Lecture Notes for

# Machine Learning in Python

⟦👨‍🏫, 👨‍💻, 🐍, 👨‍🔬⟧

Professor Eric Larson

## MLP History

# Class Logistics and Agenda

- Logistics:
  - Grading Update
  - **Next time: Flipped Module on back propagation**
- Multi Week Agenda:
  - Today: Neural Networks History, up to 1980 and Multi-layer Architectures
  - **Flipped**: Programming Multi-layer training
  - Town Hall, Lab 4 (after flipped)

# Class Overview, by topic

**Table Data Visualization**

Numpy, Pandas, Seaborn
Overviews with some in-depth discussion

**Dimension Reduction and Image Processing**

Scikit-learn, Scikit Image,
Intuition only, Some mathematics

**Linear and Logistic Regression**

Numpy, Recreate API for Scikit-learn
Detailed mathematics for simple optimization
intuition for advanced optimization

**Neural Networks and Back Prop.**

Numpy
Detailed mathematics for NN operations

**Wide and Deep Networks** | **Convolutional Networks** | **Recurrent Networks**

Keras, Tensorflow
Intuition, Detailed implement.

**Ethics in Language Models**

ConceptNet
Case studies

# Lab 3, Town Hall (if needed)

**Tyler Rablin** @Mr_Rablin · 2d

You're not grading assignments.

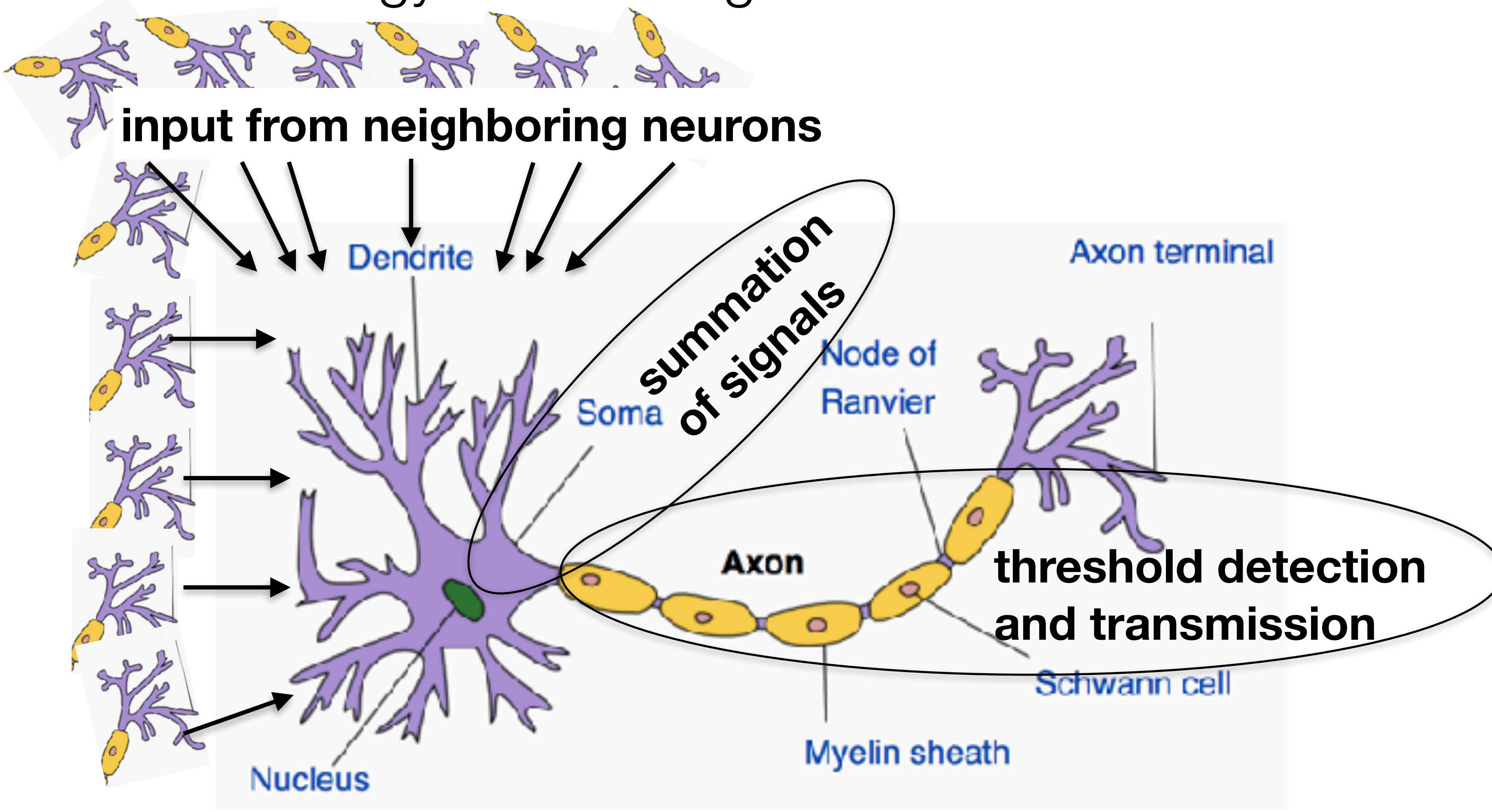
You're collecting evidence to determine student progress and pointing them towards their next steps.

Make the mental switch. It matters.

