Introduction to Robotics

Manipulation and Programming

Unit 2: Kinematics

ROTATION MATRICES

DR. ERIC CHOU

IEEE SENIOR MEMBER

Objective

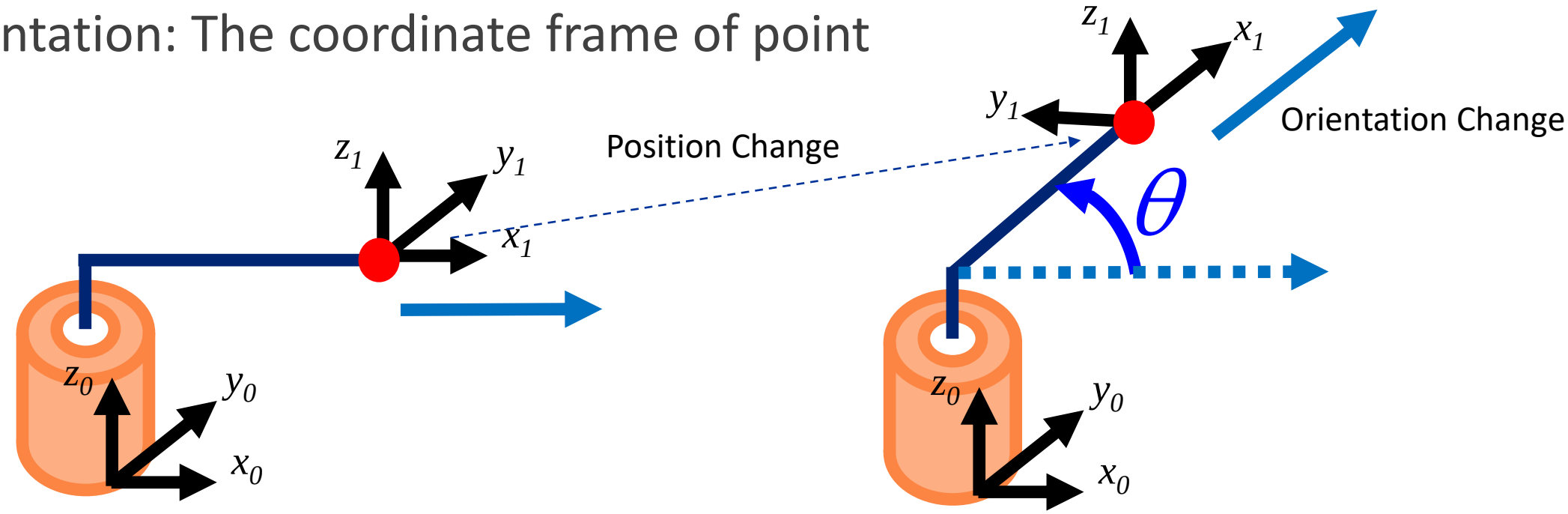
- Understand how to convert each Kinematics Diagram into rotational matrix
- Learn how to put rotational matrix to python programs

Coordination

SECTION 1

Movement and Changes by Rotation

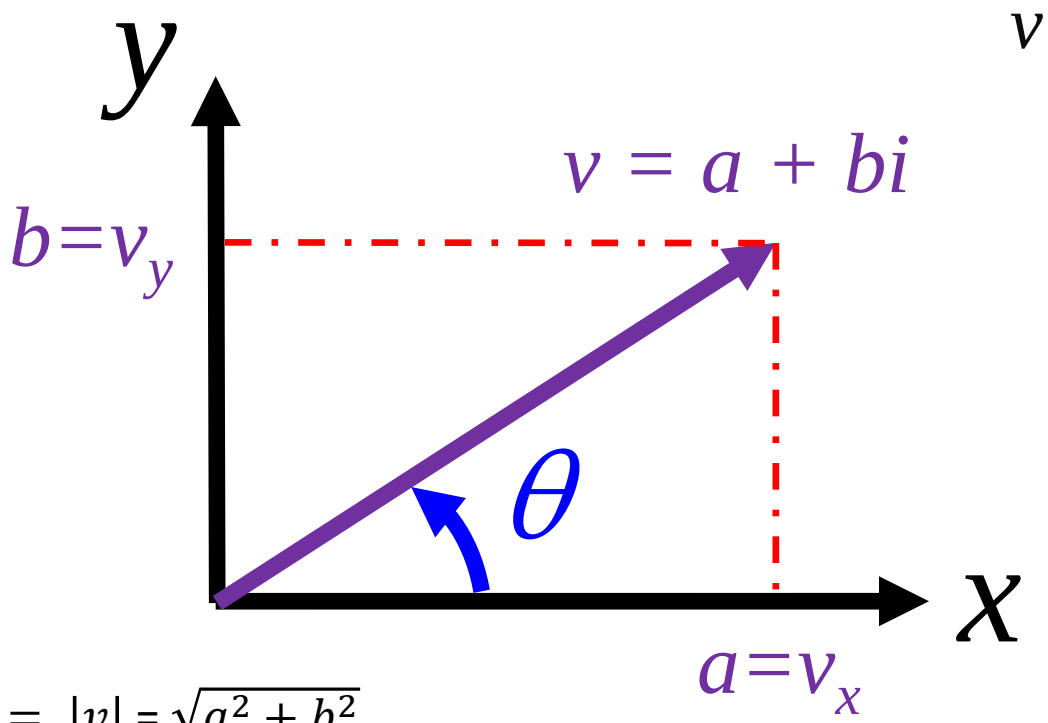
- Every time a rotation happens, position and orientation may change.
- Position: The coordinates of a point
- Orientation: The coordinate frame of point



