Equation of Motion - Pendulum Arm

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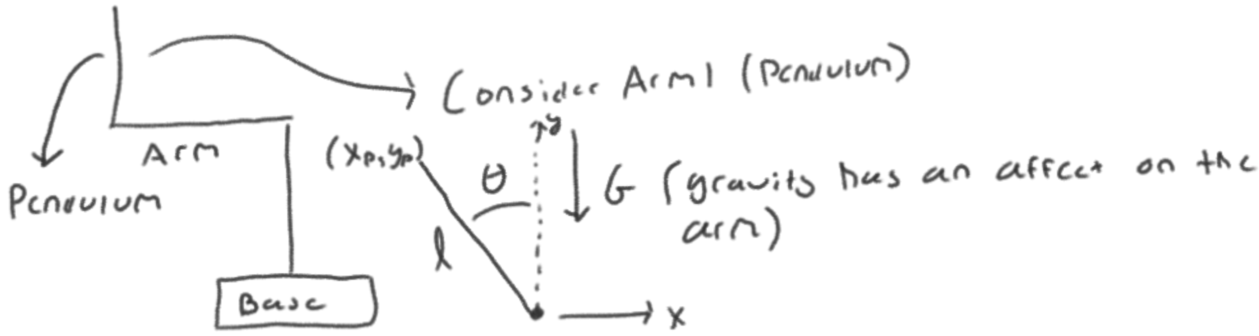

$$x_p = x - l\cos\theta \quad (1)$$
$$y_p = l\cos\theta \quad (2)$$
$$\dot{x}_p = \dot{x} - l\dot{\theta}\cos\theta$$
$$\dot{y}_p = -l\dot{\theta}\sin\theta$$

→ Potential Energy

$$V = mgy_p = mgl\cos\theta \quad \text{(potential energy)}$$

→ Kinetic energy

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}_p^2 + \dot{y}_p^2)$$
$$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[\dot{x}^2 - 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2\cancelto{1}{(\cos^2\theta + \sin^2\theta)}\right]$$

$$T = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - ml\dot{\theta}\dot{x}\cos\theta \quad \text{(Equation for kinetic energy)}$$

→ Now find the equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

$$L = T - V$$

$$L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - ml\dot{\theta}\dot{x}\cos\theta - mgl\cos\theta$$

x direction

Applied force

$$(M+m)\ddot{x} - (ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta) = F(t)$$

$$\boxed{(M+m)\ddot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta = F(t)}$$

$\underline{\theta}$ $$ml^2\ddot{\theta} - ml\ddot{x}\cos\theta + ml\cancel{\dot{x}\dot{\theta}}\sin\theta - ml\cancel{\dot{\theta}\dot{x}}\sin\theta - mgl\sin\theta = 0$$

→ Consider ml

$$\boxed{l\ddot{\theta} - \ddot{x}\cos\theta - g\sin\theta = 0}$$

Equation of Motion for the pendulum arm