PSS-1

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We seek to prove that **IV** implies **I**.
Let V be a vector space, $\{W_i\} \subset V$, γ_i be a basis of W_i, and $\cup_i \gamma_i$ is a basis of all of V.
First we must show that arbitrary sums of W_is intersect only at 0. Suppose that $\exists v \in W_i$ such that $v = \sum w_j$ for $w_j \in W_j$, $j \in 1..(i-1)$. But then we have

$$\sum_{j=1} a_j \gamma_i^j = \sum b_k \gamma_1^k + ... + \sum b_k \gamma_j^k$$

But then v can be uniquely specified as linear combinations of all γ_i^j in more than one way, contradicting that it is a basis. So there can be no non-zero intersections.
Now we must show that $v \in V$ can be represented as a sum of elements of each W_i. This is trivial. As γ_i^j is a basis, we have

$$v = \sum a_i^j \gamma_i^j = \sum_i \sum_j a_i^j \gamma_i^j = \sum_i w_i$$

for $w_i \in W_i$.