Multiple-Choice Answer Key

The following contains the answers to the multiple-choice questions in this exam.

Answer Key for AP Calculus BC Practice Exam, Section I

Question 1: C
Question 2: C
Question 3: D
Question 4: E
Question 5: A
Question 6: D
Question 7: C
Question 8: D
Question 9: D
Question 10: D
Question 11: D
Question 12: E
Question 13: D
Question 14: B
Question 15: B
Question 16: A
Question 17: B
Question 18: A
Question 19: C
Question 20: E
Question 21: A
Question 22: D
Question 23: C
Question 24: A
Question 25: C
Question 26: D
Question 27: A
Question 28: A
Question 76: A
Question 77: C
Question 78: B
Question 79: C
Question 80: E
Question 81: A
Question 82: C
Question 83: B
Question 84: C
Question 85: C
Question 86: E
Question 87: B
Question 88: B
Question 89: E
Question 90: D
Question 91: E
Question 92: B

Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

Question 1

Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

The graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.

1 : correct limits in an integral in (a), (b), or (c)

(a) $\text{Area} = \int_0^A \left(1 - x^3 - \sin\left(x^2\right)\right) dx$
$= 0.533$ (or 0.534)

2 : { 1 : integrand
1 : answer }

(b) $k = B = 0.552352$

$\int_0^A \left(1 - x^3 - k\right) dx = 0.257$ (or 0.256)

$\int_0^A \left(k - \sin\left(x^2\right)\right) dx = 0.277$ (or 0.276)

The two regions have unequal areas.

3 : { 1 : integral(s) with k value
1 : value(s) of integral(s)
1 : conclusion tied to part (a) or comparison of two integrals }

Note: Stating k value only does not earn a point.

(c) $\text{Volume} = \pi \int_0^A \left(\left(1 - x^3 + 3\right)^2 - \left(\sin\left(x^2\right) + 3\right)^2\right) dx$
$= 11.841$ (or 11.840)

3 : { 2 : integrand
1 : answer }

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Question 2

A planetary rover travels on a flat surface. The path of the rover for the time interval $0 \le t \le 2$ hours is shown in the rectangular coordinate system above. The rover starts at the point with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position of the rover change at rates given by

$$x'(t) = -12\sin\left(2t^2\right)$$
$$y'(t) = 10\cos\left(1 + \sqrt{t}\right),$$

where $x(t)$ and $y(t)$ are measured in meters and t is measured in hours.

(a) Find the acceleration vector of the rover at time $t = 1$. Find the speed of the rover at time $t = 1$.

(b) Find the total distance that the rover travels over the time interval $0 \le t \le 1$.

(c) Find the y-coordinate of the position of the rover at time $t = 1$.

(d) The rover receives a signal at each point where the line tangent to its path has slope $\frac{1}{2}$. At what times t, for $0 \le t \le 2$, does the rover receive a signal?

(a) $a(1) = \langle x''(1), y''(1) \rangle \quad \langle 19.975, -4.546 \rangle$ Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 11.678$	$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$
(b) Distance $= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 6.704$ (or 6.703)	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $y(1) = 5 + \int_0^1 y'(t)\, dt = 4.057$ (or 4.056)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10\cos\left(1+\sqrt{t}\right)}{-12\sin\left(2t^2\right)} = \frac{1}{2}$ $t = 1.072$	$2 : \begin{cases} 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

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Question 3

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
$W(32) \approx 0.5(32 - 30) + 125 = 126$

1 : answer

(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$

$W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$

3 : { 1 : left Riemann sum; 1 : approximation; 1 : answer }

(c) W' is differentiable $\Rightarrow$ W' is continuous.

$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$

By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.

2 : explanation

(d) $\dfrac{dA}{dt} = (0.3)\dfrac{2}{3}W^{-1/3} \cdot \dfrac{dW}{dt} = \dfrac{0.2}{\sqrt[3]{W}} \cdot \dfrac{dW}{dt}$

$\left.\dfrac{dA}{dt}\right|_{t=30} = \dfrac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$

3 : { 2 : $\dfrac{dA}{dt}$; 1 : answer }

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Question 4

$$f(x) = xe^{-x^2} \text{ for all real numbers } x.$$

(a) At what value of x does $f(x)$ attain its absolute maximum? Justify your answer.

(b) Find an antiderivative of f.

(c) Find the value of $\int_0^\infty x f(x)\, dx$, given the fact that $\int_0^\infty e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}$.

(a) $f'(x) = e^{-x^2} - 2x^2 e^{-x^2} = \left(1 - 2x^2\right)e^{-x^2}$

$f'(x) = 0$ when $x = -\frac{1}{\sqrt{2}}$ or $x = \frac{1}{\sqrt{2}}$

$f(x) < 0$ for $x < 0$, and $f(x) > 0$ for $x > 0$. Therefore, a maximum can only occur at a positive x-value.

$f'(x)$ changes sign from positive to negative at $x = \frac{1}{\sqrt{2}}$, so

f has a relative maximum at $x = \frac{1}{\sqrt{2}}$.

f increases on $0 < x \le \frac{1}{\sqrt{2}}$ and decreases on $\frac{1}{\sqrt{2}} \le x < \infty$.

