STATS 202C: Project 3
Cluster Sampling of the Ising/Potts Model

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Introduction

In this project, we consider the Ising model in a 256×256 lattice. We use cluster sampling and compare the results with the results from project 2, which used Gibbs sampling.

Probem 1, 2, 4

Version 1:
We plotted the sufficient statistic $H(X)$ of the current state $X(t)$ over time t and stopped when h was within ϵ distance of h^* for each value of $\beta = 0.65, 0.75, 0.85, 1.0$. The results using cluster sampling are juxtaposed with the results from project 3 in Figure 1 and Figure 2 to compare the convergence rates using different sampling methods. We see that cluster sampling gives much faster convergence than does gibbs sampling for this problem. Note that in project 2, we could not observe convergence for values of $\beta > 0.9$ in a timely manner, but using cluster sampling, convergence happens in less than 30 sweeps for $\beta = 1.0$.

Version 2: Using version 2 of cluster sampling, where we randomly pick a pixel and grow a CP around it, we observe slower convergence, as seen in Figure 4. Compared to the Gibbs sampler used in project 2, however, we still have very fast convergence, even for $\beta = 1.0$, which was not able to converge previously.

Problem 3

We also plot the number of pixels flipped during each step between the three parameters. We see in Figure 3 that the number of pixels flips fluctuates dramatically, but for $\beta = 0.65$, the number of flips when nearing convergence is 500. For $\beta = 0.75$, the number of flips near convergence is approximately 200, and for $\beta = 0.85$, the number of flips near convergence is approximately 100.

