

# Formalizing operational semantics in a proof assistant



Edoardo Marangoni  
University of Milan

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## CHAPTER 1

# Introduction



## CHAPTER 2

# Semantics

### 2.1 Introduction

In 1967, computer scientist Robert Floyd wrote, in a seminal paper [Flo67],





## CHAPTER 3

# Induction and coinduction

### 3.1 Introduction

### 3.2 Infinite datatypes

### 3.3 Infinite proofs

### 3.4 Relation with fixed points



## CHAPTER 4

# Agda

### 4.1 Dependent types

### 4.2 Termination and productivity

### 4.3 Sized types



## CHAPTER 5

# The partiality monad

### 5.1 Monads

### 5.2 Implementation



## CHAPTER 6

# The IMP language

In this chapter we will go over the implementation of a simple imperative language called **Imp**, as described in [PdAC<sup>+</sup>23]. After defining its syntax, we will give rules for its semantics and show its implementation in Agda. After this introductory work, we will discuss analysis and optimization of Imp programs.

### 6.1 Syntax

The syntax of the Imp language is straightforward, and can be described in a handful of EBNF rules.

### 6.2 Semantics

### 6.3 Implementation

### 6.4 Analysis and optimization

#### 6.4.1 Definite initialization analysis

#### 6.4.2 Pure folding





# Bibliography

- [Flo67] R. W. Floyd. Assigning meanings to programs. *Mathematical aspects of computer science*, 19, 1967.
- [PdAC<sup>+</sup>23] Benjamin C. Pierce, Arthur Azevedo de Amorim, Chris Casinghino, Marco Gaboardi, Michael Greenberg, Cătălin Hrițcu, Vilhelm Sjöberg, and Brent Yorgey. Logical foundations, 2023. [Accessed 13-09-2023].