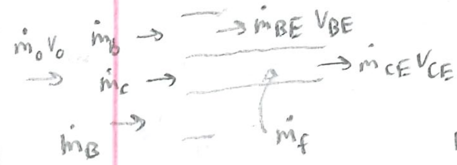
Turbofan



$\alpha = \frac{\dot m_B}{\dot m_C}$

$\dot m_o = \dot m_C + \dot m_B = \dot m_C(1+\alpha)$

$\dot m_{CE} = \dot m_C + \dot m_f$

$F_{total} = F_C + F_B = (\dot m_{CE} V_{CE} - \dot m_C V_o) + (\dot m_B(V_{BE} - V_o))$

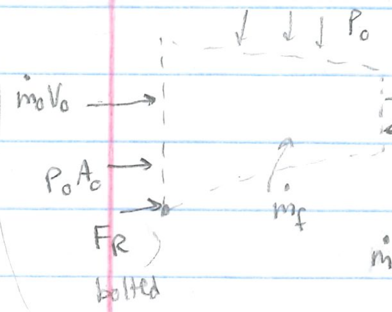
inst. $TSFC = \frac{\dot m_f}{T}$ $\left(\frac{lb_m/hr}{lb_f}\right)$

uninst. $S = \frac{\dot m_f}{F}$ $T = F(1-\phi_{inlet} - \phi_{nozzle})$

Joshua Eckels - ME411 - eqn sheet

Turbojet

$TSFC = \frac{F/\dot m_o + 2V_o}{2\eta_T h_{PR}}$



$F_R = T$

$0 = F_R + P_o A_o - P_e A_e - P_o(A_o - A_e) + \dot m_o V_o - \dot m_e V_e$

$T = (\dot m_o + \dot m_f)V_e - \dot m_o V_o + (P_e - P_o)A_e$

$\frac{T}{\dot m_o} = (1+f)V_e - V_o + \frac{(P_e-P_o)A_e}{\dot m_o}$ $f = \frac{\dot m_f}{\dot m_o}$

$\dot m_e = \dot m_f + \dot m_o$

$\eta_T = \frac{\Delta KE}{\dot Q_{in}} = \frac{\frac{1}{2}\dot m_e V_e^2 - \frac{1}{2}\dot m_o V_o^2}{\dot m_f h_{PR}}$

$\eta_P = \frac{TV_o}{\Delta KE}$ (KE → thrust)

$\eta_o = \eta_P \eta_T = \frac{TV_o}{\dot Q_{in}}$ ($\dot Q$ to KE)

Cruise-Climb

$TSFC = TSFC_{SL}\sqrt{\theta}$; $\theta = \frac{T}{T_{std}}$

$\frac{1}{W} dW = \frac{-TSFC_{SL} g}{C_L/C_D} \frac{dx}{M a_{SL}}$

$\ln\left(\frac{W_{start}}{W_{end}}\right) = \frac{\Delta X \, TSFC_{SL} g}{\frac{C_L}{C_D} M a_{SL}}$

$RF = \frac{C_L}{C_D} M a_{SL} \frac{1}{TSFC_{SL}} \frac{1}{g}$

*assumes M, C_L, C_D constant

*as $W\downarrow$, $M\rightarrow$, $P\downarrow$, $\rho\downarrow$, $C_L\rightarrow$ to keep $W = \frac{1}{2}\gamma P M^2 S C_L$

drag polar

$C_D = C_{D_o} + K_1 C_L^2 + K_2 C_L$

$\left.\frac{C_L^*}{C_D^*}\right|_{max} = \frac{1}{2\sqrt{C_{D_o} K_1} + K_2}$

$C_D^* = 2C_{D_o} + K_2 C_L^*$ | $C_L^* = \sqrt{\frac{C_{D_o}}{K_1}}$

Straight level flight

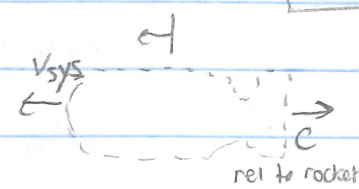
$\int \frac{1}{W} dW = \int -TSFC\, g \frac{T}{W} dt \rightarrow \ln\left(\frac{W_{start}}{W_{end}}\right) = \frac{\Delta t\, TSFC\, g}{C_L/C_D} = \frac{\Delta t}{EF}$

*assume $TSFC, C_L, C_D$ constant

$L = W, T = D$

$\frac{T}{W} = \frac{1}{C_L/C_D} = \frac{1}{L/D}$ *max $\Delta t \rightarrow \frac{C_L}{C_D}$ max

