Introduction to Robotics

Manipulation and Programming

Unit 2: Kinematics

JACOBIAN MATRIX APPLICATION – STANFORD MANIPULATOR

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Objectives

- Apply Jacobian Matrix to an example
- 1 DOF Manipulator
- 3 DOF all prismatic manipulator
- 2 DOF all rotational manipulator
- Analyze the Stanford Manipulator

Understand Jacobian

SECTION 1

Forward Kinematics

$$\begin{bmatrix} V \\ \Omega \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Jacobian

$$\begin{bmatrix} V \\ \Omega \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} = \begin{bmatrix} L \\ R \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} = \left[\begin{array}{c} \textbf{Linear Equations} \\ \hline \textbf{Angular Equations} \end{array} \right] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Kinematics Table

	Prismatic	Revolute
Linear	Rotational Effects on the Z_{i-1} direction	Rotational Effects on the Displacement Difference of Frame n and $i-1$
Rotational	Not related	Rotational Effects on the Z_{i-1} direction

Kinematics Table

	Prismatic	Revolute
Linear	$Z_{i-1} = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$Z_{i-1} \times (\Omega_n - \Omega_{i-1}) = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n - d_{i-1})$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$Z_{i-1} = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

Rotational Effect on the joint movement direction (Z) Frame $f_{i\text{-}1}$

$$Z_i = R_i^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} & & x_z \\ & & y_z \\ & & z_z \end{bmatrix}$$

The first three elements in column 3 of the Denavit-Hartenberg Matrix

Denavit-Hartenberg Method is a short cut to HTM

$$H_n^0 = \begin{bmatrix} & R_n^0 & & d_n^0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Z and $\boldsymbol{\Omega}$ (Z and OU) vectors - Prismatic

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_1 = R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_i = R_i^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_i = \begin{bmatrix} Z_{i-1} \\ \Omega_{i-1} \end{bmatrix} = \begin{bmatrix} x_z \\ y_z \\ z_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Omega_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Omega_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Both Z_i and $\boldsymbol{\Omega}_\text{i}$ are 3×1 vectors

Z and Ω (Z and OU) vectors - Revolute

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_1 = R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_i = R_i^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad J_i = \begin{bmatrix} Z_{i-1} \times (d_n - d_{i-1}) \\ Z_{i-1} \end{bmatrix}$$

$$\Omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Omega_1 = d_1 \qquad \Omega_i = d_i$$

Both Z_i and Ω_i are 3×1 vectors

Kinematics Table

	Prismatic	Revolute
Z_i	$R_i^0\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_i^0\begin{bmatrix}0\\0\\1\end{bmatrix}$
Ω_i	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	d_i

$$H_i = \begin{array}{cccc} & & Z_i & \Omega_i \\ \left[\begin{matrix} a_x \\ a_y \\ a_z \\ 0 \end{matrix}\right. & \begin{matrix} b_x \\ b_y \\ b_z \\ 0 \end{matrix} & \boxed{\begin{matrix} c_x \\ c_y \\ c_z \end{matrix}} & \left.\boxed{\begin{matrix} d_x \\ d_y \\ d_z \end{matrix}}\right] \\ & & 0 & 1 \end{array}$$

$$Z_0 = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad \Omega_0 = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

