

Lecture Notes for **Machine Learning in Python**



Professor Eric Larson **Optimization Techniques for Logistic Regression**

Class Logistics and Agenda

- Logistics: grading, guest lecture next week
- Agenda
 - Finish Logistic Regression
 - Numerical Optimization Techniques
 - Types of Optimization
 - Programming the Optimization
- **Whirlwind Lecture Alert**
 - Get an intuition, program it, maybe you don't follow every mathematical concept in lecture
 - But you know how to approach it outside lecture

Class Overview, by topic

Table Data
Visualization

Numpy, Pandas, Seaborn
Overviews with some in-depth discussion

Dimension
Reduction and
Image Processing

Scikit-learn, Scikit Image,
Intuition only, Some mathematics

Linear and
Logistic
Regression

Numpy, Recreate API for Scikit-learn
Detailed mathematics for simple optimization
intuition for advanced optimization

Neural Networks
and Back Prop.

Numpy
Detailed mathematics for NN operations

Wide and Deep
Networks

Convolutional
Networks

Recurrent
Networks

Keras, Tensorflow
Intuition, Detailed implement.

Ethics in
Language Models

ConceptNet
Case studies

Review

Objective Function:

$$l(\mathbf{w}) = \sum_i y^{(i)} \ln(g(\mathbf{x}^{(i)} \cdot \mathbf{w})) + (1 - y^{(i)}) \ln(1 - g(\mathbf{x}^{(i)} \cdot \mathbf{w}))$$

$$\underbrace{w_j}_{\text{new value}} \leftarrow \underbrace{w_j}_{\text{old value}} + \underbrace{\frac{\eta}{M} \sum_{i=1}^M (y^{(i)} - g(\mathbf{x}^{(i)} \cdot \mathbf{w})) x_j^{(i)}}_{\text{step} \quad \text{gradient}}$$

now \mathbf{x} , \mathbf{w} are vectors

$$\underbrace{\mathbf{w}}_{\text{new vect}} \leftarrow \underbrace{\mathbf{w}}_{\text{old vect}} + \frac{\eta}{M} \sum_{i=1}^M (y^{(i)} - g(\mathbf{x}^{(i)} \cdot \mathbf{w})) \cdot \mathbf{x}^{(i)}$$

weighted sum of \mathbf{x}

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \cdot \text{mean}(\mathbf{y}_{diff} \odot \mathbf{X})_{columns}$$

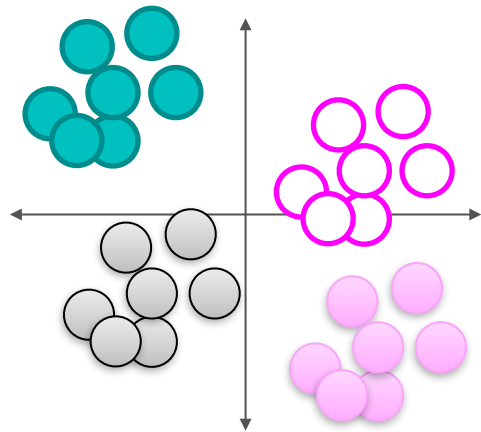
$$\mathbf{y}_{diff} = \begin{bmatrix} y^{(1)} - g(\mathbf{x}^{(1)} \cdot \mathbf{w}) \\ y^{(2)} - g(\mathbf{x}^{(2)} \cdot \mathbf{w}) \\ \vdots \\ y^{(M)} - g(\mathbf{x}^{(M)} \cdot \mathbf{w}) \end{bmatrix} = \begin{bmatrix} y_{diff}^{(1)} \\ y_{diff}^{(2)} \\ \vdots \\ y_{diff}^{(M)} \end{bmatrix}$$

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \cdot \text{mean} \left(\begin{bmatrix} y_{diff}^{(1)} \\ y_{diff}^{(2)} \\ \vdots \\ y_{diff}^{(M)} \end{bmatrix} \odot \begin{bmatrix} \leftarrow \mathbf{x}^{(1)} \rightarrow \\ \leftarrow \mathbf{x}^{(2)} \rightarrow \\ \vdots \\ \leftarrow \mathbf{x}^{(M)} \rightarrow \end{bmatrix} \right)_{\text{mean along each column}}$$

