An Investigation into Mercury and Electron Behaviour Via the Franck Hertz Experiment

Eli Cohen*
Department of Physics and Astronomy, UBC, Vancouver, BC
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This paper delves into the validation of pivotal historical contributions to quantum physics using a vacuum tube. In said vacuum tube we subject electrons to varying accelerating voltages through a thin mercury vapour to induce inelastic collisions at varying temperatures in the Franck-Hertz experiment. Due to the quantized nature of mercury's energy levels, we observe unique peaks in the current-voltage plot that are fit using an interpolating spline and a Monte Carlo method estimating uncertainties. We then determine the lowest excitation energy level of mercury to be 4.7 ± 0.5eV. Additionally, we measure the Child-Langmuir law by applying varying voltages to a vacuum tube without mercury and observing the resulting current, establishing a relationship of $I(V) \propto V^{1.48 \pm 0.06}$. These results serve to highlight the underpinnings of quantum mechanics and electron behaviour.

The Franck-Hertz experiment, conducted by James Franck and Gustav Hertz in 1914, provided crucial evidence supporting the quantization of energy levels in atoms, corroborating the Bohr model, and earning them a Nobel Prize in 1925 [1]. This historic experiment demonstrated that mercury atoms can absorb only discrete amounts of energy as they collide inelastically with electrons, leading to the emission of electromagnetic radiation with a wavelength of 2536 Angstrom from the mercury atoms only after the electron has reached sufficient kinetic energy. Moreover, by measuring the current through the tube, characteristic peaks in the current-voltage curve arise. Thus, this experiment was a groundbreaking first demonstration of quantized atomic energy levels in an electrical experiment [2]. This idea of quantized energy levels is crucial as it underpins the behaviour of particles within atoms, marking a departure from classical physics. This discrete nature of energy levels in atoms, as revealed by experiments like Franck-Hertz, has profound implications for our understanding of matter and its interactions such as electronic transitions, spectral lines, and the stability of matter [3]. It forms the basis of quantum mechanics, shaping the way we perceive and predict the behaviour of particles on both microscopic and macroscopic scales. From semiconductors to quantum computing, medical imaging, material science, and many more, quantum mechanics has found countless applications in modern technology [4]. We also concern ourselves with another important characterization of electron behaviour, the Child-Langmuir law. This phenomenon stems from space charge, the accumulation of charged particles within a confined space which acts to impede current via an electric field [5]. In this case, electrons within a vacuum tube. Under the assumptions that electrons do not scatter during their travel and that the electron velocity at the cathode is 0, Clement D. Child derived the following equation in 1912:

$$I = S\frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m_e}}\frac{V^{3/2}}{d^2} \tag{1}$$

which relates the output current, I, of a vacuum diode to the applied voltage, V. In Equation (1), S is the surface area of the anode, m_e is the mass of an electron, and d is the distance between the anode and cathode. This law and subsequent understanding of space charge was crucial in the development of vacuum tube-based amplifiers [6], however, it is also applied to modern topics such as the theory governing ion thrusters for space travel [7].

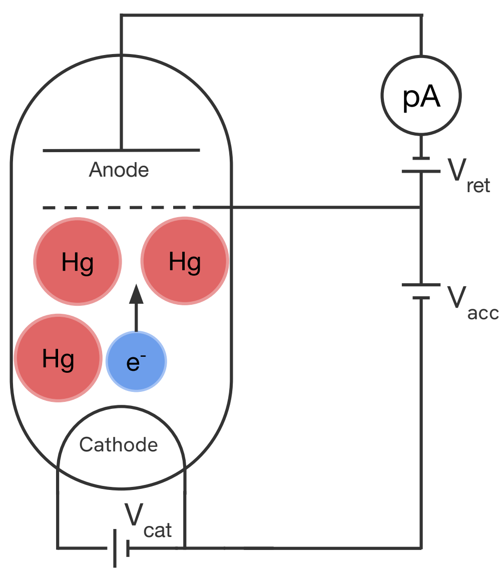

FIG. 1: A diagram of the Franck-Hertz apparatus is shown. The vacuum tube is filled with vaporized mercury. A cathode, heated by a voltage of V_{cat}, is used to launch electrons through the vapor, which are then accelerated towards the anode by a voltage of V_{acc}. If the electrons reach the anode, they are detected by a picoammeter. Additionally, a grid is present to provide a retarding voltage of V_{ret}, while still allowing high energy electrons to pass through.