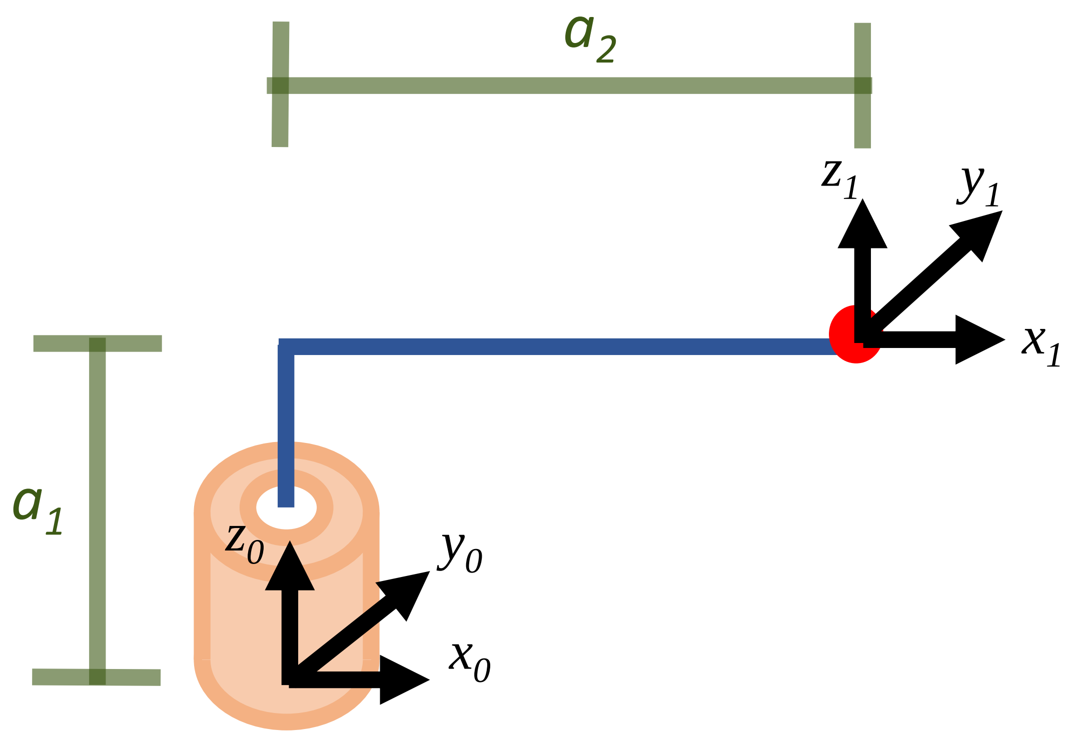
Jacobian

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

Simple Example

SECTION 2

1 DOF Example



	θ	α	r	d
1	θ_1	0	a_2	a_1

$$A_1 = \begin{bmatrix} c\theta_1 & -cs\theta_1 & 0 & a_2 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_2 s\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Jacobian - Revolute

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \Omega_1 = \begin{bmatrix} \mathrm{a}_2 c\theta_1 \\ \mathrm{a}_2 s\theta_1 \\ a_1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathrm{a}_2 C\theta_1 \\ \mathrm{a}_2 S\theta_1 \\ a_1 \end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -a_2 S\theta_1 \\ a_2 C\theta_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c\theta_1 & -cs\theta_1 & 0 & \mathrm{a}_2 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & \mathrm{a}_2 s\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

All Prismatic Joints

SECTION 3

Steps to find Jacobian Matrix

Step 1: Denavit-Hartenberg Table

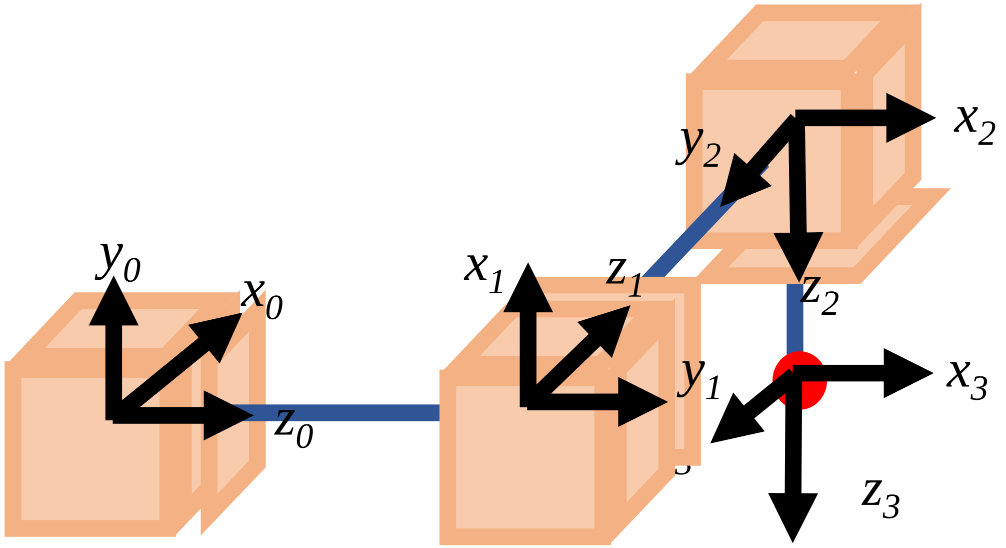
Step 2: Denavit-Hartenberg Matrix A and Serial Matrix H

Step 3: Linear Velocity Vector V and Angular Velocity Vector Ω

Step 4: Jacobian Matrix

Step 5: Verify the Velocity equations

Prismatic Manipulator



$$R_1^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_2^1 = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \quad R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^0 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