Rocket



$V_{p/grd} = V_{p/sys} + V_{sys/grd}$

$\dot m_{out} = -\frac{dm_{sys}}{dt}$

$V_{out,x} = -c + V_{sys}$

$m_{sys}\frac{dv_{sys}}{dt} + V_{sys}\frac{dm_{sys}}{dt} = -\dot m_{out} V_{out,x} \rightarrow \frac{V_{sys,f} - V_{sys,i}}{c} = \ln\left(\frac{m_{i,sys}}{m_{f,sys}}\right)$

$T = \dot m_p V_e + (P_e - P_o)A_e$

$T = \dot m_p c \rightarrow c = V_e + \frac{(P_e-P_o)A_e}{\dot m_p}$

$I_{sp} = \frac{T}{\dot m_p g} = \frac{c}{g}$ $\left(\frac{N \text{ thrust}}{N/s \text{ fuel}}\right)$

- T = constant
- a = increasing m = decreasing

MFP

$\dot m = \frac{P_t}{\sqrt{T_t}} A \cdot MFP$

$\dot m = A\frac{P_t}{\sqrt{T_t}} \underbrace{\sqrt{\frac{\gamma}{R}} M \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}}_{MFP}$

Impulse

$F_{int} = T = I_2 - I_1$

$T = P_2 A_2(1+\gamma M_2^2) - P_1 A_1(1+\gamma M_1^2)$

Compressible

$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$

$\frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/\gamma-1}$

$\frac{A_2}{A_1} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

Normal shock

$M_x P_x$, $V_x T_x$ } { $M_y P_y$, $V_y T_y$

$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2}$

$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}$

$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$

Oblique Shock

$(V_{1,t} = V_{2,t})$

$\frac{P_2}{P_1} = \frac{1+\gamma M_{1,n}^2}{1+\gamma M_{2,n}^2}$

$M_{1,n} = M_1 \sin\beta$

$\tan(\theta) = 2\cot(\beta) \cdot \left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$

$M_{2,n} = M_2 \sin(\beta - \theta)$

$M_{1,t} = M_1 \cos\beta$

$M_{2,t} = M_2\cos(\beta-\theta)$

*use $M_{1,n}$ and $M_{2,n}$ for $\frac{T_2}{T_1}$ and $\frac{P_2}{P_1}$

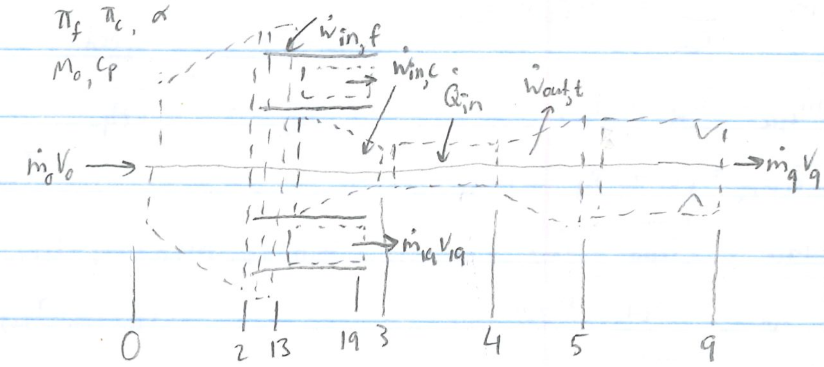
Parametric Cycle Analysis: turbofan

(turbojet $\alpha = 0$)

Given: T_{t4}, T_o, h_{PR} → Find: $F/\dot m_o, S, f, M_9, V_9, M_{19}, V_{19}$

π_f π_c, α

M_o, C_p



Diffuser 0→2

- $P_{to} = P_{t2}$, $T_{to} = T_{t2}$, $\frac{P_{to}}{P_o} = (1+0.2M_o^2)^{3.5} \rightarrow \pi_r$
- $T_{to}/T_o \rightarrow \tau_r \rightarrow T_{to}$

Fan 2→13

$\pi_f = \left(\frac{T_{t13}}{T_{t2}}\right)^{3.5} \rightarrow T_{t13}$

$w_{in,f} = \dot m_B C_p(T_{t13} - T_{t2})$

Compressor 2→3

$\pi_c = \left(\frac{T_{t3}}{T_{t2}}\right)^{3.5} \rightarrow T_{t3}$

$w_{in,c} = \dot m_c C_p(T_{t3} - T_{t2})$

Combustor 3→4

- $P_{t4} = P_{t3}$
- $\dot Q_{in} = \dot m_f h_{PR} = C_p \dot m_c(T_{t4} - T_{t3})$
- $f = \frac{\dot m_f}{\dot m_c} = \frac{C_p(T_{t4}-T_{t3})}{h_{PR}}$