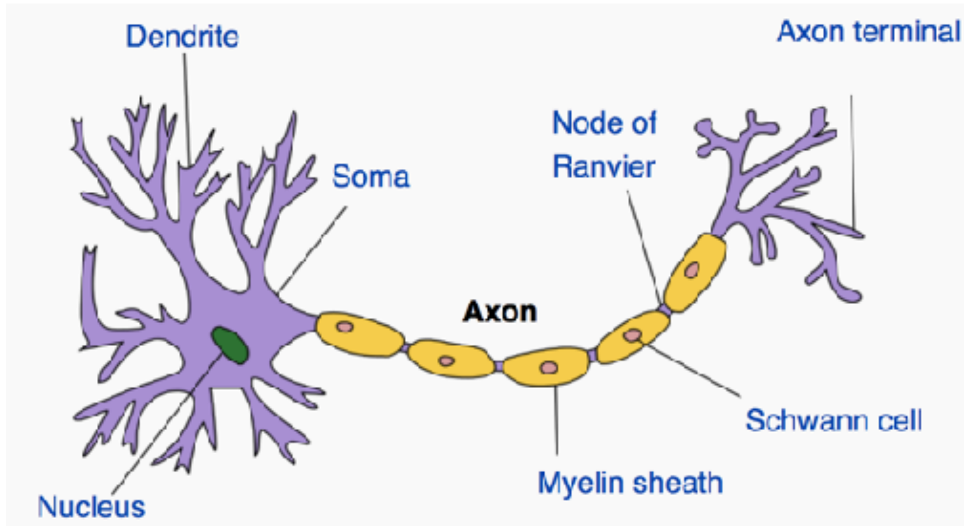
# A History of Neural Networks

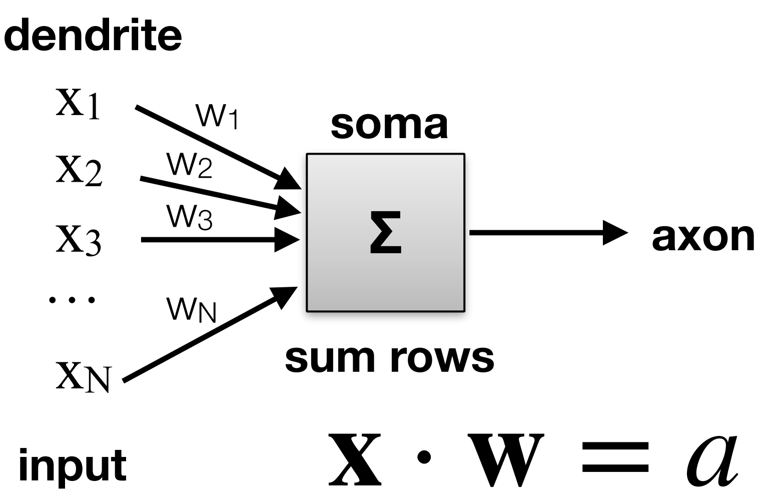
# Neurons

- From biology to modeling:



# McCulloch and Pitts, 1943




$$\mathbf{x} \cdot \mathbf{w} = a$$

for each neuron

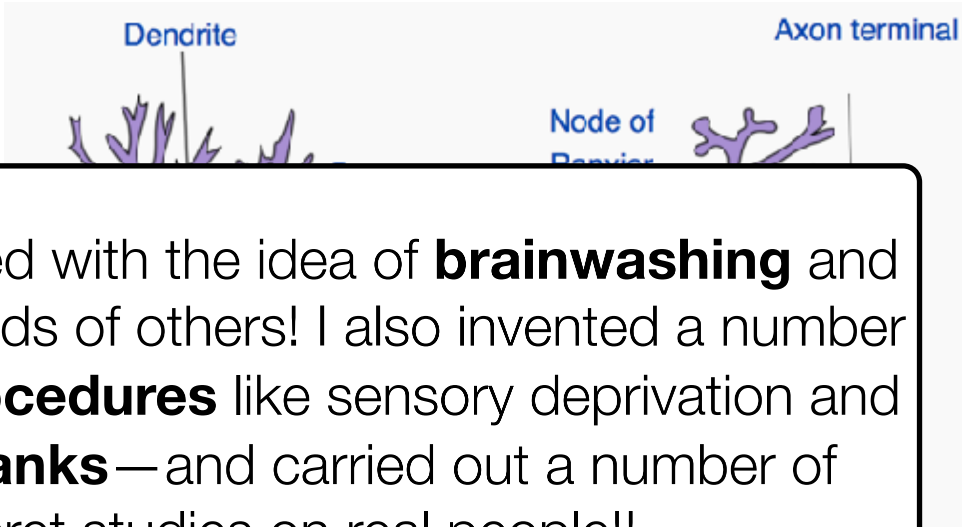
**logic gates of the mind**

Warren McCulloch

Walter Pitts

# Neurons

- McCulloch and Pitts, 1943
- Donald Hebb, 1949
  - Hebb's Law: close neurons fire together
  - neurons "learr
  - easier synaptic
  - basis of neura




I was infatuated with the idea of **brainwashing** and controlling minds of others! I also invented a number of **torture procedures** like sensory deprivation and **isolation tanks**—and carried out a number of secret studies on real people!!