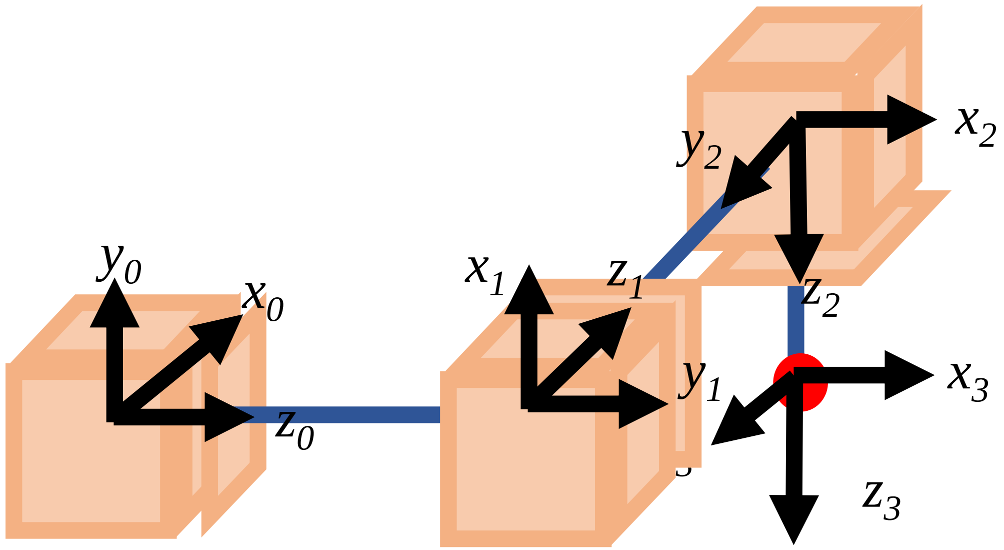
Z and Ω (Z and OU) vectors - Prismatic

$$Z_0 = \begin{bmatrix}0\\0\\1\end{bmatrix} \quad Z_1 = R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix} \quad Z_i = R_i^0\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \Omega_0 = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad \Omega_1 = \begin{bmatrix}0\\0\\0\end{bmatrix} \quad \Omega_i = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$R_0^0 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad R_1^0 = \begin{bmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix} \quad R_2^0 = \begin{bmatrix}0 & -1 & 0\\0 & 0 & -1\\1 & 0 & 0\end{bmatrix} \quad R_3^0 = \begin{bmatrix}0 & -1 & 0\\0 & 0 & -1\\1 & 0 & 0\end{bmatrix}$$

$$Z_1 = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad Z_2 = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad Z_3 = \begin{bmatrix}0\\-1\\0\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\0 & 0 & -1\\1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{d}_1\\\dot{d}_2\\\dot{d}_3\end{bmatrix}$$



