CollegeBoard

AP Calculus BC Practice Exam

From the 2012 Administration

Revised January 2013

Contents

Note: This publication shows the page numbers that appeared in the *2012–12 AP Exam Instructions* book and in the actual exam. This publication was not repaginated to begin with page 1.

Exam Instructions

The following contains instructions taken from the *2011–12 AP Exam Instructions* book.

AP® Calculus BC Exam

Regularly Scheduled Exam Date: Wednesday morning, May 9, 2012
Late-Testing Exam Date: Thursday morning, May 24, 2012

Section I: At a Glance

Total Time: 1 hour, 45 minutes
Number of Questions: 45
Percent of Total Score: 50%
Writing Instrument:
    Pencil Required

Part A:

Number of Questions: 28
Time: 55 minutes
Electronic Device:
    None allowed

Part B:

Number of Questions: 17
Time: 50 minutes
Electronic Device:
    Graphing calculator required

Section II: At a Glance

Total Time: 1 hour, 30 minutes
Number of Questions: 6
Percent of Total Score: 50%
Writing Instrument:
    Either pencil or pen with black or dark blue ink
Weight:
    The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A:

Number of Questions: 2
Time: 30 minutes
Electronic Device:
    Graphing calculator required
Percent of Section II Score:
    33.3%

Part B:

Number of Questions: 4
Time: 60 minutes
Electronic Device:
    None allowed
Percent of Section II Score:
    66.6%

Section I: Multiple Choice Booklet Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Section II: Free Response Booklet Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 \, dx$ may not be written as fnInt(X^2, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

What Proctors Need to Bring to This Exam

- Exam packets
- Answer sheets
- AP Student Packs
- *2011-12 AP Coordinator's Manual*
- This book — *AP Exam Instructions*
- School Code and Home-School/Self-Study Codes
- Extra graphing calculators

- Pencil sharpener
- Extra No. 2 pencils with erasers
- Extra pens with black or dark blue ink
- Extra paper
- Stapler
- Watch
- Signs for the door to the testing room
  - "Exam in Progress"
  - "Cell phones are prohibited in the testing room"

If you are giving the regularly scheduled AP Calculus AB or BC Exam:

- You may seat students **four feet (approximately 1.2 meters) apart** because these exams have scrambled multiple-choice sections. This allows you to test more students in fewer testing rooms.

- See page 8 for a sample seating plan, including form codes and serial numbers, that shows how exams should be distributed to ensure that students seated next to each other are not given the same form of the exam.

- Administrators and proctors must continue to be vigilant about opportunities for cheating.

If you are giving the alternate AP Calculus AB or BC Exam for late testing:

- You must seat students **no less than five feet (approximately 1.5 meters) apart** because these exams do not have scrambled multiple-choice sections.

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 42 of the *2011-12 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 41 of the *2011-12 AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks; calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use**

the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

▌Do not begin the exam instructions below until you have completed the appropriate ● General Instructions for your group.

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time.

If you are giving the regularly scheduled exam, say:

It is Wednesday morning, May 9, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Thursday morning, May 24, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

In a moment, you will open the packet that contains your exam materials.

By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2011-12 Bulletin for AP Students and Parents*. Please check to make sure you have the correct exam: Calculus AB or Calculus BC. Raise your hand if you do not have the correct exam. . . .

You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

Look at page 1 of your answer sheet and locate the dark blue box near the top right-hand corner that states, "Take the AP Exam label from your Section I booklet and place the label here.". . .

Now look at the front cover of your exam booklet and locate the AP Exam label near the top left of the cover. . . .

Carefully peel off the AP Exam label and place it on your answer sheet on the dark blue box that we just identified. . . .

Now read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

Section I is the multiple-choice portion of the exam. You may never discuss these specific multiple-choice questions at any time in any form with anyone, including your teacher and other students. If you disclose these questions through any means, your AP Exam score will be canceled. Are there any questions? . . .

You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses on your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 55 minutes for Part A. Part A questions are numbered 1 through 28. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.

🕐 Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 55 minutes, say:

Stop working on Part A and turn to page 22 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 92. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 50 minutes for Part B. You may begin.

🕐 Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should

make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 50 minutes, say:

> **Stop working and turn to page 38. You have 3 minutes to answer Questions 93–96. These are survey questions and will not affect your score. You may not go back to work on any of the exam questions. . . .**

Give students approximately 3 minutes to answer the survey questions. Then say:

> **Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. I will now collect your answer sheet.**

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. Then say:

> **Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .**

Check that each student has signed the front cover of the sealed Section I booklet. There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

> **Please listen carefully to these instructions before we take a 10-minute break. Everything you placed under your chair at the beginning of the exam must stay there. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, or textbooks about the exam during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you are not allowed to discuss the multiple-choice section of this exam. Failure to adhere to any of these rules could result in cancellation of your score. Are there any questions? . . .**
>
> 🕐 **You may begin your break. Testing will resume at _____.**

SECTION II: Free Response

After the break, say:

> **May I have everyone's attention? Place your Student Pack on your desk. . . .**
>
> **You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .**
>
> **Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .**
>
> **Now place an AP number label on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .**

CALCULUS

Read the last statement. . . .

