Joshua Eckels - CM926

ME301 - Equation sheet

$\dot{S}_{gen} \to \frac{W}{°K}$

$p\forall = nR_uT$ $\quad R_{sp} = \frac{R_u}{M}$

$pv = R_{sp}T$

$R_u = 8.314 \frac{kJ}{kmol\text{-}K}$

$W = \int \vec{F}\cdot ds$

$\dot{W} = \vec{F}\cdot\vec{V}$

$\dot{W}_{shaft} = \tau\omega$

$\dot{W}_{elec} = IV = I^2R$

$W = -\int p\,d\forall$

n	type	work
0	isobaric	$W_{in} = -p(\forall_2 - \forall_1)$
1	$p\forall = $ const	$W_{in} = p_1\forall_1 \cdot \ln\left(\frac{\forall_1}{\forall_2}\right)$
n	$p\forall^n = $ const	$W_{in} = \frac{p_2\forall_2 - p_1\forall_1}{n-1}$

Generalized Compressability

ideal gas: $Z = \frac{pv}{RT} \approx 1 = \frac{p\forall}{mRT}$

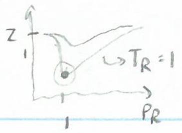
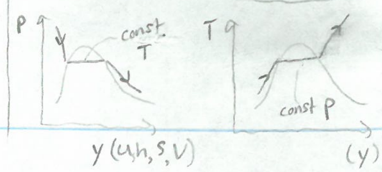


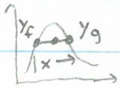
Table A-1 p1022

$P_R = P/P_{cr}$ $\quad T_R = T/T_{cr}$

$v_R' = \frac{RT_{cr}}{P_{cr}}$



quality:



$x = \frac{y - y_f}{y_g - y_f} = \frac{m_{vapor}}{m_{vap} + m_{liquid}}$

$y = y_f + x(y_g - y_f)$

coe closed: $U_{fin} - U_{init} = W_{in,net} + Q_{in,net}$

ideal gas model:	const specific heats:	temp-dependent C_p/C_v:	incompressible subs.:
$\Delta u = u_2 - u_1$	$= C_v(T_2 - T_1)$	$= \int_{T_1}^{T_2} C_v\,dT = u(T_2) - u(T_1)$	$= C_v(T_2 - T_1)$
$\Delta h = h_2 - h_1$	$= C_p(T_2 - T_1)$	$= \int C_p\,dT = h(T_2) - h(T_1)$	$= C_v(T_2 - T_1) + v(p_2 - p_1)$
$\Delta s = s_2 - s_1$ ($C_p = C_v + R_{sp}$)	$= C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$ or $= C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$	$= \int C_p \frac{dT}{T} - \int R_{sp}\frac{dp}{p}$ $= s°(T_2) - s°(T_1) - R\ln\left(\frac{P_2}{P_1}\right)$	$= C_v \ln\left(\frac{T_2}{T_1}\right)$
isentropic: $k = \frac{C_p}{C_v}$	$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ $\quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$ $\quad P_1V_1^k = P_2V_2^k$	$\frac{P_2}{P_1} = \frac{P_r(T_2)}{P_r(T_1)}$ $\quad \frac{V_2}{V_1} = \frac{v_r(T_2)}{v_r(T_1)}$	$\Delta S = 0$ $\quad T_2 = T_1$ $\quad C_v = C_p$

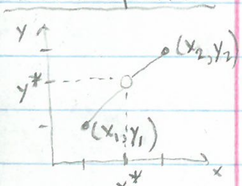
$R_{sp,air} = 0.287 \frac{kJ}{kg\,°K}$

(table A-22)

CL approximations: $v \approx v_f(T),\ u \approx u_f(T),\ s \approx s_f(T)$

$h \approx h_f(T) + v_f(T)\left[p - p_{sat}(T)\right]$

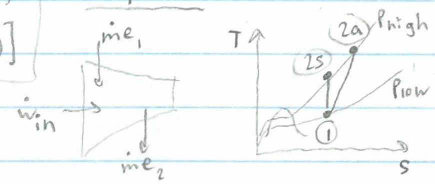
Linear interpolation: $m = \frac{y_2 - y_1}{x_2 - x_1}$