* REVTeX Support: elic@student.ubc.ca

In this paper, we seek to replicate these historic findings and conduct an experiment mirroring those conducted in 1914. We make use of a Klinger Educational commercial apparatus to conduct this experiment. This is a vacuum tube filled with mercury, subject to heating by a variable thermostat. A schematic of the apparatus is shown in FIG. 1. We use a voltage, $V_{cat} = 6.98$V, to warm the cathode and induce the maximum amount of thermionic emission of electrons without damaging the cathode. We then utilize an accelerating voltage ranging from 0 to 60V to accelerate the electrons towards a grid, and past that the anode. In the Franck-Hertz experiment, we then employ a retarding voltage V_{ret} of 1V to stop low-energy electrons that have lost their energy due to inelastic collisions. If the electron has sufficient energy it reaches the anode and is recorded as current. In the determination of the Child-Langmuir law, this retarding voltage is not applied and electrons pass without impedance.

Firstly, we discuss the Franck Hertz experiment, which involves firing electrons produced via a cathode through a vapour of mercury gas. Assuming a quantized model of mercury, electrons of sufficiently low energy collide elastically and lose little energy in the collision due to the large difference in the colliding object's masses. However, if the electrons have an energy greater than the lowest excited state of mercury, 6^3P_0, they will collide inelastically and lose a significant amount of energy. Moreover, excited mercury atoms decay quickly back to their ground state, 6^3S_0 by emission of a photon.

Unlike other implementations of the Franck-Hertz experiment that utilize the mean spacing of the minima to determine the lowest excitation level, we instead use a mean free path model. We dub how far an electron travels on average before another collision occurs, the mean free path, or λ. The electrons gain energy from our accelerating voltage over a distance of our mean free path. We denote this increase in energy δ_n. Therefore, when an electron reaches at least enough energy to transition from the ground state to the first excited state, it travels one mean-free path, gaining energy and possibly exciting higher Hg energy levels. Doubling the voltage allows electrons to undergo two collisions before reaching the grid, resulting in the energy addition effect happening twice.

We follow a derivation proposed by Rapior et al.[8] to generalize this concept. For n collisions, we calculate the energy of an electron to be

$$E_n = n(E_a + \delta_n) \tag{2}$$

where E_a is the activation energy and δ_n is the energy accumulated via the accelerating potential for one mean free path.

We also find that the mean free path is small compared to the distance separating the two grids, $\lambda << L$ and $\delta_n << E_a$. Therefore, we calculate δ_n to be