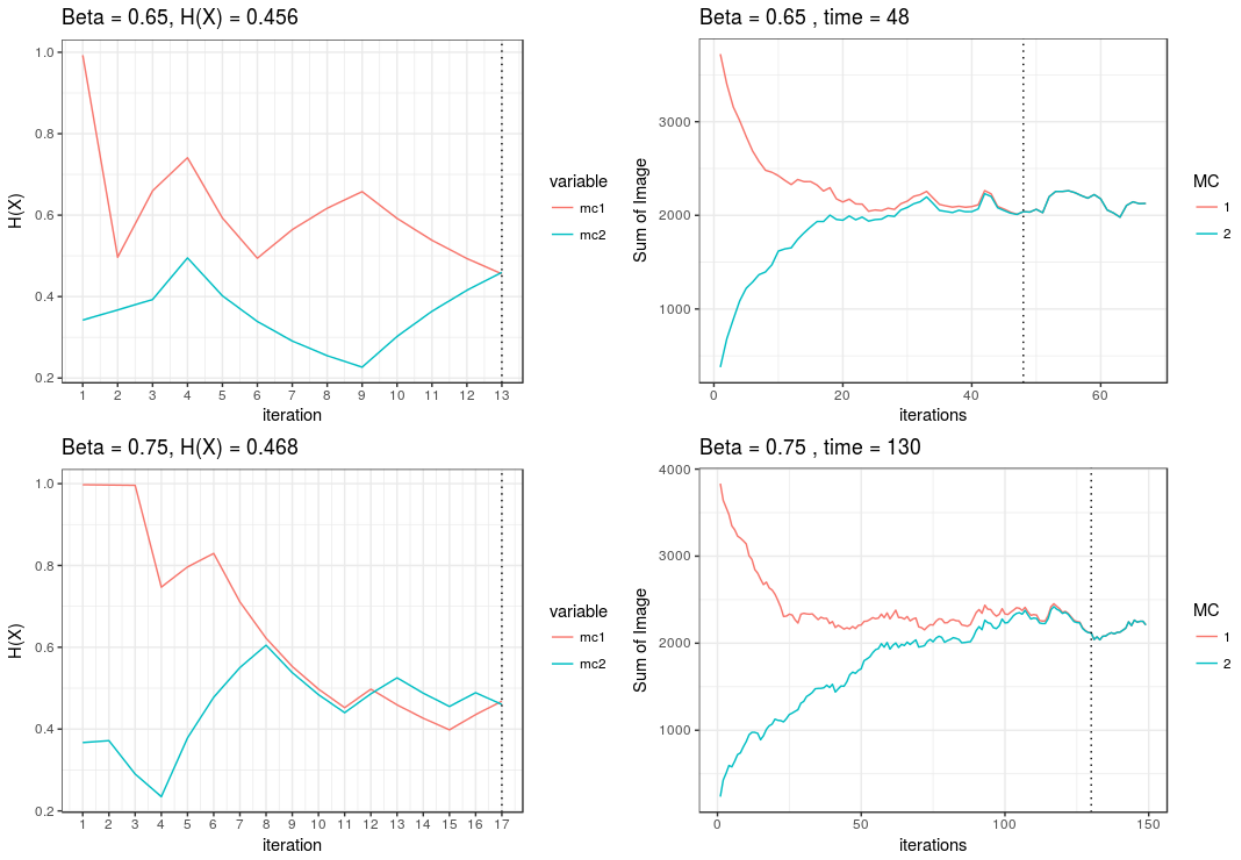

Figure 1: Using version 1 of cluster sampling, we see that the Markov Chains converge in less than 20 sweeps. Compared to the relatively fast convergence using gibbs sampling from project 2, cluster sampling is still more efficient.