$y^* = y_1 + m(x^* - x_1)$

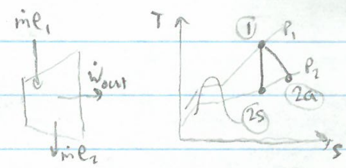
isentropic efficiency:

compressor:



$\dot{W}_{in} = \dot{m}(h_{out} - h_{in})$ $\quad \eta_{comp} = \frac{\dot{W}_{in,ideal}}{\dot{W}_{in,actual}} = \frac{\dot{m}(h_{2s} - h_1)}{\dot{m}(h_{2a} - h_1)}$

turbine: $\quad \dot{W}_{out} = \dot{m}(h_{in} - h_{out})$



$\eta_{turb} = \frac{\dot{W}_{actual}}{\dot{W}_{ideal}} = \frac{\dot{m}(h_1 - h_{2a})}{\dot{m}(h_1 - h_{2s})}$

Exergy accounting:

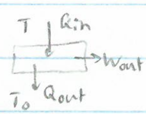
$\dot{A}_{dest} = T_0\dot{S}_{gen}$

$\frac{d}{dt}(A_{sys}) = \sum_{net}\left(1 - \frac{T_0}{T_b}\right)\dot{Q} + \left[\dot{W}_{in,net} + p_0\frac{d}{dt}(\forall)\right] + \sum_{net}\dot{m}a_{flow} - \dot{A}_{dest}$

storage: $A_{sys} = (U - U_0) + p_0(\forall - \forall_0) - T_0(S - S_0) + \frac{1}{2}mV^2 + mg(z - z_0)$

flow: $\dot{m}a_{flow} = \dot{m}\left[(h - h_0) - T_0(s - s_0) + \frac{1}{2}V^2 + g(z - z_0)\right]$

change: $A_2 - A_1 = (U_2 - U_1) + p_0(\forall_2 - \forall_1) - T_0(S_2 - S_1) + (KE_2 - KE_1) + (PE_2 - PE_1)$



$\frac{dU}{dt} = -\dot{Q}_{out,block}$

$-dU_{block} = \dot{Q}_{in}\,dt$

$\dot{W}_{out} = \eta_{best}(\dot{Q}_{in}) = \left(1 - \frac{T_0}{T}\right)\dot{Q}_{in}$

$\int \dot{W}_{out}\,dt = W_{out} = -\int_{T_i}^{T_0}\left(1 - \frac{T_0}{T}\right)mC_v\,dT$

$W_{out} = -\left[mC_v(T_0 - T_i) - mC_vT_0\ln\left(\frac{T_0}{T_i}\right)\right]$

exergetic efficiency:

$\eta_{carnot} = 1 - \frac{T_C}{T_H}$

$\eta_{II} = \frac{\eta}{\eta_{carnot}}$

$COP_{refr} = \frac{T_C}{T_H - T_C}$

$COP_{heat\ pump} = \frac{T_H}{T_H - T_C}$

turbine:

$\dot{W}_{out} = \dot{m}(h_{in} - h_{out})$

$\dot{W}_{out,max} = \dot{m}(a_{in} - a_{out}) - \dot{A}_{dest}^{\ 0}$

$\eta_{II,turb} = \frac{\dot{W}_{out}}{\dot{W}_{out,max}} = \frac{h_{in} - h_{out}}{(h_{in} - h_{out}) - T_0(s_{in} - s_{out})}$

heat exchanger:

$\eta_{II,HX} = \frac{\dot{m}_c(a_{flow,c,out} - a_{flow,c,in})}{\dot{m}_h(a_{flow,h,in} - a_{flow,h,out})}$

compressor:

$\eta_{II,comp} = \frac{\dot{W}_{best}}{\dot{W}} = \frac{\dot{m}\left[h_{out} - h_{in}\right) - T_0(s_{out} - s_{in})\right]}{\dot{m}(h_{out} - h_{in})}$

1→2 : isentropic comp.

2→3 : isothermal exp.

3→4 : isentropic exp.

4→1 : isothermal comp.