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \cdot \text{mean} \left(\begin{bmatrix} (\leftarrow \mathbf{x}^{(1)} \rightarrow) \cdot y_{diff}^{(1)} \\ (\leftarrow \mathbf{x}^{(2)} \rightarrow) \cdot y_{diff}^{(2)} \\ \vdots \\ (\leftarrow \mathbf{x}^{(M)} \rightarrow) \cdot y_{diff}^{(M)} \end{bmatrix} \right)_{\text{mean along each column}}$$








Review: One Versus Rest

Multi-class formulation



classes[   ]

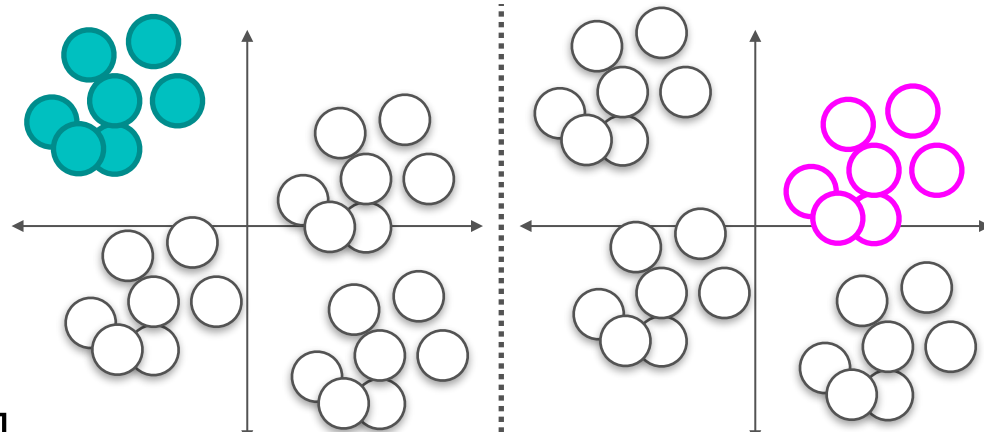
model probabilities

test data (rows)	0.91	0.0	0.78	0.53	→	0	→	
	0.53	0.5	0.32	0.91	→	3	→	
	0.01	0.6	0.98	0.01	→	2	→	
	0.05	0.83	0.21	0.81	→	1	→	
	0.85	0.02	0.33	0.02	→	0	→	
	0.63	0.11	0.09	0.33	→	0	→	
	0.01	0.32	0.12	0.88	→	3	→	
	M1	M2	M3	M4				

