USE PENCIL ONLY FOR THE ENTIRE ANSWER SHEET.

CollegeBoard AP — Answer Sheet for May 2008, Form 4EBP — PAGE 1

NAME AND EXAM AREA – COMPLETE THIS AREA AT EVERY EXAM.

To maintain the security of the exam and the validity of my AP grade, I will allow no one else to see the multiple-choice questions. I will seal the multiple-choice booklet when asked to do so, and I will not discuss these questions with anyone at any time after the completion of the section. I am aware of and agree to the AP Program's policies and procedures as outlined in the 2007-08 *Bulletin for AP Students and Parents*, including using testing accommodations (e.g., extended time, computer, etc.) only if I have been preapproved by College Board Services for Students with Disabilities.

A. SIGNATURE _____ Sign your legal name as it will appear on your college applications.

B. LEGAL NAME — Omit apostrophes, Jr., or II.

Legal Last Name—first 15 letters | Legal First Name—first 12 letters | MI

C. YOUR AP NUMBER

D. ADMIN. DAY IN MAY

E. TIME OF DAY

○ 6 AM ○ 7 AM ○ 8 AM ○ 9 AM ○ 10 AM ○ 11 AM ○ 12 PM ○ 1 PM ○ 2 PM ○ 3 PM ○ 4 PM ○ 5 PM

PLACE YOUR AP® NUMBER LABEL OR WRITE YOUR AP NUMBER HERE AT EVERY EXAM.

STUDENT INFORMATION AREA—COMPLETE THIS AREA ONLY ONCE.

I. SEX

① Male
② Female

J. CURRENT GRADE LEVEL

① Not yet in 9th grade
② 9th grade
③ 10th grade
④ 11th grade
⑤ 12th grade
⑥ No longer in high school

K. DATE OF BIRTH

Month	Day	Year
01 ○ Jan.		
02 ○ Feb.		
03 ○ Mar.		
04 ○ Apr.		
05 ○ May		
06 ○ June		
07 ○ July		
08 ○ Aug.		
09 ○ Sept.		
10 ○ Oct.		
11 ○ Nov.		
12 ○ Dec.		

L. SOCIAL SECURITY NUMBER (Optional, but preferred)

M. ETHNICITY/RACE

① American Indian or Alaska Native
② Asian, Asian American, or Pacific Islander
③ Black or African American
④ Mexican or Mexican American
⑤ Puerto Rican
⑥ Other Hispanic, Latino, or Latin American
⑦ White
⑧ Other

N. EXPECTED DATE OF COLLEGE ENTRANCE

Ⓕ Fall
Ⓦ Winter/Spring
Ⓢ Summer
Ⓤ Undecided

⓪ 2008
① 2009
② 2010
③ 2011

O. WHAT LANGUAGE DO YOU KNOW BEST?

① English
② English and another language about the same
③ Another language

P. Complete ONLY if you are a SOPHOMORE or a JUNIOR.

I want the College Board to send information about me to colleges, universities, and government scholarship programs interested in students like me.

○ Yes ○ No

F. AP EXAM I AM TAKING USING THIS ANSWER SHEET

Print exam name: _____ Print form code (e.g., 4EBP-R) from M-C booklet: _____

Fill in the oval below that corresponds to the exam that you are taking.

07 ○ U.S. History 4EBP-Q
07 ○ U.S. History 4EBP-R
13 ○ Art History
14 ○ Art: Studio Drawing
15 ○ Art: Studio 2-D Design
16 ○ Art: Studio 3-D Design
20 ○ Biology
25 ○ Chemistry
28 ○ Chinese Lang. & Culture
31 ○ Computer Science A
33 ○ Computer Science AB
34 ○ Economics: Micro
35 ○ Economics: Macro
36 ○ Eng. Language & Comp.
37 ○ Eng. Literature & Comp.
40 ○ Environmental Science
43 ○ European History
48 ○ French Language
51 ○ French Literature
53 ○ Geography: Human
55 ○ German Language
57 ○ Gov. & Pol.: U.S.
58 ○ Gov. & Pol.: Comp.
60 ○ Latin: Vergil
61 ○ Latin Literature
62 ○ Italian Lang. & Culture
64 ○ Japanese Lang. & Culture
66 ○ Calculus AB 4EBP-Q
66 ○ Calculus AB 4EBP-R
68 ○ Calculus BC 4EBP-Q
68 ○ Calculus BC 4EBP-R
75 ○ Music Theory
78 ○ Physics B
80 ○ Physics C: Mech.
82 ○ Physics C: E & M
85 ○ Psychology
87 ○ Spanish Language
89 ○ Spanish Literature
90 ○ Statistics
93 ○ World History

G. ONLINE PROVIDER CODE

H. MULTIPLE-CHOICE BOOKLET SERIAL NUMBER

743180

Q. PARENTAL EDUCATION LEVEL

Father / Male Guardian	Mother / Female Guardian	
○	○	Grade school
○	○	Some high school
○	○	High school diploma or equivalent
○	○	Business or trade school
○	○	Some college
○	○	Associate or two-year degree
○	○	Bachelor's or four-year degree
○	○	Some graduate or professional school
○	○	Graduate or professional degree

SCHOOL USE ONLY

Instructional Planning Report/Student Grade Roster ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

Fee Reduction Granted ○ Option 1 ○ Option 2

ETS USE ONLY

Exam ①	Exam ②	Exam ①	Exam ②

62699 • 00657 • TF127E3600 Q3121/06-09 mh07270

R. This section is for the survey questions in the AP Student Pack. (Do not put responses to exam questions in this section.) Be sure each mark is dark and completely fills the oval.

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ

Do not complete this section unless instructed to do so.

S. If this answer sheet is for the Chinese Language and Culture, French Language, French Literature, German Language, Italian Language and Culture, Japanese Language and Culture, Spanish Language, or Spanish Literature Exam, please answer the following questions. (Your responses will not affect your grade.)

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken? ◯ Yes ◯ No
2. Do you regularly speak or hear the language at home? ◯ Yes ◯ No

Indicate your answers to the exam questions in this section. If a question has only four answer options, do not mark option E. Your answer sheet will be scored by machine. Use only No. 2 pencils to mark your answers on pages 2 and 3 (one response per question). After you have determined your response, be sure to completely fill in the oval corresponding to the number of the question you are answering. Stray marks and smudges could be read as answers, so erase carefully and completely. Any improper gridding may affect your grade. Answers written in the multiple-choice booklet will not be scored.

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	26	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	51	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	27	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	52	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	28	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	53	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	29	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	54	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	30	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	55	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	31	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	56	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	32	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	57	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	33	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	58	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	34	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	59	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	35	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	60	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	36	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	61	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	37	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	62	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	38	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	63	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	39	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	64	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	40	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	65	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	41	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	66	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
17	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	42	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	67	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
18	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	43	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	68	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
19	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	44	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	69	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
20	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	45	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	70	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
21	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	46	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	71	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
22	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	47	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	72	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
23	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	48	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	73	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
24	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	49	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	74	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
25	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	50	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	75	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						

FOR QUESTIONS 76-151, SEE PAGE 3.

DO NOT WRITE IN THIS AREA.

Be sure each mark is dark and completely fills the oval. If a question has only four answer options, do not mark option E.

