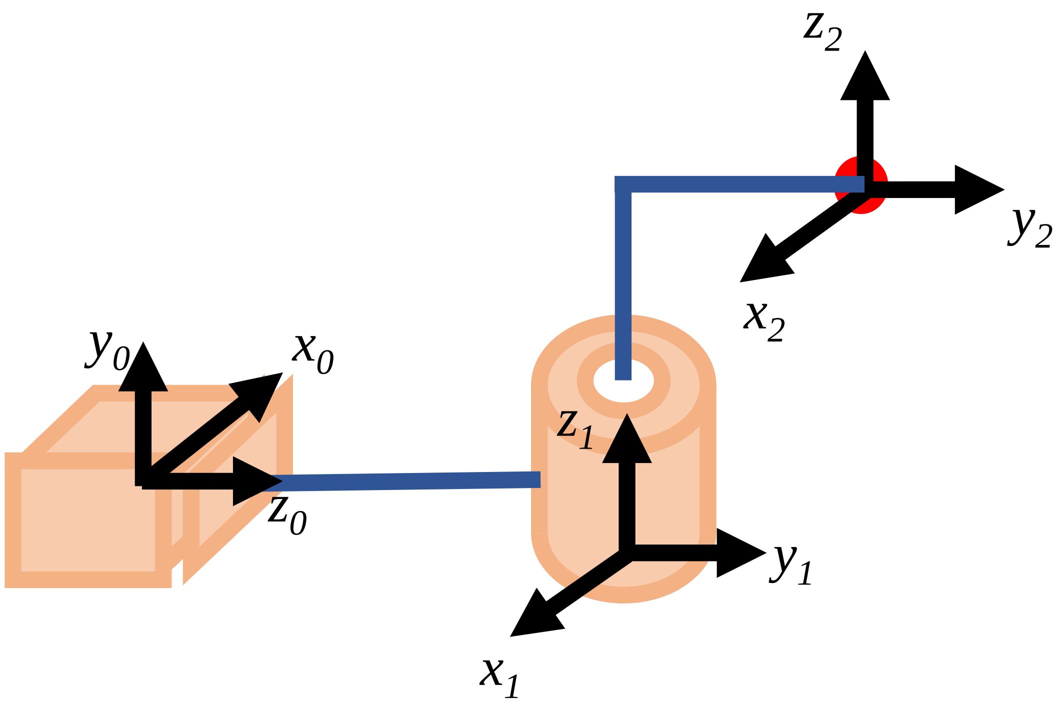
$\frac{\sqrt{1+t^2}}{t} = \frac{r+d}{y}$



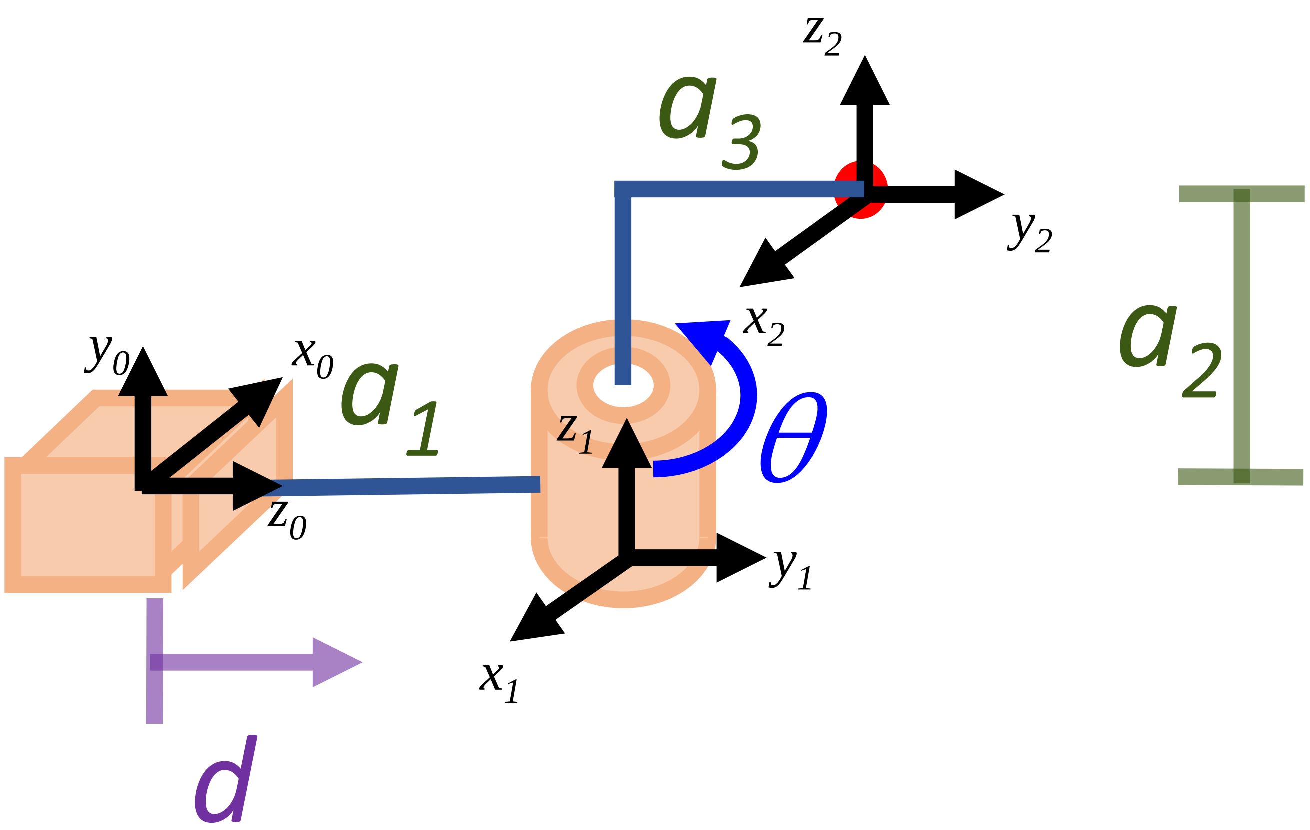
$d = \frac{y\sqrt{1+t^2}}{t} - r = \frac{y\sqrt{x^2+y^2}}{\frac{y}{x}x} - r$