Projection

SECTION 2

Projection 2D



$r = |v| = \sqrt{a^2 + b^2}$

cos(θ) = a/r

sin(θ) = b/r

$$v = [v_x, v_y] = v_x \overrightarrow{e_x} + v_y \overrightarrow{e_y} =$$

$$= [v_x, v_y] \begin{bmatrix} \overrightarrow{e_x} \\ \overrightarrow{e_y} \end{bmatrix} = [v_x, v_y] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

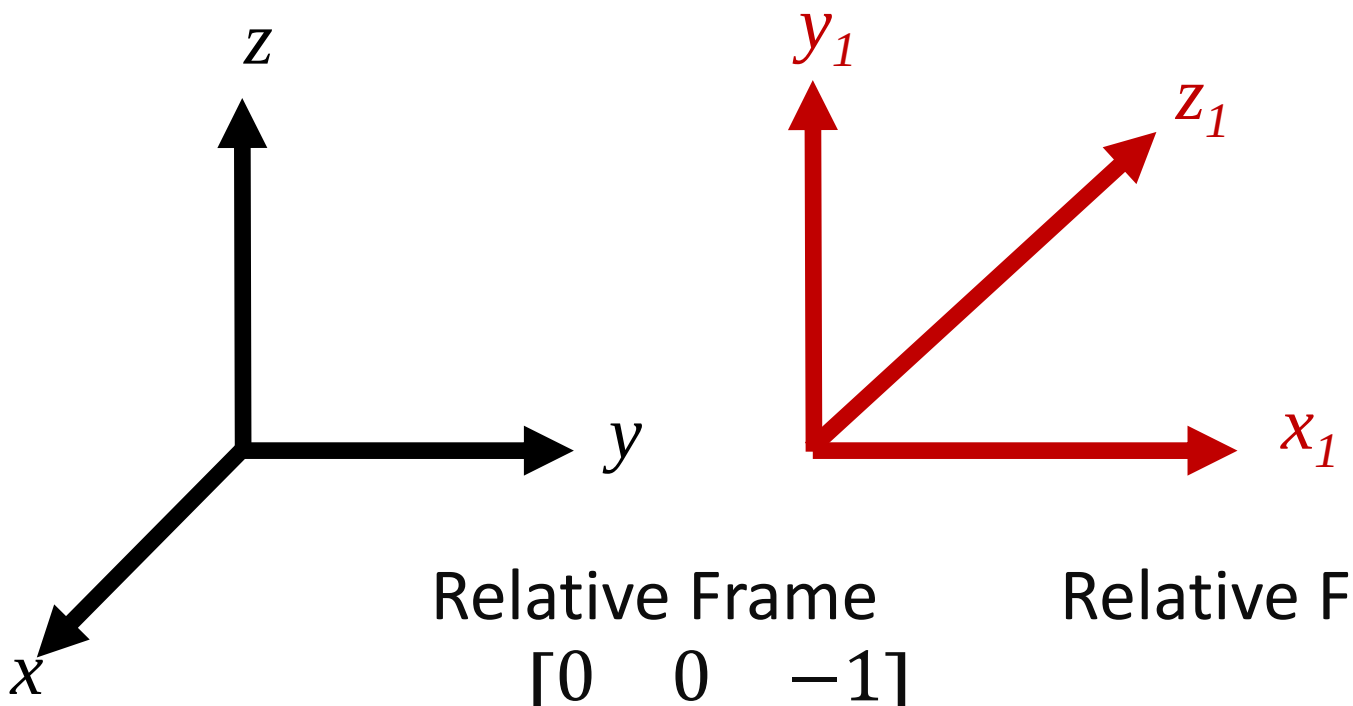
$$= [r\cos(\theta), r\sin(\theta)] = r\,[\cos(\theta), \sin(\theta)]$$

$$= r\cos(\theta) + i\,r\sin(\theta) = r\,(\cos(\theta) + i\sin(\theta))$$

$$= r\,e^{i\theta}$$

Orientation Matrix

SECTION 3



Relative Frame

$$R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Relative Frame

$$[x_1, y_1, z_1] = [x, y, z] \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Relative Frame

