Variational Approach for Bayesian Density Regression

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Overview

Problem Introduction and Setup

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Main Problem

Block 1

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Block 2

Pellentesque sed tellus purus. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Vestibulum quis magna at risus dictum tempor eu vitae velit.

Block 3

Suspendisse tincidunt sagittis gravida. Curabitur condimentum, enim sed venenatis rutrum, ipsum neque consectetur orci, sed blandit justo nisi ac lacus.

Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Variational Approach

Treatments	**Response 1**	**Response 2**
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

CAVI Updates

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Bouchard Bound for Problematic Quantity

Theorem (Mass–energy equivalence)

$E = mc^2$

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

Variational Algorithm

Input

1. Data $(y_n, x_n)_{n=1}^{N}$
2. Number of components, K
3. Prior mean, precision for coefficients vectors, $\boldsymbol{\beta}_{1:K}$
4. Prior shape, rate parameters for precision parameters, $\tau_{1:K}$

Output

1. A variational density,

$$q\left(\mathbf{Z}, \boldsymbol{\beta}, \boldsymbol{\tau}, \gamma\right) = q(\mathbf{Z})q(\boldsymbol{\beta}, \boldsymbol{\tau}, \gamma) = q(\mathbf{Z})\prod_k q(\beta_k, \tau_k)q(\gamma_k)$$

2. Fully specified by the variational parameters

Variational Algorithm

Algorithm. CAVI for Conditional Density Estimation

while *the ELBO has not converged* **do**
  for $n \in \{1, \dots, N\}$ **do**
    for $k \in \{1, \dots, K\}$ **do**
      Set $r_{nk} \propto \exp\Big\{ -\frac{1}{2}\ln(2\pi) + \frac{1}{2}\mathbb{E}[\ln \tau_k] + x_n^\intercal \mathbb{E}[\gamma_k]$
      $-\frac{1}{2}\mathbb{E}[\tau_k(y_n - x_n^\intercal \beta_k)^2] - \mathbb{E}\Big[\ln \Big(\sum_{j=1}^{K} \exp\{x_n^\intercal \gamma_j\} \Big) \Big] \Big\}$
    end
  end
  for $k \in \{1, \dots, K\}$ **do**
    for $n \in \{1, \dots, N\}$ **do**
      Set $\xi_{nk} \leftarrow \sqrt{(x_n^\intercal \mu_k - \alpha_n)^2 + x_n^\intercal \mathrm{Q}_k^{-1} x_n}$
    end
  end
  `/** Remaining Variational Updates on Next Slide **/`
end

Variational Algorithm (cont.)

Algorithm. CAVI for Conditional Density Estimation

while *the ELBO has not converged* **do**

  for $n \in \{1, \ldots, N\}$ **do**

    Set $\alpha_n \leftarrow \left[\frac{1}{2}\left(\frac{K}{2} - 1\right) + \sum_k \lambda(\xi_{nk})\mu_k^\intercal x_n\right] \Big/ \left[\sum_k \lambda(\xi_{nk})\right]$

  end

  for $k \in \{1, \ldots, K\}$ **do**

    $\mathrm{Q}_k \leftarrow \mathrm{I}_D + 2\sum_n r_{nk}\lambda(\xi_{nk})x_n x_n^\intercal$ `/* gamma_k cov */`

    $\eta_k \leftarrow \sum_n r_{nk}\left[\frac{1}{2} + 2\lambda(\xi_{nk})\alpha_n\right]x_n$

    $\mu_k \leftarrow Q_k^{-1}\eta_k$ `/* gamma_k mean */`

    $\mathrm{V}_k \leftarrow \sum_n r_{nk} x_n x_n^\intercal + \Lambda_0$ `/* beta_k cov */`

    $\zeta_k \leftarrow \sum_n r_{nk} y_n x_n + \Lambda_0 m_0$

    $m_k \leftarrow \mathrm{V}_k^{-1}\zeta_k$ `/* beta_k mean */`

    $a_k \leftarrow a_0 + N_k$ `/* tau_k shape */`

    $b_k \leftarrow b_0 + \frac{1}{2}\left[\sum_n r_{nk} y_n^2 + m_0^\intercal \Lambda_0 m_0 - \zeta_k^\intercal \mathrm{V}_k^{-1}\zeta_k\right]$ `/* tau_k rate */`

  end

  Compute ELBO using updated parameters

end

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

Thank you!