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ES214 - Msys - 12/13/18
Equation sheet

$\hat{i}$, $\hat{j}$, $\hat{k}$ — $\hat{e}_r$, $\hat{e}_\theta$, $\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dx}\cdot\frac{dx}{dt} = \frac{d\vec{r}}{dx}\cdot v_x$

$\rho(x) = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$

$\vec{v}_{A/ground} = \vec{v}_{A/B} + \vec{v}_{B/Ground}$

$\vec{a} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}v$

$a\,dx = v\,dv$

$\vec{\alpha} = \frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$

$\alpha\,d\theta = \omega\,d\omega$

$\vec{a}_{A/grd} = \vec{a}_{A/B} + \vec{a}_{B/grd}$

Dependent motion

* Datums shouldn't move
* accelerations point in same direction as datum

Circular

$\hat{e}_t = \hat{e}_\theta$
$\hat{e}_r = -\hat{e}_n$
$v_p = r\dot{\theta}$

$\vec{v}_p = v_p\hat{e}_t$ $\quad\vec{v}_p = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a}_p = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$
$= -\frac{v_p^2}{r}\hat{e}_r + \dot{v}_p\hat{e}_\theta$

restitution:

$$e = -\frac{(v_{B_n}' - v_{A_n}')}{v_{B_n} - v_{A_n}}$$

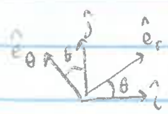
Coordinates

		r	v	a
$\hat{j}$, $\hat{i}$	Cartesian	$x\hat{i} + y\hat{j}$	$\dot{x}\hat{i} + \dot{y}\hat{j}$	$\ddot{x}\hat{i} + \ddot{y}\hat{j}$
$\hat{e}_n$, $\hat{e}_t$	Normal-tangent	—	$v\hat{e}_t$	$\dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$
$\hat{e}_r$, $\hat{e}_\theta$	radial-transverse	$r\hat{e}_r$	$\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$	$(\underbrace{\ddot{r}}_{radial} - \underbrace{r\dot{\theta}^2}_{centripetal})\hat{e}_r + (\underbrace{r\ddot{\theta}}_{tangential} + \underbrace{2\dot{r}\dot{\theta}}_{coriolis})\hat{e}_\theta$

MoI:

$I_G = \frac{1}{12}mL^2$

$I_G = \frac{1}{2}mr^2$



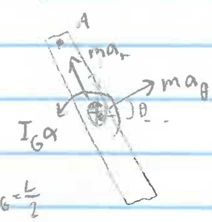
$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$
$\hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

Rotation:

$\vec{v}_B = \vec{v}_A + \vec{\omega}\times\vec{r}_{B/A}$
$\vec{a}_B = \vec{a}_A + \vec{\alpha}\times\vec{r}_{B/A} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{B/A})$
(FAR: $\vec{v}_A = \vec{a}_A = 0$, A = point of rotation)

Rate form:

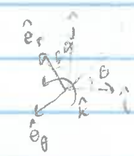
KD:



$r_{AG} = \frac{L}{2}$

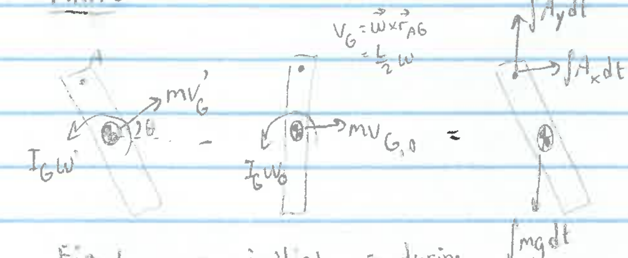
$a_r = r_{AG}\omega^2$
$a_\theta = r_{AG}\alpha$

FBD:



General: coam_A: $I_G\vec{\alpha} + \vec{r}_{G/A}\times m\vec{a}_G = \Sigma\vec{M}_A$
$I_G\vec{\alpha} + \vec{r}_{G/A}\times m(a_r\hat{e}_r + a_\theta\hat{e}_\theta) = \Sigma\vec{M}_A$

Finite

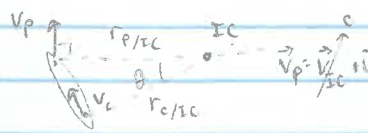


Final − initial = during

COE: $KE = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$; $W_{1\to2} = \int_{x_1}^{x_2}\vec{F}\cdot d\vec{x} + \int_{\theta_1}^{\theta_2}\vec{M}\cdot d\vec{\theta}$

$E_{spr} = \frac{1}{2}k(x-x_0)^2$

Instantaneous Center:



$\vec{v}_p = \vec{v}_{IC} + \vec{\omega}_{cp}\times\vec{r}_{p/IC}$

$\text{colm}\text{-}\hat{i}$: $ma_\theta\cos\theta - ma_r\sin\theta = A_x$ (rate)

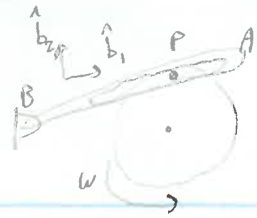
$[mv_G'\cos\theta] - [mv_{G,0}] = \int A_x dt$ (finite)

$\text{coam}\text{-}A$: $I_G\alpha + \vec{r}_{AG}\times ma_\theta = \Sigma\vec{M}_A$ (rate)

$I_G\alpha + m\left(\frac{L}{2}\right)^2\alpha = -mg\sin\theta\left(\frac{L}{2}\right)$

$[I_G\omega' + \vec{r}_{AG}\times m\vec{v}_G'] - [I_G\omega_0 + \vec{r}_{AG}\times m\vec{v}_{G,0}] = \int\Sigma\vec{M}_A dt$ (finite)

Rotating Reference frame:



$$\left.\frac{d\vec{u}}{dt}\right|_{\substack{\text{fixed}\\ \text{inertial}\\ \text{frame}}} = \left.\frac{d\vec{u}}{dt}\right|_{\substack{\text{rotating}\\ \text{frame}}} + \vec{\omega} \times \vec{u}$$

velocity: $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

acceleration: $\boxed{\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$

$$\vec{v}_{B/A} = \left.\frac{d\vec{r}_{B/A}}{dt}\right|_{\text{fixed}} = \left.\frac{d\vec{r}_{B/A}}{dt}\right|_{\text{rot}} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\frac{d\vec{r}_{B/A}}{dt} = \underbrace{\frac{dx'}{dt}\hat{b}_1 + \frac{dy'}{dt}\hat{b}_2}_{\vec{v}_{rel}}$$

$$\boxed{\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} + \vec{v}_{rel}}$$