### Statics I Equations

Average Normal Stress in Axial Loading:  $\sigma = \frac{P}{A}$ 

Average Shear Stress:  $\tau = \frac{V}{A}$ 

Stresses on Oblique Plane:  $\sigma = \frac{N}{A_1}, \quad \tau = \frac{V}{A_1}, \quad A_1 = \frac{A}{\cos(\theta)}$ 

Factor of Safey:  $FoS = \frac{Failure Load}{Allowable Load} = \frac{Failure Stress}{Design Stress}$ 

Normal Strain in Axial Loading:  $\varepsilon = \frac{\delta}{L}$ 

Hooke's Law for Axial Loading:  $\sigma = E\varepsilon$ 

Mechanical Deflection for Axial Loading:  $\delta = \frac{PL}{EA}$ 

Thermal Deflection:  $\delta_{\rm th} = \mathcal{O}(\Delta T)L$ 

Total Deflection in Axial Loading:  $\delta = \frac{PL}{EA} + \alpha(\Delta T)L$ 

Poisson's Ratio:  $v = -\frac{\mathcal{E}_y}{\mathcal{E}_x}$ 

Relating elastic properties:  $\frac{E}{2G} = 1 + v$ 

Shear of Block:  $\gamma = \frac{\delta}{h}$   $\tau = G\gamma$ 

Stress Concentration:  $\sigma_{\text{max}} = K\sigma_{\text{nom}}$ 

# Statics II - Chapter 7

Torsion:

$$\tau = \frac{T\rho}{I}$$

$$\theta = \frac{TL}{GJ}$$

$$\tau = \frac{T\rho}{J}$$
  $\theta = \frac{TL}{GJ}$   $\gamma = \frac{\rho\theta}{L}$   $\gamma = \frac{\tau}{G}$ 

2<sup>nd</sup> Moments of Area of a Circle:

$$J = \frac{\pi}{2}c^4$$

2<sup>nd</sup> Moments of Area of a Annulus:  $J = \frac{\pi}{2} \left( c_{\text{outer}}^4 - c_{\text{inner}}^4 \right)$ 

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Gears A and B mesh:

$$r_A \theta_A = r_B \theta_B$$

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$$\frac{T_A}{r_A} = \frac{T_B}{r_B}$$

Stress on Oblique Plane:

$$\sigma_n = \tau_{xy} \sin(2\alpha)$$

$$\tau_{nt} = \tau_{xy} \cos(2\alpha)$$



Power:

$$P = T\omega$$

$$1hp = 550 \text{ ft-lb/s} = 6600 \text{ in-lb/s}$$

# Statics II - Chapter 8

Bending stress in a beam:

$$\sigma = -\frac{My}{I}$$

$$\left|\sigma_{\max}\right| = \left|\frac{Mc}{I}\right| = \left|\frac{M}{S}\right|$$

Shear stress in a beam:

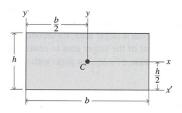
$$\tau = -\frac{VQ}{It}$$

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 where:  $Q = Ad$ 

Parallel axis:

$$I_{r} = I_{r} + Ad^{2}$$

Second moment of area:



$$I_x = \frac{bh^3}{12}$$

$$A = bh$$

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$$I_x = \frac{\pi R^4}{4}$$

$$I_{x'} = \frac{5\pi R^4}{4}$$
$$A = \pi R^2$$

Shear force and bending moment

diagram relations:

$$\frac{dV}{dx} = w$$

$$\frac{dM}{dx} = V$$

a clockwise point moment causes a positive jump in bending moment diagram

# Statics II - Chapter 10

Stress on a plane defined by rotation angle  $\theta$ 

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_{t} = \left(\frac{\sigma_{x} + \sigma_{y}}{2}\right) - \left(\frac{\sigma_{x} - \sigma_{y}}{2}\right) \cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

The state of principal stresses:

$$\sigma_{\text{max,min}} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \quad \text{where:} \quad \tan(2\theta_p) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The state of maximum shear stress:

$$\tau_{m_{\text{max}}} = \sqrt{\left(\frac{\sigma_{x} - \sigma_{y}}{2}\right)^{2} + \left(\tau_{xy}\right)^{2}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \quad \text{where:} \quad \tan(2\theta_{\tau}) = -\frac{(\sigma_{x} - \sigma_{y})}{2\tau_{xy}}$$