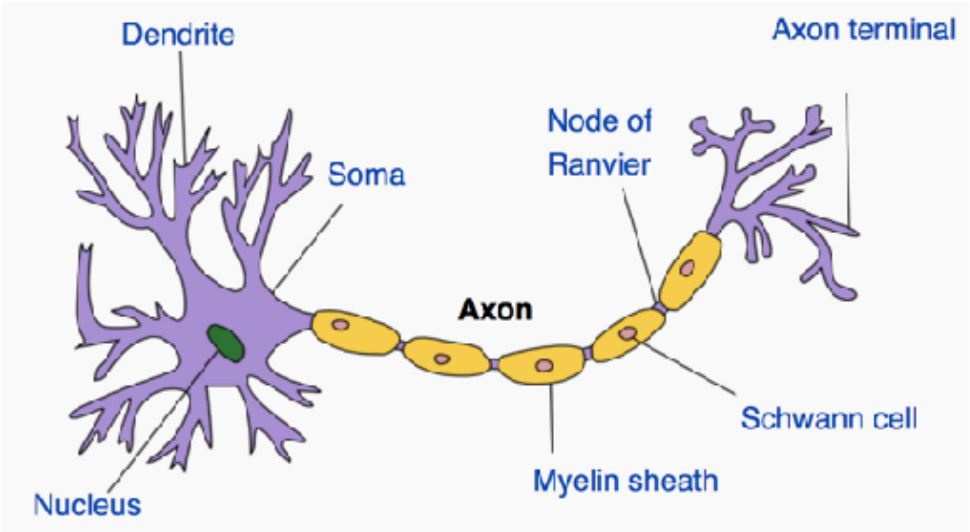
Donald O. Hebb

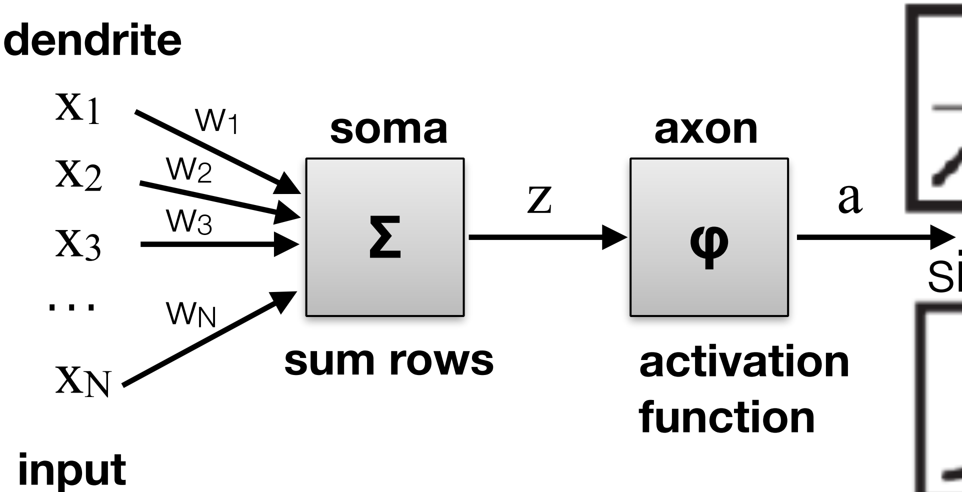
Warren McCulloch

Walter Pitts

# Rosenblatt’s perceptron, 1957


Frank Rosenblatt




hard limit

$a=-1 \quad z<0$

$a=1 \quad z>=0$

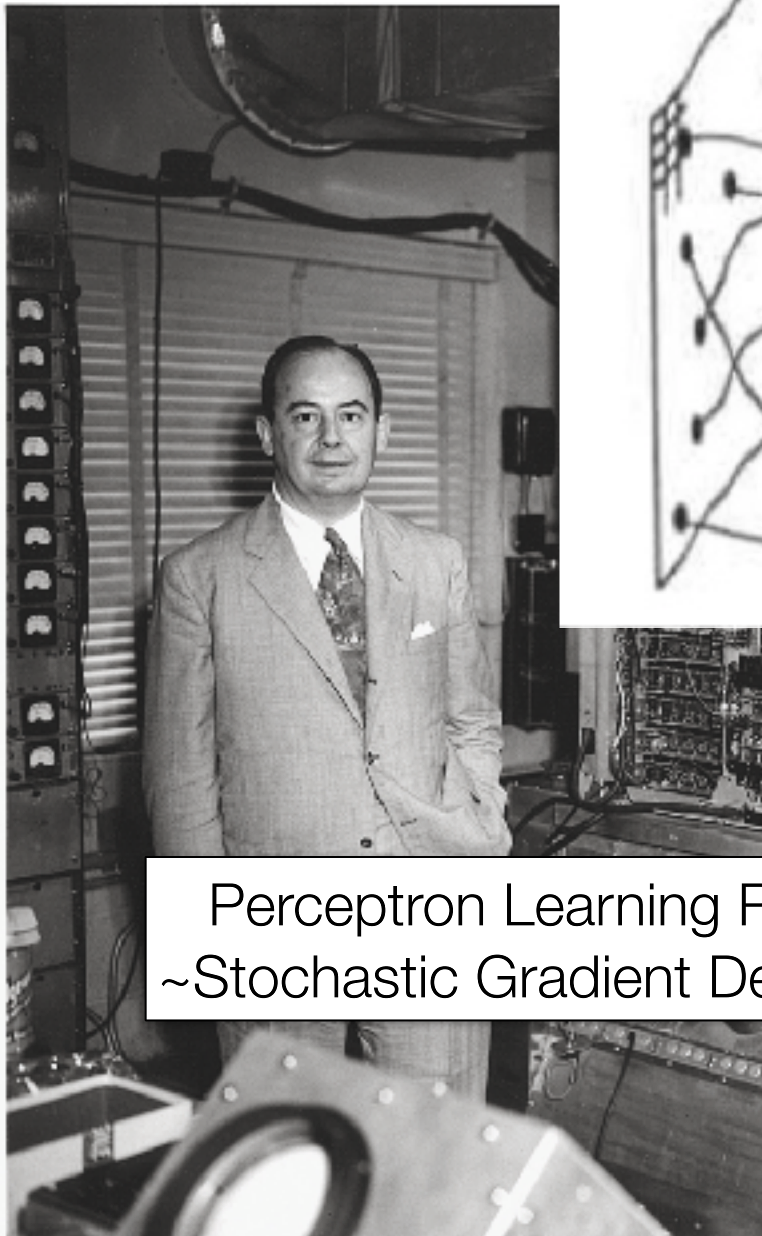
linear

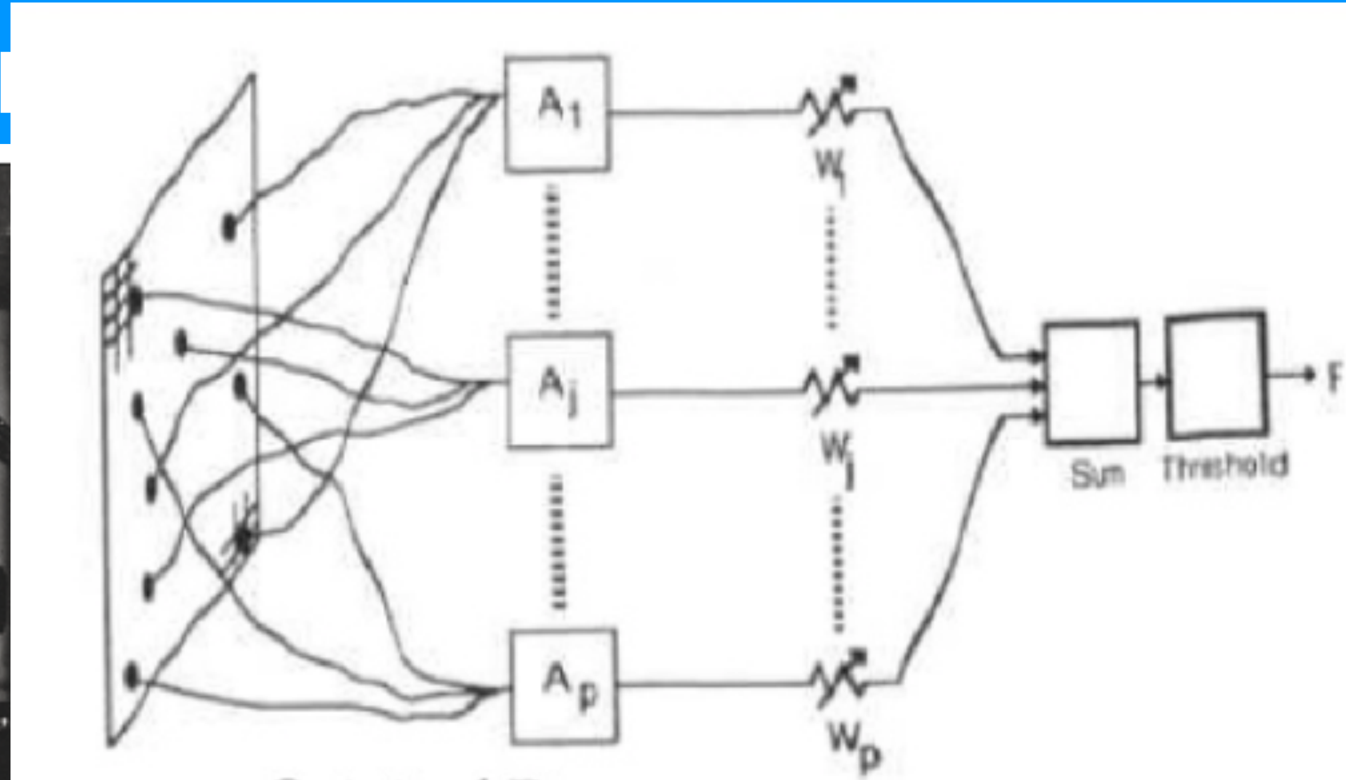
$a=z$

sigmoid

$$a=\frac{1}{1+\exp(-z)}$$

# The Mark 1 P

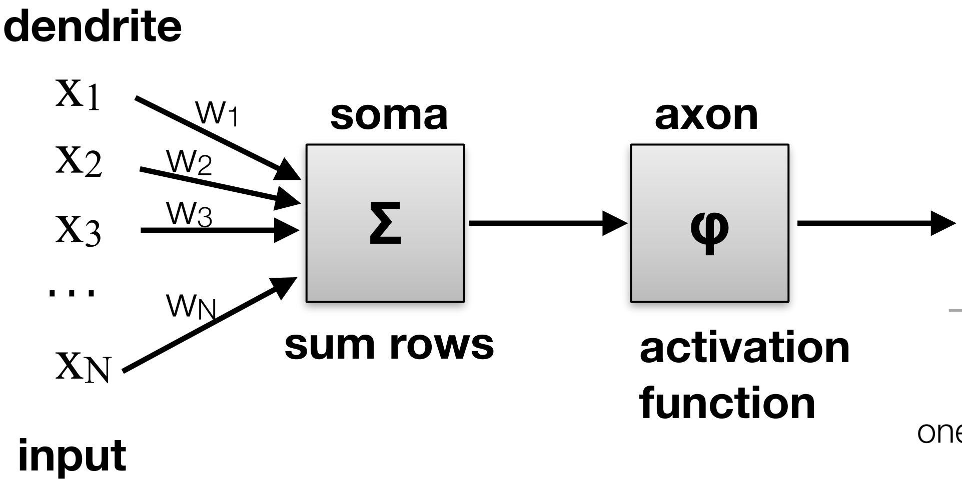





Perceptron Learning Rule:
~Stochastic Gradient Descent

# Layers Notation for Table Data

## One Neuron




**a=x**

$$\mathbf{x}^{(i)} = \begin{bmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_N \end{bmatrix}^{(i)} = [\mathbf{a}^{(1)}]^{(i)}$$

$$\mathbf{z} = \mathbf{W} \cdot \mathbf{x}^{(i)} + \mathbf{b}$$

