CBC MATHEMATICS
MATH 2412-PreCalculus
Exam Formula Sheets

- System of Equations and Matrices
  - 3 Matrix Row Operations:
    - Switch any two rows.
    - Multiply any row by a nonzero constant.
    - Add any constant-multiple row to another
- Even and Odd functions
  - Even function: $f(-x) = f(x)$ Odd function: $f(-x) = -f(x)$
- Graph Symmetry
  - x-axis symmetry: if (x, y) is on the graph, then $(x, -y)$ is also on the graph
  - y-axis symmetry: if (x, y) is on the graph, then $(-x, y)$ is also on the graph
  - origin symmetry: if (x, y)f is on the graph, then $(-x, -y)$ is also on the graph
- Function Transformations
  - <u>Stretch and Compress</u>
    - $y = af(x), \quad a > 0$ vertical: stretch $f(x)$ if $a > 1$
  - <u>Reflections</u>
    - $y = -f(x)$ reflect $f(x)$ about x-axis
    - $y = f(-x)$ reflect $f(x)$ about y-axis
  - <u>Stretch and Compress</u>
    - $y = af(x), \quad a > 0$ vertical: stretch $f(x)$ if $a > 1$
: compress $f(x)$ if $0 < a < 1$
    - $y = f(ax), \quad a > 0$ horizontal: stretch $f(x)$ if $0 < a < 1$
: compress $f(x)$ if $a > 1$
  - <u>Shifts</u>
    - $y = f(x) + k, \quad k > 0$ vertical: shift $f(x)$ up
$y = f(x) - k, \quad k > 0$: shift $f(x)$ down
    - $y = f(x + h) \quad h > 0$ horizontal: shift $f(x)$ left
$y = f(x - h), \quad h > 0$: shift $f(x)$ right

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- Formulas/Equations
  - Slope Intercept: $y = mx + b$ Point-Slope: $y - y_1 = m(x - x_1)$
  - Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$; $x_2 - x_1 \neq 0$
  - Average Rate of Change: $\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$, where $a \neq b$
  - Circle: $Circumference = 2\pi r = \pi d$, $Area = \pi r^2$
  - Triangle: $Area = \frac{1}{2}bh$
  - Rectangle: $Perimeter = 2l + 2w$, $Area = lw$
  - Rectangular Solid: $Volume = lwh$, $Surface\ Area = 2lw + 2lh + 2wh$
  - Sphere: $Volume = \frac{4}{3}\pi r^3$, $Surface\ Area = 4\pi r^2$
  - Right Circular Cylinder: $Volume = \pi r^2 h$, $Surface\ Area = 2\pi r^2 + 2\pi rh$
- General Form of Quadratic Function: $f(x) = ax^2 + bx + c$, $(a \neq 0)$
  - Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - Vertex (h, k): $h = -\frac{b}{2a}$ $k = a(h)^2 + b(h) + c$,

    or $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, or $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$
  - Axis of symmetry: $x = h$
- Vertex Form of Quadratic Function: $f(x) = a(x - h)^2 + k$ vertex (h, k)
- Polynomial function: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$
  - Polynomial graph has at most $n - 1$ turning points.
  - Remainder Theorem
    - If polynomial $f(x) \div (x - c)$, remainder is $f(c)$.
  - Factor Theorem
    - If $f(c) = 0$, then $x - c$ is a linear factor of $f(x)$.
    - If $x - c$ is a linear factor of $f(x)$, then $f(c) = 0$.

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- Rational Zeros Theorem: for polynomial function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$ having degree of at least 1 and integer coefficients with $a_n \neq 0$, $a_0 \neq 0$
  - If $\frac{p}{q}$, in lowest terms, is a rational zero of f, then p must be a factor of a_0, and q must be a factor of a_n.
- Intermediate Value Theorem(for continuous function $f(x)$)
  - If $a < b$ and if $f(a)$ and $f(b)$ have opposite signs, then $f(x)$ has at least one real zero between $x = a$ and $x = b$.
- Conjugate Pairs Theorem
  - For polynomial functions $f(x)$ with real coefficients: If $x = a + bi$ is a zero of $f(x)$, then $x = a - bi$ is also.

