Lecture Notes for

Machine Learning in Python

⟦👨‍🏫, 👨‍💻, 🐍, 👨‍🔬⟧

Professor Eric Larson

Neural Network Optimization and Activation

Class Logistics and Agenda

- Logistics
  - grading update
  - flipped model next time!
- Agenda:
  - More optimization techniques and programming examples
    - Momentum
    - Initialization
    - More activations: Tanh, ReLU, SiLU
    - Adaptive learning: AdaGrad, AdaM, etc.

Class Overview, by topic

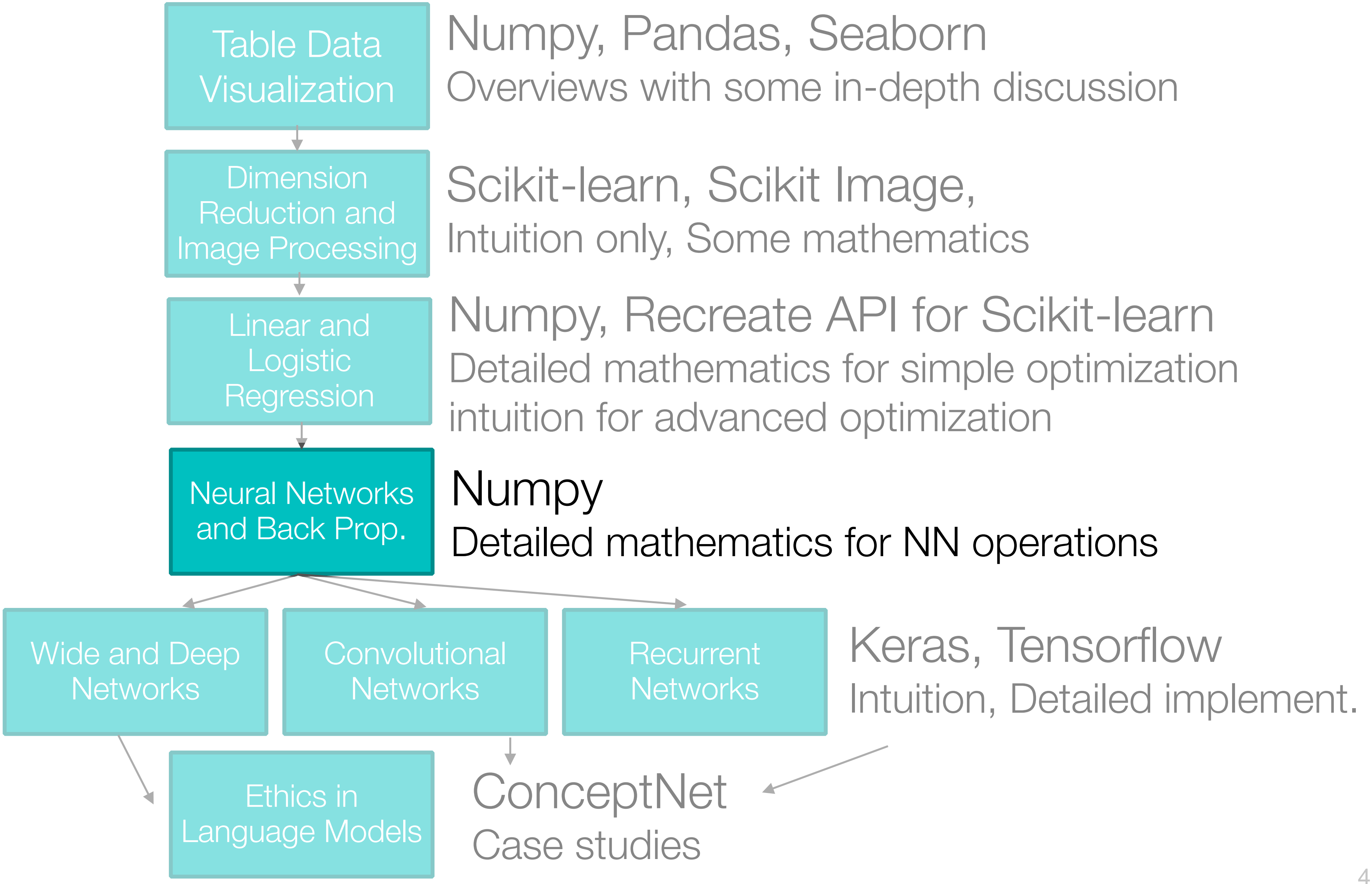


Table Data Visualization

Numpy, Pandas, Seaborn

Overviews with some in-depth discussion

Dimension Reduction and Image Processing

Scikit-learn, Scikit Image,

Intuition only, Some mathematics

Linear and Logistic Regression

Numpy, Recreate API for Scikit-learn

Detailed mathematics for simple optimization intuition for advanced optimization

Neural Networks and Back Prop.

Numpy

Detailed mathematics for NN operations

Wide and Deep Networks

Convolutional Networks

Recurrent Networks

Keras, Tensorflow

Intuition, Detailed implement.

Ethics in Language Models

ConceptNet

Case studies

Demo

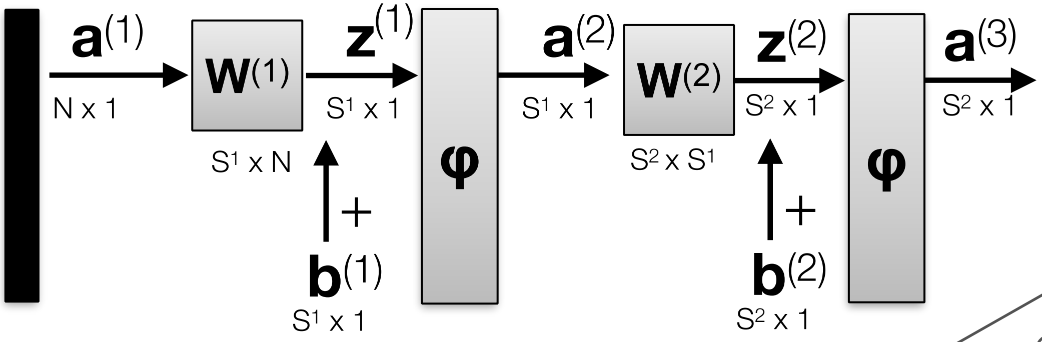
```
08a. Practical_NeuralNetsWithBias.ipynb
```

Quick Review:
Momentum
Cooling

Objective Function

My model is 10% better than our baseline.
AMAZING! That's with a metric based on how real people use the system, not just accuracy, right?
Not just accuracy, right?
imgflip.com

Changing the Objective Function



$$\mathbf{V}^{(2)} = -2(\mathbf{Y} - \mathbf{A}^{(3)}) * \mathbf{A}^{(3)} * (1 - \mathbf{A}^{(3)})$$

$$\nabla^{(2)} = \mathbf{V}^{(2)} \cdot [\mathbf{A}^{(2)}]^T$$

$$\mathbf{V}^{(1)} = \mathbf{A}^{(2)} * (1 - \mathbf{A}^{(2)}) * [\mathbf{W}^{(2)}]^T \cdot \mathbf{V}^{(2)}$$

$$\nabla^{(1)} = \mathbf{V}^{(1)} \cdot [\mathbf{A}^{(1)}]^T$$

1. Forward propagate to get **Z**, **A**
2. Get final layer gradient
3. Back propagate sensitivities
4. Update each $\mathbf{W}^{(l)}$ and $\mathbf{b}^{(l)}$

$$\mathbf{W}^{(l)} \leftarrow \mathbf{W}^{(l)} - \eta\nabla^{(l)}$$

- **Self Test**:

  True or False: If we change the cost function, $J(\mathbf{W})$, we only need to update the final layer sensitivity calculation, $\mathbf{V}^{(2)}$, of the back propagation steps. The remainder of the algorithm is unchanged.