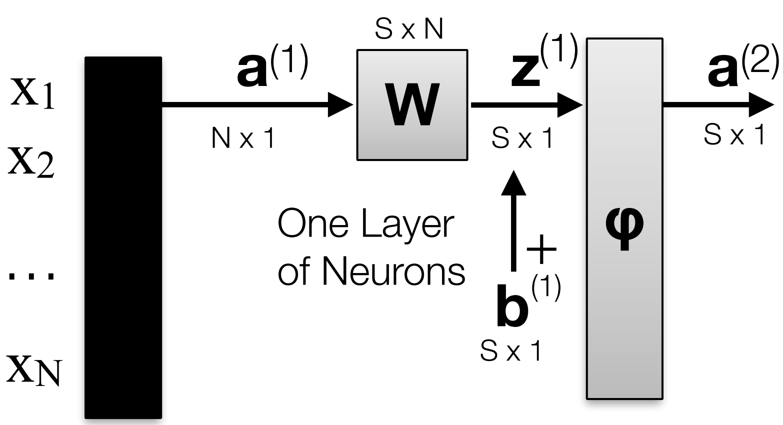
one neuron →

$$\mathbf{W} = \begin{bmatrix} \boxed{w_{1,1} \;\; \dots \;\; w_{1,N}} \\ w_{2,1} \;\; \dots \;\; w_{2,N} \\ \vdots \\ w_{S,1} \;\; \dots \;\; w_{S,N} \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_S \end{bmatrix}$$

$$[\mathbf{z}^{(1)}]^{(i)} = \mathbf{W}^{(1)} \cdot [\mathbf{a}^{(1)}]^{(i)} + \mathbf{b}^{(1)}$$

$$\mathbf{a}^{next} = \phi(\mathbf{z}^{\mathbf{current}})$$

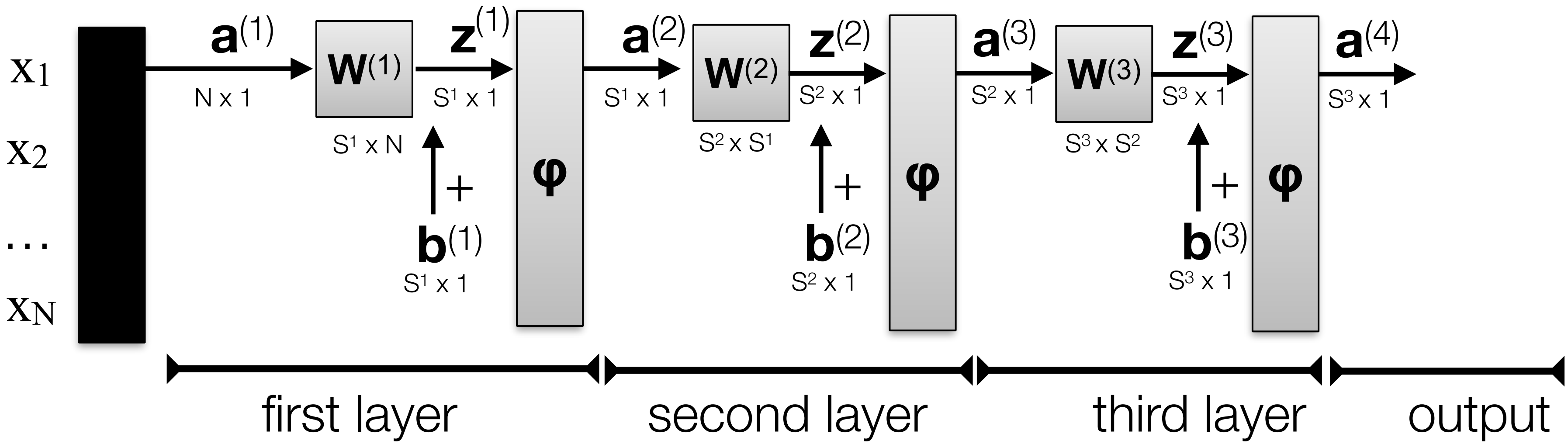
$$\mathbf{a}^{(next)} = \begin{bmatrix} \phi(z_1^{curr}) \\ \vdots \\ \phi(z_N^{curr}) \end{bmatrix} \rightarrow \mathbf{a}^{(L)} = \begin{bmatrix} \phi(z_1^{L-1}) \\ \vdots \\ \phi(z_N^{L-1}) \end{bmatrix}$$




