```
> ##########################################################
> # Host fitness for approximation to fast social information dynamics #
> ##########################################################
>
> restart :
> with(VectorCalculus) :
> with(LinearAlgebra) :
> interface(imaginaryunit = _I) :
>
> # First calculate information awareness in mutant when rare (Ams)
```

$> A := 1 - \frac{\text{sigma}}{\text{tau} \cdot c \cdot c} :$

$> dSAm := \frac{cm \cdot c}{N} \cdot (\text{tau} \cdot SUm \cdot A \cdot N) - \text{sigma} \cdot SAm :$

$> dIAm := \frac{cm \cdot c}{N} \cdot (\text{tau} \cdot IUm \cdot A \cdot N) - \text{sigma} \cdot IAm :$

$> dAm := \frac{1}{Nm} \cdot \left(\frac{cm \cdot c}{N} \cdot (\text{tau} \cdot (1 - Am) \cdot Nm \cdot A \cdot N) - \text{sigma} \cdot Am \cdot Nm \right) :$

$> Ams := solve(dAm, Am);$

$$Ams := \frac{cm\,(\tau c^2 - \sigma)}{c^2\, cm\, \tau + \sigma c - cm\, \sigma} \tag{1}$$

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>
> # Now calculate resident equilibrium
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$> dS := (b - q \cdot (S + I)) \cdot (S + I) - \frac{\text{beta} \cdot c \cdot c \cdot S \cdot I}{(S + I)} - d \cdot (1 - (1 - a) \cdot A) \cdot S + \text{gamma} \cdot I :$

$> dI := \frac{\text{beta} \cdot c \cdot c \cdot S \cdot I}{(S + I)} - (d \cdot (1 - (1 - a) \cdot A) + \text{alpha} + \text{gamma}) \cdot I :$

$> dN := (b - q \cdot N) \cdot N - d \cdot (1 - (1 - a) \cdot A) \cdot N - \text{alpha} \cdot I :$

$> sol := solve([dS, dI], [S, I]) :$

$> eql := simplify(sol[2]) :$

$> Ns := simplify(subs(eql, S + I));$

$$Ns := \frac{-\tau \beta\,(a d + \alpha - b)\, c^4 + (\alpha\,(a d + \alpha + \gamma)\,\tau + \sigma d \beta\,(-1 + a))\, c^2 - \sigma d \alpha\,(-1 + a)}{\beta c^4 \tau q} \tag{2}$$

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> # Mutant dynamics
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$> dSm := (b - q \cdot N) \cdot (Sm + \mathrm{Im}) - \frac{\text{beta} \cdot cm \cdot c \cdot Sm \cdot I}{N} - d \cdot (1 - (1 - a) \cdot Ams) \cdot Sm + \text{gamma} \cdot \mathrm{Im} :$

$> dIm := \frac{\text{beta} \cdot cm \cdot c \cdot Sm \cdot I}{N} - (d \cdot (1 - (1 - a) \cdot Ams) + \text{alpha} + \text{gamma}) \cdot \mathrm{Im} :$

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>
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> # Calculate the Jacobian

> $J := (Jacobian([dSm, dIm], [Sm, \mathrm{Im}]));$

$$J := \left[\left[-q\,N + b - \frac{\beta\,cm\,c\,I}{N} - d\left(1 - \frac{(1-a)\,cm\,(c^2\,\tau - \sigma)}{c^2\,cm\,\tau + c\,\sigma - cm\,\sigma}\right), -q\,N + b + \gamma\right], \left[\frac{\beta\,cm\,c\,I}{N}, -d\left(1 - \frac{(1-a)\,cm\,(c^2\,\tau - \sigma)}{c^2\,cm\,\tau + c\,\sigma - cm\,\sigma}\right) - \alpha - \gamma\right]\right] \quad \textbf{(3)}$$

>

> # Split the Jacobian into components F and V as follows

> $F := \langle\langle b - q\cdot N, 0\rangle | \langle b - q\cdot N, 0\rangle\rangle;$

$$F := \begin{bmatrix} -q\,N + b & -q\,N + b \\ 0 & 0 \end{bmatrix} \quad \textbf{(4)}$$

> $V := F - J;$

$$V := \left[\left[\frac{\beta\,cm\,c\,I}{N} + d\left(1 - \frac{(1-a)\,cm\,(c^2\,\tau - \sigma)}{c^2\,cm\,\tau + c\,\sigma - cm\,\sigma}\right), -\gamma\right], \left[-\frac{\beta\,cm\,c\,I}{N}, d\left(1 - \frac{(1-a)\,cm\,(c^2\,\tau - \sigma)}{c^2\,cm\,\tau + c\,\sigma - cm\,\sigma}\right) + \alpha + \gamma\right]\right] \quad \textbf{(5)}$$