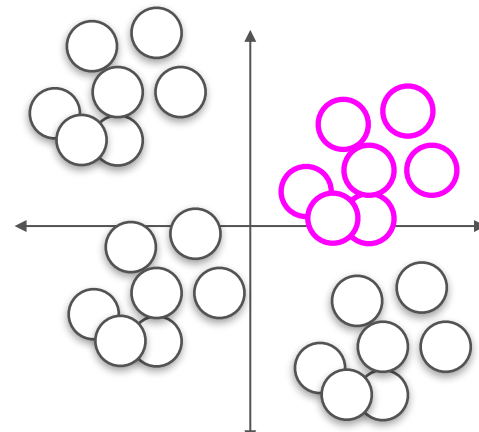
arg max

hash

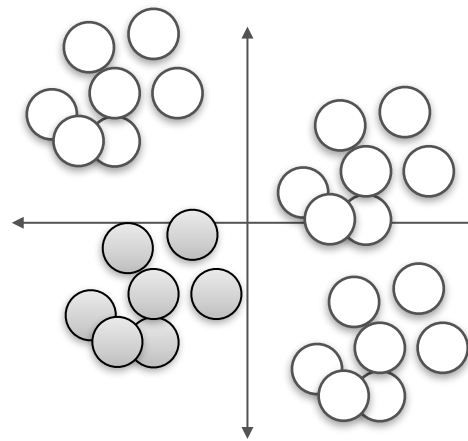
Binary formulations



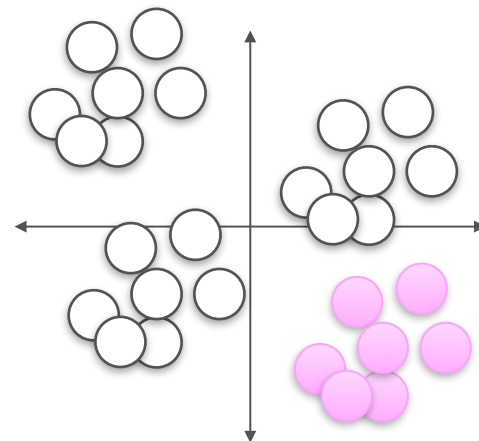
Model 1



Model 2

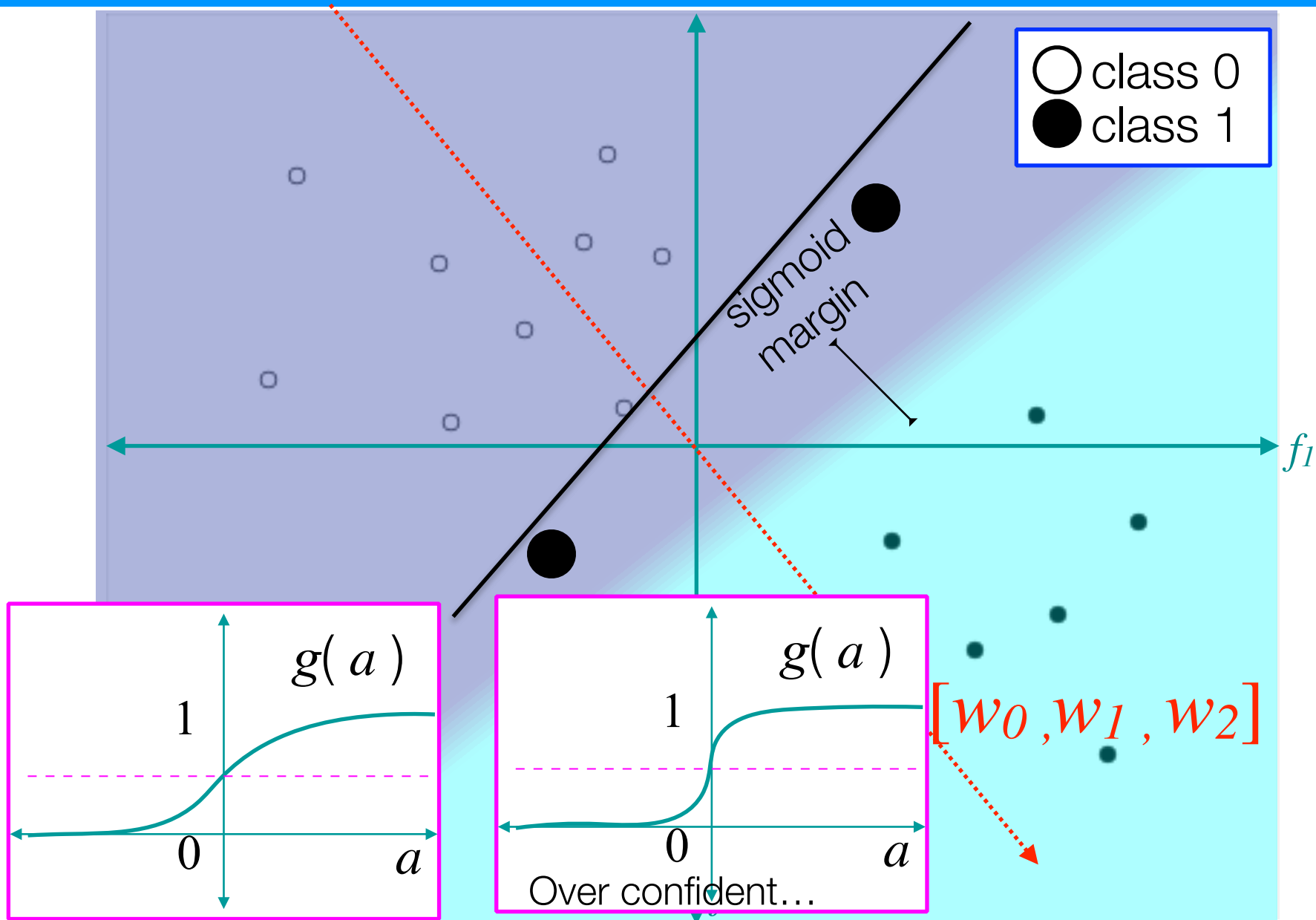


Model 3



Model 4

Regularization, Intuition



05. Logistic Regression.ipynb

“Finish”

