Econometrics III (module 5, 2023–2024)

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Assignment 3

Problem 1 "Time series bias", 10 points

Show, using a zero-mean strong AR(1) model as an example, that the OLS estimator in time series models is generally *not* conditionally unbiased. *Hint: try to replicate our proof in the random sampe case, and see when and why it breaks down.*

Problem 2 "Testing for forecast unbiasedness", 25 points

The data file FwdSpot1.dat contains monthly spot and 1-month forward exchange rates, the data file FwdSpot3.dat – monthly spot and 3-month forward exchange rates, in \$/foreign currency, for the British Pound, French Franc and Japanese Yen, for 1973:3 to 1992:8 (234 observations). Each row contains the month, the year, the spot rates for Pound, Franc, and Yen, and then the forward rates for the same three currencies. Download the data, then take logarithms of the rates.

We are interested in testing the conditional unbiasedness hypothesis that

$$\mathbb{E}_t s_{t+k} = f_{t,k}, \tag{1}$$

where s_t is the spot rate at t, $f_{t,k}$ is the forward rate for k-month forwards at t, and $\mathbb{E}_t$ denoted mathematical expectation conditional on time t information. The statement (1) says that the forward rate is a conditionally unbiased predictor of the future spot exchange rate. To test this theory, one nests (1) within the following econometric model:

$$s_{t+k} - s_t = \alpha + \beta\left(f_{t,k} - s_t\right) + e_{t+k}, \quad \mathbb{E}_t[e_{t+k}] = 0, \tag{2}$$

and test $H_0 : \alpha = 0,\ \beta = 1$. The current spot rate is subtracted to achieve stationarity. The difference $s_{t+k} - s_t$ is called *the exchange rate depreciation*, the difference $f_{t,k} - s_t$ – *the forward premium*. For the three currencies and both types of forwards, estimate (2) by OLS and test for conditional unbiasedness. Do not forget HAC variance estimation whenever appropriate; explain why it is needed or not needed. Discuss the test results.

Problem 3 "Consumption function", 5 points

The (true) consumption function

$$E\left[c_t | y_t, c_{t-1}, y_{t-1}, c_{t-2}, ...\right] = \alpha + \gamma y_t + \delta c_{t-2}$$

is estimated using OLS. Assume that c_t and y_t are jointly stationary and ergodic. Did one or did one not have to use HAC variance estimation to construct standard errors? Explain.

Problem 4 "(Non)Linear? (Non)Identifiable? (Non)OLSable?", 10 points

Investigate the following regression models for parameter identifiability and whether the parameters can be estimated by OLS.

1. $E\left[\exp\left(y\right)|x_1, x_2\right] = \beta_1 + \beta_2 \exp\left(x_1\right) + \left(1 - \beta_2\right) \ln\left(|x_2|\right)$
2. $E\left[\ln\left(y\right)|x_1, x_2\right] = \beta_1 + \beta_2\beta_3 \exp\left(x_1 x_2\right) + \ln(|x_2|^{\beta_3})$
3. $E\left[\ln(y)|x_1, x_2\right] = \beta_1 + \beta_2 \ln(x_1) \cdot x_2^{\beta_3 - \beta_2}$
4. $E\left[\ln(y)|x_1, x_2\right] = \beta_1 + \beta_2 \ln(x_1) \cdot \left(\beta_3 - \beta_2\right) x_2$
5. $E\left[\ln(y)|x_1, x_2\right] = \beta_1 + \beta_2 \ln(x_1) + \left(\beta_3 - \beta_2\right) x_2$

In those cases when (at least some) parameters are identified and OLS is applicable, show schematically how you will apply OLS and construct the standard errors.