Introduction to Robotics

Manipulation and Programming

Unit 2: Kinematics

PYTHON LAB: SOLVING THE JOINT VARIABLES

DR. ERIC CHOU

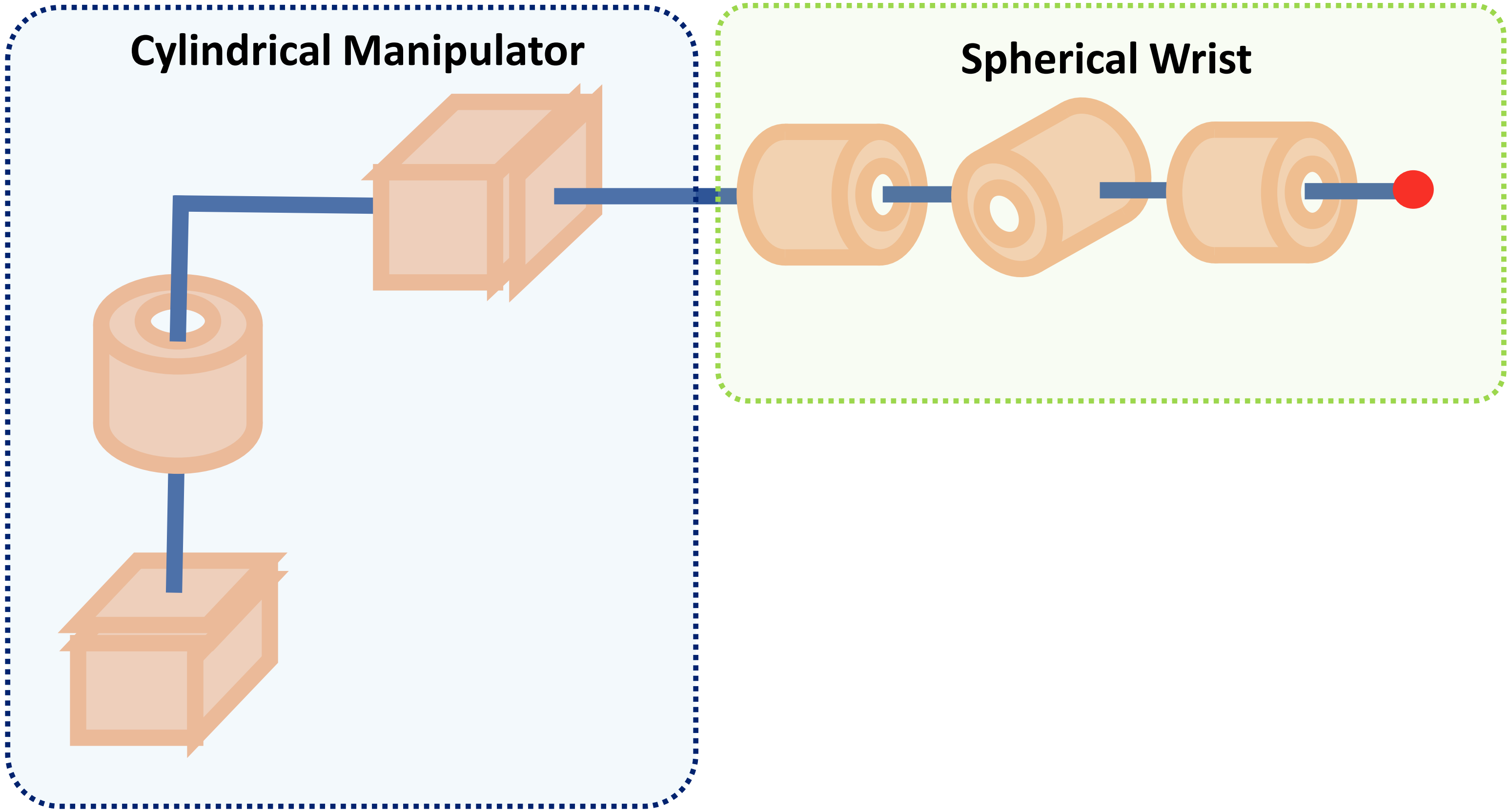
IEEE SENIOR MEMBER

Objectives

- Demonstrate how joint variables can be solved

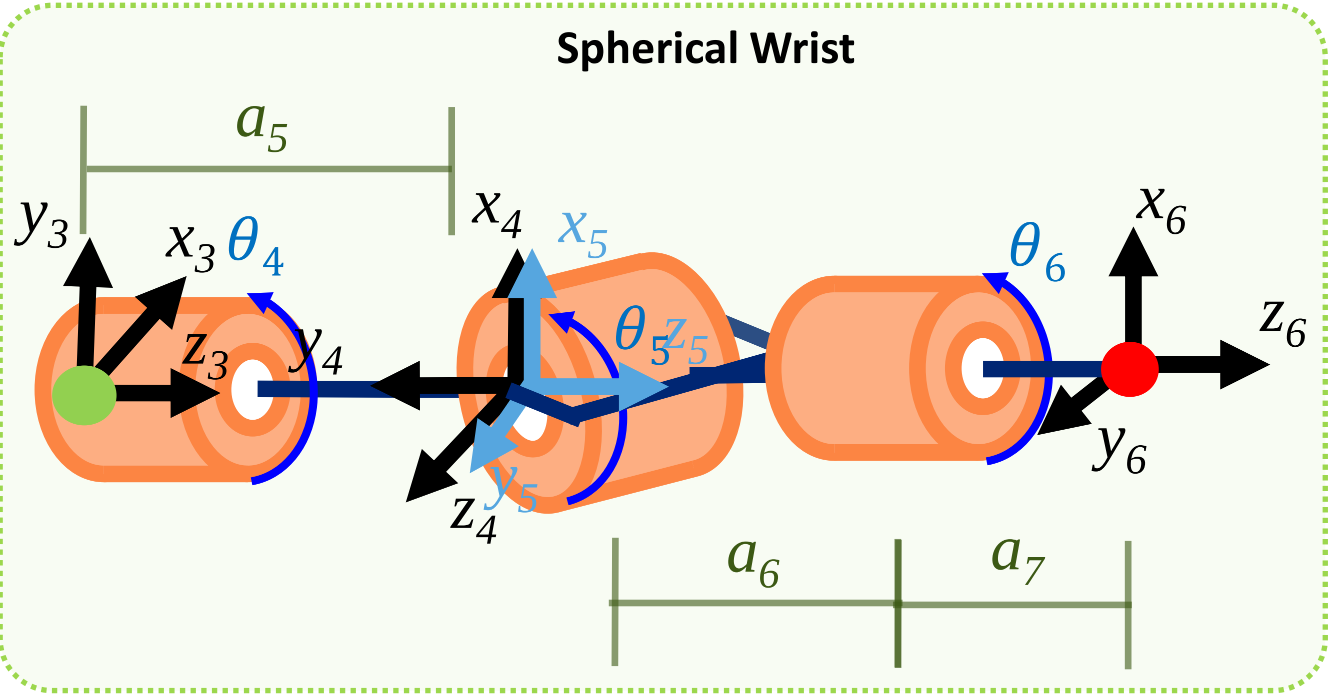
Step 4-7 and Python Example

SECTION 1


Cylindrical Manipulator
Spherical Wrist

Assumption

- The **first three joints** are entirely responsible for **POSITIONING** the end-effector, and any additional joints are responsible for **ORIENTING** the end-effector.



	θ	α	r	d
4	$90+\theta_4$	-90	0	a_5
5	θ_5	90	0	0
6	θ_6	0	0	a_6+a_7

$$H_4^3 = \begin{bmatrix} -S\theta_4 & 0 & -C\theta_4 & 0 \\ C\theta_4 & 0 & -S\theta_4 & 0 \\ 0 & -1 & 0 & a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_5^4 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C(\theta_n+90) = -S\theta_n$$
$$S(\theta_n+90) = C\theta_n$$

$$H_5^3 = \begin{bmatrix} -S\theta_4 & 0 & -C\theta_4 & 0 \\ C\theta_4 & 0 & -S\theta_4 & 0 \\ 0 & -1 & 0 & a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -S\theta_4 C\theta_5 & -C\theta_4 & -S\theta_4 S\theta_5 & 0 \\ C\theta_4 C\theta_5 & -S\theta_4 & C\theta_4 S\theta_5 & 0 \\ -S\theta_5 & 0 & C\theta_5 & a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_n^{n-1} = \begin{bmatrix} C\theta_n & -S\theta_n C\alpha_n & S\theta_n S\alpha_n & r_n C\theta_n \\ S\theta_n & C\theta_n C\alpha_n & -C\theta_n S\alpha_n & r_n S\theta_n \\ 0 & S\alpha_n & C\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_6^5 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & a_6+a_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_6^3 = \begin{bmatrix} -S\theta_4 C\theta_5 & -C\theta_4 & -S\theta_4 S\theta_5 & 0 \\ C\theta_4 C\theta_5 & -S\theta_4 & C\theta_4 S\theta_5 & 0 \\ -S\theta_5 & 0 & C\theta_5 & a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{a}_6+\mathrm{a}_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -S\theta_4 C\theta_5 C\theta_6 - C\theta_4 S\theta_6 & S\theta_4 C\theta_5 S\theta_6 - C\theta_4 C\theta_6 & -S\theta_4 S\theta_5 & -(\mathrm{a}_6+\mathrm{a}_7) S\theta_4 S\theta_5 \\ C\theta_4 C\theta_5 C\theta_6 - S\theta_4 S\theta_6 & -C\theta_4 C\theta_5 S\theta_6 - S\theta_4 C\theta_6 & C\theta_4 S\theta_5 & (\mathrm{a}_6+\mathrm{a}_7) C\theta_4 S\theta_5 \\ -S\theta_5 C\theta_6 & S\theta_5 S\theta_6 + C\theta_5 & C\theta_5 & (\mathrm{a}_6+\mathrm{a}_7) C\theta_5 + a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PRPRRR Manipulator