$\mathbf{x}^{(i)}$ One row from Table data becomes input column to model

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

# Generic Multiple Layers Notation




$$\mathbf{a}^{(L+1)} = \phi(\mathbf{z}^{(L)})$$

$\mathbf{a}^{(final)}$ size=unique classes, $C$

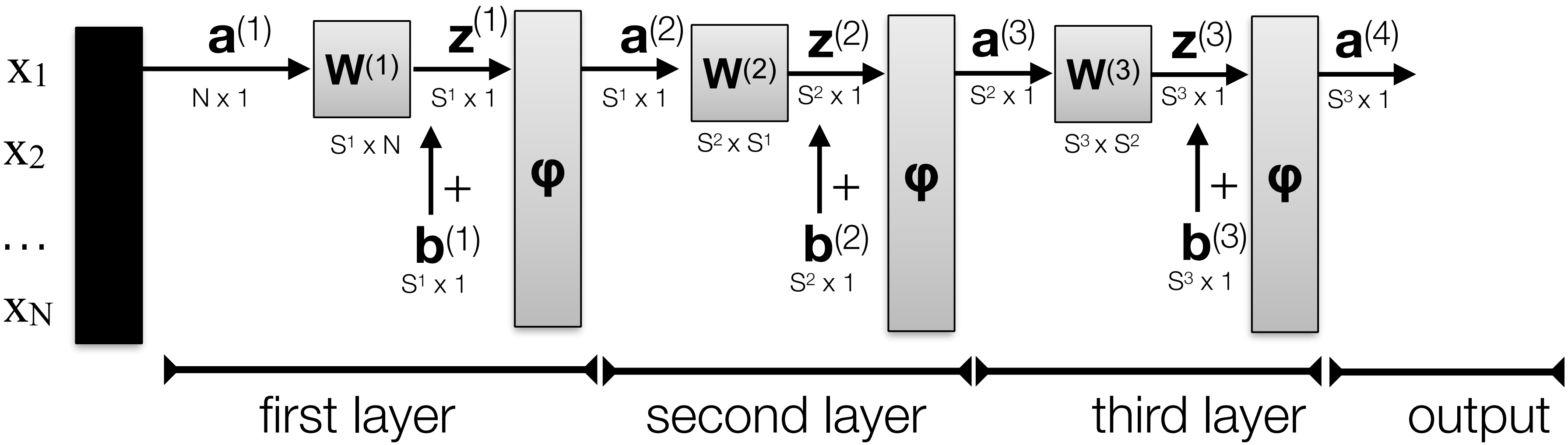
$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \mathbf{a}^{(L)} + \mathbf{b}^{(L)}$$

$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi(\mathbf{z}^{(L-1)}) + \mathbf{b}^{(L)}$$

$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi\left(\mathbf{W}^{(L-1)} \cdot \phi(\mathbf{z}^{(L-2)}) + \mathbf{b}^{(L-1)}\right) + \mathbf{b}^{(L)}$$

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

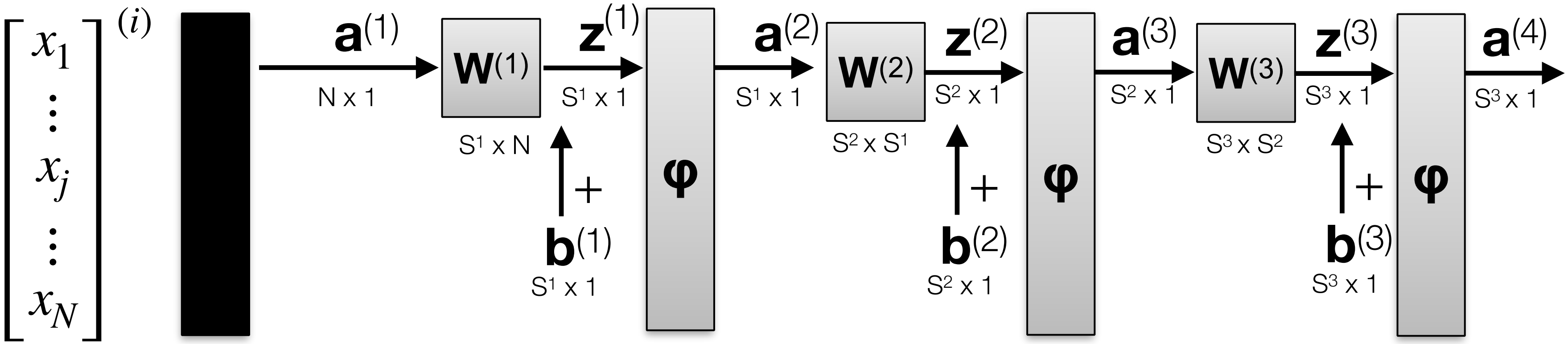
# Multiple layers notation




- **Self test**: How many parameters need to be trained in the above network?
  - A. $[(N+1) \times S^1] + [(S^1+1) \times S^2] + [(S^2+1) \times S^3]$
  - B. $|\mathbf{W}^{(1)}| + |\mathbf{W}^{(2)}| + |\mathbf{W}^{(3)}| + |\mathbf{b}^{(1)}| + |\mathbf{b}^{(2)}| + |\mathbf{b}^{(3)}|$
  - C. can't determine from diagram
  - D. it depends on the sizes of intermediate variables, $\mathbf{z}^{(i)}$

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

# Compact feedforward notation

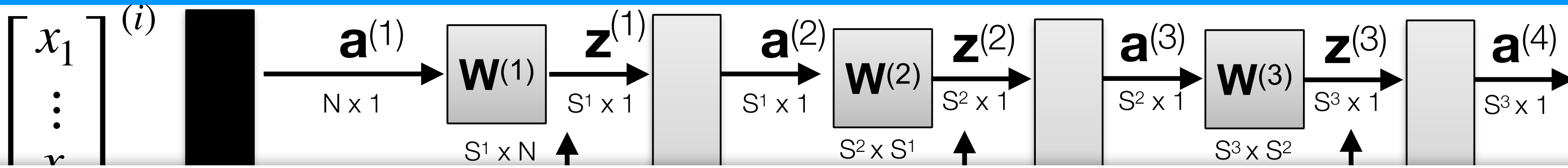



$$\mathbf{X}^T = \left[ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(1)}, \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(2)} \cdots \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(M)} \right] = \left[ \begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(1)}, \begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(M)} \right] = \mathbf{A}^{(1)}$$

Table Data

Table Data, in Neural Net Notation

# Compact feedforward notation




$$\mathbf{Z}^{(L)} = \mathbf{W}^{(L)} \cdot \mathbf{A}^{(L)} + \mathbf{b}^{(L)}$$

$$\mathbf{Z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi(\mathbf{Z}^{(L-1)}) + \mathbf{b}^{(L)}$$

$$\left[\mathbf{z}^{(L)}\right]^{(i)} = \mathbf{W}^{(L)} \cdot \left[\mathbf{a}^{(L)}\right]^{(i)} + \mathbf{b}^{(L)}$$

$$\begin{bmatrix} z_2 \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix} = \mathbf{W}^{(L)} \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix} + \mathbf{b}^{(L)}$$