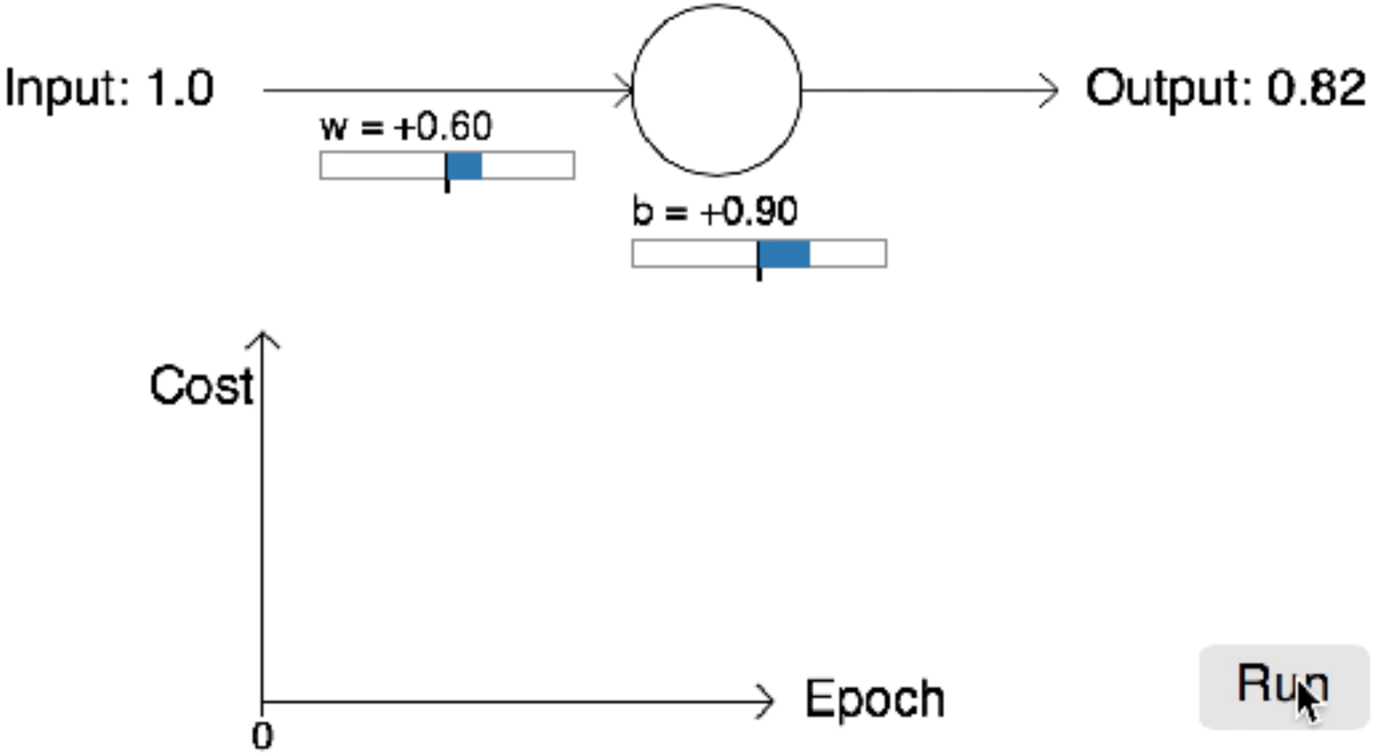
  - A. True
  - B. False

Practical Implementation of Architectures

- Mean squared error:

$$J(\mathbf{W}) = \sum_{k}^{M} (\mathbf{y}^{(k)} - \mathbf{a}^{(L)})^2$$

least squares objective,
tends to slow training initially



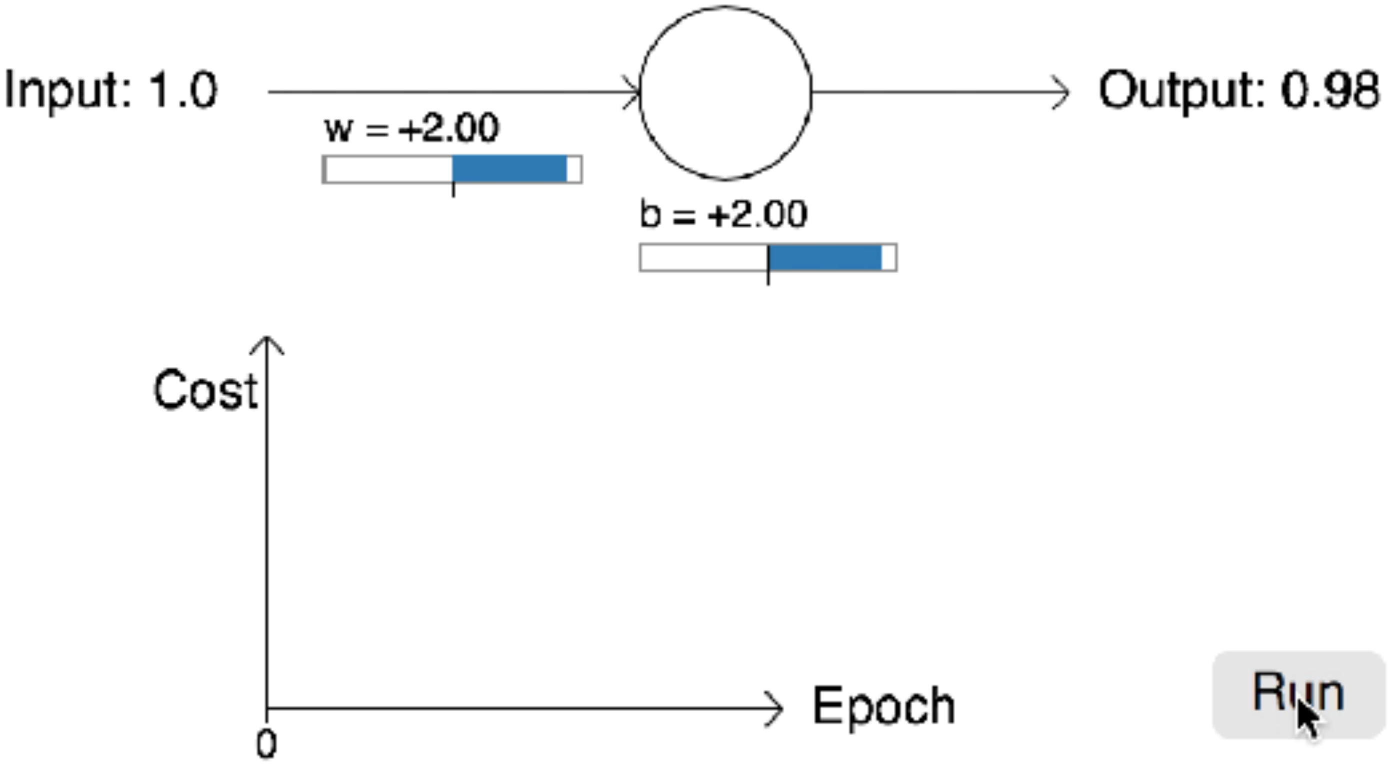
Neural Networks and Deep Learning, Michael Nielson, 2015

Practical Implementation of Architectures

- Mean squared error:

$$J(\mathbf{W}) = \sum_{k}^{M} (\mathbf{y}^{(k)} - \mathbf{a}^{(L)})^2$$

least squares objective,
tends to slow training initially



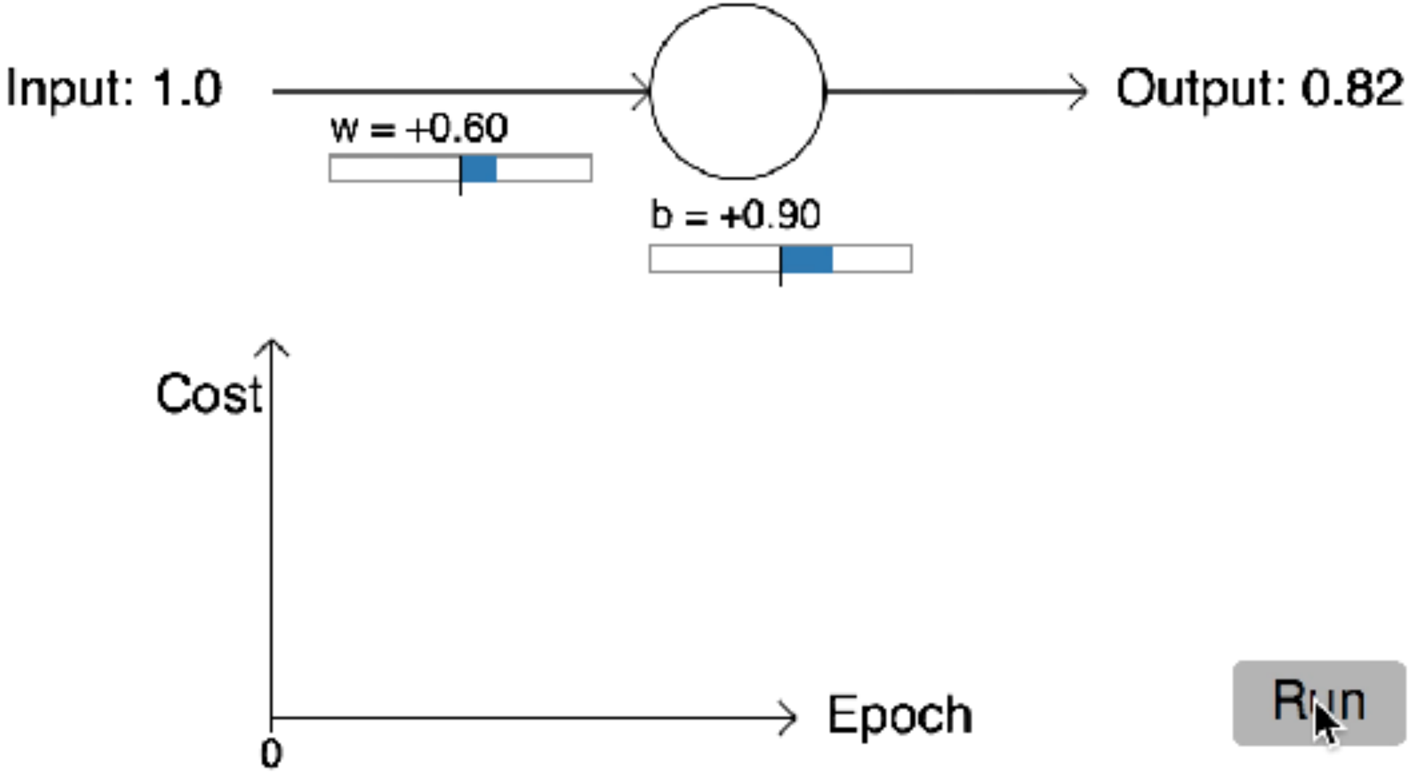
Neural Networks and Deep Learning, Michael Nielson, 2015

