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ES212 · Fluids Equation Sheet

$\tau = \mu \frac{dV}{dy}$ | $v = \frac{1}{\rho}$ | $SG = \frac{\rho}{\rho_{H_2O @ 4°C}}$ | $\gamma = \rho g$

Buoyant: $F_B = \rho g \forall_{disp}$ = weight of fluid displaced

$P_g = P_{abs} - P_{atm}$ $\qquad P_{column} = \rho g h$

$P_{vac} = P_{atm} - P_{abs}$ $\qquad$ Barometer: $P_B - P_A = -\rho g (z_B - z_A)$ $\qquad dp = -\rho V dV$

Manometer rule:
1. start at one end
2. add $\gamma_{fluid} h$ if moving ↓
3. subtract $\gamma_{fluid} h$ if moving ↑
4. keep going until at target side
5. don't add if no change in depth
6. ~~ℓ / Δ~~ Δy use Δy if slant tube

Pressure Variation normal to streamline

$$\frac{\rho V^2}{r} = \frac{dp}{dr} \qquad \int_{P_1}^{P_2} dp = \int_{r_1}^{r_2} \frac{\rho V^2}{r}\, dr \qquad P_2 - P_1 = \rho \int_{r_1}^{r_2} \frac{[V(r)]^2}{r}\, dr$$

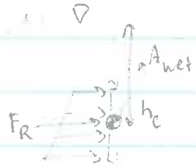
Conservation of Energy

$$\dot{W}_{in,net} + \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_{in} = \dot{m}\left(\frac{P}{\rho} + \frac{V^2}{2} + gz\right)_{out} + \underbrace{\left[\dot{Q}_{out,net} + \dot{m}(u_2 - u_1)\right]}_{\dot{E}_{mech,loss}}$$

MEB: incomp, adiabatic, reversible, SS, stream tube

Bernoulli: $\dot{W}_{in,net} = 0$

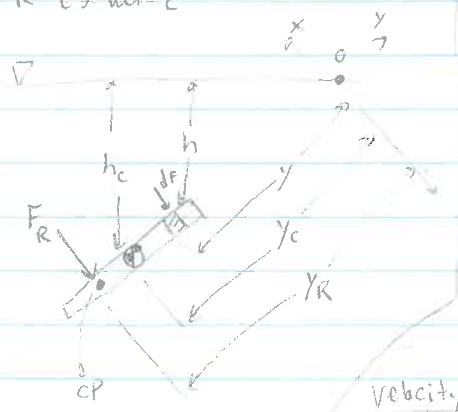
Hydrostatic Forces



$$F_R = \rho g A_{net} h_c$$

$I_{xc} = \frac{1}{12} b a^3$ $\qquad I_{yc} = \frac{1}{12} a b^3$ (a, b)

$I_{xc} = \frac{\pi R^4}{4}$



ΔP_{loss} $\left(\frac{N}{m^2}\right)$	$\Delta P_{loss} = \rho g h_L$	$\dot{E}_{mech,loss} = \frac{\dot{m}}{\rho} \Delta P_{loss}$
$\frac{\Delta P_{loss}}{\rho}$ $\left(\frac{J}{kg}\right)$	h_L (m)	$\dot{E}_{loss}$ (W)
		$\dot{W}/\dot{m}$ (J/kg)

Stagnation pressure: convert all of velocity of a flow into pressure · set $v = 0$

Static pressure: if sensor had 0 velocity relative to fluid

I_x = 2nd moment of inertia about axis through point O

I_{xc} = moment of inertia through centroid of the object

$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

Velocity Profile:

$V(r_{wall}) = 0$

$$\frac{P_{in} - P_{out}}{2L} r = \mu \frac{dV}{dr} \rightarrow V(r) = \left(\frac{P_{in} - P_{out}}{4L\mu}\right)\left(r_{wall}^2 - r^2\right)$$

$$f = \Pi_1 = \frac{\Delta P_{loss}}{\frac{1}{2}\rho V^2 \frac{L}{D}} \qquad Re = \Pi_2 = \frac{\rho V_{avg} D}{\mu} \qquad \Pi_3 = \frac{\varepsilon}{D}$$

turbulent: $\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$

laminar: $f = \frac{64}{Re}$ $\quad (Re < 2300)$

Dimensional Analysis:

1. list all important vars (k = num vars)
2. Express each in terms of prim. units (r = num primary dims)
3. # Π terms $n = k - r$
4. choose r repeating variables
5. $\Pi_1 \rightarrow$ mult dep var by all repeating vars raised to an exponent
6. Repeat for remaining non-repeating vars
7. $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_{n \ldots})$