$$\dot{x} = \dot{d}_2$$

$$\dot{y} = -\dot{d}_3$$

$$\dot{z} = \dot{d}_1$$

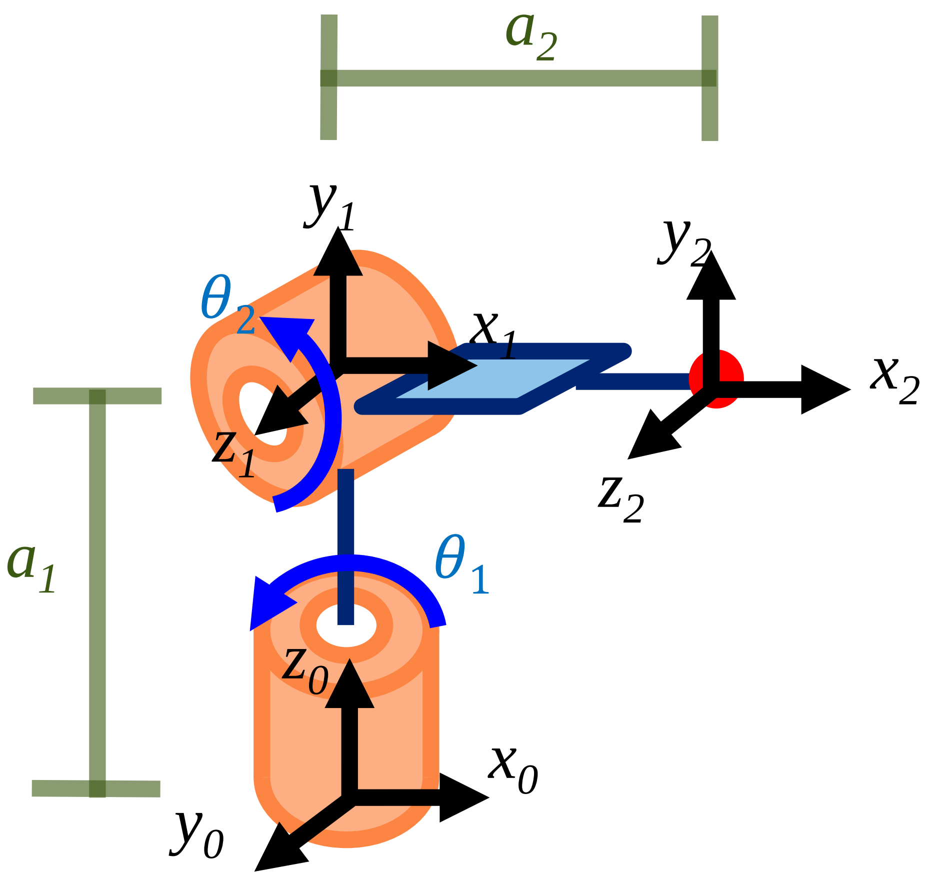
$$\omega_x = 0$$

$$\omega_y = 0$$

$$\omega_z = 0$$

All Revolute Joints

SECTION 4

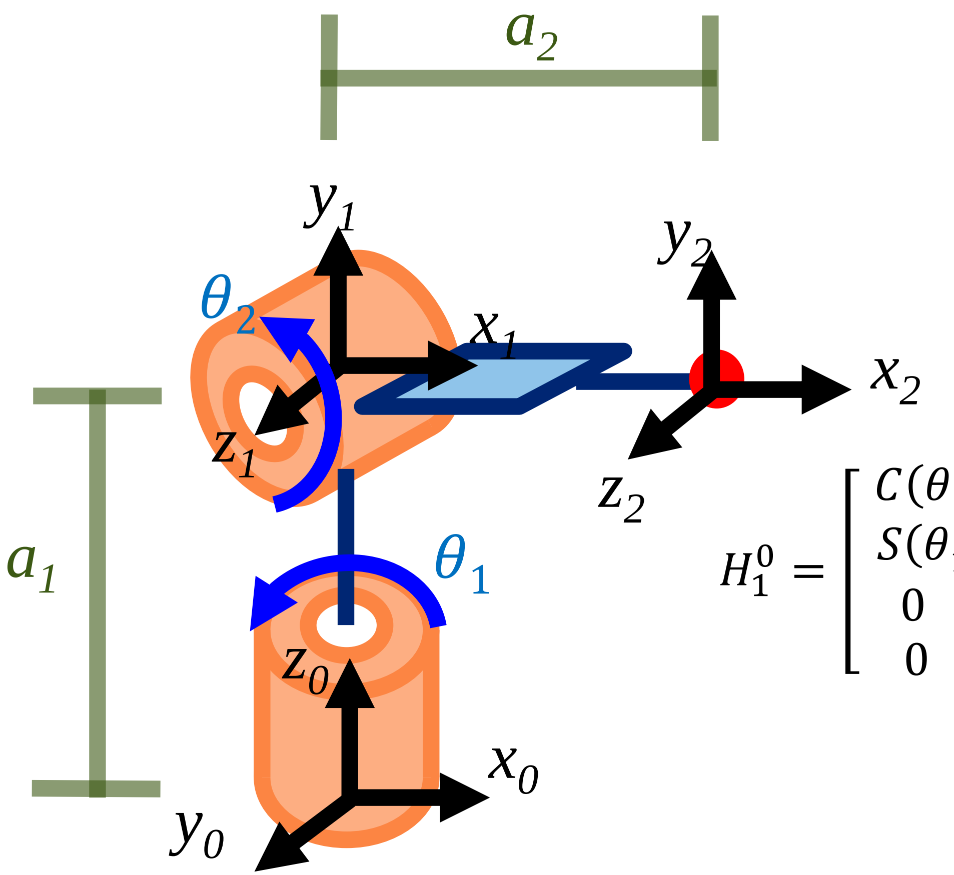


	θ	α	r	d
1	θ_1	90	0	a_1
2	θ_2	0	a_2	0

$$H_1^0 = \begin{bmatrix} C(\theta_1) & 0 & S(\theta_1) & 0 \\ S(\theta_1) & 0 & -C(\theta_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} C(\theta_2) & -S(\theta_2) & 0 & a_2C(\theta_2) \\ S(\theta_2) & C(\theta_2) & 0 & a_2S(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} C(\theta_1) & 0 & S(\theta_1) & 0 \\ S(\theta_1) & 0 & -C(\theta_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C(\theta_2) & -S(\theta_2) & 0 & a_2C(\theta_2) \\ S(\theta_2) & C(\theta_2) & 0 & a_2S(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C(\theta_1)C(\theta_2) & -C(\theta_1)S(\theta_2) & S(\theta_1) & a_2C(\theta_1)C(\theta_2) \\ S(\theta_1)C(\theta_2) & -S(\theta_1)S(\theta_2) & -C(\theta_1) & a_2S(\theta_1)C(\theta_2) \\ S(\theta_2) & C(\theta_2) & 0 & a_2S(\theta_2)+a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



	θ	α	r	d
1	θ_1	90	0	a_1
2	θ_2	0	0	a_2

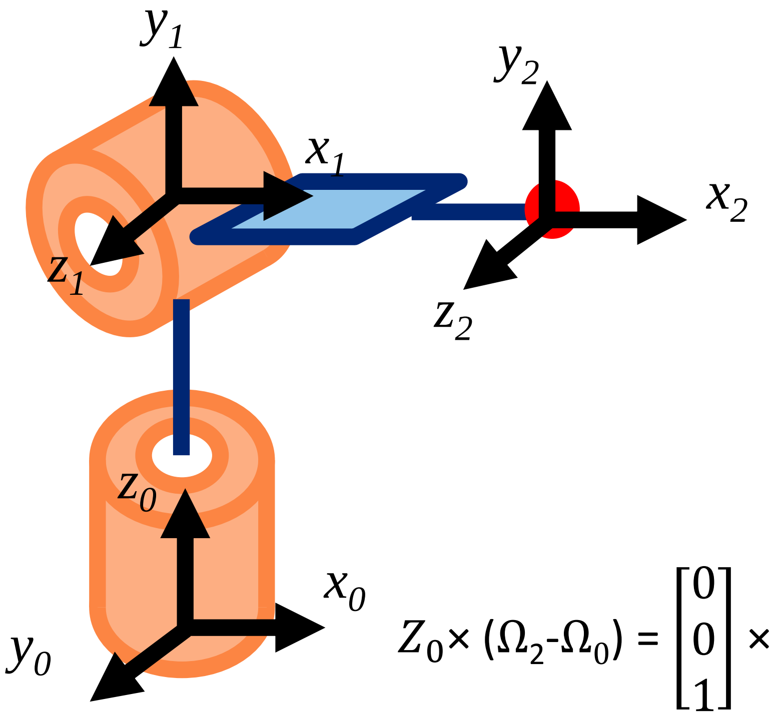
$$H_1^0 = \begin{bmatrix} C(\theta_1) & 0 & S(\theta_1) & 0 \\ S(\theta_1) & 0 & -C(\theta_1) & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \Omega_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} S(\theta_1) \\ -C(\theta_1) \\ 0 \end{bmatrix} \quad \Omega_1 = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix}$$