Programming

Vectorization

Regularization

Multi-class extension



Demo Lecture

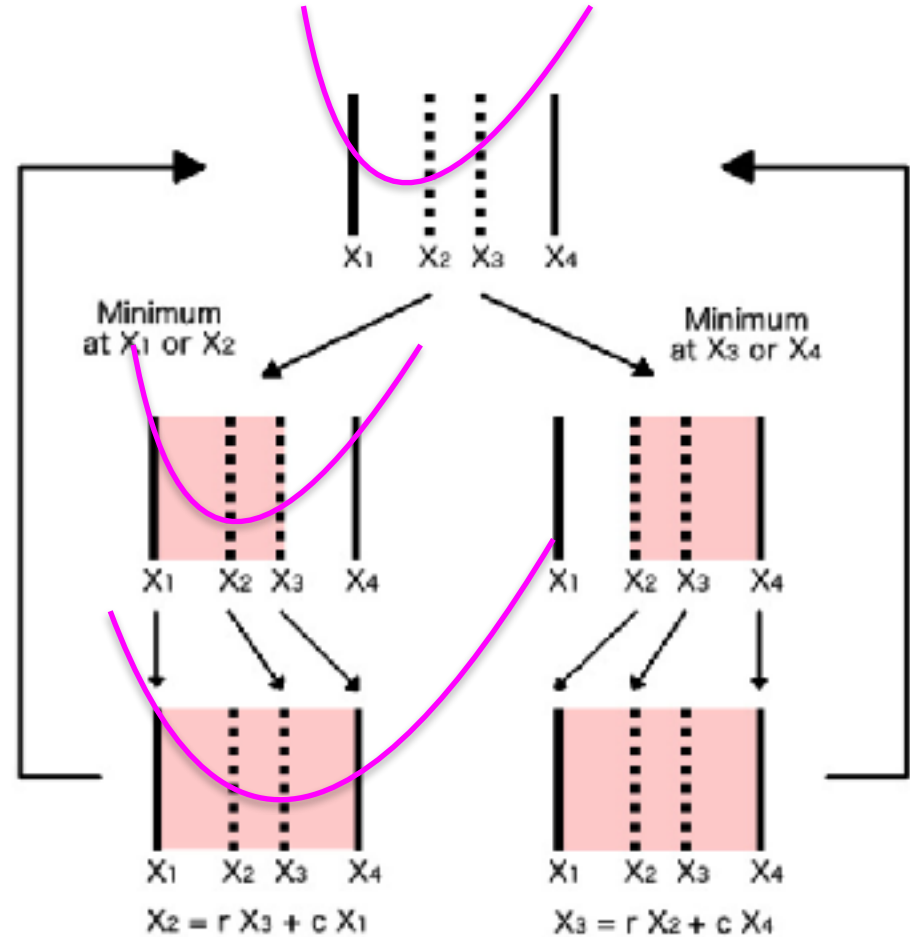
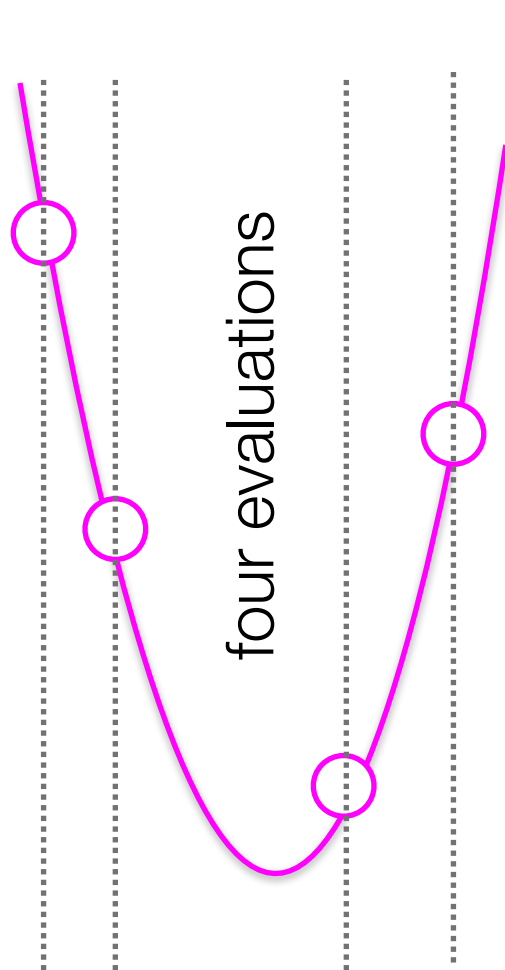
06. Optimization

Line Optimization

Intuition for the Hessian



Line Search: Intuition



How to optimally choose values to reduce required objective function evaluations?