Joshua Eckels - ME305 exam sheet
4-3-2019

Air Speed

$1.6876 \frac{ft}{s} = \text{knot}$

incompressible

$$V = \sqrt{\frac{2(P_0 - P_\infty)}{\rho}}$$

compressible

$$V = \sqrt{\left(\frac{1}{\rho}\right)\frac{2\gamma}{\gamma-1}\cdot P\left[\left(\frac{P_0-P}{P}+1\right)^{\frac{\gamma-1}{\gamma}}-1\right]}$$

		ρ	P
true:	V_T	actual	actual
equivalent:	V_{eq}	SL	actual
calibrated:	V_c	SL	SL

$C_L = \frac{L}{q_\infty S}$ $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$

$C_D = \frac{D}{q_\infty S}$

$C_m = \frac{M}{q_\infty S c}$

Pressure Coefficient

$$C_{p,i} = \frac{P_i - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \rightarrow C_l = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\,dx.$$

	C_p	C_l	C_d
$M<0.3$	$C_{p,0}$	$C_{l,0}$ (NACA charts)	$C_{d,0}$ (NACA charts)
$0.3<M<0.7$	$\frac{C_{p,0}}{\sqrt{1-M_\infty^2}}$	$\frac{C_{l,0}}{\sqrt{1-M_\infty^2}}$	$C_{d,0}$

$$C_{p,A} = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}M_A^2}\right]^{\frac{\gamma}{\gamma-1}}-1\right\}$$

$M_A = 1$

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right]^{\gamma/\gamma-1}-1\right\}$$

Boundary layer

$D = C_f q_\infty S$

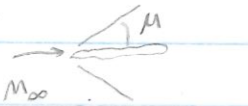
$Re_x < 500{,}000$ → laminar, → turbulent

	δ	$C_{f,x}$	$C_{f,avg}$
laminar	$\frac{5.2x}{\sqrt{Re_x}}$	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{1}{c}\int_0^c C_{f,x}(x)\,dx = \frac{1.328}{\sqrt{Re_{x=L}}}$
turbulent	$\frac{0.37x}{(Re_x)^{0.2}}$	$\frac{0.0592}{(Re_x)^{0.2}}$	$\frac{1}{c}\int_0^c C_{f,x}(x)\,dx = \frac{0.074}{(Re_{x=L})^{0.2}}$

$$C_{f,avg} = \frac{1}{c}\left[\int_0^{x_T} C_{f,x,lam}\,dx + \int_{x_T}^{c} C_{f,x,turb}\,dx\right]$$

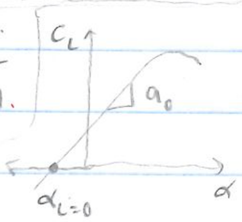
Supersonic

$\sin\mu = \frac{1}{M}$



$C_{d,w}$ = "wave drag" $= \frac{4\alpha}{\sqrt{M_\infty^2-1}}$

C_l = lift coeff = ↗ supersonic



Finite wings

$C_L = a(\alpha - \alpha_{L=0})$ $\alpha_i = \frac{C_L}{\pi e AR}$

$$a = \frac{a_0}{1+\frac{57.3a_0}{\pi e AR}}$$

$C_{D,i} = \frac{C_L^2}{\pi e AR}$ = "induced drag"

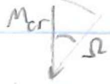
$a_0 \sim 0.1\frac{1}{deg}$

$AR = \frac{b}{c}$, e = wing eff. factor

$$C_D = \underbrace{C_{d,f} + C_{d,p}}_{C_d\text{ -NACA chart}} + C_{D,i} + C_{d,w}$$

$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L,max}}}$

- flaps → increase V_{stall} by $C_{L,max}$ ↑
  → by virtually increasing α



$\cos\Omega = \frac{M_{cr,unswept}}{M_{cr,swept}}$

→ fly faster
→ decrease lift

Thrust, Power Required

$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$ "drag polar"

$C_L^* = \sqrt{C_{D,0}\pi e AR}$

$C_D^* = 2C_{D,0}$

SLUF:

$$T_R = D = q_\infty S C_D = q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e AR} \xrightarrow{\text{subs } C_L, C_D} T_R = \frac{L}{L/D} = \frac{W}{C_L/C_D}$$

$\frac{C_L}{C_D}\Big|_{max} = \frac{C_L^*}{C_D^*}$ = min T_R

$$P_R = T_R V_\infty = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{W^2}{\frac{1}{2}\rho_\infty V_\infty S \pi e AR} = \sqrt{\frac{2W^3}{\rho_\infty S}}\cdot\left(\frac{C_D}{C_L^{3/2}}\right)$$

$\frac{C_L^{3/2}}{C_D}\Big|_{max} \rightarrow \frac{C_L^{**}}{C_D^{**}}$ = min. P_R