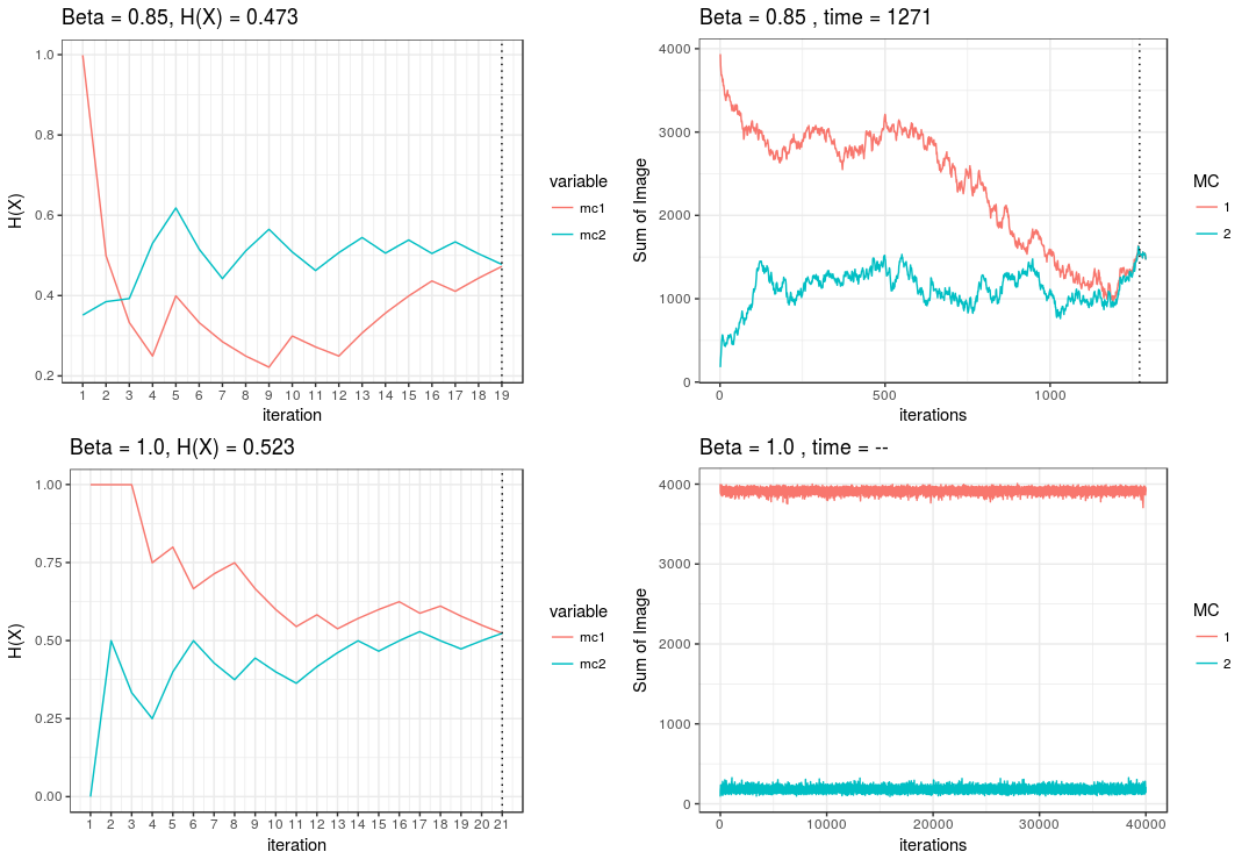

Figure 2: For these values of beta, the convergence is a little slower, but still very fast.