Using your pen, print the first, middle and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Turn to the back cover and read Item 1 under "Important Identification Information." Print the first two letters of your <u>last</u> name and the first letter of your <u>first</u> name in the boxes. Look up when you have finished. . . .

In Item 2, print your date of birth in the boxes. . . .

In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .

Read Item 4. . . .

Are there any questions? . . .

I need to collect the Student Pack from anyone who will be taking another AP Exam. You may keep it only if you are not taking any other AP Exams this year. If you have no other AP Exams to take, place your Student Pack under your chair now. . . .

While Student Packs are being collected, read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .

Collect the Student Packs. Then say:

Are there any questions? . . .

Section II also has two parts that are timed separately. You are responsible for pacing yourself, and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .

You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra piece of paper you use, be sure to write only your AP number and the number of the question you are working on. Do not write your name. Open your exam booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1's and 2's at the top of each page to assist you in monitoring their work. After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

> **Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .**
>
> **Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may <u>not</u> use your calculator. Remember to write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .**
>
> **Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may begin Part B. . . .**

🕐 Note Start Time here _____. Note Stop Time here _____. After 50 minutes, say:

> **There are 10 minutes remaining in Part B.**

After 10 minutes, say:

> **Stop working and close your exam booklet. Place it on your desk, face up. . . .**

If any students used extra paper for the free-response section, have those students staple the extra sheet/s to the first page corresponding to that question in their exam booklets. Then say:

> **Remain in your seat, without talking, while the exam materials are collected. . . .**

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box, and printed his or her initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:

> **You may not discuss these specific free-response questions with anyone unless they are released on the College Board website in about two days. You should receive your score report in the mail about the third week of July.**

If you are giving the alternate exam for late testing, say:

> **None of the questions in this exam may ever be discussed or shared in any way at any time. You should receive your score report in the mail about the third week of July.**

If any students completed the AP number card at the beginning of this exam, say:

> **Please remember to take your AP number card with you.**

Then say:

> **You are now dismissed.**

CALCULUS

All exam materials should be put in secure storage until they are returned to the AP Program after your school's last administration. Before storing materials, check the "School Use Only" section on page 1 of the answer sheet and:

- Fill in the appropriate section number circle in order to view a separate AP Instructional Planning Report (for regularly scheduled exams only) or Subject Score Roster at the class section or teacher level. See "Post-Exam Activities" in the *2011-12 AP Coordinator's Manual.*

- Check your list of students who are eligible for fee reductions and fill in the appropriate circle on their registration answer sheets.

Student Answer Sheet for
the Multiple-Choice Section

Use this section to capture student responses. (Note that the following
answer sheet is a sample, and may differ from one used in an actual exam.)

CollegeBoard

AP
2012 Answer Sheet

B123456789T

Take an AP Number label from your AP Student Pack and place the label here.

Take the AP Exam label from your Section I booklet and place the label here.

NAME AND EXAM AREA — COMPLETE THIS AREA AT EVERY EXAM.

To maintain the security of the exam and the validity of my AP score, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after the completion of the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the *2011-12 Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE
Sign your legal name as it will appear on your college applications.

Date

B. LEGAL NAME
Omit apostrophes, Jr., II.

Legal Last Name — First 15 Letters

Legal First Name — First 12 Letters

MI

USE NO. 2 PENCIL ONLY

C. YOUR AP NUMBER

D. EXAM DATE
Month

E. EXAM START TIME
AM PM

F. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER
S

G. ONLINE PROVIDER CODE

H. AP EXAM I AM TAKING USING THIS ANSWER SHEET
Print Exam Name:

Print Form: Print Form Code:

STUDENT INFORMATION AREA — COMPLETE THIS AREA ONLY ONCE.

