Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2013 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

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AP® Calculus BC Exam

SECTION I: Multiple Choice

2013

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

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Form I
Form Code 4JBP4-Q-S
68

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DO NOT seal answer sheet inside

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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9. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 2x + y$ with initial condition $f(1) = 0$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length, starting at $x = 1$?

(A) 0 (B) 1 (C) 2.75 (D) 3 (E) 6

---

10. If $\int_0^k \frac{x}{x^2 + 4}\, dx = \frac{1}{2}\ln 4$, where $k > 0$, then $k =$

(A) 0 (B) $\sqrt{2}$ (C) 2 (D) $\sqrt{12}$ (E) $\frac{1}{2}\tan\left(\ln\sqrt{2}\right)$

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14. If a and b are positive constants, then $\lim_{x\to\infty} \frac{\ln(bx+1)}{\ln(ax^2+3)} =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{2}ab$ (D) 2 (E) ∞

---

15. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(x+\frac{3}{2}\right)^n$ converges?

(A) $-\frac{5}{2} < x < -\frac{1}{2}$

(B) $-\frac{5}{2} < x \le -\frac{1}{2}$

(C) $-\frac{5}{2} \le x < -\frac{1}{2}$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

(E) $x \le -\frac{1}{2}$

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CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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GO ON TO THE NEXT PAGE.

76. Let f be a function whose derivative is given by $f'(x) = \ln\left(x^4 + 5x^3 + x^2 - 7x + 28\right)$. On the open interval $(-4, 1)$, at which of the following values of x does f attain a relative maximum?

(A) -3.623 only

(B) -0.871 only

(C) -3.623 and -3.284

(D) -3.459 and 0.581 only

(E) -3.459, -0.871, and 0.581

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GO ON TO THE NEXT PAGE.

a	$\lim_{x \to a^-} f(x)$	$\lim_{x \to a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

77. The function f has the properties indicated in the table above. Which of the following must be true?

(A) f is continuous at $x = -1$

(B) f is continuous at $x = 0$

(C) f is continuous at $x = 1$

(D) f is differentiable at $x = 0$

(E) f is differentiable at $x = 1$.

---

78. What is the area of the region in the first quadrant enclosed by the graphs of $y = \sin(2x)$ and $y = x$

(A) 0.208 (B) 0.210 (C) 0.266 (D) 0.660 (E) 0.835

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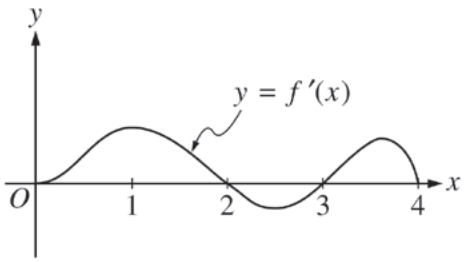
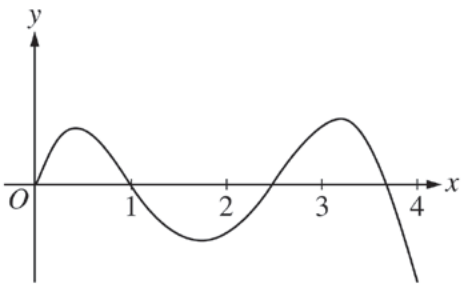


79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

(A)



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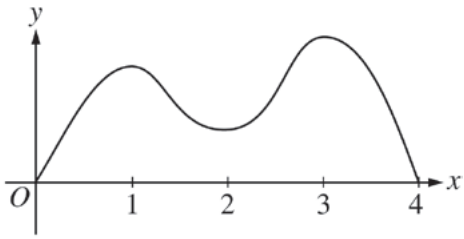

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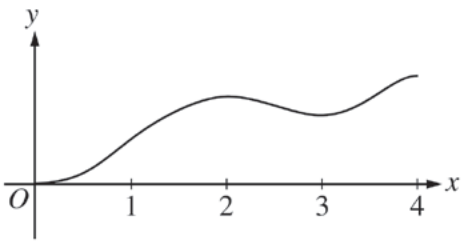

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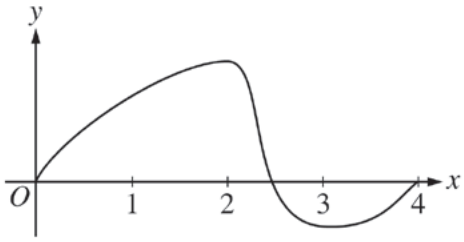
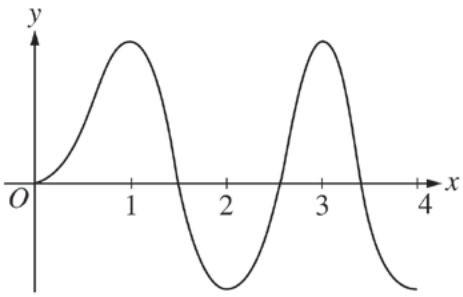


(E)



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80. The volume of a certain cone for which the sum of its radius, r, and height is constant is given by $V = \frac{1}{3}\pi r^2(10 - r)$. The rate of change of the radius of the cone with respect to time is 6. In terms of r, what is the rate of change of the volume of the cone with respect to time?