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

Relative Frame

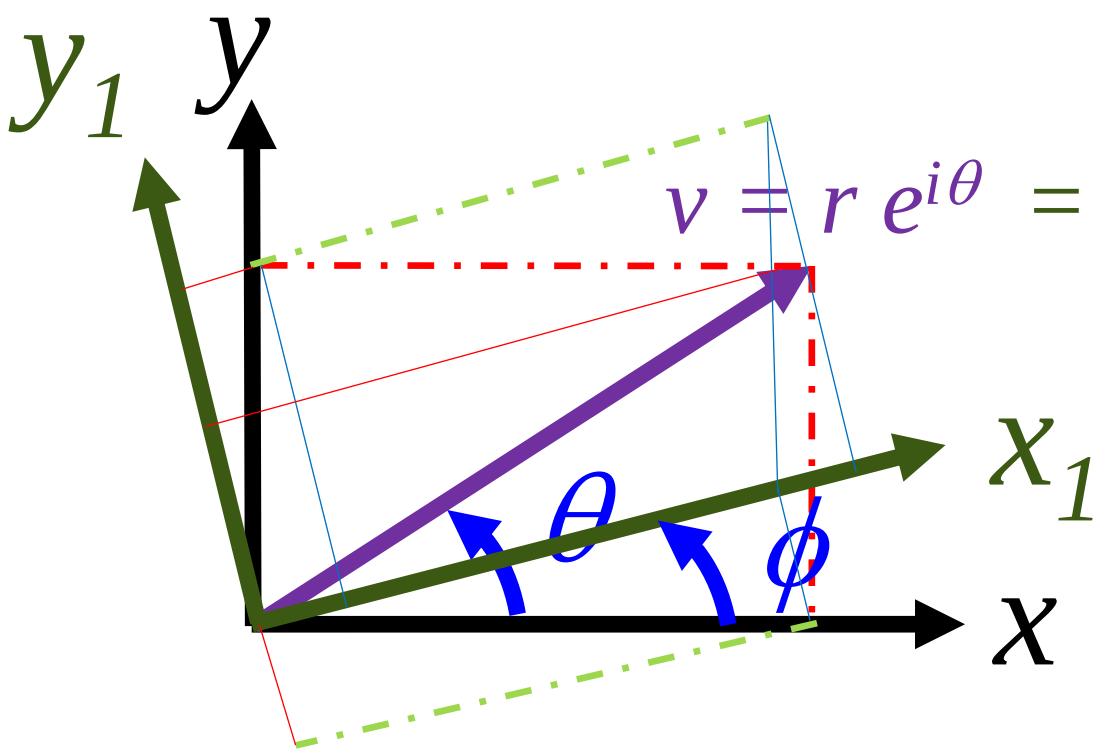
$$[x, y, z] = [x_1, y_1, z_1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

