Introduction to Robotics

Manipulation and Programming

Unit 2: Kinematics

HOMOGENOUS TRANSFORMATION MATRICES

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Objectives

- Combination of Rotational Matrix and Displacement Vector
- Python coding lab with HTM (Homogeneous Transformation Matrix)
- Denavit-Hartenberg parameters

Homogeneous Transformation Matrix

SECTION 1

Why is it called "homogeneous"?

- As mentioned before, in order to convert from Homogeneous coordinates (x, y, w) to Cartesian coordinates, we simply divide x and y by w;

$$\underset{\text{Homogeneous}}{(x, y, w)} \Leftrightarrow \underset{\text{Cartesian}}{\left(\frac{x}{w}, \frac{y}{w}\right)}$$

Homogeneous Transformation

p(x, y)

$$p = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$x' = S_x\, x$$
$$y' = S_y\, y$$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate

$$x' = x\cos(\theta) - y\, sin(\theta)$$
$$y' = x\sin(\theta) + y\, cos(\theta)$$

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translate

$$x' = x + a$$
$$y' = y + b$$

$$T = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = p' = S\,p = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Application of HTM

SECTION 2

Series of Matrix Operations

Rotation:

$$R = R_1^0 \, R_2^1 \, R_3^2$$

Displacement (Translation):

$$d \neq d_1^0 \, d_2^1 \, d_3^2$$

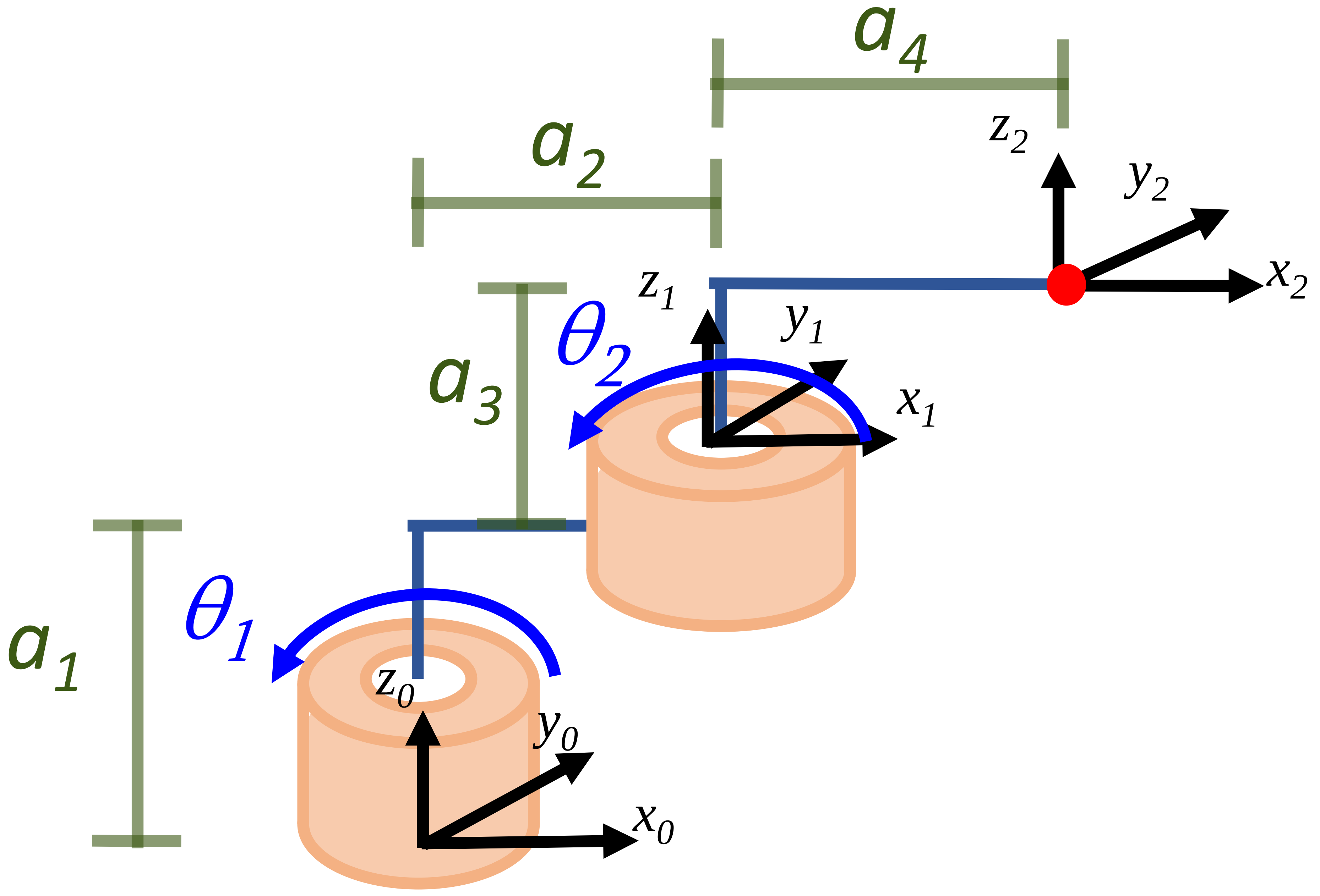
HTM

- Combination of Rotation and Translation into one matrix and this matrix is viable for series of transformation by multiplication of matrices.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = p' = H\, p = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & d_x \\ \sin(\theta) & cos(\theta) & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x\cos(\theta) - y\, sin(\theta) + d_x$$
$$y' = x\sin(\theta) + y\, cos(\theta) + d_y$$

$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$$



a_4
a_2
z_2
y_2
x_2
z_1
y_1
x_1
θ_2
a_3
θ_1
a_1
z_0
y_0
x_0

$$R = R_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^0 = \begin{bmatrix} a_2 \cos(\theta_1) \\ a_2 \sin(\theta_1) \\ a_1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_2 \cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_2 \sin(\theta_1) \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = R_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_2^1 = \begin{bmatrix} a_4 \cos(\theta_2) \\ a_4 \sin(\theta_2) \\ a_3 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_4 \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_4 \sin(\theta_2) \\ 0 & 0 & 1 & a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 \, H_2^1$$

Python Example

SECTION 3

HTM

- Rotational Matrix
- Displacement Vector
- Homogeneous Transformation Matrix