Practical Implementation of Architectures

- Our old friend, **Binary Cross entropy**

$$J(\mathbf{W}) = -\left[\mathbf{y}^{(i)}\ln([\mathbf{a}^{(L+1)}]^{(i)}) + (1-\mathbf{y}^{(i)})\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right]$$

speeds up initial training



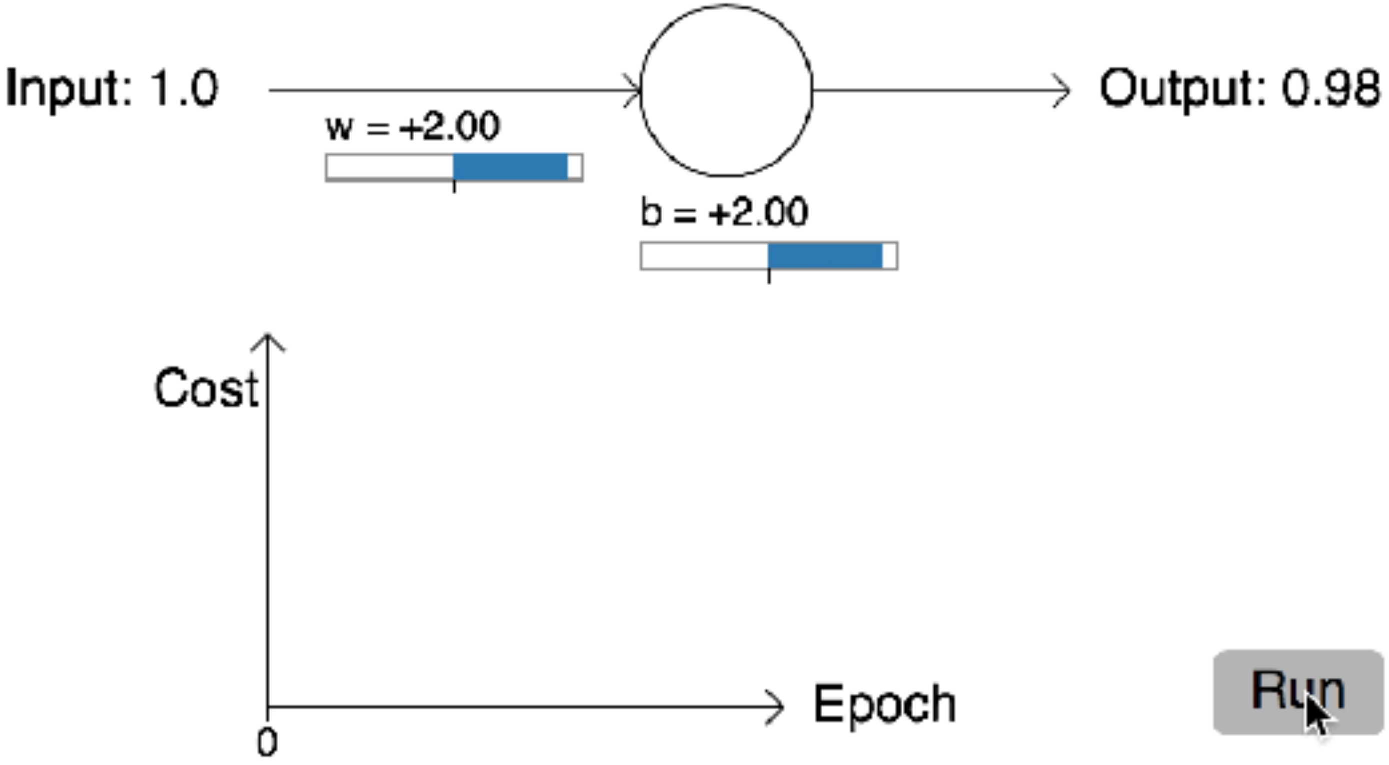
Neural Networks and Deep Learning, Michael Nielson, 2015

Practical Implementation of Architectures

- Negative of MLE: **Binary Cross entropy**

$$J(\mathbf{W}) = -\left[\mathbf{y}^{(i)}\ln([\mathbf{a}^{(L+1)}]^{(i)}) + (1-\mathbf{y}^{(i)})\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right]$$

speeds up initial training



Neural Networks and Deep Learning, Michael Nielson, 2015

Practical Implementation of Architectures

$$J(\mathbf{W}) = -\left[\mathbf{y}^{(i)}\ln([\mathbf{a}^{(L+1)}]^{(i)}) + (1-\mathbf{y}^{(i)})\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right]$$

likely to speed up initial training

$$\left[\frac{\partial J(\mathbf{W})}{\partial \mathbf{z}^{(L)}}\right]^{(i)} = -\frac{\partial}{\partial \mathbf{z}^{(L)}}\left[\mathbf{y}^{(i)}\ln([\mathbf{a}^{(L+1)}]^{(i)}) + (1-\mathbf{y}^{(i)})\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right]$$

only **a** has dependence on **z**

$$= -\left[\mathbf{y}^{(i)}\frac{\partial}{\partial \mathbf{z}^{(L)}}\left(\ln([\mathbf{a}^{(L+1)}]^{(i)})\right) + (1-\mathbf{y}^{(i)})\frac{\partial}{\partial \mathbf{z}^{(L)}}\left(\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right)\right]$$

$$= -\left[\mathbf{y}^{(i)}\frac{1}{[\mathbf{a}^{(L+1)}]^{(i)}}\left(\frac{\partial}{\partial \mathbf{z}^{(L)}}[\mathbf{a}^{(L+1)}]^{(i)}\right) + \frac{(1-\mathbf{y}^{(i)})}{1-[\mathbf{a}^{(L+1)}]^{(i)}}\left(-\frac{\partial}{\partial \mathbf{z}^{(L)}}[\mathbf{a}^{(L+1)}]^{(i)}\right)\right]$$

$$= -\left[\mathbf{y}^{(i)}\frac{1}{\cancel{[\mathbf{a}^{(L+1)}]^{(i)}}}\left(\cancel{[\mathbf{a}^{(L+1)}]^{(i)}}(1-[\mathbf{a}^{(L+1)}]^{(i)})\right) - \frac{(1-\mathbf{y}^{(i)})}{\cancel{1-[\mathbf{a}^{(L+1)}]^{(i)}}}\left([\mathbf{a}^{(L+1)}]^{(i)}\cancel{(1-[\mathbf{a}^{(L+1)}]^{(i)})}\right)\right]$$

$$= -\left[\mathbf{y}^{(i)}\left(1-[\mathbf{a}^{(L+1)}]^{(i)}\right) - (1-\mathbf{y}^{(i)})\left([\mathbf{a}^{(L+1)}]^{(i)}\right)\right]$$

$$= -\left[\mathbf{y}^{(i)} - \cancel{\mathbf{y}^{(i)}[\mathbf{a}^{(L+1)}]^{(i)}} - [\mathbf{a}^{(L+1)}]^{(i)} + \cancel{[\mathbf{a}^{(L+1)}]^{(i)}\mathbf{y}^{(i)}}\right] \qquad = [\mathbf{a}^{(L+1)}]^{(i)} - \mathbf{y}^{(i)}$$

$$\mathbf{V}^{(2)} = -2(\mathbf{Y}-\mathbf{A}^{(3)}) \odot \mathbf{A}^{(3)} \odot (1-\mathbf{A}^{(3)})$$ old update

Practical Implementation of Architectures

- Back to our old friend: **Cross entropy**

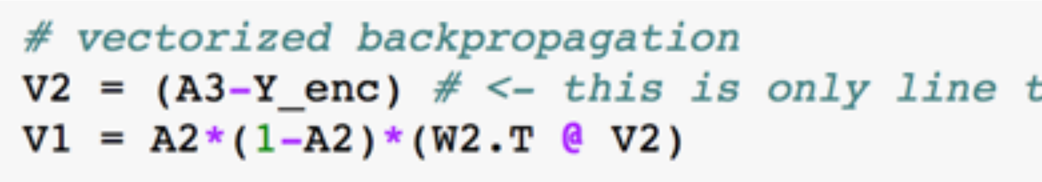
$$J(\mathbf{W}) = -\left[\mathbf{y}^{(i)}\ln([\mathbf{a}^{(L+1)}]^{(i)}) + (1-\mathbf{y}^{(i)})\ln(1-[\mathbf{a}^{(L+1)}]^{(i)})\right]$$

likely to speed up initial training

$$\left[\frac{\partial J(\mathbf{W})}{\partial \mathbf{z}^{(L)}}\right]^{(i)} = [\mathbf{a}^{(L+1)}]^{(i)} - \mathbf{y}^{(i)}$$

$$\left[\frac{\partial J(\mathbf{W})}{\partial \mathbf{z}^{(2)}}\right]^{(i)} = [\mathbf{a}^{(3)}]^{(i)} - \mathbf{y}^{(i)}$$

two layer network



```
# vectorized backpropagation
V2 = (A3-Y_enc) # <- this is only line t
V1 = A2*(1-A2)*(W2.T @ V2)
```

$$\mathbf{V}^{(2)} = \mathbf{A}^{(3)} - \mathbf{Y}$$

new update

bp-5

$$\mathbf{V}^{(2)} = -2(\mathbf{Y} - \mathbf{A}^{(3)}) \odot \mathbf{A}^{(3)} \odot (1 - \mathbf{A}^{(3)})$$ old update