$= \sqrt{x^2+y^2}$ - $(a_2 + a_3)$

Example 2: Manipulator (2 DOF)

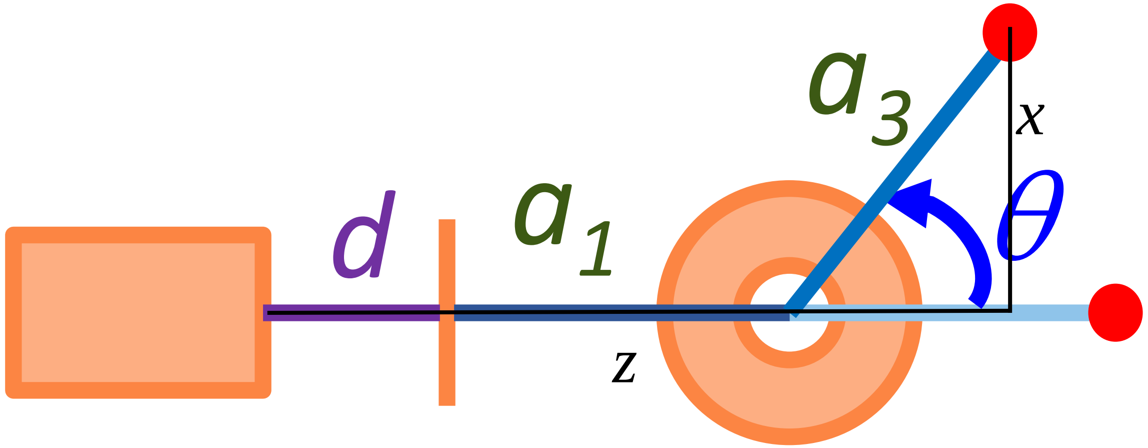


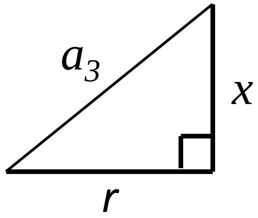
Example 2: Manipulator (2 DOF)



Given (x, z), y fixed

To find (d, θ)