Therefore, there is an absolute maximum at $x = \frac{1}{\sqrt{2}}$.

$5 : \begin{cases} 2 : f'(x) \\ 1 : \text{solves } f'(x) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(b) $\int xe^{-x^2}\, dx = -\frac{1}{2}e^{-x^2} + C$

1 : antiderivative

(c) Using integration by parts,

$u = x \qquad dv = xe^{-x^2}\, dx$

$du = dx \qquad v = -\frac{1}{2}e^{-x^2}$

$$\int x f(x)\, dx = \int x^2 e^{-x^2}\, dx = -\frac{1}{2}xe^{-x^2} + \frac{1}{2}\int e^{-x^2}\, dx$$

Therefore,

$$\begin{aligned}\int_0^\infty x f(x)\, dx &= \lim_{b\to\infty}\left[-\frac{1}{2}xe^{-x^2}\Big|_0^b + \frac{1}{2}\int_0^b e^{-x^2}\, dx\right] \\ &= \lim_{b\to\infty}\left(-\frac{1}{2}be^{-b^2}\right) + \frac{1}{2}\int_0^\infty e^{-x^2}\, dx \\ &= 0 + \frac{1}{2}\cdot\frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{4}\end{aligned}$$

$3 : \begin{cases} 2 : \text{integration by parts} \\ 1 : \text{answer} \end{cases}$

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Question 5

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

Graph of f

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(-6) = \int_{-2}^{-6} f(t)\,dt = -\int_{-6}^{-2} f(t)\,dt = -\frac{1}{2}\cdot 4\cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\,dt = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

$2 : \begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

(b) $g'(0) = f(0) = 2$

$1 : g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

$3 : \begin{cases} 1 : \text{horizontal tangent at } x = -2 \\ \quad \text{and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

$3 : \begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

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Question 6

The function f satisfies the equation

$$f'(x) = f(x) + x + 1$$

and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

(c) Find the fourth-degree Taylor polynomial for f about $x = 0$.

(d) Find $f^{(n)}(0)$, the nth derivative of f at $x = 0$, for $n \geq 2$. Use the Taylor series for f about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) - 4e^x$.

(a) $f'(0) = f(0) + 0 + 1 = 3$
An equation for the tangent line is $y = 3x + 2$.

1 : tangent line equation

(b) $f''(x) = f'(x) + 1;\ f''(0) = f'(0) + 1 = 3 + 1 = 4$

$$P_2(x) = 2 + 3x + \frac{4}{2!}x^2 = 2 + 3x + 2x^2$$

$2 : \begin{cases} 1 : f''(0) \\ 1 : \text{second-degree Taylor polynomial} \end{cases}$

(c) $f'''(x) = f''(x);\ f'''(0) = f''(0) = 4$

$f^{(4)}(x) = f'''(x);\ f^{(4)}(0) = f'''(0) = 4$

$$P_4(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4$$
$$= 2 + 3x + 2x^2 + \frac{2}{3}x^3 + \frac{1}{6}x^4$$

$2 : \begin{cases} 1 : f'''(0) \text{ and } f^{(4)}(0) \\ 1 : \text{fourth-degree Taylor polynomial} \end{cases}$

(d) $f^{(n)}(0) = 4$ for $n \geq 2$

$$f(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$$
$$4e^x = 4 + 4x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$$

Therefore, $f(x) - 4e^x = -2 - x$.

$4 : \begin{cases} 1 : f^{(n)}(0) \text{ for } n \geq 2 \\ 1 : \text{Taylor series for } f \\ 1 : \text{Taylor series for } e^x \\ 1 : \text{polynomial expression} \end{cases}$

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Scoring Worksheets

The following provides a worksheet and conversion table used for calculating a composite score of the exam.

2013 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_____	× 1.2000 =	_____
Number Correct (out of 45)		Weighted Section I Score (Do not round)

Section II: Free Response

Question 1	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 2	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 3	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 4 (AB Part)	_____ (out of 6)	× 1.0000 =	_____ (Do not round)
Question 4 (BC Part)	_____ (out of 3)	× 1.0000 =	_____ (Do not round)
Question 5	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 6	_____ (out of 9)	× 1.0000 =	_____ (Do not round)

Sum = _____
Weighted Section II Score (Do not round)

Composite Score

_____	+	_____	=	_____
Weighted Section I Score		Weighted Section II Score		Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
66-108	5
57-65	4
44-56	3
39-43	2
0-38	1

2013 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (1-2, 4-6, 10, 12-13, 17-18, 20-22, 24-25, 76-81, 83-86, 89, 91)

_____	× 1.2222 =	_____
Number Correct (out of 27)		Weighted Section I Score (Do not round)

Section II: Free Response

Question 1	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 3	_____ (out of 9)	× 1.0000 =	_____ (Do not round)
Question 4 (AB Part)	_____ (out of 6)	× 1.0000 =	_____ (Do not round)
Question 5	_____ (out of 9)	× 1.0000 =	_____ (Do not round)

Sum = _____ Weighted Section II Score (Do not round)

Composite Score

_____	+	_____	=	_____
Weighted Section I Score		Weighted Section II Score		Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Score
42-66	5
34-41	4
27-33	3
22-26	2
0-21	1

AP Calculus BC

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