I. DATE OF BIRTH
Month   Day   Year

Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec

J. SEX
Female
Male

K. CURRENT GRADE LEVEL
Pre-9th
9th
10th
11th
12th
Post-12th

L. SOCIAL SECURITY NUMBER (Optional)

M. EXPECTED DATE OF COLLEGE ENTRANCE
Fall
Winter/Spring
Summer
Undecided

2012
2013
2014
2015

N. STUDENT SEARCH SERVICE®
I want the College Board to send information about me to colleges, universities and government scholarship programs interested in students like me.
Yes   No

O. WHICH LANGUAGE DO YOU KNOW BEST?
English
English and another language about the same
Another language

P. ETHNICITY/RACE
American Indian or Alaska Native
Asian, Asian American or Pacific Islander
Black or African American
Mexican or Mexican American
Puerto Rican
Other Hispanic, Latino or Latin American
White
Other

Q. PARENTAL EDUCATION LEVEL
Father/Male Guardian   Mother/Female Guardian
Grade school
Some high school
High school diploma or equivalent
Business or trade school
Some college
Associate or two-year degree
Bachelor's or four-year degree
Some graduate or professional school
Graduate or professional degree

762000

SCHOOL USE ONLY
Section Number
1 2 3 4 5 6 7 8 9

Fee Reduction Granted
1 Option 1   2 Option 2

ETS USE ONLY
Exam

Exam

R. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this section.

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ

7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ Ⓗ Ⓘ

S. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language, or Spanish Literature Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

    ◯ Yes   ◯ No

2. Do you regularly speak or hear the language at home?

    ◯ Yes   ◯ No

Indicate your answers to the exam questions in this section. If a question has only four answer options, do not mark option E. Your answer sheet will be scored by machine. Use only No. 2 pencils to mark your answers on pages 2 and 3 (one response per question). After you have determined your response, be sure to completely fill in the corresponding circle next to the number of the question you are answering. Stray marks and smudges could be read as answers, so erase carefully and completely. Any improper gridding may affect your score. Answers written in the multiple-choice booklet will not be scored.

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74 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
75 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

ETS USE ONLY	48, 55, 62, 75, 87		R	W	O	OTHER		R	W	O
		PT02					TOTAL			
		PT03								
		PT04								

DO NOT WRITE IN THIS AREA

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

76 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
77 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
78 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
79 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
80 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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DO NOT WRITE IN THIS AREA

HOME ADDRESS AND SCHOOL AREA — COMPLETE THIS AREA ONLY ONCE.

T. YOUR MAILING ADDRESS — Use the address abbreviations from your AP Student Pack. Fill in only one circle per column. Indicate a space in your address by leaving a blank box; do not grid that column.

STREET ADDRESS (include street number, street name, apartment number, etc.)

CITY

ZIP OR POSTAL CODE

COUNTRY CODE

STATE

AK	HI	MI	NV	UT
AL	IA	MN	NY	VA
AR	ID	MO	OH	VT
AZ	IL	MS	OK	WA
CA	IN	MT	OR	WI
CO	KS	NC	PA	WV
CT	KY	ND	RI	WY
DC	LA	NE	SC	Puerto Rico
DE	MA	NH	SD	Other
FL	MD	NJ	TN	
GA	ME	NM	TX	

X. FOR STUDENTS OUTSIDE THE UNITED STATES ONLY
If the address gridded above is not complete enough for delivery of your score report, please fill in this circle and print your complete address below.

Address

City

State or Province

Country

ZIP or Postal Code

U. AREA CODE AND PHONE NUMBER

INTERNATIONAL PHONE

V. SCHOOL YOU ATTEND

SCHOOL CODE

School Name

City

State

Country

W. COLLEGE TO RECEIVE YOUR AP SCORE REPORT

COLLEGE CODE

Using the college code listed in the AP Student Pack, indicate the ONE college that you want to receive your AP score report.

College Name

City

State

Country

Y. EMAIL ADDRESS

Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION I: Multiple Choice

2012

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator
required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question   Sample Answer

Chicago is a   Ⓐ ● Ⓒ Ⓓ Ⓔ
(A) state
(B) city
(C) country
(D) continent
(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Minimum 20% post-consumer waste

Form I
Form Code 4IBP4-Q-S
68

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).
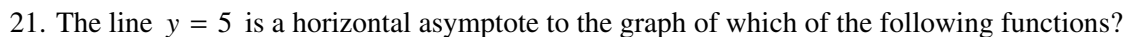
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1. If $y = \sin^3 x$, then $\dfrac{dy}{dx} =$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

2. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

Graph of f

3. The graph of f is shown above for $0 \leq x \leq 4$. What is the value of $\int_0^4 f(x)\, dx$?

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

GO ON TO THE NEXT PAGE.

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\displaystyle\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

(B) $\displaystyle\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\displaystyle\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\displaystyle\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\displaystyle\int_1^2 \left(1 + (\ln x)^2\right) dx$

5. The Maclaurin series for the function f is given by $f(x) = \displaystyle\sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\dfrac{3}{7}$ (C) $\dfrac{4}{7}$ (D) $\dfrac{13}{16}$ (E) 4

GO ON TO THE NEXT PAGE.

