Econometrics III (module 5, 2023–2024)

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Assignment 2

Problem 1 "Overlapping CI", 5 points

One can sometimes observe the following logic in empirical research. A researcher obtains confidence intervals for two parameters and argues: "Well, these two confidence intervals overlap, which means that these two parameters are equal to each other." Discuss why this logic is erroneous, and suggest a correct way of deriving such a conclusion (or its opposite) *using confidence interval construction*.

Problem 2 "Asymptotics of higher order moments", 10 points

Suppose we are interested in the skewness $\mu_3 = E[z^3]$ and kurtosis $\mu_4 = E[z^4]$ of zero mean unit variance random variable z. Under random sampling, propose analog estimators of μ_3 and μ_4, and derive their joint asymptotic distribution. Also, derive the asymptotic distribution of the analog estimator of $\zeta = \mu_3^4/\mu_4^3$.

Problem 3 "Hansen's estimator", 10 points

This is a problem from Bruce Hansen's textbook. The model is

$$y = x'\beta + e, \quad E[e|x] = 0.$$

An econometrician is worried about the impact of some unusually large values of the regressors. The model is thus estimated on the subsample for which $\|x_i\| \le c$ for some fixed c. Let

$$\tilde{\beta} = \left(\sum_{i=1}^n x_i x_i' \mathbb{I}(\|x_i\| \le c) \right)^{-1} \sum_{i=1}^n x_i y_i \mathbb{I}(\|x_i\| \le c)$$

be the OLS estimator on this subsample. Show that $\tilde{\beta}$ is consistent for β, and find the asymptotic distribution of $\sqrt{n}(\tilde{\beta} - \beta)$.

Problem 4 "Cost function", 25 points

Do Exercise 9.26(a,b) from Hansen's textbook. Then, consider the null hypothesis H_0 : $\beta_3 + \beta_4 + \beta_5 = 1$.

1. Perform an asymptotic left-sided test using a t statistic.
2. Perform an asymptotic two-sided test using a Wald statistic.

In both cases, state the alternative hypothesis and compute the p-value of the test.[1] Explain the difference between the two p-values.

[1] On p-values, re-read Section 9.7 in Hansen's textbook; see also earlier Section 5.12.