Turbine 4→5

- $w_{out,t} = \dot m_c C_p(T_{t4} - T_{t5})$
- $w_{out,t} = w_{in,f} + w_{in,c}$ → T_{t5}

$T_{t4} - T_{t5} = \alpha(T_{t13} - T_{t2}) + T_{t3} - T_{t2}$

- $\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}}\right)^{3.5}$

Nozzle 13→19, 5→9

- $P_{t9} = P_{t5}$, $T_{t9} = T_{t5}$
- $\frac{P_{t9}}{P_9} = \frac{P_{t5}}{P_o} = \frac{P_{t5}}{P_{t4}}\frac{P_{t4}}{P_{t3}}\pi_c\pi_r = (1+0.2M_9^2)^{3.5} \rightarrow M_9$
- $\frac{T_{t9}}{T_9} = (1+0.2M_9^2) \rightarrow T_9 \rightarrow a_9 \rightarrow V_9$

Overall:

$P_9 = P_{19} = P_o$

- $F_C = \dot m_c(V_9 - V_o) + (P_9 - P_o)A_9$
- $F_B = \dot m_B(V_{19} - V_o) + (P_{19} - P_o)A_{19}$
- $F/\dot m_o = \frac{(V_9 - V_o) + \alpha(V_{19} - V_o)}{1+\alpha}$
- $S = \frac{\dot m_f/\dot m_o}{(1+\alpha)F/\dot m_o} = \frac{f}{(1+\alpha)F/\dot m_o}$

Efficiency

Real turbojet

Compressor $\quad \boxed{\tau_c = \pi_c^{\frac{\gamma-1}{\gamma e_c}}}$

Turbine $\quad \boxed{\tau_t = \pi_t^{\left(\frac{\gamma-1}{\gamma}\right)e_t}}$

thermal: η_i

$$\eta_c = \frac{T_{t3i}-T_{t2}}{T_{t3}-T_{t2}} = \frac{\pi_c^{\frac{\gamma-1}{\gamma}}-1}{\tau_c - 1} \rightarrow \eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}}-1}{\pi_c^{\frac{\gamma-1}{\gamma e_c}}-1}$$

$$\eta_t = \frac{T_{t4}-T_{t5}}{T_{t4}-T_{t5i}} = \frac{1-\tau_t}{1-\pi_t^{\gamma-1/\gamma}}$$

Polytropic: e_i

$$e_c = \frac{\gamma-1}{\gamma}\left(\frac{\ln \pi_c}{\ln \tau_c}\right)$$

$$e_t = \frac{\gamma}{\gamma-1}\,\frac{\ln \tau_t}{\ln \pi_t} \rightarrow \eta_t = \frac{1-\pi_t^{\left(\frac{\gamma-1}{\gamma}\right)e_t}}{1-\pi_t^{\left(\frac{\gamma-1}{\gamma}\right)}}$$

Nozzle:

$$\eta_N = \frac{V_9^2/2}{V_{9s}^2/2}$$

$$\pi_n = \frac{P_{t9}}{P_{t7}}$$

Inlet / Diffuser:

$$\frac{P_{t2}}{P_{t0}} = \pi_d = \pi_{d,max}\,\eta_r \rightarrow \eta_r = \begin{cases} 1 & M_0 \le 1 \\ 1-0.075(M_0-1)^{1.35} & 1<M_0<5 \\ \dfrac{800}{M_0^4+935} & M_0>5 \end{cases}$$

$$\eta_d = \frac{\tau_r(\pi_d)^{\frac{\gamma-1}{\gamma}}-1}{\tau_r - 1}$$

$$\pi_d = \left[\frac{1+\frac{\gamma-1}{2}\eta_d M_0^2}{1+\frac{\gamma-1}{2}M_0^2}\right]^{\gamma/\gamma-1}$$

Burner:

$$\eta_b = \frac{(\dot m_{air}+\dot m_f)C_{p4}T_{t4} - \dot m_{air}C_{p3}T_{t3}}{\dot m_f h_{PR}} \rightarrow \frac{\dot m_f}{\dot m_0} = f = \frac{C_{p4}T_{t4}-C_{p3}T_{t3}}{\eta_b h_{PR} - C_{p4}T_{t4}}$$