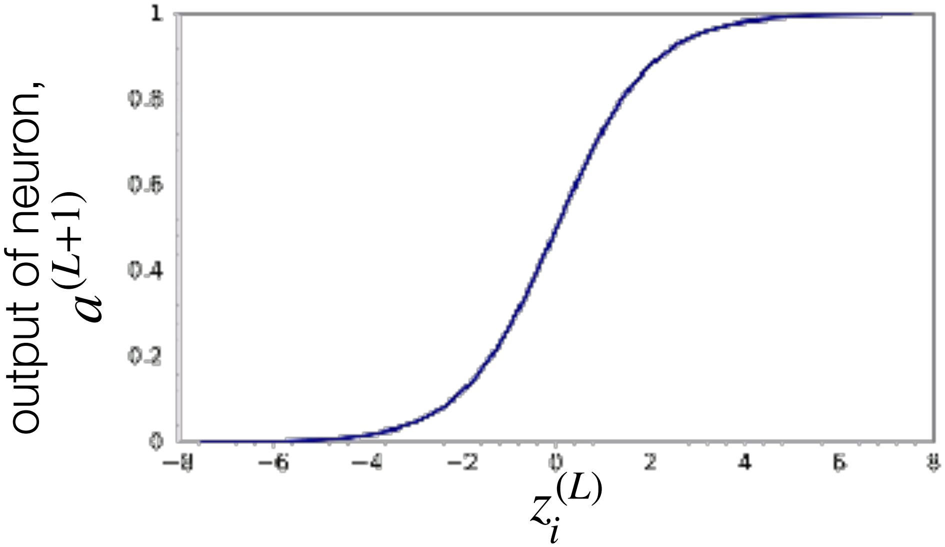
Practical Initialization of Architectures

Formative Self Test

- for adding Gaussian random variables, variances add together

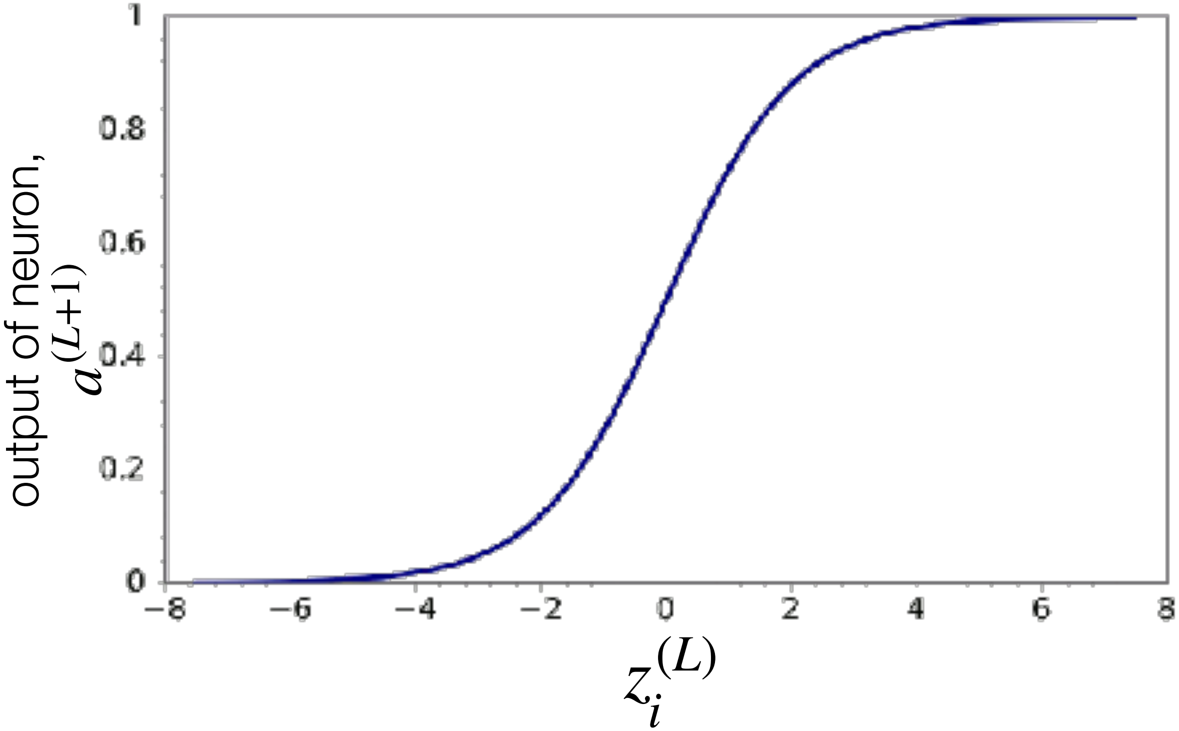
$\mathbf{a}^{(L+1)}=\phi(\mathbf{W}^{(L)}\mathbf{a}^{(L)})$ assume each element of $\mathbf{a}^{(L)}$ is Gaussian

- If you initialized the weights, $\mathbf{W}$, with too large variance, you would expect the output of the neuron, $\mathbf{a}^{(L+1)}$, to be:
  - A. saturated to “1”
  - B. saturated to “0”
  - C. could either be saturated to “0” or “1”
  - D. would not be saturated



Formative Self Test

- What is the derivative of a saturated sigmoid neuron?
    - A. zero
    - B. one
    - C. $a \times (1 - a)$
    - D. it depends



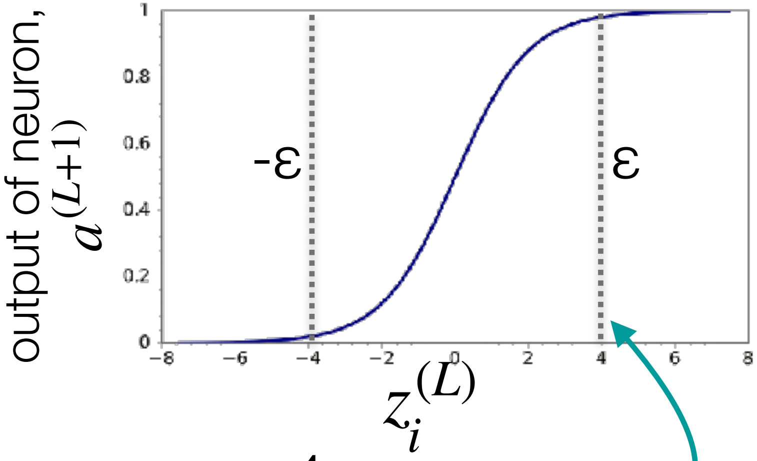
Practical Implementation of Architectures

Weight initialization: try not to saturate your neurons right away!

$$\mathbf{z}^{(L)}=\mathbf{W}^{(L)}\mathbf{a}^{(L)} \qquad z_i^{(L)} = \sum_j^{n^{(L)}} w_{ij}a_j^{(L)}$$

each row is summed before sigmoid

$$\mathbf{a}^{(L+1)}=\phi(\mathbf{z}^{(L)})$$



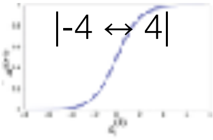
- want each $-\epsilon < z_i^{(L)} < \epsilon$ for no saturation, where $\epsilon \approx 4$
- want Std$[z_i^{(L)}] \approx 4$ which means Std$[a_i^{(L+1)}] \approx 1$
  - because $\sigma(z_i^{(L)}) = a_i^{(L+1)}$, so it will be well distributed from [0,1]
- since $z_i^{(L)} = \sum w_{ij}a_j^{(L)}$, we should squash initial weight magnitudes, w_{ij}, such that Std$[z_i^{(L)}] \approx 4$
- draw each $w_{ij} \sim \mathcal{N}(0,\alpha)$ from a Gaussian with **zero mean** and **specific standard deviation** need to solve for a good α

Glorot Weight Initialization

Xavier Glorot JMLR 2010 Yoshua Bengio
DIRO, Université de Montréal, Montréal, Québec, Canada

Goal: $\text{Std}[z_i^{(L)}] \approx 4$ in order to achieve $\text{Std}[a_i^{(L+1)}] = 1$