76	A	B	C	D	E	101	A	B	C	D	E	126	A	B	C	D	E
77	A	B	C	D	E	102	A	B	C	D	E	127	A	B	C	D	E
78	A	B	C	D	E	103	A	B	C	D	E	128	A	B	C	D	E
79	A	B	C	D	E	104	A	B	C	D	E	129	A	B	C	D	E
80	A	B	C	D	E	105	A	B	C	D	E	130	A	B	C	D	E
81	A	B	C	D	E	106	A	B	C	D	E	131	A	B	C	D	E
82	A	B	C	D	E	107	A	B	C	D	E	132	A	B	C	D	E
83	A	B	C	D	E	108	A	B	C	D	E	133	A	B	C	D	E
84	A	B	C	D	E	109	A	B	C	D	E	134	A	B	C	D	E
85	A	B	C	D	E	110	A	B	C	D	E	135	A	B	C	D	E
86	A	B	C	D	E	111	A	B	C	D	E	136	A	B	C	D	E
87	A	B	C	D	E	112	A	B	C	D	E	137	A	B	C	D	E
88	A	B	C	D	E	113	A	B	C	D	E	138	A	B	C	D	E
89	A	B	C	D	E	114	A	B	C	D	E	139	A	B	C	D	E
90	A	B	C	D	E	115	A	B	C	D	E	140	A	B	C	D	E
91	A	B	C	D	E	116	A	B	C	D	E	141	A	B	C	D	E
92	A	B	C	D	E	117	A	B	C	D	E	142	A	B	C	D	E
93	A	B	C	D	E	118	A	B	C	D	E	143	A	B	C	D	E
94	A	B	C	D	E	119	A	B	C	D	E	144	A	B	C	D	E
95	A	B	C	D	E	120	A	B	C	D	E	145	A	B	C	D	E
96	A	B	C	D	E	121	A	B	C	D	E	146	A	B	C	D	E
97	A	B	C	D	E	122	A	B	C	D	E	147	A	B	C	D	E
98	A	B	C	D	E	123	A	B	C	D	E	148	A	B	C	D	E
99	A	B	C	D	E	124	A	B	C	D	E	149	A	B	C	D	E
100	A	B	C	D	E	125	A	B	C	D	E	150	A	B	C	D	E
												151	A	B	C	D	E

ETS USE ONLY

	R	W	FS
PT1			
PT2			
PT3			
PT4			
TOT			
EQ			
TA1			
TA2			

DO NOT WRITE IN THIS AREA.

T. YOUR MAILING ADDRESS — HOME ADDRESS AND SCHOOL AREA—COMPLETE THIS AREA ONLY ONCE.

- Your grade report will be mailed to this address in July.
- Using the abbreviations given in your AP Student Pack, print your address in the boxes below. If your international address does not fit, see item V, below.
- Indicate a space in your address by leaving a blank box — do not grid that column. Only one oval is to be filled in for each column.

Street

City

STATE

1 AL, 2 AK, 3 AZ, 4 AR, 5 CA, 6 CO, 7 CT, 8 DE, 9 DC, 10 FL, 11 GA, 12 HI, 13 ID, 14 IL, 15 IN, 16 IA, 17 KS, 18 KY, 19 LA, 20 ME, 21 MD, 22 MA, 23 MI, 24 MN, 25 MS, 26 MO, 27 MT, 28 NE, 29 NV, 30 NH, 31 NJ, 32 NM, 33 NY, 34 NC, 35 ND, 36 OH, 37 OK, 38 OR, 39 PA, 40 RI, 41 SC, 42 SD, 43 TN, 44 TX, 45 UT, 46 VT, 47 VA, 48 WA, 49 WV, 50 WI, 51 WY, 52 Puerto Rico, 53 Other

ZIP OR POSTAL CODE

U. COUNTRY CODE

W. TELEPHONE NUMBER

Area Code

INTERNATIONAL TELEPHONE

If your international telephone number is longer than 10 digits, write the entire number below.

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FOR STUDENTS OUTSIDE THE UNITED STATES ONLY

V. If the address gridded above is not complete enough for delivery of your grade report, please fill in this oval and print your complete address below. ⟶ ◯

Address

Address

City — State or Province

Country — ZIP or Postal Code

X. SCHOOL YOU ATTEND

School Code

School Name, City, and State

← Make sure you have correctly entered your School Code, filled in the appropriate ovals, and completed the information below.

School Name

City

State

Y. COLLEGE TO RECEIVE YOUR AP GRADE REPORT

College Code

College Name and Address

Using the College Code list in the AP Student Pack, indicate the **one** college that you want to receive your AP Grade Report by writing in the college code number, gridding the appropriate ovals, and completing the information below.

College Name

City

State

AP® Calculus BC Exam

SECTION I: Multiple-Choice Questions

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Grade
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the ovals for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the ovals for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	(A) ● (C) (D) (E)

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

About Guessing

Many students wonder whether or not to guess the answers to questions about which they are not certain. In this section of the exam, as a correction for random guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. If you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

---

2. $\int xe^{x^2}\,dx =$

(A) $\frac{1}{2}e^{x^2} + C$ (B) $e^{x^2} + C$ (C) $xe^{x^2} + C$ (D) $\frac{1}{2}e^{2x} + C$ (E) $e^{2x} + C$

---

3. $\lim_{x \to 0} \frac{\sin x \cos x}{x}$ is

(A) -1 (B) 0 (C) 1 (D) $\frac{\pi}{4}$ (E) nonexistent

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4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n\to\infty} \frac{e}{n!} < 1$

(B) $\lim_{n\to\infty} \frac{n!}{e} < 1$

(C) $\lim_{n\to\infty} \frac{n+1}{e} < 1$

(D) $\lim_{n\to\infty} \frac{e}{n+1} < 1$

(E) $\lim_{n\to\infty} \frac{e}{(n+1)!} < 1$

---

5. Which of the following gives the length of the path described by the parametric equations $x = \sin\left(t^3\right)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2\left(t^3\right) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2\left(t^3\right) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4 \cos^2\left(t^3\right) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos\left(t^3\right) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2\left(3t^2\right) + e^{10t}}\, dt$

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$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

   I. f has a limit at $x = 2$.

   II. f is continuous at $x = 2$.

   III. f is differentiable at $x = 2$.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) I, II, and III

---

7. Given that $y(1) = -3$ and $\dfrac{dy}{dx} = 2x + y$, what is the approximation for $y(2)$ if Euler's method is used with a step size of 0.5, starting at $x = 1$?

   (A) -5 (B) -4.25 (C) -4 (D) -3.75 (E) -3.5

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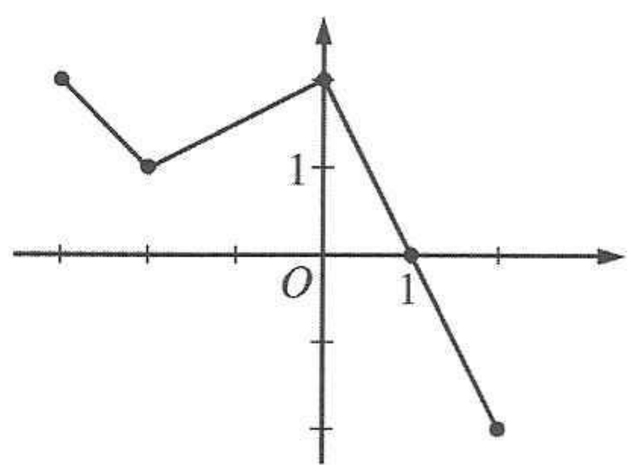
x	2	3	5	8	13
$f(x)$	6	-2	-1	3	9

8. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$, what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

(A) 6 (B) 14 (C) 28 (D) 32 (E) 50



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

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10. In the xy-plane, what is the slope of the line tangent to the graph of $x^2 + xy + y^2 = 7$ at the point $(2, 1)$?

(A) $-\frac{4}{3}$ (B) $-\frac{5}{4}$ (C) -1 (D) $-\frac{4}{5}$ (E) $-\frac{3}{4}$

---

11. Let R be the region between the graph of $y = e^{-2x}$ and the x-axis for $x \geq 3$. The area of R is

(A) $\frac{1}{2e^6}$ (B) $\frac{1}{e^6}$ (C) $\frac{2}{e^6}$ (D) $\frac{\pi}{2e^6}$ (E) infinite

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12. Which of the following series converges for all real numbers x ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

---

13. $\int_1^e \frac{x^2+1}{x}\,dx =$

(A) $\frac{e^2-1}{2}$ (B) $\frac{e^2+1}{2}$ (C) $\frac{e^2+2}{2}$ (D) $\frac{e^2-1}{e^2}$ (E) $\frac{2e^2-8e+6}{3e}$

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x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

---

15. If $f(x) = (\ln x)^2$, then $f''(\sqrt{e}) =$

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{1}{2\sqrt{e}}$ (D) $\frac{1}{\sqrt{e}}$ (E) $\frac{2}{\sqrt{e}}$

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16. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

---

17. Let h be a differentiable function, and let f be the function defined by $f(x) = h\left(x^2 - 3\right)$. Which of the following is equal to $f'(2)$?