(A) $-24\pi r$ (B) $6\pi r$ (C) $\frac{20}{3}\pi r - \pi r^2$ (D) $16\pi r - \frac{4}{3}\pi r^2$ (E) $40\pi r - 6\pi r^2$

---

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit $(°\text{F})$. If the initial temperature of the tea, at time $t = 0$ minutes, is $200°\text{F}$ and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

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82. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \leq b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \leq a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \leq a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

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x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

---

84. Let f be a function with derivative given by $f'(x) = x^3 - 5x^2 + e^x$. On which of the following intervals is the graph of f concave down?

(A) $(-\infty, 0.117)$ only

(B) $(-\infty, 1.144)$

(C) $(0.116, 2.062)$

(D) $(0.673, 2.863)$

(E) $(2.863, \infty)$

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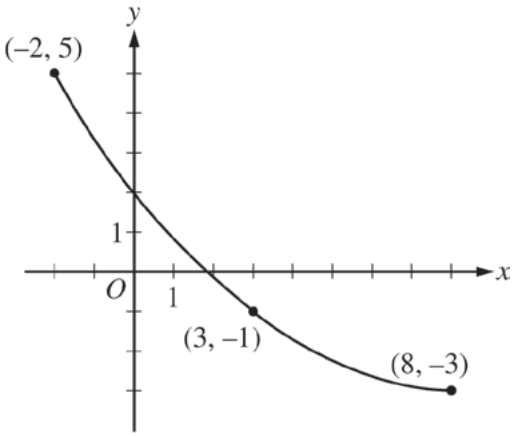

Graph of f

85. A portion of the graph of a differentiable function f is shown above. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of f at $x = 3$?

(A) $-\frac{7}{5}$ (B) $-\frac{5}{4}$ (C) $-\frac{4}{5}$ (D) $-\frac{5}{7}$ (E) $-\frac{1}{5}$

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86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
–1	4
0	3
1	1

(B)

x	$f(x)$
–1	4
0	4
1	4

(C)

x	$f(x)$
–1	4
0	5
1	6

(D)

x	$f(x)$
–1	4
0	5
1	7

(E)

x	$f(x)$
–1	4
0	6
1	7

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87. The position of a particle moving in the xy-plane is given by the parametric functions $x(t)$ and $y(t)$ for which $x'(t) = t\sin t$ and $y'(t) = 5e^{-3t} + 2$. What is the slope of the line tangent to the path of the particle at the point at which $t = 2$?

(A) 0.904 (B) 1.107 (C) 1.819 (D) 2.012 (E) 3.660

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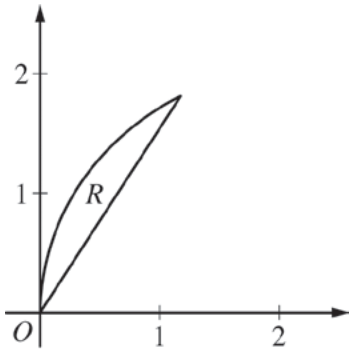

88. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 4\cos\theta$ and below by the line $\theta = 1$, as shown in the figure above. What is the area of R ?

(A) 0.317 (B) 0.465 (C) 0.929 (D) 2.618 (E) 5.819

---

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

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90. Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \dfrac{(-1)^n}{\sqrt{n} + (-1)^n}$?

I. The series is alternating.

II. $|a_{n+1}| \le |a_n|$ for all $n \ge 2$

III. $\lim_{n\to\infty} a_n = 0$

(A) None

(B) I only

(C) I and II only

(D) I and III only

(E) I, II, and III

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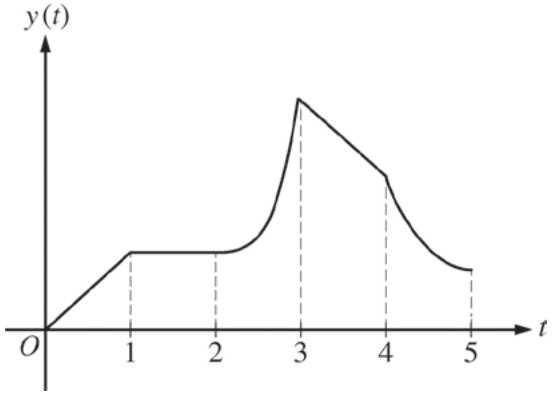

91. A particle moves along the y-axis. The graph of the particle's position $y(t)$ at time t is shown above for $0 \le t \le 5$. For what values of t is the velocity of the particle negative and the acceleration positive?