❑ Rational function: $f(x) = \frac{p(x)}{q(x)}$, $p(x)$ and $q(x)$ polynomials, but $q(x) \neq 0$.

- Vertical Asymptote: $x =$ zero of denominator in reduced $f(x)$
- Horizontal Asymptote:
  - $y = 0$ if degree of $p(x) <$ degree of $q(x)$
  - $y = \frac{leading\ coefficient\ of\ p(x)}{leading\ coefficient\ of\ q(x)}$ if degree of $p(x) =$ degree of $q(x)$
- Oblique Asymptote:
  - $y = quotient$ of $\frac{p(x)}{q(x)}$ if degree of $p(x) >$ degree of $q(x)$

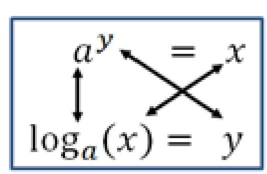
❑ Composite Function $(f \circ g)(x) = f(\,g(x)\,)$

❑ Exponential Function: $f(x) = a^x$

- If $a^u = a^v$, then $u = v$

❑ Logarithmic Function: $f(x) = \log_a(x)$

- $\log_a(1) = 0$, $\log_a(a) = 1$, $a^{\log_a(M)} = M$, $\log_a(a^p) = p$
- $\log_a(\,M \cdot N\,) = \log_a(M) + \log_a(N)$
- $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$
- $\log_a(M^p) = p \cdot \log_a(M)$
- If $\log_a(M) = \log_a(N)$, then $M = N$.



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- If $M = N$, then $\log_a(M) = \log_a(N)$.
- Change of Base formula $\log_a(M) = \frac{\log(M)}{\log(a)}$ or $\log_a(M) = \frac{\ln(M)}{\ln(a)}$

Exponential Models Formulas

- Simple Interest: $I = Prt$
- Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{n\cdot t}$
- Continuous Compounding: $A = Pe^{r\cdot t}$
- Effective Rate of Interest:

  Compounding n times per year $r_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

  Compounding continuously per year $r_{eff} = e^r - 1$
- Growth & Decay: $A(t) = A_0 e^{k\cdot t}$
- Newton's Law of Cooling: $u(t) = T + (u_0 - T)e^{k\cdot t}$
- Logistic Model: $P(t) = \frac{c}{1 + ae^{-b\cdot t}}$

Sequences and Series

- $n! = n(n-1)(n-2)\cdot \cdots \cdot (3)(2)(1)$
- $P(n, r) = \frac{n!}{(n-r)!}$ $C(n, r) = \frac{n!}{r!(n-r)!}$
- Arithmetic Sequence:

  n^{th} term $a_n = a_1 + (n-1)d$

  Sum of first n terms $S_n = \sum_{k=1}^{n}(a_1 + (k-1)d) = \frac{n}{2}(a_1 + a_n)$

  or $S_n = \sum_{k=1}^{n}(a_1 + (k-1)d) = \frac{n}{2}(2a_1 + (n-1)d)$.
- Geometric Sequence:

  n^{th} term $a_n = a_1(r)^{n-1}$

  Sum of first n terms $S_n = \sum_{k=1}^{n} a_1 r^{k-1} = a_1 \cdot \frac{1-r^n}{1-r}$ for $r \neq 0,1$
- Geometric Series: $\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r}$ if $|r| < 1$

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- Binomial Theorem:

$$(x+a)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} a^j = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} a + \cdots + \binom{n}{n-1} x a^{n-1} + \binom{n}{n} a^n$$