(A) $h'(1)$ (B) $4h'(1)$ (C) $4h'(2)$ (D) $h'(4)$ (E) $4h'(4)$

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18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

---

19. $$\int \frac{7x}{(2x-3)(x+2)}\,dx =$$

(A) $\frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

(B) $3\ln|2x-3| + 2\ln|x+2| + C$

(C) $3\ln|2x-3| - 2\ln|x+2| + C$

(D) $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

(E) $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

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20. What is the sum of the series $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$?

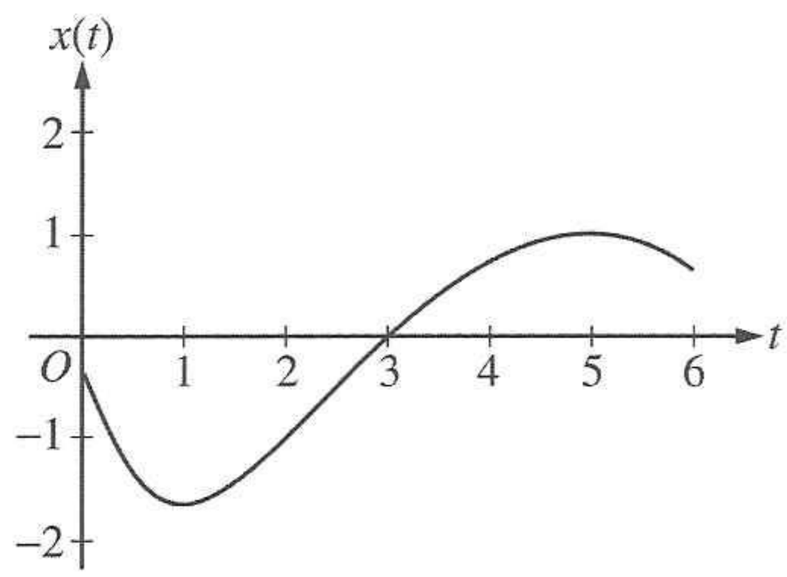
(A) $\ln 2$

(B) $\ln(1 + \ln 2)$

(C) 2

(D) e^2

(E) The series diverges.



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

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x	0	1
$f(x)$	2	4
$f'(x)$	6	–3
$g(x)$	–4	3
$g'(x)$	2	–1

22. The table above gives values of f, f', g, and g' for selected values of x. If $\int_0^1 f'(x)g(x)\,dx = 5$, then $\int_0^1 f(x)g'(x)\,dx =$

(A) –14 (B) –13 (C) –2 (D) 7 (E) 15

---

23. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

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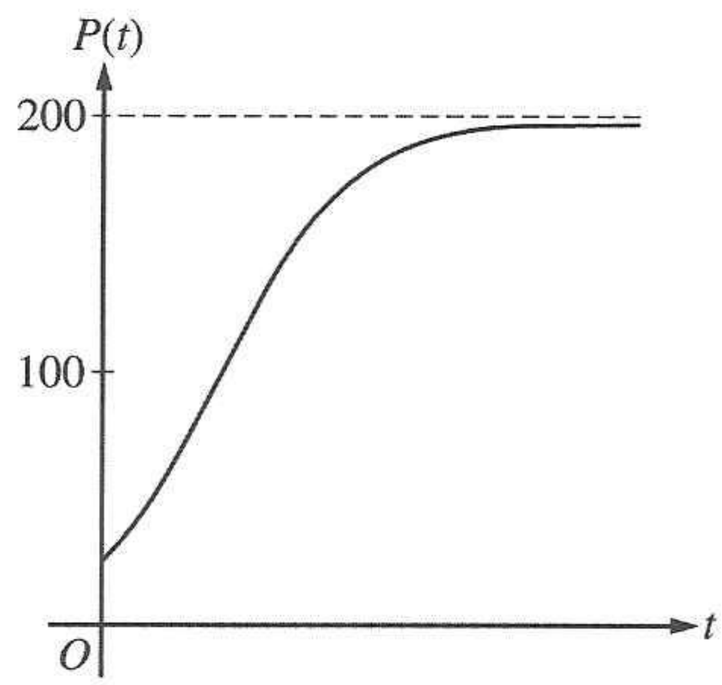

24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

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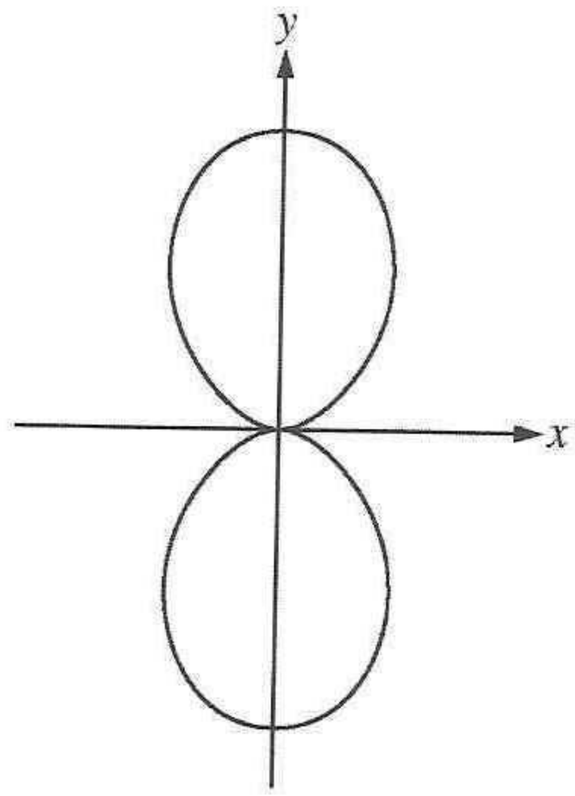

26. Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta \, d\theta$

(B) $\int_0^{\pi} \sin^2\theta \, d\theta$

(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta \, d\theta$

(D) $\int_0^{\pi} \sin^4\theta \, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta \, d\theta$

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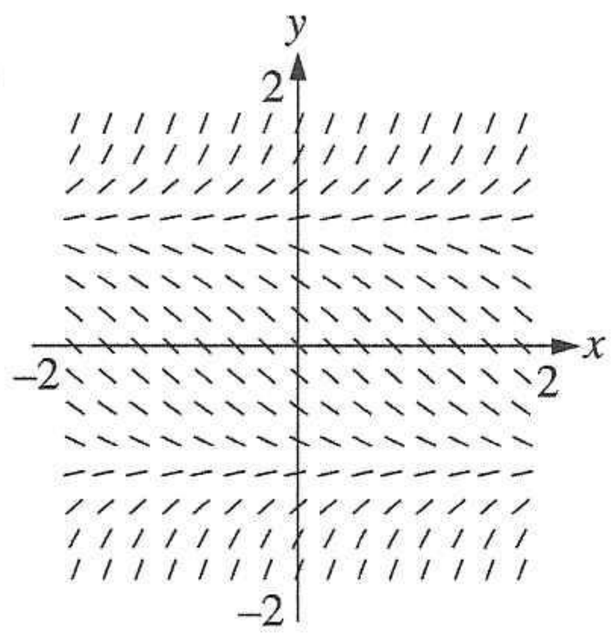
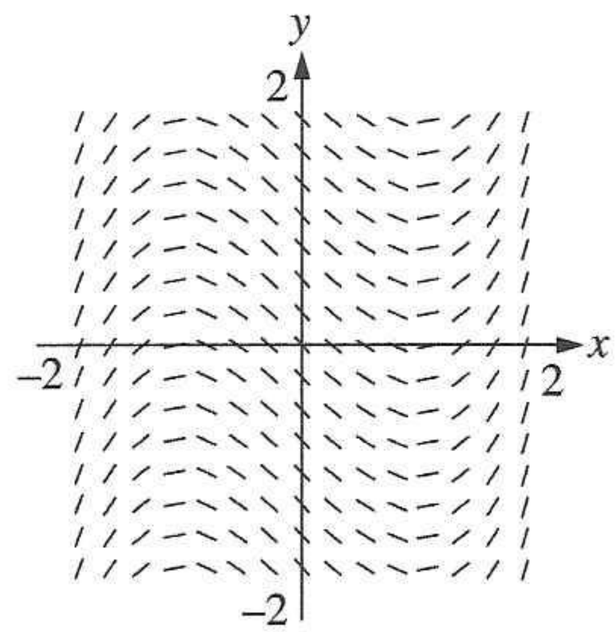
27. Which of the following could be the slope field for the differential equation $\frac{dy}{dx} = y^2 - 1$?