Variance of each layer $\text{Var}[z_i^{(L)}] = E\left[(z_i^{(L)} - E[z_i^{(L)}])^2\right] = E[(z_i^{(L)})^2] - E[z_i^{(L)}]^2$



try not to saturate

$$z_i^{(L)} = \sum_j^{n^{(L)}} w_{ij} a_j^{(L)}$$

break down feed forward by multiply in i^{th} row

$$\text{Var}[z_i^{(L)}] = \sum_j^{n^{(L)}} \underbrace{E[w_{ij}]^2\text{Var}[a_j^{(L)}] + \cancel{\text{Var}[w_{ij}]E[a_j^{(L)}]^2}}_{\text{0, if uncorrelated}} + \text{Var}[w_{ij}]\text{Var}[a_j^{(L)}] = \sum_j^{n^{(L)}} \text{Var}[w_{ij}]\text{Var}[a_j^{(L)}]$$

$$\text{Var}[z_i^{(L)}] = n^{(L)}\text{Var}[w_{ij}]\underbrace{\text{Var}[a_j^{(L)}]}_{\approx 1} = n^{(L)}\text{Var}[w_{ij}]$$

$$\boxed{\text{Std}[z_i^{(L)}] = \sqrt{n^{(L)}} \cdot \text{Std}[w_{ij}]}$$

$$\text{Std}[z_i^{(L)}] = 4 = \sqrt{n^{(L)}} \cdot \text{Std}[w_{ij}]$$

$$\text{Std}[w_{ij}] = 4 \cdot \sqrt{\frac{1}{n^{(L)}}}$$

$$w_{ij}^{(L)} \sim \mathcal{N}\left(0,\, 4 \cdot \sqrt{\frac{1}{n^{(L)}}}\right)$$

forward
from sigmoid

Understanding the difficulty of training deep feedforward neural networks

Glorot Weight Initialization

$$\text{Std}[z_i^{(L)}] = 4 = \sqrt{n^{(L)}} \cdot \text{Std}[w_{ij}]$$

$$w_{ij}^{(L)} \sim \mathcal{N}\left(0,\, 4 \cdot \sqrt{\frac{1}{n^{(L)}}}\right)$$

forward

from sigmoid

$$\mathbf{v}^{(L)} = \mathbf{a}^{(L)}(1 - \mathbf{a}^{(L)})\mathbf{W}^{(L)} \cdot \mathbf{v}^{(L+1)}$$

want to keep variance of $\mathbf{v}$ stable magnitude → stable gradient

Similar calculation for back prop.

$$\text{Var}[v_i^{(L)}] = n^{(L+1)}\text{Var}[w_{ij}]\text{Var}[v_j^{(L+1)} \cdot a_j^{(L)}(1 - a_j^{(L)})]$$

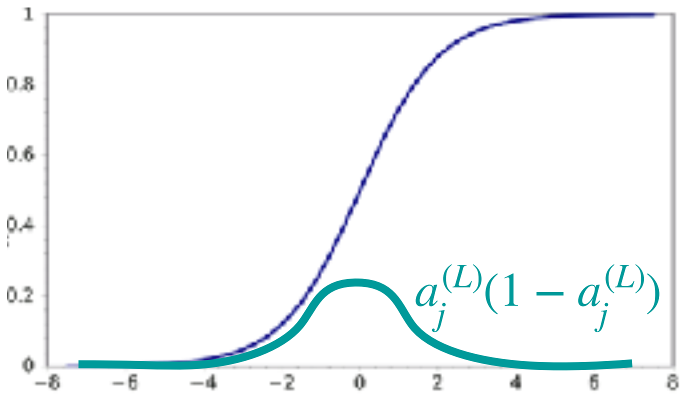
$$\text{Std}[v_i^{(L)}] = \sqrt{n^{(L+1)}} \cdot \text{Std}[w_{ij}] \cdot 0.25$$

want = 1

$$w_{ij}^{(L)} \sim \mathcal{N}\left(0,\, 4 \cdot \sqrt{\frac{1}{n^{(L+1)}}}\right)$$

backward

from sensitivity



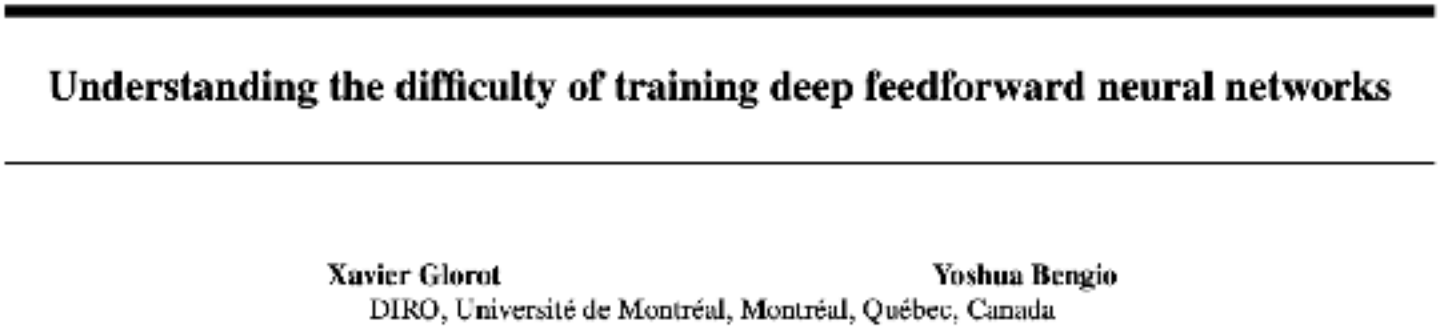
$$w_{ij}^{(L)} \sim \mathcal{N}\left(0,\, 4 \cdot \sqrt{\frac{2}{n^{(L)} + n^{(L+1)}}}\right)$$

compromise

$$w_{ij}^{(L)} \sim U\left[\pm 4\sqrt{\frac{6}{n^{(L)} + n^{(L+1)}}}\right]$$

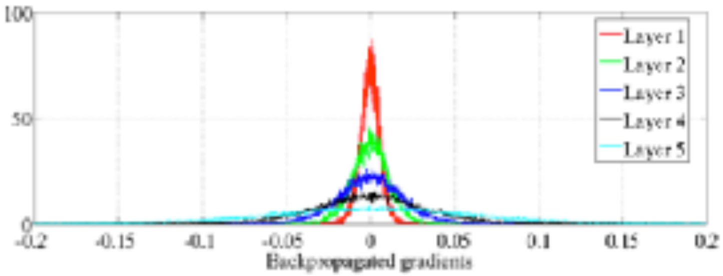
if drawn from uniform dist.

Glorot Weight Initialization



Understanding the difficulty of training deep feedforward neural networks

Xavier Glorot **Yoshua Bengio**
DIRO, Université de Montréal, Montréal, Québec, Canada



Starting gradient histograms per layer
standard initialization

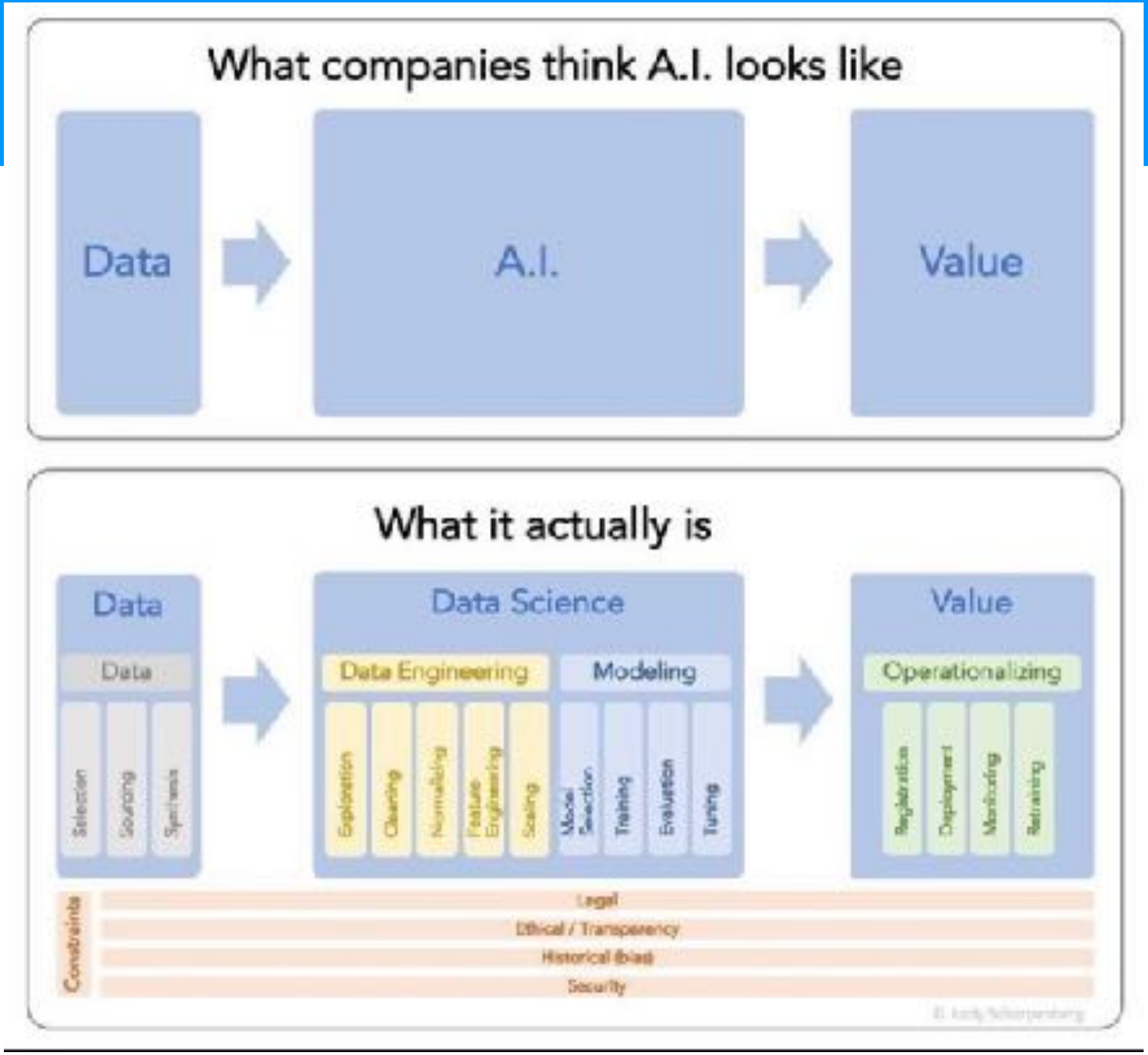
Figure 7: *Back-propagated gradients normalized histograms with hyperbolic tangent activation, with standard (top) vs normalized (bottom) initialization. Top: 0-peak decreases for higher layers.*