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2 - 3)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$

(B) $\int_{-2}^{13} u^5 \, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

7. If $\arcsin x = \ln y$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{y}{\sqrt{1 - x^2}}$

(B) $\dfrac{xy}{\sqrt{1 - x^2}}$

(C) $\dfrac{y}{1 + x^2}$

(D) $e^{\arcsin x}$

(E) $\dfrac{e^{\arcsin x}}{1 + x^2}$

GO ON TO THE NEXT PAGE.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9   (B) 68.2   (C) 114.9   (D) 116.6   (E) 118.2

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only   (B) II only   (C) III only   (D) I and III only   (E) I, II, and III

GO ON TO THE NEXT PAGE.

10. $\int_1^4 t^{-3/2} \, dt =$

(A) -1 (B) $-\dfrac{7}{8}$ (C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{2}$ (E) 1

---

11. Let f be the function defined by $f(x) = \sqrt{|x - 2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\displaystyle\lim_{x \to 2} f(x) \ne 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

GO ON TO THE NEXT PAGE.

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

(C) $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

(E) $(-1, -1)$ is a point of inflection of the graph of f.

13. What is the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3   (C) $\sqrt{3}$ (D) $\dfrac{\sqrt{3}}{2}$ (E) 0

GO ON TO THE NEXT PAGE.

14. Let k be a positive constant. Which of the following is a logistic differential equation?

(A) $\dfrac{dy}{dt} = kt$

(B) $\dfrac{dy}{dt} = ky$

(C) $\dfrac{dy}{dt} = kt(1 - t)$

(D) $\dfrac{dy}{dt} = ky(1 - t)$

(E) $\dfrac{dy}{dt} = ky(1 - y)$

GO ON TO THE NEXT PAGE.

Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt,$ which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\dfrac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\dfrac{5}{4}$ (B) 1   (C) $\dfrac{7}{4}$ (D) 2   (E) $\dfrac{21}{4}$

17. For $x > 0$, the power series $1 - \dfrac{x^2}{3!} + \dfrac{x^4}{5!} - \dfrac{x^6}{7!} + \cdots + (-1)^n \dfrac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\dfrac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$

GO ON TO THE NEXT PAGE.

Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

GO ON TO THE NEXT PAGE.

19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2}, \dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4, 2)$

(D) $(0,0)$ and $\left(4, \dfrac{2}{3}\right)$

(E) There are no such points.

20. $\displaystyle\int_0^1 \dfrac{5x+8}{x^2 + 3x + 2}\, dx$ is

(A) $\ln(8)$ (B) $\ln\left(\dfrac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

GO ON TO THE NEXT PAGE.

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x - 5}$ (D) $y = \dfrac{5x}{1 - x}$ (E) $y = \dfrac{20x^2 - x}{1 + 4x^2}$

22. The power series $\displaystyle\sum_{n=0}^{\infty} a_n (x - 3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

GO ON TO THE NEXT PAGE.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

24. Let f be a differentiable function such that $\displaystyle\int f(x)\sin x \, dx = -f(x)\cos x + \int 4x^3 \cos x \, dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

GO ON TO THE NEXT PAGE.

25. $\int_1^\infty xe^{-x^2}\,dx$ is

(A) $-\dfrac{1}{e}$ (B) $\dfrac{1}{2e}$ (C) $\dfrac{1}{e}$ (D) $\dfrac{2}{e}$ (E) divergent

GO ON TO THE NEXT PAGE.

26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2　　(B) $\dfrac{1}{2}$　　(C) 0　　(D) $-\dfrac{1}{2}$　　(E) -2

GO ON TO THE NEXT PAGE.

27. For what values of p will both series $\sum_{n=1}^{\infty} \dfrac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\dfrac{p}{2}\right)^{n}$ converge?

(A) $-2 < p < 2$ only

(B) $-\dfrac{1}{2} < p < \dfrac{1}{2}$ only

(C) $\dfrac{1}{2} < p < 2$ only

(D) $p < \dfrac{1}{2}$ and $p > 2$

(E) There are no such values of p.

GO ON TO THE NEXT PAGE.

28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\displaystyle\lim_{x \to 1} \frac{\int_1^x g(t)\, dt}{g(x) - 6}$?

(A) 0   (B) $\dfrac{1}{2}$ (C) 1   (D) 2   (E) The limit does not exist.