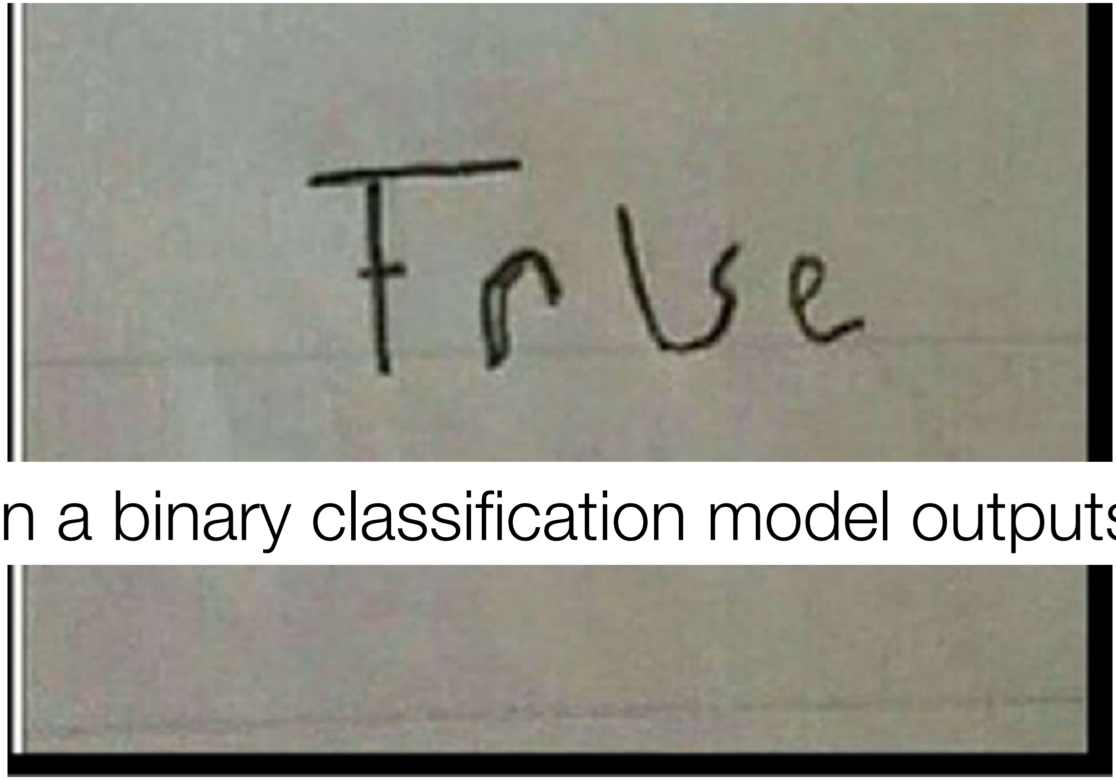
---

$$\left[ \begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(1)}, \begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(M)} \right] = \mathbf{W}^{(L)} \cdot \left[ \begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(1)}, \begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(M)} \right] + \mathbf{b}^{(L)}$$

---

$\mathbf{b}$ is broadcast added

# Training Neural Network Architectures

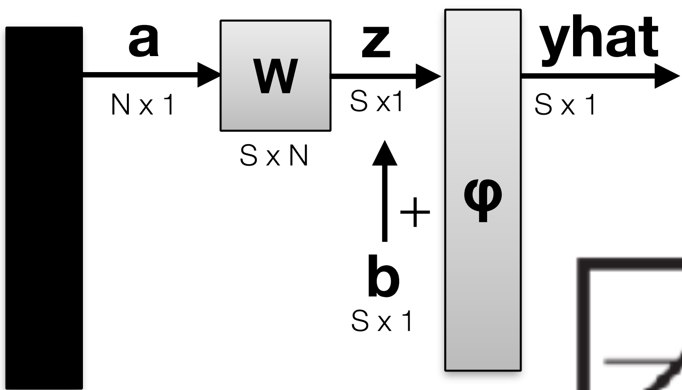



When a binary classification model outputs 0.5

# One Layer Linear Architectures

- Adaline network, Widrow and Hoff, 1960


Marcian “Ted” Hoff




==continuous output

$$\phi(\mathbf{A}\cdot\mathbf{w}) = \mathbf{A}\cdot\mathbf{w}$$

linear

Simplify Objective Function:

$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2 \longrightarrow J(\mathbf{w}) = \left\| \mathbf{Y} - \mathbf{A}\cdot\mathbf{w} \right\|^2$$

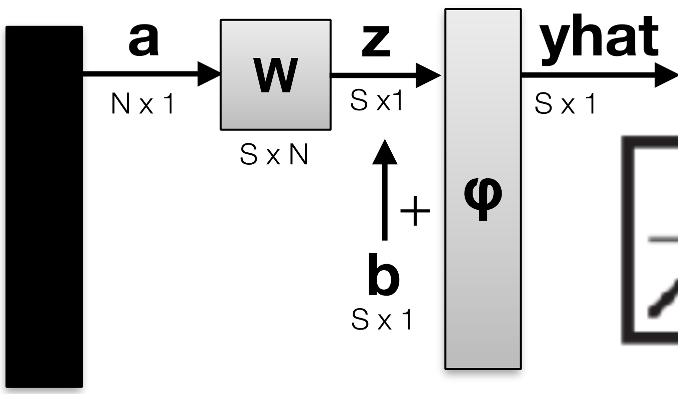
Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

We have been using the **Widrow-Hoff Learning Rule**

# One Layer Linear Architectures

Marcian “Ted” Hoff

- Adaline network, Widrow and Hoff, 1960




==continuous output

$$\phi(\mathbf{A} \cdot \mathbf{w}) = \mathbf{A} \cdot \mathbf{w}$$

linear

Bernard Widrow

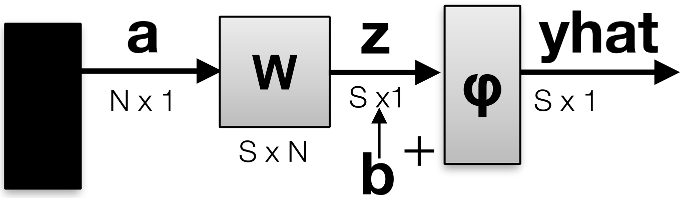
Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

For case S=1, $\mathbf{W}$ has only one row, $\mathbf{w}$
this is just **linear regression…**

$$J(\mathbf{w}) = \sum_{i=1}^{M} (y^{(i)} - \mathbf{x}^{(i)} \cdot \mathbf{w})^2$$

$$\mathbf{w} = (\mathbf{X}^T \cdot \mathbf{X})^{-1} \cdot \mathbf{X}^T \cdot y$$

# One Layer Regression to Classification




No longer continuous, $\mathbf{Y}$ is a category

ground truth $\mathbf{Y}$ is one-hot encoded!

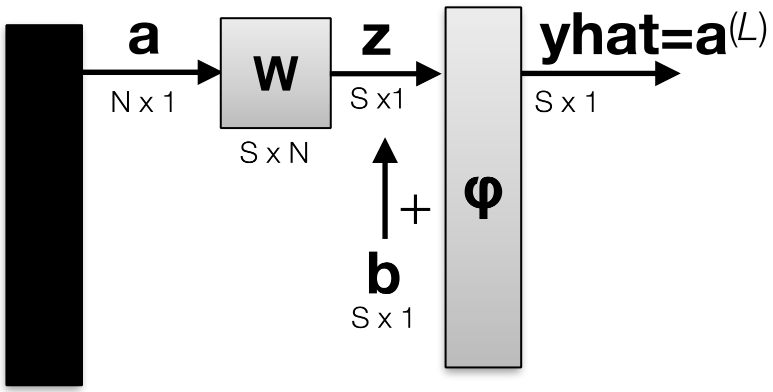
$$\mathbf{Y} = \left[ \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} \cdots \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \right] \rightarrow \left[ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{(1)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}^{(2)} \cdots \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}^{(M)} \right]$$