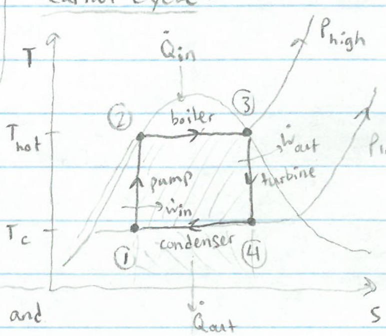
$\dot{Q}_{in} = \dot{m}\int T_h\,ds$

$\dot{Q}_{out} = -\dot{m}\int T_c\,ds$

$\dot{W}_{out,net} = \dot{Q}_{in,net} = \dot{m}\int(T_h - T_c)\,ds$ ← area box

$\eta = \frac{\dot{W}_{out,net}}{\dot{Q}_{in}} = 1 - \frac{T_c}{T_H}$

carnot cycle:



*use $\frac{\forall_2}{\forall_1} = \frac{V_r(2)}{V_r(1)}$ and

$W_{out} = \int_{\forall_2}^{\forall_3} \frac{mRT}{\forall}\,d\forall \to \frac{W_{out}}{mRT} = \ln\left(\frac{\forall_3}{\forall_2}\right)$

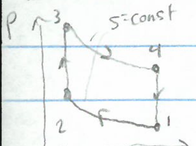
Cycle	name
1→2	comp 1
2→3	intercooler
3→4	compressor 2
4→x	regenerator
x→5	combustion
5→6	turbine 1
6→7	reheat
7→8	turbine 2
8→1	heat rejection

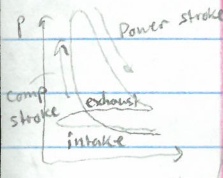
4s or 4a?

(gas-turbine)

$P_r = P_{max}/P_{min}$

Otto cycle





$\frac{g}{mol} = \frac{kg}{kmol} = \frac{lb}{lbmol} = \frac{slug}{slugmol}$

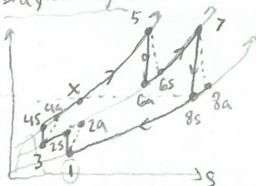
$mf = \frac{m_i}{m_{mix}}$

$nf = n_i/n_{mix}$

$M_{mix} = \frac{m_{mix}}{n_{mix}} = \sum nf_i M_i$

$p_i = \sum nf_i\, p_{mix}$

Brayton cycle:



$\eta_{cycle} = \frac{\dot{W}_{out,net}}{\dot{Q}_{x\to5} + \dot{Q}_{6\to7}}$

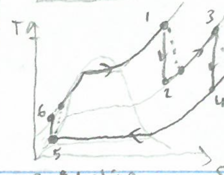
$\eta_{regen} = \frac{h_x - h_{4a}}{h_{8a} - h_{4a}}$

$bwr = \frac{\dot{W}_{in,comp}}{\dot{W}_{out,turb}}$

$r_c = V_{max}/V_{min}$

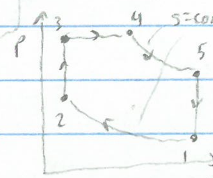
$\eta_{otto} = 1 - \frac{1}{r_c^{k-1}}$

Rankine Cycle: (vapor power cycle)



1→2	turbine
2→3	reheat
3→4	turbine
4→5	condenser
5→6	pump
6→1	superheater

Dual Cycle:



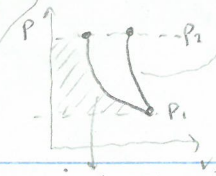
$r = \frac{V_1}{V_2}$, $3\to4: \frac{Q_{in}}{m} = (u_4 - u_3) + p(v_4 - v_3) = h_4 - h_3$

$4\to5: \frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4} \to V_4 = \left(\frac{T_4}{T_3}\right)V_2$ $\quad \frac{v_5}{v_4} = \frac{V_1}{\left(\frac{T_4}{T_3}\right)V_2} = r\left(\frac{T_4}{T_3}\right)^{-1}$

$mep = \frac{W_{out,net}}{V_{max}} \to V_1 = \frac{RT_1}{P_1}$

	nf	n	M	m	mf
N_2					
CO_2					
mix	1		—		1

Intercooling:

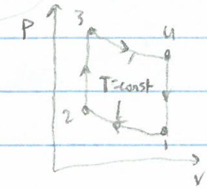


$\frac{\dot{W}}{\dot{m}} = \int_1^2 v\,dp$

*less $\dot{W}_{in}$ for isothermal compression

*increases efficiency

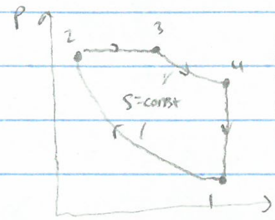
Stirling Cycle



as $r_c \to \infty$, $\eta_{stir} \to \eta_{carnot}$

$\eta_{stir} = \eta_{carnot}\left(\frac{1}{1 + \frac{\eta_{carnot}}{(k-1)\ln(r_c)}}\right)$

Diesel Cycle



$r_c = V_{max}/V_{min}$

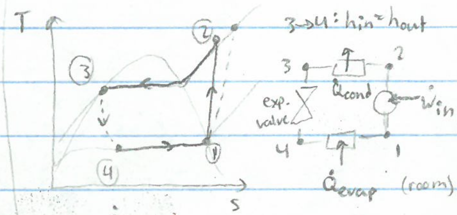
$r_e = V_{inter}/V_{min}$

(V_3/V_2) (cutoff ratio)

$\eta_{diesel} = 1 - \frac{1}{r_c^{k-1}}\left(\frac{r_e^k - 1}{k(r_e - 1)}\right)$

$\uparrow r_c = \uparrow \eta_{diesel} > \eta_{otto}$

Refrigeration cycle



$COP = \frac{\dot{Q}_{in}}{\dot{W}_{in,net}}$ $\quad COP_{carnot} = \frac{T_c}{T_H - T_c}$

$U_{mix,2} - U_{mix,1} = \sum n_i\left[\bar{u}_i(T_2) - \bar{u}_i(T_1)\right] = \sum n_i \bar{c}_{v,i}(T_{i,2} - T_{i,1})$

$H_{mix,2} - H_{mix,1} = \sum n_i\left[\bar{h}_i(T_2) - \bar{h}_i(T_1)\right] = \sum n_i \bar{c}_{p,i}(T_{i,2} - T_{i,1})$

$S_{mix,2} - S_{mix,1} = \sum n_i\left[\bar{s}_i^\circ(T_2) - \bar{s}_i^\circ(T_1) - R_u \ln\left(\frac{P_{2,mix}}{P_{1,mix}}\right)\right] = \sum n_i\left[\bar{c}_{p,i}\ln\left(\frac{T_2}{T_1}\right) - R_u\ln\left(\frac{P_2}{P_1}\right)\right]$

$\bar{u} = Mu$, $\bar{h} = Mh$, $\bar{s} = Ms$, $\bar{c}_p = Mc_p$

$\frac{kJ}{kg}\frac{kg}{kmol} = \frac{kJ}{kmol}$ $\quad u_{mix} = \sum mf_i u_i$

$\bar{u}_{mix} = \sum nf_i \bar{u}_i$

$\dot{n} = \frac{\dot{m}}{M} = \frac{kg}{s}\frac{kmol}{kg}$

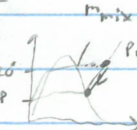
$\boxed{\text{Psychrometrics}}$ $M_a = 28.97\ \frac{kg}{kmol}$ $\quad M_v = 18.02\ \frac{kg}{kmol}$

Humidity ratio: $\omega = \frac{m_v}{m_a} = \frac{M_v P_v}{M_a P_a} = \frac{0.622\, P_v}{P_{mix} - P_v}$

Relative humidity: $\phi = \frac{nf_v}{nf_{v,sat}} = \frac{P_v}{P_{v,sat}}$

dew point $= T_{sat}(P_v)$

$m_{mix} = m_a + m_v$



$H_{mix} = m_a h_a + m_v h_v$

$\frac{H_{mix}}{m_a} = h_a + \omega h_v$

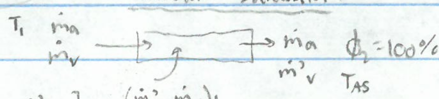
$\frac{S_{mix}}{m_a} = s_a + \omega s_v$