Demo

`08a. Practical_NeuralNetsWithBias.ipynb`

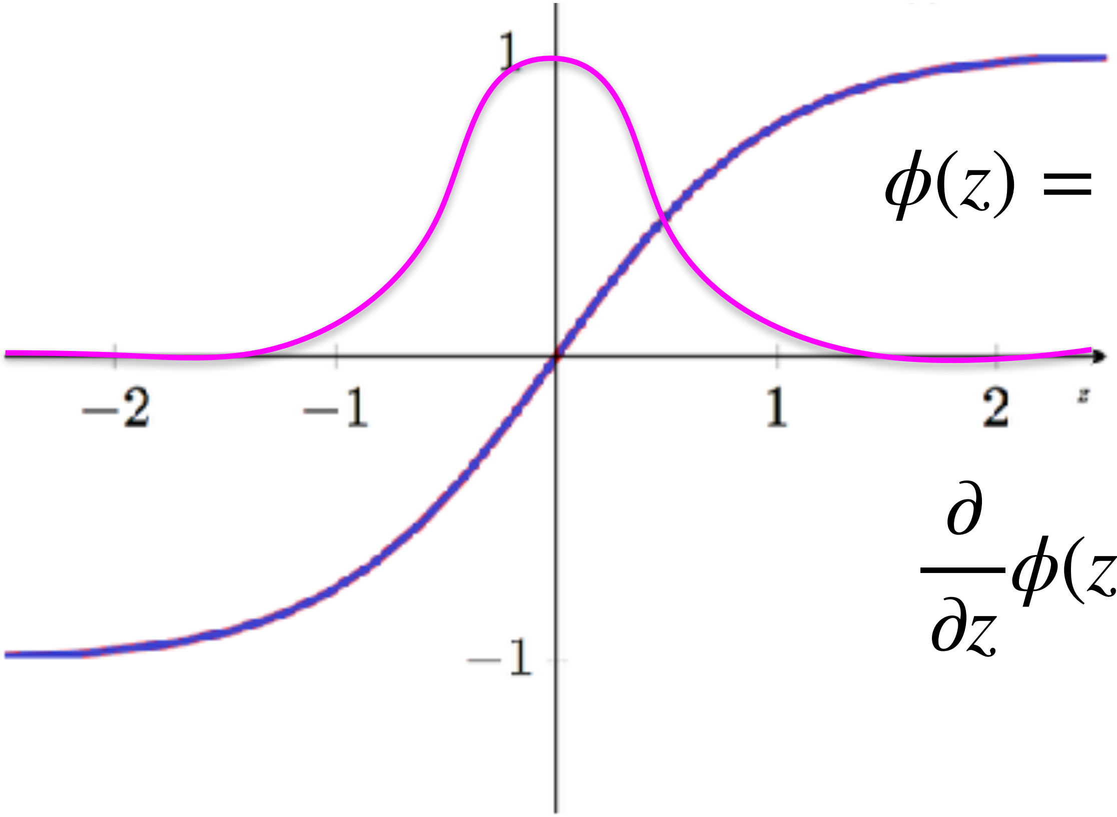
~~Momentum~~
~~Cooling~~
Cross Entropy
Smarter Weight Initialization

Beyond Sigmoid: Other Activations



New Activation: Hyperbolic Tangent

- Basically a sigmoid from -1 to 1

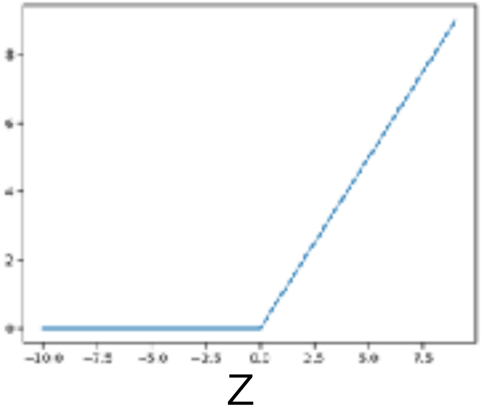


$$\phi(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$\frac{\partial}{\partial z}\phi(z) = \text{sech}^2(z)$$

New Activation: ReLU

- A new nonlinearity: **rectified linear units**



$$\phi(z) = \begin{cases} z, \text{ if } z > 0 \\ 0, \text{ else} \end{cases}$$

it has the advantage of **large gradients** and **extremely simple** derivative

$$\frac{\partial}{\partial z}\phi(z) = \begin{cases} 1, \text{ if } z > 0 \\ 0, \text{ else} \end{cases}$$

Other Activation Functions

- Sigmoid Weighted Linear Unit **SiLU** (also called Swish)
- Mixing of sigmoid, σ, and ReLU

$$\phi(z) = \sigma(z) \cdot z$$

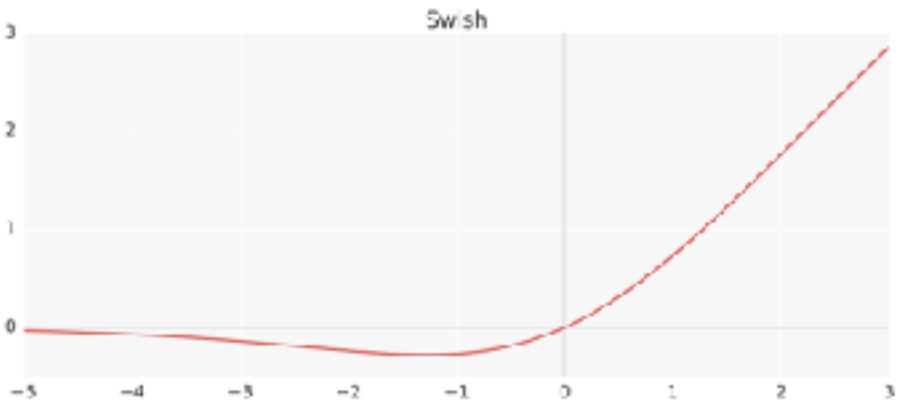


Figure 1: The Swish activation function.

$$\frac{\partial \phi(z)}{\partial z} = \frac{\partial}{\partial z}\sigma(z) \cdot z$$

$$= z \cdot \left[\frac{\partial}{\partial z}\sigma(z)\right] + \sigma(z) \cdot \left[\frac{\partial}{\partial z}z\right]$$

$$= z \cdot \sigma(z)(1 - \sigma(z)) + \sigma(z)$$

$$= z \cdot \sigma(z) + \sigma(z) \cdot (1 - z \cdot \sigma(z))$$

$$= \phi(z) + \sigma(z) \cdot (1 - \phi(z))$$

Elfwing, Stefan, Eiji Uchibe, and Kenji Doya. "Sigmoid-weighted linear units for neural network function approximation in reinforcement learning." Neural Networks (2018).

Ramachandran P, Zoph B, Le QV. Swish: a Self-Gated Activation Function. arXiv preprint arXiv:1710.05941. 2017 Oct 16

Glorot and He Initialization

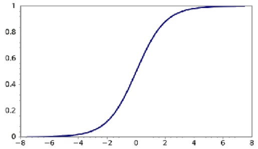
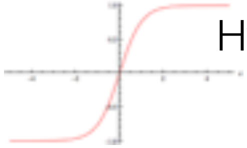
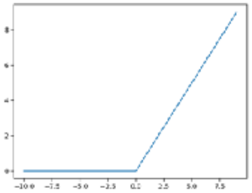
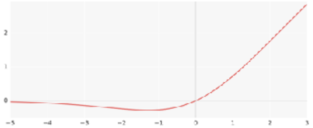
We have solved this assuming the activation output is in the range -4 to 4 (for a sigmoid) and assuming that we use Gaussian for sampling.

This range is different depending on the activation and assuming Gaussian or Uniform sampling.