$$H_2^0 = \begin{bmatrix} C(\theta_1)C(\theta_2) & -C(\theta_1)S(\theta_2) & S(\theta_1) & a_2C(\theta_1)C(\theta_2) \\ S(\theta_1)C(\theta_2) & -S(\theta_1)S(\theta_2) & -C(\theta_1) & a_2S(\theta_1)C(\theta_2) \\ S(\theta_2) & C(\theta_2) & 0 & a_2S(\theta_2) + a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} S(\theta_1) \\ -C(\theta_1) \\ 0 \end{bmatrix} \quad \Omega_2 = \begin{bmatrix} a_2C(\theta_1)C(\theta_2) \\ a_2S(\theta_1)C(\theta_2) \\ a_2S(\theta_2) + a_1 \end{bmatrix}$$



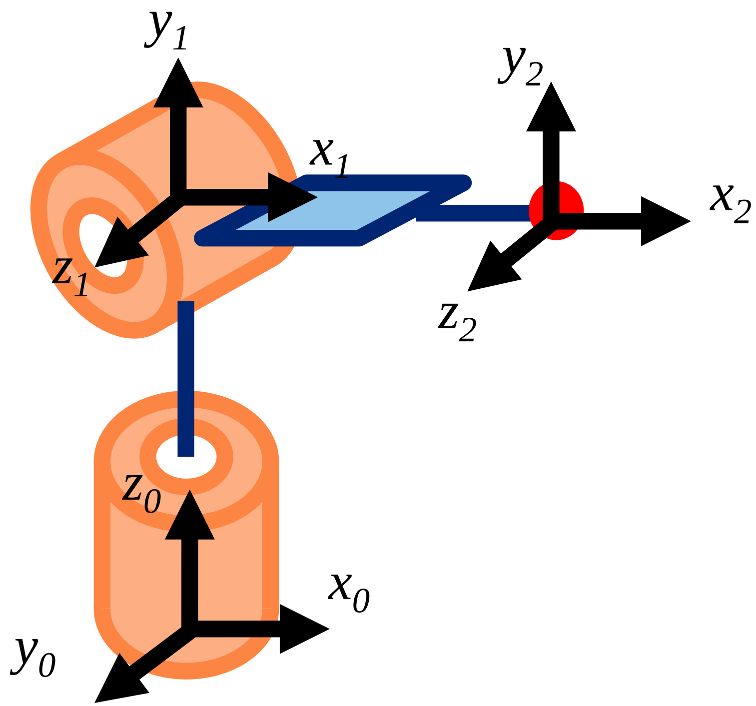
	Prismatic	Revolute
Linear	$R_{i-1}^0\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^0\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$Z_0\times(\Omega_2\text{-}\Omega_0) = \begin{bmatrix}0\\0\\1\end{bmatrix}\times\begin{bmatrix}a_2C(\theta_1)C(\theta_2)\\a_2S(\theta_1)C(\theta_2)\\a_2S(\theta_2)+a_1\end{bmatrix}$$

$$Z_1\times(\Omega_2\text{-}\Omega_1) = \begin{bmatrix}S(\theta_1)\\-C(\theta_1)\\0\end{bmatrix}\times\left(\begin{bmatrix}a_2C(\theta_1)C(\theta_2)\\a_2S(\theta_1)C(\theta_2)\\a_2S(\theta_2)+a_1\end{bmatrix}\text{-}\begin{bmatrix}0\\0\\a_1\end{bmatrix}\right)$$

$$J_1 = \begin{bmatrix}-a_2S(\theta_1)C(\theta_2)\\a_2C(\theta_1)C(\theta_2)\\0\\0\\0\\1\end{bmatrix}$$

$$J_2 = \begin{bmatrix}-a_2C(\theta_1)S(\theta_2)\\-a_2S(\theta_1)S(\theta_2)\\2a_2C(\theta_2)\\S(\theta_1)\\-C(\theta_1)\\0\end{bmatrix}$$