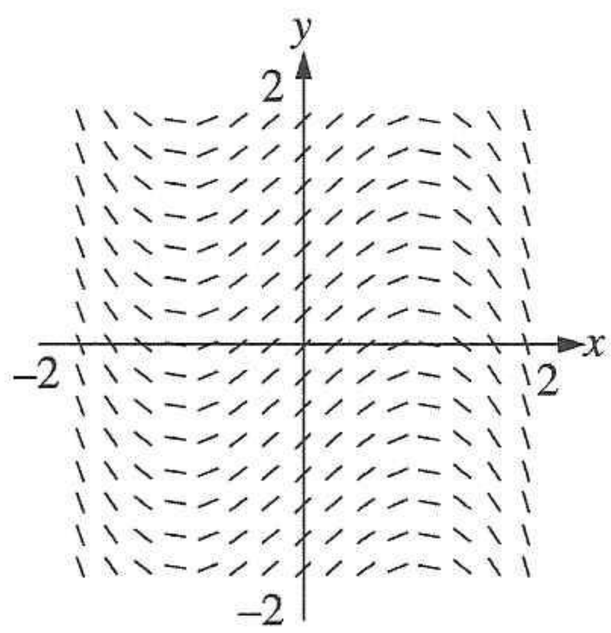
(A)



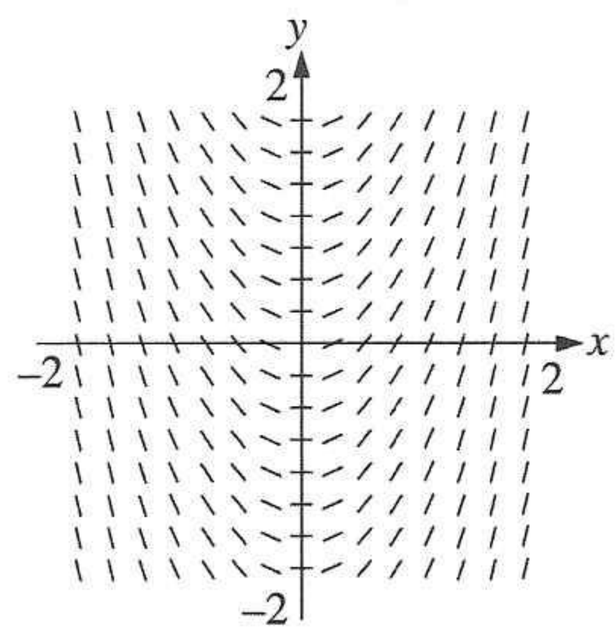
(B)



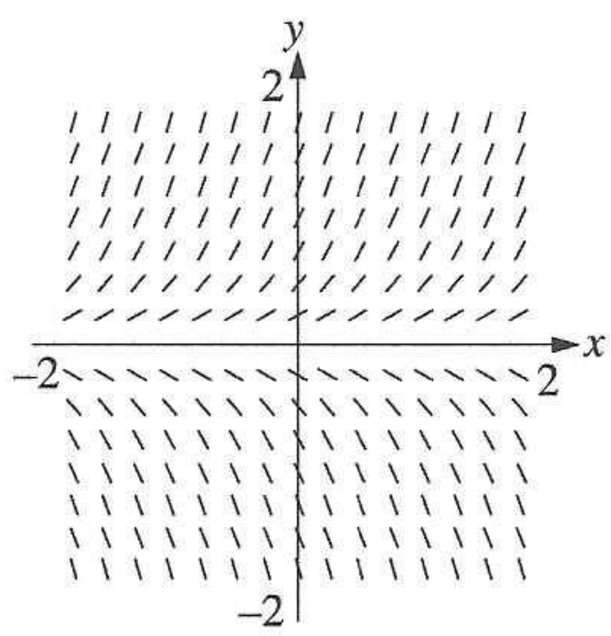
(C)



(D)



(E)



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28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) 6 (E) $6\sqrt{10}$

END OF PART A OF SECTION I

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CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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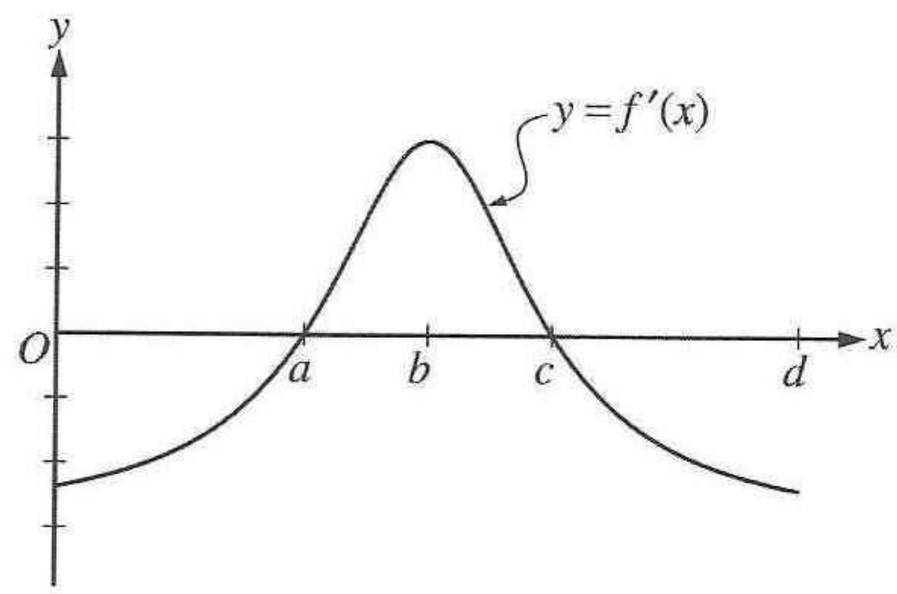

76. The graph of f', the derivative of a function f, is shown above. The domain of f is the open interval $0 < x < d$. Which of the following statements is true?

(A) f has a local minimum at $x = c$.

(B) f has a local maximum at $x = b$.

(C) The graph of f has a point of inflection at $(a, f(a))$.

(D) The graph of f has a point of inflection at $(b, f(b))$.

(E) The graph of f is concave up on the open interval (c, d).

---

77. Water is pumped out of a lake at the rate $R(t) = 12\sqrt{\dfrac{t}{t+1}}$ cubic meters per minute, where t is measured in minutes. How much water is pumped from time $t = 0$ to $t = 5$?

