

Assignment 5

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1.

$P \Rightarrow \neg Q, Q \Rightarrow \neg P$

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$ [$P \Rightarrow \neg Q$]	$\neg Q \vee \neg P$ [$Q \Rightarrow \neg P$]
T	F	F	T	T	T
T	T	F	F	F	F
F	F	T	T	T	T
F	T	T	F	T	T

$P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$\neg P \vee \neg Q$ [$P \Rightarrow \neg Q$]	$Q \vee P$ [$\neg Q \Rightarrow P$]	$(\neg P \vee \neg Q) \wedge (Q \vee P)$ [$P \Leftrightarrow \neg Q$]	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
F	T	T	T	T	F	T	T
F	F	T	F	F	F	F	F

2.

$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$

It's neither. (satisfiable)

It's not valid because there exists a $M(\text{Smoke} \Rightarrow \text{Fire})$ that doesn't belong to $M(\neg \text{Smoke} \Rightarrow \neg \text{Fire})$.

It's not unsatisfiable because $M((\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})) \neq \emptyset$.

Smoke	Fire	$\text{Smoke} \Rightarrow \text{Fire}$ [$\neg \text{Smoke} \vee \text{Fire}$]	$\neg \text{Smoke} \Rightarrow \neg \text{Fire}$ [$\text{Smoke} \vee \neg \text{Fire}$]	$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$
T	T	T	T	T

T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

$$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$$

It's neither. (satisfiable)

It's not valid because there exists a $M(\text{Smoke} \Rightarrow \text{Fire})$ that doesn't belong to $M((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$.

It's not unsatisfiable because $M((\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})) \neq \emptyset$.

Smoke	Fire	Heat	(Smoke ⇒ Fire)	(Smoke ∨ Heat)	((Smoke ∨ Heat) ⇒ Fire)	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

$$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$$

It's valid because $M(((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))) = \text{whole set}.$

S	F	H	$S \wedge H$	$S \Rightarrow F$	$H \Rightarrow F$	$(S \wedge H) \Rightarrow F$	$(S \Rightarrow F) \vee (H \Rightarrow F)$	$((S \wedge H) \Rightarrow F) \Rightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$	$((S \Rightarrow F) \vee (H \Rightarrow F)) \Rightarrow ((S \wedge H) \Rightarrow F)$	$((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$
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T	T	T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T	T
T	F	T	T	F	F	F	F	T	T	T
T	F	F	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T	T	T
F	F	T	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T

3.

(a)

Define variables: mythical, mortal, mammal, horned.

$\text{mythical} \Rightarrow \neg \text{mortal}$

$\neg \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})$

$\neg \text{mortal} \vee \text{mammal} \Rightarrow \text{horned}$

$\text{horned} \Rightarrow \text{magical}$

(b)

$\text{KB} = (\neg \text{mythical} \vee \neg \text{mortal}) \wedge$

$(\text{mythical} \vee (\text{mortal} \wedge \text{mammal})) \wedge$

$(\neg(\neg \text{mortal} \vee \text{mammal}) \vee \text{horned}) \wedge$

$(\neg \text{horned} \vee \text{magical})$

$= (\neg \text{mythical} \vee \neg \text{mortal}) \wedge (\text{mythical} \vee \text{mortal}) \wedge (\text{mythical} \vee \text{mammal}) \wedge$

$((\text{mortal} \wedge \neg \text{mammal}) \vee \text{horned}) \wedge (\neg \text{horned} \vee \text{magical})$

$= (\neg \text{mythical} \vee \neg \text{mortal}) \wedge (\text{mythical} \vee \text{mortal}) \wedge (\text{mythical} \vee \text{mammal}) \wedge$

$(\text{mortal} \vee \text{horned}) \wedge (\neg \text{mammal} \vee \text{horned}) \wedge (\neg \text{horned} \vee \text{magical})$

(c)

Use the knowledge base to prove that the unicorn is mythical:

KB:

1. $(\neg \text{mythical} \vee \neg \text{mortal})$

2a. $(\text{mythical} \vee \text{mortal})$

2b. $(\text{mythical} \vee \text{mammal})$

3a. $(\text{mortal} \vee \text{horned})$

3b. $(\neg \text{mammal} \vee \text{horned})$

4. $(\neg \text{horned} \vee \text{magical})$

Assuming unicorn is not horned, adding to KB:

5. \neg horned

We can get following:

6. \neg mammal (3b+5)

7. mortal (3a+5)

8. mythical (2b+6)

9. \neg mythical (1+7)

10. empty set (8+9)

Therefore, a unicorn is horned is proved by contradiction.

Now that we have proved that unicorn is horned, we can add horned to KB.

1. (\neg mythical \vee \neg mortal)

2a. (mythical \vee mortal)

2b. (mythical \vee mammal)

3a. (mortal \vee horned)

3b. (\neg mammal \vee horned)

4. (\neg horned \vee magical)

5. horned

Similarly, assuming unicorn is not magical, adding to KB:

6. \neg magical

7. \neg horned (4+6)

8. Empty set (5+7)

Therefore, a unicorn is magical is proved by contradiction.

We cannot prove a unicorn is mythical because no resolution can be made.