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).
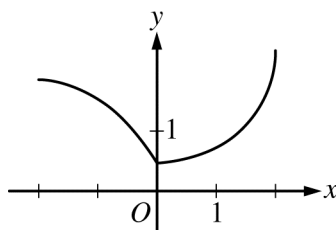
Graph of f

76. The function f, whose graph is shown above, is defined on the interval $-2 \leq x \leq 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

77. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

(A) $(0.831, 7.384)$ only

(B) $(-\infty, 0.831)$ and $(7.384, \infty)$

(C) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(D) $(-0.816, 1.430)$ and $(8.613, \infty)$

(E) $(-\infty, \infty)$
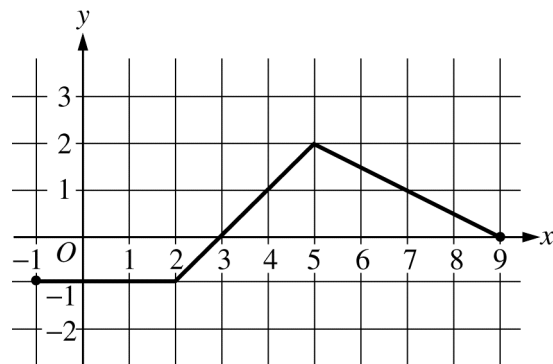
Graph of f

78. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9} (3f(x) + 2)\, dx$?

(A) 7.5   (B) 9.5   (C) 27.5   (D) 47   (E) 48.5

79. Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x - 3) + 6(x - 3)^2 + 12(x - 3)^3$

(D) $2 - (x - 3) + 3(x - 3)^2 + 4(x - 3)^3$

(E) $2 - (x - 3) + 3(x - 3)^2 + 2(x - 3)^3$

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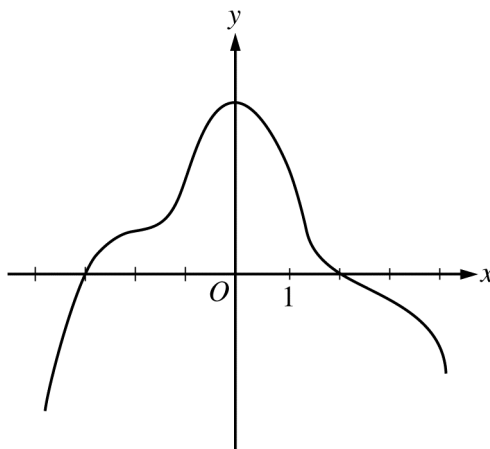
Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

   I. f has a relative minimum at $x = -3$.

   II. The graph of f has a point of inflection at $x = -2$.

   III. The graph of f is concave down for $0 < x < 4$.

   (A) I only   (B) II only   (C) III only   (D) I and II only   (E) I and III only

GO ON TO THE NEXT PAGE.

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

81. Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table above. If $f(x) = f(-x)$, what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

(A) $x = 0$ only

(B) $x = 1$ only

(C) $x = 0$ and $x = 1$

(D) $x = -1$ and $x = 1$

(E) There are no points of inflection on $-2 < x < 2$.

82. What is the average value of $y = \sqrt{\cos x}$ on the interval $0 \le x \le \dfrac{\pi}{2}$?

(A) -0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

GO ON TO THE NEXT PAGE.

83. If the function f is continuous at $x = 3$, which of the following must be true?

   (A) $f(3) < \lim\limits_{x \to 3} f(x)$

   (B) $\lim\limits_{x \to 3^-} f(x) \neq \lim\limits_{x \to 3^+} f(x)$

   (C) $f(3) = \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x)$

   (D) The derivative of f at $x = 3$ exists.

   (E) The derivative of f is positive for $x < 3$ and negative for $x > 3$.

84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

   (A) $(-0.418, 0.418)$ only

   (B) $(-1, 1)$

   (C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

   (D) $(-1.5, -1)$ and $(0, 1)$

   (E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

GO ON TO THE NEXT PAGE.

85. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(s) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of $20 \text{ miles}/\text{hour}^2$, what is the rate at which its fuel consumption is changing?

(A)   0.215 mpg per hour

(B)   4.299 mpg per hour

(C)   19.793 mpg per hour

(D)   25.793 mpg per hour

(E) 515.855 mpg per hour

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)\,dt = 0$, which of the following could be a table of values for the function f ?

(A)

x	$f(x)$
4	-4
5	-3
7	0

(B)

x	$f(x)$
4	-4
5	-2
7	5

(C)

x	$f(x)$
4	-4
5	6
7	3

(D)

x	$f(x)$
4	0
5	0
7	0

(E)

x	$f(x)$
4	0
5	4
7	6

87. Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3 - x)$, for $0 \leq x \leq 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.442   (B) 1.029   (C) 1.296   (D) 3.233   (E) 4.071

88. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

(A)

(B)

(C)

(D)

(E)