(A) 9.439 cubic meters

(B) 10.954 cubic meters

(C) 43.816 cubic meters

(D) 47.193 cubic meters

(E) 54.772 cubic meters

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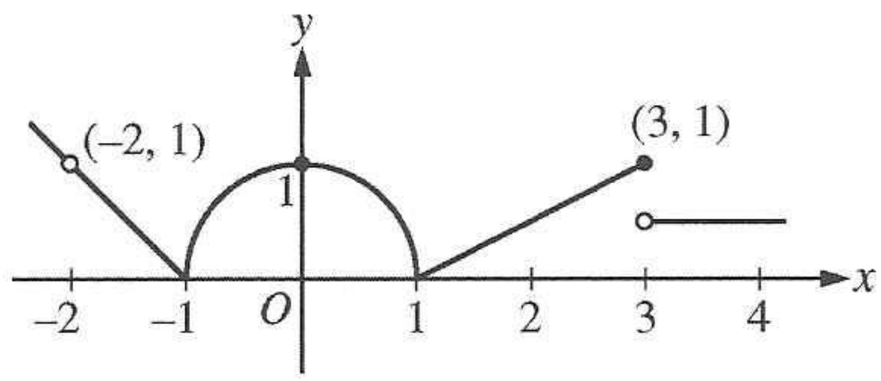

Graph of f

78. The graph of a function f is shown above. For which of the following values of c does $\lim_{x \to c} f(x) = 1$?

(A) 0 only

(B) 0 and 3 only

(C) –2 and 0 only

(D) –2 and 3 only

(E) –2, 0, and 3

---

79. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n \to \infty} a_n = k$

(B) $\int_1^n f(x)\,dx = k$

(C) $\int_1^\infty f(x)\,dx$ diverges.

(D) $\int_1^\infty f(x)\,dx$ converges.

(E) $\int_1^\infty f(x)\,dx = k$

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80. The derivative of the function f is given by $f'(x) = x^2 \cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

---

81. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$, $\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

(A) $P - Q + R - S$

(B) $P - Q - R + S$

(C) $P - Q - R - S$

(D) $P + Q - R - S$

(E) $P + Q - R + S$

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82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

---

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

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84. Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(B) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

---

85. A particle moves on the x-axis with velocity given by $v(t) = 3t^4 - 11t^2 + 9t - 2$ for $-3 \le t \le 3$. How many times does the particle change direction as t increases from -3 to 3 ?

(A) Zero (B) One (C) Two (D) Three (E) Four

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86. On the graph of $y = f(x)$, the slope at any point (x, y) is twice the value of x. If $f(2) = 3$, what is the value of $f(3)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

---

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

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88. For all values of x, the continuous function f is positive and decreasing. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$. Which of the following could be a table of values for g ?

(A)

x	$g(x)$
1	–2
2	0
3	1

(B)

x	$g(x)$
1	–2
2	0
3	3

(C)

x	$g(x)$
1	1
2	0
3	–2

(D)

x	$g(x)$
1	2
2	0
3	–1

(E)

x	$g(x)$
1	3
2	0
3	2

---

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

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x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-5	1	3	0
0	-2	0	1	1
1	0	-3	0	0.5
2	5	-1	5	2

90. The table above gives values of the differentiable functions f and g and of their derivatives f' and g', at selected values of x. If $h(x) = f(g(x))$, what is the slope of the graph of h at $x = 2$?

(A) -10 (B) -6 (C) 5 (D) 6 (E) 10

---

91. Let f be the function given by $f(x) = \int_{1/3}^{x} \cos\left(\frac{1}{t^2}\right) dt$ for $\frac{1}{3} \le x \le 1$. At which of the following values of x does f attain a relative maximum?

(A) 0.357 and 0.798 (B) 0.4 and 0.564 (C) 0.4 only (D) 0.461 (E) 0.999

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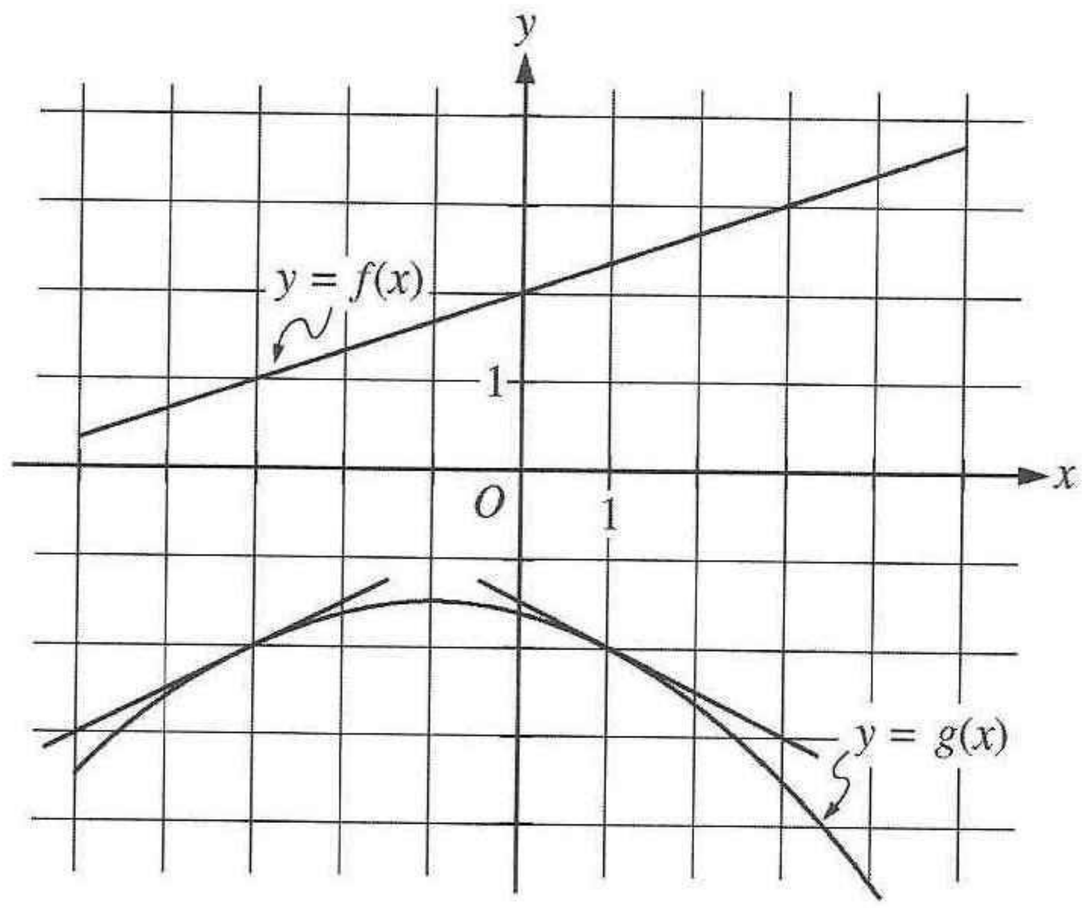

92. The figure above shows the graphs of the functions f and g. The graphs of the lines tangent to the graph of g at $x = -3$ and $x = 1$ are also shown. If $B(x) = g(f(x))$, what is $B'(-3)$?

(A) $-\frac{1}{2}$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

END OF SECTION I

AFTER TIME HAS BEEN CALLED, TURN TO THE NEXT PAGE AND ANSWER QUESTIONS 93-96.

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93. Which graphing calculator did you use during the exam?

(A) Casio 6300, Casio 7300, Casio 7400, Casio 7700, TI-73, TI-80, or TI-81
(B) Casio 9700, Casio 9800, Sharp 9200, Sharp 9300, TI-82, or TI-85
(C) Casio 9750, Casio 9850, Casio 9860, Casio FX 1.0, Sharp 9600, Sharp 9900, TI-83, TI-83 Plus, TI-83 Plus Silver, TI-84 Plus, TI-84 Plus Silver, TI-86, or TI-Nspire
(D) Casio 9970, Casio Algebra FX 2.0, HP 38G, HP 39 series, HP 40G, HP 48 series, HP 49 series, HP 50 series, TI-89, TI-89 Titanium, or TI-Nspire CAS
(E) Some other graphing calculator

94. During your Calculus BC course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus BC course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus BC course, which of the following describes the portion of testing time you were allowed to use a graphing calculator?