$$\dot{x} = -a_2 S(\theta_1) C(\theta_2) \dot{\theta}_1 - a_2\, C(\theta_1) S(\theta_2) \dot{\theta}_2$$

$$\dot{y} = a_2 C(\theta_1) C(\theta_2)\, \dot{\theta}_1 - a_2 S(\theta_1) S(\theta_2)\, \dot{\theta}_2$$

$$\dot{z} = 2a_2\, C(\theta_2)\, \dot{\theta}_2$$

$$\omega_x = S(\theta_1)\, \dot{\theta}_2$$

$$\omega_y = -C(\theta_1) \dot{\theta}_2$$

$$\omega_z = \dot{\theta}_1$$

$$J = \begin{bmatrix} -a_2 S(\theta_1) C(\theta_2) & -a_2 C(\theta_1) S(\theta_2) \\ a_2 C(\theta_1) C(\theta_2) & -a_2 S(\theta_1) S(\theta_2) \\ 0 & 2a_2 C(\theta_2) \\ 0 & S(\theta_1) \\ 0 & -C(\theta_1) \\ 1 & 0 \end{bmatrix}$$

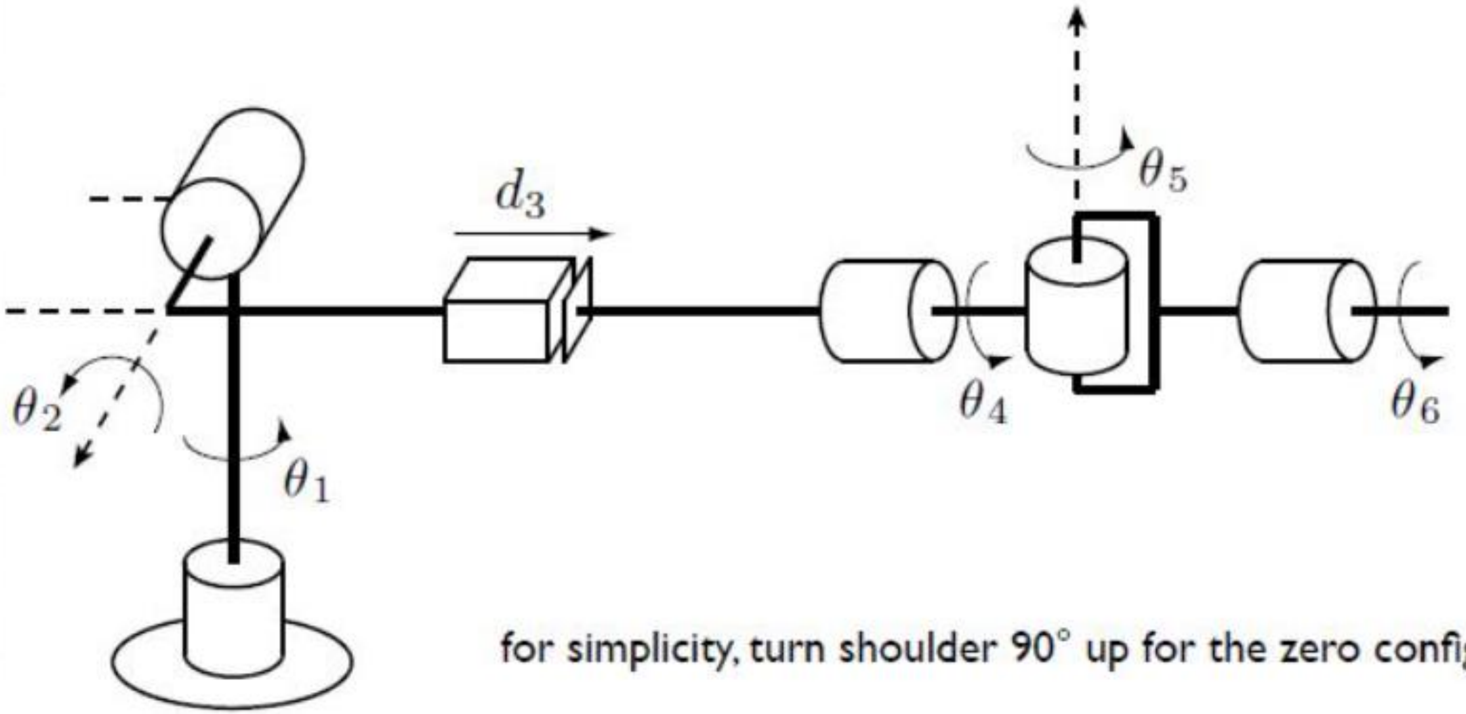
Stanford Manipulator

SECTION 5

Stanford Manipulator

Stanford Manipulator

RRPRRR



for simplicity, turn shoulder 90° up for the zero configuration

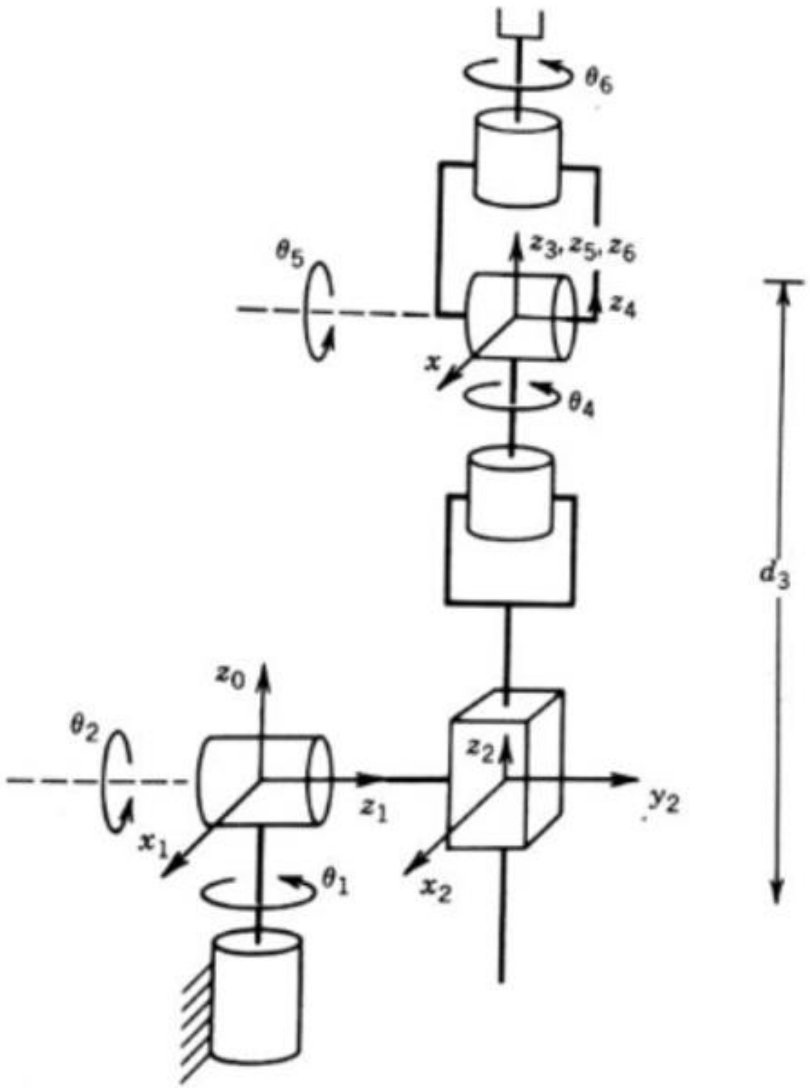


Figure 1: Stanford Manipulator

Parameters	Limitation
θ_1	[-180 180]
θ_2	[-90 90]
d_3	[1 3]
θ_4	[-180 180]
θ_5	[-25 25]
θ_6	[-180 180]

Stanford Manipulator

The DH parameters are:

Link	d_i	a_i	α_i	θ_i
1	0	0	-90	$\star$
2	d_2	0	$+90$	$\star$
3	$\star$	0	0	0
4	0	0	-90	$\star$
5	0	0	$+90$	$\star$
6	d_6	0	0	$\star$

* joint variable

$$A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Manipulator (Arm)