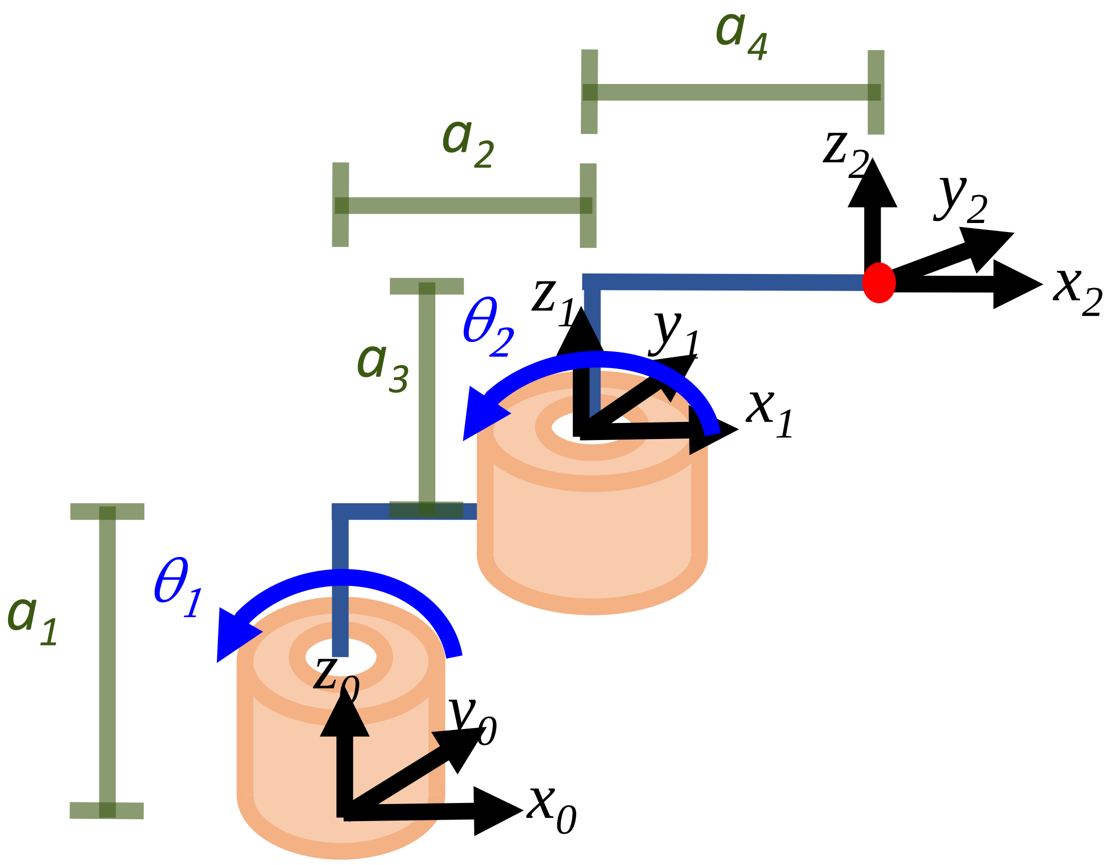


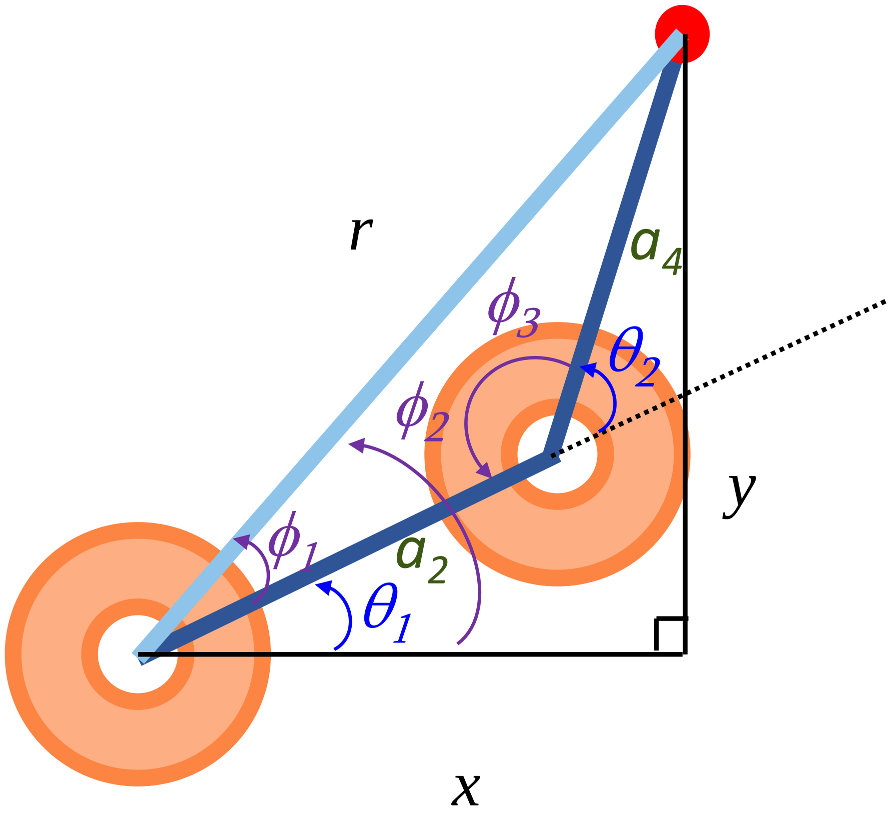


$$r = \sqrt{a_3^2 - x^2}$$

$$d = z - a_1 - \sqrt{a_3^2 - x^2}$$

Example 3: Manipulator (2 DOF)





Given (x, y)

To find (θ_1, θ_2)

$$\theta_1 = \phi_2 - \phi_1 \qquad (1)$$

$$\theta_2 = 180 - \phi_3 \qquad (2)$$

$$\phi_1 = cos^{-1}\left(\frac{a_2^2 + r^2 - a_4^2}{2a_2 r}\right) \qquad (3)$$

$$\phi_2 = tan^{-1}\left(\frac{y}{x}\right) \qquad (4)$$

$$\phi_3 = cos^{-1}\left(\frac{a_2^2 + a_4^2 - r^2}{2a_2 a_4}\right) \qquad (5)$$