Nozzle 5→9

$$\frac{P_{t9}}{P_9} = \pi_n \pi_t \pi_b \pi_c \pi_d \pi_r \frac{P_0}{P_9} \quad (\frac{P_0}{P_9} \rightarrow \text{given})$$

Work Balance:

$\dot W_{out,t} = \dot W_{in,c}$

$$\eta_m(\dot m_f+\dot m_c)C_{pt}(T_{t4}-T_{t5}) = C_{pc}\dot m_c(T_{t3}-T_{t2})$$

$$\eta_m C_{pt}(f+1)(T_{t4}-T_{t5}) = C_{pc}(T_{t3}-T_{t2})$$

Thrust:

$$F = \dot m_9 V_9 - \dot m_c V_0 + \overbrace{A_9(P_9-P_0)}^{F_{press,C}} + \dot m_B(V_{19}-V_0) + \overbrace{A_{19}(P_{19}-P_0)}^{F_{press,B}}$$

$$\frac{F}{\dot m_0} = \frac{1}{1+\alpha}\left[(1+f)V_9 - V_0 + \left(1-\frac{P_0}{P_9}\right)\frac{(1+f)V_9}{\gamma_t M_9^2} + \alpha(V_{19}-V_0) + \left(1-\frac{P_0}{P_{19}}\right)\frac{\alpha V_{19}}{\gamma_c M_{19}^2}\right]$$

$\dot m_0 = \dot m_B + \dot m_c$

$\dot m_9 = \dot m_c + \dot m_f$

$\frac{\dot m_f}{\dot m_c} = f$

$\frac{\dot m_B}{\dot m_c} = \alpha$

if $\frac{P_{t9}}{P_0} \ge \left[1+\frac{\gamma-1}{2}(1)^2\right]^{\frac{\gamma}{\gamma-1}}$ (1.893) $\rightarrow$ $M_9 = 1$, $P_{t9}/P_9 = 1.893$ "choked", $\frac{P_{t9}}{P_0} = \frac{P_{t9}}{P_9}\cdot\frac{P_9}{P_0}$

if $\frac{P_{t9}}{P_0} < 1.893 \rightarrow P_0 = P_9$ subsonic

Off-Design

$^*\tau_t = \frac{T_{t5}}{T_{t4}} \rightarrow T_{t5}$, then $f, T_{t3}, \tau_c, \pi_c, P_{t3}, P_{t4}, P_{t5}, \frac{P_{t9}}{P_9}, M_9, V_9$ $\quad$ $^*\tau_t = \text{const.}$

$C_{pc}(T_{t3}-T_{t2}) = C_{pt}\eta_m(1+f)(T_{t4}-T_{t5})$ — work balance

and $(1+f)C_{pt}T_{t4} - C_{pc}T_{t3} = f h_{PR}\eta_b$ — burner $\quad \rightarrow f, T_{t3}$

$F/\dot m_0 \rightarrow \dot m_0 \rightarrow F$

☆ (see corrected $\dot m_0$)

Corrected Parameters

R → on-design reference

$M_4 = M_{4R} = 1$, $A_4 = A_{4R}$

$$\dot m_4 = \frac{P_{t4}}{\sqrt{T_{t4}}}A_4\,MFP(M=M_4)$$

$$\dot m_{4R} = \dot m_{2R}(1+f_R)$$

$$P_{t4R} = (\pi_b \pi_c \pi_d \pi_r P_0)_R$$

$$\frac{\dot m_4}{\dot m_{4R}} = \sqrt{\frac{T_{t4R}}{T_{t4}}}\left(\frac{P_{t4}}{P_{t4,R}}\right)$$

$$\dot m_2 = \dot m_0 = \frac{\dot m_4}{(1+f)} \;☆$$

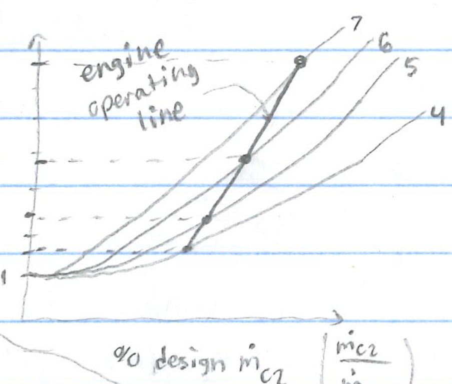
$$(F/\dot m_0)\,\dot m_0 = F$$

'corrected' mass flow

$$\dot m_{c2} = \frac{\dot m_2\sqrt{\theta_2}}{\delta_2}, \quad \theta_2 = \frac{T_{t2}}{T_{SL}}, \quad \delta_2 = P_{t2}/P_{SL}$$