(A) All or almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never

AP® Calculus BC Exam

SECTION II: Free-Response Questions

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Grade
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
3
Time
45 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
50%

Part B

Number of Questions
3
Time
45 minutes
Electronic Device
None allowed
Percent of Section II Score
50%

Instructions

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

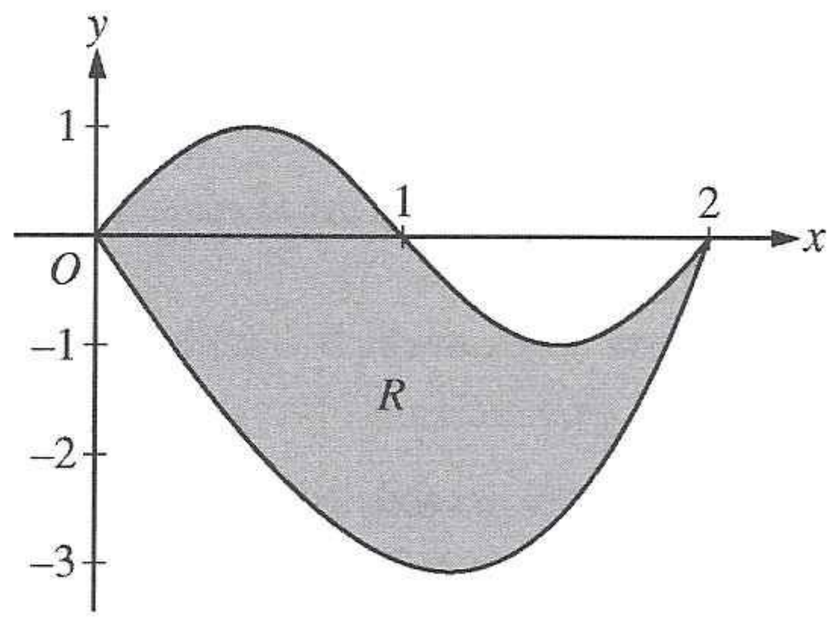
Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

3. Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

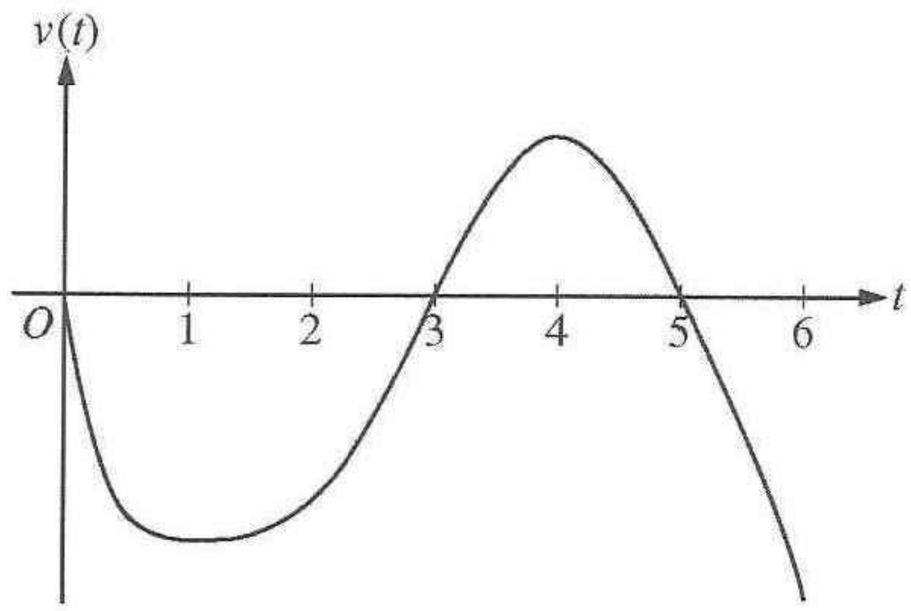
(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

   (a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

   (b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

   (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

   (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

---

5. The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for $x > 0$, and $f(1) = 7$.

   (a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

   (b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

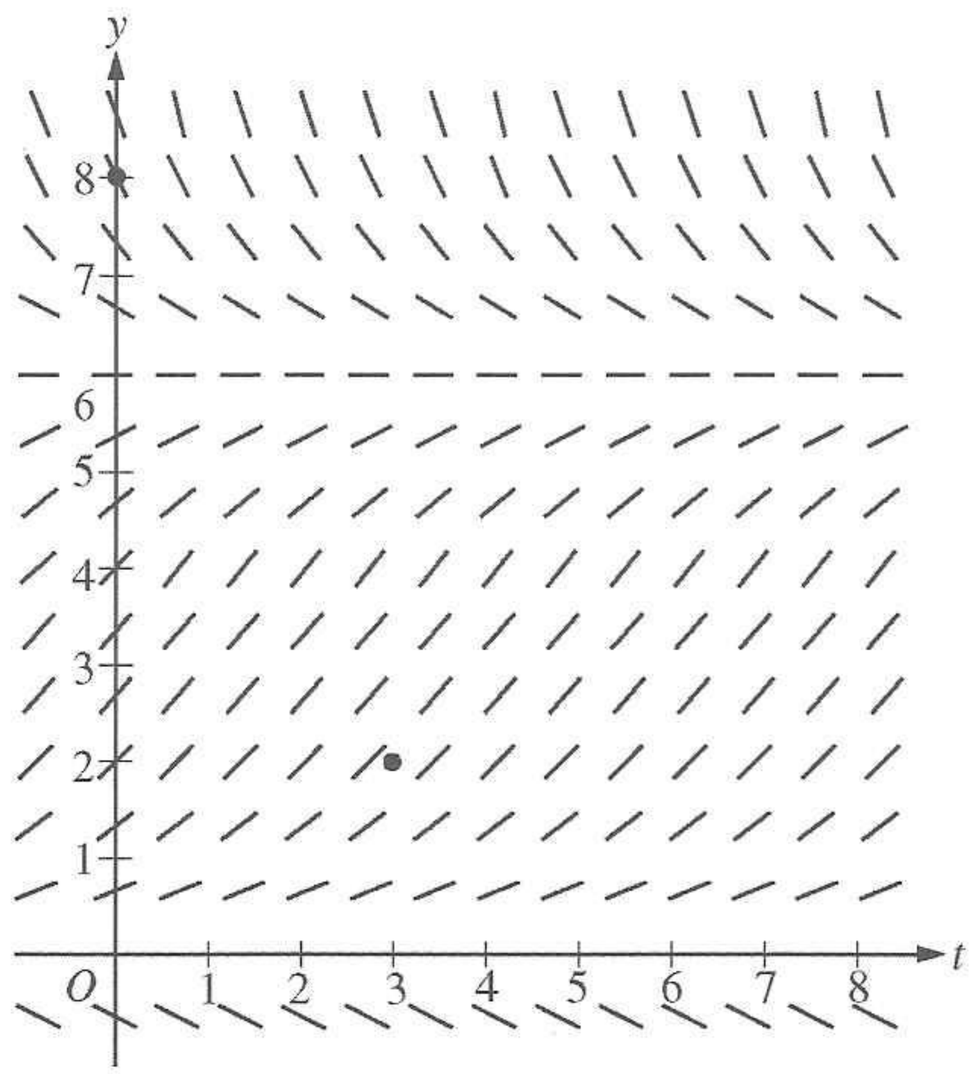
   (c) Find the value of $f(3)$.

---

6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

(Note: Use the axes provided in the exam booklet.)