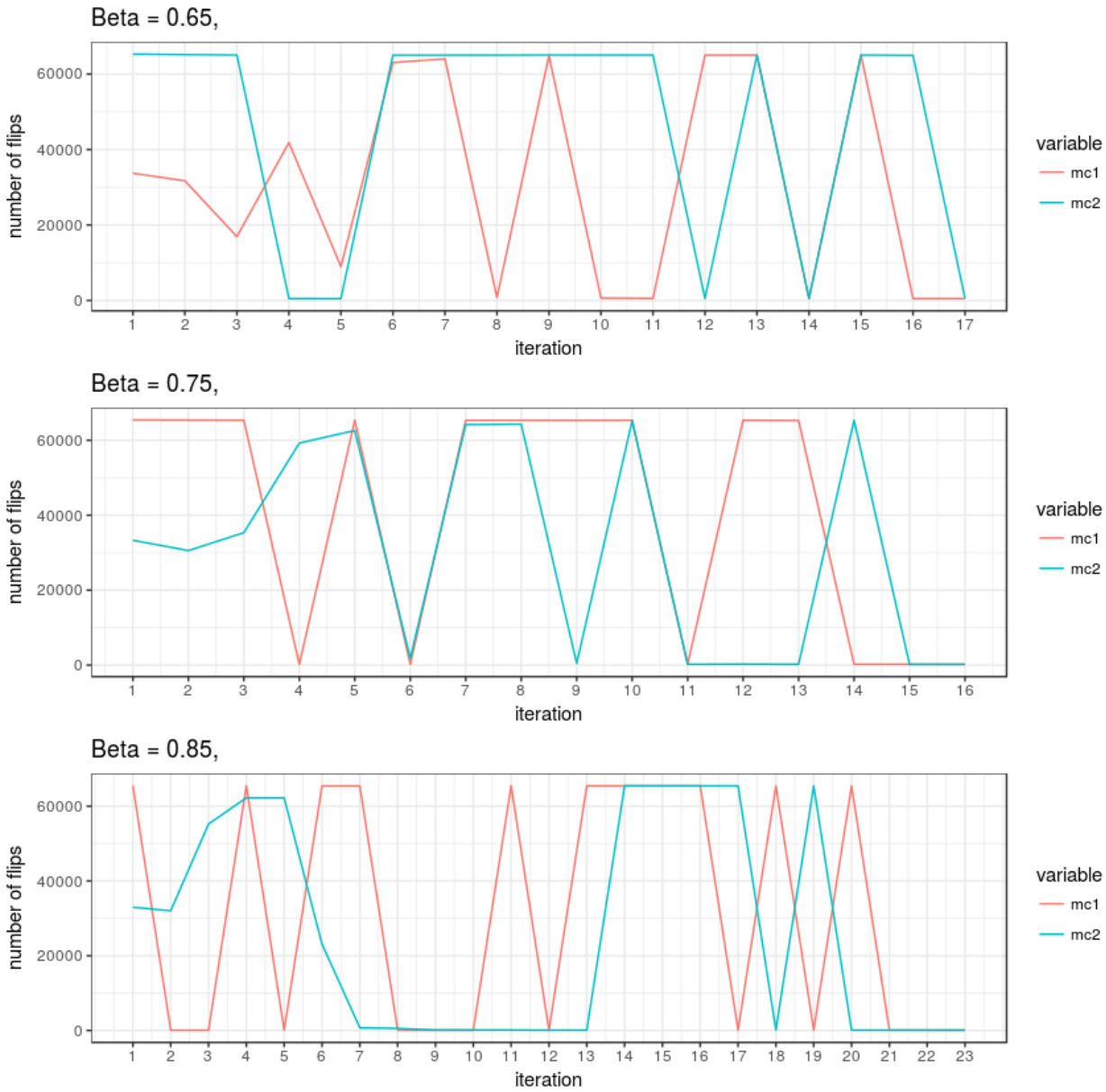

Figure 3: For these values of beta, we do not observe convergence.

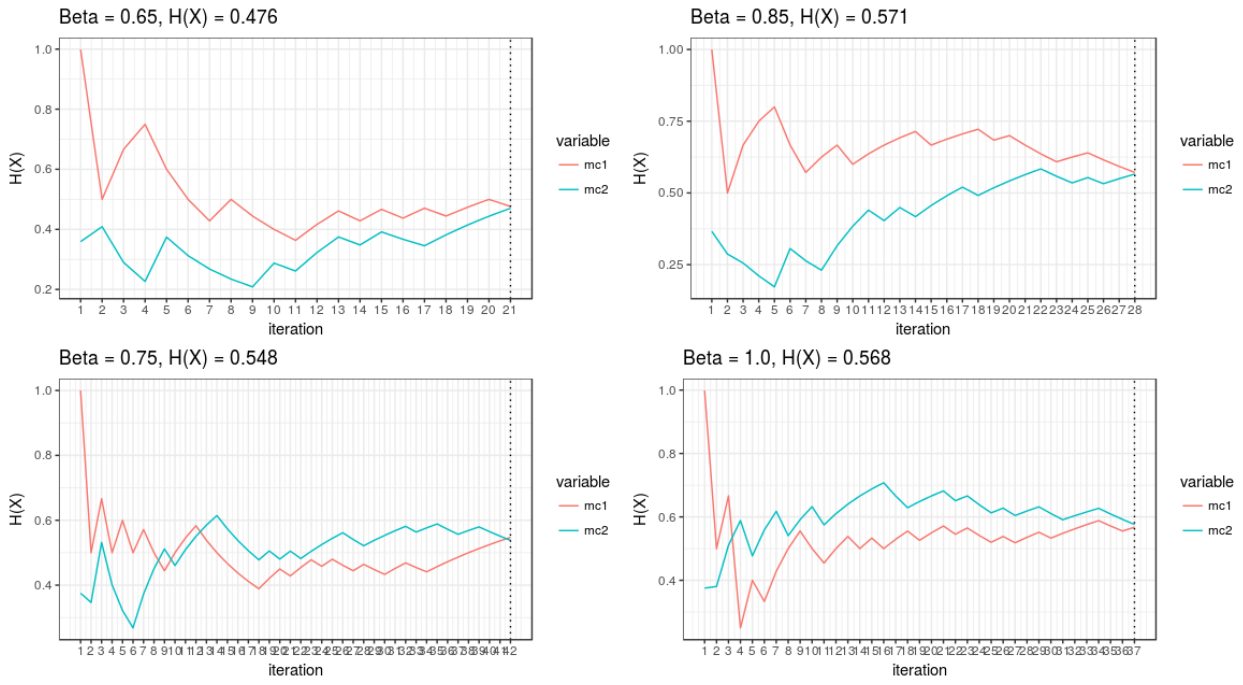

Figure 4: Using version2, we observe slower convergence. The convergence times are still significantly faster than when using Gibbs sampling.