$h_v \approx h_g(T)$

$u_v \approx u_g(T)$

$s_v \approx s_g(T) - R_{sp}\ln\left(\frac{P_v}{P_{sat}}\right)$ $\quad \frac{U_{mix}}{m_a} = u_a + \omega u_v$

adiabatic saturator



$0 = \left[\dot{m}_a h_{a,1} + \dot{m}_v h_{v,1}\right] + \left[(\dot{m}'_v - \dot{m}_v)h_{w,makeup}\right] - \left[\dot{m}_a h_{a,2} + \dot{m}'_v h_{v,2}\right]$

$0 = (\underbrace{h_{a,1}}_{h_a(T_1)} + \omega_1 \underbrace{h_{v,1}}_{h_g(T_1)}) + ((\omega' - \omega)h_f(T_{AS})) - (\underbrace{h_{a,2}}_{h_a(T_{AS})} + \omega' \underbrace{h_{v,2}}_{h_g(T_{AS})})$

$\to \omega = \frac{\left[h_a(T_{AS}) - h_a(T)\right] + \omega'\left[h_g(T_{AS}) - h_f(T_{AS})\right]}{\left[h_g(T) - h_f(T_{AS})\right]}$

$\omega' = 0.622\, P_{sat}(T_{AS})/(p - P_{sat}(T_{AS}))$

$T_{ref} = 25°C = 298°K \approx 77°F/537°R$

$P_{ref} = 1atm/101.3kPa = 14.7\ psia$

Stable: N_2, O_2, H_2, C ($\bar{h}_f^\circ = 0$)

$\boxed{\text{Combustion}}$

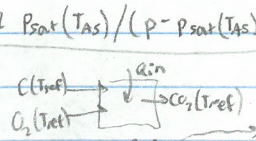
$C_xH_y + a(O_2 + 3.76N_2) \to b\,CO_2 + c\,H_2O + d\,N_2 + (e\,O_2)$

$b = x$, $d = 3.76a$, $c = \frac{y}{2}$, $a = x + \frac{y}{4}$

$AFR = \frac{\dot{n}_{air}}{\dot{n}_{fuel}}$ $\quad FAR = \frac{1}{AFR}$ $\quad \%_{air} = \frac{\text{actual } \dot{n}_{air}}{\dot{n}_{air,stoich}}$

$\lambda = \frac{FAR_{actual}}{FAR_{stoich}}$ $\quad \lambda < 1$: lean, $\lambda > 1$: rich

10% excess → $1.1a(O_2 + 3.76N_2)$



$\bar{h}(T) = \underbrace{\bar{h}_f^\circ}_{\text{enthalpy of formation}} + \underbrace{\left[\bar{h}(T_{table}) - \bar{h}(T_{ref})\right]}_{\text{adjust ref point}}$

$\frac{\dot{Q}_{in}}{\dot{n}_{fuel}} = \bar{h}_{RP} = \sum_{prod}\frac{\dot{n}_i}{\dot{n}_{fuel}}\bar{h}_i - \sum_{react}\frac{\dot{n}_i}{\dot{n}_{fuel}}\bar{h}_i$ = enthalpy of combustion

HHV → H_2O products = liquid

LHV → " = vapor

adiabatic

$\bar{u} = \bar{h} - R_uT = \bar{h}_f^\circ + (\bar{h}(T) - \bar{h}(T_{ref})) - R_uT$

$\underbrace{\sum_{prod}\frac{\dot{n}_i}{\dot{n}_{fuel}}\left[\bar{h}_f^\circ + \{\bar{h}(T) - \bar{h}(T_{ref})\}\right]_i}_{T_{max} = T_{AF}} = \underbrace{\sum_{react}\frac{\dot{n}_i}{\dot{n}_{fuel}}\left[\bar{h}_f^\circ + \{\bar{h}(T) - \bar{h}(T_{ref})\}\right]_i}_{T_1}$

adiabatic flame temperature

element	$\bar{h}_f^\circ$	$\bar{h}(T_{ref})$	$\bar{h}(T_1)$	$\bar{h}(T_2)$ …
CO_2				
N_2				
O_2				
CH_4				
⋮				

→ iterative for T_2

$\sum_{prod} - \sum_{react} = 0$? →