(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

END OF EXAM

Section I Answer Key and Percent Answering Correctly
Calculus BC

Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
1	B	97	95	93	89	81	93
2	A	96	92	87	80	67	89
3	C	88	77	67	55	36	73
4	D	95	86	74	61	40	80
5	C	94	85	76	66	40	80
6	A	81	61	52	43	35	63
7	D	90	73	58	43	23	69
8	B	89	71	56	39	26	68
9	D	96	85	74	58	33	79
10	B	90	80	72	59	39	76
11	A	75	48	34	27	21	52
12	D	64	35	25	21	16	42
13	B	75	55	40	29	23	55
14	E	63	39	32	27	22	45
15	A	85	63	45	31	17	61
16	D	62	38	27	21	16	42
17	B	79	60	45	37	28	59
18	A	59	33	23	16	13	38
19	A	61	41	30	23	18	43
20	C	49	23	16	12	9	30
21	A	58	35	23	16	10	38
22	E	31	11	7	6	5	18
23	E	80	57	42	28	15	56

Item No.	Correct Answer	Percent Correct by Grade 5	4	3	2	1	Total Percent Correct
24	A	63	37	25	19	13	41
25	B	57	27	19	13	12	35
26	D	56	35	25	19	13	38
27	A	83	66	52	41	28	64
28	D	26	5	3	2	2	13
76	D	93	84	72	56	36	77
77	D	99	98	94	87	69	93
78	C	94	81	66	49	26	74
79	D	70	45	36	27	21	49
80	E	70	56	44	33	21	54
81	B	84	68	56	41	24	65
82	E	93	78	65	51	37	75
83	B	93	82	71	59	39	77
84	A	95	86	73	54	28	78
85	C	89	74	62	49	35	71
86	C	73	42	28	19	13	47
87	D	91	68	48	31	18	65
88	A	85	60	43	29	16	59
89	E	79	55	38	24	14	54
90	D	91	79	62	44	22	71
91	D	85	66	47	29	15	61
92	B	90	73	54	33	18	67

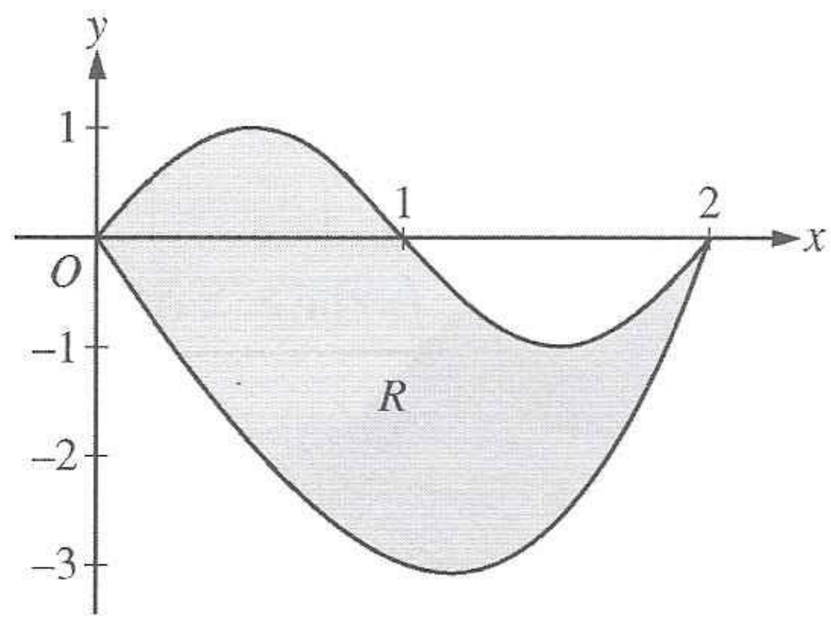
Analyzing Your Students' Performance on the Multiple-Choice Section

If you give your students the 2008 Calculus AB or Calculus BC Exam for practice, you may want to analyze the results to find overall strengths and weaknesses in their understanding of calculus. The following diagnostic worksheets will help you do this. You are permitted to photocopy and distribute them to your students for completion.

1. In each category, students should insert a check mark for each correct answer.
2. Add together the total number of correct answers for each category.
3. To compare the student's number of correct answers for a given category with the average number correct for that category, copy the number of correct answers to the "Number Correct" table at the end of the Diagnostic Guide.

In addition, under each item, the percent of AP students who answered correctly is shown, so students can analyze their performance on individual items. This information will be helpful in deciding how students should plan their study time. Please note that one item may appear in several different categories, as questions can cover different topics.

Scoring Guidelines for Calculus AB/BC Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 4$	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = 9.978$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or 8.370	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Scoring Guidelines for Calculus AB/BC Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

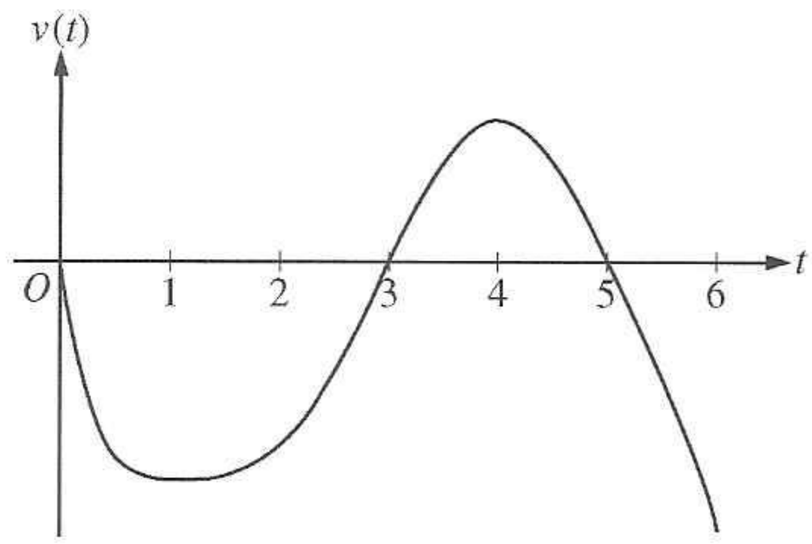
[*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Scoring Guidelines for Calculus AB/BC Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.

$$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$

$$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$

Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.

$3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$

(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.

By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.

$3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$

(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.

1 : answer with reason

(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

Scoring Guidelines for Calculus BC Question 3

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \le x \le 3$.

(a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.

(b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.

(c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4}.

(a) $P_1(x) = 80 + 128(x-2)$, so $h(1.9) \approx P_1(1.9) = 67.2$

$P_1(1.9) < h(1.9)$ since h' is increasing on the interval $1 \le x \le 3$.

$4 : \begin{cases} 2 : P_1(x) \\ 1 : P_1(1.9) \\ 1 : P_1(1.9) < h(1.9) \text{ with reason} \end{cases}$

(b) $P_3(x) = 80 + 128(x-2) + \frac{488}{6}(x-2)^2 + \frac{448}{18}(x-2)^3$

$h(1.9) \approx P_3(1.9) = 67.988$

$3 : \begin{cases} 2 : P_3(x) \\ 1 : P_3(1.9) \end{cases}$

(c) The fourth derivative of h is increasing on the interval $1 \le x \le 3$, so $\max\limits_{1.9 \le x \le 2} \left| h^{(4)}(x) \right| = \frac{584}{9}$.

Therefore, $|h(1.9) - P_3(1.9)| \le \frac{584}{9} \frac{|1.9-2|^4}{4!}$

$= 2.7037 \times 10^{-4}$

$< 3 \times 10^{-4}$

$2 : \begin{cases} 1 : \text{form of Lagrange error estimate} \\ 1 : \text{reasoning} \end{cases}$

Scoring Guidelines for Calculus BC Question 5

The derivative of a function f is given by $f'(x) = (x-3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

(c) Find the value of $f(3)$.

(a) $f'(x) < 0$ for $0 < x < 3$ and $f'(x) > 0$ for $x > 3$

Therefore, f has a relative minimum at $x = 3$.

$2: \begin{cases} 1\text{: minimum at } x = 3 \\ 1\text{: justification} \end{cases}$

(b) $f''(x) = e^x + (x-3)e^x = (x-2)e^x$

$f''(x) > 0$ for $x > 2$

$f'(x) < 0$ for $0 < x < 3$

Therefore, the graph of f is both decreasing and concave up on the interval $2 < x < 3$.