$$\delta_n = n\frac{\lambda}{L}E_a \tag{3}$$

As such, we find that the separation voltage between two

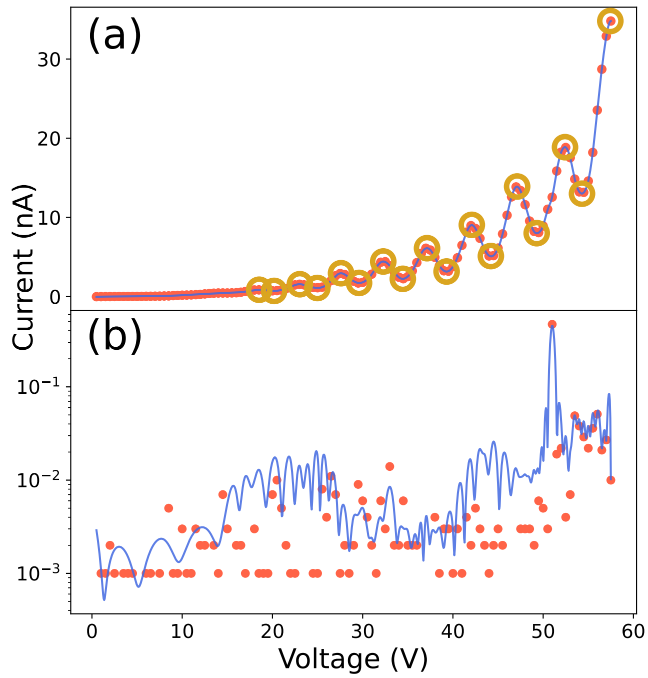

FIG. 2: In (a) we plot, in red circles, our observed current, I, measured in nA, against accelerating voltage, V, measured in Volts. Fitted to these data, in blue, is a cubic spline with a smoothing factor of 0.07. We calculate a χ^2 value of 3.14 which implies a reasonable fit to the data. Highlighted in yellow are the computationally calculated minima and maxima. In (b) we plot, in red, our observed uncertainties. In blue, we plot the standard deviation of the spline fit obtained through a Monte Carlo method for 10,000 simulations with Gaussian perturbations. Of note is the fact that the spline uncertainties are of the same order of magnitude as our original uncertainties.

minima in our voltage-current curve is

$$\Delta E(n) = E_n - E_{n-1} = \left(1 + \frac{\lambda}{L}(2n-1)\right)E_a \tag{4}$$

This equation implies that the relationship between the spacing of the minima and the separation voltage is linear. Moreover, we also obtain a way of calculating the minimum excitation voltage.

$$E_a = \Delta E(0.5) \tag{5}$$

Since this value is not directly realizable through the use of the Franck-Hertz apparatus, we seek to extrapolate from observed higher-order minima spacing using a linear fit to the minima separation and resulting current.

Using our experimental apparatus, we record Franck-Hertz curves at temperatures ranging from 115°C to 135°C, and example of which is shown in FIG. 2 (a). We use commercial software to automate the data collection process. Data are recorded from 0 to 60V in increments of 0.5V. For each data point, we require 20 seconds of current stability before 5 data points are taken and averaged.

For each temperature, we fit a third-order spline to the data. This is necessary to remove the coarseness of the spacing in voltage. To estimate the uncertainty of the interpolated values, we use a Monte Carlo method to perturb our data. We generate perturbed data in 10,000 simulations by sampling a Gaussian centered on each data with a standard deviation corresponding to the data point that was specified by our initial commercial software. These new data are also fitted with the same spline, and the standard deviation of the population of 10,000 splines is calculated. We then used this newly derived value to characterize the uncertainty of our interpolated values. A comparison of the computed and original standard deviations can be seen in FIG. 2 (b).

Since the distribution describing the original standard deviation of the data is not provided, various distributions were sampled to perturb the original data for the Monte Carlo method. These include uniform, half-Gaussian, Poisson, binomial, and geometric distributions. Our findings suggest that as long as the standard deviation of the distribution is the same as the original data, the resulting standard deviations of the interpolated values are similar. This is an extension of the Strong Law of Large Numbers [9] which implies that the sample variance will almost surely converge to the population variance.

The minima and maxima of these interpolated values are then selected using an algorithm that estimates the derivatives of our interpolated function and selects points closest to where that derivative is zero. Due to large uncertainties in the maxima as compared to the minima, we consider only the spacing between minima. This unreliability is shown in the extremely large variation in the final determination of $\Delta E(0.5)$ shown in FIG. 3.

The order of these minima are determined visually by overlaying low-temperature data, which have more pronounced minima, over higher temperature data where the signal-to-noise ratio is lower. As noted later in the paper, the minima of higher temperatures are shifted to the right, necessitating visual identification of their order using low-temperature comparisons. For example, for data collected at $115°C$, low-order minima are more prominent and are able to be extracted, but due to large currents and large uncertainties at higher voltages, higher-order minima are excluded. However, data collected at $135°C$ have extremely small peaks at low voltages, so they are excluded from the measurement, but higher-order minima are tenable. Moreover, visual analysis is required because, as the temperature increases, minima of the same order tend to occur at higher voltages. This is a product of, as discussed later in the section on slopes scaling, the mean free path of the electron tends to decrease at higher temperatures, requiring higher accelerating voltages to reach the required electron energy to excite the mercury atoms.