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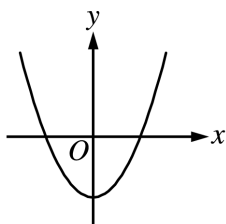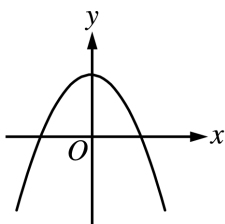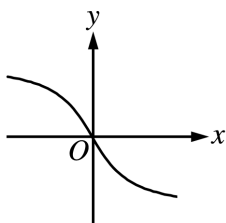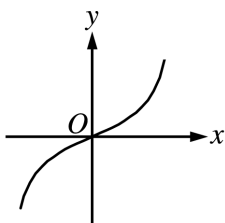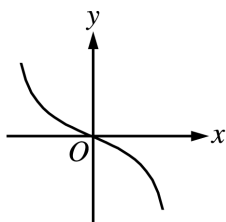
89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \dfrac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713   (B) 1.134   (C) 6.134   (D) 6.710   (E) 11.710

90. If the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\displaystyle\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\displaystyle\sum_{n=1}^{\infty} a_n = 0$

(D) $\displaystyle\sum_{n=1}^{\infty} na_n$ diverges.

(E) $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

GO ON TO THE NEXT PAGE.
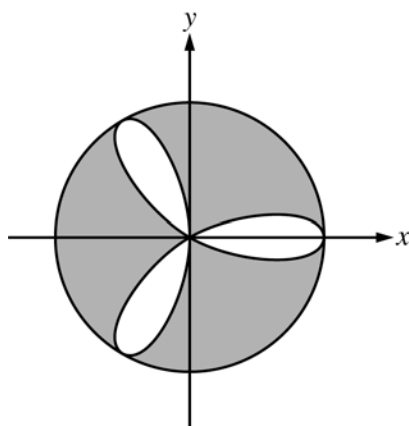
91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858   (B) 3.142   (C) 8.566   (D) 9.425   (E) 15.708

GO ON TO THE NEXT PAGE.

92. The function h is differentiable, and for all values of x, $h(x) = h(2 - x)$. Which of the following statements must be true?

   I. $\int_0^2 h(x)\, dx > 0$

   II. $h'(1) = 0$

   III. $h'(0) = h'(2) = 1$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.
_____

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**

- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**

- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

**AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND
ANSWER QUESTIONS 93–96.**

Section II: Free-Response Questions

AP® Calculus BC Exam

SECTION II: Free Response

2012

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

*   Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
*   Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2\, dx$ may not be written as fnInt(X^2, X, 1, 5).
*   Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
*   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Minimum 20% post-consumer waste

Form I
Form Code 4IBP-S2

68

CALCULUS BC
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

    (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

    (b) Use the data in the table to evaluate $\int_0^{20} W'(t)\, dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\, dt$ in the context of this problem.

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(c) For $0 \leq t \leq 20,$ the average temperature of the water in the tub is $\dfrac{1}{20}\displaystyle\int_0^{20} W(t)\, dt.$ Use a left Riemann sum

with the four subintervals indicated by the data in the table to approximate $\dfrac{1}{20}\displaystyle\int_0^{20} W(t)\, dt.$ Does this

approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \leq t \leq 25,$ the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t).$ Based on the model, what is the temperature of the water at time $t = 25$?

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2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

-6-

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

(−2, 3)

(−4, 1)

(1, 0)

O

(3, −1)
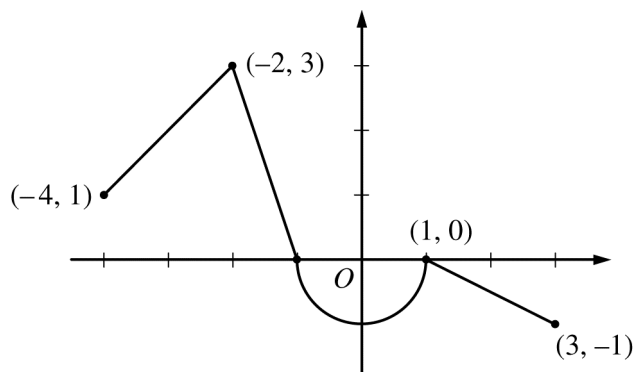
Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

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(c) Find the *x*-coordinate of each point at which the graph of *g* has a horizontal tangent line. For each of these points, determine whether *g* has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3,$ find all values of *x* for which the graph of *g* has a point of inflection. Explain your reasoning.

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x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_{1}^{1.4} f'(x)\, dx$. Use the approximation for $\int_{1}^{1.4} f'(x)\, dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\dfrac{d^2 B}{dt^2}$ in terms of B. Use $\dfrac{d^2 B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

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(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

6. The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \dfrac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\!\left(\dfrac{1}{2}\right)$ using the first two nonzero terms of this series is $\dfrac{17}{120}$. Show that this approximation differs from $g\!\left(\dfrac{1}{2}\right)$ by less than $\dfrac{1}{200}$.

_____

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

STOP

END OF EXAM

_____

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT <u>AND</u> BACK COVERS OF THE SECTION II BOOKLET.**

- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**

- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to the
multiple-choice questions in this exam.