$\Delta p \sim \left[\frac{M}{LT^2}\right]$, $V \sim \left[\frac{L}{T}\right]$, $\rho \sim \frac{M}{L^3}$, $\mu \sim \frac{M}{LT}$, $D \sim L$

choose D, V, ρ; $\Pi_1 = \Delta P_{loss} V^a \rho^b D^c$

$$\Pi_1 = \left[\frac{M}{LT^2}\right]\left[\frac{L^a}{T^a}\right]\left[\frac{M^b}{L^{3b}}\right]\left[L^c\right]$$

M: $0 = 1 + b$

L: $0 = a + c - 1 - 3b$

T: $0 = -2 - a$

$\Pi_2 = \mu V^e \rho^f D^g \rightarrow \Pi_2 = \frac{\mu}{\rho V D} \rightarrow \Pi_1 = f(\Pi_2)$

A_{ref} = surface area

Minor Losses:

$$K_L = \Pi_1 = \frac{\Delta P_{minloss}}{\frac{1}{2}\rho V^2}$$

$$h_{loss,min} = \frac{\sum K_L V^2}{2g}$$

$$\eta_{pump} = \frac{\dot{W}_{ideal}}{\dot{W}_{actual}}$$

External flow

$$C_D = \frac{F_{drag}}{\frac{1}{2}\rho V^2 A_{ref}} \qquad C_L = \frac{F_{lift}}{\frac{1}{2}\rho V^2 A_{ref}} \qquad C_p = \frac{P - P_\infty}{\frac{1}{2}\rho V^2}$$

q = dyn pres $= \frac{1}{2}\rho V_\infty^2$

$$C_D = C_L \frac{F_{drag}}{F_{lift}} \qquad P = \vec{F} \cdot \vec{V}$$

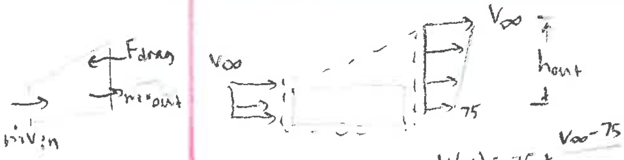
planform area (1 side)

$$C_{D,lam} = \frac{1.328}{\sqrt{Re_L}} \qquad \text{as } x \uparrow, \delta \uparrow$$

$$\delta_{lam} = \frac{4.91x}{\sqrt{Re_x}} \qquad Re < 500{,}000$$

$$C_{D,f,turb} = \frac{0.455}{[\log(Re_L)]^{2.58}} \qquad \delta_{turb} = \frac{0.37x}{(Re_x)^{1/5}} \qquad Re > 500{,}000$$

$$C_{D,f,both} = \frac{0.455}{[\log(Re_L)]^{2.58}} - \frac{1700}{Re_L}$$

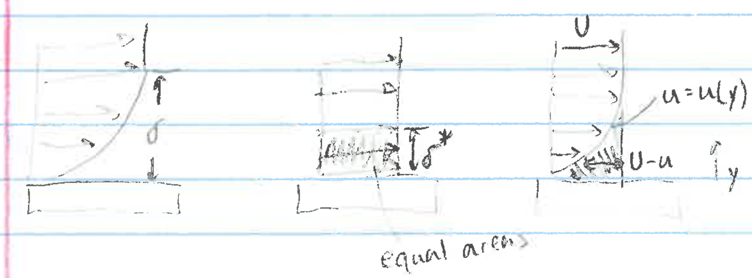


$V(y) = 75 + \frac{V_\infty - 75}{h_{out}}$

$\dot{m}V_{out} = \int_A \rho|\vec{V}_{rel}\cdot\hat{n}|\vec{V}_{abs}\,dA$

$\delta^* = \delta - h_{in}$

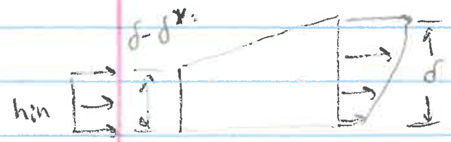
Momentum Deficit



Momentum Thickness:

height of free stream air that contains same momentum that is lost

$\theta = \frac{7}{72}\delta \qquad F_{drag} = \rho V_\infty^2 w\theta$



$\dot{m}V_{in} = \rho V_\infty^2 w(\delta - \delta^*)$

$(\frac{7}{8}\delta\ ?)$

$\dot{m}V_{out} = \rho w\int_{y=0}^{\delta}\left[V_\infty\left(\frac{y}{\delta}\right)^{1/7}\right]^2 dy$