We then calculate the difference in voltage, $\Delta E(n)$ between two minima according to Eq. (4). The uncertainties in these measurements are propagated, to the first order, through

$$\sigma_V^2 \approx \left(\frac{dI}{dV}\right)^2 \sigma_I^2 \tag{6}$$

We calculate $\frac{dI}{dV}$ using the same spline originally fit to the data. We then fit a linear function, as suggested by Eq. (4) to each temperature's data.

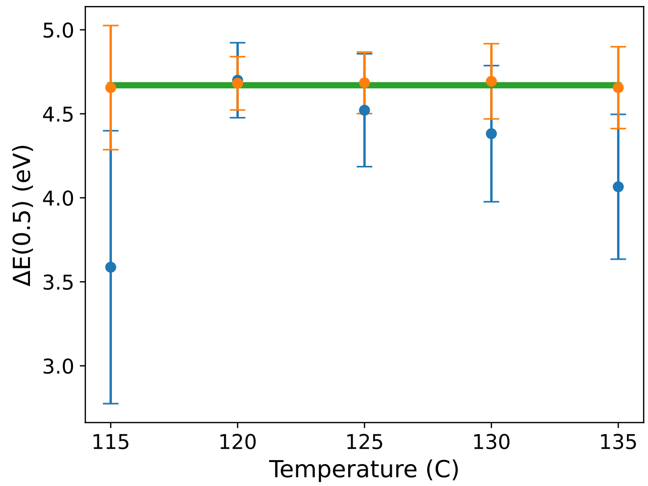

FIG. 3: The final determinations of the energy of lowest excitation state, $\Delta E(0.5)$ in eV by the minima spacing, in orange, and the maxima spacing, in blue. Of note is the large variation and larger uncertainties in the results extracted from the maxima spacing. In green is the literature value, $\Delta E(0.5) = 4.67$eV.

We then calculate a value for $\Delta E(0.5)$ for each of the temperatures using the linear fit. It should be noted, as predicted, that the slopes are of the order of the minimum 0.5 and additionally show no dependence on temperature. We calculate a mean value for the lowest excitation energy in mercury, $6^1S_0 \to 6^3P_0$ as $\Delta E(0.5) = 4.7 \pm 0.5$eV, which is in line with the value of [8] of other experimental determinations at 4.67 eV. The uncertainties of this value are extracted from each temperature's linear fit uncertainty of order 0.5 added in quadrature.

Moreover, we expect the slope of each higher temperature fit to decrease. This is due to the fact that as the temperature increases, the number density of the mercury vapour increases significantly. This leads to a subsequent decrease in the electron's free path and thus less energy gained between collisions. We then refer to Equation (4) to note that this lack of additional energy will, in turn, decrease the spacing of the minima, $\Delta E(n)$, and thus the slope of our final linear fit. Moreover, we also note that as the mean free path decreases there is also a decreased probability of the electrons gaining enough energy to excite subsequent higher energy transitions such as the transition $6^1S_0 \to 6^3P_1$ which occurs at 4.89 eV [10]. As such, proportionally more excitations should correspond to the first energy level, and the spacing of the