'corrected' blade speed

$$N_c = N/\sqrt{\theta_{in}}$$



$\frac{T_{t4}}{T_{t2}}$ = throttle setting

$\pi_c = 'C'\,\dot m_{c2} \rightarrow$ linear

$$\pi_c = \left\{\left(\frac{T_{t4}}{T_{t2}}\right)[K]+1\right\}^{\gamma/\gamma-1}$$

T_{t4}/T_{t2} limits π_c

$$'K' = \left[\eta_c\eta_m \frac{C_{pt}}{C_{pc}}(1+f)(1-\tau_t)\right]$$

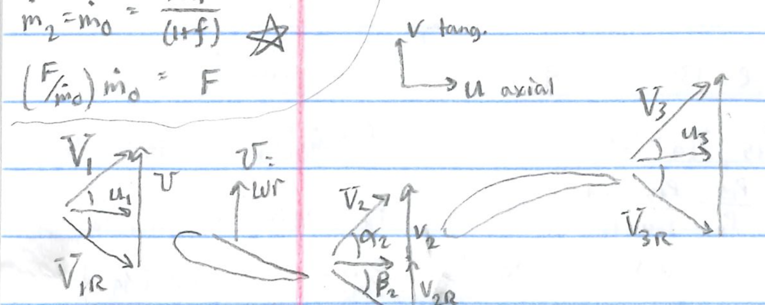
Turbo machinery

$$\dot W_{in,1\to2} = \dot m C_p(T_{t2}-T_{t1}) = \dot m r\omega(v_2 - v_1)$$

$$C_p(T_{t2}-T_{t1}) = (\omega r)^2\frac{u_1}{\omega r}\left(\tan\beta_1 - \frac{u_2}{u_1}\tan\beta_2\right)$$

$$= \upsilon^2\frac{u_1}{\upsilon}\left(\frac{u_2}{u_1}\tan\alpha_2 - \tan\alpha_1\right)$$

$$\left(\frac{P_{t3}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma e_c}} = \left[\frac{\upsilon^2}{C_pT_{t1}}\frac{u_1}{\upsilon}\left(\tan\beta_1 - \frac{u_2}{u_1}\tan\beta_2\right)+1\right]$$



Loss coefficient

$$\phi_{cr} = \frac{P_{t1R}-P_{t2R}}{\frac{1}{2}\rho_1 V_{1R}^2}$$

$$\phi_{cs} = \frac{P_{t2}-P_{t3}}{\frac{1}{2}\rho_2 V_2^2}$$

Diffusion Coefficient

$$D_r = 1 - \frac{V_{2R}}{V_{1R}} + \frac{|v_{1R}-v_{2R}|}{2\sigma V_{1R}}$$

$$D_s = 1 - \frac{V_3}{V_2} + \frac{|v_2 - v_3|}{2\sigma V_2}$$

$\sigma = \frac{c}{s}$ "solidity"

$$°R_c = \frac{T_2-T_1}{T_3-T_1}$$

$D < 0.6$

$^*\,\upsilon = V_{1R} + \upsilon$

$^*\,v_1 + v_{1R} = \upsilon$

$u_1 = V_1\cos\alpha_1 = V_{1R}\cos\beta_1$

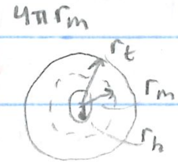
$v_1 = V_1\sin\alpha_1 = \omega r - v_{1R} = \omega r - u_1\tan\beta_1 = u_1\tan\alpha_1$

- $T_2 = T_{t2} - \frac{V_2^2}{2C_p}$
- $T_{t2} = T_{t3}$ (adiabatic)
- $T_1 = T_{1R}$, $P_2 = P_{2R}$ (relative)

$$^*\,\tfrac{1}{2}\rho_1 V_1^2 = \tfrac{1}{2}\gamma P_1 M_1^2$$

Radius:

$$r_t = r_m + \frac{A_i}{4\pi r_m}; \quad r_h = r_m - \frac{A_i}{4\pi r_m}$$



Annulus Area:

$$A_i = \frac{\dot m\sqrt{T_{ti}}}{P_{ti}\cos\alpha_i\,MFP(M_i)}$$