(A) $0 < t < 1$ (B) $1 < t < 2$ (C) $2 < t < 3$ (D) $3 < t < 4$ (E) $4 < t < 5$

---

92. If f is a function such that $f'(x) = -f(x)$, then $\int x f(x)\,dx =$

(A) $f(x)(x+1) + C$

(B) $-f(x)(x+1) + C$

(C) $\frac{x^2}{2} f(x) + C$

(D) $-\frac{x^2}{2} f(x) + C$

(E) $-\frac{x^2}{2} f(x)\left(1 + \frac{x}{3}\right) + C$

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GO ON TO THE NEXT PAGE.

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

Section II: Free-Response Questions

This is the free-response section of the 2013 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP® Calculus BC Exam

SECTION II: Free Response

2013

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
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Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code Z-3YBP2-S

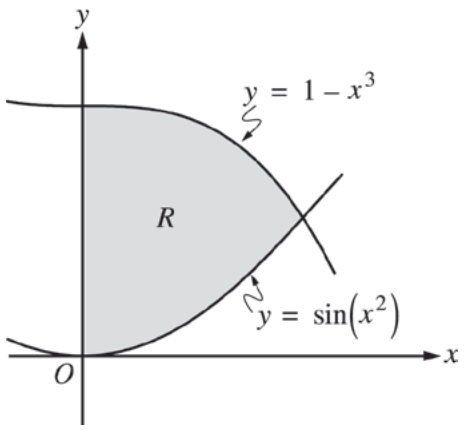
CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.



1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

   (a) Find the area of R.

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Continue problem 1 on page 5.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

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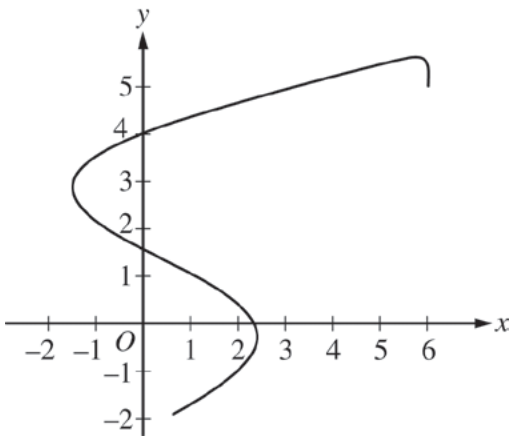

2. A planetary rover travels on a flat surface. The path of the rover for the time interval $0 \le t \le 2$ hours is shown in the rectangular coordinate system above. The rover starts at the point with coordinates $(6, 5)$ at time $t = 0$. The coordinates $(x(t), y(t))$ of the position of the rover change at rates given by

$$x'(t) = -12\sin\left(2t^2\right)$$
$$y'(t) = 10\cos\left(1 + \sqrt{t}\right),$$

where $x(t)$ and $y(t)$ are measured in meters and t is measured in hours.

(a) Find the acceleration vector of the rover at time $t = 1$. Find the speed of the rover at time $t = 1$.

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Continue problem 2 on page 7.

(b) Find the total distance that the rover travels over the time interval $0 \le t \le 1$.

(c) Find the y-coordinate of the position of the rover at time $t = 1$.

(d) The rover receives a signal at each point where the line tangent to its path has slope $\frac{1}{2}$. At what times t, for $0 \le t \le 2$, does the rover receive a signal?

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GO ON TO THE NEXT PAGE.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

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NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

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Continue problem 3 on page 15.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

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4. Consider the function f given by $f(x) = xe^{-x^2}$ for all real numbers x.

(a) At what value of x does $f(x)$ attain its absolute maximum? Justify your answer.

(b) Find an antiderivative of f.

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Continue problem 4 on page 17.

(c) Find the value of $\int_0^{\infty} x f(x)\,dx$, given the fact that $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

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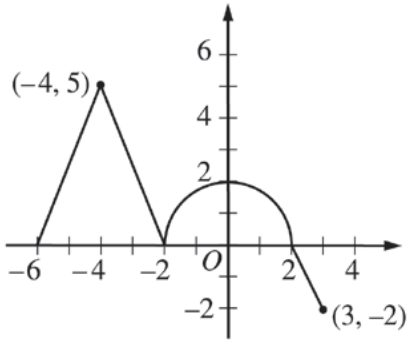

Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

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Continue problem 5 on page 19.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

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6. The function f satisfies the equation

$$f'(x) = f(x) + x + 1$$

and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

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Continue problem 6 on page 21.

(c) Find the fourth-degree Taylor polynomial for f about $x = 0$.

(d) Find $f^{(n)}(0)$, the nth derivative of f at $x = 0$, for $n \geq 2$. Use the Taylor series for f about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) - 4e^x$.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**