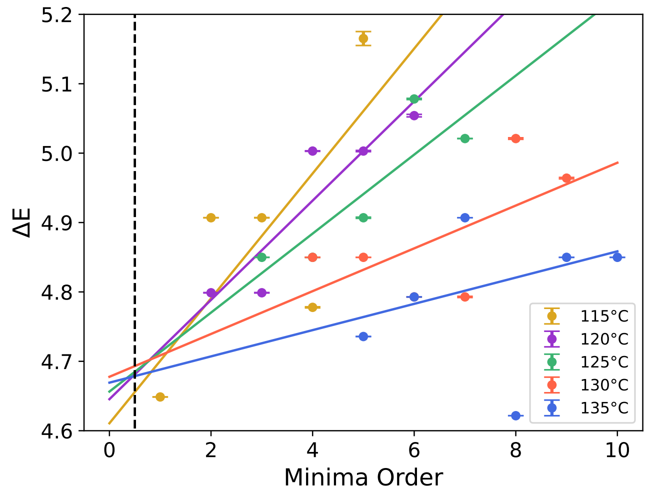

FIG. 4: A comparison of separation voltage, $\Delta E = E_n - E_{n-1}$ in Volts, and minima orders, n, at different temperatures. The corresponding solid lines are linear fits to the data for each temperature, as expected from (4). We also include a dashed line corresponding to the first excitation level $\Delta E(0.5) = E_a$. We calculate a value of $\Delta E(0.5) = 4.6 \pm 0.5$eV as the mean value of the linear fits at minima order 0.5. The high uncertainty in the linear fits may be a result of minima dependence on tube parameters such as length, shape and level of vacuum.

minima should decrease. This prediction matches our findings seen in FIG. 4.

We then turn our attention to the Child-Langmuir law. We select a temperature, 125°C, such that the mercury in the tube is not vapourized. We then increase the accelerating voltage from 0 to 60V in increments of 0.5V. The corresponding output current is measured by the picoammeter and collected using commercial software. Uncertainties are also described by our software by considering the population of the 5 data points that are averaged. The recorded data is then plotted in a log-log plot, as shown in FIG. 5 in order to extract the polynomial coefficient of the relation. We fit a linear fit to the function with slope m and extract a value of $m = 1.49 \pm 0.06$ which is in very good agreement with the Child-Langmuir law, $I(V) \propto V^{3/2}$ as seen in Equation (1). We also introduce an offset parameter, calculated as -6.14 ± 0.09, which corrects for the work function between the cathode and the surrounding environment, which in turn suppresses the emission at low voltages.

In summary, we conducted the Franck Hertz experiment for varying temperatures and inferred both the quantized nature of mercury atoms and a value for their lowest excitation level, $\Delta E(0.5) = 4.7 \pm 0.5$eV. We use a Monte Carlo method to extract uncertainties and increase the interpretability of interpolated current-voltage data. We also carried out an experiment using the vacuum tube to verify the Child-Langmuir law and extract

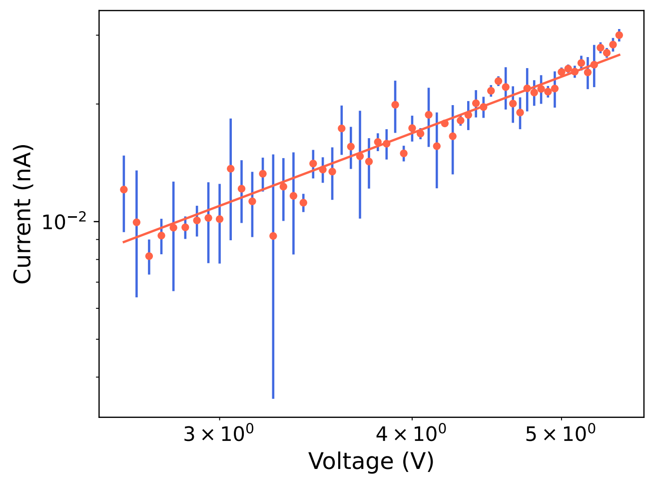

FIG. 5: A log-log plot of the current-voltage relation recorded at a tube temperature of 125±0.2°C was fitted to a linear model of the form $I(V) = mV + b$, where m is the polynomial coefficient of the relation $V \propto I^m$ and b is the log of the intercept. We plot this line in red. The calculated values for m and b were 1.49 ± 0.06 and -6.14 ± 0.09 respectively, with a χ^2 value of 0.56.

a current-voltage relation of $I(V) \propto V^{1.48 \pm 0.06}$ which is in good agreement with the scientific literature. These findings provide insight into the behaviour of electrons in vacuum tubes and the quantized nature of atomic energy levels which have had a profound historical impact in a plethora of different fields [11]. Future iterations of this experiment may seek to calculate the mean free path parameter from the slope of the linear fits to $\Delta E(n)$.

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