> # Now calculate the next-generation matrix

> $V1 := MatrixInverse(V):$

> $NG := simplify(F \cdot V1):$

>

> # Fitness is equal to the largest eigenvalue of the NG matrix minus 1

> $w := simplify(NG[1,1]) - 1;$

$$w := -\Big(\big((c^2\,\tau - \sigma)\,cm + c\,\sigma\big)\,(q\,N - b)\,\Big(-I\,\beta\,c\,(-c^2\,\tau + \sigma)\,cm^2 + \big((I\,\beta\,\sigma + N\,\tau\,(a\,d + \alpha + \gamma))\,c^2 - \sigma\,N\,(a\,d + \alpha + \gamma)\big)\,cm + \sigma\,N\,c\,(\alpha + \gamma + d)\Big)\Big) \Big/ \Big(I\,\beta\,c\,(-c^2\,\tau + \sigma)^2\,(a\,d + \alpha)\,cm^3 - 2\left(\left(\left(\left(\frac{a}{2} + \frac{1}{2}\right)d + \alpha\right)I\,\beta\,\sigma + \frac{N\,a\,d\,\tau\,(a\,d + \alpha + \gamma)}{2}\right)c^2 - \frac{\sigma\,N\,a\,d\,(a\,d + \alpha + \gamma)}{2}\right)(-c^2\,\tau + \sigma)\,cm^2 + c\,\big((I\,\beta\,(\alpha + d)\,\sigma + d\,N\,\tau\,(2\,a\,d + (a+1)\,(\alpha + \gamma)))\,c^2 - d\,N\,\sigma\,(2\,a\,d + (a+1)\,(\alpha + \gamma))\big)\,\sigma\,cm + \sigma^2\,N\,c^2\,d\,(\alpha + \gamma + d)\Big) - 1 \quad \textbf{(6)}$$

>

> # Calculate the fitness gradient

> $dw0 := simplify(subs(cm = c, diff(w, cm))):$

> $dw := simplify(subs(eql, (subs(N = Ns, dw0))));$

$$dw := \Big(-\beta\,(\alpha\,(a\,d + \alpha + \gamma)\,\tau + \sigma\,d\,\beta\,(-1 + a))\,\tau^2\,c^8 + (\alpha\,(a\,d + \alpha + \gamma)^2\,\tau^2 \quad \textbf{(7)}$$

$$+ 2\, d\, \beta\, \alpha\, \sigma\, (-1+a)\, \tau + d\, \beta^2\, \sigma^2\, (-1+a) \Big)\, \tau\, c^6 - 3\, \alpha\, \sigma\, \tau\, (-1+a) \Bigg((a\, d + \alpha$$

$$+ \gamma)\, \tau + \frac{\beta\, \sigma}{3} \Bigg)\, d\, c^4 + 3\, \alpha\, \sigma^2\, \tau\, (-1+a) \left(\frac{\alpha}{3} + \left(a - \frac{2}{3} \right) d + \frac{\gamma}{3} \right) d\, c^2$$

$$- d^2\, \alpha\, \sigma^3\, (-1+a)^2 \Bigg) \Bigg/ \Big(\beta\, c^7\, \tau^2\, \big(\tau\, \beta\, (a\, d + \alpha)\, c^4 + (-\alpha\, (a\, d + \alpha + \gamma)\, \tau - \sigma\, d\, \beta\, ($$

$$-1 + a))\, c^2 + \sigma\, d\, \alpha\, (-1+a) \big) \Big)$$

```
>
> # Calculate evolutionary stability (output hidden for brevity)
> E0 := simplify(subs(cm = c, diff(w, cm, cm))) :
> E := simplify(subs(eql, (subs(N = Ns, E0)))) :
>
>
> # Special case - 1) R0_I<1
>
> restart :
> with(VectorCalculus) :
> with(LinearAlgebra) :
> interface(imaginaryunit = _I) :
>
> A := 0 :
> Ams := 0 :
```

$$> \; dS := (b - q\cdot(S+I))\cdot(S+I) - \frac{\text{beta}\cdot c\cdot c\cdot S\cdot I}{(S+I)} - d\cdot(1-(1-a)\cdot A)\cdot S + \text{gamma}\cdot I :$$

$$> \; dI := \frac{\text{beta}\cdot c\cdot c\cdot S\cdot I}{(S+I)} - (d\cdot(1-(1-a)\cdot A) + \text{alpha} + \text{gamma})\cdot I :$$

$$> \; dN := (b - q\cdot N)\cdot N - d\cdot(1-(1-a)\cdot A)\cdot N - \text{alpha}\cdot I :$$

```
> sol := solve([dS, dI], [S, I]) :
> eql := simplify(sol[2]) :
> Ns := simplify(subs(eql, S + I));
```