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1$$

$$T_1^0 = A_1 A_2 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & -d_2 s_1 \\ s_1 c_2 & c_1 & s_1 s_2 & d_2 c_1 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & d_3 c_1 s_2 - d_2 s_1 \\ s_1 c_2 & c_1 & s_1 s_2 & d_3 s_1 s_2 + d_2 c_1 \\ -s_2 & 0 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \quad z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \quad z_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

$$O_0 = O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} -d_2 s_1 \\ d_2 c_1 \\ 0 \end{bmatrix} \quad O_3 = \begin{bmatrix} d_3 c_1 s_2 - d_2 s_1 \\ d_3 s_1 s_2 + d_2 c_1 \\ d_3 c_2 \end{bmatrix}$$

Stanford Manipulator (Hand and Wrist)

$T_4^0 = A_1A_2A_3A_4$

$T_5^0 = A_1A_2A_3A_4A_5$

$T_6^0 = A_1A_2A_3A_4A_5A_6$

```
T4 = [  c1c2c4-s1s4,    -c1s2,    -c1c2s4-s1*c4,   c1s2d3-sin1d2]
     [  s1c2c4+c1s4,    -s1s2,    -s1c2s4+c1c4,   s1s2d3+c1*d2]
     [        -s2c4,      -c2,            s2s4,          c2*d3]
     [            0,        0,               0,              1]
```

$$z_4 = \begin{bmatrix} -c_1c_2s_4 - s_1c_4 \\ -s_1c_2s_4 + c_1c_4 \\ s_2s_4 \end{bmatrix} \qquad O_4 = \begin{bmatrix} d_3c_1s_2 - d_2s_1 \\ d_3s_1s_2 + d_2c_1 \\ d_3c_2 \end{bmatrix}$$