$= \rho V_\infty^2 w\left(\frac{7}{9}\right)\delta$

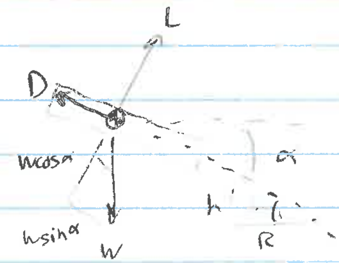
$0 = F_{drag} + \dot{m}V_{in} - \dot{m}V_{out}$

$F_{drag} = \dot{m}V_{out} - \dot{m}V_{in}$

$= \rho V_\infty^2 w\left(\frac{7}{72}\delta\right)$

$C_D \rightarrow \frac{F_{drag}}{\frac{1}{2}\rho V^2(wL)} = \left(\frac{2}{L}\right)\left(\frac{7}{72}\right)\delta$

Aircraft



$\Sigma F_{\parallel} = W\sin\alpha - D = 0$

$D = W\sin\alpha \qquad L = W\cos\alpha$

$\Sigma F_{\perp} = L - W\cos\alpha = 0$

$\frac{L}{D} = \frac{C_L}{C_D} = \frac{1}{\tan\alpha} \qquad \tan\alpha = \frac{h}{R}$

$R = h\left(\frac{C_L}{C_D}\right)$

Incompressible Flow

$c = \sqrt{kRT}$

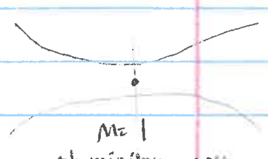
$M = \frac{V}{c}$

energy: $\frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2} \rightarrow \frac{P_2}{P_1} = \left[\frac{1 + \frac{k-1}{2}M_1^2}{1 + \frac{k-1}{2}M_2^2}\right]^{k/k-1}$

mass: $\frac{A_2}{A_1} = \frac{M_1}{M_2}\left[\frac{1 + \frac{k-1}{2}M_2^2}{1 + \frac{k-1}{2}M_1^2}\right]^{\frac{k+1}{2(k-1)}}$

Stagnation:

$\frac{P}{P_0} = \left(\frac{1}{1 + \frac{k-1}{2}M^2}\right)^{k/k-1} \rightarrow \frac{T}{T_0} = \frac{1}{1 + \frac{k-1}{2}M^2} \rightarrow \frac{\rho}{\rho_0} = \left(\frac{1}{1 + \frac{k-1}{2}M^2}\right)^{\frac{k}{k-1} - 1}$



$P_e \quad P_b \quad \cdot$ if $\frac{P_b}{P_0} \le \left[\frac{1}{1 + \frac{k-1}{2}(1)^2}\right]^{\frac{k}{k-1}}$ $M_e = 1.0$ "choked"

$\rightarrow \frac{P_e}{P_0} = \text{?} > \frac{P_b}{P_0}$

$\cdot$ if $\frac{P_b}{P_0} > \text{?}$, $M_e < 1.0$ "not choked"

(ideal gas)

isentropic flow

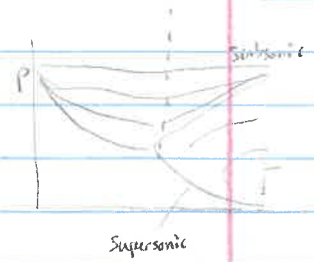
$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1}\frac{T_1}{T_2}$

$u_2 - u_1 = c_v(T_2 - T_1) \qquad h_2 - h_1 = c_p(T_2 - T_1)$

$s_2 - s_1 = c_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$

$= c_v\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$

$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$



shock:

$x \mid y \qquad \frac{P_y}{P_x} = \frac{1 + kM_x^2}{1 + kM_y^2}; \quad \frac{A_2}{A_1} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}\frac{P_1}{P_2} = 1 \quad (A_1 = A_2 \text{ for shocks})$

$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\left(\frac{2k}{k-1}\right)M_x^2 - 1}$

Kinematics

$\vec{V} = u\hat{i} + v\hat{j} \qquad \left.\frac{dy}{dx}\right|_{\text{stream line}} = \frac{v}{u}$

volumetric dilitation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \left(\frac{d\forall/dt}{\forall}\right)$

ang. def. rate: $\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

rot. rate: $\bar{\omega} = \left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\frac{1}{2}\right]\hat{k}$

vorticity: $2\bar{\omega} = \xi$

irrotational: $\xi = 0, \ \bar{\omega} = 0$

$\dot{m}_1 = \frac{P_1}{RT_1}M_1\sqrt{kRT_1}A_1$

$= \frac{P_1 M_1 A_1\sqrt{k}}{\sqrt{RT_1}}$