Problem S3: Degrees of Separation

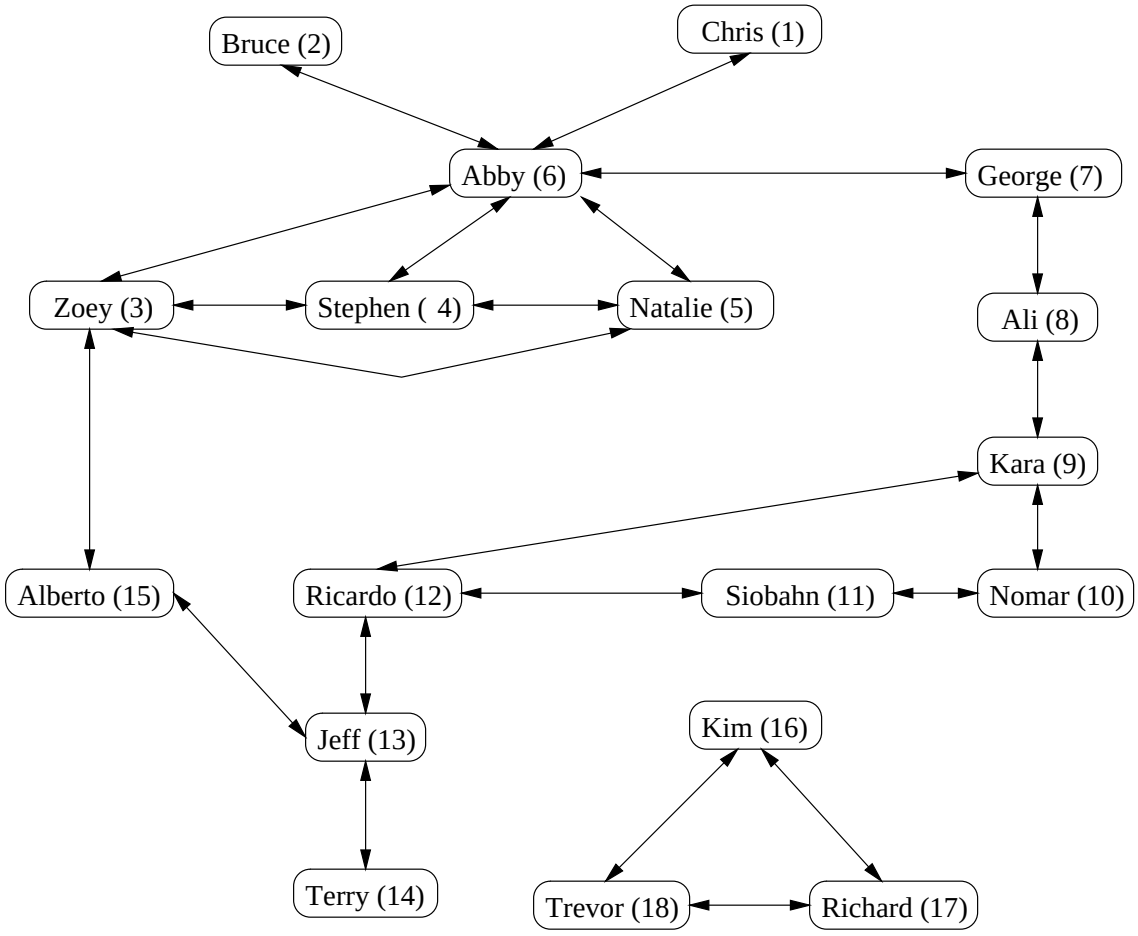
Problem Description

The main socializing tool for students today is Facebook. There are many interesting computational questions connected to Facebook, such as the "degree of separation" between two people.

For example, in the diagram below, there are many different paths between Abby and Alberto. Some of these paths are:

- Abby → Zoey → Alberto
- Abby → Natalie → Zoey → Alberto
- Abby → George → Ali → Kara → Richardo → Jeff → Alberto

The shortest path between Abby and Alberto has two steps (Abby → Zoey, and Zoey → Alberto), so we say the degree of separation is 2. Additionally, Alberto would be a friend of a friend of Abby.



You can assume an initial configuration of who is friends with who as outlined in the diagram above. You will need to store these relationships in your program. These relationships can change though, and your program needs to handle these changes. In particular, friendships can begin, possibly with new people. Friendships can end. You should be able to find friends of friends and determine the degree of separation between two people.

Input/Output Description
Your program will read in six possible commands, with the action to be performed by your program outlined below. You may assume that x and y are integers, with $x \neq y$, $x \geq 1$, $y \geq 1$, $x < 50$ and $y < 50$. You may also assume that instructions (`i`, `d`, `n`, `f`, `s`, `q`) occur one per line and parameters (zero, one or two integers) occur one per line.

- `i` x y – make person x and person y friends. If they are already friends, no change needs to be made. If either x or y is a new person, add them.
- `d` x y – delete the friendship between person x and person y.
- `n` x – output the number of friends that person x has.
- `f` x – output the number of "friends of friends" that person x has. Notice that x and direct friends of x are not counted as "friends of friends."
- `s` x y – output the degree of separation between x and y. If there is no path from x to y, output `Not connected`.
- `q` – quit the program.

Sample Interaction

Input	Output	Explanation

`i`
`20`
`10`
	(no output)	Inserting a friendship causes no output.

`i`
`20`
`9`
	(no output)	Inserting a friendship causes no output.

`n` `20`	`2`	Person 20 has two friends (10 and 9)
`f` `20`	`3`	The friends of friends of 20 are 8, 11, 12.

`s`
`20`
`6`
	`4`	The shortest path is $20 \rightarrow 9 \rightarrow 8 \rightarrow 7 \rightarrow 6$.

`q`	(no output)	Program quits.

Problem S4: Shop and Ship

Problem Description

In Doubleclickland, there are N cities ($N \leq 5,000$), with each city having various trade routes to other cities. In total, there are T trade routes ($0 \leq T \leq 25,000,000$). in Doubleclickland. For each trade route between two cities x and y, there is a transportation cost $C(x, y)$ to ship between the cities, where $C(x, y) \geq 0$, $C(x, y) \leq 10,000$ and $C(x, y) = C(y, x)$. Out of the N cities, K ($1 \leq K \leq N$) of these cities have stores with really nice pencils that can be purchased on-line. The price for each pencil in city x is P_x ($0 \leq P_x \leq 10,000$).

Find the minimal price to purchase one pencil on-line and have it shipped to a particular city D ($1 \leq D \leq N$) using the cheapest possible trade-route sequence. Notice that it is possible to purchase the pencil in city D and thus require no shipping charges.

Input Description

The first line of input contains N, the number of cities. You can assume the cities are numbered from 1 to N. The second line of input contains T, the number of trade routes. The next T lines each contain 3 integers, x y $C(x, y)$, to denote the cost of using the trade route between cities x and y is $C(x, y)$. The next line contains the integer K, the number of cities with a store that sells really nice pencils on-line. The next K lines contains two integers, z and P_z, to denote that the cost of a pencil in city z is P_z. The last line contains the integer D, the destination city.

Output Description

Output the minimal total cost of purchasing a pencil on-line and shipping it to city D.

Sample Input

```
3
3
1 2 4
2 3 2
1 3 3
3
1 14
2 8
3 3
1
```

Output for Sample Input

```
6
```