Joshua Eckels - ES205 - eqn sheet

standard form: $\tau \dot{x} + x = K f(t)$

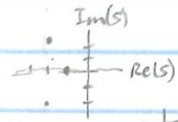
$m\ddot{x} + c\dot{x} + kx = f(t)$

$\frac{m}{k} = \frac{1}{\omega_n^2}$ $c = \frac{2\zeta}{\omega_n}$ $K = \frac{1}{k}$

$\frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = K f(t)$

$\mathcal{L}\{f''(t)\} = s^2 \cdot F(s) - s \cdot f(0) - \dot{f}(0)$

$\mathcal{L}\{f'(t)\} = s \cdot F(s) - f(0)$



poles $s = -a \pm i\omega \rightarrow \frac{1}{(s+a)^2+\omega^2}$

poles: $-2 \pm 2i \rightarrow \frac{1}{(s+a)^2+\omega^2}$

$-1 \rightarrow e^{at}$

Lab 1: $F_{in}(t) = A \cdot u(t)$

damping ratio: $\zeta = \sqrt{\frac{\ln^2(\%OS/100)}{\ln^2(\%OS/100)+\pi^2}} = \frac{\delta}{\sqrt{\delta^2+4\pi^2}}$

freq oscillation: $\omega_d = \pi / t_p$

natural freq: $\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$

static gain: $K = x_{ss}/A$

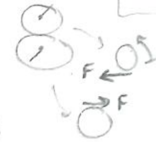
Peak time: $t_p = \frac{\pi}{\omega_d}$

Settling time: $t_s = 4/\zeta\omega_n$

% overshoot: $\%OS = \frac{x_p - x_{ss}}{x_{ss}} \cdot 100\%$

log decrement: $\delta = \frac{1}{n}\ln\left(\frac{x_1 - x_{ss}}{x_{1+n} - x_{ss}}\right) = \frac{1}{n}\ln\left(\frac{x_{ss}}{x_n - x_{ss}}\right)$

SEE: $SEE = \sqrt{\frac{\sum_i^n (x_{model,i} - x_{data,i})^2}{n-2}}$

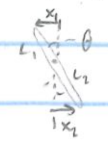
Gears:


$\omega_1 r_1 = \omega_2 r_2$

$\theta_1 r_1 = \theta_2 r_2$

$\tau_1 \omega_1 = \tau_2 \omega_2$

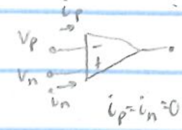
$\frac{\tau_2}{\tau_1} = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2} = \frac{n_2}{n_1}$

Lever:


$\frac{x_1}{L_1} = \frac{x_2}{L_2}$

$x_2 = L_2 \sin\theta$

$\cos\theta \approx 0$



$V_p - V_n = 0$

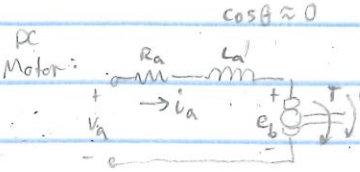
$i_p = i_n = 0$

R: $V_{12} = iR$

L: $V_{12} = L\frac{di}{dt} = Lsi$

C: $i = C\frac{dV}{dt} = VCs$

$Z_C = \frac{1}{Cs}$ $Z_L = Ls$

DC Motor:


$e_b = k_b \dot{\theta}$

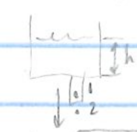
$T = k_T i_a$

$k_b = k_T$

$-V_a + i_a R_a + L_a \frac{di_a}{dt} + e_b = 0$

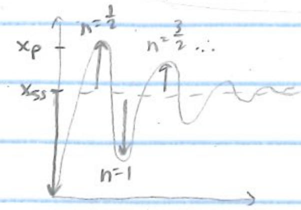
Orifice

$Q_0 = C_d A_0 \sqrt{\frac{2}{\rho}(P_1 - P_2)}$

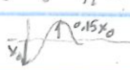


or

$Q_0 = A_0 C_d \sqrt{\frac{2}{\rho}(P_1 - P_2) + 2g(z_1 - z_2)}$



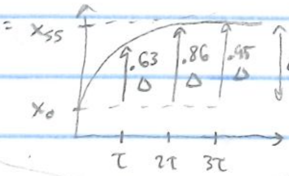
$\%OS: \delta = \frac{1}{1/2}\ln\left(\frac{x_0}{.15x_0}\right)$



$\tau \dot{x} + x = K f(t)$

$\rightarrow x(t) = x_{ss} + (x_0 - x_{ss})e^{-t/\tau}$

$z(t) = -\frac{t}{\tau} = \ln\left(\frac{x - x_{ss}}{x_0 - x_{ss}}\right)$



Heat Transfer

Conduction: $\dot{Q} = \frac{kA}{L}(T_H - T_C)$

L ~ thickness, k ~ thermal conductivity $\left(\frac{W}{mK}\right)$

Convection: $\dot{Q} = hA(T_s - T_\infty)$

$h \sim \frac{W}{m^2 K}$ heat transfer coeff. T_H T_C

Radiation: $\dot{Q} = A\varepsilon\sigma(T_s^4 - T_{surr}^4)$

$T_H - T_C$

ε ~ emissivity

$\sigma \sim 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$; $B_i = \frac{h(V/A_s)}{k}$

