Engineering phonon mediated nuclear spin interactions in trapped ion crystal

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The system studied is modelled by:

$$
\begin{aligned}
H &= \Delta_{\mathrm{M}} b^\dagger b + \Omega\left(S_x^A - S_x^B\right) + \delta\left(S_z^A - S_z^B\right) + g\left(\left(S_A^+ + S_B^+\right) b + \mathrm{h.c.}\right), \\
\mathcal{L}_\rho &= \frac{\kappa}{2}\left(2a\rho a^\dagger - \left\{a^\dagger a, \rho\right\}\right).
\end{aligned}
$$

The SDE system is:

$$
\begin{aligned}
\mathrm{d}S_x^A &= \left(2g b_{\mathrm{Im}} S_z^A - \delta S_y^A\right)\mathrm{d}t, && (1)\\
\mathrm{d}S_y^A &= \left(-2g b_{\mathrm{Re}} S_z^A + \delta S_x^A - \Omega S_z^A\right)\mathrm{d}t, && (2)\\
\mathrm{d}S_z^A &= 2g\left(b_{\mathrm{Re}} S_y^A - b_{\mathrm{Im}} S_x^A + \Omega S_y^A\right)\mathrm{d}t, && (3)\\
\mathrm{d}S_x^B &= \left(2g b_{\mathrm{Im}} S_z^B + \delta S_y^B\right)\mathrm{d}t, && (4)\\
\mathrm{d}S_y^B &= \left(-2g b_{\mathrm{Re}} S_z^B - \delta S_x^B + \Omega S_z^B\right)\mathrm{d}t, && (5)\\
\mathrm{d}S_z^B &= 2g\left(b_{\mathrm{Re}} S_y^B - b_{\mathrm{Im}} S_x^B - \Omega S_y^B\right)\mathrm{d}t, && (6)\\
\mathrm{d}b_{\mathrm{Re}} &= \left(\Delta_M b_{\mathrm{Im}} - g\left(S_y^A + S_y^B\right) - \frac{\kappa}{2} b_{\mathrm{Re}}\right)\mathrm{d}t + \sqrt{\frac{\kappa}{4}}\mathrm{d}W_x, && (7)\\
\mathrm{d}b_{\mathrm{Im}} &= \left(-\Delta_M b_{\mathrm{Re}} - g\left(S_x^A + S_x^B\right) - \frac{\kappa}{2} b_{\mathrm{Im}}\right)\mathrm{d}t + \sqrt{\frac{\kappa}{4}}\mathrm{d}W_y. && (8)
\end{aligned}
$$

With initial values that follow the distributions:

$$
\begin{aligned}
S_x^A(0) &\sim \mathcal{U}_{N/2}\left(\{1/2\};\{1\}\right), && (9)\\
S_y^A(0) &\sim \mathcal{U}_{N/2}\left(\{1/2,-1/2\};\left\{\frac{1}{2},\frac{1}{2}\right\}\right), && (10)\\
S_z^A(0) &\sim \mathcal{U}_{N/2}\left(\{1/2,-1/2\};\left\{\frac{1}{2},\frac{1}{2}\right\}\right), && (11)
\end{aligned}
$$

$$S_x^B(0) \sim \mathcal{U}_{N/2}\left(\{-1/2\};\{1\}\right), \tag{12}$$

$$S_y^B(0) \sim \mathcal{U}_{N/2}\left(\{1/2,-1/2\};\left\{\frac{1}{2},\frac{1}{2}\right\}\right), \tag{13}$$

$$S_z^B(0) \sim \mathcal{U}_{N/2}\left(\{1/2,-1/2\};\left\{\frac{1}{2},\frac{1}{2}\right\}\right), \tag{14}$$

$$b_{\mathrm{Re}} \sim \mathcal{N}\left(\mu = 0, \sigma = \sqrt{\frac{\bar{n}_{th}}{2}+\frac{1}{4}}\right), \tag{15}$$

$$b_{\mathrm{Im}} \sim \mathcal{N}\left(\mu = 0, \sigma = \sqrt{\frac{\bar{n}_{th}}{2}+\frac{1}{4}}\right). \tag{16}$$

The adiabatic elimination condition implies that:

$$0 = \Delta_M b_{\mathrm{Im}} - g\left(S_y^A + S_y^B\right) - \frac{\kappa}{2} b_{\mathrm{Re}}, \tag{17}$$

$$0 = -\Delta_M b_{\mathrm{Re}} - g\left(S_x^A + S_x^B\right) - \frac{\kappa}{2} b_{\mathrm{Im}}. \tag{18}$$

So we have that:

$$b_{\mathrm{Re}} = -\frac{g}{\Delta_{\mathrm{M}}^2 + \frac{\kappa^2}{4}}\left(\frac{\kappa}{2}\left(S_y^A + S_y^B\right) + \Delta_{\mathrm{M}}\left(S_x^A + S_x^B\right)\right), \tag{19}$$

$$b_{\mathrm{Im}} = \frac{g}{\Delta_{\mathrm{M}}^2 + \frac{\kappa^2}{4}}\left(\Delta_{\mathrm{M}}\left(S_y^A + S_y^B\right) - \frac{\kappa}{2}\left(S_x^A + S_x^B\right)\right). \tag{20}$$

If $\Delta_{\mathrm{M}} = 0$:

$$b_{\mathrm{Re}} = -\frac{2g}{\kappa}\left(S_y^A + S_y^B\right), \tag{21}$$

$$b_{\mathrm{Im}} = -\frac{2g}{\kappa}\left(S_x^A + S_x^B\right). \tag{22}$$

If $\kappa = 0$:

$$b_{\mathrm{Re}} = -\frac{g}{\Delta_{\mathrm{M}}}\left(S_x^A + S_x^B\right), \tag{23}$$

$$b_{\mathrm{Im}} = \frac{g}{\Delta_{\mathrm{M}}}\left(S_y^A + S_y^B\right). \tag{24}$$

We could scale using:

$$\Omega_{\alpha c} = \frac{N_\alpha g^2}{\sqrt{\Delta^2 + \frac{\kappa^2}{4}}}. \tag{25}$$