	Uniform	Gaussian
Tanh	$w_{ij}^{(L)} \sim \sqrt{\frac{6}{n^{(L)} + n^{(L+1)}}}$	$w_{ij}^{(L)} \sim \sqrt{\frac{2}{n^{(L)} + n^{(L+1)}}}$
Sigmoid	$w_{ij}^{(L)} \sim 4\sqrt{\frac{6}{n^{(L)} + n^{(L+1)}}}$	$w_{ij}^{(L)} \sim 4\sqrt{\frac{2}{n^{(L)} + n^{(L+1)}}}$
ReLU SiLU	$w_{ij}^{(L)} \sim \sqrt{2}\sqrt{\frac{6}{n^{(L)} + n^{(L+1)}}}$	$w_{ij}^{(L)} \sim \sqrt{2}\sqrt{\frac{2}{n^{(L)} + n^{(L+1)}}}$

Summarized by Glorot and He

Activations Summary

	Definition	Derivative	**Weight Init** *(Uniform Bounds)*
 Sigmoid	$\phi(z) = \dfrac{1}{1+e^{-z}}$	$\nabla\phi(z) = a(1-a)$	$w_{ij}^{(L)} \sim \pm 4\sqrt{\dfrac{6}{n^{(L)}+n^{(L+1)}}}$
 Hyperbolic Tangent	$\phi(z) = \dfrac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$	$\nabla\phi(z) = \dfrac{4}{(e^{z}+e^{-z})^2}$	$w_{ij}^{(L)} \sim \pm \sqrt{\dfrac{6}{n^{(L)}+n^{(L+1)}}}$
 ReLU	$\phi(z) = \begin{cases} z, \text{ if } z > 0 \\ 0, \text{ else} \end{cases}$	$\nabla\phi(z) = \begin{cases} 1, \text{ if } z > 0 \\ 0, \text{ else} \end{cases}$	$w_{ij}^{(L)} \sim \pm \sqrt{2}\sqrt{\dfrac{6}{n^{(L)}+n^{(L+1)}}}$
 SiLU	$\phi(z) = \dfrac{z}{1+e^{-z}}$	$\nabla\phi(z) = \phi(z) + \sigma(z)\cdot(1-\phi(z))$	

More Adaptive Optimization

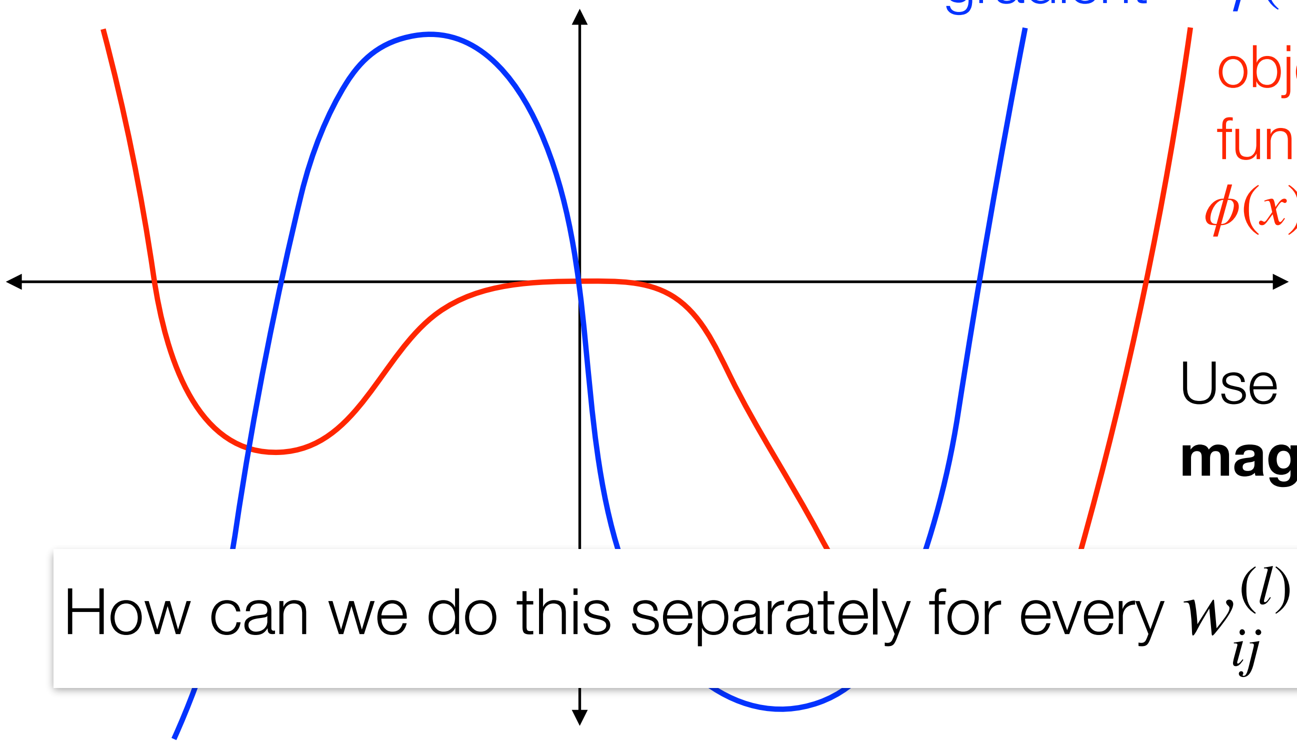
Going beyond
changing the learning rate

Be adaptive based on Gradient Magnitude?

- Decelerate down regions that are steep
- Accelerate on plateaus

gradient $\nabla\phi(x) = 16x^3 + 6x^2 + 24x$

objective function $\phi(x) = 4x^4 + 2x^3 + 12x^2$



Use **inverse** of **magnitude** of **gradient**!

How can we do this separately for every $w_{ij}^{(l)}$ in every $\mathbf{W}^{(l)}$?

Momentum: be robust to **abrupt changes** in **steepness** (accumulate inverse magnitudes)

http://www.technologyuk.net/mathematics/differential-calculus/higher-derivatives.shtml

Be adaptive based on Gradient Magnitude?

Inverse magnitude of gradient in multiple directions?

$$\mathbf{W}_{k+1} \leftarrow \mathbf{W}_k + \eta \frac{1}{\sqrt{\mathbf{G}_k + \epsilon}} \odot \nabla J(\mathbf{W}_k)$$

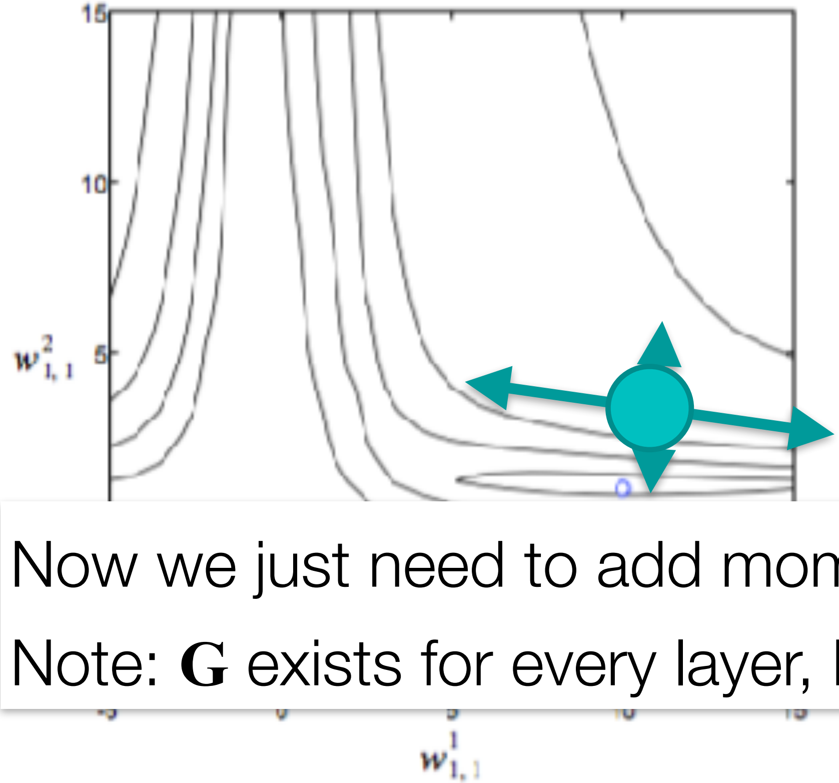
gradient

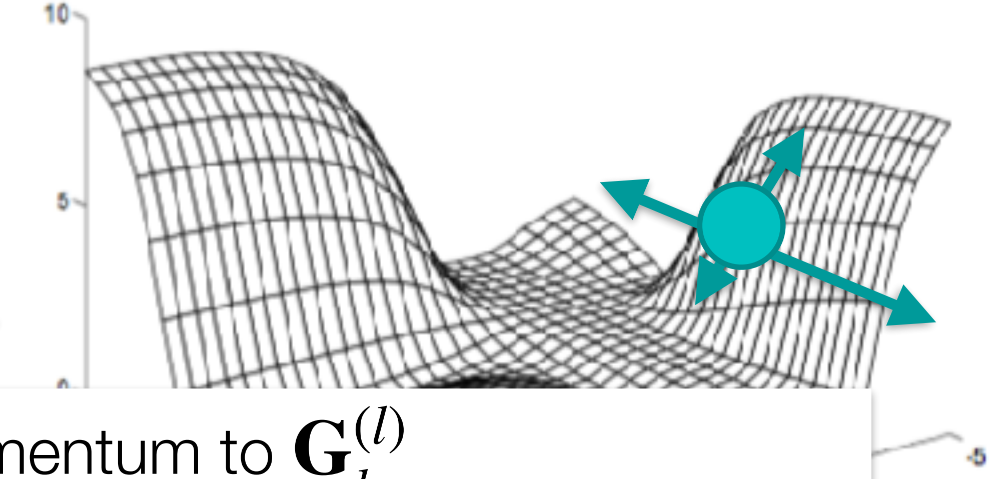
new matrix for normalizing

$$\mathbf{G}_k = \nabla J(\mathbf{W}_k) \odot \nabla J(\mathbf{W}_k)$$

same size as $\mathbf{W}$

```
G = gradW1 * gradW1
W1 += eta*gradW1 / sqrt(G+eps)
```