$mC_v \frac{dT}{dt} = \dot{Q} + \dot{W}$ $B_i < 0.1 \rightarrow$ lumped capacitance

C = thermal capacity → assume

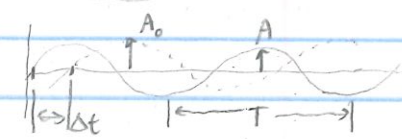
Frequency Response

$F_{in}(t) = A\sin(\omega t)$ (cos) $\rightarrow G(s) = \frac{X(s)}{F_{in}(s)} \rightarrow x_{ss}(t) = |G(i\omega)| A \sin(\omega t + \angle G(i\omega))$ (cos)

1st order: $\tau\dot{x} + x = Kf(t) \rightarrow x_p = x_{ss} = \frac{KA}{\sqrt{1+(\tau\omega)^2}}\sin(\omega t - \phi)$, where $\tan\phi = \tau\omega$

total $\rightarrow x(t) = x_h + x_p = C_1 e^{-t/\tau} + x_{ss}$

$f(t) = A\sin(\omega t)$

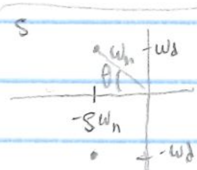


2nd order: $\frac{1}{\omega_n^2}\ddot{x} + \frac{2\zeta}{\omega_n}\dot{x} + x = K f(t) \rightarrow x_p = x_{ss} = \frac{KA}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\sin(\omega t - \phi)$, $\tan\phi = \frac{2\zeta r}{1-r^2}$ $T = \frac{2\pi}{\omega}$, $\Delta t = \frac{\phi}{\omega}$

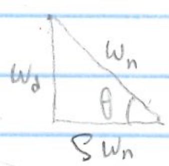
total $\rightarrow x(t) = x_h + x_p = C_1 e^{-\zeta\omega_n t}\sin(\omega_d t + \phi_1) + x_{ss}$, $r = \frac{\omega}{\omega_n}$, $r \to 1$ AR → max, $r \to \infty$ AR → 0, $\phi \to 180°$, $r = 1$ $\phi = 90°$ for all ζ, $r = 0$ AR = 1 for all ζ

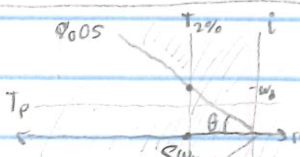
isolators: $\frac{F_T}{F_0} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}} \rightarrow r = \frac{\omega}{\omega_n} \rightarrow \omega_n = \sqrt{\frac{k_{eff}}{m}}$

force reduction: $\frac{F_T}{F_0} < 1$ when $r > \sqrt{2}$ → small k or large m



roots: $-\zeta\omega_n \pm \omega_d i$





$T_{2\%} = \frac{4}{\zeta\omega_n}$

$T_p = \frac{\pi}{\omega_d}$

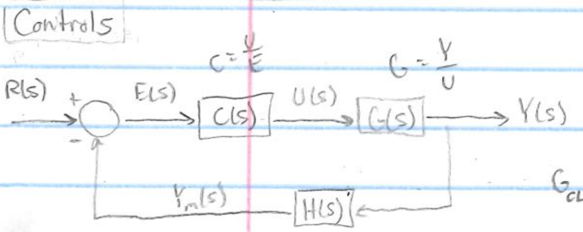
$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\cos\theta = \zeta$

$\%OS = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$

Controls

$C = \frac{U}{E}$ $G = \frac{Y}{U}$



$Y = GU = G(CE)$

$= GC(R - HY)$

$G_{CL}(s) = \frac{Y}{R} = \frac{CG}{1+CGH}$

$\frac{Y(s)}{R(s)} = \frac{1}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$

$C(s) = K_p + K_i\frac{1}{s} + K_d s$

PID controller: P, PI, PID

Misc

$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + gh_L$; $gh_L = f\frac{L}{D}\frac{V^2}{2} + \sum K_i \frac{V_i^2}{2}$

$C_d = \sqrt{\frac{1}{1+K}}$

$h = \left[h(0)^{1/2} - \frac{C_d A_0 \sqrt{2g}}{2A}t\right]^2$

height fluid in tank

Poles: $s_1 = -1 \rightarrow$ exponential decay

$s_1 = 1 \rightarrow$ exponential growth

$s_{1,2} = -1 \pm i \rightarrow$ exp. decay oscill.

$s_{1,2} = 1 \pm i \rightarrow$ exp. grow oscill.

$s_{1,2} = 0 \pm i \rightarrow$ sustained oscill.