Solar nu xsection on overdense CNB

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Solar neutrinos interact on our necessarily over-dense CNB. This write-up shows the calculation of that rate. A jupyter notebook also exists in this repo to place a limit in n-g space so as not to diminish the solar ν_e flux beyond 1% or so of its very well-known value, which parameter space is not tenable.

I. MOTIVATION

To calculate the interaction rate we want to appeal to the optical theorem,

$$\sigma = 4\pi/k\Im(f(\theta = 0)) \tag{1}$$

.

That is because in our paper we have calculated matrix elements in the very forward direction and come up with some tidy expressions (eqn 8 and 9 of the paper) for the scattering amplitudes. If, instead, we need to fully calculate the xsection from scratch we'll need to go back to the 4 diagrams and sum them with general kinematics, square them and integrate, blah blah blah. It'd be non-trivial.

II. INTRO

See [1] for a very nice elaboration of the optical theorem. Basically, one writes a plane wave $\psi = \exp^{ikz}$, decomposes in legendre polynomials and tacks on extra phases for the out-going part of the wave. The differential cross-section is calculated with $\psi^*\psi$.

$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2 \tag{2}$$

An expression is shown in that reference for $f(\theta, \phi)$ which we don't list here. But, because it contains legendre polynomials, when one calculates $\sigma_T = \int d\Omega |f(\theta, \phi)|^2$ there's a completeness/orthogonality relation that leads straight away to eqn 1.

Namely, we can hope to get at the total cross-section we need by using eqn 9 of our paper.

III. MFT IN THE HOUSE

We have the above expression 2 for the differential cross-section. The general one from Fermi's Golden Rule for 2-particles-goes-to-2-particles cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2 4E_1E_2(\Phi_1 + \Phi_2)}|M_{fi}|^2 \frac{d^3k_3}{2E_3}\frac{d^3k}{2E} * \delta(p + k_2 + k_3 + k). \tag{3}$$

1,2,3,4 are the incoming neutrino, CNB neutrino, outgoing neutrino, leftover CNB neutrino. The Φs are the fluxes of 1 and 2. Our boy MFT on pg37 of [2]integrates this out for a very light incoming particle, just as in our solar ν_e beam.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}(\frac{E_3}{ME_1})^2|M_{fi}|^2 \tag{4}$$

The M the target particle, is our CNB neutrino m_i. E_3 is our out-going neutrino k.

IV. EXTRACTING $f(\theta = 0)$

Let's write this as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}(\frac{E_3}{ME_1})^2[\Re(M_{fi})^2 + \Im(M_{fi})^2]. \tag{5}$$

If we similarly express 2 as the sum of real and imaginary parts squared and recognize that the only complex phase is held in f and M_{fi}, then we can equate the imaginary parts up to the coefficient, and evaluate at $\theta = 0$

$$\Im(f(\theta = 0)) = \frac{1}{8\pi}\frac{E_3}{mE_1}\Im(M_{fi}(\theta = 0)) \tag{6}$$

$E_3 = m_i$, $k = E_1$, $M_{fi} = [2^4 E_1 E_2 E_3 E_4]^{1/2} * T$, $E_2 = m_i, E_4 = k$

MFT's M_{fi} here is unitless. T is our script M in our paper of eqns 8 and 9. So, finally, we use the optical theorem from equation 1 and eqn9 from our paper.

$$\sigma = \frac{1}{2k^2} \cdot \text{eqn9} \cdot [2^4 E_1 E_2 E_3 E_4]^{1/2} \tag{7}$$

Where eqn9 is the expression for the imaginary part of scattering matrix M from our paper. So, finally

$$\sigma = \frac{2m_i}{k} \cdot \text{eqn9} \tag{8}$$

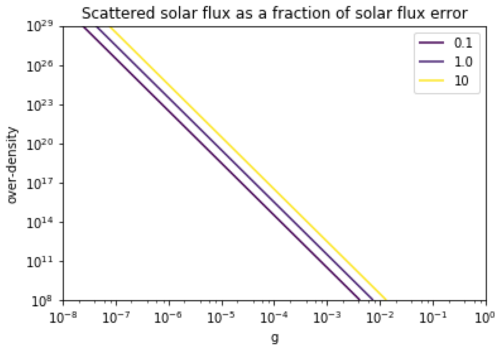

FIG. 1: The allowed region is below and left of the center line. The lines show the limit for requiring that our CNB only consumes (0.1,1.0,10.0) times 1% of the solar nu flux. The calculation of the diminishment of the solar neutrino flux is explained in the comments in the ipynb that makes this plot.

[1] https://quantummechanics.ucsd.edu/ph130a/130_notes/node441.html
[2] http://www.hep.phy.cam.ac.uk/~thomson/partIIIparticles/handouts/Handout_1_2011.pdf