Mohr's Circle: points:

$$(\sigma_{x}, -\tau_{xy})$$
 and  $(\sigma_{y}, \tau_{xy})$ 

center:  $\frac{\sigma_x + \sigma_y}{2}$ 

diameter: 
$$\phi = \sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2}$$

Thin-walled cylindrical pressure vessel:  $\sigma_h = \frac{P r}{t}$  and  $\sigma_a = \frac{P r}{2t}$ 

Beam deflection:  $\frac{d^2y}{dx^2} = \frac{M}{EI}$  and  $\frac{dy}{dx} = \theta$ 

Column buckling:  $P_{cr} = \frac{\pi^2 EI}{\left(L_{\text{eff}}\right)^2}$ 

Radius of gyration:  $r = \sqrt{\frac{I}{A}}$ 

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TABLE A-19 Beam Deflections and Slopes

Case	Load and Support (Length $L$ )	Slope at End (+△)	Maximum Deflection (+ upward)	Equation of Elastic Curve (+ upward)
1	$ \begin{array}{c cccc} y & x & P \\ \hline 0 & y_{\text{max}} \\ \theta \end{array} $	$\theta = -\frac{PL^2}{2EI}$ at $x = L$	$y_{\text{max}} = -\frac{PL^3}{3EI}$ at $x = L$	$y = -\frac{Px^2}{6EI}(3L - x)$
2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\theta = -\frac{wL^3}{6EI}$ at $x = L$	$y_{\text{max}} = -\frac{wL^4}{8EI}$ at $x = L$	$y = -\frac{wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
3	$ \begin{array}{c c} w \downarrow & y \\ \hline 0 & x \\ \hline \end{array} $	$\theta = -\frac{wL^3}{24EI}$ at $x = L$	$y_{\text{max}} = -\frac{wL^4}{30EI}$ at $x = L$	$y = -\frac{wx^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^2)$
4	$ \begin{array}{cccc}  & y & M \\  & \theta & \theta \\  & y_{\text{max}} & 0 \end{array} $	$\theta = +\frac{ML}{EI}$ at $x = L$	$y_{\text{max}} = +\frac{ML^2}{2EI}$ at $x = L$	$y = \frac{Mx^2}{2EI}$
5	y $a$ $p$ $b$ $q$	$\theta_1 = -\frac{Pb(L^2 - b^2)}{6LEI}$ at $x = 0$ $\theta_2 = +\frac{Pa(L^2 - a^2)}{6LEI}$	$y_{\text{max}} = -\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3} LEI}$ at $x = \sqrt{(L^2 - b^2)/3}$ $y_{\text{center}} = -\frac{Pb(3L^2 - 4b^2)}{48EI}$	$y = -\frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$y_{\text{max}}$ $b < a$	at $x = L$	not max 48EI	
6	$y$ $L/2$ $P$ $\theta_1$ $\theta_2$ $\theta_2$	$\theta_1 = -\frac{PL^2}{16EI}$ at $x = 0$ $\theta_2 = +\frac{PL^2}{16EI}$ at $x = L$	$y_{\text{max}} = -\frac{PL^3}{48EI}$ at $x = L/2$	$y = -\frac{Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le \frac{L}{2}$
	$\theta_1$ $\psi$	$\theta_1 = -\frac{wL^3}{24EI}$ at $x = 0$ $\theta_2 = +\frac{wL^3}{24EI}$ at $x = L$	$y_{\text{max}} = -\frac{5wL^4}{384EI}$ $\text{at } x = L/2$	$y = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
3	$ \begin{array}{c c} y & M \\ \hline 0 & \uparrow_{\text{max}} \end{array} $	$\theta_1 = -\frac{ML}{6EI}$ at $x = 0$ $\theta_2 = +\frac{ML}{3EI}$ at $x = L$	$y_{\text{max}} = -\frac{ML^2}{9\sqrt{3}EI}$ $\text{at } x = L/\sqrt{3}$ $y_{\text{center}} = -\frac{ML^2}{16EI}$	$y = -\frac{Mx}{6EIL}(L^2 - x^2)$