Dynamics of Rydberg atoms via CMFT under partial-translational invariance.

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I. MATHEMATICAL DESCRIPTION OF THE SYSTEM

We consider the hamiltonian that governs the dynamics of the Rydberg system shown in Fig. 1 with $S = 1$ and using the notation of [1] to be given by:

$$\hat{H} = \frac{\Omega_{\rm c}}{2}\sum_i(|{\rm e}\rangle\langle{\rm g}|_i + |{\rm g}\rangle\langle{\rm e}|_i) + \frac{\Omega_{\rm r}}{2}\sum_i(|{\rm e}\rangle\langle{\rm r}|_i + |{\rm r}\rangle\langle{\rm e}|_i) + \Delta\sum_i \hat{n}_{\rm rr}^{(i)} + \frac{1}{2}\sum_{ij} V_{ij}\hat{n}_{\rm rr}^{(i)}\hat{n}_{\rm rr}^{(j)}, \quad (1)$$

$$\Omega_{\rm c} = 2\pi \times 250 \text{ Hz}, \quad (2)$$

$$\Omega_{\rm r} = 2\pi \times 5.5 \text{ MHz}, \quad (3)$$

$$\Delta = 2\pi \times 11.0 \text{ MHz}, \quad (4)$$

$$C_6 = 2\pi \times 4.758 \text{ GHz}, \quad (5)$$

$$V_{ij} = \frac{C_6}{r_{ij}^6}, \text{with } r_{ij} \text{ in units of near-neighbor distance } a_{\rm lat}, \quad (6)$$

$$|{\rm e}\rangle\langle{\rm g}|_i + {\rm h.c.} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (7)$$

$$|{\rm e}\rangle\langle{\rm r}|_i + {\rm h.c.} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

$$\hat{n}_{\rm rr}^{(i)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (9)$$

$$|\psi(0)\rangle = \bigotimes_i \left(\frac{|e\rangle_i + |g\rangle_i}{\sqrt{2}}\right). \quad (10)$$

II. PARTIAL-TRANSLATIONAL INVARIANCE

The way in which we can separate our system such that we can create a partition of the systems of atoms and exploit it for constructing properly the dynamics in the cluster approach is to define a division of the lattice that depends of the distance of the elements respect to the geometric center of the lattice.

In particular, for a odd and even lattice size we can observe the separation as shown in

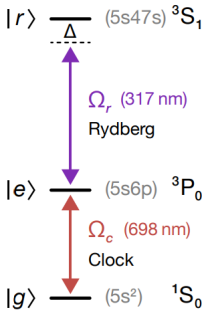

Figure 1: The diagram illustrates the atomic states (represented by black lines) and transitions (shown as colored arrows) of ^{88}Sr that are relevant for clock interrogation and Rydberg dressing. The Rabi frequencies $\Omega_{r,c}$ indicate the strength of laser coupling between the various states in the experiment, while Δ represents the detuning from the state $|r\rangle$ [2].

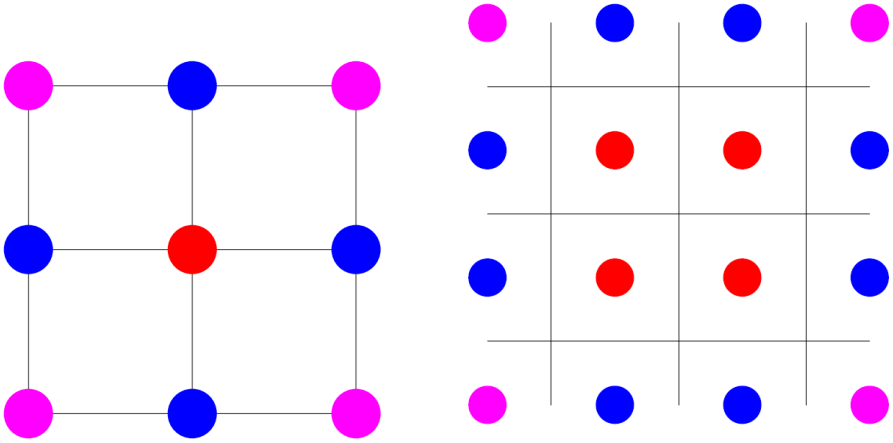

Figure 2: (Left) 3×3 and (Right) 4×4 lattice divided by equivalent elements in terms of distance to the geometrical center of the lattice that keep unchanged the interactions V_{ij}.

the Fig. 2. In general, for simulating exactly a system of size L^2, where L is the number of atoms per side, it is required to consider $O(L)$ partitions, which implies a reduction of the complexity of the cluster mean-field algorithm.

III. SINGLE-PARTICLE OBSERVABLE ANALYSIS

At first we study the convergence and consistence of the cluster method for estimating the single particle observables. In particular, we will evaluate the dynamics of the operators

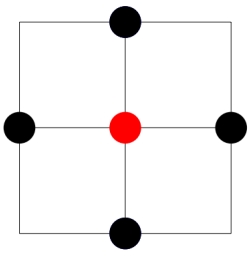

Figure 3: Crux pattern of 5 particles used to benchmark CMFT.

$\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$. For the hamiltonian and parameters shown previously we consider at first a 5-particles system composed by one central particle and 4 first neighbors in a rectangular grid as shown in the Fig. 3.

For the CMFT method we consider a cluster size n_{acut}, this variable represents the number of atoms in the cut-off. For this hamiltonian we can show that for the particle labeled with k we will have:

$$\hat{H}^{(k)}_{\text{CMFT}} = \frac{\Omega_c}{2}\sum_{i\in C_k}(|\text{e}\rangle\langle\text{g}|_i + |\text{g}\rangle\langle\text{e}|_i) + \frac{\Omega_r}{2}\sum_{i\in C_k}(|\text{e}\rangle\langle\text{r}|_i + |\text{r}\rangle\langle\text{e}|_i) + \frac{1}{2}\sum_{ij\in C_k} V_{ij}\hat{n}^{(i)}_{\text{rr}}\hat{n}^{(j)}_{\text{rr}} + \sum_{\substack{i\in C_k,\\ j\notin C_k}} V_{ij}\hat{n}^{(i)}_{\text{rr}}\left\langle\hat{n}^{(j)}_{\text{rr}}\right\rangle \quad (11)$$

$$+\Delta\sum_{i\in C_k}\hat{n}^{(i)}_{\text{rr}}. \quad (12)$$

where C_k corresponds to the cluster associated with the particle k. This cluster is constructing maximizing over V_{kj} such that we choose the elements that maximize the coupling factor between the particles k and j and $|C_k| = n_{\text{acut}}$, being C_k the set of labels such that they maximize the coupling respect k. Using all this conditions then we will have the dynamics of the single-particle operators shown in Fig. 4. (tdb)

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[1] https://easyspin.org/easyspin/documentation/spinoperators.html

[2] arXiv:2303.08078 William J. Eckner, et al (2023)