Stanford Manipulator

T5 = [(c1c2c4-s1s4)c5-c1s2s5, c1c2s4+s1c4, (c1c2c4-s1s4)s5+c1s2c5, c1s2d3-s1d2]
[(s1c2c4+c1s4)c5-s1s2s5, s1c2s4-c1c4, (s1c2c4+c1s4)s5+s1s2c5, s1s2d3+c1d2]
[-s2c4c5-c2s5, -s2s4, -s2c4s5+c2c5, c2d3]
[0, 0, 0, 1]

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix}$$

Stanford Manipulator

```
T5 = [ (c1c2c4-s1s4)c5-c1s2s5,  c1c2s4+s1c4, (c1c2c4-s1s4)s5+c1s2c5,  c1s2d3-s1d2]
     [ (s1c2c4+c1s4)c5-s1s2s5,  s1c2s4-c1c4, (s1c2c4+c1s4)s5+s1s2c5, s1s2d3+c1d2]
     [             -s2c4c5-c2s5,        -s2s4,            -s2c4s5+c2c5,        c2d3]
     [                        0,            0,                       0,           1]
```

$$z_5 = \begin{bmatrix} c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2c_5 \\ s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 \\ -s_2c_4s_5 + c_2c_5 \end{bmatrix} \qquad O_5 = \begin{bmatrix} d_3c_1s_2 - d_2s_1 \\ d_3s_1s_2 + d_2c_1 \\ d_3c_2 \end{bmatrix}$$

Stanford Manipulator

T6 = [c6c5c1c2c4-c6c5s1s4-c6c1s2s5+s6c1c2s4+s6s1c4, -c5c1c2c4+s6c5s1s4+s6c1s2s5+c6c1c2s4+c6s1c4, s5c1c2c4-s5s1s4+c1s2c5, d6s5c1c2c4-d6s5s1s4+d6c1s2c5+c1s2d3-s1d2]
[c6c5s1c2c4+c6c5c1s4-c6s1s2s5+s6s1c2s4-s6c1c4, -s6c5s1c2c4-s6c5c1s4+s6s1s2s5+c6s1c2s4-c6c1c4, s5s1c2c4+s5c1s4+s1s2c5, d6s5s1c2c4+d6s5c1s4+d6s1s2c5+s1s2d3+c1d2]
[-c6s2c4c5-c6c2s5-s2s4s6, s6s2c4c5+s6c2s5-s2s4c6, -s2c4s5+c2c5, -d6s2c4s5+d6c2c5+c2d3]
[0, 0, 0, 1]

$$O_6 = \begin{bmatrix} \mathrm{d6s5c1c2c4} - \mathrm{d6s5s1s4} + \mathrm{d6c1s2c5} + \mathrm{c1s2d3} - \mathrm{s1d2} \\ \mathrm{d6s5s1c2c4} + \mathrm{d6s5c1s4} + \mathrm{d6s1s2c5} + \mathrm{s1s2d3} + \mathrm{c1d2} \\ -\mathrm{d6s2c4s5} + \mathrm{d6c2c5} + \mathrm{c2d3} \end{bmatrix}$$

Stanford Manipulator

$$J_1 = \begin{bmatrix} z_0 \times (o_6 - o_0) \\ z_0 \end{bmatrix}, J_2 = \begin{bmatrix} z_1 \times (o_6 - o_1) \\ z_1 \end{bmatrix}$$

Joints 1,2 are revolute

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix}$$

Joint 3 is prismatic

$$J_4 = \begin{bmatrix} z_3 \times (o_6 - o_3) \\ z_3 \end{bmatrix}, J_5 = \begin{bmatrix} z_4 \times (o_6 - o_4) \\ z_4 \end{bmatrix}, J_6 = \begin{bmatrix} z_5 \times (o_6 - o_5) \\ z_5 \end{bmatrix}$$

The required Jacobian matrix **J**

$$J = \begin{bmatrix} J_1 & J_2 J_3 J_4 J_5 J_6 \end{bmatrix}$$

Inverse Velocity

- The relation between the joint and end-effector velocities:

$$\dot{X} = J(q)\dot{q}$$

- where j (m×n). If J is a square matrix (m=n), the joint velocities:

$$\dot{q} = J^{-1}(q)\dot{X}$$

- If m<n, let pseudoinverse J^{+} where

$$\dot{q} = J^{+}(q)\dot{X} \qquad J^{+} = J^{T}[JJ^{T}]^{-1}$$

Acceleration

- The relation between the joint and end-effector velocities:

$$\dot{X} = J(q)\dot{q}$$

- Differentiating this equation yields an expression for the acceleration:

$$\ddot{X} = J(q)\ddot{q} + [\frac{d}{dt}J(q)]\dot{q}$$

- Given $\ddot{X}$ of the end-effector acceleration, the joint acceleration $\ddot{q}$

$$J(q)\ddot{q} = \ddot{X} - [\frac{d}{dt}J(q)]\dot{q} \qquad \ddot{q} = J^{-1}(q)[\ddot{X} - \frac{d}{dt}J(q)]\dot{q}]$$