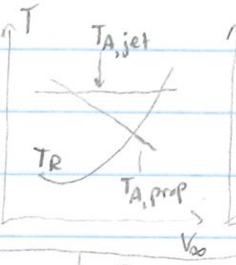
$C_L^{**} = \sqrt{3C_{D,0}\pi e AR}$

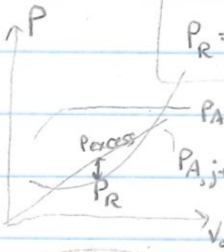
$C_D^{**} = 4C_{D,0}$

Thrust/Power Available

$\frac{P_A}{P_{A,SL}} = \frac{\rho_\infty}{\rho_{SL}}$ $\frac{T_A}{T_{A,SL}} = \frac{\rho_\infty}{\rho_{SL}}$

$V_{max,SL} > V_{max,alt}$

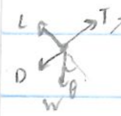




$P_{A,prop} = \eta_p P_{shaft}$

$P_{A,jet} = T_A V_\infty$

Rate of Climb



$\sin\theta = \frac{T-D}{W}$

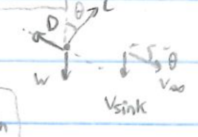
$ROC = V_\infty \sin\theta = V_\infty\left(\frac{T-D}{W}\right)$

$ROC = \frac{(P_A - P_R)}{W} \rightarrow P_{excess}$

$P_{ex,max}$ → prop: $P_{R,min}$, → jet: $T_{R,min}$

Ceiling Abs: ROC = 0
Service ceil: ROC = 100 ft/min

Glide



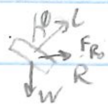
$\tan\theta = \left(\frac{1}{C_L/C_D}\right)$

$R_{max} = \frac{h}{\tan\theta_{min}} = h\frac{C_L}{C_D}\Big|_{max}$

$V_{sink} = V_\infty \sin\theta = V_\infty \frac{D}{W} = \frac{P_R}{W}$

min V_{sink} → min P_R

Turns



$L\cos\phi = W$

$n = \frac{L}{W} = \frac{1}{\cos\phi}$

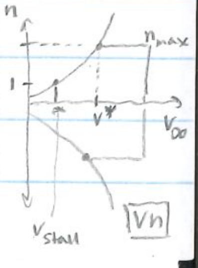
$F_R = \sqrt{L^2 - W^2}$

$R = \frac{V_\infty^2}{g\sqrt{n^2-1}} \rightarrow \omega = \frac{g\sqrt{n^2-1}}{V_\infty} = \frac{V_\infty}{R}$

	R	ω
Pull-up	$\frac{V_\infty^2}{g(n-1)}$	$\frac{g(n-1)}{V_\infty}$
Pull-down	$\frac{V_\infty^2}{g(n+1)}$	$\frac{g(n+1)}{V_\infty}$

$V^* = \sqrt{\frac{n_{max}W}{\frac{1}{2}\rho_\infty S C_L^*}}$ = tightest radius possible

$\frac{W}{S}$ = wing loading



Breguet Formulas

	fuel	units	Endurance (s)	max E	Range (m)	max R
Propellor	C or SFC	$\frac{lb_f/s}{ft\cdot lb_f/s}$	$dW = -cP_R dt$ $E = \frac{\eta_p}{c}\left(\frac{C_L^{3/2}}{C_D}\right)\sqrt{2\rho_\infty S}\left(W_e^{-1/2} - W_s^{-1/2}\right)$	$P_{R,min}$ C_L^{**}, C_D^{**}	$dW = D\frac{c}{\eta_p}dx$ $R = \eta_p\left(\frac{C_L}{C_D}\right)\frac{1}{c}\ln\left\|\frac{W_s}{W_e}\right\|$	$T_{R,min}$ C_L^*, C_D^*
Jet:	C_t or TSFC	$\frac{lb_f/s}{lb_f}$	$dW = C_t T_R dt$ $E = \left(\frac{C_L}{C_D}\right)\frac{1}{C_t}\ln\left\|\frac{W_{start}}{W_{end}}\right\|$	$T_{R,min}$ C_L^*, C_D^*	$dW = -\frac{C_t}{V_\infty}T_A dx$ $R = (W_s^{1/2} - W_e^{1/2})\frac{C_L^{1/2}}{C_D}\left(\frac{2}{C_t}\right)\sqrt{\frac{2}{\rho_\infty S}}$	$\frac{C_L^{1/2}}{C_D}\Big\|_{max} = \frac{\left[\frac{1}{3}C_{D,0}\pi e AR\right]^{1/4}}{\frac{4}{3}C_{D,0}}$

$V_\infty dt = dx$

assume

T → constant

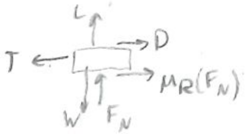
F_{eff} → constant

$V_{LO} = 1.2 V_{stall}$

$= 1.2\sqrt{\frac{2W}{\rho_\infty S C_{L,max}}}$

use $C_L = C_{L,max} \neq \cancel{C_L^*}$

Takeoff



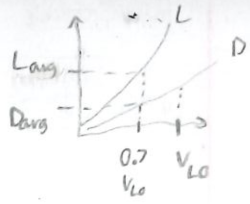
$$T - D - \mu_R(W-L) = m\frac{dV_\infty}{dt}$$

