IUT DOUALA

Spécialité/Speciality MTIN/FI2

Examen d'Algèbre linéaire/Linear Algebra Exam

Année academique/Academemic Year 2021/2022

Durée/Time :1h

corrigé

Exercice 1

1. Let's show that the vectors $x_1 = (0;1;1)$, $x_2 = (1;0;1)$, $x_3 = (1;1;0)$ form a basis of $\mathbb{R}^3$.

   il suffit de montrer que :
$$det\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix} = 2 \neq 0$$
   The coordinates of the vector $x = (1;1;1)$ in this basis
$$x = \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3$$
   donc $x = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.
2. Consider the application g defined by
$$\begin{aligned} g : \mathbb{R}^4 &\to \mathbb{R}^4 \\ (x, y, z, t) &\mapsto (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t) \end{aligned}$$
3. Let's show that g is a linear application.
   il suffit de montrer que, $\forall \alpha, \beta \in \mathbb{R}$ and $\forall u, v \in \mathbb{R}^4$, we have :
   $g(\alpha u + \beta v) = \alpha g(u) + \beta g(v)$
4. Let's determine a base and a size ker(g)
   $kerg = \{y(1, 1, 0, 0) + t(1, 0, -2, 1),\ \ y, t \in \mathbb{R}\}$
   $dimkerg = 2$
5. Let's determine a base and image size.
   $Img = \{x'(1, 0, 1) + y'(0, 1, 1),\ \ x', y' \in \mathbb{R}\}$
   $dimImg = 2$

Exercice 2

Let's consider the following system
$$(S) : \begin{cases} x + y + 2z = 5, \\ x - y - z = 1, \\ x + z = 3. \end{cases}$$

1. The matrix form associated to (S) is :

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$$

2. Let's determine the rank of A.

$$det \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix} = -1 \neq 0$$

   Then $rankA = 3$

3. Resolvons le systeme (S) par la methode de Cramer/ Let's solve the system by the cramer method.
   $\Delta_A = -1$, $\Delta_x = -3$, $\Delta_y = -2$, $\Delta_z = 0$. then,

$$S = \{(3, 2, 0)\}$$