- Trigonometry
  - Circular Measure and Motion Formulas
    - Arc Length $s = r\theta$ Area of Sector $A = \frac{1}{2} r^2 \theta$
    - Linear Speed $v = \frac{s}{t}$, $v = r\omega$ Angular Speed $\omega = \frac{\theta}{t}$
  - Acute Angle
    - $\sin(\theta) = \frac{b}{c} = \frac{opposite}{hypotenuse}$ $\cos(\theta) = \frac{a}{c} = \frac{adjacent}{hypotenuse}$ $\tan(\theta) = \frac{b}{a} = \frac{opposite}{adjacent}$
    - $\csc(\theta) = \frac{c}{b} = \frac{hypotenuse}{opposite}$ $\sec(\theta) = \frac{c}{a} = \frac{hypotenuse}{adjacent}$ $\cot(\theta) = \frac{a}{b} = \frac{adjacent}{opposite}$
  - General Angle
    - $\sin(\theta) = \frac{b}{r}$ $\cos(\theta) = \frac{a}{r}$ $\tan(\theta) = \frac{b}{a}$
    - $\csc(\theta) = \frac{r}{b}, b \neq 0$ $\sec(\theta) = \frac{r}{a}, a \neq 0$ $\cot(\theta) = \frac{a}{b}, b \neq 0$
  - Cofunctions
    - $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$, $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$, $\tan(\theta) = \cot\left(\frac{\pi}{2} - \theta\right)$
    - $\csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$, $\sec(\theta) = \csc\left(\frac{\pi}{2} - \theta\right)$, $\cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$
  - Fundamental Identities
    - $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
    - $\csc(\theta) = \frac{1}{\sin(\theta)}$ $\sec(\theta) = \frac{1}{\cos(\theta)}$ $\cot(\theta) = \frac{1}{\tan(\theta)}$
    - $\sin^2(\theta) + \cos^2(\theta) = 1$ $\tan^2(\theta) + 1 = \sec^2(\theta)$ $\cot^2(\theta) + 1 = \csc^2(\theta)$
  - Even-Odd Identities
    - $\sin(-\theta) = -\sin(\theta)$ $\cos(-\theta) = \cos(\theta)$ $\tan(-\theta) = -\tan(\theta)$
    - $\csc(-\theta) = -\csc(\theta)$ $\sec(-\theta) = \sec(\theta)$ $\cot(-\theta) = -\cot(\theta)$
  - Inverse Functions
    - $y = \sin^{-1}(x)$ means $x = \sin(y)$ where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
    - $y = \cos^{-1}(x)$ means $x = \cos(y)$ where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$
    - $y = \tan^{-1}(x)$ means $x = \tan(y)$ where $-\infty \leq x \leq \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

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- $y = \csc^{-1}(x)$ means $x = \csc(y)$ where $|x| \geq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, $y \neq 0$
- $y = \sec^{-1}(x)$ means $x = \sec(y)$ where $|x| \geq 1$ and $0 \leq y \leq \pi$, $y \neq \frac{\pi}{2}$
- $y = \cot^{-1}(x)$ means $x = \cot(y)$ where $-\infty \leq x \leq \infty$ and $0 < y < \pi$

➢ Sum and Difference Formulas
- $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$
- $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$ $\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$

➢ Half-Angle Formulas
- $\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$
- $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$
- $\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}} = \frac{1-\cos(\alpha)}{\sin(\alpha)} = \frac{\sin(\alpha)}{1+\cos(\alpha)}$

➢ Double-Angle Formulas
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$

➢ Product to Sum Formulas
- $\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
- $\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
- $\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

➢ Sum to Product Formulas
- $\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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- $\sin(\alpha) - \sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$
- $\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
- $\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

➢ Law of Sines
- $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

➢ Law of Cosines
- $a^2 = b^2 + c^2 - 2bc\cos(A)$
- $b^2 = a^2 + c^2 - 2ac\cos(B)$
- $c^2 = a^2 + b^2 - 2ab\cos(C)$

➢ Area of SSS Triangles (Heron's Formula)
- $K = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$

➢ Area of SAS Triangles
- $K = \frac{1}{2}ab\sin(C)$, $K = \frac{1}{2}bc\sin(A)$, $K = \frac{1}{2}ac\sin(B)$

➢ For $y = A\sin(\omega x - \varphi)$ or $y = A\cos(\omega x - \varphi)$, with $\omega > 0$
- Amplitude $= |A|$, Period$= T = \frac{2\pi}{\omega}$, Phase shift $= \frac{\varphi}{\omega}$