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_{L,max}$$

$$D = \frac{1}{2}\rho_\infty V_\infty^2 S\left[C_{D,0} + \phi\frac{C_L^2}{\pi e AR}\right]$$

$$\phi \le 1 = \frac{\left(\frac{16h}{b}\right)^2}{1+\left(\frac{16h}{b}\right)^2}$$

"ground effect"



$$F_{eff} = \left[T - D - \mu_R(W-L)\right]_{avg}$$

$$S_{LO} = \frac{1.44\,W^2}{g\rho_\infty S C_{L_{max}}\left[T - [D + \mu_R(W-L)]_{avg}\right]}$$

$$\left[D + \mu_R(W-L)\right]_{V_\infty = 0.7V_{LO}}$$

Landing

$$S_L = \frac{1.69\,W^2}{g\rho_\infty S C_{L_{max}}\left[D + \mu_R(W-L)\right]_{0.7V_L}}$$

$V_L = 1.3 V_{stall}$

V_L = vel. landing

V_{LO} = vel. liftoff

Sp. energy: $H_e = \frac{V^2}{2g} + h$

energy height: $h = H_e - \frac{V^2}{2g}$

Accelerated ROC

$$T - D - W\sin\theta = m\frac{dV_\infty}{dt}$$

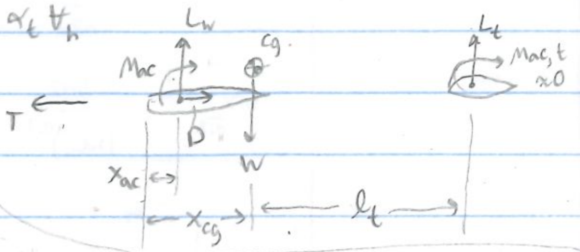
$$P_s = V\sin\theta + \frac{V}{g}\frac{dV}{dt}$$

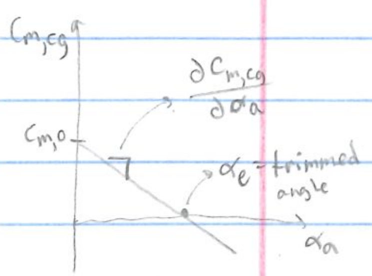
$$\frac{dH_e}{dt} = P_s = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt}$$

Trimmed flight

$C_{m,cg} = C_{L,w}(\bar{x}_{cg} - \bar{x}_{ac}) - C_{L,t}\forall_h + C_{m,ac}$; $\forall_H = \frac{l_t}{c}\cdot\frac{S_t}{S}$; $\alpha_a = \alpha - \alpha_{L=0}$; $\frac{x_{cg}}{c} = \bar{x}_{cg}$

$$C_{m,cg} = C_{m,ac} + a_w(\alpha - \alpha_{L=0})(\bar{x}_{cg} - \bar{x}_{ac}) - a_t\alpha_t\forall_h$$





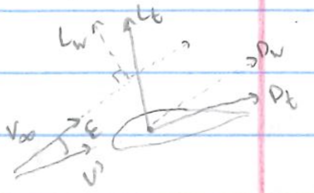
stability:

1. $\frac{\partial C_{m,cg}}{\partial\alpha_a} < 0$

2. $C_{m,0} > 0$

$$\varepsilon = \varepsilon_0 + \frac{\partial\varepsilon}{\partial\alpha_a}\alpha_a$$

$$\frac{\partial\varepsilon}{\partial\alpha_a} \approx \underbrace{\frac{(21°)a_w}{(AR)^{0.725}}\left(\frac{c}{l_h}\right)^{0.25}}$$



$$\alpha_t = \alpha_a - \varepsilon - i_T$$

ε = downwash

i_T = tail setting angle

$$\frac{\partial\alpha_T}{\partial\alpha_a} = 1 - \frac{\partial\varepsilon}{\partial\alpha_a}$$

$$\frac{\partial C_{m,cg}}{\partial\alpha_a} = a_w(\bar{x}_{cg} - \bar{x}_{ac}) - a_t\forall_h\frac{\partial\alpha_T}{\partial\alpha_a}$$

$L = L_t\cos\varepsilon - D_t\sin\varepsilon$

$D = L_t\sin\varepsilon + D_t\cos\varepsilon$

Neutral point: solve $\frac{\partial C_{m,cg}}{\partial\alpha_a} = 0$ for $\bar{x}_{cg}$:

$$\bar{x}_n = \bar{x}_{cg} = \bar{x}_{ac} + \frac{A}{1+A}\frac{l_h}{c}, \text{ where } A = \frac{a_t}{a_w}\left(1 - \frac{\partial\varepsilon}{\partial\alpha_a}\right)\frac{S_t}{S_w}$$

Static Margin: $SM = \bar{x}_n - \bar{x}_{cg}$