$$Ns := \frac{\alpha^2 + (-\beta\, c^2 + d + \gamma)\, \alpha + c^2\, \beta\, (b - d)}{\beta\, c^2\, q} \qquad \textbf{(8)}$$

```
> # Mutant dynamics
```

$$> \; dSm := (b - q\cdot N)\cdot(Sm + \mathrm{Im}) - \frac{\text{beta}\cdot cm\cdot c\cdot Sm\cdot I}{N} - d\cdot(1-(1-a)\cdot Ams)\cdot Sm + \text{gamma}\cdot\mathrm{Im} :$$

$$> \; dIm := \frac{\text{beta}\cdot cm\cdot c\cdot Sm\cdot I}{N} - (d\cdot(1-(1-a)\cdot Ams) + \text{alpha} + \text{gamma})\cdot\mathrm{Im} :$$

```
>
> # Calculate the Jacobian
> J := (Jacobian([dSm, dIm], [Sm, Im]));
```

$$J := \begin{bmatrix} -q\,N + b - \frac{\beta\,cm\,c\,I}{N} - d & -q\,N + b + \gamma \\ \frac{\beta\,cm\,c\,I}{N} & -d - \alpha - \gamma \end{bmatrix} \tag{9}$$

```
> # Split the Jacobian into components F and V as follows
> F := ⟨⟨b − q·N, 0⟩|⟨b − q·N, 0⟩⟩;
```

$$F := \begin{bmatrix} -q\,N + b & -q\,N + b \\ 0 & 0 \end{bmatrix} \tag{10}$$

```
> V := F − J;
```

$$V := \begin{bmatrix} \frac{\beta\,cm\,c\,I}{N} + d & -\gamma \\ -\frac{\beta\,cm\,c\,I}{N} & d + \alpha + \gamma \end{bmatrix} \tag{11}$$

```
> # Now calculate the next-generation matrix
> V1 := MatrixInverse(V) :
> NG := simplify(F • V1) :
>
> # Fitness is equal to the largest eigenvalue of the NG matrix minus 1
> w := simplify(NG[1, 1]) - 1;
```

$$w := -\frac{\left(\left(d + \alpha + \gamma\right) N + I\,\beta\,c\,cm\right)\left(q\,N - b\right)}{d\left(d + \alpha + \gamma\right) N + I\,c\,cm\,\beta\left(d + \alpha\right)} - 1 \tag{12}$$

```
> # Calculate the fitness gradient
> dw1 := simplify(subs(cm = c, diff(w, cm)));
```

$$dw1 := \frac{I\,\beta\,c\,(q\,N - b)\,N\,\alpha\left(d + \alpha + \gamma\right)}{\left(d\left(d + \alpha + \gamma\right) N + I\,c^2\,\beta\left(d + \alpha\right)\right)^2} \tag{13}$$

```
> # Intuitively if R0_D > 1 and alpha > 0, this is always negative (since b > q·N)
>
>
> # Special case - 2) R0_D < 1
>
> restart :
> with(VectorCalculus) :
> with(LinearAlgebra) :
> interface(imaginaryunit = _I) :
>
```

$$> A := 1 - \frac{\text{sigma}}{\text{tau}\cdot c\cdot c} :$$

> $Ams := \frac{cm\left(\tau c^2 - \sigma\right)}{c^2\, cm\, \tau + \sigma c - cm\, \sigma}$:

>

> $dS := (b - q \cdot S) \cdot S - d \cdot (1 - (1 - a) \cdot A) \cdot S$:

> $dSm := (b - q \cdot S) \cdot Sm - d \cdot (1 - (1 - a) \cdot Ams) \cdot Sm$:

> $sol := solve(dS, S)$:

> $eql := simplify(sol[2]);$

$$eql := \frac{-c^2\,(a\,d - b)\,\tau + d\,\sigma\,(-1 + a)}{c^2\,q\,\tau} \qquad \textbf{(14)}$$

> # Fitness is straightforward in this case

> $w := simplify\left(\frac{dSm}{Sm}\right);$

$$w := \frac{((q\,S + a\,d - b)\,cm - c\,(q\,S - b + d))\,\sigma - c^2\,cm\,\tau\,(q\,S + a\,d - b)}{(c - cm)\,\sigma + c^2\,cm\,\tau} \qquad \textbf{(15)}$$

> # Calcualte the fitness gradient

> $dw1 := simplify(subs(cm = c, subs(S = eql, diff(w, cm))));$

$$dw1 := -\frac{d\,\sigma\,(-1 + a)\,\left(\tau\,c^2 - \sigma\right)}{c^5\,\tau^2} \qquad \textbf{(16)}$$

> # Intuitively if R0_I > 1 & a < 1, then dw > 0 for all c

>

>

> # Special case - 3) R0_I, R0_D < 1

>

> # Trivial: cm does not feature in invasion fitness, so dw=0 for all c

>