Rotation

SECTION 4

Rotation

Vector rotate -ϕ, or Frame rotate ϕ

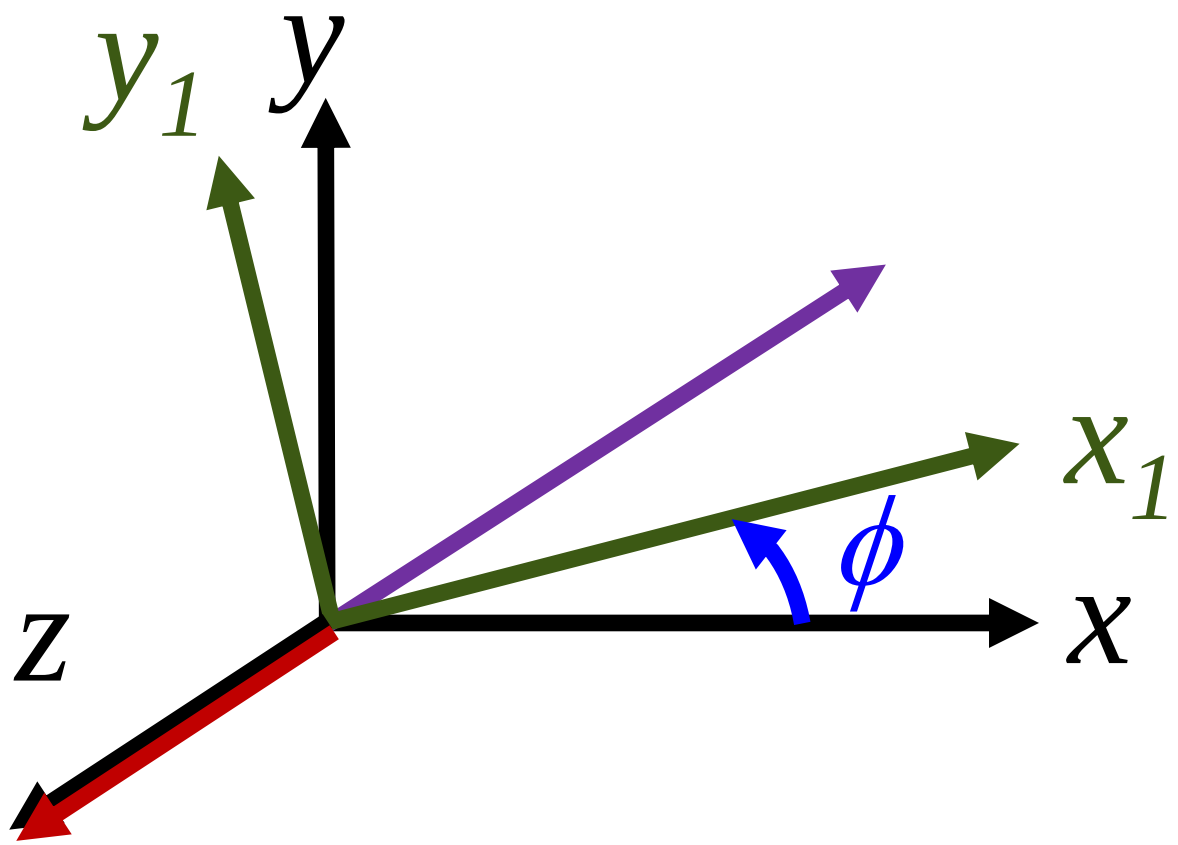


$$[X, Y] = [x, y] \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}$$

Rotational Matrix

SECTION 5

Rotational Matrix

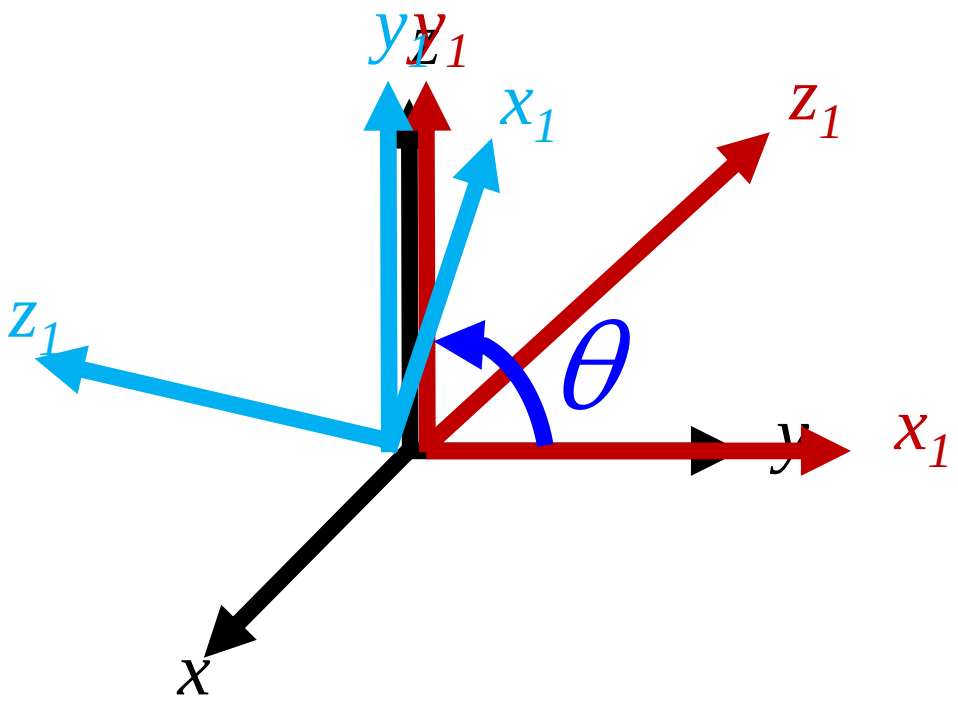


$$R_z = \begin{bmatrix} cos(\phi) & -sin(\phi) & 0 \\ sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Z = \begin{bmatrix} cos(\phi) & -sin(\phi) & 0 \\ sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} cos(\phi) & 0 & sin(\phi) \\ 0 & 1 & 0 \\ -sin(\phi) & 0 & cos(\phi) \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & sin(\phi) & cos(\phi) \end{bmatrix}$$



$$[x, y, z] = [x_1, y_1, z_1]\ \mathrm{R}_1$$

$$[x_1, y_1, z_1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} cos(\phi) & 0 & -sin(\phi) \\ sin(\phi) & 0 & cos(\phi) \\ 0 & 1 & 0 \end{bmatrix}$$

Python Coding

SECTION 6

Summary

SECTION 7