Now we just need to add momentum to $\mathbf{G}_k^{(l)}$

Note: $\mathbf{G}$ exists for every layer, but we will abuse layer notation

Common Adaptive Strategies $\mathbf{W}_{k+1} = \mathbf{W}_k - \eta \cdot \rho_k$

Adjust each element of gradient by the steepness

- AdaGrad

$$\rho_k = \frac{1}{\sqrt{\mathbf{G}_k + \epsilon}} \odot \nabla J(\mathbf{W}_k)$$

all operations are per element

where

$$\mathbf{G}_k = \gamma \cdot \mathbf{G}_{k-1} + \nabla J(\mathbf{W}_k) \odot \nabla J(\mathbf{W}_k)$$

- RMSProp

$$\rho_k = \frac{1}{\sqrt{\mathbf{V}_k + \epsilon}} \odot \nabla J(\mathbf{W}_k)$$

all operations are per element

$$\mathbf{G}_k = \nabla J(\mathbf{W}_k) \odot \nabla J(\mathbf{W}_k)$$

$$\mathbf{V}_k = \gamma \cdot \mathbf{V}_{k-1} + (1-\gamma) \cdot \mathbf{G}_k$$

- AdaDelta

$$\rho_k = \frac{\mathbf{M}_k}{\sqrt{\mathbf{V}_k + \epsilon}}$$

all operations are per element

$$\mathbf{M}_{k+1} = \gamma \cdot \mathbf{M}_k + (1-\gamma) \cdot \nabla J(\mathbf{W}_k)$$

- AdaM — $\mathbf{G}$ updates with decaying momentum of J and J^2

- NAdaM — same as Adam, but with nesterov's acceleration

None of these are **"one-size-fits-all"** because the space of neural network **optimization varies** by problem, AdaM is **popular** but **not a panacea**

Adaptive Momentum

All operations are element wise:

$$\beta_1 = 0.9,\ \beta_2 = 0.999,\ \eta = 0.001,\ \epsilon = 10^{-8}$$

$$k = 0,\ \mathbf{M}_0 = \mathbf{0},\ \mathbf{V}_0 = \mathbf{0}$$

Published as a conference paper at ICLR 2015

ADAM: A METHOD FOR STOCHASTIC OPTIMIZATION

Diederik P. Kingma*
University of Amsterdam, OpenAI

Jimmy Lei Ba*
University of Toronto

For each epoch:

update iteration $k \leftarrow k + 1$

get gradient $\nabla J(\mathbf{W}_k)$

for large k, $\hat{\mathbf{M}} \approx \mathbf{M},\ \hat{\mathbf{V}} \approx \mathbf{V}$

accumulated gradient $\mathbf{M}_k \leftarrow \beta_1 \cdot \mathbf{M}_{k-1} + (1-\beta_1) \cdot \nabla J(\mathbf{W}_k)$

accumulated squared gradient $\mathbf{V}_k \leftarrow \beta_2 \cdot \mathbf{V}_{k-1} + (1-\beta_2) \cdot \nabla J(\mathbf{W}_k) \odot \nabla J(\mathbf{W}_k)$

boost moments magnitudes
(notice k in exponent)

$$\hat{\mathbf{M}}_k \leftarrow \frac{\mathbf{M}_k}{(1-[\beta_1]^k)} \qquad \hat{\mathbf{V}}_k \leftarrow \frac{\mathbf{V}_k}{(1-[\beta_2]^k)}$$

update gradient, normalized
by second moment
similar to AdaDelta

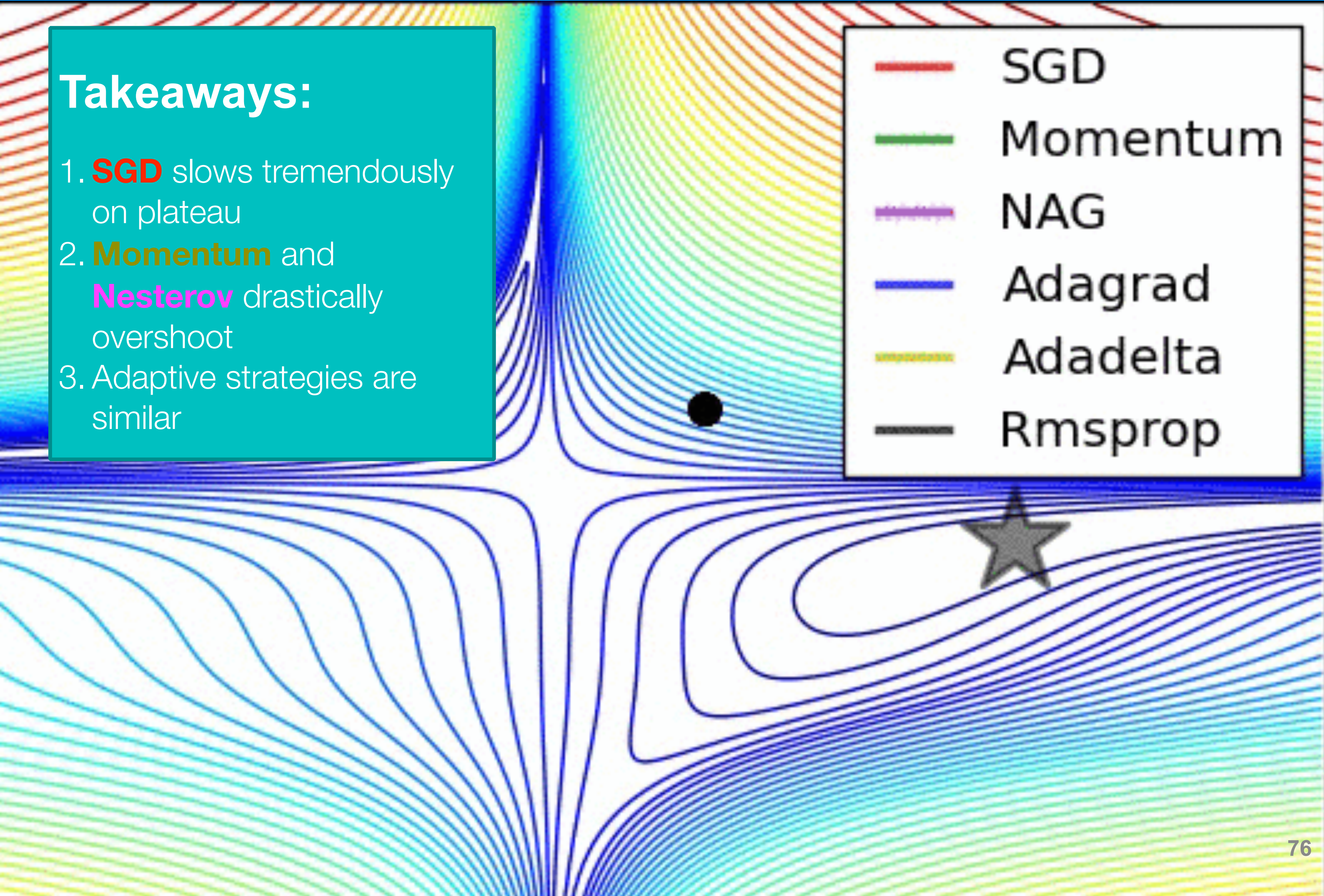
$$\mathbf{W}_k \leftarrow \mathbf{W}_{k-1} - \eta \cdot \frac{\hat{\mathbf{M}}_k}{\sqrt{\hat{\mathbf{V}}_k + \epsilon}}$$

gradient with momentum

squared magnitude normalizer

Visualization of Optimization

https://ruder.io/optimizing-gradient-descent/



Takeaways:

1. **SGD** slows tremendously on plateau
2. **Momentum** and **Nesterov** drastically overshoot
3. Adaptive strategies are similar

Demo

08a. Practical_NeuralNetsWithBias.ipynb

~~Momentum~~

~~Cooling~~

~~Cross Entropy~~

~~Smarter Weight Initialization~~

ReLU Nonlinearities

Adaptive training with AdaGrad