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0-d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$Z_0=\begin{bmatrix}0\\0\\1\end{bmatrix}\quad Z_1=\begin{bmatrix}0\\0\\1\end{bmatrix}\quad Z_2=\begin{bmatrix}C\theta_2\\S\theta_2\\0\end{bmatrix}\quad Z_3=\begin{bmatrix}C\theta_2\\S\theta_2\\0\end{bmatrix}$$

$$\Omega_0=\begin{bmatrix}0\\0\\0\end{bmatrix}\quad \Omega_1=\begin{bmatrix}0\\0\\a_1+d_1\end{bmatrix}\quad \Omega_2=\begin{bmatrix}0\\0\\a_1+a_2+d_1\end{bmatrix}\quad \Omega_3=\begin{bmatrix}(a_3+a_4+d_3)C\theta_2\\(a_3+a_4+d_3)S\theta_2\\a_1+a_2+d_1\end{bmatrix}$$

$$H_3^0=\begin{bmatrix}-S\theta_2 & 0 & C\theta_2 & (a_3+a_4+d_3)C\theta_2\\ C\theta_2 & 0 & S\theta_2 & (a_3+a_4+d_3)S\theta_2\\ 0 & 1 & 0 & a_1+a_2+d_1\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$H_4^3=\begin{bmatrix}-S\theta_4 & 0 & -C\theta_4 & 0\\ C\theta_4 & 0 & -S\theta_4 & 0\\ 0 & -1 & 0 & a_5\\ 0 & 0 & 0 & 1\end{bmatrix}\quad H_5^3=\begin{bmatrix}-S\theta_4C\theta_5 & -C\theta_4 & -S\theta_4S\theta_5 & 0\\ C\theta_4C\theta_5 & -S\theta_4 & C\theta_4S\theta_5 & 0\\ -S\theta_5 & 0 & C\theta_5 & a_5\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$H_6^3=\begin{bmatrix}-C\theta_4S\theta_5C\theta_6+S\theta_4S\theta_6 & C\theta_4S\theta_5S\theta_6+S\theta_4C\theta_6 & C\theta_4C\theta_5 & (a_6+a_7)C\theta_4C\theta_5\\ -S\theta_4S\theta_5C\theta_6-C\theta_4S\theta_6 & S\theta_4S\theta_5S\theta_6-C\theta_4C\theta_6 & S\theta_4C\theta_5 & (a_6+a_7)S\theta_4C\theta_5\\ C\theta_5C\theta_6 & -C\theta_5S\theta_6 & S\theta_5 & (a_6+a_7)S\theta_5+a_5\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$H_4^0=H_3^0H_4^3$$

$$H_5^0=H_3^0H_5^3$$

$$H_6^0=H_3^0H_6^3$$

R_3^0, R_6^3, R_6^0 can all be extracted

Step 4

Do forward kinematics on the last three joints and pull out the rotation part, R3_6

Step 5

Specify what you want the rotation matrix R0_6 to be

Step 6

Given a desired X, Y, and Z position, solve for the first three joints using the inverse kinematics equations from Step 1

Step 7

Plug in those variables and use the rotation matrix to solve for the last three joints.

Summary

SECTION 5

Summary

- This inverse kinematics calculation is to solve the joint variable values in a numerical way. The formulas are found as the next slide
- In general, there are 6 variables. There might be more than 1 answer.
- For velocity and acceleration, we still need to use Jacobian matrix to find the derivatives for the join variable.

```python
import numpy as np
E = 10

X = 5.0
Y = 0.0
Z = 3.0

a1 = 1
a2 = 1
a3 = 1
a4 = 1
a5 = 1
a6 = 1

d1 = Z -a1 -a2
T2 = np.arctan(Y/X)
d3 = np.sqrt(X**2+Y**2)-a3-a4
```

```python
R0_6 = [
     [-1.0, 0.0, 0.0],
     [0.0, -1.0, 0.0],
     [0.0, 0.0, 1.0]
    ]

R0_3 = [
     [-np.sin(T2), 0.0, np.cos(T2)],
     [np.cos(T2), 0.0, np.sin(T2)],
     [0.0, 1.0, 0.0]
    ]

R0_3inv = np.linalg.inv(R0_3)
R3_6 = np.dot(R0_3inv, R0_6)

print("R3_6=", np.matrix(R3_6))


T5 = np.arccos(R3_6[2][2])
T6 = np.arccos(-R3_6[2][0]/np.sin(T5))
T4 = np.arccos(R3_6[1][2]/np.sin(T5))
```

```
print("d1=", d1)
print("T2=", T2, "radians")
print("d3=", d3)
print("T4=", T4, "radians")
print("T5=", T5, "radians")
print("T6=", T6, "radians")
```

```
R3_6= [[ 0. -1.  0.]
 [ 0.  0.  1.]
 [-1.  0.  0.]]
d1= 1.0
T2= 0.0 radians
d3= 3.0
T4= 0.0 radians
T5= 1.5707963267948966 radians
T6= 0.0 radians
```