Answer Key for AP Calculus BC
Practice Exam, Section I

Multiple-Choice Questions	
Question #	Key
1	E
2	A
3	B
4	A
5	C
6	C
7	A
8	C
9	D
10	E
11	A
12	C
13	C
14	E
15	A
16	C
17	C
18	A
19	C
20	C
21	E
22	D

23	A
24	E
25	B
26	B
27	C
28	D
76	B
77	B
78	C
79	E
80	E
81	D
82	C
83	C
84	D
85	B
86	B
87	B
88	E
89	E
90	E
91	D
92	B

Free-Response Scoring Guidelines

The following contains the scoring guidelines
for the free-response questions in this exam.

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\, dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\, dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\dfrac{1}{20}\int_0^{20} W(t)\, dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\dfrac{1}{20}\int_0^{20} W(t)\, dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$

$\qquad = 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\, dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

(c) $\dfrac{1}{20}\int_0^{20} W(t)\, dt \approx \dfrac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$

$\qquad = \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$

$\qquad = \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\, dt$

$\qquad = 71.0 + 2.043155 = 73.043$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left. \dfrac{dx}{dt} \right|_{t=2} = \dfrac{2}{e^2}$

Because $\left. \dfrac{dx}{dt} \right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left. \dfrac{dy}{dx} \right|_{t=2} = \dfrac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055$ (or 3.054)

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \dfrac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \displaystyle\int_2^4 \dfrac{\sqrt{t+2}}{e^t}\, dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \displaystyle\int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$
$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$.
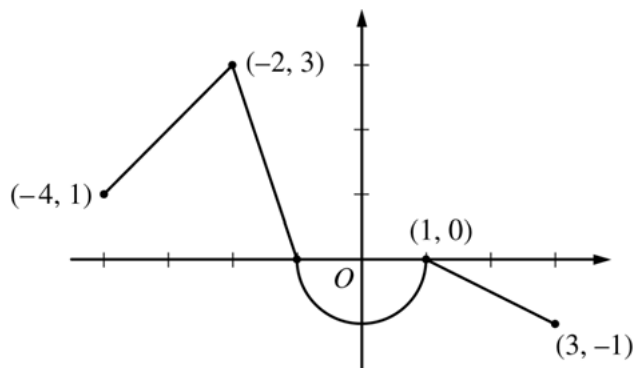
(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

Graph of f

(a) $g(2) = \int_1^2 f(t)\, dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$g(-2) = \int_1^{-2} f(t)\, dt = -\int_{-2}^1 f(t)\, dt$

$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$

$2 : \begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$

(b) $g'(x) = f(x) \implies g'(-3) = f(-3) = 2$
$g''(x) = f'(x) \implies g''(-3) = f'(-3) = 1$

$2 : \begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

Question 4

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table above.

(a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.

(b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x)\, dx$. Use the approximation for $\int_1^{1.4} f'(x)\, dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

(c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

(d) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(1.4)$.

(a) $f(1) = 15,\ f'(1) = 8$

An equation for the tangent line is
$y = 15 + 8(x - 1)$.

$f(1.4) \approx 15 + 8(1.4 - 1) = 18.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(b) $\int_1^{1.4} f'(x)\, dx \approx (0.2)(10) + (0.2)(13) = 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\, dx$

$f(1.4) \approx 15 + 4.6 = 19.6$

$3 : \begin{cases} 1 : \text{midpoint Riemann sum} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $f(1.2) \approx f(1) + (0.2)(8) = 16.6$

$f(1.4) \approx 16.6 + (0.2)(12) = 19.0$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(d) $T_2(x) = 15 + 8(x - 1) + \dfrac{20}{2!}(x - 1)^2$

$\quad = 15 + 8(x - 1) + 10(x - 1)^2$

$f(1.4) \approx 15 + 8(1.4 - 1) + 10(1.4 - 1)^2 = 19.8$

$2 : \begin{cases} 1 : \text{Taylor polynomial} \\ 1 : \text{approximation} \end{cases}$

Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0,$ when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20.$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\dfrac{d^2B}{dt^2}$ in terms of $B.$ Use $\dfrac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

(c) Use separation of variables to find $y = B(t),$ the particular solution to the differential equation with initial condition $B(0) = 20.$
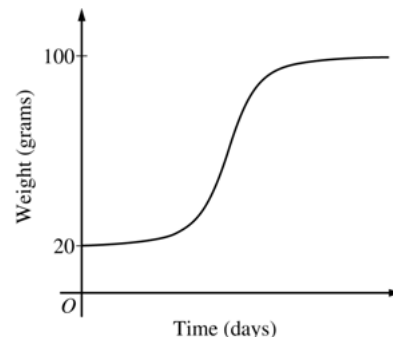
(a) $\left.\dfrac{dB}{dt}\right|_{B=40} = \dfrac{1}{5}(60) = 12$