Need objective Function, minimize MSE $J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$

$$J(\mathbf{W}) = \left| \underbrace{\left[ \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} \cdots \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \right]}_{\mathbf{Y}} - \underbrace{\left[ \begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(1)} \begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(M)} \right]}_{\hat{\mathbf{Y}}} \right|^2$$

# One Layer Classification

- Rosenblatt's perceptron, 1957



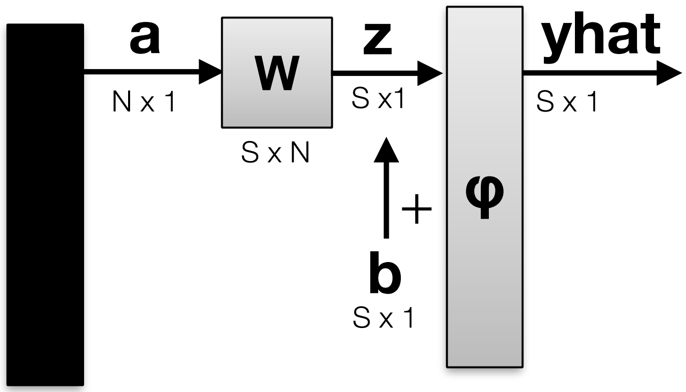


**Self Test** - If this is a binary classification problem, how large is *S*, the length of **yhat** and number of rows in **W**?

A. Can't determine
B. 2
C. 1
D. N

# One Layer Classification


- Modern Perceptron network




sigmoid

$$g(z) = \phi(z) = \frac{1}{1+\exp(-z)}$$

$$g(\mathbf{x}) = \phi(\mathbf{x}) = \frac{1}{1+\exp(-\mathbf{w}\cdot\mathbf{x})}$$

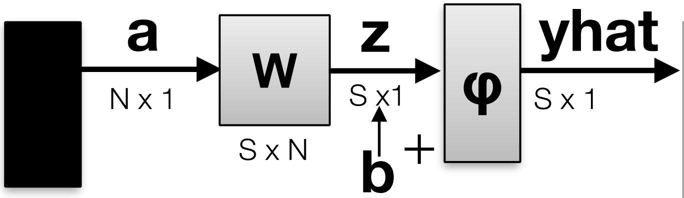
Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

For case S=1, this is just **logistic regression…** and **we have already solved this**!

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \cdot \text{mean}\left( \underbrace{(\mathbf{y} - g(\mathbf{X}\cdot\mathbf{w}))}_{\mathbf{y}_{diff}} \odot \mathbf{X} \right)_{cols}$$

What happens when S > 1 ?

# One Layer Architectures of Many Classes




$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$$

$$J(\mathbf{W}) = \left\| \mathbf{Y} - \phi\left(\mathbf{W} \cdot \mathbf{X}\right) \right\|^2$$

$$\boxed{J(\mathbf{w}_{row=1}) = \sum_i (y_1^{(i)} - \phi(\mathbf{x}^{(i)} \cdot \mathbf{w}_{row=1})^2}$$

$$\cdots$$

$$\boxed{J(\mathbf{w}_{row=C}) = \sum_i (y_C^{(i)} - \phi(\mathbf{x}^{(i)} \cdot \mathbf{w}_{row=C})^2}$$

$$\mathbf{Y} = \begin{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} & \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} & \cdots & \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \end{bmatrix} \rightarrow \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{(1)} & \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}^{(2)} & \cdots & \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}^{(M)} \end{bmatrix}$$

Each target class in **Y** can be independently optimized

$$\hat{\mathbf{Y}} = \begin{bmatrix} \begin{bmatrix} \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} & \begin{bmatrix} \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} & \cdots & \begin{bmatrix} \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} \end{bmatrix}$$

which is one versus-all!

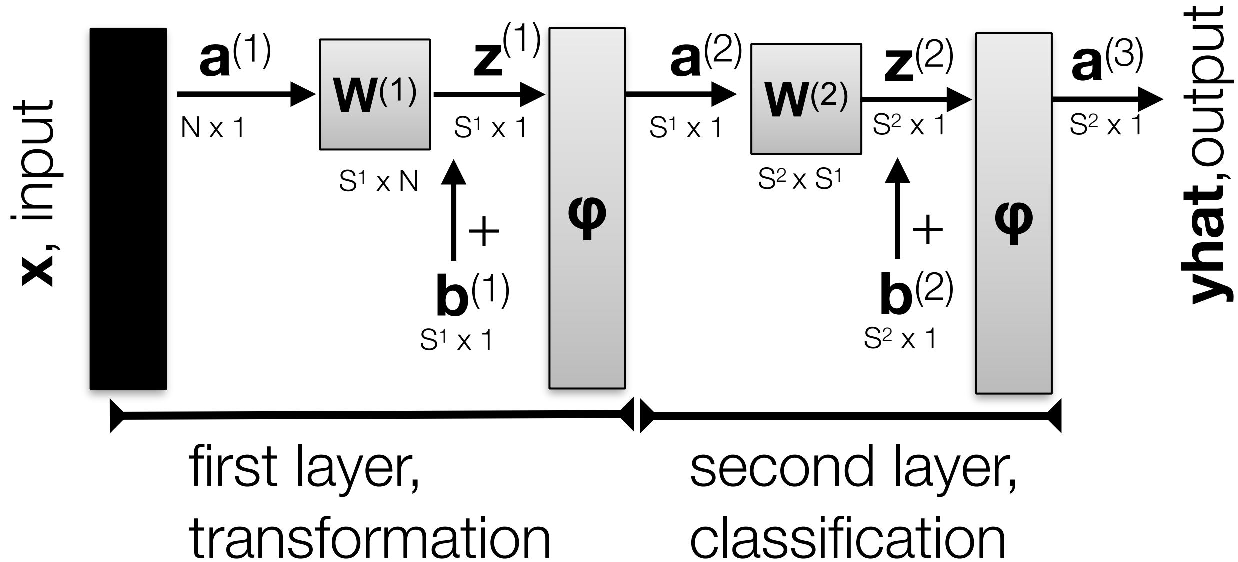
# Early Architectures: Summary

- Adaline network, Widrow and Hoff, 1960
  - linear regression, iterative updates
- Perceptron
  - *with sigmoid*: logistic regression
- One-versus-all implementation is the same as having $\mathbf{w}_{class}$ be rows of weight matrix, $\mathbf{W}$

- **But what about when we have more than one layer?**

# Moving to multiple layers…

- The multi-layer perceptron (MLP):
  - two layers shown, but could be arbitrarily many layers




each element of **yhat** is no longer independent of the rows in $\mathbf{W}^{(1)}$ so we cannot optimize using one versus all 😢