$3: \begin{cases} 2: f''(x) \\ 1: \text{answer with reason} \end{cases}$

(c) $f(3) = f(1) + \int_1^3 f'(x)\,dx = 7 + \int_1^3 (x-3)e^x\,dx$

$u = x - 3 \quad dv = e^x\,dx$

$du = dx \quad v = e^x$

$$f(3) = 7 + (x-3)e^x\Big|_1^3 - \int_1^3 e^x\,dx$$

$$= 7 + \left((x-3)e^x - e^x\right)\Big|_1^3$$

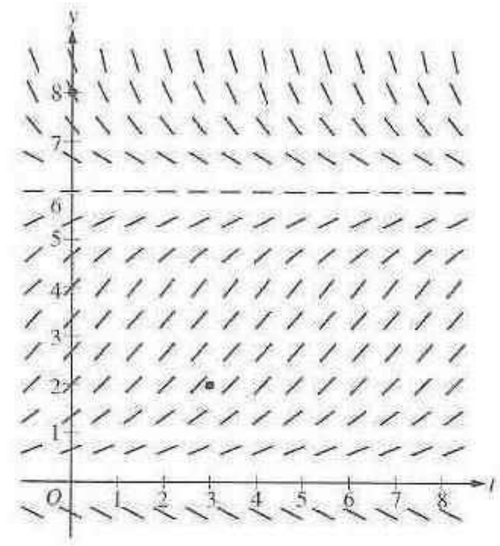
$$= 7 + 3e - e^3$$

$4: \begin{cases} 1: \text{uses initial condition} \\ 2: \text{integration by parts} \\ 1: \text{answer} \end{cases}$

Scoring Guidelines for Calculus BC Question 6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)

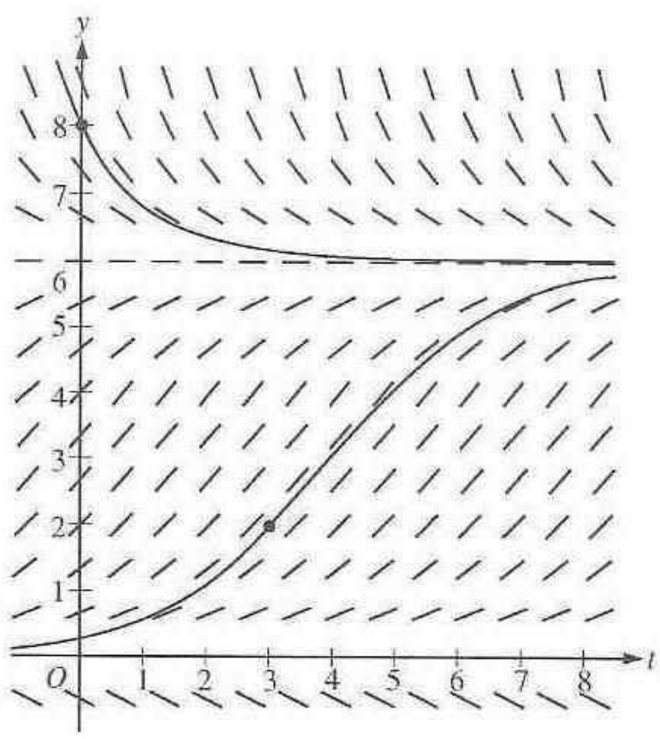


(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

(a)



$2 : \begin{cases} 1\text{: solution curve through } (0,8) \\ 1\text{: solution curve through } (3,2) \end{cases}$

(b) $f\left(\frac{1}{2}\right) \approx 8 + (-2)\left(\frac{1}{2}\right) = 7$

$f(1) \approx 7 + \left(-\frac{7}{8}\right)\left(\frac{1}{2}\right) = \frac{105}{16}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{approximation of } f(1) \end{cases}$

(c) $\frac{d^2y}{dt^2} = \frac{1}{8}\frac{dy}{dt}(6 - y) + \frac{y}{8}\left(-\frac{dy}{dt}\right)$

$f(0) = 8;\ f'(0) = \left.\frac{dy}{dt}\right|_{t=0} = \frac{8}{8}(6 - 8) = -2;$ and

$f''(0) = \left.\frac{d^2y}{dt^2}\right|_{t=0} = \frac{1}{8}(-2)(-2) + \frac{8}{8}(2) = \frac{5}{2}$

The second-degree Taylor polynomial for f about $t = 0$ is $P_2(t) = 8 - 2t + \frac{5}{4}t^2$.

$f(1) \approx P_2(1) = \frac{29}{4}$

$4 : \begin{cases} 2 : \frac{d^2y}{dt^2} \\ 1 : \text{second-degree Taylor polynomial} \\ 1 : \text{approximation of } f(1) \end{cases}$

(d) The range of f for $t \geq 0$ is $6 < y \leq 8$.

1 : answer

4.2BC—Calculus BC Scoring Worksheet

Section I: Multiple Choice

[_____ − (1/4 × _____)] × 1.2000 = _____

Number Correct (out of 45) — Number Wrong — Weighted Section I Score (If less than zero, enter zero; do not round)

Section II: Free Response

Question 1 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 2 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 3 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 4 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 5 (AB Part) _____ (out of 5) × 1.0000 = _____ (Do not round)

Question 5 (BC Part) _____ (out of 4) × 1.0000 = _____ (Do not round)

Question 6 _____ (out of 9) × 1.0000 = _____ (Do not round)

Sum = _____

Weighted Section II Score (Do not round)

AP Grade Conversion Chart
Calculus BC

Composite Score Range	AP Grade
66–108	5
55–65	4
39–54	3
31–38	2
0–30	1

Composite Score

_____ + _____ = _____

Weighted Section I Score — Weighted Section II Score — Composite Score (Round to nearest whole number)

Table 4.3BC—Calculus BC Grade Distributions

Nearly 80 percent of the AP students who took this exam earned a qualifying grade of 3 or above.

	Exam Grade	Number of Students	Percent at Grade
Extremely well qualified	5	27,964	42.8
Well qualified	4	11,469	17.5
Qualified	3	12,829	19.6
Possibly qualified	2	4,484	6.9
No recommendation	1	8,641	13.2
Total Number of Students		65,387	
Mean Grade		3.70	
Standard Deviation		1.41	

Table 4.4BC—Calculus BC Section I Scores and AP Grades

For a given range of multiple-choice scores, this table shows the percentage of students receiving each AP grade. If you have calculated the multiple-choice score (**Weighted Section I Score**) by using the formula shown in Table 4.2BC, you can use this table to figure out the most likely grade that the student would receive based only on that multiple-choice score.

Multiple-Choice Score	AP Grade					Total
	1	2	3	4	5	
46 to 54	0.0%	0.0%	0.0%	0.0%	100.0%	9.4%
37 to 45	0.0%	0.0%	0.1%	2.4%	97.5%	21.6%
28 to 36	0.0%	0.1%	7.6%	44.3%	48.1%	25.1%
19 to 27	0.5%	5.6%	64.6%	28.0%	1.3%	20.9%
10 to 18	33.0%	37.4%	29.2%	0.4%	0.0%	14.1%
0 to 9	94.3%	4.5%	1.1%	0.1%	0.0%	9.0%
Total	13.2%	6.9%	19.6%	17.5%	42.8%	100.0%

Table 4.2AB1—Calculus AB Subscore Scoring Worksheet

Section I: Multiple Choice

Questions (2, 6, 8, 9, 10, 13, 14, 15, 17, 18, 21, 25, 27, 76, 77, 78, 80, 81, 83, 85–92)

[_____ − (1/4 × _____)] × 1.1851 = _____

Number Correct (out of 27) — Number Wrong — Weighted Section I Score (If less than zero, enter zero; do not round)

Section II: Free Response

Question 1 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 2 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 4 _____ (out of 9) × 1.0000 = _____ (Do not round)

Question 5 (AB Part) _____ (out of 5) × 1.0000 = _____ (Do not round)

Sum = _____

Weighted Section II Score (Do not round)

Composite Score

_____ + _____ = _____

Weighted Section I Score — Weighted Section II Score — Composite Score (Round to nearest whole number)

AP Grade Conversion Chart
Calculus AB Subscore

Composite Score Range	AP Grade
40–64	5
29–39	4
20–28	3
12–19	2
0–11	1

Table 4.3AB1—Calculus AB Subscore Grade Distributions

More than 86 percent of the AP students who took this exam earned an AB subscore grade of 3 or above.

	Exam Grade	Number of Students	Percent at Grade
Extremely well qualified	5	32,440	49.6
Well qualified	4	15,561	23.8
Qualified	3	8,466	13.0
Possibly qualified	2	4,992	7.6
No recommendation	1	3,928	6.0
Total Number of Students		65,387	
Mean Grade		4.03	
Standard Deviation		1.21	