$\left.\dfrac{dB}{dt}\right|_{B=70} = \dfrac{1}{5}(30) = 6$

Because $\left.\dfrac{dB}{dt}\right|_{B=40} > \left.\dfrac{dB}{dt}\right|_{B=70},$ the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \dfrac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\dfrac{d^2B}{dt^2} = -\dfrac{1}{5}\dfrac{dB}{dt} = -\dfrac{1}{5} \cdot \dfrac{1}{5}(100 - B) = -\dfrac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100.$ A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \dfrac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\dfrac{dB}{dt} = \dfrac{1}{5}(100 - B)$

$\displaystyle\int \dfrac{1}{100 - B}\, dB = \int \dfrac{1}{5}\, dt$

$-\ln|100 - B| = \dfrac{1}{5}t + C$

Because $20 \le B < 100,$ $|100 - B| = 100 - B.$

$-\ln(100 - 20) = \dfrac{1}{5}(0) + C \;\Rightarrow\; -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \;\; t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

Question 6

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

(a) $\left| \dfrac{x^{2n+3}}{2n+5} \cdot \dfrac{2n+3}{x^{2n+1}} \right| = \left(\dfrac{2n+3}{2n+5} \right) \cdot x^2$

$\lim\limits_{n \to \infty} \left(\dfrac{2n+3}{2n+5} \right) \cdot x^2 = x^2$

$x^2 < 1 \implies -1 < x < 1$
The series converges when $-1 < x < 1$.

When $x = -1$, the series is $-\dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \dfrac{1}{9} - \cdots$
This series converges by the Alternating Series Test.

When $x = 1$, the series is $\dfrac{1}{3} - \dfrac{1}{5} + \dfrac{1}{7} - \dfrac{1}{9} + \cdots$
This series converges by the Alternating Series Test.

Therefore, the interval of convergence is $-1 \le x \le 1$.

$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies interior of} \\ \quad\ \text{interval of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis and interval of convergence} \end{cases}$

(b) $\left| g\left(\dfrac{1}{2}\right) - \dfrac{17}{120} \right| < \dfrac{\left(\dfrac{1}{2}\right)^5}{7} = \dfrac{1}{224} < \dfrac{1}{200}$

$2 : \begin{cases} 1 : \text{uses the third term as an error bound} \\ 1 : \text{error bound} \end{cases}$

(c) $g'(x) = \dfrac{1}{3} - \dfrac{3}{5} x^2 + \dfrac{5}{7} x^4 + \cdots + (-1)^n \left(\dfrac{2n+1}{2n+3} \right) x^{2n} + \cdots$

$2 : \begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$

Scoring Worksheets

The following provides worksheets and conversion tables
used for calculating a composite score of the exam.

2012 AP Calculus BC Scoring Worksheet

Section I: Multiple Choice

_____ × 1.2000 = _____
Number Correct   Weighted Section I Score
(out of 45)   (Do not round)

Section II: Free Response

Question 1   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Question 2   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Question 3   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Question 4   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Question 5   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Question 6   _____ × 1.0000 = _____
           (out of 9)   (Do not round)

Sum = _____
Weighted
Section II
Score
(Do not round)

Composite Score

_____ + _____ = _____
Weighted   Weighted   Composite Score
Section I Score   Section II Score   (Round to nearest
                                        whole number)

AP Score Conversion Chart
Calculus BC

Composite Score Range	AP Score
68-108	5
56-67	4
42-55	3
35-41	2
0-34	1

2012 AP Calculus BC — AB Subscore Scoring Worksheet

Section I: Multiple Choice
Questions (1, 3, 6-8, 10-12, 15, 18-19, 21, 23, 76-78, 80-89, 92)

_____ x   1.1851 =   _____
Number Correct   Weighted Section I Score
   (out of 27)   (Do not round)

Section II: Free Response

Question 1   _____ × 1.0000   = _____
                  (out of 9)   (Do not round)

Question 3   _____ × 1.0000   = _____
                  (out of 9)   (Do not round)

Question 4   _____ × 1.0000   = _____
 (AB Part: (out of 5)   (Do not round)
 Parts A and B)

Question 5   _____ × 1.0000   = _____
                  (out of 9)   (Do not round)

                          Sum = _____
                                  Weighted
                                  Section II
                                    Score
                                (Do not round)

Composite Score

_____ + _____ = _____
   Weighted   Weighted   Composite Score
Section I Score   Section II Score   (Round to nearest
                                                  whole number)

AP Score Conversion Chart
Calculus AB Subscore

Composite Score Range*	AP Score
40-64	5
31-39	4
23-30	3
17-22	2
0-16	1

AP Calculus BC