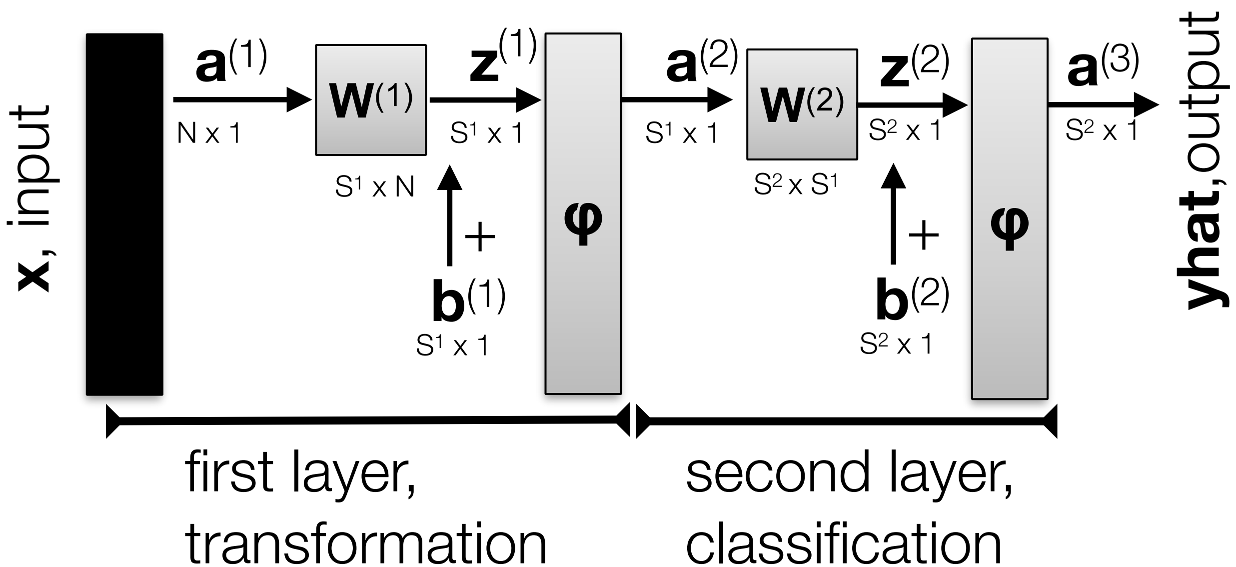

$$\begin{bmatrix}\hat{y}_1\\ \hat{y}_2\\ \vdots\\ \hat{y}_C\end{bmatrix} = \begin{bmatrix}\phi(\phi(\mathbf{z}^{(1)})\cdot \mathbf{w}^{(2)}_{row=1})\\ \phi(\phi(\mathbf{z}^{(1)})\cdot \mathbf{w}^{(2)}_{row=2})\\ \vdots\\ \phi(\phi(\mathbf{z}^{(1)})\cdot \mathbf{w}^{(2)}_{row=C})\end{bmatrix}$$

$$\boxed{\mathbf{z}^{(1)} = \mathbf{W}^{(1)}\cdot\mathbf{a}^{(1)}}$$

# Back propagation

- Optimize all weights of network at once
- Steps:
  1. Forward propagate to get all $\mathbf{Z}^{(l)}$ , $\mathbf{A}^{(l)}$
  2. Get final layer gradient
  3. Back propagate sensitivities
  4. Update each $\mathbf{W}^{(l)}$

**Back-propagation is solved in flipped assignment!!**

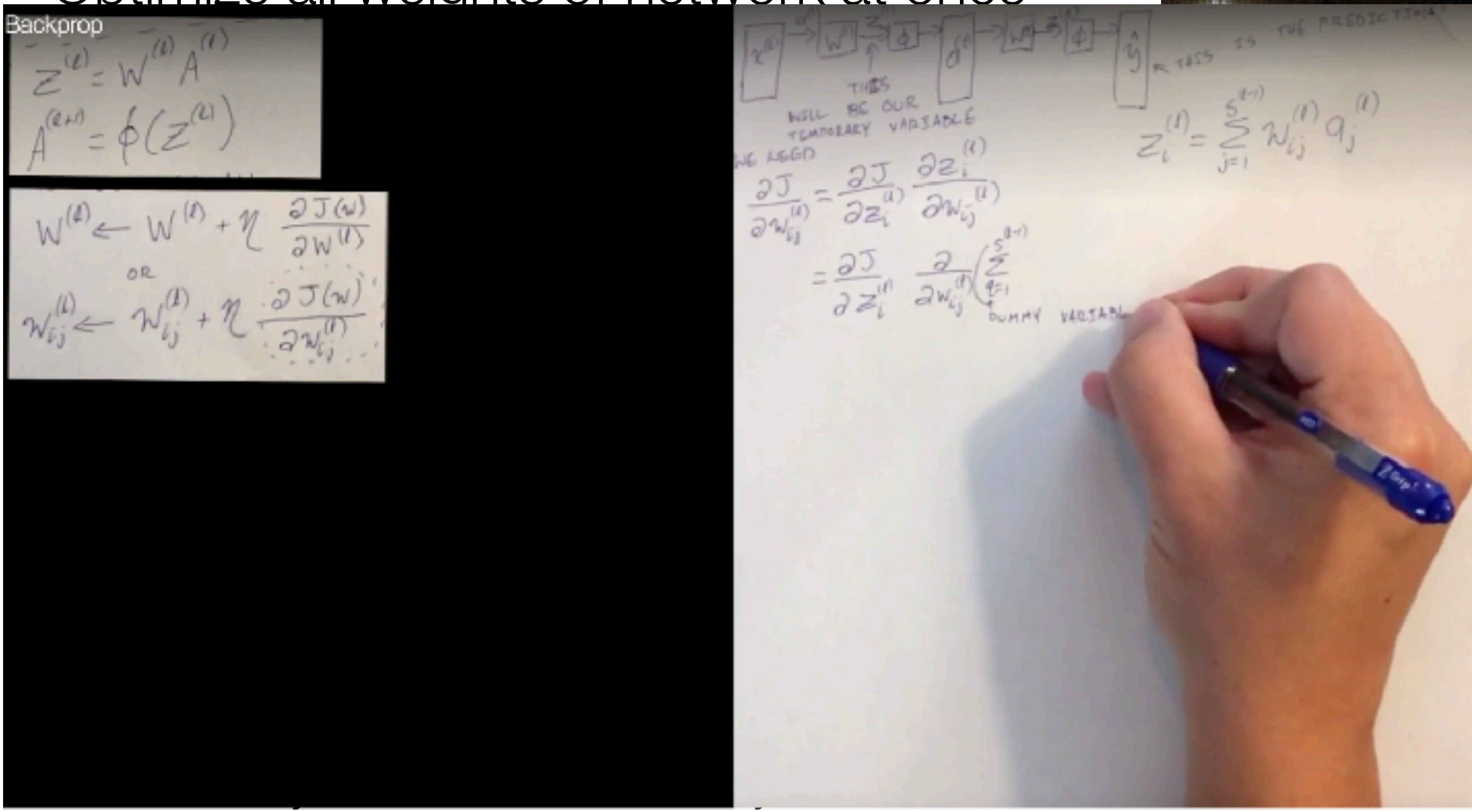



$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$$

$$w_{i,j}^{(l)} \leftarrow w_{i,j}^{(l)} - \eta \frac{\partial J(\mathbf{W})}{\partial w_{i,j}^{(l)}}$$

# Back propagation

- Optimize all weights of network at once




transformation classification